

**i Instructions****ECON4310 – Macroeconomic Theory**

This is some important information about the written exam in ECON4310. Please read this carefully before you start answering the exam.

**Date of exam:** Friday, November 23, 2018

**Time for exam:** 09.00 a.m. – 12.00 noon

**The problem set:** The problem set consists of 3 exercises (A-C) with several subquestions. They count as indicated.

**Sketches:** You may use sketches on all questions. You are to use the sketching sheets handed to you. You can use more than one sketching sheet per question. See instructions for filling out sketching sheets on your desk. It is very important that you make sure to allocate time to fill in the headings (the code for each problem, candidate number, course code, date etc.) on the sheets that you will use to add to your answer. You will find the code for each problem under the problem text. You will NOT be given extra time to fill out the "general information" on the sketching sheets (task codes, candidate number etc.).

**Access:** You will not have access to your exam right after submission. The reason is that the sketches with equations and graphs must be scanned in to your exam. You will get access to your exam within 2-3 days.

**Resources allowed:** No written or printed resources - or calculator - is allowed (except if you have been granted use of a dictionary from the Faculty of Social Sciences).

**Grading:** The grades given: A-F, with A as the best and E as the weakest passing grade. F is fail.

**Grades are given:** Friday 14 December 2018.

2

**Short Questions (40 Points)**

Answer each of the following short questions. You will only get points for correct answer with an explanation.

**Exercise A.1: (20 Points) Ricardian Equivalence**

You are advisor to the Swedish king in 1618, right at the onset of the 30-year war in Europe between protestant forces, led by Sweden, and catholic forces, led by the German emperor. The cost, per capita, of the war for the next thirty years, is 10, 000 kroner. The king has come up with three policies to finance the war:

- a Finance the war with immediate taxes of 10, 000 kroner.
- b Issue government debt, and repay that debt, including interest, in the 30 year period after the war (1648-1678).
- c Issue government debt and simply pay the interest on that government debt forever, without ever redeeming the debt itself.

Assume that the interest rate for a 30 year period is  $r = 100\%$  (so that  $1 + r = 2$ ).

Now consider Snorre Viking, a Swedish fisherman that lives from 1618 to 1678, that is, for 2 periods lasting 30 years each. By selling his fish he earns 15, 000 kroner in the first period of his life and 30, 000 kroner in the second period of his life (he gets better catching fish with experience). Snorre has utility function

$$\log(c_1) + \log(c_2)$$

**1 Exercise A1.1**

**A1.1** (6 points)

Assume that Snorre can borrow and lend freely in the financial market. How does Snorre rank policies a., b. and c., that is, which one does he like best and which one is worst for him? Explain.

**Fill in your answer here and/or on sketching paper**

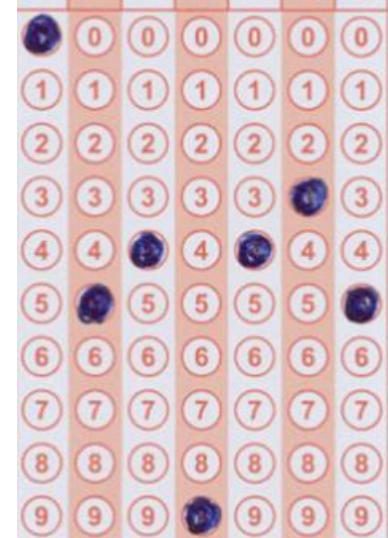
Ubesvart.

**Knytte håndtegninger til denne oppgaven?**  
Bruk følgende kode:

**0 5 4 9 4 3 5**



Oppgavekode Question code	Dato Date	Emnekode Subject code	Kandidatnummer Candidate number	Oppgavenummer Question number	Sidetal Page number
0549435	23/11-18	ECON4310	17423	A1.1	1



A1.1

Tegneområde Drawing area

Snorre's income will be reduced by 10000 kr. in the first period, while his income in the second period isn't changed. His NPV of lifetime

income is then  $15000 - 10000 + \frac{30000}{2} = 20000$ .

b) If the government takes debt at the interest rate 100%, then they have to pay back  $10000 \cdot (1+1) = 20000$  in the second period. The NPV of snorres income will then be  $15000 + \frac{30000 - 20000}{2} = 20000$

c) The government will only pay the interest rate, meaning that they tax Snorre  $10000 \cdot (r) = 10000$  in the second period.  
NPV of total income:  $15000 + \frac{30000 - 10000}{2} = 25000$

Since Snorre has the highest NPV of income in policy c, he will prefer c. He is indifferent between b and a.

↳ c breaks with Ricardian equivalence, meaning that debt today won't mean the same change in NPV of total income as tax today

## 2 Exercise A1.2

A1.2 (7 points)



Now suppose that Snorre cannot borrow any longer. How does Snorre rank policies a., b. and c., that is, which one does he like best and which one is worst for him? Explain.

**Fill in your answer here and/or on sketching paper**

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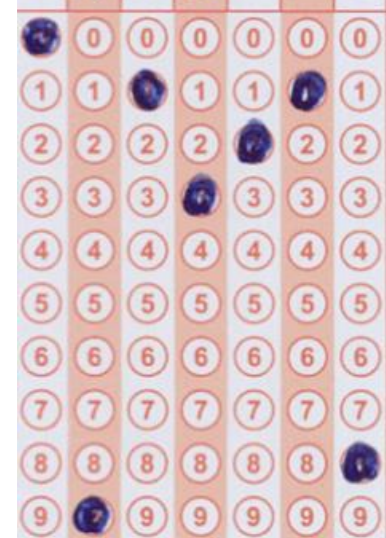
Ubesvart.

**Knytte håndtegninger til denne oppgaven?**  
Bruk følgende kode:

**0 9 1 3 2 1 8**



Oppgavekode Question code	Dato Date	Emnekode Subject code	Kandidatnummer Candidate number	Oppgavenummer Question number	Sidetal Page number
0913218	23/11-18	ECON4310	17423	A1.2	2



A1.2

Tegneområde Drawing area

~~Since~~ Since Snorre can't borrow anymore it's not the NPV of total income that matters anymore.

a) Income period 1:  $15000 - 10000 = 5000$   
income period 2: 30000.

↳ Can't use financial markets to distribute consumption over periods.

$$u = \ln(5000) + \ln(30000)$$

b) I suppose the government can use financial markets

income period 1: 15000

income period 2:  $30000 - 10000(1+r) = 10000$

$$u = \ln(15000) + \ln(10000)$$

c) income period 1 = 15000

income period 2:  $30000 - 10000 \cdot 1 = 20000$

$$u = \ln(15000) + \ln(20000)$$

He ranks them as following, cause of highest utility: policy c > policy b > policy a

### 3 Exercise A1.3

A1.3 (7 points)



Finally, suppose that Snorre eats so much healthy fish that he lives forever and that he can borrow. All other things remain the same. How does Snorre rank policies a., b. and c., that is, which one does he like best and which one is worst for him? Explain.

**Fill in your answer here and/or on sketching paper**

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Ubesvart.

