

Dynamic Games in Environmental Economics: LN1: "Business as Usual" and Agreements

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Outline

I. Dynamic Games with Emissions and Investments

"Business as usual" vs. "First Best"

II. Incomplete Contracts"

The Best Legally Binding Agreements

III. Coalitions and Participation

IV. Compliance

V. Discounting and Time Inconsistency

VI. Political Economics (and Supply-Side Regulation?)

1. Motivation - Kyoto 1997



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- The Kyoto Protocol (first commitment period):
 - 37 countries negotiated quotas
 - 5% average emission reduction (from 1990-levels)
 - 5y: 2008-2012, then 8y: 2013-2020.
- No third-party enforcement:
 - Too much emission=>make up in next period+1/3 penalty
 - Possible to exit (Canada did in 2011)
- Investments in new technology
 - Importance of technology transfer/develop recognized..
 - "*technology needs must be nationally determined, based on national circumstances and priorities*" (§114 in the Cancun Agreement, confirmed in Durban)

1. Motivation - Paris 2015



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- Similar focus on emission cuts, and not investments
- Countries had to suggest, before the Paris meeting, their "intended nationally determined contributions" (INDCs)
- Focus on transparency: The agreement calls for the U.N. Framework Convention on Climate Change to publish all national action plans on its Web site and for scientists to calculate the contributions these plans make to curbing emissions.
- A Climate Accord Based on "Global Peer Pressure" (NYT)
- Climate is "the ultimate public good."

1. Motivation - Quite Generally

- Many environmental problems are dynamic in nature
 - Pollution accumulates over *time*
- Technological solutions are also relevant
 - Takes *time* to develop
- Individuals and firms respond to policies, but countries' policies are set more or less independently
 - Must study the *game* between them
- Agreements might be made on *some* aspects (like quotas)
 - ...but not everything of interest (like investments)
- Countries that have invested (e.g., Denmark) may then be required to contribute more.
- How should such *incomplete contract* look like?
- What will it be in *equilibrium* and how can they be improved?

2.1. Why Models?

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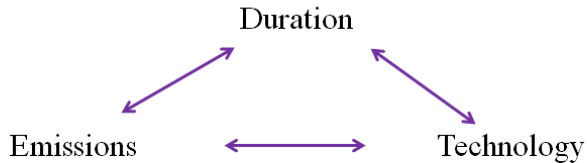
2.1. Why Models?

Emissions

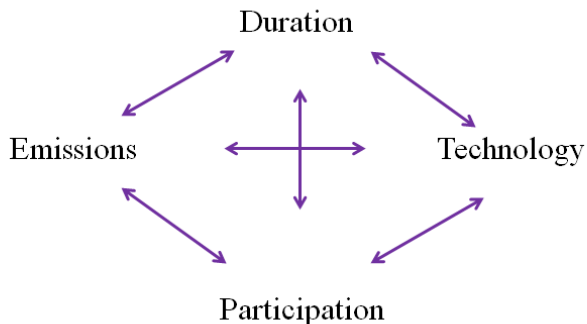


Technology

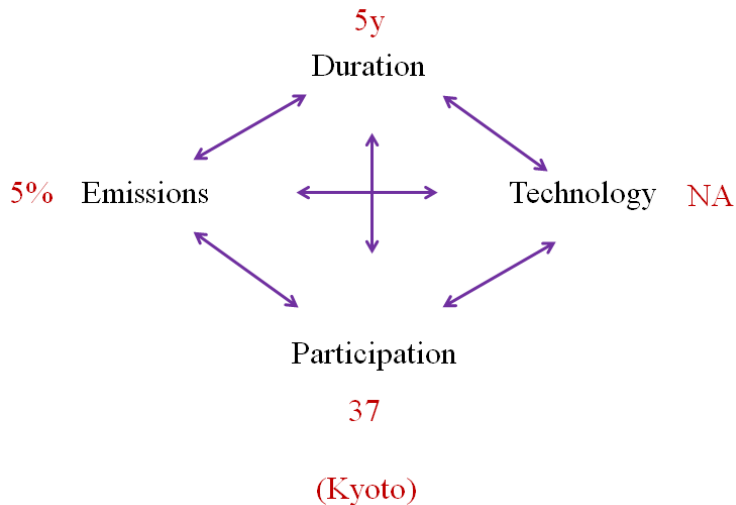
2.1. Why Models?



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2.1. Why Models?



2.2. Model - Assumptions

- All lectures:
 - Each country maximizes a well-specified objective function
 - Time of investments and emissions alternate
 - Emissions accumulate over time and across countries
 - Symmetry: All countries are quite similar
- Today's lectures:
 - Participation is taken as given
 - Compliance is taken as given
 - All actions are observable
 - Strategies are simple (Markovian)
 - Exponential discounting
 - Each country acts as one player/individual (we abstract from "domestic" politics)

2. A Model

- A model with $n + 1$ stocks:

$$V_i \equiv \sum_t \delta^t u_{i,t}, \quad i \in N \equiv \{1, \dots, n\}$$

$$u_{i,t} \equiv B_i(g_{i,t}, R_{i,t}) - C(G_t) - \kappa(r_{i,t}) + \psi \sum_{j \in N \setminus i} r_{j,t}$$

$$R_{i,t} = q_R R_{i,t-1} + r_{i,t},$$

$$G_t = q_G G_{t-1} + \sum_{i \in N} g_{i,t} + \theta_t,$$

$$\theta_t \sim F(0, \sigma^2)$$

- Continuation values,

$$V_i(G_{t-1}, \mathbf{R}_{1,t-1}) = u_{i,t}(\cdot) + \delta V_i(G_t, \mathbf{R}_{1,t})$$

$$W_i(q_G G_{t-1} + \theta_t, \mathbf{R}_{1,t}) = B_i(g_{i,t}, R_{i,t}) - C(G_t) + \delta V_i(G_t, \mathbf{R}_{1,t})$$

2. A Difficult Problem...

- If $n = 1$, the Bellman equation is:

$$V_i(G_{t-1}, \mathbf{R}_{1,t-1}) = \max_{g_{i,t}, r_{i,t}} u_{i,t}(\cdot) + \delta V_i(G_t, \mathbf{R}_{1,t})$$

- Optimal control problem, with two "controls."
- If $n > 1$, we have a "difference game" because time is discrete.
 - If time was continuous, it would be called a "differential game."
- Even with only one control (disregarding tech.), we cannot find analytical solutions in general.
- Even with quadratic utility functions, the problem is complex.

2. Two simplifications

- Reducing the number of stocks to two. With **perfect substitutes**:

$B_i(g_{i,t} + R_{i,t})$ and $y_{i,t} = g_{i,t} + R_{i,t}$, we can write:

$$u_{i,t} \equiv B_i(y_{i,t}) - C(G_t) - \kappa(r_{i,t}) + \psi \sum_{j \neq i} r_{j,t}$$

$$G_t = q_G G_{t-1} + \sum_i y_{i,t} - \sum_i R_{i,t} + \theta_t.$$

- Assume countries are more or less symmetric.
- Or: With heterogeneity only in bliss points, $\bar{y}_{i,t}$, then $B(\tilde{y}_{i,t} - \bar{y}_{i,t}) = B(y_{i,t})$ if we define $y_{i,t}$ as consumption relative to i 's bliss point.

2. Reformulated model

- Final simplification: $\kappa(r_{i,t}) = kr_{i,t}$:

$$u_{i,t} \equiv B(y_{i,t}) - C(G_t) - kr_{i,t} + \psi \sum_{j \in N \setminus i} r_{j,t}, \quad i \in N$$

$$y_{i,t} = g_{i,t} + R_{i,t}, \quad i \in N$$

$$R_{i,t} = q_R R_{i,t-1} + r_{i,t}, \quad i \in N$$

$$G_t = q_G G_{t-1} + \sum_{i \in N} y_{i,t} - R_t + \theta_t,$$

$$R_t = \sum_{i \in N} R_{i,t}$$

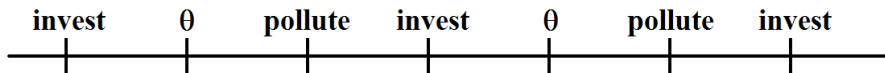
$$\theta_t \sim F(0, \sigma^2)$$

$$K \equiv k - (n-1)\psi$$

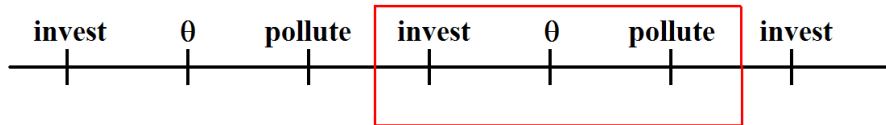
- Example Q:

$$B(\cdot) = -\frac{b}{2} (\bar{y} - y_{i,t})^2, \quad C(\cdot) = \frac{c}{2} G_t^2 \quad (\text{Q})$$

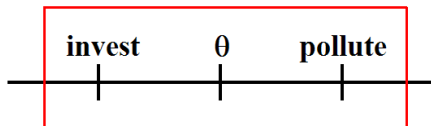
2.4. Model: Timing



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2.5. What is an Equilibrium?

- It is natural to think that policymakers do their best in response to the world.
- **Strategy:** For each country, a strategy is a rule that specifies $r_{i,t}$, and $g_{i,t}$, as a function of the history.
- **Nash equilibrium:** A collection of strategies so that each strategy is a best response to the other strategies.
 - There are many Nash equilibria in dynamic games.
- **Subgame-perfect equilibrium (SPE):** The strategies must remain Nash equilibria in every subgame.
 - There are many SPEs in dynamic games (folk theorems): See LN3.
 - This makes it difficult to make predictions.
 - The multiplicity makes it also difficult to test the theory (because: "anything goes").
- *What are natural refinements?*
 - The best SPE? (LN3!)
 - Robust SPEs?

2.5. Markov-Perfect Equilibrium (MPE)

- Mailath and Samuelsson (2006: Ch 5):
 - A strategy profile is a **Markov strategy** if they are functions of the state and time, but not of other aspects of the history
 - The strategy profile is a **Markov perfect equilibrium** if it is both Markov and a subgame-perfect equilibrium
 - A strategy profile is a **stationary Markov strategy** if they are functions of the state, but not of time or other aspects of the history
 - The strategy profile is a **stationary Markov (perfect) equilibrium** if it is both stationary Markov and a subgame-perfect equilibrium
- Maskin and Tirole (2001, JET):
 - Markov strategies depend (only) on the coarsest partition of histories that are payoff relevant
 - Two histories H and H' are payoff-irrelevant if, when other players' strategy satisfy $s_{-i}(H) = s_{-i}(H')$, then i cannot do strictly better than strategies $s_i(H) = s_i(H')$ that are not contingent on H vs H' .
 - So, states/stocks that are not payoff-relevant should not matter.

2.5. Markov-Perfect Equilibrium (MPE): Why?

- There are too many SPEs
 - hard to test and make predictions
 - many SPEs are not "renegotiation proof"
- MPE is "simplest form of behavior that is consistent with rationality" (Maskin and Tirole, 2001)
- Experimentally support in complex games (Battaglini et al 2014, Vespa 2020)
- Robust to, for example, finite time
- Meaningful to study (incomplete) contracts

3. Business as Usual

3. Business as Usual: Preliminaries

- **Theorem (1):** Continuation values can be written as $V(G_{t-1}, R_{t-1})$ and $W(q_G G_{t-1} + \theta_t, R_t)$.
- Because: *Markov strategies depend only on G_{t-1} and $R_{t-1} \equiv \sum_i R_{i,t-1}$.*
- So, same $y_{i,t}$ even if $R_{i,t}$ differ!
- $R_{i,t}$ and R_t are "public goods" even without tech. spillovers.

3. Business as Usual: Preliminaries

- **Theorem (2):** $V_R^{bau} = q_R K / n$.
- *Proof:* At the investment-stage, i solves

$$\begin{aligned} \max_{r_{i,t}} EW(q_G G_{t-1} + \theta_t, q_R R_{t-1} + \sum_{j \in N} r_{j,t}) - k r_{i,t} &\Rightarrow \\ EW_R(q_G G_{t-1} + \theta_t, R_t) = k &\Rightarrow R_t(G_{t-1}), \text{ so} \\ V^{bau}(G_{t-1}, R_{t-1}) = W(q_G G_{t-1} + \theta_t, R(G_{t-1})) \\ &\quad - \frac{K}{n} [R_t(G_{t-1}) - q_R R_{t-1}] \Rightarrow \\ &\quad V_R^{bau} = q_R K / n. \end{aligned}$$

- *Note:* Since V_R is a constant, $V_{GR} = 0$, and V_G does not depend on R .||

3. Business as Usual: Preliminaries

- **Theorem (2):** $V_G^{bau} = -q_G (1 - \delta q_R) K / n$
- *Proof:* At the emission stage,

$$B'(y_{i,t}) - C'(q_G G_{t-1} + \theta_t + \sum y_{i,t} - R_t) + \delta V_G(G, R) = 0 \quad (1)$$

- So $y_{i,t} = y_t$ is a function of $\xi_t + \theta_t$ where $\xi_t \equiv q_G G_{t-1} - R_t$, and so is G_t . Inserted, the foc for R_t comes from:

$$\max_{r_{i,t}} E \left[B(y^{bau}(\xi)) - C(G^{bau}(\xi)) + \delta V(G^{bau}(\xi), R) \right] - k r_{i,t} \quad (2)$$

which gives the foc, determining $\xi_t = \xi^{bau}$ as a constant:

$$-E \left[B'(y(\xi)) y'(\xi) - C'(G(\xi)) G'(\xi) + \delta V_G G'(\xi) \right] + \delta V_R = k. \quad (3)$$

3. Business as Usual: Preliminaries (proof continues)

- Given $\xi \equiv q_G G_{t-1} - R_t$, in the "symmetric" MPE:

$$\begin{aligned} V(G, R) = & EB(y(\xi)) - EC(G(\xi)) \\ & - \frac{k}{n} [q_G G_{t-1} - \xi - q_R R_{t-1}] \\ & + \frac{\psi(n-1)}{n} [q_G G_{t-1} - \xi - q_R R_{t-1}] \\ & + \delta V(G_t(\xi), q_G G_{t-1} - \xi) \end{aligned}$$

- Taking the derivative gives $V_G^{bau}.$

3. Business as Usual - Emission

Proposition

- The equilibrium consumption y_i^{bau} is independent of R_i , given R .
- From (1):

$$B'(y_{i,t}) = C'(q_G G_{t-1} + \theta_t + \sum y_{i,t} - R_t) + q_G (1 - \delta q_R) K / n$$

- Country i pollutes less but $j \neq i$ pollutes more if R_i is larger, fixing $R_j \forall j \neq i$. By differentiation:

$$\partial g_i^{bau} / \partial R_i = -\frac{C''(n-1) - B''}{nC'' - B''} < 0,$$

$$\partial g_j^{bau} / \partial R_i = \frac{C''}{nC'' - B''} > 0 \quad \forall j \neq i,$$

$$\frac{dG_t^{bau}}{dR_t} = \frac{B''}{nC'' - B''} < 0.$$

3. Business as Usual - Investments

Proposition

- From the derivation of V_R^{bau} , $R_t = q_G G_{t-1} - \zeta^{bau} - q_R R_{t-1}$, so:

$$\partial r_i^{bau} / \partial R_- = -q_R / n,$$

$$\partial r_i^{bau} / \partial G_- = q_G / n.$$

- If one country pollutes more, every country invests more in the next period
- If one country invests more, every other country invests less in the next period.
- There is a unique symmetric MPE.

3. Business as Usual - Example Q

Proposition

- *Under (Q),*

$$g_i^{bau}(\mathbf{R}) = \bar{y} - R_i - \frac{c(n\bar{y} + q_G G_- + \theta - R) + \delta q_G (1 - \delta q_R) k / n}{b + cn}.$$

- *Investments:*

$$\begin{aligned} r_i^{bau} = & \bar{y} - \frac{q_R}{n} R_- + \frac{q_G}{n} G_- \\ & - k \left[\frac{(b + cn)^2}{cb(b + c)n} \left(1 - \frac{\delta q_R}{n} \right) - (1 - \delta q_R) \frac{\delta q_G}{cn^2} \right]. \end{aligned}$$

3. Business as Usual - Incentives

- *Anticipating the dynamic responses:*
 - Investments decrease
 - Emissions increase
- A dynamic common pool problem that is **worse** than its static counterpart

3.2. Business as Usual vs. the First Best

- **Theorem (2):** $V_R^* = q_R K / n$.
- *Proof:* At the investment-stage, i should solve

$$\begin{aligned} V^*(G_{t-1}, R_{t-1}) &= \max_{R_t} E_{\theta} W(q_G G_{t-1} + \theta_t, R_t) - \frac{K}{n} (R_t - q_R R_{t-1}) \\ &\Rightarrow \\ V_R^* &= q_R K / n. \end{aligned}$$

- *Note:* Since V_R is a constant, $V_{GR} = 0$, and V_G does not depend on R .||

3.2. Business as Usual vs. the First Best

- **Theorem (2):** $V_G^* = -q_G (1 - \delta q_R) K / n$
- *Proof:* At the emission stage,

$$B'(y_{i,t}) - nC'(q_G G_{t-1} + \theta_t + \sum y_{i,t} - R_t) + n\delta V_G(G, R) = 0 \quad (4)$$

- So $y_{i,t} = y_t$ is a function of $\xi_t + \theta_t$ where $\xi_t \equiv q_G G_{t-1} - R_t$, and so is G_t . Inserted, the foc for R_t comes from:

$$\begin{aligned} \max_{R_t} E_\theta [& B(y^*(\xi_t + \theta_t)) - C(G^*(\xi_t + \theta_t)) + \delta V(G^*(\xi_t + \theta_t), R) \\ & - \frac{K}{n} (R_t - q_R R_{t-1}) \end{aligned}$$

which gives the foc for $r_{i,t}$, determining $\xi_t = \xi^*$ as a constant.
 $K/n =$

$$E_\theta B'(y^*(\xi + \theta_t)) (-y^{*'}) - (C'(G^*(\xi + \theta_t)) - \delta V_G) (-G^{*'}) + \delta V_R \quad (5)$$

3.2. Business as Usual vs. the First Best - proof continued

With symmetric investments:

$$\begin{aligned} V(G, R) = & E[B(y^*(\xi^* + \theta_t)) - C(G^*(\xi^* + \theta_t))] \\ & - \frac{K}{n} [q_G G_{t-1} - \xi^* - q_R R_{t-1}] \\ & + \delta V(G^*(\xi^*), q_G G_{t-1} - \xi^*) \end{aligned}$$

- Taking the derivative gives the lemma. ||

3.2. Business as Usual vs. the First Best

- The foc for $y_{i,t}$ is now: $B'(\cdot) = n(C' - \delta V_G)$. Differentiation gives:

$$y'^* = \frac{dy_{i,t}^*}{d\theta_t} \equiv -\frac{dy_{i,t}^*}{dR_t} = \frac{-C''}{nC'' - B''/n}, \text{ and} \quad (6)$$

$$G_t^{*'} = \frac{dG_t^*}{d\theta_t} = -\frac{dG_t^*}{dR_t} = 1 + ny'^* = \frac{-B''/n}{nC'' - B''/n}.$$

- The foc for $r_{i,t}$ (5) becomes

$$\begin{aligned} & E_{\theta} B'(y^*(\xi + \theta_t)) (-y'^*) + \delta V_R \\ & + (C'(G^*(\xi + \theta_t)) - \delta V_G) (1 + ny'^*) = \frac{K}{n}. \end{aligned}$$

- Combining the two:

$$E_{\theta} C' = K/n - \delta V_R + \delta V_G = \frac{K}{n} (1 - \delta q_G) (1 - \delta q_R). \quad (7)$$

- ...which pins down $g_{i,t}$, while $r_{i,t}$ becomes:

$$EB'(\cdot) = K(1 - \delta q_G). \quad (8)$$

3.2. Business as Usual vs. the First Best

- By comparing the foc's for $y_{i,t}$, we have

$$y_t^{bau} (q_G G_{t-1} - R_t + \theta_t) > y_t^* (q_G G_{t-1} - R_t + \theta_t). \quad (9)$$

- If either $\sigma^2 = 0$ or Q , then, with the equilibrium R_t , we can also see:

$$\begin{aligned} E y_t^{bau} (q_G G_{t-1} - R_t^{bau} + \theta_t) &< E y_t^* (q_G G_{t-1} - R_t^* + \theta_t) \Rightarrow \\ R_t^{bau} &< R_t^* \text{ and} \\ EG^{bau} &> EG^*. \end{aligned}$$

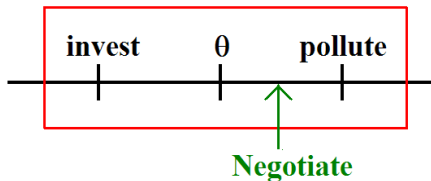
3.2. Business as Usual vs. the First Best

Proposition

- 1 *In each case, V_G and V_R are the same. Thus, when analyzing this period, it is irrelevant whether in the next period there is BAU or FB.*
- 2 *There is a unique symmetric MPE.*
- 3 *If one invests more, everyone consumes more and the others pollute more and invest less next period.*
- 4 *If one pollutes less, everyone **invests less** and **pollutes more** next period.*
- 5 *Countries pollute too much and invest too little in BAU compared to FB.*
- 6 *The dynamic common pool problem (with strategic substitutes) is **worse** than its static counterpart.*



4. Short-Term Agreements



4. Short-Term Agreements: Initial Observations

- Negotiating g_i s is equivalent to negotiating y_i s when investments are sunk.
- At the negotiation stage, the countries are identical w.r.t. y_i , regardless of differences in R_i s, just as before.
- The Nash Bargaining Solution (NBS) thus leads to the same $y_i = g_i + R_i$ for everyone.
 - So, the more a country has invested, the smaller is the negotiated quota.
- The foc for y_i coincides with the FB foc, given R .
- But what are the equilibrium (noncooperatively set) **investment** levels?

4. Short-Term Agreements: Solving the Model

- Lemma 1 and 2 hold with similar proofs as before:

$$V_G^{st} = -q_G (1 - \delta q_R) K / n \text{ and } V_R^{st} = q_R K / n.$$

- The quotas are first best, and thus given by (4). Differentiating (4) gives, as we know, (6). The foc wrt r is as in (3), where $y'(\xi)$ and $G'(\xi)$ are given by (6). This gives:

$$EC'(G) = k + \delta V_G - \delta V_R = (1 - \delta q_G) (1 - \delta q_R) K / n + (K + \psi n)$$

$$EB'(y_i) = n(n - \delta q_R) K + n^2 (n - 1) \psi$$

- The foc for $r_{i,t}$ becomes

$$-E [B'(\cdot) y' - C'(\cdot) (ny' + 1) + \delta V_G (ny' + 1)] + \delta V_R = k.$$

4. Short-Term Agreements vs. BAU

Proposition

- *There is a unique symmetric MPE.*
- *The slopes of the continuation value, V_G and V_R , are as in BAU and FB.*
- *Emission levels are ex post optimal (given R_t), but investments are smaller than at the FB, and thus consumption is also smaller.*
- *Under Q, we can also see that compared to BAU, ST leads to less emissions but also less investments:*

$$\begin{aligned}Eg^{st}(r^{st}) &= Eg^{bau}(r^{bau}) - \frac{n-1}{n(b+c)} \left(\psi(n-1) + \left(1 - \frac{\delta q_R}{n}\right) K \right) \\ r_i^{st} &= r_i^{bau} - \frac{(n-1)^2}{n(b+c)} \left(\psi(n-1) + \left(1 - \frac{\delta q_R}{n}\right) K \right).\end{aligned}$$

4. Short-Term Agreements vs. BAU

Proposition

- *If investments are important, countries can thus be worse off with ST than in BAU. Under Q:*

$$V^{st} < V^{bau} \Leftrightarrow \quad (10)$$
$$\left(\psi + \frac{K}{n}\right)^2 (n-1)^2 > (1 - \delta q_R)^2 + \frac{(b+c)(bc\sigma)^2}{(b+cn^2)(b+cn)^2}.$$

- *This condition is more likely to hold for large ψ and n , and small σ .*

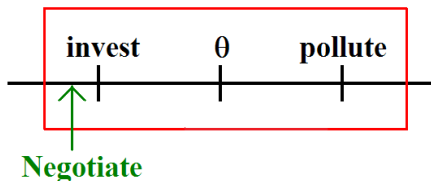
4. Short-Term Agreements vs. BAU - Lessons

- All agreements are not beneficial.
- *Intuition:*
 - Countries invest less in fear of being "held up" in future negotiations
 - Countries invest less when the problem is expected to be solved in any case (i.e.: $G^{st} < G^{bau} \Rightarrow C'(G^{st}) < C'(G^{bau})$).
 - If investments are important, this makes the countries worse off ex ante (before the investment stage)
- Agreements can be harmful ex ante - because they reduce incentives to invest.
 - If ψ is large, investments are already suboptimal so short-term agreements are then likely to be harmful.
 - If σ is large, then y respond too little under BAU, and ST is more reassuring.
- When the investments are sunk, then an agreement is always better than no agreement.



5. Long-Term Agreements

5. Long-Term Agreements



- The timing is reversed: $g_{i,t}$ is negotiated first, then i invests.
- With this timing, there is no hold-up problem in this period
- But **still underinvestments** - especially when the *next* bargaining round is near.

5.1. Long-Term Agreements - Analysis (i)

Proof:

- When the level g_i^{co} is already committed to, the first-order condition for i 's investment is:

$$k = B'(g_i^{co} + R_i) + \delta V_R \Rightarrow \quad (11)$$

$$y_i^{co} = B'^{-1}(k - \delta V_R), \quad R_i^{co} = B'^{-1}(k - \delta V_R) - g_i^{co}, \quad (12)$$

$$r_i^{co} = B'^{-1}(k - \delta V_R) - g_i^{co} - q_R R_{i,-}. \quad (13)$$

- If V_R^{co} is constant (confirmed below), the second-order condition is $B'' \leq 0$, which holds by assumption.
- If the negotiations fail, the default outcome is the non-cooperative outcome, giving everyone the same utility.
- Since the r_i s follow from the g_i s in (13), everyone understands that negotiating the g_i s is equivalent to negotiating the r_i s.

5.1. Long-Term Agreements - Analysis (ii)

- Since all countries have identical preferences w.r.t. the r_i s (and their default utility is the same) the r_i s are going to be equal for every i .
- Symmetry requires that r_i , and thus $\zeta \equiv g_i + q_R R_{i,-}$, is the same for all countries.
- This implies that $R_t = \sum_i (B'^{-1} (k - \delta V_R) - g_i^{co}) = nB'^{-1} (k - \delta V_R) - EG_t + q_G G_{t-1}$. Thus, we can write the value function as:

$$\begin{aligned} V(G_-, R_-) &= \max_{EG_i} B(B'^{-1} (k - \delta V_R)) - EC(EG_t + \theta_t) \\ &\quad - \frac{K}{n} (nB'^{-1} (k - \delta V_R) - EG_t + q_G G_{t-1} - q_R R_{i,-}) \\ &\quad + E\delta V(EG_t + \theta_t, nB'^{-1} (k - \delta V_R) - EG_t + q_G G_{t-1}) \end{aligned}$$

- Note that the Envelope theorem gives $V_{R_-} = q_R K / n$.

5.1. Long-Term Agreements - Analysis (iii)

- With this, we can also take the derivative wrt G_{t-1} to get $V_G = -q_G (1 - \delta q_R) Kn$.
- With this, the foc wrt EG_t is:

$$EC'(G) = K/n - \delta V_R + \delta V_G = (1 - \delta q_G)(1 - \delta q_R) K/n. \quad (14)$$

- This is the same pollution level as in the first best.
- Since investments are suboptimally low if $\delta q_R > 0$ or $\psi > 0$, the pollution level is suboptimally small ex post.
- Combining (14) with (11),

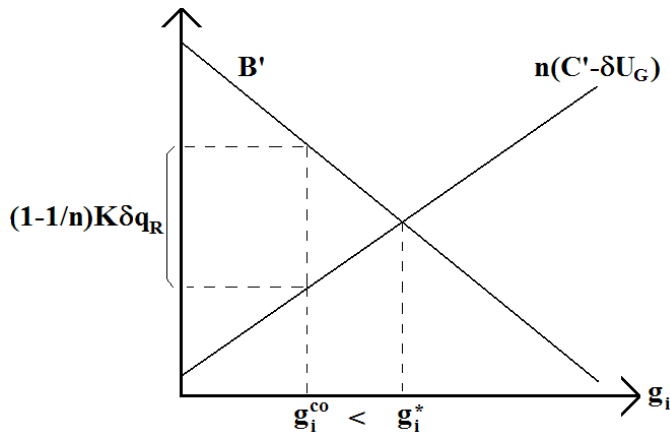
$$\begin{aligned} B'(y_i)/n - EC'(G) + \delta V_G &= (k - \delta V_R)/n - K/n + \delta V_R \\ &= \frac{1}{n}(k - K) + \frac{\delta q_R K}{n} \left(1 - \frac{1}{n}\right). \end{aligned}$$

5.1. Long-Term Agreements - Lessons

Proposition

- *There is a unique MPE.*
- *The value function is linear with Lemma 1 and 2 as before.*
- *The smaller is the quota g_i^t , the larger is $r_{i,t}$: $\partial r_{i,t} / \partial g_{i,t} = -1$ while $r_{i,t}$: $\partial r_{i,t} / \partial g_{j,t} = 0$.*
- *The negotiated $g_i + q_R R_{i,-}$ is the same for every country.*
- *Conditional on g_i^t , every country **invests too little** if either $\delta q_R > 0$ or $\psi > 0$.*
- *$EC'(G^T) = EC'(G^*)$, even if investments are too low.*
- *Thus, **quotas are too low ex post**:*
$$B'(y_i) / n - EC'(G) + \delta V_G = \frac{1}{n} (k - K) + \frac{\delta q_R K}{n} \left(1 - \frac{1}{n}\right) > 0.$$
- *If there is no uncertainty, $g_{i,t}(R) < g_{i,t}^*(R)$ if either $\delta q_R > 0$ or $\psi > 0$.*

5.1. Long-Term Agreements - Lessons

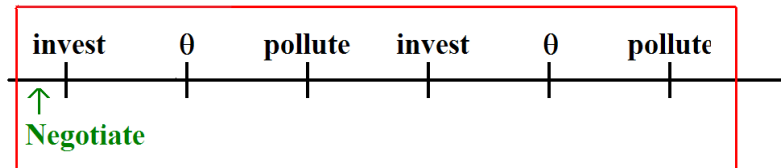


5.1. Long-Term Agreements - Lessons

Proposition

- *The quotas should&will be smaller than what is "ex post optimal" (particularly if δV_R is large, i.e., short duration).*
- *Intuition:*
 - Countries anticipate that everyone will under-invest because of the hold-up problem.
 - But everyone invests more if quotas are small.
 - Thus, they agree on small quotas since this **mitigates the underinvestment problem**.
 - The shorter is the duration, the smaller are investments, and the smaller are the optimal quotas relative to the ex post optimal emission levels.

5.2. Long-Term Agreements: Multiple Periods



5.2. Long-Term Agreements - Investments

- Given quotas $\{g_{i,t}, \dots, g_{i,T}\}$, i invests as to ensure:

$$\begin{aligned} B' \left(g_i^{lt} + R_{i,t} \right) &= k - k\delta q_R, \quad t < T, \\ B' \left(g_i^{lt} + R_{i,t} \right) &= k - K\delta q_R/n, \quad t = T. \end{aligned} \tag{15}$$

- Investments are first-best for $t < T$ iff $\psi = 0$, but they are smaller at $t = T$ if $\delta q_R > 0$.
- The smaller is the quota $g_{i,t}$, the larger is $r_{i,t}$: $\partial r_{i,t} / \partial g_{i,t} = -1$ while $r_{i,t}$: $\partial r_{i,t} / \partial g_{j,t} = 0$.
- The quotas should be smaller than what is "ex post optimal" (particularly if duration short)

5.2. Long-Term Agreements - Quotas (i)

- At the start of $t = 1$, countries negotiate emission levels for every period $t \in \{1, \dots, T\}$.
- In equilibrium, all countries enjoy the same default utilities.
- Just as before, they will therefore negotiate the quotas such that the equilibrium **investment will be the same** for all i s.
- Using (15) for each period $t < T$, this implies:

$$\begin{aligned} R_t &= \sum_i (B'^{-1}(k(1 - \delta q_R)) - g_{i,t}) \\ &= nB'^{-1}(k(1 - \delta q_R)) - E(G_t - q_G G_{t-1}) \Rightarrow \\ r_{i,t} &= (R_t - q_R R_{t-1}) / n, \quad \forall t \\ &= B'^{-1}(k(1 - \delta q_R)) - E(G_t - q_G G_{t-1}) / n \\ &\quad - q_R B'^{-1}(k(1 - \delta q_R)) + q_R E(G_{t-1} - q_G G_{t-2}) / n, \\ t &\in \{2, \dots, T-1\}. \end{aligned} \tag{16}$$

5.2. Long-Term Agreements - Quotas (ii)

- At the start of the first period, i 's continuation value can be written as:

$$\begin{aligned} V = & \max_{\{EG_t\}_t} \frac{1 - \delta^{T-1}}{1 - \delta} B(B'^{-1}(k(1 - \delta q_R))) \\ & - \sum_{t=1}^T \delta^{t-1} \left[Kr_{i,t} + C \left(EG_t + \sum_{t'=1}^t \theta_{t'} q_G^{t-t'} \right) \right] \\ & + \delta^{T-1} B(B'^{-1}(k - \delta q_R K/n)) \\ & + \delta^T V \left(EG_T + \sum_{t'=1}^T \theta_{t'} q_G^{T-t'}, R_T \right), \end{aligned} \quad (17)$$

where $r_{i,t}$ is given by (16) and R_T is given by (15).

- Theorem (2)** $V_R = q_R K/n$ and $V_G = -q_G K(1 - \delta q_R)/n$.
- Proof.* Both parts follow from (17) by applying the envelope theorem.

5.2. Long-Term Agreements - Quotas (iii)

- The first-order condition of (17) w.r.t. any EG_t , $t \in \{1, \dots, T\}$, gives:

$$EC' = (1 - \delta q_R) (1 - \delta q_G) K / n$$

- The second-order condition, $-C'' < 0$, always holds.
- Since investments are suboptimally low, the emissions are too small conditional on equilibrium technology stocks.
- Using (15),

$$\begin{aligned} B' - EC' (G) n + n\delta V_G &= (1 - \delta q_R) (k - K) \\ B' (g_i^{lt} + R_i) - EB' (g_i^* + R_i) &= (1 - \delta q_R) (n - 1) \psi \end{aligned}$$

- In Example Q, this becomes simply

$$g_i^{lt}(\mathbf{R}) = g_i^*(\mathbf{R}) - (1 - \delta q_R) (n - 1) \psi / b.$$

5.2. Long-Term Agreements - Lessons

Proposition

- *Investments are suboptimally low at $t < T$ if $\psi > 0$, and at $t = T$ if either $\psi > 0$ or $\delta q_R > 0$.*
- *The agreement becomes tougher to satisfy towards the end.*
- *Investments are larger if quotas are smaller.*
- *Therefore, the **optimal quotas are smaller** than what is ex post optimal if $\psi > 0$ in every period, and in the last period if either $\psi > 0$ or $\delta q_R > 0$.*
- *The larger is ψ , the smaller are the optimal quotas compared to what is ex post optimal.*

5.3. The Optimal Duration: The cost of the hold-up

- The optimal length T balances the cost of underinvestment when T is short and the cost of the uncertain θ 's is increasing in T .
- In period T , countries invest suboptimally, not only because of the domestic hold-up problem, but also because of the international one. When all countries invest less, u_i declines. This cost of the hold-up problem in period T , relative to any period $t < T$, can be written as:

$$\begin{aligned} H &= -\frac{b}{2} (y_{i,t} - \bar{y}_i)^2 - \frac{b}{2} (y_{i,T} - \bar{y}_i)^2 - K (r_{i,t} - r_{i,T}) (1 - \delta q_R) \\ &= -\frac{b}{2} \left(\frac{k - \delta q_R K}{b} \right)^2 + \frac{b}{2} \left(\frac{k - z}{b} \right)^2 - K \left(\frac{k - z}{b} - \frac{k - \delta q_R K}{b} \right) \\ &= \frac{\delta q_R}{b} \left(\psi + \frac{K}{n} \right) \left[\psi \left(1 - \frac{\delta q_R}{2} \right) + \frac{\delta q_R K}{2n} \right] (n-1)^2. \end{aligned}$$

- Note that H increases in ψ , n , q_R , and K , but decreases in b .

5.3. The Optimal Duration: The cost of uncertainty

- The cost of a longer-term agreement is associated with θ .
- Although EC' and thus EG_t , are the same for all periods,

$$\begin{aligned} E \frac{c}{2} (G_t)^2 &= E \frac{c}{2} \left(EG_t + \sum_{t'=1}^t \theta_{t'} q_G^{t-t'} \right)^2 \\ &= \frac{c}{2} (EG_t)^2 + \frac{c}{2} \sigma^2 \left(\frac{1 - q_G^{2t}}{1 - q_G^2} \right). \end{aligned}$$

- The last term is the loss associated with **the uncertainty**.
- For the T periods, the total present discounted value of this loss is:

$$\begin{aligned} L(T) &= \sum_{t=1}^T \frac{c}{2} \sigma^2 \delta^{t-1} \left(\frac{1 - q_G^{2t}}{1 - q_G^2} \right) = \frac{c \sigma^2}{2(1 - q_G^2)} \sum_{t=1}^T \delta^{t-1} (1 - q_G^{2t}) \\ &= \frac{c \sigma^2}{2(1 - q_G^2)} \left[\frac{1 - \delta^T}{1 - \delta} - q_G^2 \left(\frac{1 - \delta^T q_G^{2T}}{1 - \delta q_G^2} \right) \right]. \end{aligned} \quad (18)$$

5.3. The Optimal Duration: The trade-off

- If all future agreements last $\hat{T}$ periods, then the optimal T is:

$$\begin{aligned} & \min_T L(T) + \left(\delta^{T-1} H + \delta^T L(\hat{T}) \right) \left(\sum_{\tau=0}^{\infty} \delta^{\tau \hat{T}'} \right) \Rightarrow \\ & 0 = L'(T) + \delta^T \ln \delta \left(H/\delta + L(\hat{T}) \right) / \left(1 - \delta^{\hat{T}'} \right) \\ & = -\delta^T \ln \delta \frac{c\sigma^2/2}{1 - q_G^2} \left(\frac{1}{1 - \delta} - \frac{q_G^{2T+2} (1 + \ln q_G^2 / \ln \delta)}{1 - \delta q_G^2} \right) \\ & \quad - \left(-\delta^T \ln \delta \right) \left[\frac{H/\delta + L(\hat{T})}{1 - \delta^{\hat{T}'}} \right], \end{aligned} \tag{19}$$

assuming that some T satisfies (19).

- Since $\left(-\delta^T \ln \delta \right) > 0$ and the term in the brackets increases in T , the loss decreases in T for small T but increases for large T , and there is a unique T minimizing the loss.

5.3. The Optimal Duration: The optimal T

- Substituting for $\hat{T} = T$ and (18) in (19) gives:

$$\frac{H}{\delta} = \frac{c\sigma^2 q_G^2}{2(1 - q_G^2)(1 - \delta q_G^2)} \left(\frac{1 - \delta^T q_G^{2T}}{1 - \delta^T} - q_G^{2T} \left(1 + \frac{\ln(q_G^2)}{\ln \delta} \right) \right), \quad (20)$$

where the r.h.s. increases in T . $T = \infty$ is optimal if the left-hand side of (20) is larger than the right-hand side even when $T \rightarrow \infty$:

$$\frac{c\sigma^2 q_G^2}{2(1 - q_G^2)(1 - \delta q_G^2)} \leq \frac{H}{\delta}. \quad (21)$$

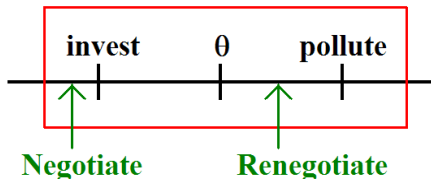
- If ψ and n are large but b is small, then H is large and (21) is more likely to hold; if (21) does not hold, the T satisfying (20) is larger.
- If c or σ^2 are larger, (21) is less likely to hold and, if it does not, (20) requires T to decrease.

5.3. The Optimal Duration: Lessons

Proposition

- 1 *There is a hold-up problem (with under-investments) for every finite agreement.*
- 2 *It is optimal to reduce/delay this cost by increasing the duration of the agreement.*
- 3 *On the other hand, the uncertainty aggregates over time.*
- 4 *The trade-off implies that the optimal duration is longer if ψ is large while σ^2 is small, for example.*
- 5 *Thus, with weak intellectual property rights, the climate treaty should last longer.*

6. Renegotiation and Updating of Agreements



- For long-term agreements, $g_i < E g_i^*$ to encourage R&D.
- This is clearly not "renegotiation proof".
- Low initial g_i^{de} still motivates r_i , because large R_i strengthen one's bargaining power when g_i^{de} is small.
- With renegotiation, $g_i \uparrow g_i^*$, at the renegotiation stage.
- This procedure implements the first best.

