

Agreements in Dynamic Games

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Outline - Day 4

- a. Games with stocks: Dynamic common-pool problems
- b. Markov-perfect equilibria as "business as usual":
- c. Short-term agreements (that are 'legally binding') and hold-up problems
- d. Long-term contracts
- e. Optimal duration

(Lecture notes permit technological spillovers (IPR/tariffs), more general functional forms, heterogeneity, and renegotiation)

Motivation

- Most environmental problems are dynamic in nature
 - Pollution accumulates over *time*
- Technological solutions are also relevant
 - Takes *time* to develop
- Countries/district often act independently
 - Must study the *game* between them
- Agreements may be 'legally binding'
- But agreements might be made on *some* aspects (like quotas)
 - ...but not everything of interest (like investments)
- Countries that have invested (e.g., Denmark) may then be required to contribute more.
- How should such an *incomplete contract* look like?
- What will it be in *equilibrium*?

a. Games with Stocks - Stochastic Games

- From Mailath and Samuelsson (2006: 174-5):
 - games [where the stage game changes from period to period] are referred to as *dynamic games* or, when stressing that the stage game may be a random function of the game's history, *stochastic games*.
 - The analysis of a dynamic game typically revolves around a set of game **states** that describe how the stage game varies
 - Each state determines a stage game
 - the appropriate formulation of the set of states is not always obvious
- In resource/environmental economics, the typical state is the **stock(s)** of resource or pollution.
- Note that the stock may or may not be "payoff relevant"

a. ... Stocks..

- We can reformulate last week's model to allow for stocks
- Consider a pollution stock $G_t = q_G G_{t-1} + \sum_j g_{j,t}$ with marginal cost C , and a technology stock $R_{i,t} = q_R R_{i,t-1} + r_{i,t}$, where the investment $r_{i,t}$ has the marginal cost K :

$$\tilde{u}_{i,t} = B(g_{i,t}, R_{i,t}) - CG_t - Kr_{i,t},$$

- If we define $c \equiv C / (1 - \delta q_G)$ and $k \equiv K (1 - \delta q_R)$, maximizing $\tilde{u}_{i,t}$ is equivalent to maximizing $u_{i,t}$, defined as: .

$$u_{i,t} = B(g_{i,t}, r_{i,t}) - c \sum g_{j,t} - kr_{i,t}.$$

- In this way, the game with stocks can be reformulated to a repeated game.
- This transformation is not possible if the stocks are "payoff relevant."

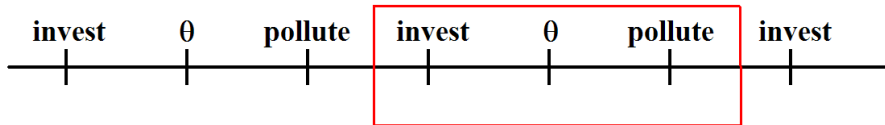
a. Games with Stocks - Markov Perfect Equilibria

- Mailath and Samuelsson (2006: Ch 5):
 - A strategy profile is a *Markov strategy* if they are functions of the state and time, but not of other aspects of the history
 - The strategy profile is a Markov perfect equilibrium if it is both Markov and a subgame-perfect equilibrium
 - A strategy profile is a *stationary Markov strategy* if they are functions of the state, but not of time or other aspects of the history
 - The strategy profile is a stationary Markov perfect equilibrium if it is both stationary Markov and a subgame-perfect equilibrium
- Maskin and Tirole (2001, JET):
 - Markov strategies depend (only) on the coarsest partition of histories that are payoff relevant
 - Two histories h and h' are payoff-irrelevant if, when other players' strategies are the same after h and h' , then i is not better off with strategies that are contingent on h vs h' .
 - So, states/stocks that are not payoff-relevant should not matter.

a. Games with Stocks - MPE - Justifications

- There are too many SPEs
 - hard to make predictions
 - many SPEs are not renegotiation proof
- MPE is "simplest form of behavior that is consistent with rationality" (Maskin and Tirole, 2001)
- Experimentally support in complex games (Battaglini et al, 2014)
- Robust to, for example, finite time.
- We will search for a 'symmetric' MPE.

