

Mechanism Design

Lecture Notes ECON5200/9200

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- A Game is "essentially" a mechanism
 - The outcome depends on the (rules of the) game
 - Comparative static: How outcome varies with the rules
 - "Reverse game theory": How the rules should vary with the outcome
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- ① Given a particular outcome/goal: What is the appropriate game?
 - ② Is it *possible* to find a game implementing the desired outcome?
 - ③ Given an objective/welfare function: What is the *best* game?

Mechanism $\Leftrightarrow$ Bayesian Game

- Take the external environment as given: The players $\{1, \dots, I\}$, the set of alternatives X , the payoffs $u_i(x, \theta_i)$, the type space $\theta_i \in \Theta_i$, and the priors (pdf over type space) $\phi : \Theta_1 \times \dots \times \Theta_I \rightarrow [0, 1]$.
- A mechanism is a set of strategies and an outcome-function:
 $\Gamma = (S_1, \dots, S_I, g(\cdot)), g : S_1 \times \dots \times S_I \rightarrow X$.
- So, the mechanism specifies who can do what and the consequences of these actions
- Interpretation: the institutions, the rules of the game, the electoral system, bargaining game, ownership, the economic system...
- The game and the external environment is together the strategic normal form Bayesian game:

$$G = [I, \{S_i\}, u_i(g(s), \theta_i), \Theta_1 \times \dots \times \Theta_I, \phi].$$

"Reverse Game Theory"

- Game theory: For a given game, find the outcomes

$$G \Rightarrow x(\theta)$$

- Mechanism design: For a given desirable outcome, find the games

$$x(\theta) \Rightarrow \Gamma \Rightarrow G$$

- The "engineering" part of economics

The Implementation Problem

- What is a desirable outcome?
- Def23B1: A *scf* is a function $f : \Theta_1 \times \dots \times \Theta_I \rightarrow X$
- Def23B2: f is *ex post efficient* if $\nexists (\theta, x)$ s.t.
 $u_i(x, \theta_i) \geq u_i(f(\theta), \theta_i) \forall i$ with strict inequality for some.
- Def23B4: Γ *implements* f if there is one equilibrium of Γ , $s^*(.)$, s.t.
 $g(s^*(\theta)) = f(\theta) \forall \theta$.
- Ambiguities:
 - Is one equilibrium enough? (Γ can have multiple equilibria: require "full implementation").
 - What is an "equilibrium"? Bayesian? Nash? Dominant? (Harder)
- ① When is f implementable?
- ② What if f is not implementable?

What is an equilibrium?

- Def23D1: s^* is a *Bayesian Nash equilibrium* of Γ if

$$E_{\theta_{-i}} [u_i (g (s_i^* (\theta_i), s_{-i}^* (\theta_{-i})), \theta_i)] \geq E_{\theta_{-i}} [u_i (g (s'_i, s_{-i}^* (\theta_{-i})), \theta_i)]$$

for all (s'_i, i, θ_i) .

- Def23C1: s^* is a *dominant strategy equilibrium* of Γ if

$$u_i (g (s_i^* (\theta_i), s'_{-i}), \theta_i) \geq u_i (g (s'_i, s'_{-i}), \theta_i)$$

for all $(s'_i, s'_{-i}, i, \theta_i)$.

- s^* is a dominant strategy equilibrium $\Rightarrow s^*$ is a Bayesian Nash equilibrium.
- f implementable in dominant strategies $\Rightarrow f$ implementable in Bayesian Nash equilibrium

Direct Revelation Mechanism

- Problem: There is a large number of mechanisms... Example:
- Def23B5: Γ is a *direct revelation mechanism* if $S_i = \Theta_i \forall i$ and $g(\theta) = f(\theta) \forall \theta$.
- Def23B6: f is *truthfully implementable* (=incentive compatible) if the direct revelation mechanism Γ has one equilibrium s^* where $s_i^*(\theta_i) = \theta_i \forall (i, \theta_i)$.
- A very *simple* mechanism: Just ask for the preferences, and implement f as if reports=messages were true.
- A very *naïve* mechanism!
- Seems very limited and hard to satisfy!?