- Results hold no matter future regimes
- Policy instruments:
 - Side transfers possible or not: Identical results
 - Permits tradable or not: Identical results
 - If Pigou taxes instead, similar results
- Heterogeneity in e.g. $R_{i,0}$ or $\bar{y}_i$, $B(y_i - \bar{y}_i)$, straightforward
- Heterogeneous k_i : Only $\min_i k_i$ should invest

Remember the Assumptions

- All lectures:
 - Each country maximizes a well-specified objective function
 - Time of investments and emissions alternate
 - Emissions accumulate over time and across countries
 - Symmetry: All countries are quite similar
- Today's lectures:
 - Participation is taken as given
 - Compliance is taken as given
 - All actions are observable
 - Strategies are simple (Markovian)
 - Exponential discounting
 - Each country acts as one player/individual (we abstract from "domestic" politics)

Further Readings

- *Details available in Harstad (2012 and 2016), which draw on 3 strands of literature:*
- *R&D and Environmental Agreements*
 - Aldy and Stavins (2005, 2007), Barrett (2005), Karp and Zhao (2009), Hong and Karp (2010), Golombek and Hoel (2005, 2006), Hoel and de Zeeuw (2009)
- *Dynamic (Differential) Games*
 - Dockner, Jorgensen, Long and Sorger (2000), Friedman (1974), Ploeg and de Zeeuw (1992), Fehrstman and Nitzan (1991), Sorger (1998), Dutta and Sundaram (1993), Dutta and Radner (2004, 2006, 2009).
- *Contracts, Hold-up, Duration, and Renegotiation Design*
 - Hart and Moore (1988), Harris and Holmstrom (1987), Gatsios and Karp (1992)
 - Aghion, Dewatripont and Rey (1994), Guriev and Kvasov (2002), Edlin and Hermalin (1996), Che and Hausch (1999).

7. Technological Spillovers

7. Technological Spillovers

- When country j has invested $r_{j,t-1}$ units in technology in period $t - 1$, country $i \neq j$ may learn and benefit from those investments.
- Parts of j 's investment may have required innovations or generated new ideas, or there may simply be traditional learning by doing or by using when j upgrades its technology stock.
- To capture such technological spillovers in a simple way, let parameter $\varphi \in [0, 1]$ be the fraction of $r_{j,t-1}$ which another country i can copy and adopt in the following period t .
- A one-period lag between the time of one country's investment and the time of diffusion to another country is reasonable.
- The evidence of Eaton and Kortum (1999) points to a diffusion lag of 6-13 years and an interval $(\frac{1}{2}, \frac{3}{4})$ for parameter φ .

7. Technological Spillovers

- Let $\underline{k}$ be the unit cost of investing in *new* technology, while the unit cost of adopting spillovers is only $(1 - \gamma) \underline{k}$, where $\gamma \geq 0$ measures the cost savings. For example, $1 - \gamma \approx 0.65$ appears as an estimate for the imitation cost, relative to new investments, in Mansfield et al. (1981:909).
- *Spillovers*: With these possibilities for adopting spillovers from $r_{j,t-1}$, country i 's cost when choosing the total upgrade $r_{i,t}$ becomes $\underline{k}r_{i,t} - \varphi\gamma\underline{k}\sum_{j \neq i} r_{j,t-1}^N$.
- To see this, note that when i adopts $r_{i,t}^A = \varphi \sum_{j \neq i} r_{j,t-1}^N$, the level of new technology must be $r_{i,t}^N = r_{i,t} - r_{i,t}^A$. The total cost of $r_{i,t}$ is thus $\underline{k}r_{j,t-1}^N + \underline{k}(1 - \gamma) r_{i,t}^A = \underline{k}r_{i,t} - \varphi\gamma\underline{k}\sum_{j \neq i} r_{j,t-1}^N$.
- Since the second term is fixed in period t , we can account for i 's imitation benefit already in period $t - 1$, exactly as represented before, if we just discount that future benefit by writing $\psi = \delta\varphi\gamma\underline{k}$.

7. Technological Spillovers

- The private investment cost is then $k = \underline{k}$ and the net social cost when investing becomes

$$K = [1 - (n - 1) \delta \phi \gamma] \underline{k}.$$

- *Intellectual property rights (IPR)*: There are several ways of adding intellectual property rights to the model.
- (i) Suppose the IPR is enforced (or judged valid) with probability α , as in Farrell and Shapiro (2008).
- (ii) Or, suppose i can imitate $(1 - \alpha) \phi r_{j,t-1}$ for free, while the remaining fraction α requires the consent of j .
- (iii) Alternatively, consider IPR as a way to raise the cost of imitation, as in Mansfield et al. (1981). For example, write the imitation cost as $[1 - \gamma (1 - \alpha)] \underline{k}$, increasing from $(1 - \gamma) \underline{k}$ to $\underline{k}$ when the IPR-policy α increases from 0 to 1.
- For any of these three alternatives, the externality would be $\psi = \delta \phi \gamma (1 - \alpha) \underline{k}$ if there were no licensing or technological trade.

7. Technological Spillovers

- *Licensing or technological trade*: The IPR policy, measured by α , inefficiently limits diffusion and thus there are gains from trade.
- Consider the moment before i can imitate j , and suppose i and j negotiate a fee permitting i to use the technology $\varphi r_{j,t-1}$.
- Assume the licensing fee is determined by the generalized Nash bargaining solution where $\beta \in [0, 1]$ is the seller's bargaining-power index.
- Suppose private purchasers of technology in country i pay an ad valorem tariff τ (or subsidy if $\tau < 0$) when buying technology.
- The tariff revenues are collected by the importing country, but it will improve the importer's terms of trade.

7. Technological Spillovers

Proposition

- With IPR modelled as (i), (ii) or (iii), i 's payoff can be written as above where the externality is:

$$\psi = \left(1 - \frac{\alpha\beta}{1 + \tau}\right) \delta\gamma\varphi k.$$

- Thus, if the strength of the IPR, given by $\alpha\beta / (1 + \tau)$, decreases, then the externality ψ increases and so does the private investment cost $k = K + (n - 1)\psi$, while K is unchanged.
- Since spillovers, fees, and revenues following $r_{j,t-1}$ are accounted for in period $t - 1$, the payoff-relevant states at the beginning of period t are still captured by the stocks G_{t-1} and $\mathbf{R}_{t-1} \equiv \{R_{1,t-1}, \dots, R_{n,t-1}\}$.

7. Technological Spillovers

Proposition

- ① *With poor patent protection:*
 - *ST-agreements are bad*
 - *T should be larger*
 - *Agreement should be more ambitious*
 - ② *Patent protection (and subsidies) should increase if T is small*
-
- Thus, trade and short-term climate treaties are *complements*
 - The ambition levels of trade and climate treaties are *substitutes*

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 - Symmetry: All countries are quite similar
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