a. Model: Timing



a. A Model

- A model with $n + 1$ stocks:

$$\begin{aligned}U_{i,t} &\equiv \sum_{\tau=t}^{\infty} u_{i,\tau} \delta^{\tau-t} \\u_{i,t} &\equiv B(g_{i,t} + R_{i,t}) - C(G_t) - kr_{i,t} \\R_{i,t} &= q_R R_{i,t-1} + r_{i,t}, \\G_t &= q_G G_{t-1} + \sum g_{i,t} + \theta_t, \quad i \in \{1, 2, \dots, n\} \\\theta_t &\sim F(0, \sigma^2)\end{aligned}$$

- Continuation values,

$$U_{i,t}(G_{t-1}, R_{1,t-1}, \dots, R_{n,t-1}), W_{i,t}(q_G G_{t-1} + \theta_t, R_{1,t}, \dots, R_{n,t})$$

a. Simplifications

- Simplifying the model. If:

$$y_{i,t} \equiv g_{i,t} + R_{i,t} \text{ and } R_t \equiv \sum R_{i,t}, \text{ then:}$$

$$u_{i,t} = B(y_{i,t}) - C(G_t) - kr_{i,t}$$

$$G_t = q_G G_{t-1} + \sum y_{i,t} - R_t + \theta_t$$

$$R_t = q_R R_{t-1} + \sum r_{i,t}$$

$$\theta_t \sim F(0, \sigma^2)$$

- Write continuation values as $U(G_{t-1}, R_{t-1})$ and $W(q_G G_{t-1} + \theta_t, R_t)$.

a. Simplifications

- **Lemma 0:** *Markov strategies depend only on G_{t-1} and $R_{t-1} \equiv \sum_i R_{i,t-1}$*
- So, same $y_{i,t}$ for all, even if $R_{i,t}$ differ.
- $R_{i,t}$ and R_t is a "public good" – even with no technological spillovers.

b. Business as Usual - Lemma 1

- **Lemma 1:** $U_R^b = q_R k / n$.
- *Proof:* At the investment-stage, i solves

$$\begin{aligned} \max_{r_{i,t}} EW(q_G G_{t-1} + \theta_t, q_R R_{t-1} + \sum r_{i,t}) - k r_{i,t} &\Rightarrow \\ EW_R(G_{t-1}, R_t) = k &\Rightarrow R_t(G_{t-1}), \text{ so} \\ U^b(G_{t-1}, R(G_{t-1})) = W(G_{t-1}, R(G_{t-1})) \\ &\quad - \frac{k}{n} [R_t(G_{t-1}) - q_R R_{t-1}] \Rightarrow \\ U_R^b &= q_R k / n. \end{aligned}$$

- *Note:* Since U_R is a constant, $U_{GR} = 0$, and U_G does not depend on R .||

b. Business as Usual - Lemma 2

- **Lemma 2:** $U_G^b = -q_G (1 - \delta q_R) k / n$
- *Proof:* At the emission stage,

$$B'(y_{i,t}) - C'(q_G G_{t-1} + \theta_t + \sum y_{i,t} - R_t) + \delta U_G(G, R) = 0 \quad (1)$$

- So $y_{i,t} = y_t$ is a function of $\tilde{\zeta}_t + \theta_t$ where $\tilde{\zeta}_t \equiv q_G G_{t-1} - R_t$, and so is G_t . Inserted, the foc for R_t comes from:

$$\max_{r_{i,t}} E[B(y(\tilde{\zeta})) - C(G(\tilde{\zeta})) + \delta U(G(\tilde{\zeta}), R_t)] - k r_{i,t} \quad (2)$$

which gives the foc, determining $\tilde{\zeta}_t = \tilde{\zeta}^b$ as a constant:

$$-E[B'(y(\tilde{\zeta})) y'(\tilde{\zeta}) - C'(G(\tilde{\zeta})) G'(\tilde{\zeta}) + \delta U_G G'(\tilde{\zeta})] + \delta U_R = k. \quad (3)$$

b. Business as Usual - L2 - Proof Continued

- In the symmetric equilibrium:

$$\begin{aligned} U(G, R) = & EB(y(\xi)) - EC(G(\xi)) \\ & - \frac{k}{n} [q_G G_{t-1} - \xi - q_R R_{t-1}] \\ & + \delta U(G_t(\xi), q_G G_{t-1} - \xi) \end{aligned}$$

