Depletion and development:  
natural resource supply with endogenous field opening

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Abstract

Supply of a non-renewable resource adjusts through two margins: the rate at which new fields are opened, and the rate of depletion of open fields. The paper combines these margins in a model in which there is a continuum of fields with varying capital costs. Opening a new field involves sinking a capital cost, and the date of opening is chosen to maximize the present value of the field. Depletion of each open field follows a Hotelling rule, modified by the fact that faster depletion reduces the amount that can ultimately be extracted. The paper establishes the equilibrium paths of output and price. In contrast to Hotelling, the long run rate of growth of prices is independent of the rate of interest, depending instead on characteristics of demand and geology. The effects of shocks, such as climate policy, are analysed.

Keywords: non-renewable resource, depletion, exhaustible, Hotelling, fossil fuel, carbon tax.

JEL classification: D9, Q3, Q4, Q5

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1. **Introduction:**

How does the supply of a non-renewable resource depend on price, and how does the market for such a resource respond to shifts in demand? These questions are important for understanding long-run issues such as the effects of climate policy on the use of fossil fuels, and short-run issues such as the behaviour of commodity prices. At one extreme, the Hotelling (1931) approach treats non-renewable resources as assets which can be depleted at any date, so that prices are linked by inter-temporal arbitrage and the rent element of price increases at the rate of interest. At the other, some industry experts use extremely low supply elasticities (the US Energy Information Administration uses short-run supply elasticity of 0.02 and long run 0.1, see Smith 2009), implying that opportunities for inter-temporal arbitrage are negligible.¹

The objective of this paper is to provide a model in which the supply of an exhaustible resource is captured in a richer manner than in the conventional Hotelling approach. The central idea is that supply can adjust through two margins, intensive and extensive. The intensive margin is the rate of depletion of existing fields (or mines). We posit a relationship between extraction costs and the rate of depletion that can vary between zero and perfect flexibility (the latter being the pure Hotelling case); this endogeneity of extraction costs breaks the rigid link between price growth and the rate of interest. The extensive margin is the development of new fields. Central to our approach is the fact that capital has to be sunk before a new field is opened, a feature that accords with reality and is a quantitatively important feature of major mining developments and oil investments in offshore and deep fields. Fields differ in capital cost per unit reserve, and it is this that produces, in equilibrium, a sequence of field openings through time.

The supply of the resource depends on choices of how fast to deplete existing fields (the intensive margin) and when to open new fields (the extensive margin). In sharp contrast to the Hotelling approach, the long-run equilibrium of the model has price increasing at a rate that is completely independent of the rate of interest. Extensive margin choices about field opening mean that the rate of price increase depends on characteristics of demand (price elasticity and growth), and characteristics of the geology and technology of supply. This is perfectly consistent with intensive margin choices that are ‘Hotelling-like’, with depletion rates on individual fields adjusting according to price growth, the rate of interest, and

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¹ Empirical tests have failed to find support for the Hotelling approach. See Chermak and Patrick (2002).
extraction costs. The combination of intensive and extensive margin effects also gives different supply responses to shocks. For example, demand reduction policies motivated by climate change may bring forward depletion of existing fields (the ‘green paradox’ noted by Sinn 2008) but will also cause postponement of the development of new fields, so that overall supply and emissions are reduced.

The model is outlined and producers’ choices of depletion rates and field opening are characterised in section 2 of the paper. In order to model the depletion of individual fields (the intensive margin) in a flexible yet tractable way we assume that extraction costs increase with the rate of depletion and are ‘iceberg’, using up the resource itself. Both these assumptions are supported in the technical literature on oil extraction which suggests that faster depletion means that less of the resource is ultimately recoverable. They are also convenient modelling simplifications which make for a tractable characterisation of the intensive margin and, by allowing aggregation over fields, facilitate analysis of aggregate resource supply in a multi-field setting.

At the extensive margin, producers decide when to sink capital in order to open a new field. For many exhaustible resources these field-specific capital costs (‘finding and development costs’) are much the largest part of costs, as discussed below. Focusing on these, our modelling approach is in contrast with much of the literature where additions to stock are typically modelled as the outcome of a continuous variable (exploration) that adds to the capacity and reduces extraction costs of the existing field (as in Pindyck 1978, Dasgupta and Heal 1979). Existing literature in which there are field set-up costs includes Hartwick et al. (1986), Holland (2003), and Livernois and Uhler (1987). Hartwick et al. assume zero extraction costs, in which case only one field is operated at any time, and Holland (2003) looks at cases where marginal extraction costs are either constant or infinite. Livernois and Uhler (1987) look at the rate of discovery of new fields with field-specific extraction costs, characterising first order conditions for the problem but doing little subsequent analysis of the equilibrium. We go beyond these models, fully integrating intensive and extensive margin choices.

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2 This is more restrictive than much of the literature, in which costs are modelled as a function of extraction and the stock of resource remaining. For example, Pindyck (1978) assumes that costs are proportional to extraction and decreasing in remaining stock. The rate of extraction is the ratio of these variables.

3 See Krautkraemer (1998) for a survey. Swierzbinski and Mendelsohn (1989) aggregate separate fields, but assuming no fixed costs and constant returns to scale in exploration and extraction.
Aggregate supply is derived by placing the field-specific choices of rate of depletion and date of field opening in the setting of a continuum of fields with different capital costs, $K$. Aggregate supply for a given price path is derived in section 3. At each date supply comes from all open fields, running down their reserves according to their optimal depletion path; new fields may also be opened, adding to the stock of open reserves. The path of supply depends on both the rate of change of price relative to the interest rate, as in the Hotelling model, and on the level of price, operating through the extensive margin and the timing of field openings. Thus, a permanent proportional price reduction postpones field opening, reducing the quantity produced in the short run, raising it in the long run, and reducing the cumulative quantity produced (i.e. total quantity depleted) at all future dates. A permanent reduction in the rate of growth of price increases production in the short run (bringing forward depletion of existing fields and, temporarily, field opening), but has a long run negative effect on cumulative quantity supplied.

