The home bias of the poor: terms of trade effects and portfolios across the wealth distribution

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Abstract

This paper documents how poorer and less educated US households hold a smaller fraction of foreign assets in their financial portfolio. This average home bias of the poor is partly due to a lower probability of participating in foreign asset markets, often attributed to fixed costs of market entry. However, portfolio shares also rise with wealth among those households that do hold foreign assets, which fixed participation costs cannot explain. I use a simple, standard two-country general equilibrium model to show that hedging of real exchange rate movements and non-financial income risk, commonly employed to explain aggregate country-level home bias, also produces non-trivial heterogeneity in portfolios across wealth levels within countries that is in line with the evidence.

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1 Introduction

It is well-documented that household portfolios become more diversified as wealth increases. Campbell (2006) and Guiso et al. (2003), for example, show that poor households are less likely to invest in risky assets. Also, many authors have found that aggregate country-portfolios have surprisingly low shares of foreign assets, the so-called “home bias in portfolios puzzle” (see Lewis (1999), and Coeurdacier and Rey (2011), for a summary of this literature). Less attention, however, has been devoted to the composition of individual household portfolios between domestic and foreign assets, and its relationship with individual wealth, partly because of the difficulty to account for indirect holdings of foreign assets via mutual funds.\footnote{Christelis and Georgarakos (2008) consider survey of consumer finances (SCF) data on direct holdings of foreign assets only, while Karlsson and Nordén (2007) focus on indirect holdings of foreign assets by Swedish Individuals via pension funds. Hau and Rey (2008a) and Hau and Rey (2008b) look directly at individual mutual fund portfolios.}

In the empirical part of this paper, I combine data from the US survey of consumer finances (SCF) with information about the portfolio composition of more than 4700 US mutual funds to derive a measure of total foreign asset holdings by US households. Similar to previous studies, I find that wealthier and more educated households are more likely to participate in foreign asset markets. However, the evidence also shows that the portfolio share of foreign assets increases with wealth even for participants. This finding is important, as it cannot be accounted for by fixed costs of participation in sophisticated asset markets, which is one of the most common explanations of investor bias towards safe and home assets.\footnote{See, for example, Christelis and Georgarakos (2008) for an application of this argument to home bias.}

In the theoretical part of the paper, I propose a first step towards an alternative explanation for investor “bias” against risky and foreign assets that does not rely on fixed costs. I use a simple, standard two country model of international trade (Cole and Obstfeld (1991)) to show that the aim of hedging fluctuations in the real exchange rate and in non-financial incomes, a key element of many recent studies of aggregate home bias in country portfolios, also implies significant variation of portfolios across wealth levels within countries. Particularly, agents with lower financial wealth optimally have a higher portfolio share of assets that hedge against fluctuations in their future non-financial income. When returns to capital and labour are positively correlated, this leads to a bias of poorer investors against equity. With home bias in consumption, however, investors are
also biased against foreign bonds, as these are a bad hedge against aggregate productivity shocks at home. Wealthy investors, whose future consumption is less dependent on labour income, care less about this hedging property than poor investors. Therefore, equilibrium portfolios vary across the wealth distribution and poorer investors have a stronger home bias than rich investors.

The intuition for these results has similarities to Baxter and Jermann (1997), who show that with income from non-marketable human capital the optimal portfolio of assets consists of two sub-portfolios, one completely diversified, the other designed to hedge against volatility of human capital returns. I show that the hedging portfolio can be dominated by safe domestic assets. And its importance relative to the diversified part of the portfolio declines as total wealth rises. My paper is related to three strands of the literature, namely research on household portfolio decisions, the “home bias” literature in international finance, and studies of heterogeneous agents models in macroeconomics.

From studies of household finances, such as Campbell (2006) or Guiso et al. (2003), I take the stylised fact that wealthier individuals have riskier and more diversified portfolios. Using the 2004 wave of the survey of consumer finances, I show how a similar relationship also holds for investments in foreign bonds and stocks. Importantly, relative to previous studies of individual foreign asset holdings in SCF data, such as Christelis and Georgarakos (2008) or Kyrchenko and Shum (2009), I am the first to consider both direct and indirect holdings of foreign assets by combining SCF data on individual mutual fund investments with information on the portfolios of more than 4700 US mutual funds provided by Morningstar. Moreover, in contrast to their studies, I estimate jointly the participation decision in the market for foreign assets and their share in the portfolios of participants. The results show how, even conditioning on participation in foreign asset markets, wealthier households have a higher portfolio share of foreign assets on average. This is important, as it suggests that the often-cited threshold effects linked to (pecuniary or non-pecuniary) fixed costs of entering sophisticated financial markets (stressed, for example, in Christelis and Georgarakos (2008)) capture only part of the economic driving forces behind observed variation in financial portfolios of US households.

While previous studies of household investments in foreign assets are entirely empirical, the second part of this paper builds on the literature on “home bias” in country portfolios and proposes a simple stochastic general equilibrium model as a first step to understand the observed pattern of investments in foreign assets by US households. Particularly, a long sequence of studies (see Lewis (1999), and Coeurdacier and Rey (2011) for surveys)
has shown that, in an open economy subject to stochastic variation in fundamentals, the resulting equilibrium comovement of asset returns and other sources of income can imply optimal portfolios that depart significantly from the naive benchmark of full diversification. For example, non-tradable goods, or a bias in consumption baskets towards locally produced goods, lead to asymmetric optimal portfolios (see e.g. Stockman and Tesar (1995)). More importantly, Cole and Obstfeld (1991) point out the importance of equilibrium terms of trade movements by showing that, even in the absence of financial diversification, the fall (rise) in the relative price of domestic goods implied by a positive (negative) productivity shock at home can lead to perfect sharing of country-specific productivity risk. Also, non-tradable risks, for example in returns to human capital, introduce country-specific hedging terms in optimal portfolios (Baxter and Jermann (1997)) that can lead to home or foreign bias depending on the covariance of returns to labour and capital (Bottazzi et al. (1996)). On this issue, Heathcote and Perri (2007) show that in models with capital, the positive correlation of investment with productivity shocks introduces negative comovement of dividends and labour income, which may help explain the large observed portfolio share of domestic equities. While most of this literature focuses on investments in equity, Coeurdacier and Gourinchas (2011) have pointed out the importance of bonds for hedging real exchange rate movements, which can potentially explain the observed home bias in bond portfolios (Tesar and Werner (1995), Burger and Warnock (2007)) as well as a larger share of home equities than in portfolios without bonds.

This paper generalises a simple two-country workhorse model from this literature (Cole and Obstfeld (1991)) to include non-insurable idiosyncratic risks to individual incomes within countries. I show how the equilibrium comovement of bond and equity returns with the terms of trade and non-diversifiable labour market risk leads to important variation in portfolios across individuals that is in line with that observed in US micro-data. Importantly, the presence of wealth heterogeneity and uninsurable idiosyncratic risk within countries makes it impossible to use standard solution methods that rely either on perturbation of aggregate variables around their non-stochastic steady state values (Devereux and Sutherland (2011)) or perfect risk-sharing (Coeurdacier and Gourinchas (2011)). I therefore use a particular model structure that allows a solution of equilibrium movements in the terms of trade and the real exchange rate independently of individual heterogeneity. This enables me to derive, in a second step, analytical portfolio shares as a function of aggregate variables and deviations of individual variables from their agent-specific condi-
tional expectations. The simplified structure allows me to identify the role of equilibrium terms of trade movements and non-diversifiable labour income risk for individual holdings of bonds and equity. However, the independence of the terms of trade from individual heterogeneity comes at the price of abstracting from capital and assuming unit-elastic preferences across domestic and foreign goods. Thus, while this study points out that the economic mechanisms used to analyse “absolute” home bias in country portfolios might also help explain the “relative” home bias of the poor, it should not be viewed as an attempt at a final answer, but rather as a first step to more general models.

Finally, this study is one of the first to include uninsurable idiosyncratic income risk within the two countries of a new open-economy macro model with non-trivial real exchange rate movements. I see this as a contribution to the small macroeconomic literature that generalises heterogeneous agents models to the open economy.\(^3\)

Section II analyses portfolio shares of foreign assets across the wealth distribution in the 2004 wave of the SCF. Section III presents a simple two country two good economy, defines the competitive equilibrium and derives the equilibrium terms of trade movements. Section IV contains the results on optimal portfolios and how they vary across the wealth distribution.

