The Choice of Certifier in Endogenous Markets*

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Abstract

This paper provides a theoretical framework for studying the consequences of the certifier's identity, the characteristics of the best certifier, and the identity of the equilibrium certifier. A certifier that cares about quality and externalities (such as an NGO) motivates firms to invest in their capacities to provide quality; a certifier concerned with the firms' profits (such as an industry association) motivates more firms to enter the market in the first place. The relative importance of externalities, investments, and entry determines the socially optimal certification authority but also the type of certifier that is most likely to enter in equilibrium. The theory's predictions are empirically testable and shed light on the variety of certifiers across markets and over time.

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1 Introduction

For markets to work efficiently, consumers must learn about the quality of the products available to them. Such learning is hard for "credence goods" where consumers care about products' effects on health, risks, and the environment, or on other public goods or externalities, such as labor conditions. Given the complexity of new goods, there is an increasing demand for third parties that can verify or certify the quality of products. On the demand side, empirical evidence suggests that consumers are willing to pay more for products with green or energy-efficiency labels.[1] On the supply side, firms are evidently making costly investments to satisfy certification requirements.[2]

But who should certify? Certifiers come in many variants, but most certifiers are one of three types. Activist or non-governmental organization (NGO) certifiers seek to reward good products (or punish bad ones) by labeling those and only those that satisfy certain criteria. The Fairtrade Labelling Organizations International eV (or "Fairtrade" for short), the Forest Stewarship Council®, the U.S. Green Building Council (the organization managing the LEED rating system), and the Green Seal are examples of such certifiers. These certifiers play a prominent role when it comes to energy efficiency and sustainable production (ranging from agriculture and forestry to household goods).

In other markets, corporate or industry associations often define "best practice" or standards that they can verify various sellers satisfy. Table 1 shows that online selling platforms, such as eBay, Airbnb, or Amazon,[3] are themselves providing certification for their most reliable sellers. Interestingly, this certification is always provided in-house even though, in principle, the platforms could delegate certification to some consumer rights association in order to boost the credibility of certification. In still other markets, for-profit certifiers provide certification services. Arguably the most prominent

[1]Eichholtz *et al.* (2010), Hainmueller *et al.* (2015), de Janvry *et al.* (2015) provide evidence of price premia associated with green labels, while Bjørner *et al.* (2004) and Hallstein and Villas-Boas (2013) measure the positive effect of green labels on consumers' willingness to pay. For evidence of consumers' responsiveness to energy-efficiency labels, see Newell and Siikamäki (2014) and Davis and Metcalf (2016).

[2]de Janvry *et al.* (2015) provide evidence on costly investments by coffee farmers to obtain fair-trade labels; Eichholtz *et al.* (2013) find large differences in energy efficiency between labeled and non-labeled buildings.

[3] For Airbnb, see the program Airbnb Plus; for eBay, see the Top Rated Seller Program; for Amazon, see a forum discussion on the Amazon's Choice badge.

Commercial certifiers	NGOs
Online platforms	**Sustainable agriculture**
Amazon, eBay, Airbnb	Fairtrade, Rainforest Alliance
Credit ratings	**Sustainable forestry and fisheries**
Moody's,	Forest Stewardship Council,
Standard & Poor's,	Marine Stewardship Council
Fitch Ratings	**Energy efficiency**
Safety certification	U.S. Green Building Council
Underwriters Laboratories	**Consumer products**
	Green Seal

Table 1: Examples of Certifiers

example is credit rating agencies. As we will show, industry and for-profit certifiers follow similar criteria when it comes to standard setting. For this reason, we place them in the same category. Table 1 lists the examples mentioned so far, together with a few others.

A third certifier type is bureaucratic or government agencies. Well-known examples are the Energy Star, the U.S. Department of Agriculture Organic Label, and the EU Ecolabel. In practice, there is a continuum of certifier types, since the decision-making boards in government programs are often composed of stakeholders from industry as well as from NGOs.[4]

What explains the variation, and what are the consequences of it? When do governments prefer to let representatives of industries or NGOs define the standards of publicly run certification programs? Which certifiers are socially optimal, and which ones establish themselves in equilibrium?

The purpose of this paper is to address all these questions head-on. We develop a purposefully stylized and simple model that can easily be built on in future research. Naturally, even the simplest such model must include consumers, firms, and a certifier. The certifier maximizes a weighted sum of

[4]To the best of our knowledge, all certifiers run by government agencies claim to work together with third parties to set up their standards. The Energy Star, for example, is a program run by the U.S. Environmental Protection Agency and U.S. Department of Energy in consultation with industry representatives (see the Environmental Protection Agency web page). The U.S. Department of Agriculture Organic Food Label and the EU Ecolabel are more inclusive: they set requirements in consultation with advisory boards composed of representatives of consumers, firms, and environmental associations (called, respectively, the National Organic Standards Board and the EU Ecolabelling Board).

firms' profits and quality. The weights depend on the certifier's type, known by everyone.

In the basic short-term version of the model, the certifier specifies a quality requirement before firms decide whether to improve the quality of their products in order to satisfy the requirement. Firms have heterogeneous costs and the more demanding the requirement is, the smaller is the mass of firms finding it worthwhile to satisfy. This prediction is in line with empirical evidence. Hui *et al.* (2018) show that when eBay replaced its "Powerseller" badge for virtuous sellers with the more demanding "Top Rated Seller" badge, the share of badged sellers dropped.[5] Elfenbein *et al.* (2015) compare different product markets on eBay and find that in markets with fewer certified products, certification commands a higher price premium. Our basic model also predicts that an NGO prefers a more demanding requirement than does the government agency, who in turn prefers a tougher requirement than does an industry association. This prediction is also in line with the evidence.[6]

In the basic model, the government always prefers to certify "in-house" rather than to delegate certification to somebody else. This preference changes in our long-term model. There, it is essential to endogenize firm types and market structure. In our long-term model, the identity of the certifier is determined before firms take actions such as investing in technology or entering a new market, while the certification standard is set after these actions. We believe this timing is reasonable as new certifiers usually take years to obtain recognition from consumers and firms, while established certifiers can, and

[5] The authors also show that the new rules led to higher quality provision only by sellers that lost their badge as a result of the change, but regained it within the next few months. This evidence lends credibility to models like ours in which sellers can specify the quality of their products.

[6] Supporting anecdotal evidence can be found in sectors in which firms can get certificates from industry-backed certifiers as well as from NGOs. Construction and forest products are two examples. In both sectors, the most commonly used NGO standard is recognized to be more stringent than the industry one (for the construction industry see the Portland Tribune; for forest products see The Washington Post and GreenBiz). Additional evidence comes from certifiers that have changed their main stakeholders: All the main credit rating agencies switched in the 1970's from an "investor pays" to an "issuer pays" business model. The former model, by focusing on investor profits, made the rating agencies' objectives comparable to those of NGOs, while the latter, by focusing on issuers' profits, made the rating agencies more similar to industry-backed certifiers. This shift has been extensively criticized as a source of widespread rating inflation: see, for example, Rivlin and Soroushian (2017).

do, change their standards on short notice.[7] Furthermore, if market conditions are subject to stochastic shocks, a certifier that could commit to a standard in advance might prefer to avoid doing so. As we are going to show, these strategic delegation concerns can make ”in-house” certification suboptimal for a government, thus reversing the basic message of the short-term model.

First, we let firms invest in their capacity to provide quality before the exact certification requirement is known. In the construction industry, for example, energy efficiency improvements require the development of new materials and construction techniques. In the healthcare sector, improvements in medical technology require long-term investments. We show that such investments are especially rewarding if the certification level is expected to be strict. When it is socially efficient to motivate investments, a certifier that emphasizes quality over profits, either in the form of an NGO or of a government agency heavily influenced by activist groups, is likely to be better. In such a case, we show governments may benefit from delegating certification authorities to NGOs.

Next, we endogenize the structure of the market. Profits are lower when the certification requirement is very demanding in our model. Thus, when it is costly for firms to enter or operate in the market, fewer firms find it optimal to enter if the certifier is an organization expected to emphasize quality over profit; more firms enter if the certifier is, instead, a profit-maximizing industry association. When entry is welfare-enhancing and, moreover, sensitive to expected profit, then everyone benefits if certification authority is delegated to the industry itself.

Altogether, we show how the optimal certification authority depends on the importance of entry, investment, and consumption externalities. Our characterization of the optimal certifier has normative policy implications, because a government can decide on the composition of a public certifier's standard-setting boards. Governments can also influence the importance of

[7] In practice, certifiers regularly revise their standards: At the time when a firm decides on an investment or whether to enter a new market, there can be uncertainty regarding the standard that will be in place when the returns on the investment/entry are reaped. For example, the standards of the LEED rating system are revised approximately every five years. The first paragraph of the LEED v4.1 web page describes the latest version of these standards as follows : “Today's version of LEED, LEED v4.1, raises the bar on building standards to address energy efficiency, water conservation, site selection, material selection, day lighting and waste reduction.”

private certifiers by requiring public procurement to prioritize products with certain labels, or by subsidizing NGOs involved in the certification process.

Finally, we endogenize the certifier type. If it is very costly to establish oneself as a certifier (due to expensive monitoring technologies, for example), then neither industries nor NGOs will volunteer to certify. If this expense declines for technological reasons, then, sooner or later, a single stakeholder will enter as a certifier. The first certifier to enter is more likely to be an NGO if investments and externalities are important, we show. In contrast, an industry association is more likely to enter first if the potential for further firm entry is important, while externalities are small. The comparative statics of this equilibrium are qualitatively similar to the comparative statics of the results we derived when the government could dictate the identity of the certifier. This similarity implies that our predictions are quite robust.

Our predictions are also testable and they align well with the examples listed in Table 1. As we explain in the concluding section, both externalities and firms' investments in their capacity to provide quality are important in agriculture and industries relying on the use of natural resources. In contrast, online selling platforms are more recent phenomena, and the potential for sellers to enter in these markets is still significant. Our theory thus predicts that the industry certifies for online selling platforms, while NGOs certify when it comes to agricultural and natural resource products, exactly as Table 1 illustrates.

Our analysis can also shed light on the evolution of certification over time. Early on, when additional firms can still be expected to enter the market, it can be beneficial for everyone to delegate certification authority to the industry itself. After the market structure is settled, at the stage when it is relatively more important to encourage the existing firms to make long-term quality investments, then the government benefits if instead NGOs certify products. When also these investments are sunk, however, the government itself would prefer to take direct control over the certification process. As an example of such an evolution, note that The National Committee for Quality Assurance, a US certifier that aims at improving the quality of healthcare by offering certification to health maintenance organizations, was founded in 1979 by two trade associations. In 1990, however, it became an independent NGO, apparently in order to improve the impact on healthcare quality.[8]

Interestingly, this dynamic is also pointing to a time inconsistency prob-

[8]See Jin (2005) for details.

lem: Even when a government would like industries to certify in order to encourage entry, or NGOs in order to encourage investments, these encouragements may have little impact if firms anticipate that the certifier's identity might change later on.

Literature Review. This paper is the first to analyze the consequences of certifiers' inability to commit ahead of time to their future requirements, and the implications that this has for the optimal certifier identity. This way, we build a bridge between the certifier literature and the literature on strategic delegation.

Several papers have, naturally, studied the effect of certification on firms' choice of product quality. Albano and Lizzeri (2001), for example, extend the seminal work of Lizzeri (1999) and characterize the profit-maximizing strategy of a certifier in an environment where product quality is endogenous. Miklós-Thal and Schumacher (2013) tackle a similar question in a dynamic setting with short-lived sellers, while Siegel (2020) shows that socially optimal standard is higher when quality becomes endogenous. On a related note, Boleslavsky and Kim (2018), Boleslavsky and Cotton (2015), Dubey and Geanakoplos (2010), and Costrell (1994) consider the way grading schemes influence students' efforts.

The effect of certification on the market structure has also been investigated. Harbaugh and Rasmusen (2018) show that a non-profit certifier can provide more information to consumers by assigning coarse grades, if one just takes into account that the equilibrium distribution of firm types will depend on the grading. Choi and Mukherjee (2019) endogenize both product quality and market structure.

These papers, however, assume that certifiers are first-movers and they focus on firms' actions *after* the certification standard is set. Thus, they ignore the possibility that firms often need to make long-term entry and investment decisions in advance, long before they know the exact certification threshold. When firms make such long-term decisions, we show how it is beneficial to delegate certification authority, just like Rogoff (1985) showed that governments may want to delegate monetary policy to a central bank. The benefits of strategic delegation has already been discussed for bargaining situations by Schelling (1956), for principal-agent settings by Aghion and Tirole (1997), for cheap-talk games by Dessein (2002), and for organizational design by Alonso *et al.* (2008). Harstad (2010) showed that the concern for bargaining power, and the concern for coalition membership, go in opposite directions when it comes to delegation in political contexts. In the present paper, the concern for entry, and the concern for investments, go in opposite directions.

With this, we add to the relatively small literature on the optimal certifier identity. Stahl and Strausz (2017) compare seller-paid certification and buyer-paid certification and show that the first variant results in ratings that are more informative. While they consider different business models, they only consider profit-maximizing certifiers. We, in contrast, consider certifiers with different objective functions. The conflict between buyers' and sellers' preferences for information disclosure is the focus of Hopenhayn and Saeedi (2019), who show that even though it is possible to separate products into multiple categories, a certifier has little to lose from sticking to the two-tiered policy considered also in our paper.[9] A few papers study competition among NGO and for-profit certifiers. Fischer and Lyon (2014) focus on certifiers that can set a single standard, while Fischer and Lyon (2019) allow for non-binary certifications but, as the other papers mentioned above, they assume the certifier makes the first move.