Section 4 moves on to the full market equilibrium, making price endogenous. The central result of the paper is that the long run rate of change of price is determined by the rate of growth of demand, the rate of technical progress, the price elasticity of demand, and a parameter summarising the geology of supply. It is completely independent of the rate of interest. While the rate of interest matters for depletion of open fields, adding extensive margin field choices means that long run price change is determined by the fundamentals of supply and demand. This is a long run result, and responses to shocks are the subject of the final two sections of the paper. These two sections draw on both analysis and simulation; the final application (section 6) is of the possible effects of climate change policy.

2. Field depletion and development:

There is a continuum of fields all of which are known at date 0, and are owned by price-taking profit maximizing agents. Each field contains one unit of the resource, but cannot produce until a field specific fixed cost $Ke^{-\theta T}$, $\theta \geq 0$, has been paid, where $e^{-\theta T}$ captures technical progress in field development that has taken place by date $t = T$, when the cost is paid. $K$ varies across fields, and we will use $K$ as the index of field types, with $K > 0$ and
running to plus infinity. The date at which a particular field is opened is endogenous, and the number (measure) of fields of type $K$ is $S(K)$.

Focusing on a particular field (i.e. taking a particular value of $K$), output at date $t$ is $xq(z)$, where $x$ is the stock remaining and $z$ is the rate of depletion, defined as the proportionate rate of decline of remaining stock, so $\dot{x} = -xz$. While $z$ is the rate of depletion of the field and $xz$ is the reduction in the stock, $xq(z)$ is the recovered output. The expression $q(z)/z \leq 1$ is the yield curve, giving the fraction of the reduction in stock that is marketable output. All current extraction costs are subsumed in this yield curve. We assume that $q(z)$ is increasing and concave in $z$; if strictly concave, increases in the rate of depletion yield less than proportionate increases in output, perhaps as too rapid pumping from an oil-field reduces the capacity of the field. Further discussion and an example are given in the next sub-section.

Profit maximization in a field with fixed cost $K$ requires that the opening date, $T$, and subsequent time paths of $z$ and $x$ are chosen to maximize the present value of profits (evaluated at date $t = 0$ with interest rate $r$),

$$PV = e^{-rT} \int_0^\infty p(T + \tau)x(\tau)q(z(\tau))e^{-r\tau}d\tau - Ke^{-(\theta + r)T}$$

subject to $\dot{x}/x = -z$, and $\dot{x}(0) = 1$, $x \geq 0$. (2)

The integral in (1) runs over dates $\tau$ measured from when the field is opened, so $t = T + \tau$, and $p(t)$ is the (exogenous) price at date $t$. We assume that, as $t \rightarrow \infty$ so $p(t) \rightarrow \hat{p}_x$, a constant exponential growth at rate less than or equal to $r$, as is necessary for the objective to be bounded.

The profit maximizing rate of depletion of an open field is given by Euler equation

$$\dot{z} = \left[ r - \frac{\hat{p}_x}{p} + z - \frac{q(z)}{q'(z)} \right] \frac{q'(z)}{q''(z)}.$$ (3)

(see appendix 1). This is a differential equation for $z$, depending on the difference between the rate of interest and rate of price increase, and also on the curvature of $q(z)$, indicating the yield loss from varying the rate of depletion. Equation (3) has stationary value $z^*$ at

$$r - \hat{p}_x = q(z^*)/q'(z^*) - z^*, \quad \text{or} \quad z^* = \zeta(r - \hat{p}_x), \quad \zeta > 0.$$ (4)

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4 Assuming each field contains one unit of resource is without loss of generality as $K$ can be interpreted as capital cost per unit capacity. The total stock of resource in fields of type $K$ is $S(K)$. 

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The function $\zeta(r - \hat{p}_x)$ summarizes the long-run relationship and has $\zeta' = -qq''/(q')^2$ which is positive by concavity of $q(z)$. The crucial point is that the function $\zeta(r - \hat{p}_x)$ is not, in general, infinitely elastic. Thus, a higher value of $(r - \hat{p}_x)$ raises the rate of depletion but does not lead to instantaneous depletion, as in the pure Hotelling case. This is because increasing the rate of depletion reduces the marginal yield and thereby increases extraction costs. If the rate of growth of prices is constant for all $t$ then $z$ simply jumps to the stationary value and remains constant.\(^5\) For more general price paths which converge to $\hat{p}_x$, concavity of $q(z)$ ensures that $z$ converges to the stationary value $z^*$ given by (4).

The profit maximizing date $T$, at which to spend $Ke^{-\theta T}$ and open the field is given by

$$\frac{\partial PV}{\partial T} = e^{-\theta T} \left[ -r \int_0^\infty pxq(z)e^{-r \tau} d\tau + (\theta + r)Ke^{-\theta T} + \int_0^\infty \hat{p}xq(z)e^{-r \tau} d\tau \right] = 0 \quad (5)$$

The intuition is that if the profile of production and costs is shifted back by $dT$, then the first term is the cost of pushing revenues further away, the second the benefit of moving costs, and the final term is the change in revenue from the fact that output $xq(z)$ is now valued at prices $dT$ later. Rearranging, the date $T$ of opening of a field of type $K$ is given by first order condition,

$$\int_0^\infty (\hat{p} - rp)xq(z)e^{-r \tau} d\tau + (\theta + r)Ke^{-\theta T} = 0 \quad (6)$$

The implications of this are most readily seen by looking at the case in which price is growing at constant rate $\hat{p}$ (with value $p_0$ at $t = 0$), so $z$ is at its stationary value $z^*$ and $x = e^{-z^*\tau}$. The present value of profits on field $K$, equation (1) is then

$$PV = p_0e^{(\hat{p} - r)T}q(z^*)\int_0^\infty e^{(\hat{p} - r - z^*)\tau} d\tau - Ke^{-\theta T}$$

$$= \frac{p_0e^{(\hat{p} - r)T}q(z^*)}{z^* + r - \hat{p}} - Ke^{-\theta T} = p_0e^{(\hat{p} - r)T}q'(z^*) - Ke^{-\theta T} \quad (7)$$

where the second line comes from integrating and using equation (4). The first and second order conditions for choice of $T$ are