2 Portfolios across the wealth distribution: evidence from the 2004 SCF

Wealthier and more educated people are more likely to invest in risky assets. This is well-documented for the US (see for example Campbell (2006) for a review and an illustration using 2001 SCF data) and a number of European countries (see Guiso et al. (2003), and Carroll (2002)). Figure 1 illustrates this stylised fact for the case of stock holdings by US households in the 2004 wave of the SCF, whose portfolio share (averaged within every decile of the financial wealth distribution to reduce noise) increases (almost) monotonically with individual financial wealth.

[Figure 1 somewhere here.]

\(^3\)Other studies in this literature, such as Mendoza et al. (2009), or Broer (2011), usually abstract from aggregate shocks and real exchange rate movements.
Equally, it is well-known that average country portfolios have surprisingly low shares of foreign assets - the “home bias in portfolios puzzle”. This has been interpreted as a consequence of a more general “local bias” of household portfolios, which overweigh local, regional, and national assets (see e.g. Campbell (2006)). But compared to the portfolio shares of risky assets in general, or of domestic equity more in particular, there is relatively little evidence on the home bias of individual households and its determinants. For the case of Sweden, Calvet et al. (2007) conclude that international diversification possibilities exist, but are usually exploited only by wealthier individuals, who have a higher share of investments in mutual funds (with an average portfolio share of 25 percent for foreign assets). In contrast, Karlsson and Nordén (2007) find no evidence that individuals with higher net worth or income take different investment decisions in the Swedish defined contribution pension system, although they do find men, public employees, and less educated individuals to invest more in funds dominated by domestic assets. Using SCF data, Christelis and Georgarakos (2008) and Kyrychenko and Shum (2009) find a positive effect of financial wealth, as well as other proxies of investor sophistication, for the likelihood to enter foreign asset markets. Similar to their studies, I document the determinants of foreign asset holdings of US households and their evolution across the wealth distribution using the US survey of consumer finances (SCF), which includes information on the US dollar value of households’ holdings of “bonds issued by foreign governments or companies” and “stock in a company headquartered outside of the United States”. In contrast to their studies, however, I also include a measure of foreign assets held via mutual funds. Particularly, I derive a measure of total foreign asset holdings by summing to individuals’ direct investments in foreign equity and bonds the reported value of their mutual fund shares in equity, bond and combination funds multiplied by the average portfolio weight of foreign bonds and equity in each type of fund. Figure 2 plots the resulting foreign

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4 Question codes x7638 and x7641. An obvious problem of this measure is that it does not refer to non-dollar assets, but to assets issued by foreign issuers, in foreign currency and US dollars.

5 In other words, I do not consider pension funds. One reason for this is that individuals’ decisions on pension fund investments are taken under a very different set of constraints than other investment decisions. Also, most shares in pension funds are not actively managed as a part of regular portfolio decisions. However, both these arguments do not apply to individual mutual fund investments.

6 To my knowledge, these average portfolio shares of mutual funds are not readily available from published sources. But Morningstar kindly provided data on portfolio shares of non-US assets for more than 4700 US mutual funds, not including funds of funds. From this I calculated weighted averages for portfolio shares of foreign bonds and equity for the three categories of funds for the year 2003. Since equity (bond) funds seem to often not report zero foreign bond (equity) holdings, I made an adjustment
asset portfolio shares (averaged within every decile of the financial wealth distribution) as a function of individual financial wealth. The first fact to observe is that the share of foreign assets in the portfolio of US households is significantly smaller than that of stocks for all values of financial wealth. It is important to note, however, that the values in figure 2 are not directly comparable to the share of foreign assets in aggregate country portfolios. Important for the question of this paper, figure 2 shows that the portfolio share of foreign assets, like that of stocks in figure 1, increases monotonically across deciles of the financial wealth distribution. Richer households thus seem to have lower home bias on average.

[Figure 2 somewhere here.]

The evidence presented in Figure 2, however, raises several questions. First, there are at least two potential sources of error in the way I measure individual portfolios. One arises from households under- or misreporting their foreign asset holdings. But since there is evidence of variation in foreign asset shares across mutual funds at least for equities (Hau and Rey (2008a) and Hau and Rey (2008b)), another source of error is the use of average mutual fund portfolios, which could lead to a bias in the results. An appendix argues that both kinds of measurement error are likely to bias any positive relationship by setting missing observations to zero for all funds that reported portfolio shares summing to at least 99.5 percent. The resulting sample included around 2800 observations for shares of international equity and slightly less for bonds. Using this sample, the average US equity mutual fund invested 17.1 percent in foreign shares, while the average bond fund (disregarding funds of government / municipal bonds) invested 3.6 percent abroad. Combination funds invested on average 10.7 percent in non-US assets.

7Both the deciles and the averages take account of the fact that the SCF oversamples parts of the population, by applying the weights suggested by Kennickell (1999), and the multiple imputation procedure used for the SCF. This is because, to eliminate inconsistencies and missing values, the SCF imputes some values from the other information provided by a household. However, rather than simply reporting one best guess for the imputed values, the SCF provides 5 draws per observation from the distribution of the missing values conditional on observables.

8The latter are usually simply the ratio of a country’s foreign equity holdings to its domestic market capitalisation. The measure in figure 2 differs from this as it includes bond holdings in the numerator, and use the SCF measure of gross financial wealth as the denominator of the ratio. The later includes a large range of non-equity assets such as insurance contracts, liquid retirement funds, and other assets and liabilities (such as deposits and loans) that net out across households in aggregate measures of wealth. Moreover, the aggregate shares of foreign assets in the country portfolio cannot directly be read from the graph. The ratios of foreign to total assets of the implied weighted aggregate portfolio are 2.75, 4.08, 3.99 percent for bonds, equities and their total respectively.
between portfolio shares and financial wealth towards zero. This is because off-shore investments for tax evasion are likely to make underreporting more severe for foreign assets, and average mutual fund portfolio shares are likely to under-represent the foreign asset holdings by wealthy households’ if these systematically choose mutual funds with higher foreign exposure. A second question is whether the rise in average portfolio shares across the wealth distribution could merely be due to a higher participation rate of wealthy individuals in the foreign asset market, rather than a rise in individual portfolio shares of participants as they become richer. The literature often motivates such a pattern by fixed costs of entering sophisticated financial markets (see e.g., for the case of foreign assets, Christelis and Georgarakos (2008)). An appendix presents a simple model that shows that this implies a non-linear relationship between financial wealth and participation, in the form of a threshold value of assets below which individuals do not hold any foreign assets. Optimal portfolios above the threshold value, however, would not be affected by sunk fixed costs. Thus, any variation in portfolio shares above the threshold value has to be attributed to other factors.

Finally, one might suspect that financial wealth simply captures the effects of other important variables, such as education, age, or income, on portfolios. In this case we would expect an analysis that controls for these variables to yield significantly different results. To investigate these issues in detail, I perform a more formal econometric analysis. Specifically, and contrary to previous studies, I estimate jointly the probability of participation and the optimal portfolio share of participants with the Heckman (1979) method, conditioning on other variables that were found to be important for portfolio decisions of individuals in previous research. To be precise, I estimate the parameters of the following 2 equation system

\[
SHARE = \begin{cases} 
\alpha + \beta_1 \ln(FIN) + \beta_2 \ln(INCOME) + \epsilon_1 & \text{if } H > 0 \\
0 & \text{otherwise}
\end{cases}
\] (1)

with

\[
H = a + b_1 FIN_2 + b_2 FIN_3 + b_3 FIN_4 + b_4 AGE + b_5 AGE^2 + b_6 AGE^3 \\
+b_7 COL + b_8 BLACK + b_9 HISP + \epsilon_2
\] (2)