Our analysis also aims at predicting which type of certifier is more likely to arise in different markets. Bonroy and Constantatos (2014) and Baron (2011) take a similar positive approach; yet the dynamic aspect of our analysis is absent in their models.[10]

Outline. In Section 2, we present the workhorse model and explain why NGOs prefer the most demanding certification requirements, while industry associations prefer the least stringent requirements. In Section 3, we allow firms to invest in their capacities to provide quality, before we present conditions under which the government prefers to delegate certification authority to an NGO. In Section 4, the market size is endogenous since firms can enter. Here, we characterize the equilibrium market size as a function of the certifier's identity and we describe the optimal certification authority. In Section 5, even the certifier type is endogenous. Section 6 concludes by returning to the anecdotal evidence. A number of extensions are discussed in Appendix A; all proofs are contained in Appendix B.

[9]In general, the buyers' optimal information disclosure is rarely studied. Exceptions are Roesler and Szentes (2017) and Harbaugh and Rasmusen (2018). See Dranove and Jin (2010) for a review of the earlier literature.

[10]Our positive analysis is somewhat related to Biglaiser (1993), who looks at incentives to operate as a middleman between sellers and buyers and to provide buyers with information about the product.

2 The Basic Analysis

2.1 The Basic Model

This section presents a simple workhorse model that can be built on in the following sections. The model intends to capture the key trade-offs when it comes to product quality and certification requirements in the short term. The model consists of consumers, firms, and a single certifier.

Consumers: There is a mass one of homogeneous consumers. Each consumer can buy a unit of a good to get $u(x) = v(x) - p(x)$, where $v(x)$ is the value of a good with quality x and $p(x)$ is the price of such a good. The consumer gets a utility of 0 from not buying. The value function v is assumed to be simply $v(x) = v_0 + x$. Our results would stay unchanged, qualitatively, if consumers were heterogeneous in that only a fraction of them cared about quality.[11]

Firms: There is a mass $m > 1$ of heterogeneous firms. Each firm produces up to one unit of a good. To produce a good of quality x_i, firm i incurs the cost $\tilde{q}_i x_i^\alpha$, where $\alpha \in (1,2)$.[12] We assume that the distribution of each $\tilde{q}_i$ is uniform and i.i.d. over the interval $[q, q + \Delta_q]$. Consequently, if we order firms according to the $\tilde{q}_i$'s, we can write $\tilde{q}_i = q + i\Delta_q/m$ for $i \in [0, m]$. We let firm i's profit be denoted by $\pi_i(x_i) = \mathbb{1}_i (p(x_i) - \tilde{q}_i x^\alpha)$, where $p(x_i)$ denotes firm i's revenue, while $\mathbb{1}_i \in \{0, 1\}$ measures the equilibrium production level of firm i. In Section 3, we endogenize the cost of providing quality, as captured by parameter q, and, in Section 4, we endogenize the market size, m.

The certifier: In many markets, consumers cannot observe the product quality directly. Hence, consumers benefit if a third party can guarantee that the product is of a certain quality. We will refer to this third party as the certifier.

[11] For example, a unit of good of quality x could be worth $v(x)$ for some consumers, and v_0 for others.

[12] Our analysis stays essentially unchanged if the cost took the form $q_0 + \tilde{q}_i x^\alpha$, where $q_0 \in [0, v_0]$. Also, we focus on the case in which $\alpha \in (1,2)$, since the outcome will be trivial otherwise: for $\alpha \geq 2$ all certifiers set a standard such that a mass 1 of firms get a certificate.

The certifier determines and announces a requirement, $\underline{x}$, and certifies all goods with this or higher quality. Such a binary pass/fail standard is adopted by the majority of labels: The directory Ecolabel Index lists 291 labels that adopt a pass/fail standard and only 97 that provide multi-tiered standards.[13]

After learning $\underline{x}$, each firm i decides on x_i. Since quality is costly, each firm finds it optimal to either satisfy the requirement exactly, or to provide zero quality. That is, in equilibrium, $x_i \in \{0, \underline{x}\} \, \forall i$. We let $n(\underline{x}) \in [0, m]$ measure the equilibrium mass of firms that satisfy the requirement $\underline{x}$.

Certification fees are analyzed in Appendix A, but here we simplify by assuming that certification is offered at no cost to the firms. After all, only a minority of labels operate for-profits in reality: The Ecolabel Index lists 349 labels that define themselves as NGOs, government labels, or as run by an industry association, while only 97 define themselves as for-profit organizations.

Payoffs: The payoffs for consumers and firms are described already. The certifier's objective function may take several aspects into account. One extreme variant is that the certifier chooses $\underline{x}$ to simply maximize aggregate profit. This profit is a function of $\underline{x}$:

$$\Pi(\underline{x}) \equiv \int_0^m \pi_i(\underline{x}) di.$$

This objective may be reasonable if we have in mind a trade association, consisting of the firms in the market. This association may want to define best practice, as discussed in the Introduction.

Another special case arises if the certifier maximizes the aggregate quality:

$$U(\underline{x}) \equiv \int_0^{n(\underline{x})} \underline{x} di = n(\underline{x})\, \underline{x}.$$

This objective function may be reasonable if the certifier is an activist group or an NGO. The aggregate quality $U(\underline{x})$ may represent the industry's total reduction in emissions, pesticides, the use of child labor, or cruel animal treatments.

In many cases, the quality may well be a public good enjoyed by all citizens. Then, if one buyer's marginal value of $\underline{x}$ stays normalized at one,

[13] The data can be obtained from the Ecolabel Index web page.

the entire *social* value of quality can be $U(\underline{x})$ multiplied by a large number, $s > 1$. However, it always holds that $\arg\max sU(\underline{x}) = \arg\max U(\underline{x})$, so the quality-maximizing $\underline{x}$ is independent of s. Furthermore, as long as each consumer is infinitely small, the level of s will not influence one consumer's decision since s measures the externality on the other consumers. It is therefore inconsequential to exclude externalities associated with quality from the consumers' objective function above, since these would not affect the individual decisions.[14]

That said, the externality does matter for a government or bureaucracy, B, which may take social welfare into account when B sets the requirement. If B is the certifier, B maximizes a weighted sum of aggregate profits and aggregate quality:

$$W_b(\underline{x}) \equiv bU(\underline{x}) + (1-b)\,\Pi(\underline{x}).$$

Here, it is natural that parameter b is increasing in the externality s.[15]

In the following, we let the NGO or activist certifier be referred to as A, associated with the weight a placed on quality and the weight $1-a$ placed on profit. The industrial or commercial certifier is referred to as C, placing weight c on quality and $1-c$ on profit. The reader is welcome to simplify by assuming $a = 1$ and $c = 0$, but for our results it is sufficient to assume:

$$1 \geq a > b > c \geq 0.$$

In addition to discussing the effect of whether the certifier is A, B, or C, we can also allow for a more general certifier identity, D. This certifier maximizes $dU(\underline{x}) + (1-d)\,\Pi(\underline{x})$, where $d \in [0,1]$ can be different from a, b, and c. As we have discussed already, a government often delegates the certification process to a committee consisting of all kinds of representatives. The committee places various weights on profits vs. quality, depending on its composition. With this, the choice of certifier, as measured by d, may be fine-tuned by B, as a function of b.

The reader may question whether consumer surplus should also be reflected in the certifier's objective function. It may well be, but the next

[14]The analysis is also unchanged if there are negative externalities associated with consuming low-quality products, as we explain in Appendix A. If the negative externality is, for example, $s(1-v(\underline{x}))$, decreasing in the quality $\underline{x}$, the constant s can be ignored and we can re-define this externality as a positive externality when a buyer consumes a product with large $\underline{x}$.

[15]Note that $sU(\underline{x}) + \Pi(\underline{x}) = [bU(\underline{x}) + (1-b)\,\Pi(\underline{x})]\,\frac{1}{1-b}$ if $s \equiv \frac{b}{1-b} \Leftrightarrow b \equiv \frac{s}{1+s} \in (0,1)$.

Figure 1: The timing of the game

Section proves that, in our model, both individual and aggregate consumer surplus are constant (and independent of $\underline{x}$) whenever $n(\underline{x}) \in [0,1]$. In that case, the constant will not affect anyone's choice of $\underline{x}$, and it is thus inconsequential to let the certifier ignore consumer surplus. In Appendix A, we discuss how the model can be modified so that the consumer surplus becomes relevant.

Timing: Figure 1 illustrates the timing of the game. The identity of the certifier is known throughout the game. The certifier specifies $\underline{x}$ before all firms simultaneously and independently decide whether to raise their firm-specific qualities enough to be certified. Thereafter, the consumers observe the products that are certified and those that are not, before each consumer buys at most one product. The prices clear the market. This timing leads to a unique subgame-perfect equilibrium that we can find when we solve the game by backward induction.[16]

2.2 The Market Equilibrium

Since the game is solved by backward induction, we start by determining the market prices as a function of $\underline{x}$ and of n. Suppose, for a start, that $n \in (0,1)$, so that the mass of products that have been certified is smaller than the mass of consumers. In this case, some consumers will purchase certified products, while others will not. The consumers purchasing non-certified products benefit from the fact that $m > 1$. This inequality implies that competition among the firms drives profits to zero for the firms producing non-certified products. Therefore, their price will be $p(0) = 0$, and the consumer surplus is $u(0) = v_0$. Hence, every consumer is guaranteed at least this consumer surplus.

[16]If, instead, the standard where decided on at the same time as the firms committed to quality levels, then there would be multiple equilibria as the certifier and the firms would essentially play a coordination game.

However, there is a scarcity of the certified product when $n < 1$, and competition among the consumers implies that each of them is willing to pay up to $p(\underline{x})$, given by $v(\underline{x}) - p(\underline{x}) = u(0)$. It follows that $p(\underline{x}) = \underline{x}$. So, as we said above, the consumer surplus is v_0, regardless of $\underline{x}$.

Given the equilibrium price $p(\underline{x})$, the profit of certified firm i is:

$$\pi_i(\underline{x}) = \underline{x} - (q + i\Delta_q/m)\,\underline{x}^{\alpha}.$$

Now, consider the stage at which firms decide on whether to seek certification. Firm i benefits from meeting the quality requirement if no more than a mass one of firms meet the threshold and if:

$$\underline{x} - (q + i\Delta_q/m)\,\underline{x}^{\alpha} \geq 0 \Leftrightarrow i \leq \frac{m}{\Delta_q}\left(\frac{1}{\underline{x}^{\alpha-1}} - q\right).$$

With this, the equilibrium mass of certified firms is:

$$n(\underline{x}) = \frac{m}{\Delta_q}\left(\frac{1}{\underline{x}^{\alpha-1}} - q\right) \in (0, 1),$$

under the condition $\frac{m}{\Delta_q}\left(\frac{1}{\underline{x}^{\alpha-1}} - q\right) \in (0, 1)$. As we prove below, this condition will require that $\Delta_q > mq\left(\frac{2(\alpha-1)}{2-\alpha}\right)$. Therefore, we assume that firm heterogeneity is sufficiently large so that some but not all firms seek certification: that is, we assume $\Delta_q > mq\left(\frac{2(\alpha-1)}{2-\alpha}\right)$. We explain the consequences of relaxing this assumption below: see the *Remark on Heterogeneity.*

In line with the evidence mentioned in the Introduction (see the discussion of Elfenbein *et al.* (2015)), $n(\underline{x})$ decreases with the requirement $\underline{x}$. It is also intuitive that $n(\underline{x})$ decreases in the cost, q, while $n(\underline{x})$ increases in the mass of firms, m. Finally, the aggregate profit and the aggregate quality can be written, respectively, as:

$$\Pi(\underline{x}) = \int_0^{n(\underline{x})} \pi_i(\underline{x})di = \frac{\underline{x}^{\alpha} m}{2\Delta_q}\left(\frac{1}{\underline{x}^{\alpha-1}} - q\right)^2 \text{ and} \tag{1}$$

$$U(\underline{x}) = n(\underline{x})\,\underline{x} = \frac{m}{\Delta_q \underline{x}^{\alpha-1}}\left(\underline{x} - q\underline{x}^{\alpha}\right).$$

2.3 The Equilibrium Requirement

At the certification stage, any certifier D prefers the requirement

$$\underline{x}_d = \arg\max_{\underline{x}} dU(\underline{x}) + (1-d)\,\Pi(\underline{x}).$$

Given the explicit formulae for $U(\underline{x})$ and $\Pi(\underline{x})$, it is straightforward to derive our first benchmark result.

Proposition 1 *The equilibrium requirement is larger if $d \in [0,1]$ is large:*

$$\underline{x}_d = \left(\frac{(2-\alpha)(1+d)}{q\left(1+\sqrt{1-\alpha(2-\alpha)(1-d^2)}\right)} \right)^{\frac{1}{\alpha-1}}, \tag{2}$$

implying that $\underline{x}_d$ is increasing in d, and that

$$n(\underline{x}_d) = \frac{mq}{\Delta_q}\left(\frac{1+\sqrt{1-\alpha(2-\alpha)(1-d^2)}}{(2-\alpha)(1+d)} - 1 \right) \in (0,1).$$

Consequently,

$$\underline{x}_a > \underline{x}_b > \underline{x}_c > 0 \text{ and } 0 < n(\underline{x}_a) < n(\underline{x}_b) < n(\underline{x}_c) < 1.$$

In other words, A prefers a more demanding certification requirement than does B, and B prefers a more demanding requirement than does C. These differences are in line with the evidence discussed in the Introduction (see footnote 6).

More generally, the larger the weight a certifier places on quality relative to profit, the higher the requirement is. The explanation is the following. The scarcity of certified products implies that the firms capture the consumers' value of quality. Thus, the industry's aggregate revenue includes the aggregate quality, $U(\underline{x})$. If certifier C aimed at maximizing revenues alone, it would indeed prefer $\underline{x}_1 = \arg\max_{\underline{x}} U(\underline{x})$. However, certifier C emphasizes the weight on profit, and profit equals revenues minus the cost. Since the cost increases in $\underline{x}$, it follows that C prefers $\underline{x}_c < \arg\max_{\underline{x}} U(\underline{x})$. Intuitively, then, a convex combination of $U(\underline{x})$ and $\Pi(\underline{x})$ implies that B's preferred $\underline{x}$ is between $\underline{x}_a$ and $\underline{x}_c$.