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\(^5\) Derivation of the optimal depletion rate is simple in this case as the integral in (1) is

$$\int_0^\infty p_0xq(z)e^{\hat{p}(z - r)\tau} d\tau = p_0xq(z)/[r + z - \hat{p}]$$

and condition (4) comes from setting the derivative of this with respect to $z$ equal to zero.
\[
\frac{\partial PV}{\partial T} = (\hat{p} - r) p_0 e^{(\hat{p} - r)T} q'(z^*) + (\theta + r) Ke^{-(\theta + r)T} = 0,
\]
(8)

\[
\frac{\partial^2 PV}{\partial T^2} = - (\theta + \hat{p})(\theta + r) Ke^{-(\theta + r)T} < 0,
\]
(9)

(where the second order condition is evaluated at \( \frac{\partial PV}{\partial T} = 0 \)). If \( \theta + r > 0 \), the second order condition requires that \( \hat{p} + \theta > 0 \), and we assume this to be satisfied. From the first order condition, an interior solution requires \( r > \hat{p} \), as already assumed; if not it would pay to postpone entry indefinitely getting the dual benefit of later capital expenditure and higher present value of revenue flow.\(^6\) These conditions mean that the higher is \( K \) the later the field is opened, since
\[
dT\dk = -\frac{\partial^2 PV / \partial T \partial K}{\partial^2 PV / \partial T^2} \quad \text{and} \quad \frac{\partial^2 PV}{\partial T \partial K} = (\theta + r) e^{-(\theta + r)T} > 0,
\]
implying that
\[
dT\dk = \frac{1}{(\theta + \hat{p})K} > 0.
\]
The implication is that, with a continuum of fields differing only in capital cost per unit reserve, low \( K \) fields will be opened first.

### 2.1 The rate of depletion: discussion

Our modeling of extraction costs and depletion is grounded in the technical literature on resource depletion, particularly in the oil sector. In this literature the benchmark assumption is that output from a field follows an exponential rate of decline (Adelman 1990, 1993); in our framework this would mean constant \( z \).\(^7\) Varying the rate of depletion has a cost primarily by its impact on total recoverable reserves. This variation is typically achieved by altering the rate of water or gas injection which pressurizes the well, and its effects are geology dependent; Nystad (1985, 1987) categorises fields as ‘Hotelling’, ‘intermediate’, and ‘geosensitive’, in increasing order according to loss of recoverable reserves from faster depletion. We capture this in the relationship \( q(z) \).

Understanding these relationships is facilitated by a particular functional form that will be used in simulations later in the paper. Suppose that \( q(z) \) takes the form
\[
\begin{aligned}
z > b\lambda, & \quad q(z) = a(z - b\lambda)^{1-\lambda}, \quad \text{with} \ a > 0, \ b \geq 0, \ \text{and} \ \lambda \leq 1: \\
z < b\lambda, & \quad q(z) = 0.
\end{aligned}
\]
(10)

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\(^6\) And, with prices endogenous, competitive equilibrium would not exist, see Holland (2003).

\(^7\) A constant \( z \) means exponential decline in remaining stock \( x \), and hence in output \( q(z)x \).
With this specification the Euler condition (3) and long-run value of the rate of depletion are,

$$\dot{z} = \left(\frac{z - b\lambda}{\lambda}\right) \left[\frac{\bar{p}}{p} - r + \frac{\lambda (z - b)}{1 - \lambda}\right], \quad z^* = b + \frac{(1 - \lambda)(r - \bar{p}_\infty)}{\lambda}. \quad (11)$$

Examples are given in figure 1. The key parameter is $\lambda$ which captures the extent to which faster depletion leads to loss of reserves, and hence also the extent to which optimal depletion is sensitive to price. The pure Hotelling case is $\lambda = 0$, (solid line in figure 1) in which the rate of depletion is infinitely sensitive to the gap between $\bar{p}$ and $r$, so continuing extraction over an interval of time is possible only if these are equal. At the other extreme, as $\lambda \to 1$ the optimal rate of depletion is equal to $b$, and completely independent of the rate of price increase or rate of interest (the long-dashed line has $\lambda = 0.95$). This is consistent with Adelman’s (1990) view that the rate of depletion from a particular reservoir is quite insensitive to price, and well approximated by a constant exponential rate of decline (at rate $b$ in this specification). For cases with intermediate degrees of ‘geosensitivity’ the extraction path is more tilted towards the present the larger is $\bar{p} - \hat{p}$.

Figure 1: Examples of extraction costs, $q(z)$

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8 Parameter $b$ must be large enough that $q(z) \leq z$ for all $z$. Yield is diminishing with $z$ for $z > b$. 
While this paper deals with supply coming from many fields, it is worth briefly connecting our modeling of depletion with the standard model of market equilibrium with a single field. If demand for the resource is iso-elastic, \( Q_D = Dp^{-\eta}e^{\sigma t} \) where \( D \) is a constant, \( \eta \) is the price elasticity of demand, and \( g \) the exogenous rate of growth of demand, then along the equilibrium path supply, \( q(z)x \), must change at rate \( g - \eta \dot{p} \). The rate of change of supply is simply \( \dot{x} / x = -z^* \), so, using (11), the equilibrium rate of growth of price is

\[
\dot{p} = \frac{\lambda (b + g) + (1 - \lambda) r}{\lambda \eta + 1 - \lambda}.
\]  

(12)

This is a simple generalization of the Hotelling model, in which the role of the interest rate depends on \( \lambda \). \( \lambda = 0 \) gives the pure Hotelling case, and when \( \lambda > 0 \) the rate of price increase is greater the faster the growth of demand, \( g \), the smaller the price elasticity, \( \eta \), and the larger the base rate of depletion, \( b \).