The Revelation Principle

Proposition

- $\exists \Gamma$ implementing $f \Leftrightarrow f$ is truthfully implementable

- *Proof* (Dominant strategies): If Γ implements f

$$\begin{aligned} u_i(g(s_i^*(\theta_i), s'_{-i}), \theta_i) &\geq u_i(g(s'_i, s'_{-i}), \theta_i) \Rightarrow \\ u_i(g(s_i^*(\theta_i), s_{-i}^*(\theta_{-i})), \theta_i) &\geq u_i(g(s'_i, s_{-i}^*(\theta_{-i})), \theta_i) \Rightarrow \\ u_i(g(s_i^*(\theta_i), s_{-i}^*(\theta_{-i})), \theta_i) &\geq u_i(g(s_i^*(\hat{\theta}_i), s_{-i}^*(\theta_{-i})), \theta_i) \Rightarrow \\ u_i(f(\theta), \theta_i) &\geq u_i(f(\hat{\theta}_i, \theta_{-i}), \theta_i). \end{aligned}$$

- Good: *Not* difficult to find Γ
- Bad: Difficult to find implementable f !

The Gibbard-Satterthwaite Theorem

Suppose X is finite and $|X| \geq 3$, preferences are strict and $f(\Theta) = X$.

Proposition

- *The scf f is implementable in dominant strategies iff f is dictatorial.*
- The scf f is *dictatorial* if there exists an i such that for all $\theta \in \Theta$
$$f(\theta) \in \{x \in X : u_i(x, \theta_i) \geq u_i(y, \theta_i) \forall y \in X\}$$
- The simplest proof builds on Arrow's impossibility theorem.
- Proven independently by Gibbard '73 and Satterthwaite '75.

Vickrey-Clarke-Groves Mechanism

- Quasilinear case / transferable utility

$$u_i(x, \theta_i) = v_i(k, \theta_i) + t_i$$

- Project $x = (k, t_1, \dots, t_I)$, $X = \{k \in K, \sum t_i \leq 0\}$
- Note: f specifies k and transfers as a function of θ
- Def23B2 implies: f is ex post efficient only if $k(\cdot)$ is efficient, i.e.

$$\sum v_i(k(\theta), \theta_i) \geq \sum v_i(k', \theta_i) \forall (\theta, k').$$

- Prop23C4:** Suppose k is efficient. Then f is implementable in dominant strategies if

$$t_i(\theta) = \left[\sum_{j \neq i} v_j(k(\theta), \theta_j) \right] + h_i(\theta_{-i}),$$

no matter the functions $h_i(\theta_{-i})$.

- Example: $I = 2$, $h_i(\theta_j) = -v_j(k(\theta_j, \underline{\theta}), \theta_j)$; second-price auction.

- *Proof* by contradiction. If truth is *not* a dominant strategy, then it is possible that

$$\begin{aligned}
 v_i \left(k \left(\hat{\theta}_i, \theta_{-i} \right), \theta_i \right) + t_i \left(\hat{\theta}_i, \theta_{-i} \right) &> \\
 v_i \left(k \left(\theta \right), \theta_i \right) + t_i \left(\theta \right) &\Leftrightarrow \\
 v_i \left(k \left(\hat{\theta}_i, \theta_{-i} \right), \theta_i \right) + \sum_{j \neq i} v_j \left(k \left(\hat{\theta}_i, \theta_{-i} \right), \theta_j \right) &> \\
 v_i \left(k \left(\theta \right), \theta_i \right) + \sum_{j \neq i} v_j \left(k \left(\theta \right), \theta_j \right) &\Rightarrow \\
 \sum v_j \left(k', \theta_j \right) &> \sum v_j \left(k \left(\theta \right), \theta_j \right),
 \end{aligned}$$

so $k(\theta)$ is *not* ex post efficient.

VCG with Budget Balance

- **Assume A:** If $V = \{v : K \rightarrow \mathbb{R}\}$ then $\{v_i(\cdot, \theta_i) : \theta_i \in \Theta_i\} = V$.
- **Prop23C5:** If $k(\cdot)$ is efficient and **A**, it is implementable if and only if, for some $h_i(\theta_{-i})$:

$$t_i(\theta) = \left[\sum_{j \neq i} v_j(k(\theta), \theta_j) \right] + h_i(\theta_{-i}),$$

- Def23B2 also implies: f is ex post efficient iff $k(\cdot)$ is efficient and $\sum t_i(\theta) = 0 \forall \theta$
- **Prop23C6:** If f is ex post efficient and **A**, it is not implementable in dominant strategies.
- Relax assumption: If one v_i is constant, then ex post efficient f is implementable in dominant strategies

The Expected Externality Mechanism / AGV

- Relax equilibrium concept: If f is ex post efficient (and types independent), then f is implementable in Bayesian Nash equilibrium.
- ...with the Expected Externality Mechanism (d'Aspremont and Gerard-Varet 79, Arrow 79):

$$t_i(\theta) = E_{\theta_{-i}} \left[\sum_{j \neq i} v_j(k(\theta), \theta_j) \right] + h_i(\theta_{-i}),$$

- Trick is to carefully select $h_i(\theta_{-i})$

$$h_i(\theta_{-i}) = \frac{-1}{I-1} \sum_{j \neq i} \left[E_{\theta_{-j}} \left[\sum_{l \neq j} v_l(k(\theta), \theta_l) \right] \right]$$

- Problem in all above mechanism: Participation constraints

Myerson-Satterthwaite Theorem -83

- Suppose seller's and buyer's valuation of the good has a nonempty intersection.
- $\nexists f$ which is ex post efficient, implementable in Bayesian Nash equilibrium, and which gives every type nonnegative expected gains from participation.
- Easy to check in example with $\{\underline{c}, \bar{c}\}$ and $\{\underline{v}, \bar{v}\}$
- **OPTIMAL MECHANISMS:**
 - Often, the ideal f is not implementable
 - The second-best problem: Maximize a given objective, s.t. (IC), (IR) and (BB)
 - This is often easier than to detect implementable f

Incomplete Contracts - What is that?

- Some acts are contractible, others are not
- Are non-contractible actions suboptimal?
 - What do they depend on?
 - How do they depend on the contractible acts?
- How can we influence non-contractible acts by carefully selecting the contractible ones?

Incomplete Contracts - Examples

- Firms: Ownership is contractible, effort is not
- Trade: Quantity contractible, investments not
- Finance: Repayment is partly contractible, risk-taking not
- Climate: Emission is contractible, R&D is not
- EU: Voting rules are contractible, local policies are not

Incomplete Contracts - General setting

- ① Agents I may contract on q
- ② Actions x_i may also be feasible, $i \in I$
- ③ Agents negotiate a decision $k \in K$ and t_i
 - Payoff $V_i(x; q, k) + t_i - \psi_i(x_i)$
 - What is the equilibrium x_i ?
 - How does x_i depend on q ?
 - What is the optimal q ?
 - Can we find q s.t. x is first-best?

Incomplete Contracts - Bilateral example

- Let $K = \{yes, no\}$
- Let $\Phi_i(x_1, x_2; q) = V_i(x_1, x_2; q, no)$
- Let $\alpha_i \in [0, 1]$ be i 's "bargaining power"
- Suppose $k = yes$ will be ex post optimal
- Bargaining surplus is

$$\sum_i V_i(.) - \sum_i \Phi_i(.).$$

- Agent 1 receives $\Phi_1(.) + \alpha_1 \cdot \text{Bargaining-surplus}$

Incomplete Contracts - Hold-up problems

- When choosing x_i , the foc is

$$\alpha_i \left(\frac{\partial V(.)}{\partial x_i} - \frac{\partial \Phi(.)}{\partial x_i} \right) + \frac{\partial \Phi_i(.)}{\partial x_i} = \psi'_i(x_i)$$

- First-best x_i is

$$\frac{\partial V(.)}{\partial x_i} \equiv \sum_{j \in I} \frac{\partial V_j(.)}{\partial x_i} = \psi'_i(x_i^*).$$

- Why is $x_i \neq x_i^*$? When is $x_i = x_i^*$?

Incomplete Contracts - Solutions

- Consider assets $\{a_1, a_2\}$ but $\Phi_i(.) = 0$ unless i owns both.
- Suppose $\Phi'_1(x_1, x_2, q) = \Phi'_1(x_1, \{a_1, a_2\}) > 0$, $V_{12} > 0$
- Then... non-integration is dominated
 - If $\Phi'_i(x_1, x_2, \{a_1, a_2\}) < 0$, non-integration better
- Complementary assets should be owned together
- If only one invests, that agent should own both assets
- If only one invests, that agent should have all bargaining power
- The agent having an outside option/market should own