Here, SHARE is the portfolio share of foreign assets, equal to the ratio of total foreign assets to FIN, the SCF definition of gross financial wealth, and INCOME is the sum of salaries, wages and income or losses from a professional practice, business, limited partnership, or farming. H is an indicator variable that captures the probability of participation in foreign asset markets. This probability is a function of a set of dummies \(FIN_x\) that
capture financial wealth, taking the value 1 when total financial assets of the household fall in the (weight-adjusted) $x$th quartile, as well as a third degree polynomial in AGE (equal to the age of the household head in years minus 15), a dummy variable “COL” that equals 1 when the household head holds a college degree, and dummies that take the value 1 when she reports her race to be black or hispanic. Only when $H$ is above a threshold, normalised to 0, do agents participate in foreign asset markets and we observe the variable SHARE, their portfolio share of foreign assets. Conditional on participation the portfolio share is a function of income and financial wealth. The errors $\epsilon_1$ and $\epsilon_2$ are assumed to follow a joint normal distribution. The equations are estimated jointly with full maximum-likelihood adjusted for sampling weights. Identification is achieved by restricting the effects of financial wealth to be linear in logs in (1), and constant within quartiles in (2), which I take to be a proxy for different possible participation thresholds.\footnote{I also estimated an alternative specification that included income quartiles in the participation equation. While in the presence of fixed costs of entering foreign asset markets we would expect financial wealth to determine the participation threshold and not income, current income could act as a proxy for future financial wealth. However, the income quartile dummies turned out to be insignificant, so I excluded them from the final specification.}

Table 1 to 3 report the results of estimating equations (1) and (2) for three asset classes.\footnote{Again, an additional complication for the estimation is the use in the SCF of multiple imputations for missing values. To account for this, I estimate the same model for each of the 5 implicates separately and then aggregate the estimation results. For the coefficients and standard errors reported in Table 1 to 3, I use the formulae suggested in the SCF codebook (http://www.federalreserve.gov/PUBS/oss/oss2/2004/codebk2004.txt). For the $\chi^2$ statistics I report a simple average of the 5 values.}

To put the evidence on foreign asset holdings into context, table 1 confirms the well-known results in Campbell (2006) or Guiso et al. (2003) about the determinants of household investments in stocks (domestic or foreign-issued): more educated and richer US households are more likely to participate in stock markets (although the latter effect is not strictly monotone across quartiles of the financial wealth distribution), while households with a head who reports her race as black participate less on average. Interestingly, there are no significant age effects. Importantly, and in line with figure 1, even among participants in the stock markets the portfolio share increases strongly with financial wealth, while there is no significant effect of non-financial income.
Table 2 presents equivalent estimation results for the portfolio share of total foreign asset holdings, including those held indirectly via mutual funds. The effect of financial wealth is significant (at the 1 percent level) in both equations. Ceteris paribus, individuals in the bottom quartile of the financial wealth distribution are least likely, and those in the top quartile most likely to invest in foreign assets, although, again, the effect is not monotone. Rather, there is a jump between the first and second quartile of the financial wealth distribution that can be interpreted as suggestive evidence of a threshold value of assets beyond which a rise in wealth has only a small effect on the probability of participation. Like in the case of total stock holdings, there is a positive effect of a college degree, and negative race effects, for the probability of participation. Importantly for the issue of this paper, the estimation results for equation (1) show that higher financial wealth increases significantly the portfolio share of foreign assets even of participants, which cannot be attributed to fixed costs. Again, the effect of rising non-financial income on the portfolio share of foreign assets is negative but insignificant.

To show how the evidence for foreign assets is not just a special case of the stylised fact of increasing equity holdings across the wealth distribution, Table 3 shows the results for foreign bond holdings. Overall, the same results as for total foreign assets hold in this sub-category. Thus, households in the upper quartile of the wealth distribution are again significantly more likely to hold any foreign bonds.\(^\text{11}\) Also, there are positive education and negative race effects. And importantly, although the coefficient on financial wealth in equation 1 is smaller, in line with the lower portfolio share of foreign bonds on average, there is a significant increase of the bond portfolio share across the wealth distribution even for participants.

This section has shown that individual portfolio shares of foreign assets increase with financial wealth. There is some evidence of a threshold of financial wealth for participation in foreign asset markets, consistent with fixed participation costs. But fixed costs cannot explain the significant positive relationship between portfolio shares and financial wealth.

\(^{11}\)Note that no households in the lower quartile holds bonds, which reduces the number of quartile dummies in table 3 to 2.
for participants. The next section presents a simple model of the international economy, where general equilibrium movements in the relative price of home and foreign goods can make home assets better hedges against income fluctuations, and thus lead to the observed pattern of portfolios: poor individuals have a stronger taste for home bonds as in general equilibrium their real payoffs hedge against volatile endowments, which are their dominant source of income.

3 A two country heterogeneous agents endowment economy

This section aims to make a first step to bridge the gap between the classical literature on aggregate home bias in country portfolios, and more recent, mainly empirical studies of home bias in household portfolios. Particularly, it aims to show how some of the mechanisms that are prominent in the former might help explain results such as those presented in the previous section, where I showed higher financial wealth to have a significantly positive effect both on the probability of participating in foreign asset markets, and on the portfolio shares of participants.

One of the main results of the international macroeconomics literature on aggregate home bias is that, apart from differences in mean returns and fixed costs of investing in financial markets, it is the desire to hedge fluctuations in the real exchange rate and in non-diversifiable sources of income, such as wages, that determines optimal portfolios across domestic and foreign assets (see, for example, Coeurdacier and Rey (2011)). Moreover, in the general equilibrium of an international economy hit by shocks to its fundamentals, these hedging concerns can imply significant departures from the benchmark of identical, fully diversified portfolios across countries (Lucas (1982)). In this section, I propose a model that includes several features that have been found to help explain aggregate home bias in portfolios, such as consumption baskets dominated by domestic goods (Stockman and Tesar (1995)), non-diversifiable income risk (Baxter and Jermann (1997)), and investment opportunities in both bonds and equities (Coeurdacier and Gourinchas (2011)).

To be able to analyse variation in portfolios across individuals, I add to this setup two elements from the macroeconomic literature on heterogeneous agents economies: borrowing constraints and idiosyncratic income risk. Unfortunately, the inclusion of these two elements implies that the standard methods to derive aggregate country-portfolios can
not, or only with difficulty, be used as they rely either on perturbation of the model’s aggregate variables around their non-stochastic steady state values, potentially very different to expected values of individual variables (Devereux and Sutherland (2011)), or perfect insurance (Coeurdacier and Gourinchas (2011)). The strategy of this paper is therefore to make a number of simplifying assumptions that allow to separate the portfolio problem of households from the general equilibrium movements in real exchange rates and the terms of trade. Particularly, I use a two-period version of a standard two-country model in the home bias literature, first proposed by Cole and Obstfeld (1991). Its assumption of unit-elastic consumption baskets, together with symmetry in preferences and portfolios, conveniently allows me to solve for the general equilibrium terms of trade prior to the portfolio decision of households, for which I then derive a solution that is linear in the deviations of variables from their individual expected values in a second step.\textsuperscript{12} The resulting setup enables me to analyse the evolution in portfolios across three asset categories: home and foreign bonds, which I also call “safe assets”, as well as stocks, or “risky assets”. However, the simple character of the model comes at the price that it is unable to analyse home bias in equity. This is because, with unit-elastic consumption baskets, equilibrium terms of trade movements equalise the real returns to home and foreign stocks for any realisation of aggregate risk in the model. So the portfolio composition between home and foreign stocks is not well defined. A later section of the paper provides a tentative generalisation of the model to account for this issue.

3.1 The general environment

I consider an economy with two countries, home (H) and foreign (F). Both countries are completely symmetric. In particular, in each country there is a large number of agents with unit mass. Individual agents are indexed by $h, f$ at home and abroad respectively. They live for two periods, and receive endowments of a country-specific perishable good $H$ or $F$.

Agents’ preferences are described by a von Neumann-Morgenstern utility function with constant relative risk aversion $\sigma$ over a Cobb-Douglas aggregate with symmetric bias

\textsuperscript{12}I also abstract from capital accumulation, partly because of the two-period nature of the model. I comment further on this below.
towards domestic goods.

\[ U_k = U(c_k) + \beta E[U(c'_k)] \]  
\[ U(c_k) = \frac{c_k^{1-\sigma} - 1}{1 - \sigma} \]  
\[ c_h = c_{h,H}^{\theta} e_{f,F}^{1-\theta} \]  
\[ c_f = c_{f,F}^{\theta} e_{f,H}^{1-\theta} \]  
\[ \theta > \frac{1}{2}, \sigma > 1 \]  

where \( c_k \) is the consumption basket of agent \( k \) and \( c_{k,I} \) denotes consumption by agent \( k \) of good \( I \) for \( k \in \{ h, f \}, I \in \{ H, F \} \). In line with many papers on home bias, I assume \( \sigma > 1 \). More generally, notation is as follows: capital letters \( H,F \) denote country-specific variables or goods, small letters \( h,f \) denote individual variables that can vary across agents of country \( H,F \). First subscripts denote agents or countries, second subscripts goods. Second period values of a variable \( x \) are denoted as \( x' \), their probability distribution as \( \Psi^x \).

### 3.2 Heterogeneity and uncertainty

Heterogeneity of agents within the same country comes from differences in endowments. More precisely, agents in country \( K \) receive individual endowments \( \epsilon_k, \epsilon'_k \) of their specific good in period 1 and 2 respectively. Initial endowments \( \epsilon_k \) are known at the beginning of period 1 before agents choose consumption and portfolios. Income inequality in country \( K \) is summarised by the distribution of period 1 endowments across agents \( \Psi^\epsilon_K \), which is common knowledge.

\( \epsilon'_k \), the endowment of individual \( k \) in period 2, is the product of two terms: an “individual endowment share” \( \epsilon'_k \), and a country-specific “aggregate endowment” \( Y'_K \).

\[ \epsilon'_k = \epsilon'_k \ast Y'_K \]  

“Idiosyncratic risk” is given by the probability distribution of \( \epsilon'_k \), the period 2 endowment shares of individual \( k \), which I denote \( \Psi^\epsilon'_K \). For simplicity I assume that second period endowments are i.i.d. across agents and independent of all aggregate variables. Also I normalise expected period 2 individual endowment to 1, \( \int \epsilon'_k \Psi^\epsilon'_K = 1 \). By the iid assumption and the law of large numbers this means the sum of realised endowment shares is
always 1 and aggregate period 2 output in country K simply equals \( Y'_{K} \).\(^{13}\)

"Aggregate risk" is summarized by the probability distribution of \( Y'_{H} \) and \( Y'_{F} \), the aggregate endowments in period 2, denoted \( \Psi'_{H}, \Psi'_{F} \). I assume that these are identically distributed, and independent of individual random variables, but possibly correlated among each other.

I assume that all period 2 random variables are log-normally distributed:
\[
(\hat{e}'_h, \hat{e}'_f, \hat{Y}'_H, \hat{Y}'_F)' \sim N( (\overline{e}'_h, \overline{e}'_f, \overline{Y}'_H, \overline{Y}'_F)', \Sigma),
\]
where a hat denotes natural logarithms \( \hat{z} = \ln(z) \) and \( \Sigma \) is a matrix with diagonal entries \( V_{e'h}, V_{e'f}, V_{H}, V_{F} \) whose only non-zero off diagonal entry is the covariance of aggregate log-endowments \( \text{Cov}_{HF} \). Symmetry and the normalisation of individual endowments imply \( \overline{e}'_h = \overline{e}'_f = 1, V_{e'h} = V_{e'f} = V_{e}, V_{H} = V_{F} = V \) etc.

### 3.3 Incomplete asset markets and borrowing constraints

I impose the simplest structure of asset markets that allows me to analyse two kinds of trade-offs in optimal portfolios: the choice between safe and risky assets on the one hand, and between home and foreign assets on the other.

Like Huggett (1993), agents trade “IOUs” that are in zero net supply and denominated in domestic goods. These are “safe” assets in the sense that for 1 unit of H goods invested today, IOUs in H always pay \( R_{bH} \) units of good H next period (where “b” stands for “bonds”). Equivalently, foreign IOUs pay \( R_{bF} \) units of F goods.

In contrast to Huggett (1993)’s economy, however, agents can also trade shares in national mutual funds, and are allowed to buy shares and IOUs from foreigners. Shares are also in zero net supply, and risky in the sense that their payoffs are proportional to the stochastic aggregate endowment. Thus, for some constant \( R_{sH} \), the return on home shares is \( R_{sH}Y'_{H} \) per unit of H goods invested, equivalently for F. One obvious implication of the exogenous incompleteness of asset markets is that individual claims to future endowments are non-tradable, and that the resulting risk thus is non-diversifiable.

I denote h’s holdings of home and foreign IOUs by \( a^h_{h,H} \) and \( a^h_{h,F} \) respectively, and her holdings of shares by \( a^s_{h,H} \) and \( a^s_{h,F} \). Asset quantities are denoted in endowment goods of the owner. So if h holds a portfolio \( a^h_{h,H}, a^h_{h,F} \), she owns \( a^h_{h,H} \) units of H IOUs and \( a^h_{h,F} \) units of F IOUs, where \( p \) is the relative price of home goods. I denote the vector of returns as \( \overline{R} \), the vectors of assets held by individuals in H, F as \( \overline{a}_h, \overline{a}_f \), and the total value, in

\(^{13}\)For the derivation of a law of large numbers for continuum economies, see Uhlig (1996).
terms of their domestic good, of their assets at the end of period 1 as $a_h, a_f$. I assume both IOUs and shares have zero default probability. Consistent with this, agents can credibly promise to repay only in units of their income - so borrowing contracts are always written in the endowment good of the issuer. This means agents can issue only domestic assets, but invest both at home and abroad. One consequence of the no-default assumption are individual borrowing constraints: agents in country K can only issue IOUs and mutual fund shares up to maximum amounts $B^e_K, B^s_K$. In particular, and similar to for example Coeurdacier and Gourinchas (2011), I assume that agents can only sell claims amounting up to a fraction $\delta_i$ of their expected period two endowment$^{14}$

$$a^e_{k,K} \geq B^e_K = -\delta^e E[\epsilon'_k] \frac{R^e_K}{R^e}$$

$$a^b_{k,K} \geq B^b_K = -\delta^b E[\epsilon'_k] \frac{R^b_K}{R^b}$$

3.4 The problem of a typical household

A typical home household h maximises expected lifetime utility by choosing in period 1 consumption and a vector of assets $\bar{a}_h$ subject to her budget constraint, borrowing constraints for domestic assets and the non-negativity of foreign asset holdings, taking as given the relative price of home goods (in units of the foreign good) $p$ this period and the vector of returns $\bar{R}$. h’s problem is thus given as:

$$\max_{c_h, \epsilon'_h, \bar{a}_h} \frac{\epsilon_h^{1-\sigma} - 1}{1 - \sigma} \cdot \frac{1}{1 - \sigma} + \beta E\left\{\frac{\epsilon'_h^{1-\sigma} - 1}{1 - \sigma}\right\}$$

$^{14}$The “natural” limit to total borrowing in riskless assets would equal the present discounted value of minimum future income $B_K = b_K \frac{\epsilon_{K, min}}{R}$, which is the highest amount agents can repay for sure. But with log-normal endowments there is a positive probability of having endowment realisations arbitrarily close to 0, such that this formulation does not lead to a non-zero borrowing limit. The problem can be avoided by introducing a positive non-stochastic minimum endowment level for all agents in a country. This can be chosen such that the resulting natural borrowing limit equals the sum of $B^b_K$ and $B^s_K$ above.
Subject to the constraints

\[
\begin{align*}
    c_h &= \epsilon_h - \sum_{i \in \{b, s\}} a_{i,H}^h - \sum_{j \in \{b, s\}} a_{i,F}^h \\
    c_h' &= \frac{\epsilon_h' + R^b_{H}a_{i,H}^b + R^s_{H}a_{i,H}^s + (R^b_{F}a_{i,F}^b + R^s_{F}a_{i,F}^s)\nu'}{p'_{H}} \\
    a_{i,H}^i &\geq B_{H}^i, \text{ for } i \in \{b, s\} \\
    a_{i,F}^j &\geq 0, \text{ for } j \in \{b, s\} \\
    \epsilon_h' &= \epsilon'_H 
\end{align*}
\]

where \( p_H = \theta^{-\theta}(1 - \theta)^{(1-\theta)p^{1-\theta}} \) is the home consumption price index. The problem of a typical foreign household is symmetric.

### 3.5 Definition of competitive equilibrium

A competitive equilibrium is

1. **A Consumption Allocation:**
   
   For every agent \( k \), a consumption sequence of both goods for both periods: \( c_{k,H}, c_{k,F}, c'_{k,H}, c'_{k,F} \), where \( c'_{k,j} \) is a random variable depending on the realisation of period 2 uncertainty.

2. **A set of Portfolios:**
   
   For every agent \( k \), a vector \( \overline{a_k} \) specifying holdings of all assets in the economy at the end of period 1.\(^{15}\)

3. **A Price System**, consisting of
   
   - \( p, p' \), the relative prices of \( F \) goods in terms of \( H \) goods in period 1 and 2, where \( p' \) is a random variable with distribution \( \Psi^p' \).
   - \( \overline{R} \), the vector of asset returns.

such that

\(^{15}\)Summed across all agents individual quantities imply an aggregate consumption allocation for consumption of good \( K \) in country \( J \), \( C_{J,K} = \int c_{j,K} d\psi_j, C'_{J,K} = \int c'_{j,K} d\psi'_j \), as well as a country portfolio of gross and net asset holdings, and a net asset position once net holdings of all assets in a country are summed at period 1 prices.
1. Agents allocate their funds optimally across goods in period 2 given a particular realisation $p'$. 

2. The allocation solves every household’s problem (12) in period 1 given a relative price $p$, a distribution $\Psi^{p'}$, and rates of return $\overline{R}$. 

3. Markets clear: 
   - for goods: $\int c_{h,H}dh + c_{f,H}df = Y_H$, $\int c_{h,F}dh + c_{f,F}df = Y_F$ in both periods 
   - and assets: $\int a^i_{h,j}dh + \int p a^i_{f,j}df = 0$, $\forall i \in \{b, s\}$, $J \in \{H, F\}$ (each asset is in zero net supply) 

4. The distribution of the future relative price $\Psi^{p'}$ is consistent with the joint distribution of random variables $e^{h}_{h}', e^{f}_{f}', Y^{H}_{H}', Y^{F}_{F}'$, and individual asset holdings at the end of period 1. 

Note that optimal portfolios in this environment depend on the distribution of future relative prices $\Psi^{p'}$. But the latter depends on expenditure patterns tomorrow, and thus on savings and portfolio decisions today. In other words, the model has a complicated circular relationship between savings and portfolio decisions on the one hand, and the process for market clearing relative prices $\Psi^{p'}$ on the other.\textsuperscript{16} As the next section shows, the assumption of identical homothetic preferences for all agents within a particular country implies that a country’s demand for goods only depends on their relative price and aggregate country-level resources (net asset returns plus endowments), and not on the distribution of individual savings or endowments. However, individual uncertainty and heterogeneity still matter for aggregate savings and thus net asset positions. The symmetry of the model then ensures that even the aggregate net asset positions do not matter for excess demands, so aggregate endowments tomorrow completely determine aggregate demand for goods and thus market-clearing prices. 

\textsuperscript{16}This is similar to the recursive framework with capital accumulation presented by Krusell and Smith (1998), where agents need to know the law of motion for the joint distribution of individual asset holdings and (aggregate and idiosyncratic) shocks, as this determines aggregate savings and thus the returns to capital tomorrow.
3.6 How unit-elasticity implies terms of trade movements independent from heterogeneity

To derive the equilibrium process of tomorrow’s relative price $p'$, note first that identical homothetic preferences imply identical expenditure shares for all agents in the same country:

$$c_{h,H} = \theta s_h, \quad c_{h,F} = (1 - \theta) \frac{1}{p} s_h \quad \forall h$$

(13)

$$c_{f,H} = (1 - \theta) p s_f, \quad c_{f,F} = \theta s_f \quad \forall f$$

(14)

where $s_k$ denotes expenditure of agent $k$ in domestic goods. The linearity of the demand functions allows us to sum across agents and express excess demand for goods as a function of aggregate expenditure. But since expenditure shares on goods differ between countries, the market clearing price could potentially depend on relative country wealth and thus portfolio decisions. However, note that, from symmetry, claims on domestic output sold to foreigners $\nu$ are the same in both countries. Since therefore the share of claims to foreign output bought by domestic agents equals $1 - \nu$ in both countries, excess demand for $H$ goods becomes

$$\int \theta s_h dh + \int (1 - \theta) p s_f df$$

$$= \theta [\nu Y_H + (1 - \nu) p Y_F] + (1 - \theta)[(1 - \nu) Y_H + \nu p Y_F]$$

(16)

which yields the market clearing price

$$p = \frac{Y_H}{Y_F} \forall \Psi_F^\theta, \Psi_H, \Psi_F, \Psi_H$$

(17)

(18)

So the relative price is independent of the within-country heterogeneity in the economy. This is essential, as it allows me to solve for the optimal portfolios in closed form.

Note how another, perhaps in the present context less desirable, feature of unit-elastic demand for goods is that claims to country-endowments, or national mutual fund shares, must have equal stochastic consumption payoffs in equilibrium. For $h$ agents these are

$$\frac{R_{H}^h Y_H p_H'}{p_H} = \frac{R_{F}^h Y_F p_H'}{p_H} = \frac{R^{\theta h} Y_H Y_F}{p_H}$$

(19)
where I set the period 1 relative price of goods to 1 for simplicity and impose $R^*_{H} = R^*_{F} = R^*$ from symmetry. So agents are always indifferent between home and foreign mutual fund shares. In this sense, the equilibrium portfolio is never unique with international trade in shares. An appendix discusses conditions for uniqueness and existence of equilibrium.

4 Optimal portfolios

Asset holdings differ across individuals for two reasons: first, although the distributions of their future endowment income are the same (due to the i.i.d. assumption), agents differ in wealth due to differences in period 1 income. To smooth consumption, richer agents, with higher current income, save more than poorer agents. Second, poor agents, with low or negative savings, have tomorrow’s consumption determined largely by tomorrow’s endowment income. Thus, they prefer assets that are good hedges against fluctuations of endowment income, to limit consumption volatility. Aggregate home supply shocks reduce the relative price of home goods, and thus the real returns to home bonds. This makes home bonds better hedges against aggregate home endowment risk than foreign assets or mutual fund shares. Richer agents, whose consumption is mainly determined by asset returns, care relatively less about this hedging, and thus have a lower portfolio share of home bonds.

Note that we can define portfolio shares in two ways, namely as a share of financial wealth, or of total wealth including the present value of claims to future endowments (see also Cochrane (2007), section 2.4). Since the optimality conditions that describe household portfolios are linear in log total wealth, the model naturally yields expressions for wealth portfolio shares. Corresponding to the empirical section of this paper, Proposition 1 then considers financial portfolio shares, defined as a proportion of gross financial assets. Specifically, I start by showing that wealth portfolios are the sum of 2 sub-portfolios: first, a ”hedge portfolio”, designed to optimally sell off individual endowment risk, and thus more important for individuals whose expected endowment income is high relative to financial claims. And second, a ”diversified portfolio” that reflects relative returns as well as optimal hedging of real exchange rate fluctuations, and is independent of the level of wealth. For a given wealth portfolio share, financial portfolio shares change as a function of the relative importance of endowment and financial wealth in total wealth. This effect introduces another source of heterogeneity in portfolios across the distribution of financial wealth.
Since real payoffs to home and foreign mutual fund shares are always equalised by equilibrium terms of trade movements, in this section I call both of them shares in an “international mutual fund”. Also, from symmetry, the local currency returns to home and foreign bonds \( R_H, R_F \) have to be equal. This allows me to drop superscripts on returns and asset quantities in this section: returns on home and foreign IOUs are from now on simply denoted as \( R_H = R_F = R_B \), those on shares as \( R_S \), and \( h \)’s corresponding asset holdings as \( a_{h,H}, a_{h,F}, a_{h,S} \). I concentrate on the portfolios of home agents. Symmetry implies that the exact same results hold for the foreign country.

### 4.1 Unconstrained portfolios

Assume that there is at least one individual in the home country with non-binding borrowing and short-selling constraints. Imposing the equilibrium relative price as a function of output, we can write the four elements of her portfolio as follows

\[
\begin{align*}
\text{Real endowment} & : \quad e'_h Y^\theta_H Y^{1-\theta}_F \\
\text{Real share return} & : \quad a_{h,S} R_S Y^\theta_H Y^{1-\theta}_F \\
\text{Real return to foreign IOUs} & : \quad a_{h,F} R_B Y^\theta_H Y^{1-\theta}_F \\
\text{Real return to home IOUs} & : \quad a_{h,H} R_B Y^{\theta-1}_H Y^{1-\theta}_F 
\end{align*}
\]

(20)

The first thing to note is that share returns co-move perfectly with endowments. Thus, although consumers cannot hedge their idiosyncratic endowment risk, which is orthogonal to all returns, they can short-sell shares to hedge aggregate endowment risk. Furthermore, as \( \theta > \frac{1}{2} \) rises to 1, the consumption value of home IOU returns becomes less and less volatile for home agents. This is why home bias in consumption leads to home bias in bonds when agents are sufficiently risk-averse. To see this more in detail, I take a log-approximation to marginal utility and use the log-normality of random variables to solve the consumer’s arbitrage conditions for wealth portfolio shares as a function of the
parameters of the model. This yields

\[
\tilde{a}_{h,F} = \frac{r - y - r_S}{\sigma V} + \frac{1}{2} + \frac{(1 - \theta)(\sigma - 1)}{\sigma} \quad (21)
\]

\[
\tilde{a}_{h,S} = \frac{2r_S + y - r}{\sigma V} - \tilde{e}_{h}^f \quad (22)
\]

\[
\tilde{a}_{h,H} = \frac{r - y - r_S}{\sigma V} + \frac{\theta(\sigma - 1)}{\sigma} \quad (23)
\]

Return differential \hspace{2cm} RER hedge \hspace{2cm} Endowment hedge

where a tilde denotes ratios with respect to total wealth \( w = \tilde{e}_{h}^f Y_H + a_h \), \( V \) is the variance of aggregate log-output at home and abroad, and \( y, r_k \) are expected growth rates and log returns respectively. Note that, since expected endowments are equal across individuals, the portfolio share of endowment wealth \( \tilde{e}_{h}^f \) falls as total wealth rises. In line with the intuition, the optimal portfolio consists of two parts: one that hedges against non-diversifiable risk, and another that is perfectly diversified as a function of relative returns and real exchange rate risk. Given the perfect correlation of equity returns and aggregate endowments, aggregate endowment risk can be hedged perfectly through a negative position in shares that is equal to \( -\tilde{e}_{h}^f \) and thus proportional to the relative weight of endowments in total wealth. In other words, in a portfolio of bonds and stocks, bonds do not help to hedge endowment risk. The diversified portfolio, on the other hand, reflects relative returns and hedging of real exchange rate risk, but is independent of endowment risk. Specifically, all portfolio shares increase linearly in risk-weighted expected return differentials, the first term in equations (21) to (23). Hedging against real exchange rate risk is incorporated in the second terms in the expressions for bond portfolio shares \( \tilde{a}_{h,F}, \tilde{a}_{h,H} \). To understand these, note that for low risk-aversion (\( \sigma < 1 \)), agents would actually prefer assets that have high real returns when consumption is relatively cheap (the substitution effect dominates). So the portfolio share of foreign bonds, whose real returns are high when the terms of trade and home consumption prices are relatively low, would exceed that of home bonds. In line with the standard assumption in the literature on home bias, this paper assumed risk-aversion of \( \sigma > 1 \). In this case, agents prefer assets that have high returns when consumption is relatively expensive, to limit consumption volatility. From (23), this implies \( \tilde{a}_{h,H} > \frac{1}{2} > \tilde{a}_{h,F} \). Note that for \( \sigma = 1 \), both effects cancel, and agents choose an equally weighted bond-portfolio \( \tilde{a}_{h,F} = \tilde{a}_{h,H} = \frac{1}{2} \).

In summary, the optimal unconstrained portfolio consists of a short-position in stocks to hedge aggregate endowment risk, a long bond position dominated by home bonds in order
to a hedge against real exchange rate risk, and a portfolio that reflects relative expected returns. The latter do not have a closed form solution in this model. But it is easy to see that in equilibrium we have to have that

\[ r_S + y - r > 0 \]  

(24)

In other words, rich agents, with small endowment weights \( \tilde{e}_h \), have to have incentives to hold positive equity positions, as otherwise there would be an oversupply of shares sold to hedge against endowment risk. This immediately implies that \( \tilde{a}_{h,S} \) rises with wealth.

### 4.2 Constrained portfolios

Individuals can only sell off a fraction \( \delta^* < 1 \) of their endowment wealth. Since the level of the optimal stock position falls to \( \tilde{e}_h = 1 \) as we move down the wealth distribution, there is a strictly positive cutoff value of wealth below which the borrowing constraint on shares is binding. The portfolio shares of investors with \( w < w^* \) that are constrained in their share position but hold both home and foreign bonds can derived from their arbitrage condition as before, yielding

\[
\begin{align*}
\tilde{a}_{h,S} &= -\delta \tilde{e}_h \\
\tilde{a}_{h,F} &= \frac{1}{2} + (1 - \theta)(\sigma - 1) - \frac{1}{\sigma} \delta \tilde{e}_h \\
\tilde{a}_{h,H} &= \frac{1}{2} + \theta(\sigma - 1) - \frac{1}{\sigma} \delta \tilde{e}_h
\end{align*}
\]

(25) (26) (27)

Agents with low wealth therefore have a constant negative position in shares. The bond portfolio consists of a diversified subportfolio, as before, but now also includes positions that hedge the remaining endowment risk. Since home and foreign bonds have identical hedging properties with respect to aggregate endowments, the hedge portfolio has identical negative positions in home and foreign bonds, equal to half of the endowment risk that remained after short-selling the maximum amount of shares. The diversified portfolio again overweighs home bonds. Since \( \tilde{a}_{h,H} > \tilde{a}_{h,F} \), total portfolios are thus biased towards home bonds. Also, both the shares of home and foreign bonds decrease as total wealth \( w \) falls, with a slope of \( \frac{1}{2} \). Taken together, this implies that, when moving down the wealth distribution, at some positive wealth level \( w^{**} < w^* \) the short-selling constraint on foreign bonds will start to bind. Moreover, given the smaller level and identical slope of the foreign asset share, holdings of home bonds are positive at \( w^{**} \). Moving further
down the distribution of total wealth, individuals continue to have 0 foreign bonds and a constant short position in shares, but decrease their home bond holdings, which eventually become negative.

4.3 The home bias of the poor

The previous section showed how the linearity of first-order conditions in log-consumption led to simple expressions for portfolio shares as a fraction of total net wealth, including endowment wealth. In order to compare the theoretical results to the empirical evidence presented previously, this section considers financial portfolio shares, defined as a fraction of gross financial assets.

Proposition 1 The poorest investors with positive gross financial assets hold home bonds only. Moving up the wealth distribution, the financial portfolio share of home bonds falls, while those of stocks and foreign bonds rise, converging to the portfolio shares of the diversified portfolio. So poorer agents have both stronger home bias, and a stronger bias in favour of safe assets.

Proof

Agents with low wealth $w < w^{**}$ are constraint by both the short-selling constrained for foreign bonds, and the borrowing limit for shares. But from (27), there are investors with $w = w^{**} - \epsilon$ that hold positive amounts of home bonds. Their portfolio share of home bonds is thus 1.

Multiplying all portfolio shares in (25) - (27) by the ratio of total wealth to gross financial assets $\tilde{e}'_h Y_H + a_{h,F} + a_{h,H} + a_{h,S}$, and imposing $\tilde{a}_{h,S} = -\delta \tilde{e}'_h$, we get an expression for the share of IOUs in the financial portfolio of investors with total wealth $w$ between $w^{**}$ and $w^*$, who have unconstrained holdings of home and foreign bonds but are borrowing constrained in shares.

\[
\tilde{a}_{h,F}^{fin} = \frac{1}{2} + \frac{(1 - \theta)(\sigma - 1)}{\sigma} + \left[ \frac{1}{2} + \frac{(1 - \theta)(\sigma - 1)}{\sigma} \right] - \frac{1}{2}(1 - \delta)\tilde{e}_h^{fin}
\] (28)

\[
\tilde{a}_{h,H}^{fin} = \frac{1}{2} + \frac{\theta(\sigma - 1)}{\sigma} + \left[ \frac{1}{2} + \frac{\theta(\sigma - 1)}{\sigma} \right] - \frac{1}{2}(1 - \delta)\tilde{e}_h^{fin}
\] (29)

where $\tilde{e}_h^{fin} = \frac{Y_H \tilde{a}_{h,F} + a_{h,F} + a_{h,H}}{a_{h,F} + a_{h,H}}$ falls as gross asset holdings rise. From $\theta > \frac{1}{2}$ the portfolio share of home bonds thus falls, while that of foreign bonds rises, with gross assets.
Equivalently, for $w > w^*$, the financial portfolio shares of unconstrained investors are

$$a_{h,f}^{fin} = \frac{r - y - r_S}{\sigma V} + \frac{1}{2} + \frac{(1 - \theta)(\sigma - 1)}{\sigma}$$  \hspace{1cm} (30)$$

$$a_{h,s}^{fin} = 2\frac{r_s + y - r}{\sigma V} - e_{h}^{fin}$$  \hspace{1cm} (31)$$

$$a_{h,H}^{fin} = \frac{r - y - r_S}{\sigma V} + \frac{1}{2} + \frac{\theta(\sigma - 1)}{\sigma}$$  \hspace{1cm} (32)$$

So for unconstrained investors, the portfolio share of stocks rises, while the others are constant. For large wealth levels, all portfolio shares are thus equal to those in the diversified portfolio. ■

Proposition 1 shows that this simple economic environment is able to replicate the observed structure of individual asset holdings across the wealth distribution: Poor individuals do not participate in the markets for foreign or risky assets. And even beyond the value of wealth that makes participation worthwhile, the portfolio shares of foreign and risky assets continue to increase as wealth rises.

### 4.4 Generalising the model

The theoretical model of this paper made several simplifying assumptions. Specifically, it abstracted from capital investment, assumed homothetic preferences across goods, and looked at a simplified two-period version of the economy. This section discusses how the results translate to a more general environment.

Coeurdacier and Rey (2011), and similarly Coeurdacier et al. (2010), study aggregate country-portfolios in the general equilibrium of a two-country economy that is similar to the one analysed here, but takes into account capital investment, non-homothetic preferences and allows for more than two periods. Rather than heterogeneity in wealth levels and idiosyncratic income risk, however, the model has two representative agents, one for each country. Nevertheless, for the special case of very small deviations of individual wealth levels from the non-stochastic steady state mean, we can use their approximate solution to gain insights about individual portfolios in a more generally setting. The authors show how, like in the model of this paper, optimal portfolios comprise subportfolios that hedge against real exchange rate fluctuations and non-diversifiable labour income risk, respectively. Again, agents use bonds, not stocks, to hedge exchange rate risk, with home bond positions that (for risk-aversion $\sigma > 1$) exceed those of foreign bonds and are
increasing in consumption home bias $\theta$. The sub-portfolio that hedges against endowment risk, however, differs from that in the simple model of this paper. Particularly, with non-homothetic preferences, home and foreign stocks cease to be equivalent. Moreover, capital investment rises in periods with high productivity and high wages. The implied fall in dividends makes home stocks a good hedge against labour income risk. This implies a hedge portfolio with long stock positions biased towards home stocks, in contrast to the endowment model of this paper that featured a short position in stocks to hedge endowment risks. Also, hedging using stocks is not perfect as it was in the endowment case. Rather, agents hold an additional long position in home (foreign) bonds to hedge against the remaining positive (negative) comovement of real labour income with the terms of trade that results when the elasticity of substitution between home and foreign goods is relatively low (high). For sufficiently small heterogeneity in financial wealth and small idiosyncratic risk, the covariance structure of the terms of trade, labour income and asset returns should remain approximately unchanged. As long as portfolios are linear in wealth, the results should therefore be a guide also to individual portfolios, implying home bias of the poor: poor investors have relatively high expected labour income, and thus a portfolio dominated by hedging concerns, for which they invest mainly in home stocks and (for the case of a low elasticity of substitution) home bonds. This would explain relative home bias of the poor. In contrast to the simpler model of this paper, it does not imply, however, a bias of poor investors towards safe assets, as labour income risk is hedged through long, rather than short, positions in stocks. I leave a rigorous and quantitative investigation of a more general environment like this to future research.

5 Conclusion

In this paper I have shown that, according to the Survey of Consumer Finances, wealthier US Households invest a higher share of their portfolio in international assets. This result is due both to the fact that poorer households are less likely to participate in foreign asset markets, and to lower portfolio shares of foreign assets for poorer participants. Importantly, the latter observation is not explained by fixed costs of participating in foreign asset markets. So I constructed a simple two country model with incomplete markets and income heterogeneity that can account for this finding. Agents in the model receive stochastic endowments of a country-specific tradable good which are affected by idiosyncratic and country-specific shocks. Agents are prevented from access to a complete
set of asset markets but can trade in riskless assets and in stocks. Assuming log-normal returns, I derived asset portfolios as a function of total investor wealth. Poorer individuals’ consumption is mainly determined by endowment income. Relative to richer individuals, they therefore have a bias against equity, which has real payoffs that co-move strongly with individual endowments. But poorer home agents also have a relative bias in favour of home vs. foreign bonds, since home bonds are a good hedge against aggregate volatility in the supply of home goods, which have a stronger weight in their consumption.

More research into the determinants of individual portfolio decisions should be carried out in order to assess the welfare consequences of poorer households’ non-participation in sophisticated financial markets, which are likely to be larger in a model with fixed costs of participation than in the model of this paper. More generally, it would be interesting to see whether the positive results of this paper also hold in more general environments with capital investment, non-homothetic preferences, or demand shocks. I leave this, however, to future research.
Appendix 1: Measurement error in foreign assets

The measure of total foreign asset holdings used in the empirical part of this study potentially suffers from at least two kinds of measurement error. First, the responses of households to questions on their asset holdings are accurate only insofar individuals both know the accurate dollar value of their assets, and truthfully report it. Since I only look at portfolio investments (in other words I disregard directly owned foreign companies), market values of investments are in principle available, and individuals should report their dollar values at current exchange rates. This may be a strong assumption not only as individuals might not be aware of up-to-date market values for long-term investments or exchange rates, but also, for example, if some of them underreport systematically offshore investments used to evade tax payments. In the latter case, however, the resulting measurement error would tend to dilute the correlation between wealth and the foreign asset share of the portfolio. So a rejection of the Null hypothesis of no relation would be less likely in the presence of this kind of measurement error. To see this, suppose all individuals were to invest x percent of their foreign asset holdings in unreported offshore vehicles. In this case, the difference between true portfolio shares $\tilde{a}_{true}$ and those calculated from reported asset holdings can easily be shown to be $\frac{x}{1-x}(\tilde{a}_{true})(1 - \tilde{a}_{true})$. Thus portfolio shares of foreign assets calculated from individual reports are always smaller than the true shares, and the difference is greatest for intermediate portfolios. As we see foreign asset shares rising from zero to single-digit percentages in Figure 2, the bias will increase along the wealth distribution.

A second source of measurement error results from the use of average portfolio shares in the imputation of households’ indirect foreign asset holdings via mutual funds. If rich individuals systematically invest in funds with different exposures to foreign assets, this might distort the observed wealth effect on total foreign assets. But again, this error is likely to dampen the observed relationship between wealth and the portfolio share of foreign assets as long as richer individuals choose funds with a higher share of foreign assets. Using average mutual fund portfolios then introduces measurement error that is negative for rich individuals, positive for poor individuals. This biases the wealth effect estimated from observed data towards zero. The bias will be even stronger when richer individuals also have a higher portfolio share of mutual funds. So again, we would be less likely to reject the null of no wealth effect on portfolios in the presence of measurement error, than we would be without it.
7 Appendix 2: Fixed costs and home bias

This section shows that higher costs of investing in foreign assets alone cannot explain the relative home bias of poorer market participants found in the data. Consider the 2 period problem of a home investor that receives a stochastic share \( e \) of aggregate home endowment \( Y_H \), and can invest in home bonds at a return \( R_H \), or foreign assets, yielding \( R_F \) units of foreign currency for bonds and \( R_S Y_F \) for shares. Assume \( e, Y_H, Y_F \) are independent log-normal random variables. To abstract from the general equilibrium terms of trade movements at the basis of the results in the main text, assume that the exchange rate \( S \), defined as the price of foreign currency in units of the home currency, is simply also a mean zero independent log-normal variable. In addition, assume that to buy \( a^* \) units of foreign assets, the investor has to pay a cost of \( K = k_0 + k_1 a^* \), i.e. there are fixed and proportional costs of investing abroad. The investor’s problem can be expressed as

\[
\max_{c_b, c_s, \pi} \frac{c_b^{1-\sigma} - 1}{1 - \sigma} + \beta E \lambda \left\{ \frac{c_b^{1-\sigma} - 1}{1 - \sigma} \right\}
\]

Subject to the constraints

\[
c_h = eY_H - a^b_{h,H} - \sum_{j \in \{b, s\}} a^j_{h,F}
\]

\[
c'_h = e'Y'_H + R^b_H a^b_{h,H} + \frac{S'}{S} (R^b_F a^b_{h,F} + R^s_F Y'_F a^s_{h,F}) - K
\]

where \( K = 0 \) if the investor has zero foreign asset holdings.

The problem can be seen as a two-stage decision. First, the investor determines the optimal portfolios with and without foreign assets; then she compares expected utility for both and decides weather or not to hold foreign assets.

For simplicity consider binary portfolios where the investor either invests in shares or bonds. Given log-normality and independence, and approximating marginal utility from investing in foreign assets at zero real returns (\( R^b_F = R^b_H = \frac{S'}{S} = 1 \)), the approximate share of foreign bonds in a diversified portfolio is

\[
\tilde{a}_f = \frac{1}{2} + \frac{r^b_F - k_1 - r^b_H}{\sigma Var_s}
\]

where lower case letters denote logs and \( Var_s \) is the variance of the log exchange rate.

Equivalently, the portfolio share of foreign shares is

\[
\tilde{a}_s = \frac{1}{2} + \frac{r^s_F - k_1 - r^b_H}{\sigma (Var_F + Var_s)}
\]
So optimal portfolios are a function of risk aversion, excess returns and the variances of payoffs. But importantly, they do not include individual wealth. Also, proportional costs simply show up as a proportionate reduction of returns that affects all portfolios equally. So portfolio do not change with wealth among participants. Fixed costs on the other hand mean that only investors with a large enough portfolio diversify into foreign assets, where for investment in foreign bonds say, the threshold value of total assets is defined as that for which losses from fixed costs exactly offset those from sub-optimal portfolios

\[ E[u(e' - aR_H^h)] = E[u(e' - a^*R_H^h - a^*_h(S'^h/R_H^F - k_1) - k_0)] \]  
\(37\)

Note that the cost structure assumed in this appendix comprises the case of costly acquisition of a fixed amount of information on any given asset. As information, once acquired, is non-exclusive in its use, the corresponding cost structure has different values of \(k_0\) for different foreign assets, and \(k_1 = 0\) for all of them. However, the assumption that costs are linearly affine in the size of investment does not capture a scenario where agents can acquire additional information on a given asset at non-zero marginal cost. As the marginal returns to information are increasing in investment size, wealthier individuals might find it optimal in this setting to acquire more information. This can induce variation of portfolios across individuals with different wealth levels. However, this result does not survive when poor agents can pool their foreign investments in a mutual fund, or if they can copy wealthier individuals’ investment strategies.

8 Appendix 3: Existence and uniqueness of equilibrium

In section 3, I showed that the equilibrium relative price of goods is independent of heterogeneity and the allocation of assets. As long as agents have some preference for both goods \((0 < \theta < 1)\), (18) thus describes a non-empty, single-valued mapping from the two-dimensional space of aggregate endowments into a market-clearing price. In other words a market-clearing price of goods always exists and is unique for any combination of \(Y_H, Y_F\).

The excess demands for assets are the sum of the quantities solving (12), integrated across the distribution of unconstrained agents in both countries, plus maximum borrowing multiplied by the measure of constrained agents. For example, for Home IOUs, remembering
that these can only be issued by home agents and that asset quantities are denoted in terms of domestic goods for home and foreign agents, we get

\[ a_H = \int a_{h,H} d\Psi_H^r + p \int a_{f,H} d\Psi_F^r \]

\[ = \int_{e_h^*}^{\infty} B_h^b \Psi_H^r + \int_{e_h^*}^{\infty} a_{h,H} d\Psi_H^r + p \int_0^{\infty} a_{f,H} d\Psi_F^r \]

where \( e^* \) is that value of first period endowments for which \( B_h^b \), the maximum level of home bond issuance, exactly solves the first order condition for home bonds.

Under financial autarky, existence of an equilibrium price vector \( \bar{R} = (\bar{R}_b, \bar{R}_s^H) \) is easy to prove by a fixed point argument. Local uniqueness of both consumption allocation and portfolios can also be shown.

However, global uniqueness is more difficult to prove as individual asset demands are not necessarily monotone in relative returns. Two special cases where the equilibrium can be shown to be globally unique are when agents only trade in either bonds, or shares, but not both, and either \( \sigma \leq 1 \) (substitution effects dominate income effects) or \( B_j^k = \infty \) (unconstrained issuance of assets). This is because with one asset only, total excess demand shows no inter-asset substitution effects. Then, for \( \sigma < 1 \), all individual asset demands, and therefore total excess demand for assets, are monotone in returns as the substitution effect dominates. For \( \sigma > 1 \) savers may have decreasing asset demand (as the income effect dominates). But borrowers’ asset demand is always increasing in returns, with an elasticity higher than that of savers at optimal borrowing levels as long as everybody faces the same period 2 uncertainty. So if all borrowers are unconstrained the total excess demand is again upward sloping in returns, and the equilibrium globally unique. However, even with only one asset, when a lot of borrowers are constrained, there may be multiple equilibria, as the non-monotonous asset demands of savers can dominate total excess demand.

With more than 1 asset, possibly traded across countries, the equilibrium is not generally globally unique. But conditions for global uniqueness can be derived for example by imposing the gross substitution property on the system of individuals’ arbitrage equations. For the analysis here this is not a problem, however, as I only look at interior portfolios, given an equilibrium vector of returns \( \bar{R} \). I do not solve for the equilibrium explicitly, which will be a function of the particular specification of distributions and borrowing constraints in both countries.
References


Broer, T. 2011. *Domestic or global imbalances? Rising inequality and the fall in the US current account*, IIES.


9 Figures and Tables
Figure 1: Portfolio share of domestic and foreign stocks (decile average) across the financial wealth distribution

Figure 2: Portfolio share of total foreign assets (decile average) across the financial wealth distribution
Table 1: Heckman model for participation and portfolio share of stocks

<table>
<thead>
<tr>
<th>Equation (1)</th>
<th>Equation (2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>ln(FIN)</td>
<td>FIN₂</td>
</tr>
<tr>
<td>ln(INCOME)</td>
<td>FIN₃</td>
</tr>
<tr>
<td>const</td>
<td>FIN₄</td>
</tr>
<tr>
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<td>1.39</td>
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<tr>
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<th>AGE³</th>
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<th>HISP</th>
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<td>-0.19</td>
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</table>

| No of obs   | 4519 | Censored: | 2686 |
| χ²(2)       | 183.2|           |      |

The dependent variable is the portfolio share of stocks; FIN is the SCF measure of total gross financial wealth; INCOME the sum of salaries, wages and income or losses from a professional practice, business, limited partnership, or farming; AGE the age of the household head in years; FINₓ a dummy variable that takes the value 1 when financial wealth falls in the (weight-adjusted) xth quartile of the cumulative distribution; and COL (BLACK,HISP) dummy variables that equal 1 if the head of the household has a college degree (or reports her race as black, or hispanic, respectively). Numbers in italics are standard errors.
Table 2: Heckman model for participation and portfolio share of foreign assets

<table>
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<th>AGE$^2$</th>
<th>AGE$^3$</th>
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The dependent variable is the portfolio share of foreign bonds and stocks, domestic or foreign. For other variable definitions, see table 1. Numbers in italics are standard errors.
Table 3: Heckman model for participation and portfolio share of foreign bonds

<table>
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<th>AGE</th>
<th>AGE2</th>
<th>AGE3</th>
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The dependent variable is the portfolio share of foreign bonds. For other variable definitions, see table 1. The selection equation has dummies for only two quartiles of the financial wealth distribution because no households in the first quartile hold foreign bonds. Numbers in italics are standard errors.