2.2 Field development: discussion

The field owner’s objective, equation (1), was written in terms of a field of size one \( (x(0) = 1) \) developed at cost \( K \). Setting the size of each field at unity is a normalization, and the key measure is size per unit capital cost. Furthermore, \( K \) can be thought of as the expected sunk cost, rather than the actual one. The role of \( K \) is to induce a sequence of dates of field openings, and the realization of \( K \) plays no role in the model. This important source of uncertainty is therefore consistent with our framework.

The empirical counterpart of \( K \) in the oil sector is ‘finding and development’ (F&D) costs per barrel. Having risen sharply in recent years these are now the largest part of the sector’s costs, with global average of $21 per barrel over the period 2006-09 (EIA 2011); they are of course field specific and in some cases go much higher (e.g. US F&D costs on offshore projects were $64 per barrel in 2006-08). These costs are several times greater than other production costs (‘lifting’ costs), running at global average of $11 per barrel (EIA 2011). Furthermore, from an economic standpoint some elements of lifting cost should probably be classified as F&D; for example, some capital equipment may be highly specific to a field but is rented by the firm and counted as ‘lifting’ not F&D costs.
3. Aggregate supply

We now move from depletion and development decisions on a single field to aggregate supply from all fields. In this section we derive aggregate supply given an exogenous price path, endogenising price in the next section.

The economy contains a continuum of fields, indexed by their capital costs, $K$. While fields vary in $K$, we assume that all have the same yield function, $q(z)$. Equation (6) gives the date at which a field of type $K$ is opened, and we now invert this relationship to give the type of field opened at date $T$, $K(T)$. Since $dT/dK > 0$ fields are opened in sequence, with low capital cost fields opened first. Using (6) we write this explicitly as

$$K(T) = \frac{e^{\int_0^T (r - \hat{p}(z))x(t, T)e^{-r(t-T)} dt}}{(r + \hat{p})} \int_{-\infty}^T (e^{r(t-T)} dt),$$

where $x(t, T) = \exp\left[-\int_t^0 z(\chi)d\chi\right].$ (13)

Integration is now defined to run over $t$ (rather than $\tau = t - T$), and $x(t, T)$ denotes the stock remaining at date $t$ in a field opened at date $T$.

The measure of fields of type $K$ is $S(K)$. The total number of fields that are open at date $t$ is therefore $\int_{-\infty}^t \hat{K}(T)S(K(T))dT$, i.e. the integral over all previous dates of the set of field types that opened at each date, $\hat{K}(T)$, times the number of fields of type $K$, $S(K(T))$.

We define open reserves at date $t$, $R(t)$, as the stock remaining in fields that are open, i.e.

$$R(t) \equiv \int_{-\infty}^t \hat{K}(T)S(K(T))x(t, T)dt.$$ (14)

$R$ moves according to differential equation

$$\dot{R} = \hat{K}S(K) - zR,$$ (15)

derived by differentiating (14) with respect to $t$ and using $x(t, t) = 1$ and $\dot{x} = -zx$. Notice that, since $q(z)$ is assumed to be same in all fields, so too is the optimal value of $z$. The interpretation is straightforward; open reserves change as new fields are opened at rate $\hat{K}S(K)$ and existing ones are depleted at rate $z$.

Total output at each date is the sum of current extraction from all open fields. Once again, the fact that all open fields are identical except for the scalar difference in the size of stock remaining, makes this aggregation over open fields straightforward. Total supply, $Q_s$, is simply the yield from depletion of the stock of open reserves,
The final building block is the relationship $S(K)$, giving the number of fields (or total capacity) associated with each value of $K$. A convenient form is iso-elasticity, with $S(K) = sK^{\sigma-1}$. Total reserves are proportional to $s$, and vary with field type according to $\sigma$. This may be positive or negative, but we shall generally interpret results taking $\sigma < 0$, which means that the remaining resource stock is finite, while $\sigma > 0$ means it is infinite. This relationship can easily be given a micro-foundation. The size distribution of oil fields is well approximated by a power law (see Laherrere 2000). If the elasticity of capital costs with respect to field size is less than unity and greater than the absolute value of the exponent in the power law, then the relationship $S(K) = sK^{\sigma-1}$ with $\sigma < 0$ follows (see appendix 2).

This completes characterization of the supply side of the model, given a price path $p(t)$. To summarize, supply is characterized by three variables. The first is $z$, the rate of depletion, this inducing values of $x(t,T)$ in each open field. The second is $K(T)$, the time path of field openings, and the third is $R(t)$, the stock left in open fields this, together with the rate of depletion, determining supply, $Q_s$. $z$ and $K$ are forward looking decision variables that can jump in response to a shock, although $K$ can only jump upwards (capital costs in field openings are sunk). $R$ is a state variable, depending on the past history of field opening.

Insight comes from looking at the case where $\hat{p}$ is constant for all future dates and hence $z = z^*$. The path of field openings is then, integrating (13) and using (4) (as in equation (8))

$$K(T) = \frac{p_0e^{(\theta+\hat{\rho})T}q'(z^*)(r-\hat{p})}{(r+\theta)}.$$  \hspace{1cm} (17)

This establishes that $K$ is proportional to initial price $p_0$, and the growth of $K$ is constant at rate $\hat{K} = \hat{\rho} + \theta$. If $S(K) = sK^{\sigma-1}$ the differential equation for open reserves, (15), becomes

$$\dot{R} = s\hat{K}K^{\sigma-1} - z^* R,$$ \hspace{1cm} (18)

which, with $z^*$ and $\hat{K}$ constant, has explicit solution,

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\[ K \text{ runs to plus infinity; the stock remaining is finite iff } \sigma < 0, \text{ since } \int_{K^\infty} S(K)dK = s \int_{K^\infty} K^{\sigma-1}dK = s \left[ \frac{K^{\sigma}}{\sigma} \right]_{K^\infty} \]
\[ R = \frac{s\hat{K}K^\sigma}{z^* + \sigma \hat{K}} + e^{-z^* t} \left[ R_0 - \frac{s\hat{K}K_0^\sigma}{z^* + \sigma \hat{K}} \right] \]  