Remark on Heterogeneity. Note that n is large when d is small, as when $d = c$. Nevertheless, it is easy to check that we always have $n(\underline{x}_c) < 1$ if $\Delta_q > mq\left(\frac{2(\alpha-1)}{2-\alpha}\right)$. If this inequality fails, it is still true that $\underline{x}_a \geq \underline{x}_b \geq \underline{x}_c$, but these inequalities can then bind. In particular, if $\Delta_q < mq\left(\frac{2(\alpha-1)}{2-\alpha}\right)$, the heterogeneity between the firms is so small that a mass $n(\underline{x}_c) = 1$ of firms prefer to satisfy the requirement when the requirement $\underline{x}_c$ maximizes $\Pi(\underline{x})$. A further reduction in $\underline{x}$ would reduce aggregate profit, however, since then a larger number of certified products would eliminate the scarcity of certified products and thus profits would decline. If Δ_q is reduced further, then, eventually, $n(\underline{x}_b) = 1$, and, therefore, $\underline{x}_b = \underline{x}_c$. Finally, if $\Delta_q < mq\left(\frac{\alpha-1}{2-\alpha}\right)$, even $\underline{x}_a$ implies $n(\underline{x}_a) = 1$. In this case, $\underline{x}_a = \underline{x}_b = \underline{x}_c$.

2.4 The Optimal Certifier

While A emphasizes the quality and C the profit, B maximizes a weighted sum of the two. If B represents a benevolent planner, the socially optimal certifier is a certifier D, associated with weight $d(b)$, so that B's objective function is maximized. In this model, $d(b) = b$, that is, B clearly prefers to certify directly. Doing that leads to the best outcome, from B's point of view, while any other choice would imply that the equilibrium $\underline{x}$ would be either too small or too high.

From now on, we let B's (strict) preference ordering over certifiers' identities be represented by the binary relation $\succeq$ ($\succ$). So, we have $B \succeq A$ if and only if $W_b(\underline{x}_b) \geq W_b(\underline{x}_a)$, for example.

Proposition 2 *The optimal certifier's identity is given by the function* $d(b) = b$. *Consequently,* $B \succeq A$ *and* $B \succeq C$.

Our basic model takes as given the firms' types, represented by q, and the market structure, represented by m. In the following two sections, we endogenize both q and m by allowing for corporate investments and the entry of firms. These possibilities can dramatically change the choice of certifier, we show.

3 Endogenous Cost of Quality

We now endogenize the firms' costs of improving the product quality. After all, corporations often make long-term investment decisions that influence their ability to provide quality in the future. Consider, for example, the installation of a new technology that is more environmentally friendly or the establishment of a supply chain in countries that are expected to treat employees better.

To model such investments in the simplest way, we now introduce the possibility that each firm may invest in order to reduce the future cost of providing quality. Each firm invests at the beginning of the game, *after* learning the identity of the certifier, but *before* the certifier has determined the requirement, $\underline{x}$. The firms' average choice of investment is publicly observed, and the rest of the game is as in the previous section.

Each firm i can invest any amount $y^i \in [0, q]$. The investment reduces the above cost parameter q to $q - y^i$, but the cost of y^i is $k\left(y^i\right)^2/2$.

To ensure that each firm's objective function is concave, we assume:

$$k > \frac{1}{\Delta_q q^{\frac{\alpha}{\alpha-1}}} \Longleftrightarrow \Delta_q > \frac{1}{k q^{\frac{\alpha}{\alpha-1}}}.$$

We assume that the possibility to invest arises with probability $\psi_q \in [0, 1]$, so that the model in the previous section is a special case with $\psi_q = 0$. One may interpret ψ_q as the development stage of the industry, so that ψ_q is large for emerging markets with plenty of unexploited investment opportunities.

3.1 Equilibrium Investments

In this Section, we consider the equilibrium level of investment in the presence of an arbitrary certifier, D. Each firm has rational expectations regarding the future quality requirement and how it will depend on all the firms' investments. However, because there is a continuum of firms, each of them finds it impossible to influence the future decision on $\underline{x}_d$.

For any given $\underline{x}_d$, a firm's expected profit is (in line with eq. (1)):

$$\frac{\underline{x}_d^{\alpha}}{2\Delta_q}\left(\underline{x}_d^{1-\alpha} - \left(q - y^i\right)\right)^2 - k\frac{\left(y^i\right)^2}{2}.$$

The first-order condition for y^i is satisfied when y^i equals

$$y_d \equiv \frac{\underline{x}_d - q\underline{x}_d^{\alpha}}{\Delta_q k - \underline{x}_d^{\alpha}}, \text{ where } \underline{x}_d = \underline{x}_d^{q}\left(y_d\right),$$

where we have written the equilibrium threshold, as it is given by (2), as a function, $\underline{x}_d^q\left(y_d\right)$, to emphasize that $\underline{x}_d$ will depend on the quality cost. The previous quality cost, q, is now replaced by $q - y_d$. The second-order condition is $\Delta_q > \underline{x}_d^{\alpha}/k$. When this inequality holds, all firms invest the same quantity, y_d, and this quantity depends on d. Given the equilibrium investments, the $\tilde{q}_i$'s will now be distributed uniformly and i.i.d. on the interval $[q - y_d, q - y_d + \Delta_q]$.

On the one hand, a larger investment leads to a higher standard for any given d. On the other hand, we show in the proof of the next proposition that the equilibrium is nevertheless unique and that a larger d leads to a larger y_d, and thus to a larger $\underline{x}_d$. This result implies that firms invest more when the certifier is A than when it is B, and when it is B than when it is C.

Proposition 3 *There exists a unique equilibrium outcome. In equilibrium, all firms invest the same amount, and the expected ex ante investment, $\mathbb{E}y_d$, is an increasing function of the expected $\underline{x}$, and of d. Consequently,*

$$\mathbb{E}y_a > \mathbb{E}y_b > \mathbb{E}y_c.$$

The proposition states that firms invest the most in their capacities to provide quality when the certifier is A, and the least when the certifier is C. This fact should be taken into account by B when it is socially efficient that firms invest.

3.2 The Optimal Certifier with Investments

For any given level of $\underline{x}$, the equilibrium investment is always smaller than the socially optimal investment level. The intuition for this claim is as follows. Because each firm invests to maximize profits, a marginally larger investment level has only a second-order effect on profits. However, a larger investment increases the mass of firms that will satisfy the quality requirement, and aggregate quality thus increases, as well. This is a first-order benefit, suggesting that social welfare increases with investment.

To motivate firms to invest, B may prefer to delegate certification authority to a certifier that places a larger weight on quality, since the equilibrium $\underline{x}$ will then be larger. The larger $\underline{x}$ encourages the firms to invest. Naturally, the benefit of increasing d so that it is greater than b is larger when it is more likely that the firms can invest (i.e., when ψ_q is larger). When the benefit (b) of quality is large, it is especially important to motivate firms to invest in their capacities to provide quality. Thus, a larger b makes it more attractive to delegate to a certifier with a large d. The next proposition formalizes these remarks.

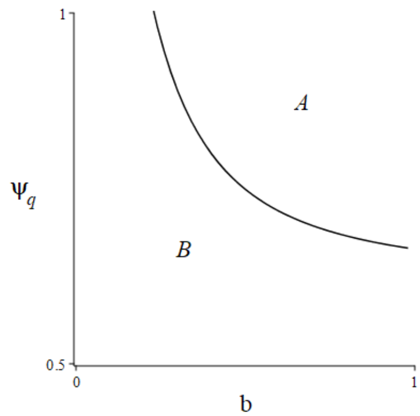

Figure 2: Optimal certifier with endogenous cost of quality

Proposition 4

(i) The optimal certifier is given by a function $d_q(\psi_q, b) > b$*, increasing in both arguments.*

(ii) Consider any pair $\underline{D}$ *and* $\overline{D}$ *associated with* $\overline{d} > \underline{d} \geq b$*. (E.g.,* $\underline{D} = B$ *and* $\overline{D} = A$*.) If* $\overline{D} \succeq \underline{D}$ *for some* (ψ_q, b)*, then* $\overline{D} \succ \underline{D}$ *for all* $(\psi'_q, b') > (\psi_q, b)$*.*[17]

Part (i) of the proposition is explained already. Part (ii) says that A is more likely to be the optimal certifier if investments are important and externalities are large. Figure 2 illustrates the parameter space under which B benefits from delegating the certifying responsibility to A.[18]

4 Endogenous Market Structure

While in the previous section we endogenized the firms' types, in this section we endogenize the market structure, as represented by the mass of firms. After all, although some firms may have entered the market years ago, the willingness of new firms to enter will hinge on the profit they expect.

[17]We follow the convention that the two vectors be ranked as $(\psi'_q, b') > (\psi_q, b)$ if and only if $\psi'_q \geq \psi_q$ and $b' \geq b$ with at least one strict inequality.

[18]For Figure 2, we set: $a = 1$, $\Delta_q = 3.5$, $\alpha = 1.3$, $k = 0.3$, and $q = 1$.

To capture this idea, we now allow for entry, *after* firms learn the identity of the certifier, but *before* the remaining game proceeds exactly as analyzed above. Suppose a mass $\underline{m}$ of firms are in the market already, while another mass $\Delta_m = \overline{m} - \underline{m}$ might enter.[19] Each firm faces an entry cost e_i, distributed uniformly and i.i.d. with density σ_m on interval $\left[\underline{e} - \frac{1}{2\sigma_m}, \underline{e} + \frac{1}{2\sigma_m}\right]$. Each firm compares the entry cost with the expected profit. In equilibrium, firm i enters the market if and only if the expected profit is weakly larger than e_i. After each firm has decided whether to enter, the timing of the remaining game is exactly as analyzed in the previous section.

We assume that the possibility to enter arises with probability $\psi_m \in [0, 1]$, so that the model in the previous section is a special case with $\psi_m = 0$. One may interpret ψ_m as the development stage of the industry, so that ψ_m is large for recent industries where many firms are yet to enter.

4.1 Equilibrium Entry

As the analysis above has shown, the equilibrium level of m does not influence any firm's investment decision or the certifier's choice of a quality threshold. The expected profit for each firm takes into account the option of investing later. This profit will be independent of m as long as $n(\underline{x}) < 1$. Consequently, depending on the expected quality requirement, which, in turn, depends on the certifier's characteristic d, there will be a threshold $e(d)$ such that firm i enters if and only if $e_i \leq e(d)$. The expected mass of firms will be given by

$$\mathbb{E}m_d = \underline{m} + \Delta_m \psi_m Pr\left(e_i \leq e(d)\right).$$

When certifier C sets $\underline{x}_c$ such as to maximize (ex post) profit, it is intuitive that entry is larger if the certifier is C rather than A, or, more generally, if $d \in [0, 1]$ is small.

Proposition 5 *The expected equilibrium mass of firms, $\mathbb{E}m_d$, is a decreasing function of the expected $\underline{x}$, and thus of d. Consequently,*

$$\mathbb{E}m_a \leq \mathbb{E}m_b \leq \mathbb{E}m_c,$$

with strict inequalities as long as $\underline{m} < \mathbb{E}m_b < \overline{m}$.

[19] In line with our previous assumption, we assume $\Delta_q > \overline{m}q\left(\frac{2(\alpha-1)}{2-\alpha}\right)$.

In order to focus on the most interesting parameter region, we assume in the following that:

$$\underline{m} < \mathbb{E}m_a < \mathbb{E}m_b < \mathbb{E}m_c < \overline{m}. \tag{3}$$

This assumption holds if the firm facing the smallest possible entry cost always prefers to enter, while the firm facing the highest possible entry cost will never enter, regardless of the identity of the certifier.

4.2 The Optimal Certifier with Entry and Investments

Entry is valuable to everyone in our model. If more firms have entered the market, there is a larger mass of firms that may find it affordable to satisfy any quality requirement. When a larger mass of goods are certified, the total quality and the total profit increase. Thus, every stakeholder in the model has an incentive to encourage entry.[20]

For potential entrants, entry is attractive when the expected profit is large, and profits will be large when the certifier places a small weight d on quality relative to profits. Thus, it can be beneficial to delegate certification authority to C, placing a small weight on quality, if entry is very important. Encouraging entry is indeed important when σ_m is large, for example, since then a small change in $\underline{x}$ (or, equivalently, in d) dramatically influences the mass of firms. Thus, the larger σ_m is, the smaller the optimal d is.

If σ_m is instead small, so that the variance in the entry cost is large, then it will mainly be the realization of the random entry cost that determines whether firms enter, and the effect of d will be minor. In this case, it will be relatively more important to motivate firms to invest, and a larger d will therefore be optimal. In this case, $D = A$ can be preferred to B and C.

A larger Δ_m reduces the optimal d for similar reasons. However, if b is large, B's weight on quality is larger, which makes it optimal for B to delegate strategically by increasing d, since a larger d raises the equilibrium $\underline{x}$ and thus firms' quality investments. Proposition 6 formalizes these remarks.[21]

[20] This preference may change if entry itself is associated with negative externalities, for example because firms that fail to sell to the consumers in this market can continue to produce by exporting their low-quality products to other markets. We discuss this possibility in Appendix A.

[21] The proof of the following result assumes that B does not internalize the firms' entry costs when ranking certifiers. However, the result would continue to hold (although the proof would have to be modified) also if B does internalize these costs.