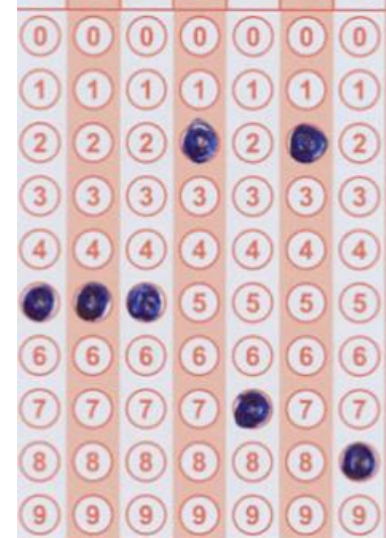
**Knytte håndtegninger til denne oppgaven?**

Bruk følgende kode:

**5 5 5 2 7 2 8**



Oppgavekode Question code	Dato Date	Emnekode Subject code	Kandidatnummer Candidate number	Oppgavenummer Question number	Sidetal Page number
5552728	23/11-18	ECON4310	17423	A1.3	3



Tegneområde Drawing area

A1.3.

Here I suppose Snorre may use financial markets again, meaning

that it's again the NPV of lifetime income that matters.

~~NPV of income  $\sum_{t=0}^{\infty} \frac{5000}{(1+r)^t}$~~

a) and b) will have the same NPV of total income.

c) will have the same NPV assuming

that  $\lim_{T \rightarrow \infty} \frac{D_{T+1}}{\prod_{s=1}^T (1+r_s)} = 0$ . This just means

that the government ~~terminal~~ terminal debt in the infinite horizon case has to be 0.

↳ Transversality condition.

↳ In the infinite living HH case, Ricardian equivalence again holds for all policies, meaning that the HH knows that extra government spending just means future taxes, and will save accordingly, so that consumption in all periods is the same.

#### 4 Exercise A.2

Exercise A.2: (10 Points) Permanent Technology shocks in Real business cycle model and



**Consumption Response**

Consider a simple two-period model of labor supply, as we have seen in lectures, where we assume that utility is separable in consumption and labor supply:

$$\begin{aligned} \max_{\{c_0, c_1, h_0, h_1, a_1\}} \quad & \log c_0 - \phi \frac{h_0^{1+\theta}}{1+\theta} + \beta [\log c_1 - \phi \frac{h_1^{1+\theta}}{1+\theta}] \\ \text{s. t.} \quad & \\ & c_0 + a_1 = w_0 h_0 + (1 + r_0) a_0 \\ & c_1 = w_1 h_1 + (1 + r_1) a_1 \end{aligned}$$

for given  $a_0 = 0$ . Assume  $r_0, r_1$  are exogenously given. We know the household has the following intertemporal labor supply condition:

$$\beta \frac{\phi h_1^\theta}{\phi h_0^\theta} = \frac{w_1}{(1+r_1)w_0},$$

and the solution for  $h_0$  is given by:

$$\phi h_0^{1+\theta} \left[ 1 + \left( \frac{w_1}{(1+r_1)w_0} \right)^{1+\frac{1}{\theta}} \beta^{-\frac{1}{\theta}} \right] = (1 + \beta).$$

Suppose there is a permanent change to wages at the beginning of time 0: both wages in the first and second period increase by 10%. Then this household will take advantage of this opportunity and consume more in  $c_0$  by 10%.

True or false?

Fill in your answer here and/or on sketching paper

Ubesvart.

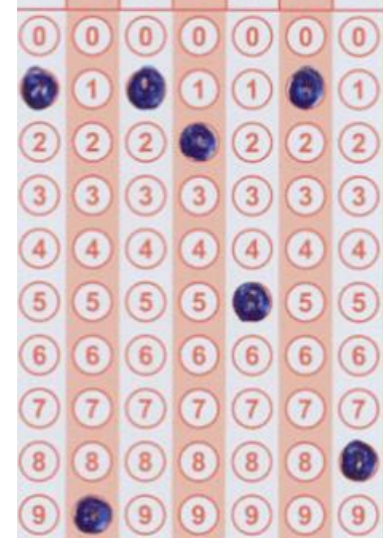
Knytte håndtegninger til denne oppgaven?

Bruk følgende kode:

**1 9 1 2 5 1 8**



Oppgavekode Question code	Dato Date	Emnekode Subject code	Kandidatnummer Candidate number	Oppgavenummer Question number	Sidetal Page number
1912518	23/11-18	ECON4310	17423	A2.	4



Tegneområde Drawing area

A2. TRUE

Here, the relative wage doesn't change  
This means that labour supply

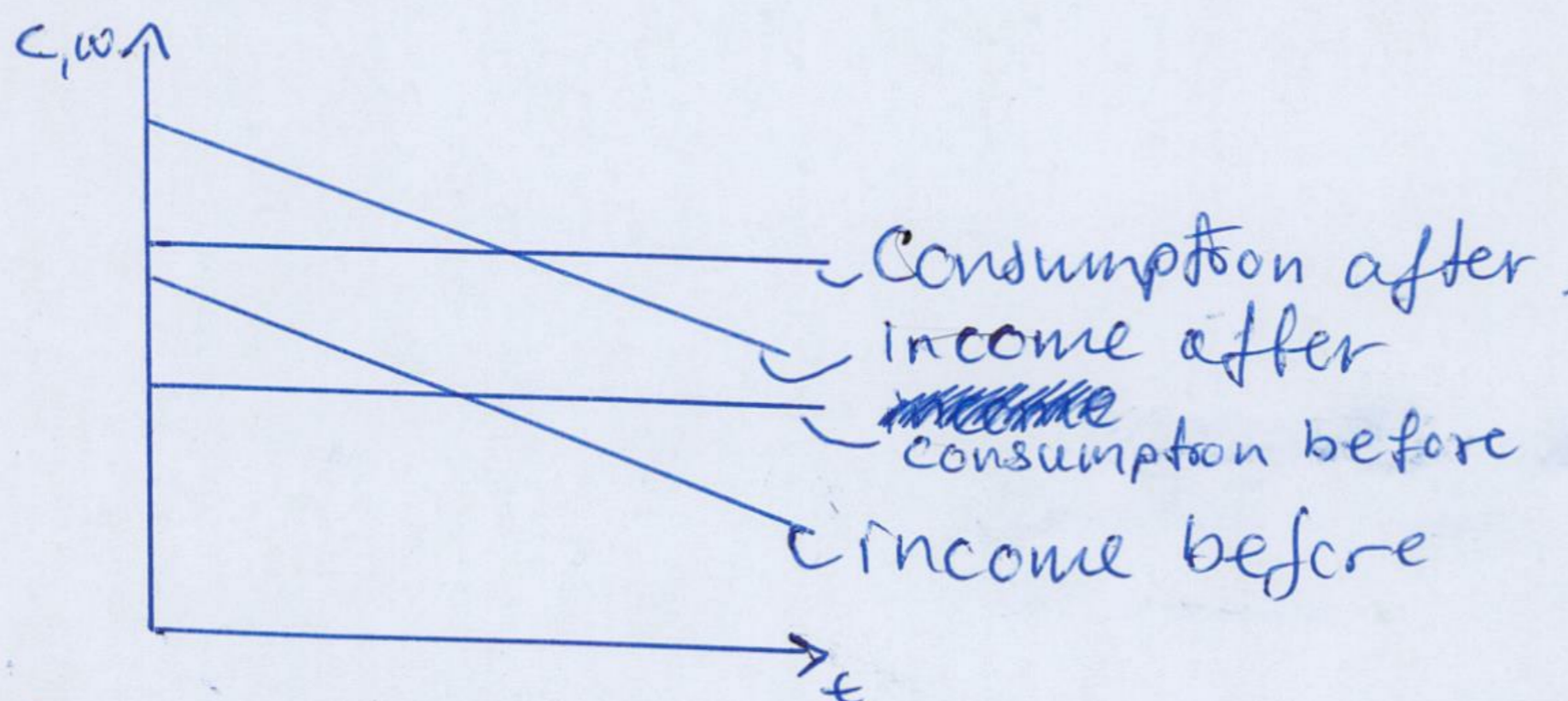
in the two periods doesn't change.