(19)

where \( K_0 \) and \( R_0 \) are the values of \( K \) and \( R \) at date zero. The effect of these initial values goes to zero with \( e^{-z^* t} \), so \( R \) converges to value given by \( R / K^\sigma = s\hat{K} / (z^* + \sigma \hat{K}) \). The long run rate of change of open reserves is therefore \( \dot{R} = \sigma \hat{K} = \sigma(\dot{p} + \theta) \) so, with \( \sigma < 0 \), open reserves decline exponentially. Furthermore, since \( Q_s = q(z^*) R \), output also declines at rate \( \dot{Q} = \sigma(\dot{p} + \theta) \). We summarize these properties as follows.

**Proposition 1:**

If price grows at constant rate \( \dot{p} \) at all dates and \( \dot{p} + \theta > 0 \), \( r > \dot{p} \), then:

i) \( z \), the rate of depletion of each field is constant, and is faster the larger is \( r - \dot{p} \) (equation (4)).

ii) Fields open in increasing order of their sunk cost per unit reserve, \( K \). Field opening is such that \( K \) is proportional to \( p_0 \) and \( K \) increases at rate \( \dot{K} = \dot{p} + \theta \) (equation (17)).

If, additionally, the number of fields of type \( K \) is \( S(K) = sK^{\sigma-1} \), with \( \sigma < 0 \) (corresponding to a finite stock of the resource) then:

iii) The rate of growth of open reserves and of supply converge asymptotically to \( \dot{Q}_S = \dot{R} = \sigma(\dot{p} + \theta) < 0 \).

iv) On the long run (asymptotic) growth path \( R \) and \( Q \) are given by

\[ R = \frac{(\dot{p} + \theta)sK^\sigma}{z^* + \sigma(\dot{p} + \theta)}, \quad Q_S = q(z^*) R. \]  

(20)

One comparative dynamic result can be noted at this stage. A higher price level, \( p_0 \), is associated with more fields having been opened at each date (higher \( K \), (17)) and, if \( \sigma < 0 \), lower open reserves and less supply at each date (20). From (17) and (20) the long run elasticity of supply with respect to price level is therefore \( \sigma \). The intuition behind this negative supply elasticity is that a higher level of prices means that more fields have been opened and (partially) depleted so current output is lower. Of course, this is a comparative change across asymptotic growth paths. The effects of shocks to these paths are discussed in section 6, after establishing equilibrium prices.
4. Market equilibrium:

We now go to the full market equilibrium, adding demand and endogenising price. The demand curve is assumed to have constant price elasticity $\eta \geq 0$, exogenous rate of growth $g$, and level parameter $D$,

$$Q_D = D p^{-\eta} e^{\delta t}, \quad \text{so} \quad \dot{Q}_D = g - \eta \dot{p}. \quad (21)$$

The equilibrium price path comes from equating $Q_D$ to $Q_S$. Section 3 established that if price is growing at a constant rate the long run rate of growth of supply is constant at

$$\dot{Q}_S = \sigma (\dot{p} + \theta) \quad \text{(proposition 1).}$$

Equating this with the rate of growth of demand, the equilibrium rate of growth of price is

$$\dot{p} = \frac{g - \sigma \theta}{\eta + \sigma}. \quad (22)$$

Recalling that $\sigma$ is the (asymptotic) price elasticity of supply, this expression links a demand shift (demand growth $g$) to price change via elasticities of supply and demand in the usual way. A number of points are noteworthy.

First, in contrast to the standard Hotelling approach, the equilibrium rate of price increase is independent of the rate of interest. A higher interest rate means faster depletion of existing fields and a Hotelling-like result follows if the extensive margin is completely fixed (no new fields open, and supply response comes only from altering depletion of existing fields, equation (12)). However, once the extensive margin is included in the supply response the long run rate of growth of price depends on demand and supply elasticities in a familiar way, and not at all on the interest rate.

Second, the necessary condition for our characterization of the date of field opening to be a profit maximum is that $\dot{p} + \theta > 0$ (section 2). With $\dot{p}$ given endogenously by (20), this condition could fail for two distinct reasons. One is that $g$ is substantially negative (with denominator of (22) positive) in which case demand is falling too fast to support the positive price growth necessary to induce delay in field opening.10 The other is that $\eta + \sigma < 0$ (with numerator of (22) positive). This could arise if $\sigma < 0$ in which case, as already noted, the long run price elasticity of supply is negative. We impose the condition that $\eta + \sigma > 0$, failing which the second order condition for field opening is not satisfied.

10 A high value of $\theta$, the rate of technical change on $K$, supports postponement of field opening.
Long run equilibrium values of other variables in the system follow directly from the price growth given by (22) together with proposition 1. The long run rates of growth of open fields, open reserves, and output are

\[
\hat{K} = \frac{g + \eta \theta}{\eta + \sigma}, \quad \hat{Q} = \frac{\sigma(g + \eta \theta)}{\eta + \sigma}.
\]

(23)

The initial price equates supply and demand so, using (17) and (20) in (21) at \( T = 0 \), \( p_0 \) satisfies

\[
p_0^{-1(\eta + \sigma)} = \left( \frac{s}{D} \right) \left( \hat{p} + \theta \right)q(z^*) \left[ q'(z^*)(r - \hat{p}) \right]^\sigma.
\]

(24)

The following proposition summarizes these properties of the long run equilibrium.

**Proposition 2:**

On the long run (asymptotic) path the rate of growth of price is independent of the rate of interest, and given by \( \hat{p} = (g - \sigma \theta)/(\eta + \sigma) \). The rate of depletion is constant and output is declining at rate \( \sigma(g + \eta \theta)/(\eta + \sigma) \). The elasticity of the equilibrium price with respect to the level of demand is \( 1/(\eta + \sigma) \).