- Taking the derivative wrt G_{t-1} gives the lemma.||
- Simplify further:
 - Skip t -subscript: $G \equiv G_t$, $G_{t-1} \equiv G_-$, $G_{t+1} \equiv G_+$, etc.
 - **Example Q** (which we will stick to today):

$$B(y_{i,t}) = -\frac{b}{2} (\bar{y} - y_{i,t})^2, \quad C(G_t) = \frac{c}{2} G_t^2 \quad (\text{Q})$$

b. Business as Usual - Emissions

Proposition

- *The FOC at the g -stage can be written as:*

$$g_i^b(\mathbf{R}) = \bar{y} - R_i - \frac{c(n\bar{y} + q_G G_- + \theta - R) + \delta q_G (1 - \delta q_R) k / n}{b + cn}$$

- *If i has a good technology, i pollutes less. Thus, other countries pollute more.*
- *The FOC at the r -stage can be written as:*

$$\begin{aligned} r_i^b = & \bar{y} - \frac{q_R}{n} R_- + \frac{q_G}{n} G_- \\ & - k \left[\frac{(b + cn)^2}{cb(b + c)n} \left(1 - \frac{\delta q_R}{n} \right) - (1 - \delta q_R) \frac{\delta q_G}{cn^2} \right] \end{aligned}$$

- *If i pollutes a lot, every country is subsequently investing more.*

b. Business as Usual - Emissions

- *Anticipating this:*
 - Investments decrease
 - Emissions increase
- A dynamic common pool problem that is **worse** than its static counterpart

b. First Best

Proposition

- U_G and U_R are as in Lemmata 1 and 2 (try to prove yourself).
- The FB FOC at the g -stage can be written as:

$$g_i^*(\mathbf{R}) = \bar{y} - R_i - n \frac{c(n\bar{y} + q_G G_- + \theta - R) + \delta q_G (1 - \delta q_R) k / n}{b + cn^2}$$

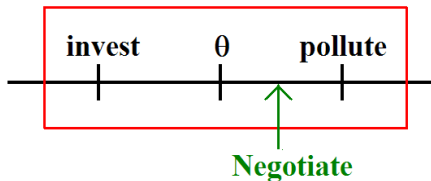
- The FB FOC at the r -stage can be written as:

$$\begin{aligned} r_i^* = \bar{y} - \frac{q_R}{n} R_- + \frac{q_G}{n} G_- \\ - k(1 - \delta q_R) \left(\frac{1 - \delta q_G}{cn^2} + \frac{1}{b} \right). \end{aligned}$$

- By comparison,

$$r_i^b < r_i^* \text{ and } g_i^b(\mathbf{R}^b) > g_i^*(\mathbf{R}^b) > g_i^*(\mathbf{R}^*).$$

c. Short-Term Agreements



c. Short-Term Agreements: Initial Observations

- Negotiating g_i s is equivalent to negotiating y_i s when investments are sunk.
- At the negotiation stage, the countries are identical w.r.t. y_i , regardless of differences in R_i s, just as before.
- The Nash Bargaining Solution (NBS) thus leads to the same $y_i = g_i + R_i$ for everyone.
 - So, the more a country has invested, the smaller is the negotiated quota.
- The FOC for y_i coincides with the FB FOC, given R .
- But what are the equilibrium (noncooperatively set) investment levels?

c. Short-Term Agreements

Proposition

- *There is a unique symmetric MPE: U_G and U_R are as above.*
- *The g_i^{st} 's are FB, given $\mathbf{R}$, but investments are smaller than FB.*
- *In fact, it is easy to see that:*

$$r_i^{st} = r_i^b - \frac{(n-1)^2}{n(b+c)} \left(1 - \frac{\delta q_R}{n} k \right) < r_i^b.$$

- *With severe underinvestment (e.g., n large), countries can thus be **worse off with ST** than with BAU. $U^b > U^{st}$ iff:*

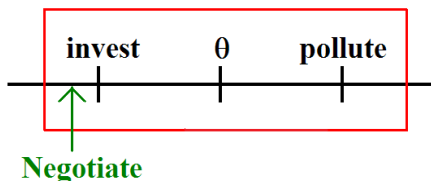
$$k^2 \left(1 - \frac{1}{n} \right)^2 > (1 - \delta q_R)^2 + \frac{(b+c)(bc\sigma)^2}{(b+cn^2)(b+cn)^2}.$$