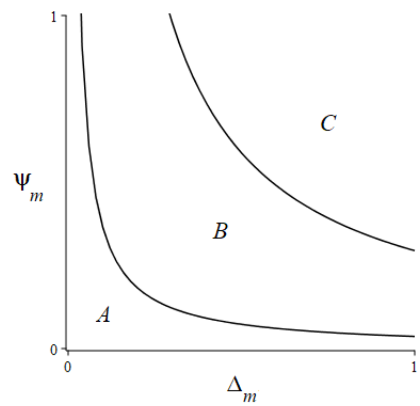

Figure 3: Optimal certifier with endogenous market structure

Proposition 6

(i) The optimal certifier is given by a function $d_m(\sigma_m, \Delta_m, \psi_m, -b)$, decreasing in all its arguments.

(ii) Consider any pair $\underline{D}$ and $\overline{D}$ characterized by $\underline{d}$ and $\overline{d} > \underline{d}$. If $\underline{D} \succeq \overline{D}$ for some $(\sigma_m, \Delta_m, \psi_m, -b)$, then $\underline{D} \succ \overline{D}$ for all $(\sigma'_m, \Delta'_m, \psi'_m, -b') > (\sigma_m, \Delta_m, \psi_m, -b)$.

Part (ii) implies that B is less likely to prefer that A certifies, and more likely to prefer that C certifies, if entry is of significant importance, while the benefit of quality is limited. In line with this result, Figure 3 illustrates the parameter spaces under which it is better to delegate certification authority to A, B, and C, respectively.[22]

With all this, our theory can explain when governments prefer to delegate certification authority to private certifiers emphasizing profits, as a way to encourage entry, or to NGOs that instead focus on quality, as a way to motivate firms to invest in quality. Although the results can be interpreted normatively, they also provide testable empirical predictions in cases when governments can influence the identity of the certifier. In the next section, we discuss a dynamic interpretation of this prediction before we endogenize the decision to establish oneself as a certifier.

[22]For Figure 3, we set: $a = 1$, $b = 0.5$, $c = 0$, $\Delta_q = 3.5$, $\alpha = 1.3$, $k = 0.3$, $q = 1$, $\psi_q = 1$, $m = 1.1$, $\underline{e} = 0.0055$, and $\sigma_m = 434$.

5 Endogenous Certifier Identity

5.1 The Equilibrium Certifier

If B delegates in order to maximize B's objective function, then the results above are not only normative but are also positive predictions for the delegation of certifier responsibility, and for how the certifier's identity will systematically vary across industries and over time.

In reality, the government may be unable to dictate the identity of the future certifier. Establishing oneself as a product certifier is costly, for example because of expensive monitoring technology. Hence, certifiers must pay an entry cost, just as firms had to pay a cost to enter the market in the previous section. To follow this line of thought, we investigate in this section how the various stakeholders' incentives to enter as a certifier vary across industries and with the industry's stage of development. Interestingly, we will show that the certifier that enters first in equilibrium can be the same as the certifier that is socially optimal: Qualitatively, our predictions are the same regardless of whether B appoints the certifier (as above) or whether we consider the equilibrium with entry (as below).

To investigate the incentive to certify, note that if there is no certifier, then firms provide zero quality and profits are zero. Therefore, any certifier D, characterized by $d \in [0, 1]$, receives some positive payoff, denoted V_d, for being present in the market as a certifier. The cost of establishing oneself as a certifier is, say, $\kappa + \kappa_d$. We permit this cost to consist of a common component, κ, and a component that varies with d, κ_d.

Consider, for instance, two potential certifiers, $\underline{D}$ and $\overline{D}$, characterized by parameters $\underline{d}$ and $\underline{d} < \overline{d}$, respectively. (For example, $\overline{D} = A$ and $\underline{D} = C$.) If the common cost κ is very high, then no one enters as a certifier. If κ is gradually reduced, then, sooner or later, one of the stakeholders will find it optimal to enter. For example, if $V_{\underline{d}} - \kappa_{\underline{d}} < \kappa < V_{\overline{d}} - \kappa_{\overline{d}}$, then $\overline{D}$ is willing to enter but $\underline{D}$ is not. We will say that $\overline{D}$ "can certify first" if $V_{\overline{d}} - \kappa_{\overline{d}} \geq V_{\underline{d}} - \kappa_{\underline{d}}$ and that $\overline{D}$ "certifies first" if $V_{\overline{d}} - \kappa_{\overline{d}} > V_{\underline{d}} - \kappa_{\underline{d}}$.

Note that every other stakeholder benefits when one stakeholder certifies. Thus, the benefit for one stakeholder to enter and out-compete an existing certifier is smaller than the benefit from entering if there is no other certifier in the market already. Once $\overline{D}$ enters, $\underline{D}$ is no longer willing to enter when $\kappa = V_{\underline{d}} - \kappa_{\underline{d}}$, but is willing only when κ is below an even lower threshold. This discouragement effect motivates us to limit attention to the situation with a single certifier when the entry cost is high.

The value V_d shifts if the other parameters of the model are adjusted, and this shift in V_d varies systematically with d. For example, and in contrast to the analysis we have provided so far, the spillover s (described in Section 2.1) can now be important because a larger s increases the importance of quality. It is thus intuitive that a larger s increases A's but not C's willingness to establish itself as the certifier.

To be consistent with Section 4, the proof of the following result assumes that when a potential certifier considers whether to enter, it does not internalize the producing firms' subsequent entry cost. This assumption describes private certifiers such as industry certifiers, NGOs, and hybrid ones. To the extent that private certifiers care about firms' costs, they should not care about costs incurred by firm that are still outside the industry.

Proposition 7 *Consider any pair of potential certifiers, $\underline{D}$ and $\overline{D}$, characterized by quality weights $\underline{d}$ and $\overline{d} > \underline{d}$, respectively.*

(i) Consider the situation with a fixed market structure. If $\overline{D}$ can certify first for some (ψ_q, s), then $\overline{D}$ certifies first for all $\left(\psi'_q, s'\right) > (\psi_q, s)$.

(ii) Consider the situation in which additional firms can enter. If $\underline{D}$ can certify first for some $(\sigma_m, \Delta_m, \psi_m, -s)$, then $\underline{D}$ certifies first for all $(\sigma'_m, \Delta'_m, \psi'_m, -s') > (\sigma_m, \Delta_m, \psi_m, -s)$.

Part (i) implies that, if the market structure (m) is fixed, then A is more likely to certify first if firms' are more likely to invest, or if the spillover associated with quality is large. C is less likely to certify first in these circumstances. This implies, in turn, that if A can certify first in the basic model without investments, then A is definitely certifying first in the model with investments (but without entry).

Part (ii) similarly implies that when the market structure is endogenous, then C is the most likely stakeholder to certify first if it is likely that several new firms can enter, or if the spillover is small. This implies, in turn, that if C can certify first in the model without entry, then C is definitely certifying first if entry is possible. Similarly, if A can certify first in the model with entry, then A is definitely certifying first when no further entry can be expected.

These predictions are qualitatively consistent with the normative results above, and with the predictions we derived when the certifier was directly appointed by B. With this consistency, our analysis provides robust testable predictions for how the certifier's identity varies across markets and over time, and for how this identity influences the entry of new firms, the firms' investments in quality, the certification requirement, and the number of firms that end up meeting this requirement.

5.2 The Choice of Certifier Over Time

The analysis above suggests the following development of optimal and equilibrium certification. In a new market, when it is still possible and likely that new firms can enter (in that ψ_m is large), it is socially desirable that certification be delegated to a stakeholder that places a large weight on profits. At this stage, an industry association may itself be the best provider of certifications. The industry association is also most willing to pay any fixed operational cost, according to Proposition 7. At a later stage, the market structure might be developed (and the probability for further entry, ψ_m, might be small), but firms may still take actions that enhance their capacities to provide quality (in that ψ_q is large). At this stage, it would be socially optimal that the certifier placed a large weight on quality and, therefore, it would be optimal that an NGO will be responsible for certifications. (This will also be the equilibrium if the "fixed cost", discussed in the previous subsection, can be reclaimed once a certifier ends its business.) Finally, when further investments seem less likely (in that ψ_m is small), the government prefers to take control of defining the certification requirement.[23]

This development points to an underlying time inconsistency problem when it comes to certification. While Figure 3 and Proposition 6 describe how it is optimal to delegate authority for the long run, in order to motivate both entry and investments, the firms may anticipate that the government will later prefer to delegate certification to another stakeholder. In that case, it is simply not sufficient to let an industry association certify today, when the firms expect that the threshold will be more demanding (and profits lower) in the future. Similarly, for large long-term investments to be attractive, it is not sufficient to let an NGO certify today, if firms expect that governments

[23]If the governments learn about the externality over time, however, the dynamics can be quite different: Then, the government may prefer to delegate to an NGO as soon as one learns about the externality (f.ex, an environmental problem) and the possibilities to reduce the harm by making additional investments. Analogously, the government may want to switch to industry certifications if one learns about the possibilities (or benefits) of additional entry.

will later take control and relax the quality requirement. In other words, entry or investments can be negatively affected if the government is unable to commit today to the type of certifier in the future.

6 Conclusion

To sustain something else than markets for lemons, it is essential that the buyer be able to trust that the seller's product quality is high. This paper provides an analysis of the role of the certifier. To focus on the most important long-term consequences, we endogenize the firms' ability to provide quality, the market structure, and also the certifier's identity. The assumptions of the model are motivated by empirical observations, as discussed in the Introduction. Thus, the resulting normative recommendations and our positive predictions deserve careful scrutiny as well.

As discussed in the Introduction, the certifier's identity varies systematically across industries. Table 1 shows that NGOs are active as certifiers of food (organic food and fair-trade products), natural resources (forests and fisheries), and energy efficiency. The qualities of such products are associated with important environmental externalities. Moreover, it is important that firms make long-term investments to be able to provide high-quality products. When externalities and investments are important, the optimal certifier, as well as the equilibrium certifier, is an NGO, according to our predictions: See Propositions 4, 6, and 7. These predictions match well with the evidence in Table 1.

For the more recent online-selling-platform industry, the certifier is instead associated with the industry itself, as summarized in Table 1. Since this industry is new and, to a large extent, still emerging, the market structure is still evolving and entry remains important. According to our theory, the optimal certifier, as well as the equilibrium certifier, is then an industry association: See Propositions 6 and 7, and the illustration in Figure 3.

A serious empirical investigation is naturally beyond the scope of the present paper. However, our theoretical predictions, combined with the anecdotal evidence, suggest that such empirical research will be important for our understanding of the certifier.

Theoretically, our model is stylized and tractable for pedagogical reasons, and also because such a workhorse model can then be used as a vehicle for future generalizations. Appendix A shows how our workhorse model can be extended to permit certification fees, negative externalities of entry, and nontrivial consumer surplus.[24] Other generalizations should be explored in future research. While we have limited attention to binary licenses, motivated by the empirical regularity of this type of licensing, future research should allow for a ladder of quality thresholds.[25] While we have limited attention to the first certifier that establishes itself when the certifiers' entry costs decline, future research should explore the equilibrium that emerges when the entry costs are sufficiently low to permit multiple certifiers to be active simultaneously.[26] Some of these aspects are already discussed in research that is complementary to ours, as discussed in the above literature review, but future research should combine the complementary approaches in order to arrive at a deeper understanding of certification. After all, credible communication between sellers and buyers is crucial for markets for products other than lemons.

[24]Lizzeri (1999) and Faure-Grimaud *et al.* (2009), among others, treat the price of certificates as a strategic variable. Matthews and Postlewaite (1985) shows that informative certification need not be desirable for consumers.

[25]For a discussion of multi-tiered certification see, for example, Farhi *et al.* (2013).

[26]The presence of multiple certifiers, in turn, raises novel issues. For instance, a proliferation of certificates can generate confusion among consumers. See Heyes *et al.* (2020) for the first model in which consumers have to exert effort in order to learn the exact meaning of each label.

Appendix A: Fees, Negative Externalities, and Consumer Surplus

Our workhorse model is designed to be stylized and simple so that it can then be built on in several directions. Above, we showed how the basic model can permit both entry and long-term investments in the capacities to provide quality. In this appendix, we discuss a number of other possible extensions.

A.1 Private Certifiers and Certification Fees

If firms must pay some (endogenous) fee f to be certified, firm i is willing to pay at most $\underline{x} - (q + i\Delta_q/m)\,\underline{x}^\alpha$, implying that the total number of firms willing to pay f is

$$m\frac{\underline{x} - q\underline{x}^\alpha - f}{\underline{x}^\alpha \Delta_q},$$

and the total revenue from collecting the fees is

$$fm\frac{\underline{x} - q\underline{x}^\alpha - f}{\underline{x}^\alpha \Delta_q}.$$

With this, it is easy to take first-order conditions and derive the revenue-maximizing f and $\underline{x}$, and then to compare this $\underline{x}$ with the $\underline{x}$'s derived in Section 2.3. For a given f, the revenue-maximizing $\underline{x}$ is

$$(1-\alpha)\,\underline{x}^{-\alpha} + \alpha f \underline{x}^{-\alpha-1} = 0 \Rightarrow \underline{x} = \frac{f\alpha}{\alpha - 1},$$

if we assume that the following second-order condition holds:

$$-\alpha\,(1-\alpha)\,\underline{x}^{-\alpha-1} - \alpha(1+\alpha)f\underline{x}^{-\alpha-2} < 0 \Rightarrow (1+\alpha)f\underline{x}^{-1} > \alpha - 1 \Rightarrow \underline{x} < \frac{(1+\alpha)f}{\alpha - 1},$$

which it clearly does locally when $\underline{x} = \frac{f\alpha}{\alpha-1}$. The first-order condition is then sufficient.

For a given $\underline{x}$, the revenue-maximizing f is

$$f = \frac{\underline{x} - q\underline{x}^\alpha}{2}.$$

When we combine the two conditions, we find that the revenue-maximizing $\underline{x}$ is equal to $\underline{x}_0$, derived above. When $c = 0$, we had

$$\underline{x}_c = \frac{(\underline{x}_c - q\underline{x}_c^\alpha)\alpha}{2(\alpha - 1)} = \left(\frac{2-\alpha}{q\alpha}\right)^{\frac{1}{\alpha-1}}.$$

Consequently, allowing the industry to set $\underline{x}$ is equivalent to letting a private revenue-maximizing certifier set $\underline{x}$, in our model. It is intuitive that a revenue-maximizing certifier, who captures the firms' profits, also prefers to set $\underline{x}$ so as to maximize these profits.

Here, the revenue-maximizing $\underline{x}_0$ trades off the benefit of larger revenue per licensed firm with the fact that a smaller mass of firms seek certifications when $\underline{x}_0$ is large. Thus, this $\underline{x}_0$ is smaller than the quality requirement that would have maximized f alone, which would have been $\underline{x} = \left(\frac{1}{q\alpha}\right)^{\frac{1}{\alpha-1}}$. In fact, $\underline{x} = \left(\frac{1}{q\alpha}\right)^{\frac{1}{\alpha-1}}$ is also larger than $\underline{x}_a$, it is easy to see. This comparison suggests that $f_a > f_b > f_c$, when a certifier associated with $d \in \{a, b, c\}$ sets the fee f_d. However, A and B also care about the total quality that is provided, and the total number of firms seeking certification is smaller when f is large. Thus, for any given $\underline{x}$, both A and B would have preferred a smaller fee than the revenue-maximizing fee, as is described above.

When this effect is taken into account, it is unclear whether A and B prefer a fee that is higher or lower than the fee preferred by C.

What does seem to be clear, however, is that when the certifier's weight d on quality is larger, $\underline{x}$ is larger and $n(\underline{x})$ is smaller, just as in our analysis above.

Suppose that also other certifiers (such as A and B) collect fees and place some weight on these revenues. When this weight is larger, A and B will be more similar to C and the preferred quality requirement will be smaller. If B, but not A, places a larger weight on collecting the fees, B should be more likely to prefer C to be the certifier instead of A. If A, but not B, places a larger weight on collecting fees, A becomes more similar to C (and thus to B) when setting the quality requirement, and B should be more likely to prefer A to be the certifier instead of C, everything else equal.

A.2 Negative Externalities

In our model, a larger weight on quality was justified if high quality had a social value beyond what individual purchasers could be willing to pay for. This situation is reasonable for many types of goods, such as when the good's characteristics or production process determines environmental impacts or labor conditions. As explained already, positive externalities associated with consuming high-quality products is equivalent to negative externalities associated with consuming low-quality products, because we are assuming that every consumer purchases exactly one good. Under this assumption, it also seems reasonable to ignore any externalities associated with entry itself, since many firms do not produce in the equilibrium we have described.

More realistically, however, demand is not as inelastic as we have assumed, and firms that fail to sell in our market may have the opportunity to export their products to other markets and still generate externalities. In this situation, entry itself generates externalities, and it matters a great deal if these externalities are negative or positive. If they are negative, because production leads to emissions, for example, then it is no longer clear that it is socially desirable to encourage entry.

It would be simple to extend our model to this situation. If the externalities were negative and significant, it would be optimal to discourage entry, and this effect is achieved by letting the certifier place higher weights on quality. These externalities are thus having the same impact as the externalities that are already in our model: everything else equal, such externalities would raise the weight the optimal certifier places on quality. When the externalities are large, the government would be more likely to prefer to delegate certification authority to an NGO than to the industry itself.

A.3 Consumer Surplus

It is straightforward to extend our model to make consumer surplus relevant. Above, consumer surplus was constant (and thus irrelevant) whenever the number of certified firms was smaller than the number of consumers, and this condition, we showed, holds in the model when the certifier does not care about consumer surplus itself.

That said, it is easy to see that the consumer surplus could be larger if one reduced the quality requirement so much that the mass of certified firms becomes larger than the mass of consumers. A certifier that places a large weight on consumer surplus may thus be inclined to lower the quality requirement as a way to intensity competition among certified firms. This effect implies that such a certifier's objective function may not be single-peaked in the quality requirement, and we thus have to compare the values at different local peaks to determine the equilibrium threshold. While this exercise will generate a few additional interesting results, the intuition for the above qualitative findings, as described by Propositions 1–6, seems quite robust, in our view.

Appendix B: Proofs

Proof of Proposition 1. We first derive the expression for $\underline{x}_d$. As discussed in Section 2.2, $\underline{x}_d$ maximizes $W_d(\underline{x}) \equiv dU(\underline{x})+(1-d)\Pi(\underline{x})$. By the arguments in Section 2.2, for $\underline{x} \geq q^{\frac{1}{1-\alpha}}$, no firm gets a certificate ($n(\underline{x}) = 0$), hence $W_d(\underline{x}) = 0$. For $\underline{x} \leq \left(\frac{\Delta_q}{m} + q\right)^{\frac{1}{1-\alpha}}$, a mass 1 of firms gets a certificate ($n(\underline{x}) = 1$). In this case, $W_d(\underline{x}) \leq \underline{x} - (q + \frac{\Delta_q}{2m})\underline{x}^\alpha$.[27] Assumption $\Delta_q > \frac{2(\alpha-1)}{2-\alpha}mq$ implies that $\underline{x} - (q + \frac{\Delta_q}{2m})\underline{x}^\alpha$ is an increasing function of $\underline{x}$, as long as $\underline{x} < \left(\frac{\Delta_q}{m} + q\right)^{\frac{1}{1-\alpha}}$. If $\underline{x} \in \left(\left(\frac{\Delta_q}{m} + q\right)^{\frac{1}{1-\alpha}}, q^{\frac{1}{1-\alpha}}\right)$, then $n(\underline{x}) \in (0, 1)$, and

$$W_d(\underline{x}) = (1 + d)\underline{x}^{2-\alpha} - 2q\underline{x} + q^2(1 - d)\underline{x}^2.$$

The first-order condition with respect to $\underline{x}$ for this function corresponds to

$$(2 - \alpha)(1 + d)\underline{x}^{2(1-\alpha)} - 2q\underline{x}^{1-\alpha} + (1 - d)\alpha q^2 = 0.$$

This condition holds if and only if $\underline{x} \in \left\{\left(\frac{\chi_1(d)}{q}\right)^{\frac{1}{\alpha-1}}, \left(\frac{\chi_2(d)}{q}\right)^{\frac{1}{\alpha-1}}\right\}$, where

$$\chi_1(d) \equiv \frac{(2-\alpha)(1+d)}{1+\sqrt{1-\alpha(2-\alpha)(1-d^2)}}, \text{ and } \chi_2(d) \equiv \frac{(2-\alpha)(1+d)}{1-\sqrt{1-\alpha(2-\alpha)(1-d^2)}}.$$

The second-order condition corresponds to

$$\left(\frac{(2-\alpha)(1+d)}{q}\right)^{\frac{1}{\alpha-1}} > \underline{x}.$$

The second-order condition is satisfied if $\underline{x} = \left(\frac{\chi_1(d)}{q}\right)^{\frac{1}{\alpha-1}}$, and is violated if $\underline{x} = \left(\frac{\chi_2(d)}{q}\right)^{\frac{1}{\alpha-1}}$. Assumption $\Delta_q > \frac{2(\alpha-1)}{2-\alpha}mq$ ensures that $\left(\frac{\chi_1(d)}{q}\right)^{\frac{1}{\alpha-1}} > \left(\frac{\Delta_q}{m} + q\right)^{\frac{1}{1-\alpha}}$. Straightforward algebra ensures that $\left(\frac{\chi_1(d)}{q}\right)^{\frac{1}{\alpha-1}} < q^{\frac{1}{1-\alpha}}$. We are left with two candidate values for corner solutions: $\left(\frac{\Delta_q}{m} + q\right)^{\frac{1}{1-\alpha}}$, and

[27] $W_d(\underline{x}) = \underline{x} - (q + \frac{\Delta_q}{2m})\underline{x}^\alpha$ if firms with cost $\tilde{q}_i \in [q, q + \frac{\Delta_q}{m}]$ set $x_i = \underline{x}$, and all other firms set $x_i = 0$. If instead a different subset of firms (with mass 1) set $x_i = \underline{x}$, then $W_d(\underline{x}) < \underline{x} - (q + \frac{\Delta_q}{2m})\underline{x}^\alpha$.

$q^{\frac{1}{1-\alpha}}$. As $\underline{x} - (q + \frac{\Delta_q}{2m})\underline{x}^{\alpha} = (1+d)\underline{x}^{2-\alpha} - 2q\underline{x} + q^2(1-d)\underline{x}^2$ for $\underline{x} = \left(\frac{\Delta_q}{m} + q\right)^{\frac{1}{1-\alpha}}$, we can rule out the first candidate corner solution. Note also that $W_d\left(q^{\frac{1}{1-\alpha}}\right) = 0 < W_d\left(\left(\frac{\chi_1(d)}{q}\right)^{\frac{1}{\alpha-1}}\right)$, hence we can rule out the second candidate as well. We conclude that $\underline{x}_d = \left(\frac{\chi_1(d)}{q}\right)^{\frac{1}{\alpha-1}}$.

We show next that $\frac{d\underline{x}_d}{d(d)} > 0$:

$$\frac{d\underline{x}_d}{d(d)} > 0 \Leftrightarrow \chi_1'(d) > 0 \Leftrightarrow$$
$$\sqrt{1-\alpha(2-\alpha)(1-d^2)} > \alpha(2-\alpha)(1+d) - 1. \tag{4}$$

If $\alpha(2-\alpha)(1+d)-1 < 0$, then (4) holds. Suppose instead that $\alpha(2-\alpha)(1+d)-1 \geq 0$. Then (4) holds if $1-\alpha(2-\alpha)(1-d^2) > (\alpha(2-\alpha)(1+d)-1)^2$. A few steps of algebra are sufficient to verify that the last inequality holds. To conclude the proof of part (i) of the proposition, note that

$$n(\underline{x}_d) = \frac{m}{\Delta_q}\left(\frac{q}{\chi_1(d)} - q\right) =$$
$$\frac{mq}{\Delta_q}\left(\frac{1+\sqrt{1-\alpha(2-\alpha)(1-d^2)}}{(2-\alpha)(1+d)} - 1\right).$$

The rest of the proposition follows immediately. ■

Proof of Proposition 2. The proof is immediate. ■

In order to prove Proposition 3, we first establish four lemmata.

Lemma 1 *In the subgame in which investment is possible, there exists an equilibrium in which all firms invest the same amount.*

Proof. We construct an equilibrium in which all firms invest the same amount. Suppose firm i expects standard $\underline{x}$. If, following investment y^i, the firm anticipates raising quality to $\underline{x}$ with some probability in the interval

$(0,1)$ (and expects a mass of firms not larger than 1 to be doing the same, so that $p(\underline{x}) = \underline{x}$), then firm i's expected profits are

$$\pi^q(\underline{x}, y^i) \equiv \frac{\underline{x}^\alpha}{2\Delta_q} \left(\underline{x}^{1-\alpha} - q + y^i\right)^2 - \frac{k(y^i)^2}{2}.$$

Let $y^*(\underline{x})$ be defined, implicitly, by $\frac{\partial \pi^q(\underline{x},y)}{\partial y}|_{y=y^*(\underline{x})} = 0$. Hence, for any $\underline{x} \neq (\Delta_q k)^{\frac{1}{\alpha}}$:

$$y^*(\underline{x}) = \frac{\underline{x} - q\underline{x}^\alpha}{\Delta_q k - \underline{x}^\alpha}. \tag{5}$$

Following the notation introduced in Section 3.1, we denote with $\underline{x}_d^q(y)$ the standard set in equilibrium by certifier D, as long as all firms invest y. Note that $\Delta_q > mq\left(\frac{2(\alpha-1)}{2-\alpha}\right)$ implies $\Delta_q > m(q-y)\left(\frac{2(\alpha-1)}{2-\alpha}\right)$, for any $y \in [0, q]$. Hence, Proposition 1 implies that, for any $y \in [0, q)$:

$$\underline{x}_d^q(y) = \left(\frac{\chi_1(d)}{q-y}\right)^{\frac{1}{\alpha-1}}, \tag{6}$$

where $\chi_1(\cdot)$ is defined in the proof of Proposition 1. Equation (6) is equivalent to:

$$y = q - \chi_1(d)\left(\underline{x}_d^q(y)\right)^{1-\alpha}. \tag{7}$$

Replacing y with $y^*(\underline{x}_d^q(y))$, (7) becomes:

$$t(d, \underline{x}_d^q(y)) \equiv \Delta_q k(q - \chi_1(d)(\underline{x}_d^q(y))^{1-\alpha}) - (1-\chi_1(d))\underline{x}_d^q(y) = 0. \tag{8}$$

Next, we show that, for any $d \in [0,1]$, there exists a unique $\underline{x}_d^* \in \left(0, q^{\frac{1}{1-\alpha}}\right)$ such that $t(d, \underline{x}_d^*) = 0$. The following remarks will prove useful:
(i) for any $d \in [0,1]$, and any $\underline{x} > 0$, $\frac{d^2 t(d,\underline{x})}{d\underline{x}^2} < 0$;
(ii) for any $d \in [0,1]$, $\lim_{\underline{x}\to 0^+} t(d, \underline{x}) = -\infty$; and
(iii) for any $d \in [0,1]$, $t\left(d, q^{\frac{1}{1-\alpha}}\right) > 0$.
Remarks (i) and (ii) are immediate. Remark (iii) follows from assumption $k > \frac{1}{\Delta_q q^{\frac{\alpha}{\alpha-1}}}$, as shown:

$$k > \frac{1}{\Delta_q q^{\frac{\alpha}{\alpha-1}}} \Leftrightarrow (1-\chi_1(d))\left(k - \frac{1}{\Delta_q q^{\frac{\alpha}{\alpha-1}}}\right) > 0 \Leftrightarrow t\left(d, q^{\frac{1}{1-\alpha}}\right) > 0,$$

where the first equivalence holds as $\chi_1(d) < 1$, for any $d \in [0,1]$, and the second equivalence amounts to rearranging terms.

Remarks (i)-(iii) together imply that, for any $d \in [0,1]$, there exists a unique $\underline{x}_d^* \in \left(0, q^{\frac{1}{1-\alpha}}\right)$ such that $t(d, \underline{x}_d^*) = 0$.

We show that there exists an equilibrium in which every firm invests $y^*(\underline{x}_d^*)$. Note first that $y^*(\underline{x}_d^*) \in (0,q)$ (call this Remark (iv)).[28] Furthermore, Proposition 1 ensures that if every firm, except possibly firm i, invests $y^*(\underline{x}_d^*)$, then $n(\underline{x}_d^*) \in (0,1)$ (call this Remark (v)).

Next, we show that if firm i anticipates standard $\underline{x} \in (0, q^{\frac{1}{1-\alpha}})$, and anticipates a mass of firms not larger than 1 getting a certificate, then firm i's optimal choice of investment is $y^*(\underline{x})$ (call this Remark (vi)). Let

$$h(z, \underline{x}, y^i) \equiv z(\underline{x} - (q - y^i + \frac{\Delta_q}{2} z)\underline{x}^\alpha) - \frac{k(y^i)^2}{2}.$$

If firm i anticipates a standard $\underline{x} \in (0, q^{\frac{1}{1-\alpha}})$, and anticipates a mass of firms not larger than 1 getting a certificate, then firm i's profits equal:

$$\begin{cases} h(0, \underline{x}, y^i) & \text{if } \arg\max_{z \in [0,1]} h(z, \underline{x}, y^i) = 0; \\ \pi^q(\underline{x}, y^i) & \text{if } \arg\max_{z \in [0,1]} h(z, \underline{x}, y^i) \in (0,1); \\ h(1, \underline{x}, y^i) & \text{if } \arg\max_{z \in [0,1]} h(z, \underline{x}, y^i) = 1; \end{cases}$$

which is equivalent to:

$$\begin{cases} h(0, \underline{x}, y^i) & \text{if } y^i \leq q - \underline{x}^{1-\alpha}; \\ \pi^q(\underline{x}, y^i) & \text{if } y^i \in (q - \underline{x}^{1-\alpha}, q - \underline{x}^{1-\alpha} + \Delta_q); \\ h(1, \underline{x}, y^i) & \text{if } y^i \geq q - \underline{x}^{1-\alpha} + \Delta_q. \end{cases}$$

Note that $\frac{\partial^2 h(1,\underline{x},y^i)}{\partial (y^i)^2} = \frac{\partial^2 h(0,\underline{x},y^i)}{\partial (y^i)^2} = -k$. Furthermore, $h(0, \underline{x}, y^i) = \pi^q(\underline{x}, y^i)$ for $y^i = q - \underline{x}^{1-\alpha}$, and $h(1, \underline{x}, y^i) = \pi^q(\underline{x}, y^i)$ for $y^i = q - \underline{x}^{1-\alpha} + \Delta_q$. Hence, continuity of $\pi^q(\underline{x}, y^i)$ with respect to y^i implies that firm i's profits are a continuous function of y^i, for any $y^i \in [0,q]$.

Note that $\left(0, q^{\frac{1}{1-\alpha}}\right) \subset (0, (k\Delta_q)^{\frac{1}{\alpha}})$. Hence, $\frac{\partial^2 \pi^q(\underline{x},y^i)}{\partial (y^i)^2} = \frac{\underline{x}^\alpha}{\Delta_q} - k < 0$. As $\pi^q(\underline{x}, y^i) \geq \max\{h(1, \underline{x}, y^i), h(0, \underline{x}, y^i)\}$, for any $\underline{x} > 0$, and any $y_i \in [0,d]$, then firm i's profits are a strictly concave function of y^i, for any $y^i \in [0,q]$.

[28] Remark (iv) holds as $\underline{x}_d^* \in \left(0, q^{\frac{1}{1-\alpha}}\right)$, and $k > \frac{1}{\Delta_q q^{\frac{\alpha}{\alpha-1}}}$.

Remark (vi) follows. Remarks (iv)-(vi) establish the existence of an equilibrium in which all firms invest the same amount. ■

Lemma 2 *In the subgame in which investment is possible, there exists a unique equilibrium outcome.*

Proof. We show that the equilibrium outcome we described in the proof of Lemma 1 is the unique equilibrium outcome of the subgame in which investment is possible.

In equilibrium, if the mass of firms expected to get a certificate is larger than 1, then all firms invest 0. Yet, Proposition 1 ensures that, if all firms invest 0, then there is no equilibrium in which the mass of firms expected to get a certificate is larger than 1. We conclude that in any equilibrium the mass of firms expected to get a certificate is not larger than 1.

Let $\underline{x} \in \left(0, (k\Delta_q)^{\frac{1}{\alpha}}\right)$. Remark (vi) implies that the equilibrium outcome we have found is the unique equilibrium outcome in which $\underline{x} \in \left(0, q^{\frac{1}{1-\alpha}}\right)$. Furthermore, if $\underline{x} \in \left[q^{\frac{1}{1-\alpha}}, (k\Delta_q)^{\frac{1}{\alpha}}\right)$, then $y^*(\underline{x}) < 0$. Hence, the optimal investment level is 0. As, for any $d \in [0,1]$, $\underline{x}_d^q(0) = \left(\frac{\chi_1(d)}{q}\right)^{\frac{1}{\alpha-1}} < q^{\frac{1}{1-\alpha}}$, we conclude that there exists no equilibrium in which $\underline{x} \in \left[q^{\frac{1}{1-\alpha}}, (k\Delta_q)^{\frac{1}{\alpha}}\right)$.

Consider now $\underline{x} = (k\Delta_q)^{\frac{1}{\alpha}}$. In this case, $\frac{\partial \pi^q(\underline{x}, y^i)}{\partial y^i} = k\left(x^{1-\alpha} - q\right) < 0$, for any $y^i \in [0, q]$. Hence, also in this case, the optimal investment level is 0. By an argument analogous to the one in the last paragraph, we conclude that there exists no equilibrium in which $\underline{x} = (k\Delta_q)^{\frac{1}{\alpha}}$.

Finally, let $\underline{x} > (k\Delta_q)^{\frac{1}{\alpha}}$. Then, $\frac{\partial^2 \pi^q(\underline{x}, y^i)}{\partial (y^i)^2} = \frac{\underline{x}^\alpha}{\Delta_q} - k > 0$. Hence, in equilibrium, either all firms invest 0, or a positive mass of firms make an investment sufficiently large to ensure that they get a certificate with probability 1. The first case can be ruled out as $\underline{x}_d^q(0) < (k\Delta_q)^{\frac{1}{\alpha}}$. The second case can be ruled out as follows. As $\frac{\partial^2 h(1, \underline{x}, y^i)}{\partial (y^i)^2} = -k < 0$, in equilibrium, all firms that make a positive investment must invest the same amount. In equilibrium, $\underline{x}$ should maximize the certifier's objective function, conditional on the firms that invested 0 not getting the certificate (we know that for $\underline{x} > (k\Delta_q)^{\frac{1}{\alpha}} > q^{\frac{1}{1-\alpha}}$, a firm that invests 0 never gets a certificate). Proposition 1, ensures that, for any $d \in [0,1]$, the optimal standard is such that firms that make a positive investment get a certificate with probability smaller than 1, thus yielding a contradiction. This observation implies that the equilibrium we have found is the unique one, and concludes the proof of the lemma. ■

Lemma 3 *For any $d \in [0,1]$, $\underline{x}_d^*$ is increasing in d.*

Proof. We know from the proof of Lemma 1 that $\frac{\partial t(d,\underline{x})}{\partial \underline{x}}|_{\underline{x}=\underline{x}_d^*} > 0$, for any $d \in [0,1]$. Hence, it is sufficient to show that $\frac{\partial t(d,\underline{x})}{\partial d} < 0$, for any $\underline{x} \in \left(0, q^{\frac{1}{1-\alpha}}\right)$, and any $d \in [0,1]$. This is the case as, for any $d \in [0,1]$,

$$\frac{\partial t(d,\underline{x})}{\partial d} < 0 \Leftrightarrow \left(\frac{\underline{x}}{\Delta_q k} - \underline{x}^{1-\alpha}\right)\chi_1'(d) < 0,$$

and the latter inequality holds as $\chi_1'(d) > 0$, and

$$\underline{x} < q^{\frac{1}{1-\alpha}} \Rightarrow \underline{x} < (\Delta_q k)^{1/\alpha} \Leftrightarrow \frac{\underline{x}}{\Delta_q k} - \underline{x}^{1-\alpha} < 0.$$

■

Lemma 4 *For any $d \in [0,1]$, $\frac{dy^*(\underline{x})}{d\underline{x}}|_{\underline{x}=\underline{x}_d^*} > 0$.*

Proof. Note that (5) implies:

$$\begin{aligned}\frac{dy^*(\underline{x})}{d\underline{x}} > 0 \Leftrightarrow \frac{\left(1 - q\alpha\underline{x}^{\alpha-1}\right)\left(\Delta_q k - \underline{x}^{\alpha}\right) + \alpha\underline{x}^{\alpha-1}\left(\underline{x} - q\underline{x}^{\alpha}\right)}{\left(\Delta_q k - \underline{x}^{\alpha}\right)^2} > 0 \Leftrightarrow \\ \left(1 - q\alpha\underline{x}^{\alpha-1}\right)\Delta_q k - \left(1 - q\alpha\underline{x}^{\alpha-1}\right)\underline{x}^{\alpha} + \alpha q^{\alpha}\left(1 - q\underline{x}^{\alpha-1}\right) > 0 \Leftrightarrow \\ f(\underline{x}) \equiv \left(\underline{x}^{1-\alpha} - q\alpha\right)\Delta_q k + (\alpha - 1)\,\underline{x} > 0.\end{aligned}$$

As $(\Delta_q k)^{\frac{1}{\alpha}} > q^{\frac{1}{1-\alpha}}$, then $f'(\underline{x}) = (\alpha - 1)(1 - \underline{x}^{-\alpha}\Delta_q k) < 0$, as long as $\underline{x} \in \left(0, q^{\frac{1}{1-\alpha}}\right)$. Thus, in light of Lemma 3, to prove this lemma it suffices to show that $f(\underline{x}_1^*) > 0$. If $\underline{x}_1^* \leq (\alpha q)^{\frac{1}{1-\alpha}}$, then it is immediate that $f(\underline{x}_1^*) > 0$. Suppose instead that $\underline{x}_1^* > (\alpha q)^{\frac{1}{1-\alpha}}$. Then:

$$f(\underline{x}_1^*) > 0 \Leftrightarrow \Delta_q k < \frac{(\alpha - 1)\,\underline{x}_1^*}{q\alpha - \left(\underline{x}_1^*\right)^{1-\alpha}}.$$

Note also that $t(1, \underline{x}_1^*) = 0$ (see the proof of Lemma 1 for the definition of $t(\cdot,\cdot)$). Condition $t(1, \underline{x}_1^*) = 0$ is equivalent to:

$$\Delta_q k = \frac{(\alpha - 1)\,\underline{x}_1^*}{q - (2 - \alpha)\left(\underline{x}_1^*\right)^{1-\alpha}}. \tag{9}$$

We now show that:

$$\frac{(\alpha-1)\,\underline{x}_1^*}{q-(2-\alpha)\left(\underline{x}_1^*\right)^{1-\alpha}}<\frac{(\alpha-1)\,\underline{x}_1^*}{q\alpha-\left(\underline{x}_1^*\right)^{1-\alpha}}.$$

This inequality is equivalent to:

$$\frac{q\alpha-\left(\underline{x}_1^*\right)^{1-\alpha}}{q-(2-\alpha)\left(\underline{x}_1^*\right)^{1-\alpha}}<1.$$

As (9) implies $q-(2-\alpha)\left(\underline{x}_1^*\right)^{1-\alpha}>0$, then

$$\begin{aligned}\frac{q\alpha-\left(\underline{x}_1^*\right)^{1-\alpha}}{q-(2-\alpha)\left(\underline{x}_1^*\right)^{1-\alpha}}&<1\Leftrightarrow\\ q\alpha-\left(\underline{x}_1^*\right)^{1-\alpha}<q-(2-\alpha)\left(\underline{x}_1^*\right)^{1-\alpha}&\Leftrightarrow\\ q^{\frac{1}{1-\alpha}}&>\underline{x}_1^*.\end{aligned}$$

As $q^{\frac{1}{1-\alpha}}>\underline{x}_d^*$, for any $d\in[0,1]$ (see the proof of Lemma 1), the lemma holds. ■

Proof of Proposition 3. The proposition follows from Lemmata 1–4, and the remark that there exists a unique equilibrium outcome of the subgame in which investment is not possible (see Proposition 1). ■

Proof of Proposition 4. Let

$$\begin{aligned}U^q(\underline{x},y)&\equiv\frac{m\underline{x}^{1-\alpha}}{\Delta_q}(\underline{x}-(q-y)\underline{x}^{\alpha}),\\ \Pi^q(\underline{x},y)&\equiv m\pi^q(\underline{x},y),\text{ and}\\ W_b^q(\underline{x},y)&\equiv bU^q(\underline{x},y)+(1-b)\left(\Pi^q(\underline{x},y)-\frac{ky^2}{2}\right),\end{aligned}$$

where $\pi^q(\cdot,\cdot)$ is defined in the proof of Lemma 1. If D is the certifier, then Certifier B's payoff equals:

$$\psi_q W_b^q(\underline{x}_d^*,y_d)+(1-\psi_q)W_b(\underline{x}_d). \tag{10}$$

It is easy to check that (10) is a continuous function of d. Hence, compactness of the interval $[0,1]$ ensures that a weight d that maximizes (10) exists.

It is also easy to check that it is (generically) the case that $W_b^q(\underline{x}_d^*, y_d) \neq (1-\psi_q)W_b(\underline{x}_d)$. Hence, it is generically true that the maximand is unique. This maximand is a function $d_q(\psi_q, b)$.
We show next that $d_q(\psi_q, b) > b$. First, we show that $\frac{\partial^2 \Pi^q(\underline{x},y)}{\partial \underline{x}^2}|_{\underline{x}=\underline{x}_d^*} < 0$, for any $y \in [0,q)$, and any $d \in [0,1]$:

$$\frac{\partial^2 \Pi^q(\underline{x},y)}{\partial \underline{x}^2}|_{\underline{x}=\underline{x}_d^*} < 0 \Leftrightarrow$$
$$\frac{\partial^2 \pi^q(\underline{x},y)}{\partial \underline{x}^2}|_{\underline{x}=\underline{x}_d^*} < 0 \Leftrightarrow$$
$$\left(\frac{\sqrt{2-\alpha}}{\sqrt{\alpha}(q-y)}\right)^{\frac{1}{\alpha-1}} > \underline{x}_d^* \Leftarrow$$
$$\left(\frac{\sqrt{2-\alpha}}{\sqrt{\alpha}(q-y)}\right)^{\frac{1}{\alpha-1}} > \underline{x}_1^* \Leftrightarrow$$
$$(\alpha-1)^2 > 0.$$

Note that the left arrow follows from Lemma 3.
Next, we show that $\frac{\partial^2 U^q(\underline{x},y)}{\partial \underline{x}^2}|_{\underline{x}=\underline{x}_d^*} < 0$, for any $y \in [0,q)$, and any $d \in [0,1]$:

$$\frac{\partial^2 U^q(\underline{x},y)}{\partial \underline{x}^2}|_{\underline{x}=\underline{x}_d^*} < 0 \Leftrightarrow (2-\alpha)(1-\alpha)(\underline{x}_d^*)^{-\alpha} < 0 \Leftrightarrow \alpha \in (1,2).$$

We conclude that $\frac{\partial^2 W_b^q(\underline{x},y)}{\partial \underline{x}^2}|_{\underline{x}=\underline{x}_d^*} < 0$, for any $d \in [0,1]$, and any $y \in [0,q)$. A marginal increase in d increases marginally both $\underline{x}_d^*$ (see Lemma 3), and y_d (see Lemma 4). As $\frac{d\underline{x}_d^*}{d(d)} > 0$, while, for any $y \in [0,q)$, $\frac{\partial^2 W_b^q(\underline{x},y)}{\partial \underline{x}^2}|_{\underline{x}=\underline{x}_d^*} < 0$, and $\frac{\partial W_b^q(\underline{x},y)}{\partial \underline{x}}|_{\underline{x}=\underline{x}_b^*} = 0$, then fixing $y \in [0,q)$, a marginal increase in $\underline{x}_d^*$ weakly increases $W_b^q(\underline{x}_d^*, y)$, as long as $d \leq b$. For any $d \in [0,1]$, holding $\underline{x}_d^*$ fixed, a marginal increase in y_d strictly increases $W_b^q(\underline{x}_d^*, y_d)$, as it leaves profits unchanged, while increasing the mass of firms that gets a certificate, thus increasing $U^q(\underline{x}_d^*, y_d)$. Setting $y = 0$, the arguments just presented imply that also $W_b(\underline{x}_d)$ is an increasing function of d, as long as $d \in [0,b)$ (and a decreasing function of d, as long as $d \in (b,1]$). Hence, $d_q(\psi_q, b) > b$.
Next, we prove part (ii) of the proposition. Standard arguments ensure that the rest of part (i) follows from part (ii).
Consider a pair $\underline{D}$ and $\overline{D}$ characterized by $\underline{d} \geq b$, and $\overline{d} > \underline{d}$, respectively. Assume $\overline{D} \succeq \underline{D}$, for some (ψ_q, b). As we just established that $W_b(\underline{x}_d)$ is an

increasing function of d, as long as $d \in [0, b)$ (and a decreasing function of d, as long as $d \in (b, 1]$), then $W_b(\underline{x}_{\underline{d}}) > W_b(\underline{x}_{\overline{d}})$. Hence, $\overline{D} \succeq \underline{D}$ implies $W_b^q(\underline{x}_{\underline{d}}, y_{\underline{d}}) > W_b^q(\underline{x}_{\overline{d}}, y_{\overline{d}})$. Thus, $\overline{D} \succ \underline{D}$ for all $(\psi'_q, b) > (\psi_q, b)$.
Next, we establish that $\overline{D} \succ \underline{D}$ for all $(\psi_q, b') > (\psi_q, b)$. First, we show that, for any $d \in [0, 1]$,

$$\frac{dU^q(\underline{x}_d^*, y_d)}{d(d)} = \frac{\partial U^q(\underline{x}_d^*, y)}{\partial y}|_{y=y_d} \frac{dy_d}{d(d)} + \frac{\partial U^q(\underline{x}, y_d)}{\partial \underline{x}}|_{\underline{x}=\underline{x}_d^*} \frac{d\underline{x}_d^*}{d(d)} > 0.$$

For any $d \in [0, 1]$, it is immediate to check that $\frac{\partial U^q(\underline{x}_d^*, y)}{\partial y}|_{y=y_d} > 0$, while Lemma 4 establishes that $\frac{dy_d}{d(d)} > 0$, and Lemma 3 establishes that $\frac{d\underline{x}_d^*}{d(d)} > 0$. Furthermore,

$$\frac{\partial U^q(\underline{x}, y_d)}{\partial \underline{x}}|_{\underline{x}=\underline{x}_d^*} > 0 \Leftrightarrow \underline{x}_d^* < \left(\frac{2-\alpha}{q - y_d}\right)^{\frac{1}{\alpha-1}},$$

for any $d \in [0, 1]$, thus $U^q(\underline{x}_{\overline{d}}^*, y_{\overline{d}}) > U^q(\underline{x}_{\underline{d}}^*, y_{\underline{d}})$. It is also easy to check that $U(\underline{x}_{\overline{d}}) > U(\underline{x}_{\underline{d}})$. Hence, either

$$\psi_q \left(\Pi^q(\underline{x}_{\overline{d}}^*, y_{\overline{d}}) - \frac{k\,(y_{\overline{d}})^2}{2}\right) + (1-\psi_q)\Pi(\underline{x}_{\overline{d}}) \geq$$
$$\psi_q \left(\Pi^q(\underline{x}_{\underline{d}}^*, y_{\underline{d}}) - \frac{k\,(y_{\underline{d}})^2}{2}\right) + (1-\psi_q)\Pi(\underline{x}_{\underline{d}}),$$

and therefore $\overline{D} \succ \underline{D}$ for all $(\psi_q, b') > (\psi_q, b)$, or, else, the last highlighted inequality is violated, in which case $\overline{D} \succ \underline{D}$ for all values of ψ_q. This concludes the proof of part (ii) of the proposition. ∎

Proof of Proposition 5. A firm's expected profits, net of the entry cost, equal

$$\mathbb{E}\pi^q(d) \equiv \psi_q \pi^q(\underline{x}_d^q(y_d), y_d) + (1-\psi_q)\pi^q(\underline{x}_d^q(0), 0),$$

where $\pi^q(\cdot, \cdot)$ is defined in the proof of Lemma 1. We show next that, for any $d \in (0, 1]$, and any $y \in [0, y_1]$, $\pi^q(\underline{x}_d^q(y), y)$ is a decreasing function of d. Note, first, that:

$$\frac{d\pi^q(\underline{x}_d^q(y), y)}{d(d)} = \frac{\partial \pi^q(\underline{x}, y)}{\partial \underline{x}}|_{\underline{x}=\underline{x}_d^q(y)} \frac{d\underline{x}_d^q(y)}{d(d)}.$$

Equation (6) ensures that, for any $d \in [0,1]$, and any $y \in [0,q)$, $\frac{d\underline{x}_d^q(y)}{d(d)} > 0 \Leftrightarrow \chi_1'(d) > 0$, where $\chi_1(d)$ is defined in the proof of Proposition 1. We showed in the proof of Proposition 1 that, indeed, $\chi_1'(d) > 0$ for any $d \in [0,1]$. Hence, we conclude that $\frac{d\underline{x}_d^q(y)}{d(d)} > 0$ for any $d \in [0,1]$, and any $y \in [0,q)$. Furthermore,

$$\frac{\partial \pi^q(\underline{x}, y)}{\partial \underline{x}}|_{\underline{x}=\underline{x}_d^q(y)} < 0 \Leftrightarrow$$
$$\alpha(\underline{x}_d^q(y))^{\alpha-1}((\underline{x}_d^q(y))^{1-\alpha} - q + y)^2 + 2(1-\alpha)((\underline{x}_d^q(y))^{1-\alpha} - q + y) < 0.$$

For any $y \in [0, y_1]$, we showed in the proof of Lemma 1 that $(\underline{x}_d^q(y))^{1-\alpha} - q > 0$, and therefore $(\underline{x}_d^q(y))^{1-\alpha} - q + y > 0$. Hence, for any $y \in [0, y_1]$, the last highlighted inequality is equivalent to:

$$\alpha(\underline{x}_d^q(y))^{\alpha-1}((\underline{x}_d^q(y))^{1-\alpha} - q + y) + 2(1-\alpha) < 0 \Leftrightarrow$$
$$\underline{x}_d^q(y) > \left(\frac{2-\alpha}{\alpha(q-y)}\right)^{\frac{1}{\alpha-1}}.$$

As $\underline{x}_0^q(y) = \left(\frac{2-\alpha}{\alpha(q-y)}\right)^{\frac{1}{\alpha-1}}$ (see the proof of Lemma 1), and $\underline{x}_d^q(y) > \underline{x}_0^q(y)$ for any $d \in (0,1]$ (see the proof of Lemma 3), then the last highlighted inequality holds, for any $d \in (0,1]$.
Finally, note that by the envelope theorem, $\frac{d\pi^q(\underline{x}_d^q(y),y)}{d(d)}|_{y=y_d} = \frac{d\pi^q(\underline{x}_d^q(y),y_d)}{d(d)}$. We have thus established that both $\pi^q(\underline{x}_d^q(y_d), y_d)$, and $\pi^q(\underline{x}_d^q(0), 0)$ are decreasing functions of d, hence $\mathbb{E}\pi^q$ is a decreasing function of d. The proposition follows. ■

Proof of Proposition 6. Let:

$$\mathbb{E}u^q(d) \equiv \frac{1}{m}\left(\psi_q U^q(\underline{x}_d^*, y_d) + (1-\psi_q)U(\underline{x}_d)\right),$$

where $U^q(\cdot,\cdot)$ is defined in the proof of Proposition 4, while $\underline{x}_d^*$ is defined in the proof of Lemma 1. In equilibrium, for any $d \in [0,1]$, $e(d) = \mathbb{E}\pi^q(d)$ (see the proof of Proposition 5 for a definition of $\mathbb{E}\pi^q(\cdot)$). Let:

$$w_b^q(d) \equiv b\mathbb{E}u^q(d) + (1-b)\mathbb{E}\pi^q(d).$$

Certifier B's objective function is thus:

$$W_b^m(d) \equiv \underline{m}w_b^q(d) + \psi_m\Delta_m\sigma_m \int_{\underline{e}-\frac{1}{2\sigma_m}}^{\mathbb{E}\pi^q(d)} \left(w_b^q(d) - (1-b)e\right) de =$$

$$\underline{m}w_b^q(d) +$$

$$\psi_m\Delta_m\sigma_m \left(\mathbb{E}\pi^q(d) - \underline{e} + \frac{1}{2\sigma_m}\right)\left(w_b^q(d) - (1-b)\frac{\mathbb{E}\pi^q(d) + \underline{e} - \frac{1}{2\sigma_m}}{2}\right).$$

It is easy to check that W_b^m is a continuous function. Hence, a weight $d \in [0,1]$ that maximizes (10) in the interval $[0,1]$ exists. Moreover it is (generically) the case that $\psi_m\sigma_m \left(\mathbb{E}\pi^q(d) - \underline{e} + \frac{1}{2\sigma_m}\right)\left(w_b^q(d) - (1-b)\frac{\mathbb{E}\pi^q(d)+\underline{e}-\frac{1}{2\sigma_m}}{2}\right) \neq w_b^q(d)$. Hence, it is generically true that the maximand is unique. This maximand is a function $d_m(\sigma_m, \Delta_m, \phi_m, -b)$. We prove here part (ii) of the proposition (the rest of part (i) is an immediate consequence of part (ii)). Consider a pair $\underline{D}$ and $\overline{D}$ characterized by $\underline{d}$, and $\overline{d} > \underline{d}$, respectively. Suppose $\underline{D} \succeq \overline{D}$ for some $(\sigma_m, \Delta_m, \psi_m, -b)$. Note that

$$W_b^m(\underline{d}) = \underline{m}w_b^q(\underline{d}) +$$

$$\psi_m\Delta_m\sigma_m \left[\left(\mathbb{E}\pi^q(\overline{d}) - \underline{e} + \frac{1}{2\sigma_m}\right)\left(w_b^q(\underline{d}) - (1-b)\frac{\mathbb{E}\pi^q(\overline{d}) + (\underline{e} - \frac{1}{2\sigma_m})}{2}\right) + \right.$$

$$\left.\left(\mathbb{E}\pi^q(\underline{d}) - \mathbb{E}\pi^q(\overline{d})\right)\left(w_b^q(\underline{d}) - (1-b)\frac{\mathbb{E}\pi^q(\underline{d}) + \mathbb{E}\pi^q(\overline{d})}{2}\right)\right].$$

Hence, if $w_b^q(\underline{d}) \geq w_b^q(\overline{d})$, then

$$W_b^m(\underline{d}) \geq \underline{m}w_b^q(\overline{d}) +$$

$$\psi_m\Delta_m\sigma_m \left[\left(\left(\mathbb{E}\pi^q(\overline{d}) - \underline{e}\right) + \frac{1}{2\sigma_m}\right)\left(w_b^q(\overline{d}) - (1-b)\frac{\mathbb{E}\pi^q(\overline{d}) + (\underline{e} - \frac{1}{2\sigma_m})}{2}\right) + \right.$$

$$\left.\left(\mathbb{E}\pi^q(\underline{d}) - \mathbb{E}\pi^q(\overline{d})\right)\left(w_b^q(\overline{d}) - (1-b)\frac{\mathbb{E}\pi^q(\underline{d}) + \mathbb{E}\pi^q(\overline{d})}{2}\right)\right] =$$

$$W_b^m(\overline{d}) + \psi_m\Delta_m\sigma_m \left(\mathbb{E}\pi^q(\underline{d}) - \mathbb{E}\pi^q(\overline{d})\right)\left(w_b^q(\overline{d}) - (1-b)\frac{\mathbb{E}\pi^q(\underline{d}) + \mathbb{E}\pi^q(\overline{d})}{2}\right) >$$

$$W_b^m(\overline{d}).$$

Thus, if $w_b^q(\underline{d}) \geq w_b^q(\overline{d})$, then $\underline{D} \succ \overline{D}$ for all values of σ_m, Δ_m, and ψ_m.

Suppose, instead, that $w_b^q(\underline{d}) < w_b^q(\overline{d})$. It is immediate to verify that, in this case, if $\underline{D} \succeq \overline{D}$ for some Δ_m and ψ_m, then $\underline{D} \succ \overline{D}$ for any $\Delta'_m > \Delta_m$, and for any $\psi'_m > \psi_m$. Note also that

$$\frac{dW_b^m(d)}{d\sigma_m} = \frac{1}{\sigma_m}\left(W_b^m(d) - \underline{m}w_b^q(d) - \frac{\psi_m \Delta_m b}{2}\mathbb{E}u^q(d)\right).$$

As $W_b^m(\underline{d}) \geq W_b^m(\overline{d})$, $w_b^q(\underline{d}) < w_b^q(\overline{d})$, and $\mathbb{E}u^q(\underline{d}) < \mathbb{E}u^q(\overline{d})$, we conclude that $\frac{dW_b^m(\overline{d})}{d\sigma_m} < \frac{dW_b^m(\underline{d})}{d\sigma_m}$. This, in turn, implies $\underline{D} \succ \overline{D}$, for any $\sigma'_m > \sigma_m$.
Next, we show that $\underline{D} \succ \overline{D}$ for any $b' < b$. First, note that:

$$W_b^m(d) = b\left(\underline{m} + \psi_m\Delta_m\left(\sigma_m\left(\mathbb{E}\pi^q(d) - \underline{e}\right) + \frac{1}{2}\right)\right)\mathbb{E}u^q(d) + $$
$$+ (1-b)\left(\underline{m}\mathbb{E}\pi^q(d) + \psi_m\Delta_m\sigma_m\left(\mathbb{E}\pi^q(d) - \underline{e} + \frac{1}{2\sigma_m}\right)^2\right).$$

Note that the the term multiplying $(1-b)$ is larger for $d = \underline{d}$ than for $d = \overline{d}$, that is:

$$\underline{m}\mathbb{E}\pi^q(\underline{d}) + \psi_m\Delta_m\sigma_m\left(\mathbb{E}\pi^q(\underline{d}) - \underline{e} + \frac{1}{2\sigma_m}\right)^2 >$$
$$\underline{m}\mathbb{E}\pi^q(\overline{d}) + \psi_m\Delta_m\sigma_m\left(\mathbb{E}\pi^q(\overline{d}) - \underline{e} + \frac{1}{2\sigma_m}\right)^2.$$

Hence, either $\underline{D} \succ \overline{D}$ for any b, or else $\underline{D} \succeq \overline{D}$ for some b implies $\underline{D} \succ \overline{D}$, for any $b' < b$. This concludes the proof of part (ii) of the proposition. ■

Proof of Proposition 7.
(i) Consider first the model without firm entry. Let:

$$v_d^q \equiv ds\mathbb{E}u^q(d) + (1-d)\mathbb{E}\pi^q(d),$$

where $\mathbb{E}u^q(\cdot)$ and $\mathbb{E}\pi^q(\cdot)$ are defined, respectively, in the proofs of Proposition 6 and Proposition 5. The payoff of certifier D from being in the market is mv_d^q.
Suppose $v_{\overline{d}}^q \geq v_{\underline{d}}^q$ for some (ψ_q, s). In the proof of Proposition 4, we showed that, for any $d \in [0,1]$, $U^q(\underline{x}_d^*, y_d)$ is increasing in both its arguments (we

defined $U^q(\cdot,\cdot)$ in the same proof, and $\underline{x}_d^*$ in the proof of Lemma 1), and $U(\underline{x}_d)$ is increasing in $\underline{x}_d$. Lemma 4 ensures that y_d is increasing in d, for any $d \in [0,1]$. Hence, $v^q_{\overline{d}} > v^q_{\underline{d}}$, for any $(\psi_q, s') > (\psi_q, s)$. Next, we prove that, for any $d \in [0,1]$, $\frac{d^2 \mathbb{E} v^q_d}{d(d) d\psi_q} > 0$, or, equivalently,

$$\frac{d}{d(d)}\left(dsU^q(\underline{x}_d^*, y_d) + (1-d)\left(\Pi^q(\underline{x}_d^*, y_d) - \frac{mk\,(y_d)^2}{2}\right)\right) > \frac{d}{d(d)}\left(dsU(\underline{x}_d) + (1-d)\Pi(\underline{x}_d)\right), \tag{11}$$

where $\Pi^q(\cdot,\cdot)$ is defined in the proof of Proposition 4. By the envelope theorem: $\frac{d}{d(d)}\left(dsU(\underline{x}_d) + (1-d)\Pi(\underline{x}_d)\right) = sU(\underline{x}_d) - \Pi(\underline{x}_d)$. As y_d maximizes firms' profits given $\underline{x}_d^*$, the envelope theorem also ensures:

$$\frac{d}{d(d)}\left(dsU^q(\underline{x}_d^*, y_d) + (1-d)\left(\Pi^q(\underline{x}_d^*, y_d) - \frac{mk\,(y_d)^2}{2}\right)\right) = sU^q(\underline{x}_d^*, y_d) - \Pi^q(\underline{x}_d^*, y_d) + \frac{mk\,(y_d)^2}{2} + ds\frac{\partial U^q(\underline{x}_d^*, y_d)}{\partial (y_d)}.$$

Hence, (11) is equivalent to:

$$sU^q(\underline{x}_d^*, y_d) - \Pi^q(\underline{x}_d^*, y_d) + \frac{mk\,(y_d)^2}{2} + ds\frac{\partial U^q(\underline{x}_d^*, y_d)}{\partial y_d} > sU(\underline{x}_d) - \pi(\underline{x}_d).$$

As we discussed above, $\frac{\partial U^q(\underline{x}_d^*, y_d)}{\partial y_d} > 0$. Moreover, $\frac{mk(y_d)^2}{2} > 0$. Thus, the last highlighted inequality holds if:

$$sU^q(\underline{x}_d^*, y_d) - \Pi^q(\underline{x}_d^*, y_d) > sU(\underline{x}_d) - \Pi(\underline{x}_d).$$

As, for any $d \in [0,1]$, $U^q(\underline{x}_d^*, y_d) > U(\underline{x}_d)$, then the last highlighted inequality holds for any $s \geq 1$, as long as:

$$U^q(\underline{x}_d^*, y_d) - \Pi^q(\underline{x}_d^*, y_d) > U(\underline{x}_d) - \Pi(\underline{x}_d). \tag{12}$$

In order to show that (12) holds, we show that $U^q(\underline{x}_d^*, y_d) - \Pi^q(\underline{x}_d^*, y_d)$ is increasing in y, for any $y \in [0,q)$. In the proof of Lemma 1, we showed that

$\underline{x}_d^* = \left(\frac{\chi_1(d)}{q-y_d}\right)^{\frac{1}{\alpha-1}}$, where $\chi_1(d)$ is defined in the proof of Proposition 1. A few steps of algebra ensure:

$$U^q(\underline{x}_d^*, y_d) - \Pi^q(\underline{x}_d^*, y_d) = \frac{2m}{\Delta_q}(\chi_1(d))^{\frac{2-\alpha}{\alpha-1}}(1-\chi_1(d))^2\,(q-y_d)^{\frac{\alpha-2}{\alpha-1}}.$$

Note that $(q-y)^{\frac{\alpha-2}{\alpha-1}}$ is increasing in y, for any $y \in [0,q)$, and $\frac{2m}{\Delta_q}(\chi_1(d))^{\frac{2-\alpha}{\alpha-1}}(1-\chi_1(d))^2 > 0$. We have thus proved that (12) holds. Thus, for any $d \in [0,1]$, $\frac{d^2 v_d^q}{d(d)d\psi_q} > 0$. It follows that $v_{\overline{d}}^q \geq v_{\underline{d}}^q$ for any $\left(\psi_q', s\right) > (\psi_q, s)$. This concludes the proof of part (i).

(ii) Consider now the model with firm entry. In this model, the payoff of certifier D from being in the market is equal to $\mathbb{E}m_d v_d^q$.

Suppose $\mathbb{E}m_{\underline{d}} v_{\underline{d}}^q = \mathbb{E}m_{\overline{d}} v_{\overline{d}}^q$ for some $(\sigma_m, \Delta_m, \psi_m, -s)$. In the proof of Proposition 5, we showed that $\frac{d\mathbb{E}\pi^q(d)}{d(d)} < 0$, for any $d \in [0,1]$. In the proof of Proposition 4, we showed that $\frac{d\mathbb{E}u^q(d)}{d(d)} > 0$, for any $d \in [0,1]$. Hence, $\mathbb{E}m_{\underline{d}} v_{\underline{d}} = \mathbb{E}m_{\overline{d}} v_{\overline{d}}$ implies $\mathbb{E}m_{\underline{d}}\mathbb{E}u^q(\underline{d}) < \mathbb{E}m_{\overline{d}}\mathbb{E}u^q(\overline{d})$. It follows that $\mathbb{E}m_{\underline{d}} v_{\underline{d}} > \mathbb{E}m_{\overline{d}} v_{\overline{d}}$, for any $(\sigma_m, \Delta_m, \psi_m, -s') > (\sigma_m, \Delta_m, \psi_m, -s)$.

Furthermore,

$$\mathbb{E}m_d = \underline{m} + \psi_q \Delta_m \sigma_m \left(\mathbb{E}\pi^q(d) - \underline{e} + \frac{1}{2\sigma_m}\right). \tag{13}$$

As $\mathbb{E}m_{\underline{d}} > \mathbb{E}m_{\overline{d}}$, then $\mathbb{E}m_{\underline{d}} v_{\underline{d}}^q = \mathbb{E}m_{\overline{d}} v_{\overline{d}}^q$ implies $v_{\underline{d}}^q < v_{\overline{d}}^q$. Hence $\mathbb{E}m_{\underline{d}} v_{\underline{d}}^q > \mathbb{E}m_{\overline{d}} v_{\overline{d}}^q$ for any $(\sigma_m, \Delta_m', \psi_m', -s) > (\sigma_m, \Delta_m, \psi_m, -s)$.

Fix $(\sigma_m, \Delta_m, \psi_m, -s)$ to be such that $\mathbb{E}m_{\underline{d}} v_{\underline{d}}^q = \mathbb{E}m_{\overline{d}} v_{\overline{d}}^q$. By equation (13), this is equivalent to:

$$\left(\underline{m} + \psi_q \Delta_m \sigma_m \left(\mathbb{E}\pi^q(\underline{d}) - \underline{e} + \frac{1}{2\sigma_m}\right)\right) v_{\underline{d}}^q =$$
$$\left(\overline{m} + \psi_q \Delta_m \sigma_m \left(\mathbb{E}\pi^q(\overline{d}) - \underline{e} + \frac{1}{2\sigma_m}\right)\right) v_{\overline{d}}^q.$$

Suppose

$$(\mathbb{E}\pi^q(\underline{d}) - \underline{e}) v_{\underline{d}}^q \leq (\mathbb{E}\pi^q(\overline{d}) - \underline{e}) v_{\overline{d}}^q.$$

Then, as $\mathbb{E}\pi^q(\underline{d}) > \mathbb{E}\pi^q(\overline{d})$, we should conclude that $v_{\underline{d}}^q < v_{\overline{d}}^q$, and, furthermore, $\mathbb{E}m_{\underline{d}} v_{\underline{d}}^q < \mathbb{E}m_{\overline{d}} v_{\overline{d}}^q$, which contradicts our assumption. We conclude that $(\mathbb{E}\pi^q(\underline{d}) - \underline{e}) v_{\underline{d}}^q > (\mathbb{E}\pi^q(\overline{d}) - \underline{e}) v_{\overline{d}}^q$. It follows that $\mathbb{E}m_{\underline{d}} v_{\underline{d}} > \mathbb{E}m_{\overline{d}} v_{\overline{d}}$, for any $(\sigma_m', \Delta_m, \psi_m, -s) > (\sigma_m, \Delta_m, \psi_m, -s)$. This observation concludes the proof of part (ii) of the proposition. ■

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