Comparing across long run equilibrium paths, parameters \( s, D \) and \( r \) determine the levels of variables, while other parameters also influence rates of growth and decline. For example, routine calculation indicates that a higher demand parameter, \( D \), lower \( s \), or higher \( r \) is associated with higher price and lower supply at all dates on the long-run path. The higher price is intuitive, and is associated with higher \( K \) at each date; this means that more has been depleted on the transition to the long run path, giving the lower level of output. A higher value of the rate of growth, \( g \), or the rate of technical progress, \( \theta \), is associated with a lower value of \( K \) at each date. Field opening is postponed in anticipation of future demand or technical improvement. Correspondingly, current output is higher and price lower (as low \( K \) is associated with large \( S(K) \)); the rate of growth of price and rate of fall of output are larger.

These comparative dynamics are hard to interpret (and are not very policy relevant) as they are the outcome of a long-run process which generally involves a transitional dynamic. We now turn, therefore, to the effect of shocks and the short and medium run price and quantity responses that they create.
5. Responses to shocks

Shocks create a new long run path to which the model converges, but adjustment is slow because open reserves are determined by the past history of field opening. We focus on demand shocks, looking at changes in both the level of demand, $D$, and the rate of growth, $g$. Of course, these shocks may be induced by policy changes such as carbon taxation which create a wedge between consumer and producer prices. We proceed in two stages, looking first at the supply response, i.e. taking a price shock as given, and then turning to the full equilibrium response to shifts in the demand curve.

5.1 Price shocks and supply.

Suppose that an unanticipated upwards jump in $p$ occurs at date 0 and lifts the price path equi-proportionately at all future dates. Since this is a price level (not growth) effect it has no effect on the rate of depletion (intensive margin, equations (3), (4)), in which price enters only in the form of future price growth. However, an increase in $p_0$ affects the extensive margin through the timing of field openings, causing an equi-proportionate increase in $K$ as given by equation (17). An upwards jump in $K$ means that a discrete number of new fields are opened as the shock occurs but, if $\sigma < 0$, fewer fields are opened at every date thereafter. This initial jump and subsequently lower rate of field opening works through into the stock of open reserves and hence output through equation (19). $R$ jumps and then converges asymptotically to $R / K^\sigma = \tilde{K} / (z^* + \sigma \tilde{K})$; the right hand side of this expression is unchanged, but since $K^\sigma$ is lower at each date, so too is $R$. Output is proportional to open reserves, so a permanent proportional price increase elicits a positive short to medium run supply response which turns negative as fewer new fields are being opened. While the short run price elasticity of supply is positive, the long run supply elasticity is negative (if $\sigma < 0$), as discussed above. Since field openings are brought forward, cumulative supply (cumulated from the date of the shock) and hence total resource extracted is increased by a positive price level shock.

We illustrate these effects, for a downwards price jump, on figure 2a in which the horizontal axis is time, solid lines give variables on the initial path, and dashed lines give variables with a 20% lower price at all dates. The price fall causes a pause in field opening
(the shift in $K$, top left panel). During this pause open reserves fall, as does output. Once field openings resume $K$ is lower and $S(K)$ is higher ($\sigma < 0$), so more capacity is opened at each date. Open reserves recover and overtake what they otherwise would have been. Combining the short run reduction in output and long run increase, the effect is to decrease cumulative output (and hence total stock depleted) at all dates. We summarize these effects in proposition 3.

**Proposition 3:**

A permanent proportionate change in the price ($\hat{\rho}$ constant and unchanged) has no effect on the rate of depletion or the long run rate of growth of supply. A price increase brings forward the opening of fields. Supply increases before eventually falling below what it otherwise would have been (with long run price elasticity of supply of $\sigma$). Cumulative supply is increased at all dates. A price decrease has reverse effects, leading to a reduction in cumulative supply at all dates.

A change in the rate of growth of price affects both the intensive and the extensive margin. At the intensive margin, a permanent increase in price growth causes an immediate and permanent fall in the rate of depletion, $z$ (equation (4)). Slower depletion means less supply from a given quantity of open reserves but more open reserves at all future dates, so a short run reduction in supply is followed by higher supply in future, the Hotelling-like response that would be expected.

The extensive margin now operates in a similar manner to the intensive as higher future prices creates an incentive to postpone field opening. Field opening is reduced (or ceases altogether) for a period, and then resumes at a faster rate, since $\hat{K} = \hat{\rho} + \theta$. The tension between these forces can be seen by using equation (4), $z^* + r - \hat{\rho} = q(z^*)/q'(z^*)$, in equation (17) to give

$$K(T) = \frac{P_0 e^{(\theta + \hat{\rho})T}q'(z^*)(r - \hat{\rho})}{(r + \theta)} = P_0 e^{(\theta + \hat{\rho})T}\left[ q(z^*) - z^* q'(z^*) \right] \frac{(r - \hat{\rho})}{(r + \theta)}$$

and differentiating with respect to $T$ giving

$$\frac{dK(T)}{d\hat{\rho}} = \frac{P_0 e^{(\theta + \hat{\rho})T}T}{(r + \theta)} \left[ q(z^*) - z^* q'(z^*) \right] T - z^* q''(z^*) \frac{dz^*}{d\hat{\rho}}.$$  

$$\left(26\right)$$
This expression is negative for small $T$ (since $q'' < 0$ and $dz^*/d\hat{p} < 0$) and positive for large $T$, when the first term in the square brackets comes to dominate. There is therefore a period in which field openings are reduced (or cease altogether), following which more fields are opened at each date and the new path overtakes the old.