- *Short-term agreements are always worse when the duration is sufficiently short (i.e., when $\delta q_R \rightarrow 1$ and $\sigma \rightarrow 0$).*

c. Short-Term Agreements vs. BAU - Lessons

- Short term agreements can be harmful!
- *Intuition:*
 - Countries invest less in fear of being "hold up" in future negotiations
 - Countries invest less when the problem is expected to be solved in any case (i.e.: $G^{st} < G^b \Rightarrow C'(G^{st}) < C'(G^b)$)
 - If investments are important, this makes the countries worse off ex ante (before the investment stage)
- Agreements can be harmful ex ante - because they reduce incentives to invest.
 - But when investments are sunk, the agreement is always better than BAU.

d. Long-Term Agreements ($T=1$)



- The timing is reversed: g_i is negotiated first, then i invests.
- With this timing, there is no hold-up problem in this period.
- But **still underinvestments** - especially when the *next* bargaining round is near.

d. Long-Term Agreements - Investments

Proposition

- *When the level g_i^{lt} is already committed to, the first-order condition for i 's investment is:*

$$k = b \left(\bar{y} - g_i^{lt} - r_i - q_R R_{i,-1} \right) + \delta U_R.$$

- *Investments decrease in the emission quota.*
- *In the FB FOC, the term δU_R is multiplied with n . Thus:*

$$r_i(g_{i,t}) = r_i^*(g_i) - (n-1)\delta U_R$$

- *Investments are suboptimally small, especially if n and δ are larger.*

d. Long-Term Agreements - Quotas

Proposition

- As before, U_R and U_G are given by Lemma 1 and 2.
- A larger g_i does not affect y_i , but only r_i and G . Thus, FOC:

$$k - nEC' \left(G^{lt} \right) + n\delta U_R - n\delta U_G = 0.$$

- The FB FOC is the same! Thus: $EC' \left(G^{lt} \right) = EC' \left(G^* \right)$.
- But since $r_i^{lt} (g_i) < r_i^* (g_i)$, the *emission levels are suboptimally small "ex post"*, given the equilibrium technology level:

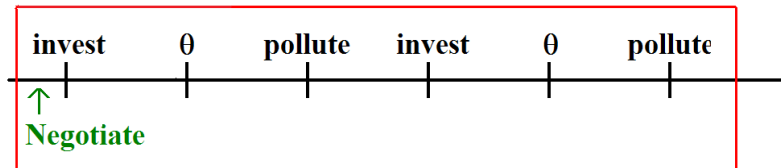
$$g_i^{lt} < Eg_i^* \left(\mathbf{R}^{lt} \right).$$

d. Long-Term Agreements - Lessons

Proposition

- *The quotas should&will be smaller than what is "ex post optimal" (particularly if δU_R is large, i.e., short duration).*
- *Intuition:*
 - Countries anticipate that everyone will under-invest because of the hold-up problem.
 - But everyone invests more if quotas are small.
 - Thus, they agree on small quotas since this mitigates the underinvestment problem.
 - The shorter is the duration, the smaller are investments, and the smaller are the optimal quotas relative to the ex post optimal emission levels.

e. Long-Term Agreements ($T > 1$)



e. Long-Term Agreements: The Optimal Time Horizon

Proposition

- ① For every $t < T$, investments are FB (conditional on $g_{i,t}^{lt}$):

$$k = b \left(\bar{y} - g_{i,t}^{lt} - R_{i,t} \right) + \delta q_R k.$$

- ② There are underinvestments only at the end of the commitment period (when $t = T$, the FOC is as when $T = 1$).
- ③ To remove/postpone the hold-up problem, a larger T is better.
- ④ But if future $g_{i,t}^{lt}$'s cannot be conditioned on the θ_τ 's, $\tau \leq t$, it is costly to commit to quotas for far away t 's.
- ⑤ The optimal T^* solves this trade-off.
- If $\sigma \downarrow 0$, then $T^* \uparrow \infty$.
 - With technological spillovers, $T \uparrow$.