Since wages increases by 10% in both periods, this makes it possible to

increase consumption in both periods

by 10%, meaning that  $c_0$  increases by 10%.

Illustration:



Note that the HIF still might want to distribute consumption over periods by saving, since we don't know about the original wages, but that compared to the original case, since the relative wages doesn't change, there will just be a parallel jump.

### 5 Exercise A.3

Exercise A.3: (10 Points) Consumption and Saving with Worker Heterogeneity



Consider households' optimal intertemporal consumption choice in a two-period model. Suppose there are two types of workers in the economy, type A with constant wages  $w_A$  over time and type B with constant wages  $w_B$ , with  $w_B = (1 + 10\%)w_A$ . Both of them begin with 0 initial assets. Households have preferences  $U = \sum_{t=0}^1 \beta^t u(c_t)$  where  $\beta \in (0, 1)$  is the discount factor and the momentary utility function is

$$u(c_t) = \frac{c_t^{1-\theta} - 1}{1-\theta}, \theta > 1.$$

Also, assume the risk-free interest rate  $r$  is constant. Assume  $\beta(1+r) < 1$ .

Denote the optimal consumption for type-A household as  $(c_0^A, c_1^A)$ , and  $(c_0^B, c_1^B)$  for type-B household. Then we know type-B workers will have relatively lower consumptions in the first period, i.e.,  $c_0^B < (1 + 10\%)c_0^A$ .

True or false?

Fill in your answer here and/or on sketching paper

Ubesvart.

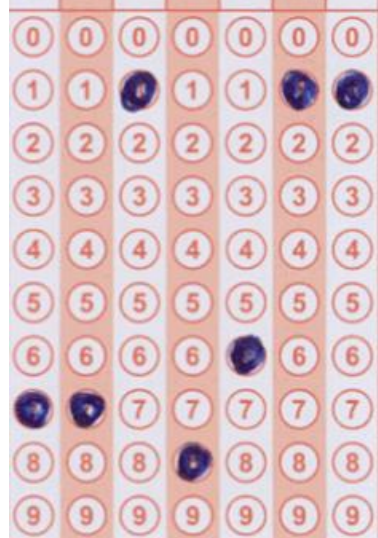
Knytte håndtegninger til denne oppgaven?

Bruk følgende kode:

**7 7 1 8 6 1 1**



Opgavekode Question code	Dato Date	Emnekode Subject code	Kandidatnummer Candidate number	Opgavenummer Question number	Sidetal Page number
7718611	23/11-18	ECON4310	17423	A3.	5



Tegneområde Drawing area

A3: FALSE

Since this is a CRRS, the % they save <sup>borrow</sup> of their income

does not change relative to their income.

They will both have the same consumption euler equation:

$$\beta(1+r)u'(c_1) = u'(c_0)$$

$$u'(c) = c^{-\theta}$$

$$\Rightarrow \beta(1+r)^{\frac{1}{\theta}} c_0 = c_1, \text{ where } c_0 > c_1$$

$\Rightarrow$  They are both borrowing.

$$\text{CRRS: } - \frac{u''(c) \cdot c}{u'(c)} = - \frac{-\theta \cdot c^{-\theta-1} \cdot c}{e^{-\theta}} = - \frac{-\theta c^{-\theta}}{c^{-\theta}}$$

$$= \theta$$

Meaning that  $c_0^B = (1+10\%)c_0^A$



3

**A Four Period Model**

For the entire question, the interest rate is  $r = 0$ . First consider a household that lives for four periods. It has utility function

$$\log(c_1) + \log(c_2) + \log(c_3) + \log(c_4)$$

and income in the four periods of  $y_1 = 10,000$ ,  $y_2 = 10,000$ ,  $y_3 = 50,000$  and  $y_4 = 10,000$

**1 Exercise B.1**

(5 Points)

Compute the optimal consumption choices  $(c_1, c_2, c_3, c_4)$

Fill in your answer here and/or on sketching paper

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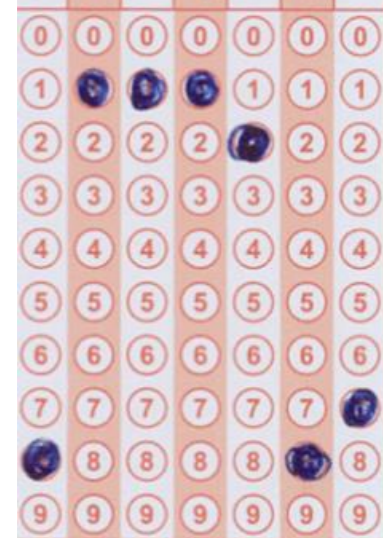
Ubesvart.

**Knytte håndtegninger til denne oppgaven?**  
Bruk følgende kode:

**8 1 1 1 2 8 7**



Oppgavekode Question code	Dato Date	Emnekode Subject code	Kandidatnummer Candidate number	Oppgavenummer Question number	Sidetal Page number
8111287	23/11-18	ECON4310	17423	B.1	6



B.1.

Tegneområde Drawing area

We can see from the utility function that the HH cares just as much about the future as today.

Since the interest rate is 0 and we got ln-utilities, the best way of distributing consumption, is to have equal levels of consumption in every period.

$$\text{Total income} = 10000 \cdot 3 + 50000 = 80000.$$

Distributing this equally over the periods gives 20000 to use on consumption in every period. Given that one unit of consumption costs 1  $y$ :

$$C_1 = 20000$$

$$C_2 = 20000$$

$$C_3 = 20000$$

$$C_4 = 20000.$$

This means that they borrow 10000 in period 1 and 2, then pay back  $20000(1+0)$  in period 3, and then saves 10000 for period 4, which yields with interests;  $(-10000)(1+0) = -10000$

## 2 Exercise B.2

(10 Points)



Suppose the household cannot borrow. Now what are the optimal consumption choices?

Now consider two members of the same dynasty that both live for two periods. Children have utility function

$$\log(c_3) + \log(c_4)$$

and parents have the utility function

$$\log(c_1) + \log(c_2) + V(b)$$

where  $b$  are the bequests left to the children and  $V(b)$  is the maximal utility children can obtain when given bequests  $b$ . Income of parents is  $(y_1, y_2) = (10,000, 10,000)$  and that of children is  $(y_3, y_4) = (50,000, 10,000)$ .

Fill in your answer here and/or on sketching paper

Ubesvart.

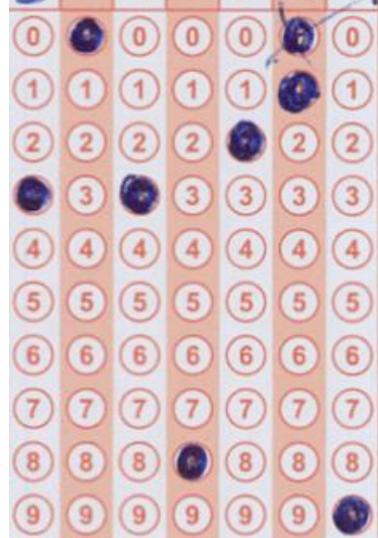
Knytte håndtegninger til denne oppgaven?

Bruk følgende kode:

**3 0 3 8 2 1 9**



Oppgavekode Question code	Dato Date	Emnekode Subject code	Kandidatnummer Candidate number	Oppgavenummer Question number	Sidetail Page number
3038219	23/11-18	ECON4310	17423	B2.	7



B2:

Tegneområde Drawing area

Since the parents can't borrow, <sup>and children</sup> their budgets are constrained to having a positive net saving.

As I stated in B1, they would prefer to have the same consumption in every period, meaning that the parents would like to borrow 10000 in each period and leave negative bequest to their children.

But since they are not allowed to borrow, only save, their budget will be the same as their income.

$$\rightarrow C_1 = 10000$$

$$C_2 = 10000.$$

The children would now like to distribute the remaining 60000 on the 4 periods.

Since they are allowed to ~~borrow~~ save, they will save 20000 in period 3:

$$C_3 + a_4 = W_3 \Leftrightarrow C_3 = 50000 - 20000 = 30000$$

$$C_4 = W_4 + (1+r)a_4 \Rightarrow C_4 = 10000 + (1+0)20000$$

$$C_4 = 30000.$$

### 3 Exercise B.3

(10 Points)



Solve the maximization problem of the children to obtain  $V(b)$ , that is, solve

$$V(b) = \max_{c_3, c_4} \log(c_3) + \log(c_4)$$
$$s. t.$$
$$c_3 + c_4 = 60,000 + b$$

Fill in your answer here and/or on sketching paper

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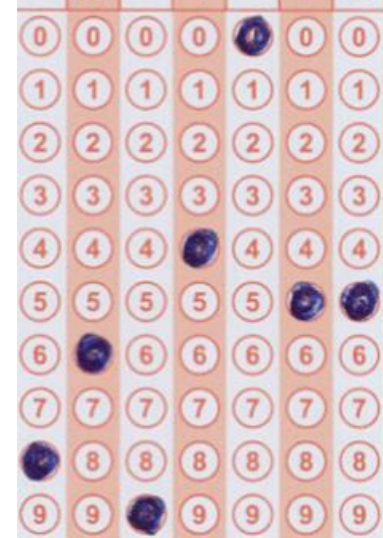
Ubesvart.

**Knytte håndtegninger til denne oppgaven?**  
Bruk følgende kode:

**8 6 9 4 0 5 5**



Oppgavekode Question code	Dato Date	Emnekode Subject code	Kandidatnummer Candidate number	Oppgavenummer Question number	Sidetal Page number
8694055	23/11-18	ECON4310	17423	B3.	8



Tegneområde Drawing area

$$\max_{C_3, C_4} \ln(C_3) + \ln(C_4)$$

$$\text{s.t. } C_3 + C_4 = 60000 + b$$

$$\mathcal{L}: \ln(C_3) + \ln(C_4) + \lambda(60000 + b - C_3 - C_4)$$

$$\text{FOC}_{C_3}: \frac{1}{C_3} = \lambda \dots \dots \dots \textcircled{1}$$

$$\text{FOC}_{C_4}: \frac{1}{C_4} = \lambda \dots \dots \dots \textcircled{2}$$

$$\text{FOC}_{\lambda}: C_3 + C_4 = 60000 + b \dots \dots \dots \textcircled{3}$$

$$\text{From } \textcircled{1} \text{ and } \textcircled{2} \Rightarrow C_3 = C_4 \dots \dots \dots \textcircled{4}$$

$$\textcircled{4} \text{ into } \textcircled{3} \Rightarrow 2C_3 = 60000 + b$$

$$\Leftrightarrow C_3 = 30000 + \frac{b}{2}$$

$$\Rightarrow C_4 = 30000 + \frac{b}{2}$$

Again: consumption in both periods should be the same, and bequest should be equally distributed between them.

4 **Exercise B.4**

(15 Points)

Use your answer from the previous question to solve the parents' maximization problem. *Allow bequests to be negative.*

**Fill in your answer here and/or on sketching paper**

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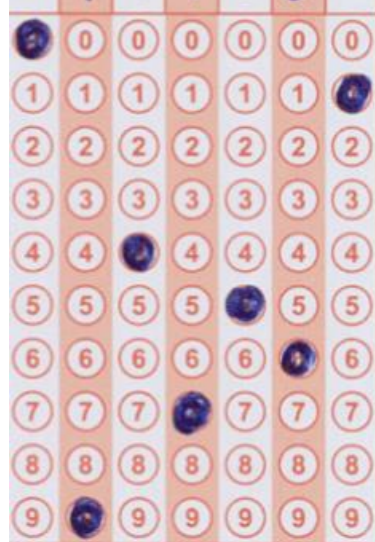
Ubesvart.

**Knytte håndtegninger til denne oppgaven?**  
Bruk følgende kode:

**0 9 4 7 5 6 1**



Oppgavekode Question code	Dato Date	Emnekode Subject code	Kandidatnummer Candidate number	Oppgavenummer Question number	Sidetal Page number
0947564	23/11-18	ECON4310	17423	B4	9



Tegneområde Drawing area

$$\begin{aligned} \text{Max } & \ln(c_1) + \ln(c_2) + V(B) \\ \text{s.t. } & 80000 = c_1 + c_2 + c_3 + c_4 \end{aligned}$$

$$\begin{aligned} \mathcal{L}: & \ln(c_1) + \ln(c_2) + \ln(c_3) + \ln(c_4) \\ & + \lambda (80000 - c_1 - c_2 - c_3 - c_4) \end{aligned}$$

$$\text{FOC } c_1 : \frac{1}{c_1} = \lambda \quad \dots \quad \textcircled{1}$$

$$\text{FOC } c_2 : \frac{1}{c_2} = \lambda \quad \dots \quad \textcircled{2}$$

$$\text{FOC } c_3 : \frac{1}{c_3} = \lambda \quad \dots \quad \textcircled{3}$$

$$\text{FOC } c_4 : \frac{1}{c_4} = \lambda \quad \dots \quad \textcircled{4}$$

$$\text{FOC } \lambda : 80000 = c_1 + c_2 + c_3 + c_4 \quad \dots \quad \textcircled{5}$$

$$\text{From } \textcircled{1}, \textcircled{2}, \textcircled{3}, \textcircled{4} \Rightarrow c_1 = c_2 = c_3 = c_4 \quad \dots \quad \textcircled{6}$$

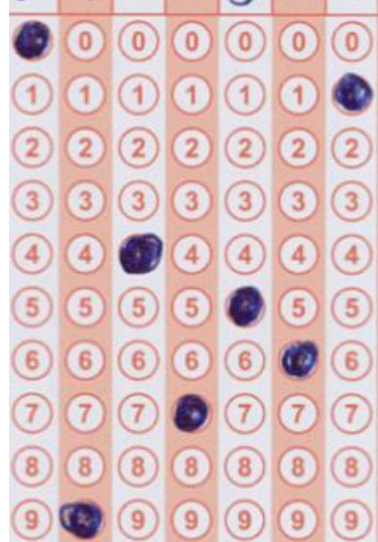
⑥ into ⑤ yields:

$$80000 = 4c_1$$

$$\Leftrightarrow 20000 = c_1 = c_2 = c_3 = c_4.$$



Oppgavekode Question code	Dato Date	Emnekode Subject code	Kandidatnummer Candidate number	Oppgavenummer Question number	Sidetall Page number
0947561	23/11-18	ECON4310	17423	B4	10



Tegneområde Drawing area

Solving for bequest from task  
B3:

$$C_3 = 30000 + \frac{b}{2}$$

$$\Rightarrow 20000 = 30000 + \frac{b}{2}$$

$$\Leftrightarrow -10000 = \frac{b}{2}$$

$$\Leftrightarrow -20000 = b.$$

So, the parents leaves behind 20000  
in debt for their children in optimum.

This is because the parents cares  
just as much about their own well being  
as their childrens. Since they know  
their children will be richer, they  
will therefore take up debt and  
pass this on to their children.

### 5 Exercise B.5

(10 points)



Repeat question B.4, but now assume that bequests cannot be negative (that is, assume a constraint of the form  $b \geq 0$ ). You don't have to do any calculations, but you have to explain your answer.

**Fill in your answer here and/or on sketching paper**

Since the parents can't leave negative bequest, they will now be constrained to having their consumption equal to their wage, since their wage is lower than the next generation. This means that we are back in the case of task B2, where  $c_1$  and  $c_2$  is equal to 10000 and  $c_3$  and  $c_4$  is equal to 30000.

To take this question to a realistic setting: The parents cares equally much about their childrens well being as their own, as long as their children is having less wage than them selves. At once the parents believes that the children will have less income than theirselves, they will leave on positive bequest. But as long as the parents believes that their children is having more wage than themselves, they will not take any of the money from their children.

Another way to look upon this, is that the children should not be "punished" by their parents because they have a better marginal productivity for their labour than their parents. Therefore the government imposes a rule that all generations needs to have positive terminal net savings.

Besvart.

**Knytte håndtegninger til denne oppgaven?**

**3 3 0 6 8 8 4**

Bruk følgende kode:

**6 Exercise B.6**

(10 Points)

Bequests still cannot be negative. Now suppose the government increases taxes in period 2 by 5,000 and gives back 5,000 in subsidies in period 3. What is the optimal consumption allocation now. Again you don't have to do any calculations, but you have to explain your answer.

**Fill in your answer here and/or on sketching paper**

The parents will now have a NPV of life time income equal to  $20000 - 5000 = 15000$ . Since they want to distribute the consumption equally over their lifetime, they will now use 7500 of their total income in period 1 and 7500 in period 2. Since the parents are not allowed to leave negative bequest, they will not be able to borrow money and make the children pay. I here assume that the parents can have negative net saving (borrow) inside their generation, since the task only says that they can't have negative bequest for their children. If I have misunderstood here, the consumption in period 1 will be 10000 and consumption in period 2 will be 5000.

The children on the other hand will now have  $50000 + 5000 + 10000 = 65000$  in total lifetime income. Same in NPV, since there is no impatience and interest rate. This means that they also will distribute this equally over the lifetime, meaning that they will spend 32500 in period 3 and 32500 in period 4.

One logical explanation behind this tax, is that the government cares more about future generations than they do about the current one. Another one is that the government explains their tax with ricardian equivalence, so that the NPV of the infinitely living HH does not change, but forgot to think about the fact that generations dies. They should therefore have allowed for negative bequest equal to the taxation which is later given as a lump sum subsidy.

Besvart.

**Knytte håndtegninger til denne oppgaven?**

**1 0 6 2 7 3 9**

Bruk følgende kode:



4

### A real business cycle model

Consider a representative household of a closed economy. The household has a planning horizon of two periods and is endowed with the following preferences over consumption,  $c$ ,

$$U = u(c_1) + \beta E u(c_2(s_2)),$$

with the following marginal utility

$$u'(c) = c^{-\gamma}, \gamma \geq 1.$$

The variable  $s_2$  denotes the state of the economy in the second period which follows the stochastic process

$$s_2 = \begin{cases} s_G, & \text{with prob. } p \\ s_B, & \text{with prob. } 1 - p, \end{cases}$$

and the household conditions the consumption,  $c_2(s_2)$ , in the second period on the state,  $s_2$ . Assume the household's labor supply is exogenous and always equal to 1.

*Labour market assumptions:*

Assume that in each period and in each state of the economy,  $s_t$ , there is a linear (in labor  $n_t$ ) production technology of the form

$$y_t(s_t) = A_t(s_t)n_t(s_t),$$

and the labor market is assumed to be perfect competitive. Assume the labor productivity in the first period is given by  $A_1 = A$ , and the labor productivity is higher in the good state of the second period,

$$A_2(s_G) = A + A(1-p)\epsilon > A_2(s_B) = A - A p \epsilon, \quad \epsilon > 0, A > 0, 0 < p < 1,$$

than in the bad state of the second period. The wages are denoted as  $w_1$ ,  $w_2(s_G)$  and  $w_2(s_B)$ .

*Asset market assumptions:*

Assume the household does have access to a risk-free asset,  $a_2$ , and the associated interest rate is denoted as  $r_2$ .

### 1 Exercise C.1

(5 Points)

Find the equilibrium wages,  $w_1$ ,  $w_2(s_G)$ , and  $w_2(s_B)$ , and show that the expected wages in the second period is the same as wage in in the first period.

Fill in your answer here and/or on sketching paper

Ubesvart.

Knytte håndtegninger til denne oppgaven?

Bruk følgende kode:

**0 2 1 5 7 5 7**



Oppgavekode Question code	Dato Date	Emnekode Subject code	Kandidatnummer Candidate number	Oppgavenummer Question number	Sidetall Page number
0215757	23/11-18	ECON4310	17423	C.1	17

0	0	0	0	0	0
1	1	1	1	1	1
2	2	2	2	2	2
3	3	3	3	3	3
4	4	4	4	4	4
5	5	5	5	5	5
6	6	6	6	6	6
7	7	7	7	7	7
8	8	8	8	8	8
9	9	9	9	9	9

Tegneområde Drawing area

C.1.

$$w_1 = \frac{\partial y_1}{\partial n_1} = \frac{\partial (A_1 \cdot n_1)}{\partial n_1} = A_1 = A$$

$$w_2(S_G) = \frac{\partial y_2(S_G)}{\partial n_2(S_G)} = \frac{\partial ((A + A(1-p)E)n_2(S_G))}{\partial n_2(S_G)} =$$

$$= A + A(1-p)E = A_2(S_G) > A$$

$$w_2(S_B) = \frac{\partial y_2(S_B)}{\partial n_2(S_B)} = \frac{\partial ((A - ApE)n_2(S_B))}{\partial n_2(S_B)} =$$

$$= A - ApE = A_2(S_B) < A$$

## 2 Exercise C.2

(5 Points)



Write down the state-by-state budget constraints for the household.

**Fill in your answer here and/or on sketching paper**

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Ubesvart.

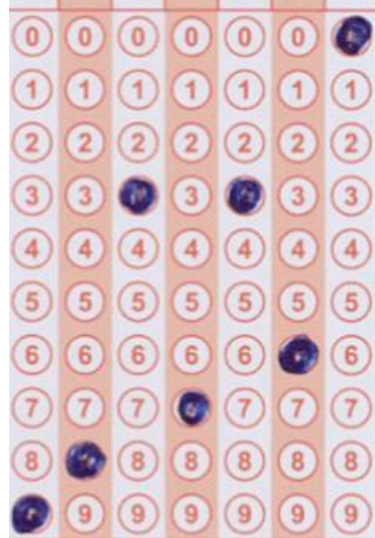
**Knytte håndtegninger til denne oppgaven?**

Bruk følgende kode:

**9 8 3 7 3 6 0**



Oppgavekode Question code	Dato Date	Emnekode Subject code	Kandidatnummer Candidate number	Oppgavenummer Question number	Sidetal Page number
9837360	23/11-18	ECON4310	17423	C.2	12



Tegneområde Drawing area

C.2

$$C_1 + a_2 = W_1$$

$$C_2(S_G) = W_2(S_G) + (1+r_2)a_2$$

$$C_2(S_B) = W_2(S_B) + (1+r_2)a_2$$

3 **Exercise C.3**

(10 Points)



Let  $(\lambda_1, \lambda_2(s_G), \lambda_2(s_B))$  denote the Lagrange multipliers of the state-by-state budget constraints. State the representative agent's Lagrangian. (Note that the expected utility for the second period is the summation of utility across good and bad states, weighted by probability, i.e.,  $\mathbb{E}u(c_2(s_2)) = pu(c_2(s_G)) + (1 - p)u(c_2(s_B)).$ )

Fill in your answer here and/or on sketching paper

---

Ubesvart.

Knytte håndtegninger til denne oppgaven?  
Bruk følgende kode:

**4 4 4 1 6 4 6**

Oppgavekode Question code	Dato Date	Emnekode Subject code	Kandidatnummer Candidate number	Oppgavenummer Question number	Sidetall Page number
4441646	23/11-18	ECON4310	17423	C.3.	13

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1	1	1	1	1	1	1
2	2	2	2	2	2	2
3	3	3	3	3	3	3
4	4	4	4	4	4	4
5	5	5	5	5	5	5
6	6	6	6	6	6	6
7	7	7	7	7	7	7
8	8	8	8	8	8	8
9	9	9	9	9	9	9

Tegneområde Drawing area

C.3

$$\begin{aligned} L: & u(c_1) + \lambda_1(w_1 - a_2 - c_1) \\ & + \beta p u(c_2(s_A)) + \lambda_2(s_A)(w_2(s_A) + (1+r_2)a_2 - c_2(s_A)) \\ & + \beta(1-p)u(c_2(s_B)) + \lambda_2(s_B)(w_2(s_B) + (1+r_2)a_2 - c_2(s_B)) \end{aligned}$$

#### 4 Exercise C.4

(10 Points)



Derive the optimality conditions with respect to consumption,  $(c_1, c_2(s_G), c_2(s_B))$  and savings,  $a_2$  by using multipliers.

Fill in your answer here and/or on sketching paper

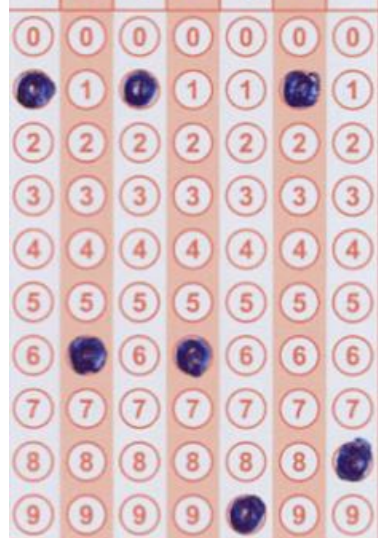
Ubesvart.

**Knytte håndtegninger til denne oppgaven?**

Bruk følgende kode:

**1 6 1 6 9 1 8**

Opggavekode Question code	Dato Date	Emnekode Subject code	Kandidatnummer Candidate number	Opgavenummer Question number	Sidetall Page number
1616918	28/11-18	ECON4310	17423	C4	14



C4)  
 max  $\lambda$  wrt  $c_1, c_2(SG), c_2(SB)$   
 and  $a_2$ .

Tegneområde Drawing area

FOC:  
 $c_1$  :  $u'(c_1) = \lambda_1 \dots \dots \dots \textcircled{1}$

FOC  
 $c_2(SG)$  :  $\beta \rho u'(c_2(SG)) = \lambda_2(SG) \dots \dots \dots \textcircled{2}$

FOC  
 $c_2(SB)$  :  $\beta(1-\rho)u'(c_2(SB)) = \lambda_2(SB) \dots \dots \dots \textcircled{3}$

FOC  
 $a_2$  :  $\lambda_2(SG)(1+r_2) + \lambda_2(SB)(1+r_2) = \lambda_1 \dots \dots \dots \textcircled{4}$

5 **Exercise C.5**

(10 Points)



Derive the stochastic consumption Euler equation ( it only involves with  $c_1$ ,  $c_2(s_2)$ ,  $\beta$  and  $r_2$  and No multipliers).

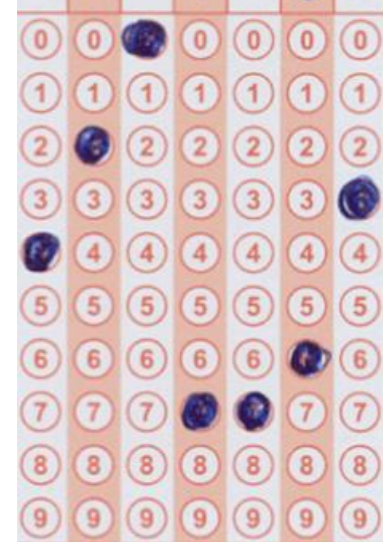
Fill in your answer here and/or on sketching paper

Ubesvart.

**Knytte håndtegninger til denne oppgaven?**  
Bruk følgende kode:

**4 2 0 7 7 6 3**

Oppgavekode Question code	Dato Date	Emnekode Subject code	Kandidatnummer Candidate number	Oppgavenummer Question number	Sidetal Page number
4207763	23/11-18	ECON4310	17423	C5	15



C5:

Tegneområde Drawing area

Substituting  $\lambda_1$  from (1),  $\lambda_2(s_G)$  from (2) and  $\lambda_2(s_B)$  from (3) into (4):

$$\beta(\cancel{p})u'(c_2(s_G))(1+r_2) + \beta(1-p)u'(c_2(s_B))(1+r_2) = u'(c_1)$$

$$\Leftrightarrow (1+r_2)\beta(pu'(c_2(s_G)) + (1-p)u'(c_2(s_B))) = u'(c_1)$$

Substituting  $pu'(c_2(s_G)) + (1-p)u'(c_2(s_B))$  with  $E(u'(c_2(s_2)))$ :

$$(1+r_2)\beta(E(u'(c_2(s_2)))) = u'(c_1)$$

$$\Leftrightarrow \frac{u'(c_1)}{E[u'(c_2(s_2))]} = (1+r_2)\beta$$

## 6 Exercise C.6

(10 Points)



For (f) and (g), assume that the asset  $a_2$  is available in zero supply. What is the household's optimal choice of  $a_2$  in the equilibrium? What are the household's optimal choices of consumption? Can the household fully smooth consumption? i.e., are  $c_1$ ,  $c_2(s_G)$  and  $c_2(s_B)$  equal?

Fill in your answer here and/or on sketching paper

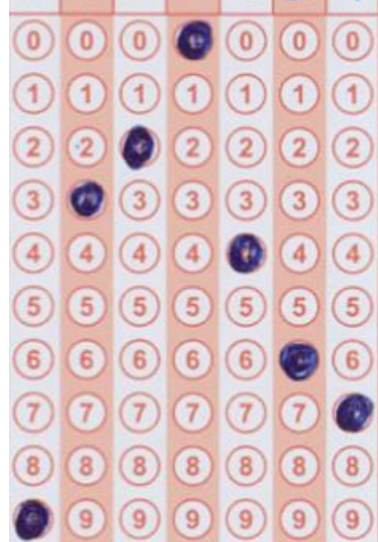
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Ubesvart.

Knytte håndtegninger til denne oppgaven?  
Bruk følgende kode:

**9 3 2 0 4 6 7**

Oppgavekode Question code	Dato Date	Emnekode Subject code	Kandidatnummer Candidate number	Oppgavenummer Question number	Sidetal Page number
9320467	23/11-18	ECON4310	17423	C6.	16



Tegneområde Drawing area

C6.  
If there is 0 supply of  $a_2$ , then the optimal choice of  $a_2$  for the

HH is  $a_2 = 0$ .

This means that the HH can't smooth the consumption by saving, meaning that the optimal choice for consumption is to use all income in a period/state on consumption. From the period by period <sup>State</sup> budget constraints we then get: <sup>State</sup>

$$C_1 = W_1$$

$$C_2(S_G) = W_2(S_G)$$

$$C_2(S_B) = W_2(S_B)$$

$$C_1 = A$$

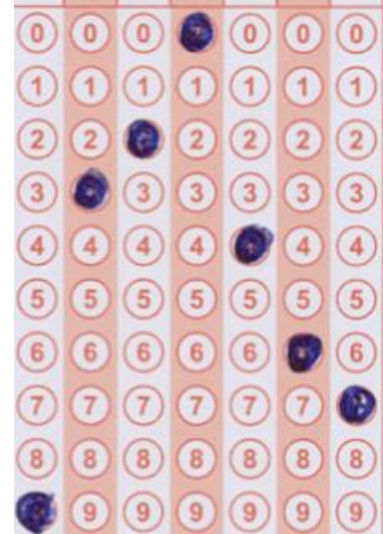
$$C_2(S_G) = A + A(1-p)E > A$$

$$C_2(S_B) = A - Ape < A$$

$$C_2(S_G) > C_1 > C_2(S_B)$$

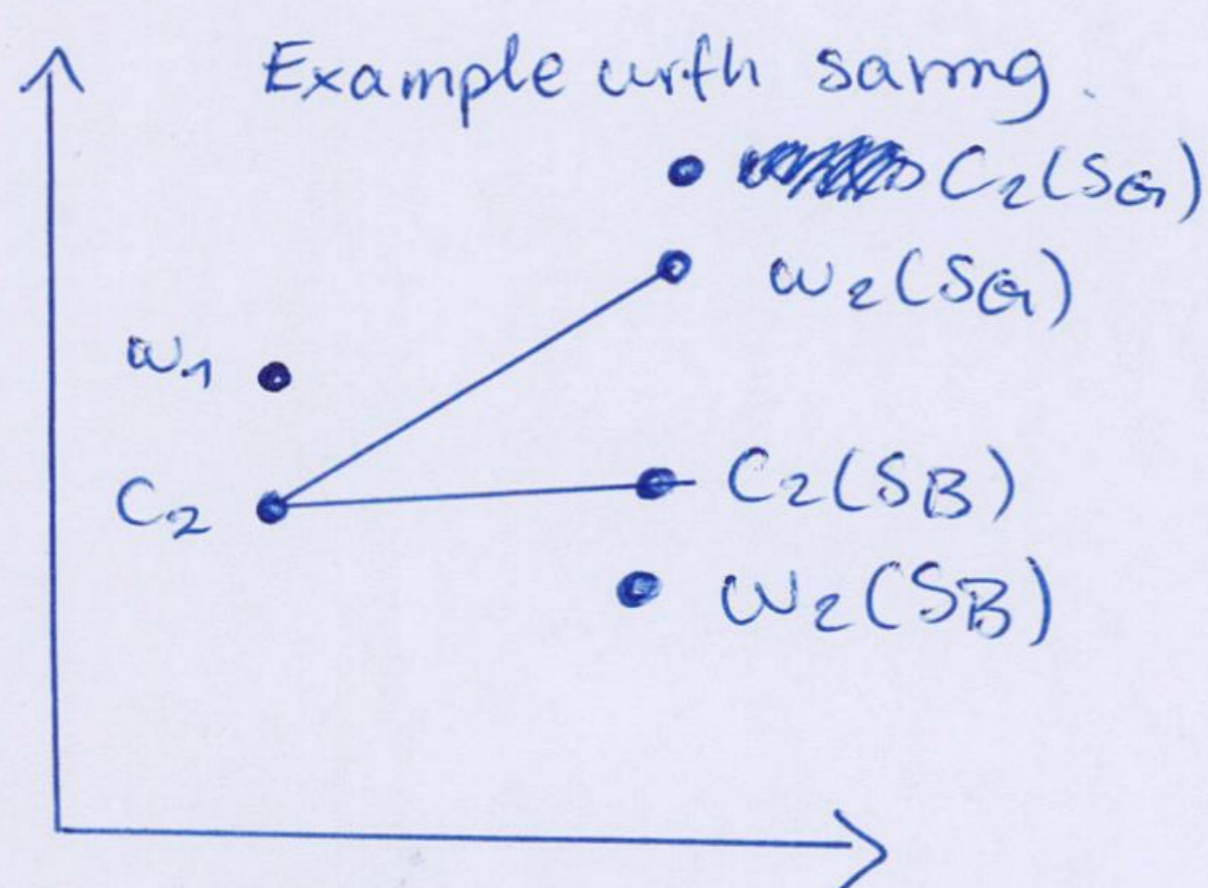
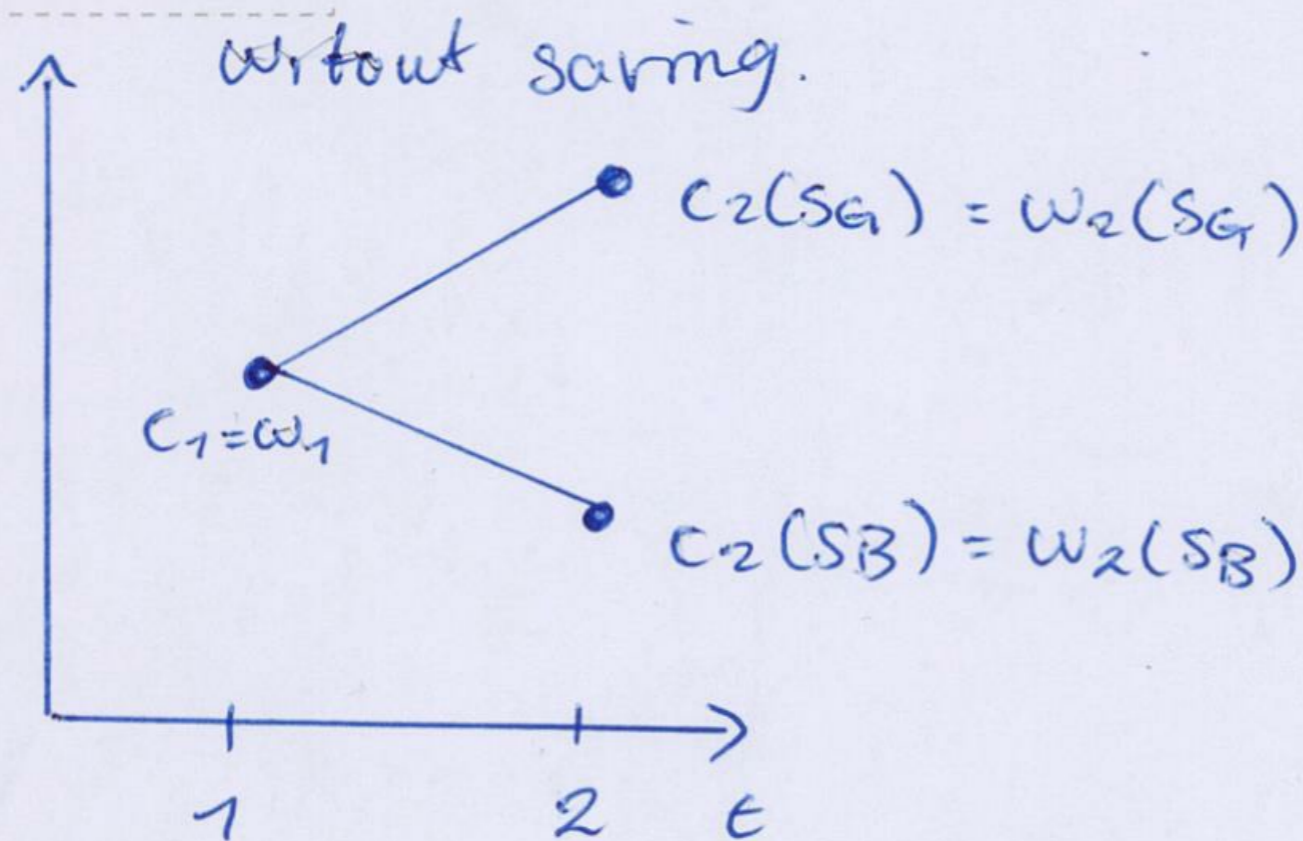


Oppgavekode Question code	Dato Date	Emnekode Subject code	Kandidatnummer Candidate number	Oppgavenummer Question number	Sidetal Page number
9320467	23/11/18	ECON4310	17423	C6	17



Tegneområde Drawing area

Illustration:



(This is not answering the question, just further explanation of the idea of the model.)

The reason why the HHH wants to save, is because they are risk averse. This means the bad state in period 2 hurts more than the good state benefits.  
 ↳ By saving they can smooth the expected utility.

7 Exercise C.7

(10 Points)



Is the equilibrium interest rate  $r_2$  higher or lower than  $r_{RN} \equiv \frac{1}{\beta} - 1$ ? Why? (Hint: do it step by step: (1) use the budget constraint to link consumption and wages; (2) use the Euler equation and the result,  $u'(w_1) \leq \mathbf{E}[u'(w(s_2))]$ , which comes from the Jensen's inequality.)

Fill in your answer here and/or on sketching paper

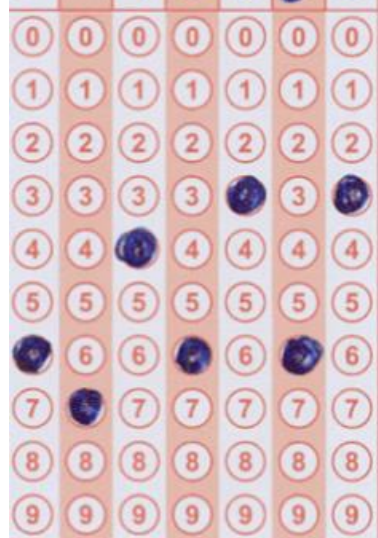
Ubesvart.

Knytte håndtegninger til denne oppgaven?  
Bruk følgende kode:

**6 7 4 6 3 6 3**



Oppgavekode Question code	Dato Date	Emnekode Subject code	Kandidatnummer Candidate number	Oppgavenummer Question number	Sidetal Page number
6746363	23/11-18	ECON4310	77423	C7.	18



Tegneområde Drawing area

C7.

Without saving, the consumption is equal to the wages, and

we can therefore substitute the wages into the utility function:

$$u(w_1) + \beta E u(w_2(s_2))$$

The Euler equation I solved for in task

$$CS: \frac{u'(c_1)}{E[u'(c_2(s_2))] = (1+r_2)\beta.$$

Substituting  $c_1$  with  $w$ :

$$\frac{u'(w_1)}{E[u'(w_2(s_2))] = (1+r_2)\beta$$

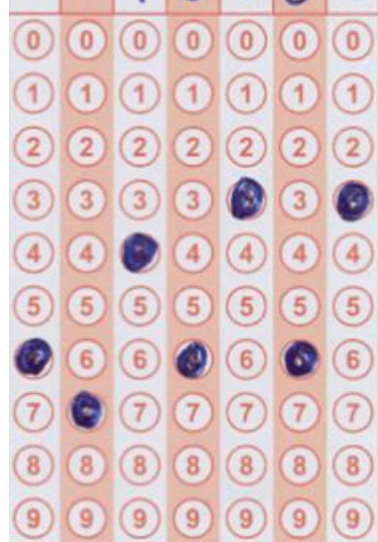
if (from Jensen's inequality)  $u'(w_1) \leq E[u'(w_2(s_2))]$ , then the Euler equation only holds with equality if both sides are  $\leq 1$ .

Plotting in the risk neutral interest rate yields:

$$(1 + \frac{1}{\beta} - 1)\beta \Leftrightarrow \frac{1}{\beta} \cdot \beta = 1.$$



Oppgavekode Question code	Dato Date	Emnekode Subject code	Kandidatnummer Candidate number	Oppgavenummer Question number	Sidetal Page number
6746363	23/11	ECON4310	17423	C7.	19



Tegneområde Drawing area

For  $(1+r_2)B \leq 1$ , then  $r_2$   
has to be lower ~~than~~  $(\frac{1}{B} - 1)$   
or equal to

Example :  $r_2 = \frac{1}{2B} - 1$

$$(1 + \frac{1}{2B} - 1)B = \frac{B}{2B} = \frac{1}{2} < 1.$$

The logic behind this is that if the HH is risk neutral, the Good state benefits them just as much as the bad state hurts them. Therefore the HH ~~in~~ which is risk averse would like an ~~insurance~~ insurance. But this is costly, making the risk premium, which here is the interest rate, less than the risk neutral one.