Figure 2b illustrates for a permanent reduction in $\dot{p}$. This increases the rate of depletion and brings forward field opening, giving the $K$ crossing that we noted in equation (26), (top left hand panel). The top right and bottom left panels give the paths of $R$ and $QS$, giving initial path (solid), intensive margin only ($K$ constant, short dash) and full adjustment (long dash). Faster depletion alone (short dash) gives a fall in open reserves at all dates, associated with higher output in the short run and lower output in the long run. Combining this with the change in field openings (long dash), the effect is magnified with a larger output increase in the short run, but a sharper fall in the long run. Cumulative output is raised for a short period, but then permanently reduced as lower prices have a major impact in reducing field openings (bottom left panel). We summarize results in proposition 4:

**Proposition 4:**

A permanent increase in the rate of growth of price tilts production to the future. Depletion of existing fields is slowed down, and opening of new fields postponed. Supply is reduced for a period, after which it overtakes its previous level. The converse holds for a permanent decrease in the rate of price growth.
Figure 2a: Price decrease

Solid line: original path. Dashed line: new path.

Figure 2b. Slower price growth

Short dash: Intensive margin, $K$ constant, $z$ adjusts.
Long dash: Intensive and extensive margin, $K$ and $z$ adjust.
5.2 Demand shocks and equilibrium responses:

Consider a change in the level of demand at all dates, i.e. a shift in $D$. We know from section 4 that there is no effect on long rate rates of growth of $p$, $Q_S$, or $R$, or on the level of $z$, although there is a change in the price level. If there were no extensive margin effects (the path of $K$ held constant) then there would be no short-run effects either; all quantities would be unaffected and the demand change would be shifted wholly to the price level. However, as seen in the previous sub-section, the extensive margin depends on the level of prices as well as their rate of change; a change in the price level changes the timing of field opening, this changing supply and inducing a transitional dynamic response.

Figure 3a illustrates the effect of a permanent decrease in demand ($D$ falling to 75% of its previous value), with all variables now expressed relative to the initial path. The top right hand panel gives the price path. The short dashed line gives the price path in the absence of extensive margin effects: a one-off drop to $0.866 = 0.75^{1/\eta}$ of its previous value. Including extensive margin effects, the long dashed line indicates a larger ultimate price fall, asymptoting to $0.68 = 0.75^{1/(\eta+\sigma)}$ of its previous value. As we saw in the preceding section, a price fall leads to postponement of field opening; a pause (top left), and resumption with $K$ smaller and $S(K)$ larger. This means that supply falls and then overtakes its previous path (bottom left). This now has a feedback effect on price; price drops abruptly as demand falls, increases when supply is falling, and then falls to its asymptotic path (top right). The main message concerns the equilibrium path of supply, particularly cumulative supply (bottom right). Without the extensive margin, a demand change would have no effect whatsoever on output. With the extensive margin operating, a reduction in demand cuts supply in the short run, raises it in the long run, and has a negative impact on the cumulative quantity extracted and supplied to all dates.
Figure 3a: Decrease in demand: relative to constant growth path

Solid line: original path. Dashed line: new path.

Figure 3b: Slower growth of demand: relative to constant growth path

Short dash: Intensive margin, $K$ constant, $z$ adjusts:
Long dash: Intensive and extensive margin, $K$ and $z$ adjust.
A permanent change in demand growth affects the long run growth of variables as well as transitional dynamics. Long run growth rates can be found explicitly (appendix table 1); a reduction in the rate of growth of demand gives a lower long run rate of price increase and a less rapid decline of output. The full dynamic story is illustrated in figure 3b. Following the reduction in demand growth inter-temporal substitution creates an incentive to shift both depletion and field opening from the future to the present, but this is combined with a price level effect that deters field opening. If adjustment were to take place only at the intensive margin, then the path of supply would be unambiguously tilted towards the present (short dashes); price growth is slower, the rate of depletion faster, and the increase in present supply leads to an immediate fall in price. The extensive margin of field opening responds both to this fall in the price level, and to the slow future growth of prices. The combined effect is to slow the rate of field opening and push opening new capacity into the future giving the U-shaped path of output (bottom left). In the short run, the faster extraction of open fields dominates and supply increases. In the medium run supply is lower because open fields have been depleted faster and because fewer new fields have been opened. In the long run supply turns up, because the high S(K) field types, opening of which was postponed, are coming on stream. Looking at cumulative supply, we see that adding the extensive margin effect mitigates the shift in supply towards the present; cumulative supply is raised for a shorter period, beyond which it is associated with larger reductions in cumulative output and cumulative stock of resource extracted.

6. Carbon taxation with endogenous field opening:

The equilibrium impact of climate change measures such as a carbon tax depend on both the demand and supply responses of fossil fuels. Much of the climate change literature has concentrated on demand reduction, while Sinn (2008) has used a simple model of resource depletion to argue that supply conditions may create a ‘green paradox’; carbon taxes or other measures to reduce demand might be ineffective or, if they are expected to become more severe in future, have perverse effects, bringing forwards extraction from the far future to the nearer future. How does this work when both extensive and intensive margin effects are present?

Policy measures that lead to permanent proportionate demand reduction cause an initial fall in output and permanently lower level of the cumulative quantity of the resource
supplied (section 5.2, figure 3a). This is in contrast to the case when the extensive margin effect is absent, in which policy has no effect on quantities produced, the difference being that a lower price delays field opening and postpones production. Policy measures that reduce the rate of growth of demand bring forward extraction from existing fields, this raising current output. However, this is offset by the price level effect which postpones field opening. Output therefore falls faster, and the cumulative output increase is smaller, and positive for a shorter period of time (figure 3b)

Demand shifts could be implemented by a tax on resource use, such as an emissions tax. For a proportionate decrease in demand at all dates this would require a constant ad valorem tax (with iso-elastic demand). If the rate of growth of demand is to be reduced, the tax rate would need to increase exponentially. Figure 4 looks at an alternative case in which an emissions tax is imposed at date 0 and then held constant in perpetuity, (therefore declining relative to the resource price). As before, short-dashed lines give the effect through the intensive margin alone, and long-dashed lines the full response. The producer price falls on impact, but then converges back to its previous level (as the relative value of the tax diminishes). This reduces the rate of extraction, giving the short run fall in supply followed by long run increase. However, when the extensive margin operates (long dashes) the producer price fall generates a period in which no new fields are opened, and hence a much larger fall in supply. As usual, this is a postponement of field opening, so supply rises in future. Once again, the key point is that the price level effect of demand reduction policy postpones field openings and thereby has a negative impact on supply in the near future. This is sufficiently large that cumulative output is reduced at all dates.
Figure 4: Constant specific resource tax: relative to constant growth path

Open fields, $K(t)$

Price, $p(t)$

Supply, $Q_s(t)$

Cumulative supply prop. change

Short dash: Intensive margin, $K$ constant, $z$ adjusts:
Long dash: Intensive and extensive margin, $K$ and $z$ adjust.

7. Concluding comments

The paper has developed a model of the supply of a non-renewable resource in which the empirically compelling fact that large sunk costs are associated with the development of new mines or fields is put centre stage. The model encompasses both depletion of existing fields and the development of new fields, thereby providing a modest step towards greater reality. New insights come from the approach. The most fundamental is that while the rate of interest may matter for depletion rates and short run transitional dynamics, it has no impact on the long run behaviour of resource prices; long run price growth depends on demand and underlying supply considerations (the geology of available fields). The approach also provides perspective on some ‘paradoxes’ that have gained recent attention. For example, emissions taxes may tend to bring forward depletion of existing resources, but they also discourage the development of new fields, so are likely to have to the desired effect of
pushing production into the future, reducing cumulative output and any associated stock of emissions.

The approach suggests a number of extensions and applications. For example, we have assumed throughout that (following a shock) future price paths are known with certainty and that owners of fields will postpone opening until the date at which the present value of the field is maximized. Allowing price uncertainty and placing the field opening decision in a stochastic context is clearly important. Lags in opening fields will introduce a more complex dynamic response to shocks. The development of substitutes provides a further supply margin. On the applied side, the model provides a relatively tractable framework for thinking about a number of practical and policy issues. The paper discusses some of the issues to do with fossil fuel supply and climate change, but the model also provides a framework for analysis of rent taxes (royalties, production sharing arrangements and corporate income taxes) which have to balance the need to capture rent with incentives for field development.
Appendix 1:
Substituting (2) in (1) gives $PV = \int_0^\infty pxq(-\dot{x}/x)e^{-rt}d\tau$.

The Euler-Lagrange equation is $\frac{d}{d\tau} \left[ p\dot{x}(-\dot{x}/x)e^{-rt} \right] = p\dot{q} + q\dot{x}/x e^{-rt}$, giving equation (3) of the text.

Appendix 2:
Fields vary in capital cost $K$, with the number of fields of type $K$ denoted $S(K)$. This can be derived from the following set up. Suppose that fields are ordered by size, $s$, with $m(s)$ fields of size $s$, $m' < 0$. $m(s)$ follows a power law, so $m(s) = s^a$, $a < 0$. The total capacity of fields of size $s$ is $sm(s) = s^{1+a}$. The capital cost of a field of size $s$ is $k(s)$, and we suppose $k(s) = s^\kappa$, $0 < \kappa < 1$, so costs are increasing and strictly concave in field size; the capital cost of one unit of capacity on a field of size $s$ is $s\kappa^{-1}$, i.e. $K = s^{\kappa-1}$. Since the capacity associated with fields of size $s$ is $S = s^{1+a}$, we have, eliminating $s$, $S(K) = K^{(1+a)/(\kappa-1)}$. Thus, $\sigma - 1 = (1+\alpha)/(\kappa-1)$ and hence $\sigma = (\kappa + \alpha)/(\kappa-1)$, which is negative if $\kappa < 1$ and $\kappa + \alpha > 0$.

Appendix 3:
Parameter values, figures 2, 3, and 4:
$r = 0.02; g = 0.005; \eta = 2; \sigma = -1.25; a = 0.1; b = 0.005; \lambda = 0.5$.
Long run equilibrium $\hat{p} = 0.067$ (exogenous in figures 2 and 3).
Figure 2a: initial price $p_0$ reduced by 20%. Figure 2b: $\hat{p}$, halved to 0.0025
Figure 3a: demand, $D$, cut by 25%. Figure 3b: growth rate $g$ halved to 0.0025
Figure 4: Constant specific tax at 30% of initial price (e.g. carbon price $50$, oil price $70$, 0.43 tonnes of CO$_2$ per barrel of oil).

Table 1: Asymptotic growth rates for a reduction in the rate of growth of demand:
$g_I$, initial growth of demand; $g_N$, new growth of demand. $g_I < g_N$

<table>
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<th>Initial, $g_I$</th>
<th>New, $g_N$</th>
<th>New, $g_N$</th>
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<tbody>
<tr>
<td></td>
<td>Intensive margin only</td>
<td>Intensive &amp; extensive margin</td>
<td></td>
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<tr>
<td>$\hat{K}$</td>
<td>$g_I + \eta\theta / (\eta + \sigma)$</td>
<td>$g_I + \eta\theta / (\eta + \sigma)$</td>
<td>$g_N + \eta\theta / (\eta + \sigma)$</td>
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<tr>
<td>$\hat{Q}_S$</td>
<td>$\sigma(g_I + \eta\theta) / (\eta + \sigma)$</td>
<td>$\sigma(g_I + \eta\theta) / (\eta + \sigma)$</td>
<td>$\sigma(g_N + \eta\theta) / (\eta + \sigma)$</td>
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<tr>
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<td>$g_N - \eta\hat{p}$</td>
<td>$g_N - \eta\hat{p}$</td>
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<tr>
<td>$\hat{p}$</td>
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<td>$g_N - \sigma\theta + \sigma(g_N - g_I) / \eta$</td>
<td>$g_N - \sigma\theta / \eta + \sigma$</td>
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References:


