CONTINGENT TRADE AGREEMENTS

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Abstract

How can trade motivate environmental conservation? I develop a model that combines trade-specific investments (e.g., deforestation) with environmental externalities. Traditional trade agreements raise investments and thus deforestation. Nevertheless, a contingent trade agreement (CTA), where default tariffs can vary with changes in the production capacity (e.g., forest cover), can be designed so as to motivate conservation. The model permits many products, countries, and collaborators. A simple calibration suggests that growth and liberalization can cause Brazil's agricultural area to expand by 27%, but this expansion can be avoided if the EU and the US offer a CTA.

The tension between international trade and environmental concerns is intensifying. Already in 1999, tens of thousands of protesters demonstrated in Seattle and criticized trade negotiators for betraying environmental and social values.[1] They requested that trade should be limited rather than liberalized. When the Transatlantic Trade and Investment Partnership (TTIP) agreement was negotiated, protesters said they expected 250,000 demonstrators to turn out in Germany because "TTIP threatens environmental and consumer protection" (*The Guardian*, Sept. 17, 2016).

Today – in 2023 – the European Union struggles with a recent agreement negotiated with Mercosur. The agreement was negotiated in 2019, but deforestation in the Brazilian Amazon has continued to increase every year since that time.[2] Thus, the agreement is still not ratified, and it has been opposed by countries such as France, Germany, Netherlands, Belgium, Ireland, Austria, and Luxembourg.[3] Empirical evidence does suggest that trade agreements can be associated with deforestation,[4] contributing to climate change (IPCC, 2021).

To deal with the conflict, trade agreements often include sustainability requirements. For instance, the EU's agreements include so-called trade and sustainable development (TSD) chapters. A major challenge is that threats and conditions might not be credible ex post (Hsiao, 2022). After the land-use-change has already taken place, it will be in everyone's interest to trade rather than to impose costly sanctions. Ferrari et al. (2021) confirm that the EU provisions have had little effect. Trade has generally failed the environment, according to the UN (2019) and the World Bank (2019).

Consequently, "Member states and the European Parliament are looking for trade concessions to be made conditional on compliance with a wider range of sustainable development criteria."[5] In May, 2020, France and the Netherlands made a novel policy initiative. In a so-called "non-paper," they first admit a "lack of progress in compliance" with the TSD chapters, before they propose that "Parties should introduce, where relevant, staged implementation of tariff reduction linked to the effective implementation of TSD provisions and clarify what conditions countries are expected to meet for these reductions, including the possibility of withdrawal of those specific tariff lines in the event of a breach of those provisions."[6] In 2022, the EU confirmed that also trade unions and NGOs "supported the use of trade agreements

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[1] About the 1999 WTO negotiations in Seattle, *The New York Times* wrote (Oct. 13, 1999) that 50,000 demonstrators were expected and, underlying the protests, there "is a fundamental disagreement about the proper role of the trade organization."

[2] See: http://terrabrasilis.dpi.inpe.br/app/dashboard/deforestation/biomes/legal_amazon/rates

[3] In April, 2023, "details are [still] missing":
https://www.reuters.com/world/americas/brazil-hopes-conclusion-eu-mercosur-trade-deal-this-year-2023-04-23/

[4] Abman and Lundberg (2020), for instance, document that deforestation levels peak around the ratification date for regional trade agreements. For earlier papers verifying the connection between trade liberalization and deforestation in the tropics, see, for example, Barbier (2000), Faria et al. (2016), or Pendrill et al. (2019).

[5] *Financial Times*, Sept. 21, 2020. The article is available here: https://www.ft.com/content/b508b3b1-999f-4528-a0d2-f1b37f0e0b87. In July, 2022, the EU confirmed that "as the world's biggest trading bloc, [it] needs to be ambitious ... when designing additional autonomous instruments to support global climate action, the fight against biodiversity loss and deforestation". See:
https://www.europarl.europa.eu/doceo/document/INTA-RD-734209_EN.pdf

[6] The non-paper is available here: https://nl.ambafrance.org/Non-paper-from-the-Netherlands-and-France-on-trade-social-economic-effects-and

to uphold implementation of [non-trade] agreements, by linking removal of tariffs to implementation."[7] The non-paper is brief and specifies neither exactly how one can achieve staged implementation and withdrawal of tariff lines, nor the extent to which such a design may motivate conservation.

This Paper

Section 2 presents a model with trade-specific investments and environmental externalities. The model is intentionally designed so as to capture negative interactions between free trade agreements (FTAs) and conservation. The purpose of the paper is to show that – even in this situation – environmental conservation can be motivated by a "contingent trade agreement" (CTA).

In the model, the parties invest in production capacity before the market clears. In the business-as-usual (BAU) scenario, tariff levels are set noncooperatively after the investment stage. Each tariff is set so as to improve the country's terms of trade. This behavior is consistent with the literature and empirical evidence.[8] An implication, in my model, is that tariffs are larger when investments have been larger. The analysis builds on two complementary assumptions.

I. In the South (S), the investment decision is made by the government, taking into account how tariffs and prices will respond. This assumption is reasonable when S exports agricultural products that necessitate land use change and deforestion.[9] To illustrate the effect of this assumption, investments in production capacity are made by private price-taking actors in the North (N). In both countries, investments are reduced when the tariffs are expected to be positive in BAU. In addition, S limits investments for two reasons: A lower production capacity raises the equilibrium price for S's product, and the lower capacity induces N to set a lower tariff.

Section 3 establishes benchmark results: (1) Trade agreements have larger influence on investments when the investments are private, (2) free trade causes deforestation, (3) larger gains from trade cause more deforestation, and (4) larger environmental damage reduces the value of the FTA.

More generally, a bilateral trade agreement (BTA) does allow countries to negotiate fixed tariffs (or border taxes) before investments adjust. If environmental damages are large, the optimal tariffs are high. But the tariffs are distortionary and every renegotiation-proof BTA causes more deforestation than does BAU, I show.

II. Section 4 assumes that default tariffs can be contingent on the capacity levels. Brazil's forest cover, for instance, is observable and verifiable, and changes are monitored by satellites (UN, 2019).

Assumptions I and II complement each other: The contingency has *no* role to play when capacity investments are made by private actors that take prices and tariffs as given. This observation may explain why the usual trade agreements are not CTAs. But with public influence over the capacity level,

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[7]See https://circabc.europa.eu/ui/group/e9d50ad8-e41f-4379-839a-fdfe08f0aa96/library/95aafa87-8d69-4f1e-9ce6-a4e5416ba444/details

[8]Tariffs are motivated by the terms-of-trade effects in Bagwell and Staiger (2004) and Ludema and Mayda (2013), for instance. Broda et al. (2008) and Bagwell and Staiger (2011) present empirical support for the terms-of-trade motive.

[9]Burgess et al. (2019) have documented that governmental policies are determining deforestation rates in the Brazilian Amazon.

as when the government monitors land-use change, S will pay attention to how the capacity will influence the terms of trade. To motivate conservation, S's terms of trade must be more attractive when the forest cover is large, and less attractive when the capacity to produce beef is large.

The CTA exploits the fact that there is more than one way of splitting the gains from trade: If one country's tariff increases, terms-of-trade effects imply that this country obtains more of the gains, while the other loses. The CTA lets the point on the Pareto frontier be contingent on S's capacity (or forest cover). It is reasonable to require the tariffs to be Pareto optimal, i.e., credible and renegotiation proof, so that there is no other tariff pair that both countries prefer ex post, no matter the realized forest cover. There is a limit for how large the tariff can be before the parties want to renegotiate it, and thus there is a limit to what the CTA can achieve. To achieve more, the CTA can make *both* tariffs contingent on S's capacity, so that S's tariff could be positive as long as S's forest cover remains large.

The main contribution of this paper is to show how all the negative effects from traditional trade agreements on conservation, and the corollaries are reversed when countries can negotiate a CTA: (1) The CTA can only influence investments when they are public, (2) more can be conserved under the CTA than with BAU, (3) larger gains from trade makes it possible to conserve more, and (4) larger environmental damage raises the value of this trade agreement.

Even in the absence of environmental damage, the CTA is strictly better than any FTA and BTA. With free trade, S invests less than private investors would. The CTA can motivate S to invest more by letting the terms of trade be more attractive to S when the capacity is large than when it is small.

Section 5 (and the Appendix) allows for more than two products and two countries, and countries can be of different sizes. In all these cases, the CTA can motivate S to conserve or invest.

A serious calibration of the model is beyond the scope of this paper, but Section 6 illustrates the possibility by matching the predictions of the model (with all generalizations) with the empirically reasonable 20% beef tariff, and the modest export fraction (25% of beef, 63% of soy) that we see in Brazil. The calibration indicates that Brazil exports to five major trading blocks, which is in line with other calibrated trade models (e.g., Ossa, 2011). The calibration predicts that trade liberalization can cause a 4.8% increase in the agricultural area, but that a CTA between Brazil and the EU can avoid the increase. If demand from three trading blocks (i.e., Asia) doubles, the agricultural area will increase by 27%, and the forest will diminish accordingly – with free trade. However, the increase is below 18%, 10%, or 1%, respectively, if the CTA is offered by the EU, the EU and the US, or by three trading partners.

Literature

In the traditional literature on trade and the environment, countries may reduce environmental standards to become competitive (Markusen, 1975) or to specialize in their comparative advantages. The South may have a comparative advantage in environmentally damaging production because of policies (Pethig, 1976). Dasgupta et al. (1978) studied depletion rates of exhaustable resources in open economies.[10] If

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[10]Relatedly, Hillman and Van Long (1983) studied a country depleting a resource at the same time as it was importing

countries in the South struggle with an open-access problem, and are unable to control extraction rates, then trade can worsen the problem and cause depletion (Chichilnisky, 1994; Brander and Taylor, 1997; Karp et al., 2001). Shapiro (2021) finds that dirty goods tend to face lower tariffs.[11]

A novelty of my model is that the environmental damage follows from up-front investments in capacity (such as land-use change). Thus, I connect with the literature on irreversible trade-specific investments. When countries produce similar products, Krugman (1987) showed how protectionism can facilitate investments in the less competitive country. With different products, the increased investments have been referred to as the "dynamic gains from trade" (Baldwin, 1992), and they can also influence subsequent negotiations (McLaren, 1997). McLaren and Bond and Park (2002) find, as I do, that the equilibrium tariff is larger if the investment level is larger. Empirical investigations have confirmed that trade infliuences capacity investments.[12] Baier and Bergstrand (2007) find there to be a ten-year adjustment period from liberalization to increased trade. After the adjustment, trade doubles, in line with my Proposition 2.

McLaren considered investments by firms, and Ossa (2011) the relocation of firms, but Bond (2006) and Guriev and Klimenko (2015) study investments made by a government. These scholars refer to investments in infrastructure such as transportation facilities, roads, ports, pipelines, or electricity grids. With such kinds of investments, environmental externalities are naturally abstracted from. By permitting environmental externalities, I combine this strand of literature with the trade–environmental one.

Based on the standard trade–environmental literature, scholars have recommended unilaterally imposed trade sanctions (Barrett, 1997), border taxes (Ludema and Wooton, 1994; Keen and Kotsogiannis, 2014), or climate clubs (Nordhaus, 2015). However, Copeland et al. (2022:126) note that these alternatives "fall short of the full optimum" because of the ex post distortions and, when the resource is non-renewable, the commitment problems: "Once a forest is cut down, the benefits of maintaining trade restrictions are substantially reduced. If those developing plantations anticipate this, then trade restrictions may have little deterrent value." Hsiao (2022) shows that these drawbacks are quantitatively important. Here, too, I illustrate the drawbacks of border taxes in Section 3.3.

My main contribution is to show that both the ex post distortions and the commitment problems are avoided when countries exploit the fact that there are multiple ways of distributing the gains from trade. When the production capacity is a payoff-relevant stock, as here, it becomes possible to set tariffs contingent on out-of-equilibrium stock levels in a way that is renegotiation proof. With this possibility, I characterize treaty designs that motivate environmental conservation more effectively than what seems to be possible in the earlier literature.

Because the CTA ties tariffs to the stock level, the analysis adds to the literature on shallow vs. deep

extracted amounts from another country. If there is a (lower) risk of trade disruption, then the country conserves more (less) of its own resource. With a larger number of jurisdictions, depletion can be larger also because prices will be less sensitive to one's own supply (Markusen, 1981).

[11]Trade can also raise income levels, and because of the environmental Kuznets curve, the outcome can be a cleaner environment (Antweiler et al., 2001; Copeland and Taylor, 2004). My contribution to this literature is to show how, even when we abstract from these effects, trade agreements can be designed so as to motivate conservation.

[12]For example, Juhasz (2018) shows that the 1803–1815 protectionist period in the French Empire influenced the production capacities in mechanized cotton spinning.

integration.[13] Even if there is no damage, free trade fails to implement the first best in my model, because S withholds capacity investment to influence terms of trade. In fact, the increase in capacity following trade liberalization is here comparable to the increase following privatization. When privatization is infeasible, I show that the CTA guarantees deeper and more efficient integration than can any traditional shallow trade agreement. "Deep agreements have been very controversial", explain Maggi and Ossa (2022:1), who show that they can be undesirable when there are *local* environmental externalities. With *international* externalities, however, the type of deep integration permitted by the CTA can motivate more conservation than what we can expect with shallow integration, I show.

Issue linkages occur in practice. Berger et al. (2013) finds that military support is connected to trade, for example. However, the theoretical literature on issue linkages often relies on punishments that are not renegotiation proof (Ederington, 2002; Limão, 2005). Importantly, I do not permit a commitment to, or enforcement of, ex post suboptimal policies. Instead, the CTA takes advantage of the fact that the Pareto frontier consists of more than one point, and the selected point can be a function of the remaining resource stock.

A companion/working paper (Harstad, 2023) verifies that the benefits of the CTA survive in a dynamic model with an infinite time horizon and if export subsidies are available. That paper, however, assumes linear utility and nondistortionary tariffs, and it does not permit tariffs in a "business as usual" scenario or settings with traditional agreements. None of the generalizations in Section 5 is considered in Harstad (2023), so that model cannot be calibrated in a reasonable way.

2. THE MODEL

Demand. There are two countries, the North (N) and the South (S). Each country produces a unique good that is sold and consumed in both countries. Let $c_{ij} > 0$ measure country i's consumption level of country j's good, where $i, j \in \{N, S\}$.

A representative consumer enjoys the following consumption utility:

$$U_i = c_{i0} + \sum_{j \in \{N,S\}} u_{ij}(c_{ij}), \, i \in \{N, S\}. \tag{1}$$

When c_{i0} is a freely traded numeraire good that can be used as currency, we have a general equilibrium model. Assume the endowments are so large that $c_{i0} \geq 0$ never binds.

I follow Maggi and Rodríguez-Clare (2007), or Bond and Park (2002), in assuming quadratic $u_{ij}(c_{ij})$ with bliss point v_{ij}:

$$u_{ij}(c_{ij}) = -\frac{(v_{ij} - c_{ij})^2}{2a_j}. \tag{2}$$

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[13] Although the terms-of-trade motive is still important, studies of deep integration also consider behind-the-border policies (Antras and Staiger, 2012), lobbies and process standards (Maggi and Ossa, 2022), or concentrate on principles such as reciprocity and nondiscrimination (Bagwell and Staiger, 1999) to prevent bilateral opportunism through "concession erosion" (Bagwell and Staiger, 2016).

The Appendix, and Section 5, allow for many goods and countries and heterogeneous country sizes.

Supply. Markets are competitive, but total consumption is limited by the production capacity, X_i:

$$\sum_{j \in \{N,S\}} c_{ji} \leq X_i, \ \forall i \in \{N,S\}. \tag{3}$$

A key assumption in the model is that S's government decides on X_S before the good is traded by the consumers. This assumption is reasonable in certain important situations. When S produces beef, the amount is limited by S's amount of agricultural land, X_S, determined by S's policy regarding land use change, deforestation, and the monitoring of illegal logging.

To illustrate the effect of this assumption, suppose X_N is decided on competitively by price-taking private investors. This assumption is not crucial: Thanks to the quasi-linear (1), the market outcome for S's good – and thus the paper's main results – is independent of the market for N's good. We could assume X_N to be set by N's government, or X_N could be exogenous. By assuming X_N is invested in by perfectly competitive price-taking private investors, we can compare the outcomes for the two goods in order to shed light on the effect of our key assumption, i.e., that S, or S's policy, determines X_S. To make the comparison clean, I will assume that the countries are similar in other respects. In Online Appendix B, I explain how the results would modified if a government could use investment taxes, export tariffs, or both.

The following is not needed for the analysis, but it is straightforward to permit a marginal production cost, $\kappa_S \geq 0$, when beef is produced on a unit of the land, a marginal cost of clearing the forest and converting it to agriculture, $\varkappa_S$, and a marginal value of the lumber, ν_S. S's decision on X_S will depend on the net marginal cost $k_S \equiv \kappa_S + \varkappa_S - \nu_S$. Assume $k_S \geq 0$, so that S will never clear land that is not used for beef production. This assumption implies that (3) will bind in equilibrium.

N may face transport cost $t_{NS} \geq 0$ when importing a unit from S.[14]

Analogously, the marginal costs of increasing the capacity and produce X_N sum to $k_N \geq 0$, and S's marginal cost of transporting N's good is $t_{SN} \geq 0$.

If capacity investments are irreversible, then we must also require $X_i \geq X_i^0$, where $X_i^0 \geq 0$ is the initial capacity. This inequality will not bind, and the level of X_i^0 will have no impact on the results, under the assumption that X_i^0 is weakly smaller than the noncooperative business-as-usual (BAU) level.[15]

Externalities and Payoffs. A representative consumer maximizes (1) subject to the budget constraint:

$$c_{i0} + p_i c_{ii} + (p_j + \tau_i + t_{ij}) c_{ij} \leq y_i, \ \forall \{i,j\} = \{N,S\}, \tag{4}$$

taking as given the export prices p_i and p_j, and i's tariff τ_i and income y_i (all measured relative to the price of the numeraire). If e_i is an exogenous endowment, the national income is:

$$y_i = e_i + \tau_i c_{ij} + (p_i - k_i) X_i, \ \forall \{i,j\} = \{N,S\}. \tag{5}$$

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[14] When prices can adjust, it's irrelevant whether the exporter or importer is responsible for paying the transport cost.

[15] That is, I will henceforth assume $X_i^0 \leq X_i^{BAU} \ \forall i \in \{N,S\}$, where X_i^{BAU} is characterized by Proposition 2. Even in a dynamic model, $X_S^0 < X_S^{BAU}$ can be reasonable if parameters changed over time in a way that makes the new BAU level for X_S larger than in the past: see Section 6.

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S X_S (agric land) τ_S (South's tariff)   Markets clear

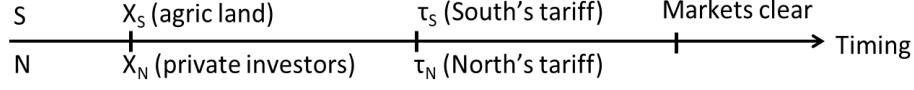N X_N (private investors) τ_N (North's tariff)   → Timing

Figure 1: *The timing of the noncooperative game*

In (5), the second term is the country's tariff revenue, and the last term measures the profit. In S, the profit is earned by, or redistributed to, the consumers. That is, it will not matter whether land is owned and beef is produced by S's citizens or its government. In N, the competitive private sector invests to the point when $p_i = k_i$, i.e., there is zero profit in equilibrium, so (5) holds also in this case.

I will permit externalities associated with production and/or capacity expansion. If S clears the forest to produce more beef, we lose biodiversity, carbon sinks, and the homes of indigenous tribes. The cost to S is internalized by S (this cost can be included in S's total capacity expansion cost, k_S). The expected environmental damage experienced by N is given by the function $d_N(X_S)$.[16]

Symmetrically, S may face the damage $d_S(X_N)$ when N invests, or produces. For instance, N's production might contribute to climate change. (Total investment equals total production, because (3) will bind in equilibrium.)[17]

Each function $d_i(\cdot)$ is assumed to be weakly increasing and weakly convex. If it happens to be linear, then

$$d_i(X_j) = d_i' X_j, \tag{6}$$

for some constant marginal damage $d_i' \geq 0$. If $d_i(X_j)$ is nonlinear, the marginal damage d_i' is a function of X_j.[18] The welfare in country i is the consumption utility (which includes the income from the tariff and profit), minus the damage:

$$U_i - d_i(X_j), \ \{i,j\} = \{N,S\}.$$

Timing. The timing of the noncooperative game is illustrated in Fig. 1. The capacity level, such as the stock of agricultural land, adjusts slowly, but tariffs can easily adjust to stock changes. Thus, tariffs are set after the X_i's. S takes into account the effects on future tariffs and prices when determining

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[16] With "expected damage", I permit a possible catastrophe if X_S exceeds threshold or tipping point $\widetilde{X}$, as in Maggi and Staiger (2022). They quote Pindyck (2021). Pindyck writes (p. 26): "We don't know where a "tipping point," if there is one, might lie." To capture this uncertainty, suppose $d_N(X_S) = \underline{d}X_S + h\Pr\left(X_S \geq \widetilde{X}\right)$, where h is the expected additional harm above threshold $\widetilde{X}$. If $\widetilde{X} \sim U\left[\mathrm{E}\widetilde{X} - 1/2\sigma, \mathrm{E}\widetilde{X} + 1/2\sigma\right]$, then $d_N(X_S) = \underline{d}X_S + \left(1/2 + \sigma\left(X_S - \mathrm{E}\widetilde{X}\right)\right)h$, so the marginal expected damage is $\underline{d} + \sigma h$.

[17] There may or may not be an externality from the investments in X_N on country N. If there is, then we may either assume that N's government incentivizes the investors to internalize the harm on N (for example, k_N may include the domestic Pigouvian CO_2 tax), or we can permit an externality that is not internalized. In the latter case, the analysis remains unchanged if $d_S(X_N)$ is interpreted as the global externality from X_N.

[18] If $d_i(\cdot)$ is quadratic, then the second-order derivative is constant ($d_i'' \geq 0$) and we can write:

$$d_i(X_j) = d_i' X_j + d_i'' X_j^2/2.$$

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X_S, but N's investors take prices and tariffs as given. At the end, price-taking consumers make their decisions, the market clears, and payoffs are realized. By solving the game by backward induction, we characterize the subgame-perfect equilibrium (SPE).

Extensions. After deriving the SPE, agreements on free trade and border taxes are considered. Section 4 presents the main results – on CTAs. Section 5 allows for many products and countries, and heterogeneous country sizes; Section 6 shows that the model is still simple to calibrate. Online Appendix B considers policy instruments such as investment taxes and export tariffs. The results hold, qualitatively, with multiple periods, as shown in an earlier working paper.

3. TRADE, TARIFFS, AND TRADITIONAL TREATIES

First Best. With transferable utilities, social efficiency requires the sum of payoffs to be maximized. As a consequence, the marginal benefit, minus the total marginal cost, must equal zero for each country:

$$\frac{v_{ii} - c_{ii}}{a_i} - \left(k_i + d'_j\right) = \frac{v_{ji} - c_{ji}}{a_i} - \left(k_i + d'_j + t_{ji}\right) = 0 \quad \forall \{i,j\} = \{N,S\}. \tag{7}$$

If there were no damage, (7) would require:

$$c_{ii} = \overline{v}_{ii} \equiv v_{ii} - a_i k_i \text{ and } c_{ji} = \overline{v}_{ji} \equiv v_{ji} - a_i k_i - a_i t_{ji},$$

where the gains from trade, captured by $\overline{v}_{ji}$, decrease in the transport cost t_{ji} and increase in the value of lumber (since that, as explained, contributes to a smaller k_i). With damage, (7) requires

$$c_{li} = \overline{v}_{li} - a_i d'_j \quad \forall \{i,j\} = \{N,S\}, l \in \{N,S\}. \tag{8}$$

With the constraint $X_i \geq X_i^0$, the first-best is

$$X_i^{FB}\left(d'_j\right) = \max\left\{X_i^0, \overline{v}_{ii} + \overline{v}_{ji} - 2a_i d'_j\right\}. \tag{9}$$

The constraints $c_{ii} \geq 0$ and $c_{ji} \geq 0$ will never bind, one can show, if $\overline{v}_{ij} > 0 \ \forall i,j \in \{N,S\}$, and $\overline{v}_{SS} > \overline{v}_{NS}/3.$[19]

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[19] If $\overline{v}_{SS} < \overline{v}_{NS}/3$, S will only produce for the export market. In (9), d'_j is a constant if the harm is linear. With nonlinear damage, d'_j, on the right-hand side of (9), is a function of X_i. If the function is quadratic, with marginal damage $d'_i + d''_i X_j$, then

$$X_i^{FB}(\cdot) = \max\left\{X_i^0, \overline{v}_{ii} + \overline{v}_{ji} - 2a_i \frac{(\overline{v}_{ii} + \overline{v}_{ji})d''_j + d'_j}{1 + 2a_i d''_j}\right\}.$$

3.1. Business as Usual

The Market Equilibrium. To derive the SPE by backward induction, we first solve the consumers' problem. When consumers maximize (1), subject to (4), demand for i's good is:

$$c_{ii} = v_{ii} - a_i p_i \text{ and} \tag{10}$$

$$c_{ji} = v_{ji} - a_i \left(p_i + \tau_j + t_{ji} \right), \forall \{i, j\} = \{N, S\}.$$

Note that (10) can be consistent with the first best, (7), only when $\tau_j = 0$. If $\tau_j \neq 0$, marginal net benefits will differ for the two sets of consumers. However, a larger τ_j reduces the equilibrium price p_i. When (3) binds,

$$p_i = \frac{v_{ii} + v_{ji} - X_i}{2a_i} - \frac{\tau_j + t_{ji}}{2}. \tag{11}$$

Best-response Tariffs. The lower import price implies that j's terms of trade improve when τ_j increases. Country j trades off this benefit with the distortions that follow when j's consumption falls because of the tariff. Country j's optimal tariff maximizes the consumer surplus plus the tariff revenues. The Appendix verifies that j's optimal τ_j is given by the following best response to X_i:

$$\tau_j^{BR} \left(X_i \right) = \frac{\overline{v}_{ji} - \overline{v}_{ii} + X_i}{3a_i}. \tag{12}$$

When (10)–(12) are combined, it is easy to check that when $c_{ji} > 0$, the right-hand side of (12) is positive. As is standard (and discussed by Dixit, 1985, for instance) and empirically supported (Broda et al., 2008), the tariff is smaller if export is elastic. After X_i is determined, i's export is elastic when a_i is large.

Furthermore, $\tau_j^{BR} \left(X_i \right)$ increases in X_i. Intuitively, if X_i is large, j imports a lot, and it is more important for j to improve its terms of trade. This logic is both robust and well known (since Bond and Park, 2002).

Equilibrium Capacity. In country N, private price-taking and tariff-taking investors invest in X_N as long as $p_N \geq k_N$. Because p_N decreases in X_N, equilibrium X_N ensures that this inequality binds. With the definitions in (8), (11) implies that the market response to an expected tariff is:

$$X_N^{MR} \left(\tau_S \right) = \overline{v}_{NN} + \overline{v}_{SN} - a_N \tau_S. \tag{13}$$

In line with the first welfare theorem, the first best is implemented by a perfect market: $X_N^{MR} (0) = X_N^{FB} (0)$, when there is no tariff and no damage.

In (13), τ_S is actually the *expected* tariff, which N's investors take as given. It is intuitive that it is less profitable to invest in X_N if S's tariff is expected to be large.

Expectations are rational, so equilibrium pair (X_N, τ_S) satisfies both (12) and (13), as illustrated by the top-right intersection in Fig. 2.

EXAMPLE 1: All figures are drawn for $\overline{v}_{ji} = \overline{v}_{ii} = a_i = a_j = 1$, $i, j \in \{N, S\}$, implying $\tau_S^{BAU} = 1/2$.
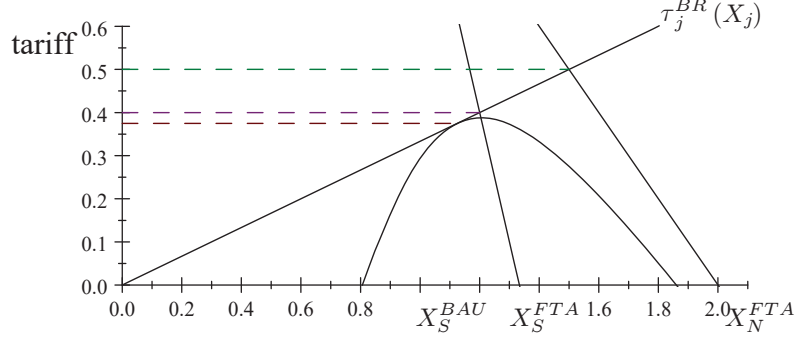
10

Figure 2: *S's indifference curve is tangent with N's best-response curve.*

More generally, combining (12) and (13), the BAU levels become:

$$X_N^{BAU} = \overline{v}_{NN} + \frac{\overline{v}_{SN}}{2} \text{ and } \tau_S^{BAU} = \frac{\overline{v}_{SN}}{2a_N}. \tag{14}$$

Equilibrium $\left(X_N^{BAU}, \tau_S^{BAU}\right)$ is also a Nash equilibrium if investors invest in X_N at the same time as (instead of before) τ_S is set. After all, N's investors do not attempt to influence τ_S.

In contrast, when setting X_S, S takes into account that a larger X_S increases τ_N and reduces p_S.

Even if we fixed τ_N, S would prefer to limit X_S in order to raise p_S. For a given τ_N, the Appendix shows that S's best response, X_S, to τ_N is:

$$X_S^{BR}(\tau_N) = \overline{v}_{SS} + \frac{\overline{v}_{NS} - a_S \tau_N}{3}. \tag{15}$$

Note that $X_S^{BR} < X_N^{MR}$, and $\partial X_S^{BR}/\partial \tau_N > \partial X_N^{MR}/\partial \tau_S$, if a_S and a_N are similar. The two functions are drawn in Fig. 2, together with $\tau_N^{BR}(X_S)$, for Example 1. The free-trade levels are given by $X_N^{FTA} = X_N^{MR}(0)$ and $X_S^{FTA} = X_S^{BR}(0)$.

More generally, if X_S and τ_N were decided on simultaneously, the Nash equilibrium would be:

$$X_S^{NASH} = \overline{v}_{SS} + \frac{\overline{v}_{NS}}{5} \text{ and } \tau_N^{NASH} = \frac{2\overline{v}_{NS}}{5a_S}. \tag{16}$$

With the timing in Fig. 1, S will also take into account the effect on τ_N. If S limits X_S further, N will set a smaller τ_N. The smaller τ_N contributes to a higher p_S. In equilibrium, X_S is small, relative to X_N, both because S takes into account the effect on the price, and because S attempts to motivate the trading partner to reduce the tariff.

Appendix B verifies that also if S can use production taxes or export tariffs, equilibrium X_S is limited because S attempts to improve its terms of trade.

PROPOSITION 1: *The SPE outcomes for X_N and τ_S are given by (14), while X_S and τ_N are:*

$$X_S^{BAU} = \overline{v}_{SS} + \frac{\overline{v}_{NS}}{8} \text{ and } \tau_N^{BAU} = \frac{3\overline{v}_{NS}}{8a_S}. \tag{17}$$

11

If there is no damage, each $X_i^{BAU} < X_i^{FB}(0)$ because $j \neq i$ cannot commit to $\tau_j = 0$. In addition, X_S^{BAU} is limited further because S internalizes the effect on the price (remember, $X_S^{BR}(\cdot) < X_S^{MR}(\cdot)$) and the effect on the tariff (so, $X_S^{BAU} < X_S^{BR}(\tau_N^{BAU})$). Thus, there are three reasons for X_S to be smaller than the first-best level if there is no damage.

Of course, if the marginal damage is sufficiently large, then $X_i^{FB} < X_i^{BAU}$. After all, equilibrium X_i^{BAU} does not vary with the damage.

3.2. *Free Trade*

An FTA is here defined as a commitment to zero tariffs before capacity investments are made. As observed above, the equilibrium (10) can coincide with the first best (7) only if $\tau_j = 0$. In addition, with a FTA, the equilibrium capacities will be larger than in BAU. For both reasons, the total consumer surplus increases after liberalization.

PROPOSITION 2: *Compared to BAU, the FTA increases X_N, X_S, and consumer surpluses:*

$$X_N^{FTA} = \overline{v}_{NN} + \overline{v}_{SN} > \overline{v}_{NN} + \frac{\overline{v}_{SN}}{2} = X_N^{BAU},$$

$$X_S^{FTA} = X_S^{BR}(0) = \overline{v}_{SS} + \frac{\overline{v}_{NS}}{3} > \overline{v}_{SS} + \frac{\overline{v}_{NS}}{8} = X_S^{BAU},$$

$$\left(U_N^{FTA} + U_S^{FTA}\right) - \left(U_N^{BAU} + U_S^{BAU}\right) = \frac{1}{8}\frac{\overline{v}_{SN}^2}{a_N} + \frac{1}{8}\frac{133}{144}\frac{\overline{v}_{NS}^2}{a_S}.$$

When the capacity is endogenous, it is easy to check that trade of the privately provided good doubles when we move from BAU to the FTA: c_{SN} increases from $\overline{v}_{SN}/2$ to $\overline{v}_{SN}$, while c_{NS} increases from $3\overline{v}_{NS}/8$ to $2\overline{v}_{NS}/3$.[20] These increases are in line with the empirical evidence from Baier and Bergstrand (2007), who show that trade doubles with the FTA, after a 10-year phase-in period (these years may be necessary to build the capacity).

Regarding the increases in capacity, consider, first, private investments in X_N. With a commitment to $\tau_S = 0$, N's investors expect higher demand, a higher price, and a larger return on a unit of capacity. As a result, the equilibrium capacity is $X_S^{FTA} = X_N^{MR}(0) > X_N^{BAU}$. As noted already, equilibrium X_N is first best if there is no tariff and no damage, so $X_S^{FTA} = X_N^{FB}(0)$. Consequently, if all investments were private, and there were no damage, the FTA would implement the first best.

Next, consider S's capacity. Even if $\tau_N = 0$, S limits X_S in order to improve its terms of trade: $X_S^{FTA} = X_S^{BR}(0) < X_S^{FB}(0)$. In fact, according to my model, S has a stronger incentive to manipulate its terms of trade when S exports a lot, as it does when $\tau_N = 0$. This prediction is empirically supported.[21] From (13) and (15), we see that $X_i^{MR}(\cdot)$ is a steeper function than is $X_i^{BR}(\cdot)$. This comparison explains why the FTA leads to a larger increase in X_N than in X_S, if $\overline{v}_{NS} = \overline{v}_{SN}$.

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[20] These numbers are found by combining X_i and τ_j, from Propositions 1 and 2, with (10) and (11). S's import doubles, but N's import increases by only 78% because S has a stronger incentive to limit X_S (to raise p_S) when N imports more.

[21] Copeland et al. (2022:121) survey the literature and write: "Several papers have found that reductions in tariffs tend to increase the use of non-tariff barriers."

COROLLARY 1: *The FTA has a greater influence on X_i when investments are private than when they are public.*

Nevertheless, the FTA leads to a larger X_S for two reasons. (i) When τ_N is no longer an increasing function of X_S, S no longer needs to limit X_S to keep τ_N from being raised to a level that is higher than τ_N^{BAU}. This effect corresponds to an increase from X_S^{BAU} to $X_S^{BR}\left(\tau_N^{BAU}\right)$. (ii) When τ_N is reduced, N demands more, the price (p_S) increases, and so does the return from making land available to agriculture. This effect corresponds to the increase from $X_S^{BR}\left(\tau_N^{BAU}\right)$ to $X_S^{BR}(0)$.

COROLLARY 2: *Compared to BAU, the FTA leads to a larger X_S (i.e., $X_S^{FTA} > X_S^{BAU}$).*

When there is no damage, the FTA is always valuable. Then, N's capacity is first best, and S's capacity is closer to the first best than with BAU. For the FTA to implement the first best in both markets, S must also privatize the investment decision. Interestingly, the increase in capacity following trade liberalization is comparable, in magnitude, to the effect of privatization.

Larger gains from trade, measured by $\overline{v}_{ji}$, lead to increases in X_i^{BAU}, X_i^{FTA}, and $X_i^{FTA} - X_i^{BAU}$.

COROLLARY 3: *If the gains from trade increase, X_S^{FTA} increases.*

It is easy to see that if the damages are linear and in line with (6), the value of the FTA is positive if and only if:

$$\frac{1}{2}\overline{v}_{SN}d_S' + \frac{5}{24}\overline{v}_{NS}d_N' < \frac{1}{8}\frac{\overline{v}_{SN}^2}{a_N} + \frac{1}{8}\frac{133}{144}\frac{\overline{v}_{NS}^2}{a_S}. \tag{18}$$

Hence, the FTA might not be valuable if the increase in X_i causes damage.

COROLLARY 4: *Compared to BAU, the value of the FTA is smaller if the damage is larger.*

3.3. Tariff Agreements

Now, the analysis in Section 3.2 is generalized in that the tariffs are not necessarily zero, although they are fixed from the beginning. Although free trade maximizes the sum of payoffs *ex post,* after the X_i's have been decided on, positive tariffs reduce capacity investments and the environmental externalities.

In fact, it is frequently argued that a border tax can be useful in order to reduce the environmental externality. While border taxes may be set noncooperatively, the best case for them (relative to BAU or FTA) permits the taxes to maximize the sum of payoffs. To evaluate the best case for border taxes, this section studies bilateral trade or tariff agreements (BTAs) where countries negotiate fixed tariffs (that maximize the sum of payoffs) before capacity investments are made. With Assumption 1, N and S will agree on tariffs that are optimal ex ante. Otherwise, the game is as before, and it can be solved by backward induction.

PROPOSITION 3: *If fixed tariffs are negotiated at the beginning of the game, the optimal levels are:*

$$\tau_S^* = \min\left\{\tau_S^{BAU}, d_S'\right\} \ and \ \tau_N^* = \min\left\{\tau_N^{NASH}, \frac{3d_N' - \overline{v}_{NS}/a_S}{5}\right\}. \tag{19}$$

Given these tariffs, equilibrium capacity levels follow from (13) and (15).[22]

Importantly, but perhaps unsurprisingly, price-taking investors in N should face a Pigouvian-like tariff.

More interestingly, the tariff facing S should be *less* than the Pigouvian level. There are two reasons for this result. First, S is voluntarily limiting X_S to improve its terms of trade. If d_N' is small, $X_S^{FTA} < X_S^{FB}$, and then (19) verifies that it would have been better to subsidize (rather than to tax) trade in S's good. Second, the comparison of (13) and (15) shows that a decrease in the tariff has a smaller effect when the capacity is decided on by the government instead of by private investors. As explained before Corollary 1, the intuition for this difference is that S benefits more from raising p_S (by withholding X_S) if S produces a lot (as when τ_N is small). Consequently, the effect from τ_N on X_S will be relatively small compared to the ex post distortions from the tariff.

The essence of the corollaries continues to hold: A given tariff reduction influences X_N more than it reduces X_S (Corollary 1), but trade liberalization (i.e., a lower τ_N) does raise X_S (Corollary 2). And, because S's voluntary reduction in X_S (compared to $X_S^{FB}(0)$) is larger when S exports a lot, a larger $\overline{v}_{NS}$ reduces τ_N^*. It follows that larger gains from trade cause more deforestation (Corollary 3). Hence, more liberalization (i.e., a lower τ_N) is socially optimal only if the damage is small (Corollary 4).

REMARK 1—RENEGOTIATION-PROOFNESS: The justification for min operators in (19) is that $\tau_S^* > \tau_S^{BAU}$ or $\tau_N^* > \tau_N^{NASH}$ would be ex post Pareto dominated. If the countries agreed on $\tau_S > \tau_S^{BAU}$, then, in equilibrium, $X_N = X_N^{MR}(\tau_S) < X_N^{BAU}$, implying that $\tau_S > \tau_S^{BR}(X_N)$. Thus, *both* countries would strictly benefit if they agreed to reduce τ_S. In other words, $\tau_S > \tau_S^{BAU}$ would not be renegotiation proof. Similarly, $\tau_N > \tau_N^{NASH}$ would not be renegotiation proof.[23]

Note that, with the restrictions that $\tau_S \in \left[0, \tau_S^{BAU}\right]$ and $\tau_N \in \left[0, \tau_N^{NASH}\right]$, a BTA cannot induce $X_i \leq X_i^{BAU}$, $i \in \{N, S\}$.

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[22] Unless the marginal damages are constant, they are functions of the stocks. E.g., with quadratic damage functions, and marginal damage $d_i' + d_i'' X_j$, we get:

$$\tau_S^* = \frac{d_S' + (\overline{v}_{NN} + \overline{v}_{SN}) d_S''}{1 + a_N d_S''} \ and \ \tau_N^* = \frac{3d_N' + (3\overline{v}_{SS} + \overline{v}_{NS}) d_N'' - \overline{v}_{NS}/a_S}{5 + a_S d_N''}.$$

[23] Three remarks are in order: (1) If $\tau_N > \tau_N^{NASH}$, defined by (16), then $X_S = X_S^{MR}(\tau_N) < X_S^{NASH}$. With such a small X_S, both N and S benefit from reducing the tariff from τ_N to $\tau_N^{BR}\left(X_S^{MR}(\tau_N)\right)$. (2) In principle, a second justification for min operators in (19) is that constraint $X_N \geq X_N^0$ implies that there is no point of setting τ_S so large that $X_N^{MR}(\tau_S) < X_N^0$, since a smaller tariff will reduce ex post distortions, without affecting X_N. Similarly, there is no point of setting τ_N so large that $X_S^{BR}(\tau_N) < X_S^0$. However, given the assumption $X_i^0 \leq X_i^{BAU}$, the renegotiation-proofness constraint binds before constraints $X_N^{MR}(\tau_S) \geq X_N^0$ and $X_S^{BR}(\tau_N) \geq X_S^0$. (3) I do not permit side transfers at the renegotiation stage. If the parties could renegotiate using side transfers, then every strictly positive tariff would be renegotiated (since it is distortionary ex post). This is inconsistent with real-world facts. Even if we did allow for side payments at the renegotiation stage, however, the effect of the initially set tariff on investments is exactly as in my analysis if we let country i have all the bargaining power when τ_i is renegotiated. This is in line with the assumption by Guriev and Klimenko (2015:1833), who write: "During each period, parties can renegotiate the previously concluded agreements. All the bargaining power belongs to the home country." If both countries have strictly positive bargaining power when τ_i is renegotiated, the effects of τ_i on X_j are quantitatively different, but the results will hold, qualitatively.

4.1. *Contingency and Credibility*

In general, traditional trade agreements cannot implement the first best. If there is no damage, X_N^{FTA} is first best but X_S^{FTA} is inefficiently small. With damage, the second-best tariff characterized by Proposition 3 trades off the deadweight loss from unequal marginal benefits with the effect on conservation.

In the trade agreements considered above, N and S committed to zero or fixed tariffs before capacity investments. In BAU, equilibrium tariffs were contingent on X_i. When X_i is verifiable, N and S may be able to consider *how* the tariffs should be contingent on X_i. If the contingent tariffs are unattractive to S when X_S is large, S can be motivated to keep X_S small.

To be realistic, we must require the contingent tariffs to be credible, or renegotiation proof, as discussed in Remark 1. If, at some X_i, $\tau_j^{CTA}(X_i) > \tau_j^{BR}(X_i)$, then both N and S benefit if j's tariff is reduced. Such a contingency would not be credible.

In addition, one may argue that a pair of tariffs is not renegotiation proof if $\tau_N \tau_S > 0$, because with $\tau_N \tau_S > 0$ it would be possible, ex post, to reduce both tariffs in a way that would make both countries better off. If only τ_j is positive, and $\tau_j \leq \tau_j^{BR}(X_i)$, the pair is Pareto optimal in that no other tariff pair can make both parties better off. In line with Bond and Park (2002:397), then: "no renegotiation takes place over the life of the agreement because the payoff of the two parties is always on the utility possibility frontier."

DEFINITION 1: *A contingent trade agreement (CTA) specifies $\tau_j \in \left[0, \tau_j^{BR}(X_i)\right]$, with $\tau_N \tau_S = 0$, for every $X_S \geq X_S^0$, $X_N \geq X_N^0$, and $i, j \in \{N, S\}$.*

The main point of this paper is to show that Corollaries 1–6 are all reversed when the countries can sign a CTA.

The reversal of Corollary 1 follows from Definition 1. The investors in N are assumed to be tariff- and price-takers. As before, N's equilibrium capacity will be characterized by $X_N^{MR}(\cdot)$, as a function of the expected and equilibrium tariff, and not of any hypothetical tariff at an out-of-equilibrium X_N. The socially optimal τ_S, given this market response, is as given by Proposition 3. The contingency has no role to play when investors are price-takers.

In contrast, when S decides on X_S, S takes into account how tariffs vary with X_S. Thus, S can be induced to select $X_S \neq X_S^{FTA}$, even if $\tau_N^{CTA}(X_S) = 0$, if the contingent tariffs are less attractive at other capacity levels.

COROLLARY 1^{CTA}: *The contingency can influence X_i when investments are public, but not when they are private.*

Consequently, there is no loss (of generality) from letting the CTA tariffs be contingent only on X_S.

4.2. *CTA with Free Trade*

As noted already, the first best requires that there be no tariff on the equilibrium path. Thus, we start by characterizing what is feasible with an agreement, CTA^0, which is restricted in that trade must be free when X_S takes its equilibrium value, X_S^{CTA}.

Feasible CTA^0s. The free-trade requirement is essentially equivalent to a requirement that only τ_N, and not τ_S, will be contingent on X_S. To see this, note that S generally prefers a larger τ_S after X_N has been decided on. If we require $\tau_S \left(X_S^{CTA} \right) = 0$, it can only be harder to implement X_S^{CTA} if we permit $\tau_S > 0$ when $X_S \neq X_S^{CTA}$. Thus, the best CTA^0, in this situation, allows τ_S to be independent of X_S. This independence implies that there is no linkage between the two markets. In fact, the analysis in this subsection is unchanged if τ_S is fixed at any other level, not necessarily zero. (In particular, the socially optimal non-contingent τ_S is characterized by Proposition 3.)

PROPOSITION 4: *Every $X_S \geq X_S^0$ can be implemented by a CTA^0 if and only if $X_S \in \left[\underline{X}, \overline{X} \right]$, where*

$$\underline{X} = X_S^{FTA} - \overline{v}_{NS}\sqrt{10}/6 = \overline{v}_{SS} - \frac{\overline{v}_{NS}}{3}\left(\sqrt{\frac{5}{2}} - 1 \right) \approx \overline{v}_{SS} - 0.19\overline{v}_{NS},$$

$$\overline{X} = X_S^{FTA} + \overline{v}_{NS}\sqrt{10}/6 = \overline{v}_{SS} + \frac{\overline{v}_{NS}}{3}\left(\sqrt{\frac{5}{2}} + 1 \right) \approx \overline{v}_{SS} + 0.86\overline{v}_{NS}.$$

To implement $X_S^{CTA} \geq X_S^0$, it must be that for every other $X_S \geq X_S^0$, $\tau_N^{CTA}(X_S)$ must be so large that S prefers the pair $\left(X_S^{CTA}, \tau_N = 0 \right)$ to $\left(X_S, \tau_N^{CTA}(X_S) \right)$. To motivate S to conserve $X_S^{CTA} < X_S^{FTA}$, the tariff on S's export must be larger when X_S is large. Fig. 3(a) illustrates that multiple out-of-equilibrium tariff functions can implement $X_S = \underline{X}$. To motivate S to increase the production capacity, if that is socially optimal, the tariff must be larger as long as X_S remains small (Fig. 3(b)).

There is a lower and an upper boundary on implementable X_S's because of the renegotiation constraint. For $X_S < \underline{X}$, there is no pair $\tau_N \in \left[0, \tau_N^{BR}(X_S) \right]$ and $\tau_N' \in \left[0, \tau_N^{BR}\left(X_S^{BAU} \right) \right]$ so that S prefers (X_S, τ_N) to $\left(X_S^{BAU}, \tau_N' \right)$, as can be seen from Fig. 3.

It is easy to check, however, that the feasibility set $\left[\underline{X}, \overline{X} \right]$ includes X_S^{BAU}, X_S^{FTA}, and every X_S implementable by the BTA in Section 3.3. Thus, all these traditional trade agreements are dominated by some CTA^0.

The Optimal and Equilibrium CTA^0. Given the possibilities described by Proposition 4, it is straightforward to characterize an optimal agreement. If $X_S^{FB} \in \left[\underline{X}, \overline{X} \right]$, N and S will find it optimal to sign a CTA with $\tau_N^{CTA}\left(X_S^{FB} \right) = 0$. Consequently, the CTA implements the first-best allocation and production
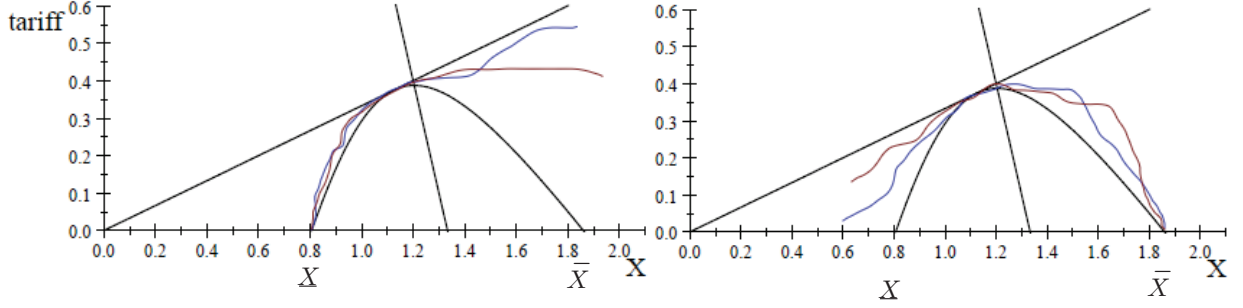
Figure 3: *Multiple credible tariff schedules can support the lowest possible X_S.*

of X_S if $\underline{X} \leq X_S^{FB} \leq \overline{X}$. With (9), these inequalities can be written as follows if the damage is linear:[24]

$$\frac{0.07}{a_S} \leq d_N' \leq \frac{0.60}{a_S}.$$

If the damage is so small that $X_S^{FB} > \overline{X}$, then the optimal and equilibrium CTA^0 implements $X_S = \overline{X}$ with free trade. If, instead, the damage is so large that $X_S^{FB} < \underline{X}$, the optimal and equilibrium CTA^0 implements $X_S = \underline{X}$ with free trade. In either case, the CTA^0 leads to larger payoffs than do the FTA and any other fixed-tariff agreement.

The rest of this subsection considers the case in which X_S is suboptimally large with BAU (i.e., $X_S^{FB} < X_S^{BAU}$). In this situation, all corollaries are reversed with the CTA:

COROLLARY 2^{CTA}: *The CTA^0 implements a smaller X_S than with BAU (i.e., $X_S^{CTA} < X_S^{BAU}$).*

Next, note that $\underline{X}$ decreases, while $\overline{X}$ increases, in $\overline{v}_{NS}$. Intuitively, if the gains from trade ($\overline{v}_{NS}$) increase, S has more to lose from a large τ_N. The potential loss makes S willing to select an X_S that is very different from X_S^{FTA}, if that is necessary to obtain the most attractive terms of trade. This willingness reverses the essence of Corollary 3.

COROLLARY 3^{CTA}: *If the gains from trade increases, X_S^{CTA} decreases.*

Because the CTA^0 can motivate more conservation than BAU (in contrast to the FTA), the value of the CTA^0 is larger if the damage is large, so that conservation is more valuable. The insight of Corollary 4 is thus reversed:

COROLLARY 4^{CTA}: *If the damage is larger, the value of the CTA^0 is larger.*

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[24]When these inequalities hold, CTA^0 implements the first best in *both* markets if there is no damage from N's good, i.e., if $d_S' = 0$. However, note that the first inequality fails when $d_N' = 0$. In this case, $\overline{X}_S < X_S^{FB}(0)$, and the first best cannot be implemented by CTA^0 in this simple model. The CTA can implement $X_S^{FB}(0)$ when we introduce cross contingency (Section 4.3) or multiple products (Section 5.1).

The CTA permits carrots as well as sticks. That is, S can be motivated to select a socially desirable X_S not only by the threat that S will otherwise face a larger tariff on its own product, as studied above, but also by the possibility to set a positive tariff on the goods imported from N. As discussed above, S's terms of trade are improved with $\tau_S > 0$. To take advantage of this carrot, or cross contingency, we will now establish and draw on a linkage between the two markets.

Feasible CTAs. By definition, the CTA allows the positive τ_S to be conditioned on X_S. Thus, S can be induced to stay with $X_S^{CTA} < \underline{X}$, to be allowed $\tau_S\left(X_S^{CTA}\right) > 0$, if the tariff τ_S is smaller at alternative X_S's. In other words, the range of X_S's that can be supported by a CTA is larger when we permit a contingent $\tau_S > 0$ on the equilibrium path. As in Section 3.3, it is not credible with $\tau_S > \tau_S^{BAU}$.

PROPOSITION 5: *Every $X_S \geq X_S^0$ can be implemented by a CTA if and only if $X_S \in \left[\underline{X}\left(\tau_S\right), \overline{X}\left(\tau_S\right)\right]$, where:*

$$\underline{X}\left(\tau_S\right) = \overline{v}_{SS} - \frac{\overline{v}_{NS}}{3}\left(\sqrt{\frac{5}{2} + \frac{a_S}{\overline{v}_{NS}^2}\left(12\overline{v}_{SN} - 15a_N\tau_S\right)\tau_S} - 1\right),$$

$$\overline{X}\left(\tau_S\right) = \overline{v}_{SS} + \frac{\overline{v}_{NS}}{3}\left(\sqrt{\frac{5}{2} + \frac{a_S}{\overline{v}_{NS}^2}\left(12\overline{v}_{SN} - 15a_N\tau_S\right)\tau_S} + 1\right).$$

For every τ_S, N's capacity follows from $X_N = X_N^{MR}\left(\tau_S\right)$.

If the permitted τ_S increases marginally from 0, S's benefit from the CTA increases, and S accepts a larger range of X_S's. Consequently, $\underline{X}\left(\tau_S\right)$ decreases, and $\overline{X}\left(\tau_S\right)$ increases.

From Section 3, we know that S does not prefer an arbitrarily large τ_S. It is easy to check that $\underline{X}\left(\tau_S\right)$ is minimal, and $\overline{X}\left(\tau_S\right)$ maximal, at

$$\tau_S^M \equiv \arg\min_{\tau_S} \underline{X}\left(\tau_S\right) = \arg\max_{\tau_S} \overline{X}\left(\tau_S\right) = \frac{2}{5}\frac{\overline{v}_{SN}}{a_N} \Rightarrow$$

$$\underline{X}^M \equiv \min_{\tau_S} \underline{X}\left(\tau_S\right) = \overline{v}_{SS} - \frac{\overline{v}_{NS}}{3}\left(\sqrt{\frac{5}{2} + \frac{12}{5}\frac{a_S}{a_N}\frac{\overline{v}_{SN}^2}{\overline{v}_{NS}^2}} - 1\right) \approx \overline{v}_{SS} - 0.40\overline{v}_{NS} \text{ if } \overline{v}_{SN}^2/a_N \approx \overline{v}_{NS}^2/a_S,$$

$$\overline{X}^M \equiv \max_{\tau_S} \overline{X}\left(\tau_S\right) = \overline{v}_{SS} + \frac{\overline{v}_{NS}}{3}\left(\sqrt{\frac{5}{2} + \frac{12}{5}\frac{a_S}{a_N}\frac{\overline{v}_{SN}^2}{\overline{v}_{NS}^2}} + 1\right) \approx \overline{v}_{SS} + 1.07\overline{v}_{NS} \text{ if } \overline{v}_{SN}^2/a_N \approx \overline{v}_{NS}^2/a_S.$$

If $\tau_S > \tau_S^M$, the tariff is not maximizing S's payoff from τ_S relative to $\tau_S = 0$, and thus the set of implementable X_S's is smaller when $\tau_S^{CTA} > \tau_S^M$ than when $\tau_S = \tau_S^M$. For Example 1, where $\tau_S^M = 0.4$ and $\tau_S^{BAU} = 0.5$, Fig. 4 illustrates how $\underline{X}\left(\tau_S\right)$ and $\overline{X}\left(\tau_S\right)$ vary with $\tau_S \in \left[0, \tau_S^{BAU}\right]$. The vertical axis measures $\tau_N - \tau_S$, so τ_S is measured on the negative vertical axis.[25]

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[25] The τ_S that motivates the smallest X_S, τ_S^M, is less than τ_S^{BAU}. The intuition for why $\tau_S^M < \tau_S^{BAU}$ is that when τ_S is reduced below τ_S^{BAU}, X_N increases, and the larger X_N increases the cost for S if τ_S is lowered as a consequence of $X_S \neq X_S^{CTA}$.
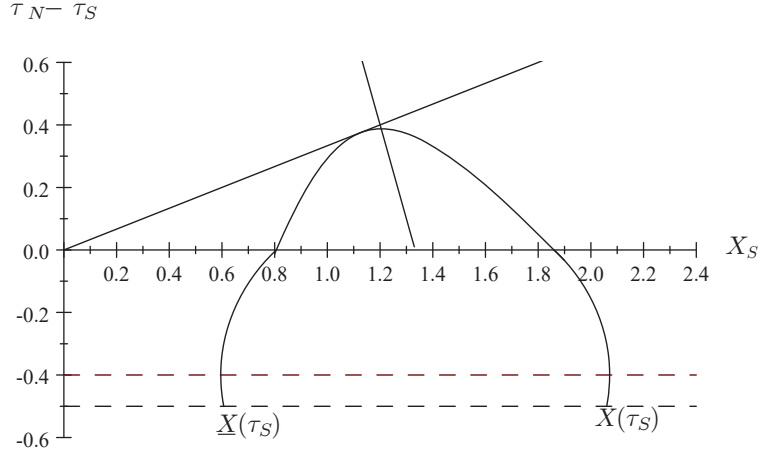
Figure 4: *For every $\tau_N - \tau_S$, the CTA can implement every X_S within S's indifference curve.*

The Optimal and Equilibrium CTA. When N and S can use side transfers when they negotiate the CTA, the equilibrium CTA will be the optimal one, which maximizes the sum of payoffs. Because a small τ_N is both minimizing ex post distortions, and preferred by S, the optimal CTA ensures that $\tau_N = 0$, in equilibrium. The remaining question regards the level of τ_S.

If $X_S^{FB} \in \left[\underline{X}(d_S'), \overline{X}(d_S') \right]$, there is no trade-off. X_S^{FB} can be implemented with $\tau_S^* = d_S'$, which is optimal according to Proposition 3. Interestingly, this scenario is more likely with a large $d_S' \in \left(0, \tau_S^M\right)$. In this case, the large damage from N's production justifies a large tariff. With the (out-of-equilibrium) threat that this tariff will be reduced if $X_S \neq X_S^{FB}$, S becomes more willing to stick with X_S^{FB}.

If $X_S^{FB} \notin \left[\underline{X}(d_S'), \overline{X}(d_S') \right]$, there is a trade-off. Even if the CTA *can* implement $\underline{X}^M$ with $\tau_S = \tau_S^M$, this is suboptimal, no matter how much damage N faces (unless $d_S' = \tau_S^M$). Instead, the optimal τ_S trades off the distortion in the market for N's good with the value of conserving X_S. If $\tau_S > \tau_S^*$ increases, the marginal distortion increases, but the marginal impact on $\underline{X}(\tau_S)$ declines. When $\tau_S \to \tau_S^M$, $\partial \underline{X}(\tau_S)/\partial \tau \to 0$. Consequently, the optimal τ_S^{CTA} is less than τ_S^M, if $\tau_S^* < \tau_S^M$.

PROPOSITION 6:

(i) If $X_S^{FB} \in \left[\underline{X}(\tau_S^*), \overline{X}(\tau_S^*) \right]$, the optimal CTA implements $\tau_S^{CTA} = \tau_S^*$, $\tau_N = 0$, X_S^{FB}, and $X_N^{MR}(\tau_S^*)$.

(ii) If $X_S^{FB} \notin \left[\underline{X}(\tau_S^*), \overline{X}(\tau_S^*) \right]$, the optimal τ_S^{CTA} is strictly between τ_S^* and τ_S^M.

(iii) Assume (a) $X_S^{FB} < \underline{X}(\tau_S^*)$ and (b) $d_S' < \tau_S^M$. With the optimal CTA, $X_S^{CTA} \in \left(X_S^{FB}, \underline{X}_S \right)$. If d_N' or d_S' increases, the optimal τ_S^{CTA} increases, and both X_N^{CTA} and X_S^{CTA} decrease.

Assumptions (a) and (b) hold if the damage from S's capacity expansion is large, while the damage from N's production is small. In this case, the trade-off is the following. On the one hand, by increasing τ_S above τ_S^*, $\underline{X}(\tau_S)$ is reduced, and more can be conserved. On the other hand, the larger τ_S creates ex post distortions in the market for N's product, and it reduces the incentives to invest in X_N. The optimal

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τ_S^{CTA} trades off these two concerns. If d_S' increases, the *cost* of a given $\tau_S > d_S'$ is smaller, and it becomes socially optimal to increase τ_S. In other words: A larger damage associated with N's product makes it optimal to conserve more in country S if $X_S^{FB} < \underline{X}(d_S')$. If, instead, d_N' increases, then, everything else equal, the *benefit* from raising τ_S is larger, while the cost of raising τ_S is unchanged. Again, it becomes socially optimal to increase τ_S.

Assumptions (a) and (b) are relaxed in the Appendix, where I derive the optimal CTA quite generally.

5. GENERALIZATIONS

Before the model can be calibrated in a reasonable way, it is necessary to permit a number of generalizations.

5.1. *Multiple Goods*

The scope for the CTA is strengthened if countries produce multiple types of goods. If each good is distinct, and satisfies a unique term in the quasilinear utility function, then each market can be modeled as in Section 2, and analyzed as in Section 3. The larger the number of goods is, the larger the gains from trade are.

With Section 4's CTA, S can be offered low tariffs on several goods contingent on the socially desirable conservation level. Even if S's other goods are privately provided, S is willing to limit the expansion of the agricultural sector (X_S) if that is necessary to avoid revenue losses in the private sector. To implement a socially desirable X_S^{CTA}, S will be offered zero tariff on all its products when $X_S = X_S^{CTA}$, but higher tariffs on all products if $X_S \neq X_S^{CTA}$.

Just as in Section 4.3, the CTA is further strengthened if we permit contingent positive tariffs on the goods that S import. The larger the number of goods that S imports from N is, the larger the potential loss for S is if S selects an $X_S \neq X_S^{CTA}$ that will lead to a reduction in the tariff levels on S's imported goods. Therefore, the feasibility set expands when there are many products: See Online Appendix C for details.

5.2. *Multiple Consumers*

The proofs in the Appendix permit the mass of consumers to be m_N in N, and m_S in S. If we fix the ratio m_N/m_S, then capacity levels in BAU, the FTA, and the CTA's boundaries, $\underline{X}$ and $\overline{X}$, are all proportional to $m \equiv m_N + m_S$: When the population sizes double, these capacity levels double.

One way of learning about the effect of the relative sizes is to fix m, and consider changes in $r_N \equiv m_N/m \in (0,1)$. For Example 1, where $m = 2$, Fig. 5(a) illustrates that when r_N increases, then X_S^{FTA}
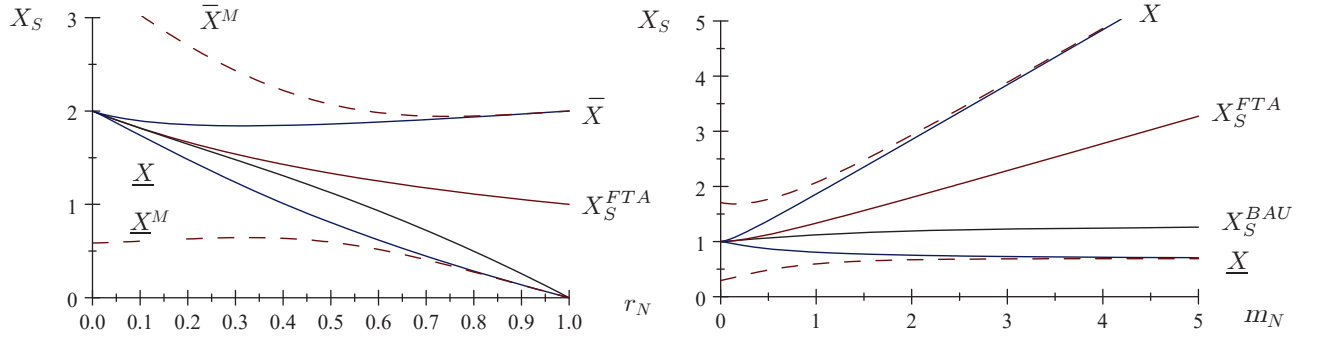
Figure 5: *Cross-contingency is especially important if (a) r_N is small or (b) m_N is small.*

decreases towards the monopoly quantity (which is 1). The intuition is that when S mainly produces for the international market, S becomes more willing to withhold X_S in order to raise the price. When $r_N \uparrow 1$, $\tau_N^{BAU} \uparrow \infty$, so $X_S^{BAU} \downarrow 0$, in line with the holdup problem.

When r_N is large, it is more expensive for S that τ_N is large, and S accepts a larger range of X_S's in return for $\tau_N = 0$. Therefore, $\underline{X}$ decreases, while $\overline{X}$ increases, in r_N.

The flip side of this logic is that when r_N is *small*, N's tariff is less important for how much the CTA can motivate S to conserve. That is, the "stick" $\tau_N > 0$ is less effective. In this case, it is instead more effective to use the "carrot" by allowing S to introduce a tariff on the import from N. This instrument is especially effective when r_N is small, because when S is the main market for N's product, S's tariff has a large influence on the equilibrium price on N's product.

This insight is confirmed in Fig. 5(b), where m_S is fixed while m_N increases along the horizontal axis. A larger m_N leads to a larger X_S^{BAU} and X_S^{FTA} because the total number of consumers increases.[26] Nevertheless, $\underline{X}$ decreases in m_N. That is, the CTA succeeds in conserving more if m_N is large. The intuition is that, when m_N is large, it is more important for S to maintain $\tau_N = 0$.

5.3. Multiple Countries

The analysis in the Appendix allows for multiple countries. This is relevant in the EU–Mercosur context, because Mercosur exports to many countries. To include them, I henceforth assume that "our" northern country, N, is just one of n countries importing S's good. Each of them exports a unique good to all the others, just as modeled in Section 2.

The Appendix allows the countries to include different masses of consumers. For the sake of illustration, assume here that all n countries are identical and with consumer mass 1, like S's consumer mass. If

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[26]Naturally, X_S^{FTA} approaches the monopoly quantity when m_N is large. If m_N is relatively small, S produces more than the monopoly quantity to reduce the domestic price.
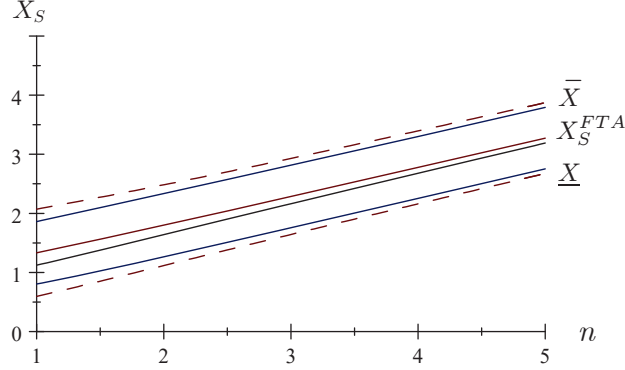
Figure 6: *CTA^0 continues to motivate more conservation than with BAU, also if n is large.*

there were no damage, the first best would be $X_S = \overline{v}_{SS} + n\overline{v}_{NS}$. With free trade, equilibrium X_S^{FTA} is as in Fig. 5(b), if just m_N is replaced by n.

Appendix C allows all countries to set their tariffs strategically. Here, suppose that the $n-1$ other importers, except for our country, N, trade freely with S. In this case, the BAU outcome converges to the FTA outcome if n is large. The reason is that τ_N^{BAU} decreases in n, because N can influence the price less by its tariff when n is large.

With the lower equilibrium tariff, and with multiple buyers of beef, one may at first guess that a unilateral tariff is less effective in securing conservation (this is the finding by by Hsiao, 2022, for instance). Surprisingly, CTA^0 can motivate *more* conservation relative to BAU when n is large, even if CTA^0 is signed bilaterally between S and N only, and even if it implements free trade in equilibrium. Fig. 6 illustrates how X_S^{BAU}, X_S^{FTA}, $\underline{X}$, $\overline{X}$, $\underline{X}^M$, and $\overline{X}^M$ increase in n. A careful look at the figure (and the Appendix) discloses that when n increases, $X_S^{FTA} - X_S^{BAU}$ decreases (in line with the discussion above), but $X_S^{BAU} - \underline{X}$ increases, so the CTA has a larger effect (relative to BAU) when n is large.

To understand the intuition for this result, consider, again, Fig. 2. When n is large, τ_N^{BR} is flatter, as a function of X_S, than when $n = 1$. Therefore, S's indifference curve is flatter at X_S^{BAU}, and thus S finds it inexpensive to reduce X_S from X_S^{BAU}. In other words, S is willing to reduce X_S by quite a lot, relative to X_S^{BAU}, in return for a decrease in N's tariff.

The cross-contingency discussed by Proposition 5 is less important when n is large, however. Because the tariffs are ex post distortionary (in that marginal utilities are unequal), it is generally less desirable to introduce a tariff at home than to eliminate the tariff abroad. This is especially true when n is large, because the distortions from the tariff are very large when there are many other importers that can purchase the good. Thus, the added value of cross-contingency (the red dashed lines) is smaller when n is large.
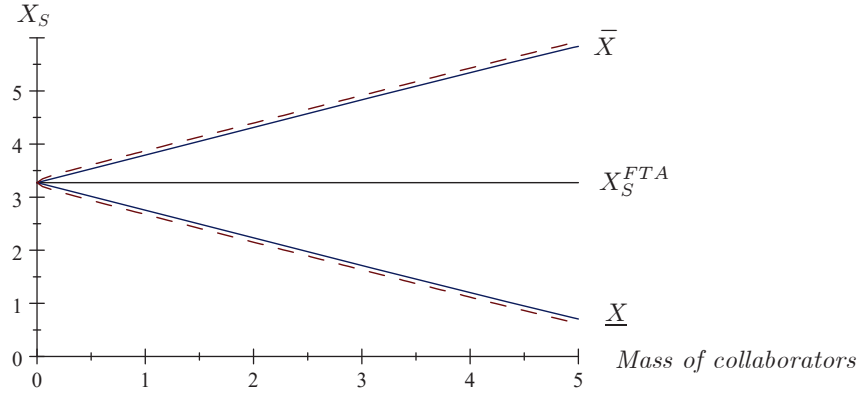
Figure 7: *The larger is the mass in countries offering the CTA, the more can be conserved.*

5.4. *Multiple Collaborators*

Naturally, the CTA can achieve more conservation if multiple countries collaborate in offering preliminary tariffs contingent on S's capacity to produce. The formulae derived in the Appendix allow the CTA-participant to include any mass of consumers. If two countries collaborate by jointly offering S low tariffs contingent on its capacity remaining low, the effect of the CTA^0 is equivalent to N having a larger mass, in this model. With cross contingency, S is permitted to introduce a positive tariff on two imported goods, contingent on X_S, when two beef importers collaborate on the CTA in addition to S.

Fig. 7 fixes the mass of S's foreign consumers at 5 (relative to S's mass of consumers). Then, $X_S^{FTA} = 3.27$. The (blue) solid curves show that $\underline{X}$ declines, and $\overline{X}$ increases, if the mass of consumers included in the CTA-participating importers increases. The (red) dashed lines illustrate $\underline{X}^M$ and $\overline{X}^M$. As discussed in the previous subsection, the additional effect from cross contingency is small when n is as large as 5, but the additional effect increases somewhat with the number of CTA-collaborators, because S can then introduce a positive tariff on a larger number of goods.

6. CALIBRATION

It is beyond the scope of this paper to provide a serious calibration and quantitative analysis. However, the formulae in the Appendix rest on few parameters, they permit all extensions in Section 5, and they state predictions for equilibrium tariff levels and export ratios. When these predictions are matched with empirical observations, we can estimate the parameters in the model. With the parameters, we can proceed by deriving the quantitative effects of trade liberalization and of the CTA.

Calibration. To provide a vague idea of the promise for such an exercise, consider the agricultural export from Brazil. Brazil and the EU are major trading partners. Most of the EU's consumed soy is

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imported from Brazil, and 81% of the EU's beef import is from Mercosur (36% from Brazil).[27]

As a start, I begin by assuming that there are n identical importing blocks with equal population masses. Brazil's (S's) mass can be different. I will require that the equilibrium BAU tariff in each trading partner be 20%. After all, the tariff on high-quality beef from Brazil is 20% in Europe,[28] 24% in the US,[29] 30% in India,[30] and 15% in China[31] and Russia.[32] A tariff on 20% is also consistent with numbers from the World Bank, the WTO, and earlier calibrations.[33]

Further, let's require that the BAU export fraction be 43%. This number is a weighted average of the soy export fraction (63%)[34] and the beef export fraction (25%),[35] where the weights reflect the values of the two sectors (the value of the soy production is 35% and the value of the beef production is 38% of Brazil's total agricultural production). This number is also similar to the total agricultural export fraction, according to the Brazilian government.[36] The transportation costs are very low,[37] so I will ignore them, but I take into account that existing non-tariff barriers on food seem to be about 20% (Cadot et al., 2018).

Appendix D shows that when the model is calibrated to match these numbers, then, approximately, $n \approx 5$ and $r_S \equiv m_S/m \approx 1/2$, where m_S is the mass of S's consumers buying S's product, and m is the total mass of consumers buying S's product. Conversely, if $n = 5$ and $r_S = 1/2$, the BAU equilibrium predicts that the tariffs are 21% and S exports 45% of its own production.

The result that there are 5 major importing blocks, in addition to Brazil's market, is in line with Ossa (2011).[38] The result $r_S = 1/2$ seems high, but reflects the assumption that only S produces beef. In reality, much of the demand in the importing countries is saturated by other/domestic producers. In this light, it is not that odd if Brazil contains half of the consumers with demand not yet saturated by other producers.

Results. With these numbers, some of the model's predictions are shown in Table 1. That is, moving

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[27] https://estadisticas.mercosur.int/

[28] https://trade.ec.europa.eu/access-to-markets/en/results?product=0201300031&origin=BR&destination=DE

[29] https://hts.usitc.gov/current

[30] https://www.exportgenius.in/india-import-duty/hscode-0201.30

[31] https://www.wto.org/english/res_e/statis_e/daily_update_e/tariff_profiles/CN_e.pdf

[32] https://www.fas.usda.gov/data/russia-russia-mulls-replacing-its-beef-trqs-tariff-2022

[33] See Table 3 and 4 in Ossa (2011). According to the World Bank, the weighted average tariff for food products from Brazil was 23.34 % in 2019:
https://wits.worldbank.org/CountryProfile/en/Country/WLD/StartYear/1988/EndYear/2019/TradeFlow/Import/Indicator/AHS-WGHTD-AVRG/Partner/BRA/Product/all-groups
According to the WTO, the simple average tariff lines on agricultural products are 12.2% for China, 15.2% for EU, 17.1% for Japan, and 7.5% for the US:
https://www.wto.org/english/res_e/statis_e/daily_update_e/tariff_profiles/BR_E.pdf
However, the marginal tariff is higher: The WTO writes that the final bound is 35.4%: https://www.wto.org/english/thewto_e/countries_e/brazil_e.htm#statistics

[34] https://apps.fas.usda.gov/newgainapi/api/Report/DownloadReportByFileName?fileName=Oilseeds%20and%20Products%20Update_Brasilia_Brazil_07-01-2021.pdf

[35] https://apps.fas.usda.gov/newgainapi/api/Report/DownloadReportByFileName?fileName=Livestock%20and%20Products%20Annual_Brasilia_Brazil_08-15-2021.pdf

[36] In a typical month, such as in January, 2021, the agricultural export fraction was 41.76% according to
https://www.gov.br/agricultura/pt-br/assuntos/politica-agricola/todas-publicacoes-de-politica-agricola/agrofoco/2021

[37] It is 3% on Brazilian pork: https://usdabrazil.org.br/wp-content/uploads/2021/11/Livestock-and-Products-Annual_Brasilia_Brazil_08-15-2021.pdf

[38] Ossa (2011:125) writes: "I focus on Brazil, China, the European Union, India, Japan, and the United States since these countries are typically considered to be the main players in GATT/WTO negotiations."

from BAU to free trade (with no non-tariff barriers) increases X_S by almost 5%. If one of the five importing blocks agrees on a CTA^0 with S, however, the entire increase can be avoided. (In fact, the CTA^0 can motivate more conservation than in BAU.)

X_S^{FTA}/X_S^{BAU}	$\underline{X}/X_S^{BAU}$
1.048	0.963

Table 1: With free trade, X_S increases by 4.8%, but the CTA can prevent the increase.

Doubled Demand From Asia. From now, I refer to the above level of X_S^{BAU} as X_S^0, because I will consider a change in parameters. After all, the increase in X_S, following trade liberalization, does not capture how fast Brazil's exports have increased over the last few years.[39] Because of economic growth in Asia, and other factors, the mass of relevant consumers in the importing countries, relative to S's mass of consumers, has increased sharply.

To understand the effects of a similar development in the next few years, suppose that the mass of relevant consumers in each of three of the five importing blocks doubles in size. (The number of relevant consumers in the EU, the US, and in Brazil is held constant.) This doubling is not unreasonable: Beef export from Brazil to China more than doubled between 2018 and 2020.[40] As a consequence, Brazil's total beef exports increased from USD 5.3b in 2015 to 8.1b in 2020.[41]

I consider seven scenarios: (F) free trade, (1^0) the EU (which now has 1/13 of the relevant consumers) offers CTA^0, (1) the EU offers a CTA with cross-contingency, (2^0) the EU and the US (with 2/13 of the consumers) coordinate on CTA^0, (2) the EU and the US coordinate on a CTA with cross-contingency on the export from both the EU and the US, (3^0) the EU, the US, and one of the third importers (with doubled consumer mass) coordinate on CTA^0, and (3) same as with (3^0), but with cross contingency.

When I permit cross-contingency, I let S export a second (privately provided) good, in addition to beef, as discussed in Section 5.1. For Brazil's export, manufacture is as important as agriculture.[42] Analogously, each beef importer exports two types of goods to the rest of the world. For simplicity, every good has the same value/characteristics as S's beef, except that they are privately provided.[43]

The effects on $\underline{X}$, as a function of the number of collaborators, relative to X_S^0, are derived in Appendix C and presented here:

(F)	(1^0)	(1)	(2^0)	(2)	(3^0)	(3)
X_S^{FTA}/X_S^0	$\underline{X}/X_S^0$	$\underline{X}^M/X_S^0$	$\underline{X}/X_S^0$	$\underline{X}^M/X_S^0$	$\underline{X}/X_S^0$	$\underline{X}^M/X_S^0$
1.27	1.18	1.14	1.10	1.03	0.93	0.82

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[39] Brazil's beef production is expected to increase sharply according to:
https://www.ers.usda.gov/amber-waves/2019/july/brazil-once-again-becomes-the-world-s-largest-beef-exporter/
[40] https://apps.fas.usda.gov/newgainapi/api/Report/DownloadReportByFileName?fileName=Livestock%20and%20
Products%20Annual_Brasilia_Brazil_08-15-2021.pdf
[41] https://apps.fas.usda.gov/newgainapi/api/Report/DownloadReportByFileName?fileName=Livestock%20and%20
Products%20Annual_Brasilia_Brazil_08-15-2021.pdf
[42] https://www.wto.org/english/thewto_e/countries_e/brazil_e.htm#statistics
[43] That is, after the growth in Asia, the EU includes 5/13 of the consumers demanding EU's products, while Brazil includes 1/13.

Table 2: X_S can increase by 27%, or by only 10% if the EU and the US offer a CTA^0.

Thus, with free trade, scenario (F), and larger demand from Asia, the model predicts that X_S will increase by 27%. The increase in X_S falls by 9 percentage points in scenario (1^0), where one country offers CTA^0, and by 13 percentage points with cross contingency, i.e., scenario (1). Consequently, the EU's action alone can have a significant effect. If the EU acts in collaboration with the US, most of the expansion can be avoided. With three or more collaborators, X_S does not increase.

Such collaboration is highly relevant. In January, 2021, Bruce Babbitt, leading a group of US climate leaders, outlined and submitted an *Amazon Protection Plan* to the new Biden Administration. The heart of the proposal involves making the avoidance of deforestation central to future trade agreements. They write that the US government should be "working with Europe, Japan, China and other major economies to align international efforts and thereby spread globally the policies outlined above."[44]

Avoiding the expansion is important because the increase in X_S is associated with deforestation. Almost 60% of Brazil's area is forest: The forest area is 4.97m km^2, and the level of the agricultural area, X_S^0, is 2.37m km^2.[45] An increase in X_S by 27% means that the agricultural area will increase by 0.64m km^2, which amounts to 13% of the remaining forest area. The total value of Brazil's agricultural production[46] (USD87.5b) is USD369 per hectare, about 0.9% of the conservation value (per hectare) estimated by Franklin and Pindyck (2018). Franklin and Pindyck (2018:166) distinguish between the Amazon's direct value, indirect value (as a carbon stock), option value (because of its biodiversity), and existence value, and sum the valuations to almost USD40,000 per hectare.

In practice, the loss of forest tends to be even larger than the increase in the agricultural area, because not all the former forest area continues to be productive for agriculture: Between 2010 and 2018, X_S increased by 0.51m km^2, but the decline in forest cover was more than twice that amount: 1.25m km^2.[47] If an increase in X_S leads to twice as much forest loss, we can expect that 26% of the remaining forest will disappear with growing demand and free trade. According to the results above, however, most (all) of this deforestation can be avoided if two northern countries offer the CTA^0 analyzed in this paper.

7. RELEVANCE

As discussed in the Introduction, the EU, the UN, and the World Bank are concerned with the potential conflict between trade liberalization and environmental conservation. Nigel Purvis, the former

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[44] The action plan is here:
https://climateprincipals.org/wp-content/uploads/2021/01/Amazon-Protection-Plan-Final_Climate-Principals.pdf
[45] See, respectively, https://data.worldbank.org/indicator/AG.LND.FRST.K2?locations=BR and
https://data.oecd.org/agrland/agricultural-land.htm
[46] https://data.worldbank.org/indicator/NV.AGR.TOTL.KD?locations=BR
[47] https://data.worldbank.org/indicator/AG.LND.FRST.K2?locations=BR

US climate negotiator, has also admitted that trade is "unintentionally creating a financial incentive for criminals to set fire to the Amazon and convert it into farmland."[48]

There is a large literature on trade agreements and environmental problems. Based on this literature, scholars recommend policies, such as border taxes, that trade off distortions and the environmental benefits. In this paper, I make a connection to the literature on trade-specific investments. When investments in production capacity are payoff-relevant, tariffs can be contingent on them. For the tariffs to be credible, or renegotiation proof, payoffs must always be on the Pareto frontier. Nevertheless, a contingent trade agreement can motivate environmental conservation and still guarantee free trade in equilibrium. These results are important because they show that even when traditional trade agreements lead to resource depletion, such as deforestation, it need not be so. Clever agreements exploit the gains from trade and use the gains to motivate conservation rather than exploitation.

CTAs are politically and empirically relevant. They draw on the fact that there is more than one way of splitting the gains from trade; a fact also emphasized in the literature on "fair trade" (Dragusanu et al., 2014). For tropical forests, we already have verifiable and rather precise measures of forest cover, thanks to satellite monitoring.[49]. As mentioned in the Introduction, the formalization of the CTA in this paper is an interpretation of the proposal by France and the Netherlands in May 2020. The present analysis provides a first exploration of how a contingent trade agreement might be implemented, and of how much conservation it might motivate. The combined framework that I provide is surprisingly tractable, and the model can be calibrated. One simple but intriguing preliminary finding is that the CTA can prevent much of the deforestation that will otherwise arise in the Brazilian Amazon, even if Brazil has a growing number of export markets.

The theory abstracted from dynamic considerations, technological progress, and political forces. Serious calibration exercises and empirical investigations on the promise of CTAs have not even started. The simplicity of the theory leaves the door open for future research, but also the possibility that this tractable model can be successfully generalized and developed in a number of important directions.

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[48] *The New York Times*, January 29, 2021: https://www.nytimes.com/2021/01/29/climate/biden-amazon-deforestation.html

[49] As stated by the UN (2019, Ch. 6:56): "The monitoring systems have been improved to the point of offering daily real-time data, constituting one of the most important tools for the fight against deforestation in Brazil."

Abman, Ryan, and Lundberg, Clark (2020): "Does Free Trade Increase Deforestation? The Effects of Regional Trade Agreements," Journal of the Association of Environmental and Resource Economists 7(1): 35-72.

Antràs, Pol, and Staiger, Robert W. (2012): "Trade Agreements and the Nature of Price Determination," American Economic Review 102(3): 470-76.

Antweiler, Werner; Copeland, Brian R., and Taylor, M. Scott (2001): "Is Free Trade Good for the Environment?" American Economic Review 91(4): 877-908.

Bagwell, Kyle, and Staiger, Robert W. (1999): "An Economic Theory of GATT," American Economic Review 89(1): 215-48.

Bagwell, Kyle, and Staiger, Robert W. (2004): The Economics of the World Trading System. Cambridge: MIT press.

Bagwell, Kyle, and Staiger, Robert W. (2010): "Backward stealing and forward manipulation in the WTO," Journal of International Economics 82(1): 49–62.

Bagwell, Kyle, and Staiger, Robert W. (2011): "What Do Trade Negotiators Negotiate About? Empirical Evidence from the World Trade Organization," American Economic Review 101(4): 1238-73.

Bagwell, Kyle, and Staiger, Robert W. (2016): "The Design of Trade Agreements," in K. Bagwell and R. Staiger (Eds.), Handbook of Commercial Policy 1: 435-529.

Baier, Scott L., and Bergstrand, Jeffrey H. (2007): "Do free trade agreements actually increase members' international trade?" Journal of International Economics 71(1): 72-95.

Baldwin, Richard E. (1992): "Measurable Dynamic Gains from Trade," Journal of Political Economy 100(1): 162-74.

Barbier, Edward B. (2000): "Links Between Economic Liberalization and Rural Resource Degradation in the Developing Regions," Agricultural Economics 23(3): 299-310.

Barrett, Scott (1997): "The Strategy of Trade Sanctions in International Environmental Agreements," Resource and Energy Economics 19(4): 345-61.

Berger, Daniel; Easterly, William; Nunn, Nathan; and Satyanath, Shanker (2013): "Commercial Imperialism? Political Influence and Trade During the Cold War," American Economic Review 2013, 103(2): 863–96.

Bond, Eric W. (2006): "Transportation Infrastructure Invesements and Trade Liberalization," Japanese Economic Review 57(4): 483-500.

Bond, Eric W., and Park, Jee-Hyeong (2002): "Gradualism in Trade Agreements with Asymmetric Countries," Review of Economic Studies 69(2): 379–406.

Brander, James A., and Taylor, Scott M. (1997): "International Trade and Open Access Renewable Resources: the Small Open Economy Case," Canadian Journal of Economics 30(3): 526-52.

Broda, Christian; Limão, Nuno, and Weinstein, David (2008): "Optimal Tariffs and Market Power: The Evidence," American Economic Review 98(5): 2032–65.

Burgess, Robin; Costa, Francisco JM, and Olken, Benjamin A. (2019): "The Brazilian Amazon's Double Reversal of Fortune," working paper.

Cadot, Olivier; Gourdon, Julien, and van Tongeren, Frank (2018): "Estimating Ad Valorem Equivalents of Non-Tariff Measures: Combining Price-Based and Quantity-Based Approaches", OECD Trade Policy Papers, No. 215, OECD Publishing, Paris: https://doi.org/10.1787/f3cd5bdc-en

Chichilnisky, Graciela (1994): "North-south Trade and the Global Environment," American Economic Review 84(4): 851-74.

Copeland, Brian R.; Shapiro, Joseph S.; and Taylor, Scott M. (2022): "Globalization and the environment," in G. Gopinath, E. Helpman, and K. Rogoff (eds.), Handbook of International Economics 5: 61-146. Elsevier.

Copeland, Brian R., and Taylor, Scott M. (2004): "Trade, Growth and the Environment," Journal of Economic Literature 42(1): 7-71.

Dasgupta, Partha; Eastwood, Robert; and Heal, Geoffrey (1978): "Resource Management in a Trading Economy," Quarterly Journal of Economics 92(2): 297-306.

Dixit, Avinash (1985): "Tax policy in open economies," in Alan J. Auerbach, and Martin Feldstein (eds.), Handbook of Public Economics 1: 313-374. Elsevier.

Dragusanu, Raluca; Giovannucci, Daniele; and Nunn, Nathan (2014): "The Economics of Fair Trade," Journal of Economic Perspectives 28(3): 217–36.

Ederington, Josh (2002): "Trade and Domestic Policy Linkage in International Agreements," International Economic Review 43(4): 1347-68.

Faria, Weslem Rodrigues, and Almeida, Alexandre Nunes (2016): "Relationship Between Openness to Trade and Deforestation: Empirical Evidence from the Brazilian Amazon," Ecological Economics 121: 85-97.

Ferrari, Alessandro; Fiorini, Matteo; Francois, Joseph; Hoekman, Bernard; Lechner, Lisa Maria; Manchin, Miriam, and Santi, Filippo (2021): "EU Trade Agreements and Non-Trade Policy Objectives, " EUI Working Paper RSC 2021/48.

Franklin Jr, Sergio L., and Pindyck, Robert S. (2018): "Tropical Forests, Tipping Points, and the Social Cost of Deforestation," Ecological Economics 153: 161-71.

Guriev, Sergei, and Klimenko, Mikhail (2015): "Duration and Term Structure of Trade Agreements," The Economic Journal 125 (589): 1818–49.

Harstad, Bård (2023): "Trade and Trees." Mimeo, Stanford GSB.

Hillman, Arye L., and Van Long, Ngo (1983): "Pricing and Depletion of an Exhaustible Resource when there is Anticipation of Trade Disruption," Quarterly Journal of Economics 98(2): 215-33.

Hsiao, Allan (2022): "Coordination and Commitment in International Climate Action: Evidence from Palm Oil," mimeo, Princeton University.

IPCC (2021): "Climate Change 2021: The Physical Science Basis." Contribution of Working Group I to the Sixth Assessment Report of the Intergovernmental Panel on Climate Change [Masson-Delmotte, V., P. Zhai, A. Pirani, S. L. Connors, C. Péan, S. Berger, N. Caud, Y. Chen, L. Goldfarb, M. I. Gomis, M. Huang, K. Leitzell, E. Lonnoy, J.B.R. Matthews, T. K. Maycock, T. Waterfield, O. Yelekçi, R. Yu and B. Zhou (eds.)]. Cambridge University Press.

Juhász, Réka (2018): "Temporary Protection and Technology Adoption: Evidence from the Napoleonic Blockade," American Economic Review 108(11): 3339-76.

Karp, Larry; Sacheti, Sandeep, and Zhao, Jinhua (2001): "Common Ground Between Free-Traders and Environmentalists," International Economic Review 42(3): 617-48.

Keen, Michael, and Kotsogiannis, Christos (2014): "Coordinating Climate and Trade Policies: Pareto Efficiency and the Role of Border Tax Adjustments," Journal of International Economics 94: 119-28.

Krugman, Paul (1987): "The Narrow Moving Band, the Dutch Disease and The Competitive Consequences of Mrs. Thatcher: Notes on Trade in the Presence of Dynamic Scale Economies," Journal of Development Economics 27(1-2): 41-55.

Limão, Nuno (2005): "Trade policy, cross-border externalities and lobbies: do linked agreements enforce more cooperative outcomes?" Journal of International Economics 67(1): 175-199.

Ludema, Rodney D., and Mayda, Anna Maria (2013): "Do Terms-of-trade Effects Matter for Trade Agreements? Theory and Evidence from WTO Countries," Quarterly Journal of Economics 128(4): 1837-93.

Ludema, Rodney D., and Wooton, Ian (1994): "Cross-Border Externalities and Trade Liberalization: The Strategic Control of Pollution," Canadian Journal of Economics 27(4): 950-66.

Maggi, Giovanni (2016): "Issue Linkage," in K. Bagwell and R. Staiger (Eds.), Handbook of Commercial Policy 1: 513-64.

Maggi, Giovanni, and Ossa, Ralph (2022): "The Political Economy of International Regulatory Cooperation," mimeo.

Maggi, Giovanni, and Rodríguez-Clare, Andrés (2007): "A Political-Economy Theory of Trade Agreements," American Economic Review 97(4): 1374-1406.

Maggi, Giovanni, and Staiger, Robert W. (2021): "International Climate Agreements and the Scream of Greta," mimeo.

Markusen, James R. (1975): "International Externalities and Optimal Tax Structures," Journal of International Economics 5(1): 15-29.

Markusen, James R. (1981): "Trade and the Gains from Trade with Imperfect Competition," Journal of International Economics 11(4): 531-51.

McLaren, John (1997): "Size, Sunk Costs and Judge Bowker's Objection to Free Trade," American Economic Review 87(3): 400-20.

Nordhaus, William (2015): "Climate Clubs: Overcoming Free-riding in International Climate Policy," American Economic Review 105(4): 1339-70.

Ossa, Ralph (2011): "A "New Trade" Theory of GATT/WTO Negotiations," Journal of Political Economy 119(1): 122-52.

Pendrill, Florence; Persson, U. Martin; Godar, Javier; Kastner, Thomas; Moran, Daniel; Schmidt, Sarah, and Wood, Richard (2019): "Agricultural and Forestry Trade Drives Large Share of Tropical Deforestation Emissions," Global Environmental Change 56: 1-10.

Pethig, Rüdiger (1976): "Pollution, Welfare, and Environmental Policy in the Theory of Comparative Advantage," Journal of Environmental Economics and Management 2(3): 160-69.

Pindyck, Robert S. (2021): "What We Know and Don't Know about Climate Change, and Implications for Policy," Environmental and Energy Policy and the Economy, vol. 2.

Shapiro, Joseph S. (2021): "The Environmental Bias of Trade Policy," Quarterly Journal of Economics 136(2): 831-86.

UN (2019): Global Assessment Report on Biodiversity and Ecosystem Services of the Intergovernmental Science-Policy Platform on Biodiversity and Ecosystem Services. E. S. Brondizio, J. Settele, S. Díaz, and H. T. Ngo (editors). IPBES secretariat, Bonn, Germany.

World Bank (2019): "Trade Integration as a Pathway to Development?" Semiannual Report of the Latin America and Caribbean Region.

Notation. A generic country is indexed by i, j, or l. Typically, $j \neq i$.

Let $\overline{v}_{il} \equiv v_{il} - (k_l + t_{il})\, a_l$ measure the socially optimal c_{il}, all costs are taken into account, when t_{il} is the transport cost from l to i and there is no damage. When prices adjust, it does not matter whether the exporter or the importer is responsible for paying the transport cost. If we let the transport cost be paid by the importer, $\overline{p}_i \equiv p_i - k_i$ measures a producer's net profit, per unit of capacity (i.e., the price minus the net marginal production cost).

Generalization. The set of countries is $I = \{S, N, 1..., n-1\}$. These countries do not produce the same goods that N and S produce.[50] A country i has a mass m_i of consumers, where $m \equiv \sum_{i \in I} m_i$ is the total mass, and $r_i \equiv m_i / m$ is the relative size of $i \in I$. For averages, write $\overline{v}_{Ai} \equiv \sum_{l \in I} r_l \overline{v}_{li}$. I will also use $\overline{v}_{-S} \equiv \sum_{l \in I \backslash S} r_l \overline{v}_{li}$, although I henceforth find it unnecessary to include I in the summation subscripts. With these definitions, the following lemma confirms that we can henceforth simplify the notation by ignoring the k_i's and the t_{il}'s in the proofs.

LEMMA 1: *Every individual utility (1) can be written as:*

$$
\begin{aligned}
U_i &= \overline{U}_i + \kappa_i, \text{ where} \\
\overline{U}_i &\equiv -\sum_l \frac{(\overline{v}_{il} - c_{il})^2}{2a_l} - \sum_{l \neq i} \overline{p}_l c_{il} + \overline{p}_i \frac{\sum_{l \neq i} m_l c_{li}}{m_i}, \\
\overline{v}_{il} &\equiv v_{il} - (k_l + t_{il})\, a_l, \\
\overline{p}_i &\equiv p_i - k_i, \text{ and} \\
\kappa_i &\equiv e_i / m_i - \sum_l \left[v_{il} (k_l + t_{il}) - a_l (k_l + t_{il})^2 / 2 \right],
\end{aligned}
$$

where κ_i is a constant that is henceforth ignored.

PROOF: With (2), a binding (3), (4), and (5), (1) is:

$$
\begin{aligned}
U_i &= -\sum_l \frac{(v_{il} - c_{il})^2}{2a_l} + \frac{e_i}{m_i} - \sum_l p_l c_{il} + p_i \frac{\sum_l m_l c_{li}}{m_i} - k_i \frac{\sum_l m_l c_{li}}{m_i} - \sum_l t_{il} c_{il} \\
&= \frac{e_i}{m_i} - \sum_l \left[\frac{(v_{il} - c_{il})^2 + 2a_l (k_l + t_{il}) c_{il}}{2a_l} \right] - \sum_l \overline{p}_l c_{il} + \overline{p}_i \frac{\sum_l m_l c_{li}}{m_i} \\
&= \kappa_i - \sum_l \frac{(\overline{v}_{il} - c_{il})^2}{2a_l} - \sum_{l \neq i} \overline{p}_l c_{il} + \overline{p}_i \frac{\sum_{l \neq i} m_l c_{li}}{m_i}. \quad \|
\end{aligned}
$$

In Sections 2-4, we have $m = 2$ and $r_N = r_S = 1/2$ because we impose:

ASSUMPTION 2:

$$ n = 1 \text{ and } m_N = m_S = 1. \tag{A2} $$

---

[50] Because of the quasi-linear utility function, the market for the goods produced by these countries is characterized independently of the market for N's good and for S's good.

PROOF OF PROPOSITION 1: As announced, in addition to country N and S, we begin by permitting $n-1$ other countries that are passive in that their tariffs are fixed and reflected by $\overline{v}_{li}$.[51] In Appendix C, all tariffs are set strategically.

The Market Equilibrium. Let $\{i,j\} = \{N, S\}$, while l can be any country. The consumption levels of i's product are as follows for each consumer in l (including i), and $j \neq i$:

$$c_{li} = v_{li} - a_i \left(p_i + t_{li} \right) = \overline{v}_{li} - a_i \overline{p}_i \text{ and } c_{ji} = v_{ji} - a_i \left(p_i + t_{ji} + \tau_j \right) = \overline{v}_{ji} - a_i \left(\overline{p}_i + \tau_j \right). \quad (20)$$

When (3) binds,

$$\overline{p}_i = \frac{\sum_l m_l \overline{v}_{li} - a_i m_j \tau_j - X_i}{a_i m}, \quad (21)$$

which can be written as (11) under (A2).

Equilibrium Tariff. Anticipating (20) and (21), j sets τ_j to maximize j's surplus from the market for i's good, s_{ji}:

$$s_{ji} = -m_j a_i \left(\overline{p}_i + \tau_j \right)^2 / 2 - m_j \overline{p}_i \left(\overline{v}_{ji} - a_i \left(\overline{p}_i + \tau_j \right) \right). \quad (22)$$

The first-order condition (f.o.c.) with respect to (w.r.t.) τ_j is (note that the second-order condition (s.o.c.) holds):

$$-m_j a_i \left(\overline{p}_i + \tau_j \right) + a_i m_j \overline{p}_i - m_j \left[a_i \left(\overline{p}_i + \tau_j \right) + \left(\overline{v}_{ji} - a_i \left(2 \overline{p}_i + \tau_j \right) \right) \right] \frac{\partial \overline{p}_i}{\partial \tau_j} = 0 \Leftrightarrow$$

$$-a_i \left(\overline{p}_i + \tau_j \right) + a_i \overline{p}_i - \left[a_i \left(\overline{p}_i + \tau_j \right) + \left(\overline{v}_{ji} - a_i \left(2 \overline{p}_i + \tau_j \right) \right) \right] \left(-\frac{m_j}{m} \right) = 0 \Leftrightarrow$$

$$-\tau_j + \left(\frac{\overline{v}_{ji}}{a_i} - \frac{\sum_l m_l \overline{v}_{li} - a_i m_j \tau_j - X_i}{a_i m} \right) r_j = 0 \Leftrightarrow \quad (23)$$

$$\tau_j = \tau_j^{BR} \left(X_i \right) \equiv \frac{r_j}{a_i} \frac{\overline{v}_{ji} - \overline{v}_{Ai} + X_i / m}{1 - r_j^2}. \quad (24)$$

When we combine (21) and (24), $\overline{p}_i$ becomes :

$$\overline{p}_i = \frac{\overline{v}_{Ai}}{a_i} - \frac{X_i}{a_i m} - \frac{r_j^2}{a_i} \frac{\overline{v}_{ji} - \overline{v}_{Ai} + X_i / m}{1 - r_j^2} = \frac{\overline{v}_{Ai} - r_j^2 \overline{v}_{ji} - X_i / m}{a_i \left(1 - r_j^2 \right)}. \quad (25)$$

Equilibrium Capacity. When investors are price-takers, and expect tariff τ_i, the market response is that investments increase as long as $\overline{p}_i \geq 0$. From (21):

$$X_i^{MR} \left(\tau_j \right) = \left(\overline{v}_{Ai} - a_i r_j \tau_j \right) m. \quad (26)$$

Expectations are rational, so the combination of (24) and (26) characterizes the BAU equilibrium in a country with private investments. For $i = N$, this gives:

$$\tau_S^{BAU} = \tau_S^{BR} \left(X_N^{BAU} \right) \equiv \frac{r_S}{a_N} \frac{\overline{v}_{SN} - \overline{v}_{AN} + \left(\overline{v}_{AN} - a_N r_S \tau_S \right)}{1 - r_S^2} \Leftrightarrow$$

$$\tau_S^{BAU} = \frac{r_S \overline{v}_{SN}}{a_N} \text{ and } X_N^{BAU} = \left(\overline{v}_{AN} - r_S^2 \overline{v}_{SN} \right) m. \quad (27)$$

---

[51] That is, if country $l \notin \{N, S\}$ has tariff τ_{li} on good i, l's demand is $\overline{v}_{li} - a_i \overline{p}_i$ if $\overline{v}_{il} \equiv v_{il} - \left(k_l + t_{il} + \tau_{li} \right) a_l$.

As explained in Section 2, S maximizes $U_S - d_S(X_N)$. It follows that S selects X_S to maximize S's surplus from the market for S's good, s_{SS}, which is the sum of S's consumer surplus from c_{SS} and S's profit:

$$s_{SS} = m_S \left[-\frac{(\bar{v}_{SS} - c_{SS})^2}{2a_S} - \bar{p}_S c_{SS} \right] + \bar{p}_S X_S \tag{28}$$

$$= m_S a_S \bar{p}_S^2 / 2 + \bar{p}_S (X_S - m_S \bar{v}_{SS}). \tag{29}$$

The f.o.c. of s_{SS} w.r.t. X_S is, given (21):

$$\bar{p}_S + (m_S a_S \bar{p}_S + X_S - m_S \bar{v}_{SS}) \left(\frac{\partial \bar{p}_S}{\partial X_S} \right) = 0 \Leftrightarrow \tag{30}$$

$$\bar{p}_S + (m_S a_S \bar{p}_S + X_S - m_S \bar{v}_{SS}) \left(-\frac{1}{ma_S} - \frac{m_N}{m} \frac{\partial \tau_N}{\partial X_S} \right) = 0. \tag{31}$$

If τ_N was fixed, $\partial \tau_N / \partial X_S = 0$, and (31) would simplify to:

$$\bar{p}_S - \frac{m_S}{m} \bar{p}_S - \frac{X_S}{ma_S} + \frac{m_S \bar{v}_{SS}}{ma_S} = 0 \Leftrightarrow$$

$$\frac{\sum_l m_l \bar{v}_{lS} - a_S m_N \tau_N - X_S}{ma_S} (1 - r_S) - \frac{X_S}{ma_S} + \frac{m_S \bar{v}_{SS}}{ma_S} = 0 \Leftrightarrow$$

$$(2 - r_S) m_S \bar{v}_{SS} + (1 - r_S) \left(\sum_{l \neq i} m_l \bar{v}_{lS} - a_S m_N \tau_N \right) = (2 - r_S) X_S \Leftrightarrow$$

$$m_S \bar{v}_{SS} + \frac{1 - r_S}{2 - r_S} m (\bar{v}_{-S} - a_S r_N \tau_N) = X_S.$$

So, S's best response to a fixed τ_N would be:

$$X_S^{BR}(\tau_N) = m_S \bar{v}_{SS} + \frac{1 - r_S}{2 - r_S} m (\bar{v}_{-S} - a_S r_N \tau_N), \text{ and, with zero tariff:} \tag{32}$$

$$X_S^{FTA} \equiv X_S^{BR}(0) = m_S \bar{v}_{SS} + \frac{1 - r_S}{2 - r_S} m \bar{v}_{-S}. \tag{33}$$

N's tariff is endogenous with BAU. From (24), $\partial \tau_N / \partial X_S = r_N / m a_S (1 - r_N^2)$. Thus, (31) becomes:

$$\bar{p}_S + (m_S a_S \bar{p}_S + X_S - m_S \bar{v}_{SS}) \left(-\frac{1}{ma_S} - \frac{m_N}{m} \frac{r_N}{ma_S (1 - r_N^2)} \right) = 0 \Leftrightarrow$$

$$ma_S (1 - r_N^2 - r_S) \bar{p}_S - (X_S - m_S \bar{v}_{SS}) = 0 \Leftrightarrow \tag{34}$$

$$ma_S (1 - r_N^2 - r_S) \left(\frac{\bar{v}_{AS} - r_N^2 \bar{v}_{NS} - X_S/m}{a_S (1 - r_N^2)} \right) - (X_S - m_S \bar{v}_{SS}) = 0 \Leftrightarrow$$

$$(1 - r_N^2 - r_S) \left(\frac{\sum_l m_l \bar{v}_{lS} - m r_N^2 \bar{v}_{NS} - X_S}{1 - r_N^2} \right) - (X_S - m_S \bar{v}_{SS}) = 0 \Leftrightarrow$$

$$X_S^{BAU} = m_S \bar{v}_{SS} + \frac{1 - r_N^2 - r_S}{2 - 2r_N^2 - r_S} (\bar{v}_{-S} - r_N^2 \bar{v}_{NS}) m. \tag{35}$$

S's Payoffs. Combining (25) and (35), we can write:

$$\bar{p}_S = \frac{\bar{v}_{AS} - r_N^2 \bar{v}_{NS}}{a_S (1 - r_N^2)} - \frac{1/ma_S}{1 - r_N^2} \left[m_S \bar{v}_{SS} + \frac{1 - r_N^2 - r_S}{2 - 2r_N^2 - r_S} (\bar{v}_{-S} - r_N^2 \bar{v}_{NS}) m \right]$$

$$= \frac{\bar{v}_{AS} - r_N^2 \bar{v}_{NS} - r_S \bar{v}_{SS}}{a_S (2 - 2r_N^2 - r_S)}. \tag{36}$$

33

And, from above, where (34) established,

$$X_S - m_S \overline{v}_{SS} = \left(1 - r_N^2 - r_S\right) m a_S \overline{p}_S,$$

we get from (29) and (36):

$$
\begin{aligned}
s_{SS}^{BAU} &= m_S a_S \overline{p}_S^2/2 + \left(1 - r_N^2 - r_S\right) m a_S \overline{p}_S^2 = \frac{m a_S}{2}\left(2 - 2r_N^2 - r_S\right)\left(\frac{\overline{v}_{AS} - r_N^2 \overline{v}_{NS} - r_S \overline{v}_{SS}}{a_S\left(2 - 2r_N^2 - r_S\right)}\right)^2 \\
&= \frac{m}{2a_S}\frac{\left(\overline{v}_{AS} - r_N^2 \overline{v}_{NS} - r_S \overline{v}_{SS}\right)^2}{2 - 2r_N^2 - r_S} = \frac{m}{2a_S}\frac{\left(\overline{v}_{-S} - r_N^2 \overline{v}_{NS}\right)^2}{2 - 2r_N^2 - r_S}.
\end{aligned}
\tag{37}
$$

When all foreign countries share the value $\overline{v}_{NS}$, s_{SS}^{BAU} becomes

$$\frac{m v_{NS}^2}{2a_S}\frac{\left(1 - r_S - r_N^2\right)^2}{2 - r_S - 2r_N^2}.$$

Equilibrium when $n = 1$. When $n = 1$,

$$s_{SS}^{BAU} = \frac{m\overline{v}_{NS}^2}{2a_S}\frac{\left(r_N - r_N^2\right)^2}{2 - 2r_N^2 - (1 - r_N)} = \frac{m\overline{v}_{NS}^2}{2a_S}\frac{r_N^2\left(1 - r_N\right)^2}{(1 - r_N)(1 + 2r_N)} = \frac{m\overline{v}_{NS}^2}{2a_S}\frac{r_N^2\left(1 - r_N\right)}{1 + 2r_N},
\tag{38}$$

and with (A2):

$$s_{SS}^{BAU} = \frac{\overline{v}_{NS}^2}{16a_S}.
\tag{39}$$

S's Product. With (35), and $r_S = 1 - r_N$,

$$X_S^{BAU} = m_S \overline{v}_{SS} + \frac{1 - r_N^2 - (1 - r_N)}{2 - 2r_N^2 - (1 - r_N)}\left(r_N - r_N^2\right)m\overline{v}_{NS} = m_S \overline{v}_{SS} + \frac{r_N\left(1 - r_N\right)}{1 + 2r_N}m_N \overline{v}_{NS}.$$

The tariff (24) becomes

$$
\begin{aligned}
\tau_N^{BAU} &= \tau_N^{BR}\left(X_S^{BAU}\right) = \frac{r_N}{a_S}\frac{\overline{v}_{NS} - \overline{v}_{AS} + X_S/m}{1 - r_N^2} = \frac{r_N}{a_S}\frac{\overline{v}_{NS} - r_N \overline{v}_{NS} + (1 - r_N)r_N^2 \overline{v}_{NS}/(1 + 2r_N)}{1 - r_N^2} \\
&= \frac{r_N}{a_S}\frac{(1 - r_N)(1 + 2r_N)\overline{v}_{NS} + (1 - r_N)r_N^2 \overline{v}_{NS}}{(1 - r_N)(1 + r_N)(1 + 2r_N)} = \frac{1 + r_N}{1 + 2r_N}\frac{r_N \overline{v}_{NS}}{a_S}.
\end{aligned}
$$

From (36),

$$
\begin{aligned}
\overline{p}_S &= \frac{r_N \overline{v}_{NS} + (1 - r_N)\overline{v}_{SS} - r_N^2 \overline{v}_{NS} - (1 - r_N)\overline{v}_{SS}}{a_S\left(2 - 2r_N^2 - (1 - r_N)\right)} = \frac{r_N \overline{v}_{NS}/a_S}{1 + 2r_N}, \text{ so} \tag{40} \\
\overline{p}_S + \tau_N &= \frac{2 + r_N}{1 + 2r_N}\frac{r_N \overline{v}_{NS}}{a_S}. \tag{41}
\end{aligned}
$$

N's Product. (24) and (26) both holds when

$$
\begin{aligned}
\tau_S &= \tau_S^{BR}\left(X_N\right) \equiv r_S \frac{\left(\overline{v}_{SN} - \overline{v}_{AN}\right)/a_N + X_N/m a_N}{1 - r_S^2} \text{ and} \\
X_N &= X_N^{MR}\left(\tau_S\right) = m v_{AN} - a_N m_S \tau_S, \text{ so}
\end{aligned}
$$

$$
\begin{aligned}
X_N^{BAU} &= v_{AN}m - a_N m_S\left[\frac{r_S}{a_N}\frac{\overline{v}_{SN} - \overline{v}_{AN} + X_N^{BAU}/m}{1 - r_S^2}\right] \Leftrightarrow \\
X_N^{BAU} &= \left(1 - r_S^2\right)m v_{AN} - m_S r_S\left(\overline{v}_{SN} - \overline{v}_{AN}\right) = m v_{AN} - m r_S^2 \overline{v}_{SN} \\
&= m\left(r_N \overline{v}_{NN} + r_S \overline{v}_{SN}\right) - m r_S^2 \overline{v}_{SN} = m_N \overline{v}_{NN} + r_N m_S \overline{v}_{SN}, \text{ and} \tag{42}
\end{aligned}
$$

34

$$\tau_S^{BAU} = \frac{r_S}{a_N} \frac{\overline{v}_{SN} - \overline{v}_{AN} + X_N^{BAU}/m}{1 - r_S^2}$$

$$= \frac{r_S}{a_N} \frac{(\overline{v}_{SN} - r_N\overline{v}_{NN} - r_S\overline{v}_{SN}) + (r_N\overline{v}_{NN} + r_S\overline{v}_{SN}) - r_S^2\overline{v}_{SN}}{1 - r_S^2} = \frac{r_S\overline{v}_{SN}}{a_N}. \tag{43}$$

With (A2), the equations simplify to Proposition 1.

PROOF OF PROPOSITION 2: *N's Product.* In equilibrium, $\overline{p}_N = 0$, and the payment for the product is simply a transfer from one country to the other. Thus, for every given τ_S, the total BAU surplus associated with N's product follows from (1), (2), and (20). When $n = 1$,

$$s_N(\tau_S) \equiv s_{NN} + s_{SN} = -\frac{m_N a_N}{2}\overline{p}_N^2 - \frac{m_S a_N}{2}(\overline{p}_N + \tau_S)^2 = -\frac{m_S a_N}{2}\tau_S^2, \tag{44}$$

when $\overline{p}_N = 0$. From (43), $\tau_S = \frac{r_S\overline{v}_{SN}}{a_N}$, so

$$s_N^{BAU} = -\frac{m_S a_N}{2}\left(\frac{r_S\overline{v}_{SN}}{a_N}\right)^2 = -\frac{mr_S^3\overline{v}_{SN}^2}{2a_N}.$$

Under the FTA, $\tau_S = 0$, and $s_N(\tau_S) = 0$, so:

$$s_N^{FTA} - s_N^{BAU} = \frac{mr_S^3\overline{v}_{SN}^2}{2a_N}, \text{ which is } \frac{\overline{v}_{SN}^2}{8a_N} \text{ under (A2)}.$$

S's Product. From (22) and (28), (40), and (41), we find the total BAU surplus associated with S's product. When $n = 1$,

$$s_S^{BAU} \equiv s_{NS} + s_{SS} = -m_N a_S(\overline{p}_S + \tau_N)^2/2 - m_S a_S\overline{p}_S^2/2 \tag{45}$$

$$= -\frac{m_N a_S}{2}\left(\frac{2 + r_N}{1 + 2r_N}\frac{r_N\overline{v}_{NS}}{a_S}\right)^2 - \frac{m_S a_S}{2}\left(\frac{r_N\overline{v}_{NS}/a_S}{1 + 2r_N}\right)^2$$

$$= -\frac{m}{2a_S}\left(\frac{r_N\overline{v}_{NS}}{1 + 2r_N}\right)^2\left(r_N(2 + r_N)^2 + r_S\right) = -\frac{m}{2a_S}\left(\frac{r_N\overline{v}_{NS}}{1 + 2r_N}\right)^2\left(1 + 3r_N + 4r_N^2 + r_N^3\right).$$

Next, consider the surplus for any fixed τ_N. When $n = 1$, (32) becomes:

$$X_S^{BR}(\tau_N) = \frac{m(\overline{v}_{AS}(1 - r_S) + r_S\overline{v}_{SS}) - a_S(1 - r_S)m_N\tau_N}{2 - r_S} = m_S\overline{v}_{SS} + \frac{m_N r_N\overline{v}_{NS}}{1 + r_N} - \frac{a_S r_N m_N\tau_N}{1 + r_N}.$$

From (21), we have:

$$\overline{p}_S = \frac{m_N\overline{v}_{NS} - a_S m_N\tau_N - (X_S - m_S\overline{v}_{SS})}{ma_S} = \frac{m_N\overline{v}_{NS} - a_S m_N\tau_N}{(1 + r_N)ma_S} = \frac{r_N}{1 + r_N}\left(\frac{\overline{v}_{NS}}{a_S} - \tau_N\right) \tag{46}$$

$$\overline{p}_S + \tau_N = \frac{r_N\overline{v}_{NS}/a_S + \tau_N}{1 + r_N}. \tag{47}$$

Substituting (46) and (47) into (45), we get:

$$s_S(\tau_N) = -\frac{m_N a_S}{2}\left(\frac{r_N\overline{v}_{NS}/a_S + \tau_N}{1 + r_N}\right)^2 - \frac{m_S a_S}{2}\left(\frac{r_N}{1 + r_N}\left(\frac{\overline{v}_{NS}}{a_S} - \tau_N\right)\right)^2$$

$$= -\frac{ma_S}{2}\left(\frac{r_N\overline{v}_{NS}/a_S}{1 + r_N}\right)^2 - \frac{m_N r_N^2\overline{v}_{NS}\tau_N}{(1 + r_N)^2} - \frac{a_S m_N(1 + r_S r_N)}{2}\left(\frac{\tau_N}{1 + r_N}\right)^2. \tag{48}$$

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With free trade,

$$s_S^{FTA} \equiv s_S(0) = -\frac{ma_S}{2}\left(\frac{r_N\overline{v}_{NS}/a_S}{1+r_N}\right)^2.$$

By comparison, the gains from liberalizing trade for S's good is:

$$
\begin{aligned}
s_S^{FTA} - s_S^{BAU} &= \frac{m}{2a_S}\left(\frac{r_N\overline{v}_{NS}}{1+2r_N}\right)^2\left(r_N(2+r_N)^2 + (1-r_N)\right) - \frac{ma_S}{2}\left(\frac{r_N\overline{v}_{NS}/a_S}{1+r_N}\right)^2 \\
&= \frac{m(r_N\overline{v}_{NS})^2}{2a_S}\left(\frac{r_N(2+r_N)^2 + 1 - r_N}{(1+2r_N)^2} - \frac{1}{(1+r_N)^2}\right).
\end{aligned}
$$

If $r_N = 1/2$,

$$s_S^{FTA} - s_S^{BAU} = \frac{1}{8}\frac{133}{144}\frac{\overline{v}_{NS}^2}{a_S} = \frac{133}{1152}\frac{\overline{v}_{NS}^2}{a_S}.$$

PROOF OF PROPOSITION 3: When the damage is taken into account, the ex ante socially optimal fixed τ_S solves:

$$\max_{\tau_S} s_N(\tau_S) - d_S\left(X_N^{MR}(\tau_S)\right),$$

where $s_N(\tau_S)$ is given by (44) when $n = 1$. With (26), the f.o.c. is (note that s.o.c. holds):

$$-m_S a_N \tau_S + d_S'(X_N)(a_N m_S) = 0 \Leftrightarrow \tau_S = d_S'(X_N). \tag{49}$$

Similarly, the optimal τ_N solves:

$$\max_{\tau_N} s_S(\tau_N) - d_N\left(X_S^{BR}(\tau_N)\right),$$

and with (32) and (48), the f.o.c. is (again, s.o.c. holds):

$$
\begin{aligned}
-\frac{m_N r_N^2 \overline{v}_{NS}}{(1+r_N)^2} - \frac{a_S m_N(1+r_S r_N)}{(1+r_N)^2}\tau_N - d_N'(X_S)\left(\frac{\partial X_S^{BR}}{\partial \tau_N}\right) &= 0 \Leftrightarrow \\
-m_N r_N^2 \overline{v}_{NS} + (1+r_N)a_S r_N m_N d_N'(X_S) &= a_S m_N(1+r_S r_N)\tau_N \Leftrightarrow \\
r_N \frac{(1+r_N)d_N'(X_S) - r_N\overline{v}_{NS}/a_S}{1+r_S r_N} &= \tau_N.
\end{aligned}
$$

PROOF OF PROPOSITION 4: Without cross-contingency, it suffices to consider S's surplus from S's product, as a function of X_S and τ_N. When (21) is substituted in (28), and we define $Z \equiv X_S - m_S\overline{v}_{SS}$ and $Y_\tau \equiv \sum_{i\neq S} m_i\overline{v}_{iS} - a_S m_N \tau_N$, s_{SS} can be written as:

$$
\begin{aligned}
s_{SS}(X_S, \tau_N) &= \frac{m_S a_S}{2}\left(\frac{mv_{AS} - a_S m_N \tau_N - X_S}{ma_S}\right)^2 + \left(\frac{mv_{AS} - a_S m_N \tau_N - X_S}{ma_S}\right)(X_S - m_S\overline{v}_{SS}) \\
&= \frac{m_S a_S}{2}\left(\frac{Y_\tau - Z}{ma_S}\right)^2 + \frac{Y_\tau - Z}{ma_S}Z = \frac{1}{2ma_S}\left(r_S(Y_\tau - Z)^2 + 2(Y_\tau - Z)Z\right). \tag{50}
\end{aligned}
$$

To implement X_S^{CTA}, we must have

$$s_{SS}\left(X_S^{CTA}, \tau_N^{CTA}(X_S^{CTA})\right) \geq s_{SS}\left(X_S', \tau_N^{CTA}(X_S')\right) \quad \forall X_S' \geq X_S^0. \tag{51}$$

Three observations help us to simplify (51). (1) For the CTA to be credible, $\tau_N^{CTA}(X_S) \in \left[0, \tau_N^{BR}(X_S)\right]$. To implement X_S^{CTA}, $\tau_N^{CTA}(X_S^{CTA}) = 0$ is both ex post efficient, and it helps to satisfy (51), because S prefers the lowest tariff among the credible alternatives $\tau_N^{CTA}(X_S) \in \left[0, \tau_N^{BR}(X_S)\right]$. (2) The right-hand

side (r.h.s.) of (51) is smallest when $\tau_N^{CTA}(X_S') \leq \tau_N^{BR}(X_S')$ binds. (3) When S considers a deviation X_S' accompanied with $\tau_N^{CTA}(X_S') = \tau_N^{BR}(X_S')$, the proof of Proposition 1 established that S prefers X_S^{BAU}, inducing $s_{SS}^{BAU} = s_{SS}(X_S^{BAU}, \tau_N^{BR}(X_S^{BAU}))$, which is characterized already: see (37). With (1)-(3), (50), and $Y_0 \equiv mv_{-S}$, (51) can be simplified to:

$$\frac{1}{2ma_S}\left(r_S(Y_0 - Z)^2 + 2(Y_0 - Z)Z\right) \geq s_{SS}^{BAU} \Leftrightarrow$$
$$Z^2(2 - r_S) - 2ZY_0(1 - r_S) - r_S Y_0^2 + 2ma_S s_{SS}^{BAU} \leq 0, \tag{52}$$

which binds when:

$$Z = \frac{Y_0(1 - r_S)}{2 - r_S} \pm \frac{1}{2(2 - r_S)}\sqrt{[2Y_0(1 - r_S)]^2 - 4(2 - r_S)\left(2ma_S s_{SS}^{BAU} - r_S Y_0^2\right)}.$$

With $Y_0 \equiv mv_{-S}$, $X_S = m_S \overline{v}_{SS} + Z$, and (37), (52) require:

$$X_S \in [\underline{X}, \overline{X}], \text{ where}$$
$$\underline{X} \equiv m_S \overline{v}_{SS} + \frac{Y_0(1 - r_S)}{2 - r_S} - \frac{1/2}{2 - r_S}\sqrt{[2Y_0(1 - r_S)]^2 - 4(2 - r_S)\left(2ma_S s_{SS}^{BAU} - r_S Y_0^2\right)}$$
$$= m\left[r_S \overline{v}_{SS} + \frac{1 - r_S}{2 - r_S}\overline{v}_{-S} - \frac{\overline{v}_{-S}}{2 - r_S}\sqrt{1 - \frac{2(2 - r_S)a_S}{m\overline{v}_{-S}^2}s_{SS}^{BAU}}\right] \tag{53}$$
$$= m\left[r_S \overline{v}_{SS} + \frac{1 - r_S}{2 - r_S}\overline{v}_{-S} - \frac{\overline{v}_{-S}}{2 - r_S}\sqrt{1 - \frac{2(2 - r_S)a_S}{m\overline{v}_{-S}^2}\left[\frac{m}{2a_S}\frac{(\overline{v}_{-S} - r_N^2 \overline{v}_{NS})^2}{2 - 2r_N^2 - r_S}\right]}\right],$$
$$\overline{X} \equiv m\left[r_S \overline{v}_{SS} + \frac{1 - r_S}{2 - r_S}\overline{v}_{-S} + \frac{\overline{v}_{-S}}{2 - r_S}\sqrt{1 - \frac{2(2 - r_S)a_S}{m\overline{v}_{-S}^2}s_{SS}^{BAU}}\right]$$
$$= m\left[r_S \overline{v}_{SS} + \frac{1 - r_S}{2 - r_S}\overline{v}_{-S} + \frac{\overline{v}_{-S}}{2 - r_S}\sqrt{1 - \frac{2(2 - r_S)a_S}{m\overline{v}_{-S}^2}\left[\frac{m}{2a_S}\frac{(\overline{v}_{-S} - r_N^2 \overline{v}_{NS})^2}{2 - 2r_N^2 - r_S}\right]}\right].$$

If $n = 1$, then $r_N = 1 - r_S$ and $\overline{v}_{-S} = (1 - r_S)\overline{v}_{NS}$. With (38), (53) becomes:

$$\underline{X} = m\left[r_S \overline{v}_{SS} + \frac{(1 - r_S)^2}{2 - r_S}\overline{v}_{NS} - \frac{(1 - r_S)}{2 - r_S}\overline{v}_{NS}\sqrt{1 - \frac{2(2 - r_S)a_S}{(1 - r_S)^2 \overline{v}_{NS}^2 m}\left[s_{SS}^{BAU}\right]}\right] \tag{54}$$
$$= m\left[r_S \overline{v}_{SS} + \frac{(1 - r_S)^2}{2 - r_S}\overline{v}_{NS} - \frac{(1 - r_S)}{2 - r_S}\overline{v}_{NS}\sqrt{1 - \frac{(2 - r_S)r_S}{3 - 2r_S}}\right].$$

If also $m = 2$ and $r_S = 1/2$, as under (A2), then, with (39):

$$\underline{X} = \overline{v}_{SS} + \frac{\overline{v}_{NS}}{3} - \frac{2}{3}\overline{v}_{NS}\sqrt{1 - \frac{6a_S}{\overline{v}_{NS}^2}\left[s_{SS}^{BAU}\right]} \tag{55}$$
$$= \overline{v}_{SS} + \frac{\overline{v}_{NS}}{3} - \frac{2}{3}\overline{v}_{NS}\sqrt{1 - \frac{6a_S}{\overline{v}_{NS}^2}\left[\frac{\overline{v}_{NS}^2}{16a_S}\right]} = \overline{v}_{SS} + \frac{\overline{v}_{NS}}{3} - \frac{\overline{v}_{NS}}{6}\sqrt{10} \approx \overline{v}_{SS} - 0.19\overline{v}_{NS}.$$

Similarly,

$$\overline{X} = \overline{v}_{SS} + \overline{v}_{NS}\left(\frac{1}{3} + \frac{1}{6}\sqrt{10}\right) \approx \overline{v}_{SS} + 0.86\overline{v}_{NS}.$$

PROOF OF PROPOSITION 5: With $\tau_S^{CTA}\left(X_S^{CTA}\right)$, $X_N = X_N^{MR}\left(\tau_S^E\right)$, where $\tau_S^E = \tau_S^{CTA}\left(X_S^{CTA}\right)$ is the equilibrium and expected tariff in S when the CTA implements X_S^{CTA}.

For S to prefer X_S^{CTA}, we must have:

$$s_{SS}\left(X_S^{CTA}, \tau_N^{CTA}\left(X_S^{CTA}\right)\right) + s_{SN}\left(X_N^{MR}\left(\tau_S^E\right), \tau_S^E\right) \geq \tag{56}$$
$$s_{SS}\left(X_S', \tau_N\left(X_S'\right)\right) + s_{SN}\left(X_N^{MR}\left(\tau_S^E\right), \tau_S^{CTA}\left(X_S'\right)\right) \quad \forall X_S' \geq X_S^0.$$

Four observations help to simplify (56): (1) For the CTA to be credible, $\tau_N^{CTA}\left(X_S\right) \in \left[0, \tau_N^{BR}\left(X_S\right)\right]$ and $\tau_S^{CTA}\left(X_S\right) \in \left[0, \tau_S^{BR}\left(X_N\right)\right]$. (2) The r.h.s. of (56) is most likely to hold when $\tau_N^{CTA}\left(X_S\right) \leq \tau_N^{BR}\left(X_S\right)$ binds. (3) The r.h.s. of (56) is most likely to hold when $\tau_S^{CTA}\left(X_S\right) = 0$. (4) The most attractive deviation for S is X_S^{BAU}. Given (1)-(4), (56) simplifies to:

$$s_{SS}\left(X_S, 0\right) \geq s_{SS}^{BAU} - \Delta_{SN}\left(\tau_S^E\right), \text{ where} \tag{57}$$
$$\Delta_{SN}\left(\tau_S^E\right) \equiv s_{SN}\left(X_N^{MR}\left(\tau_S^E\right), \tau_S^E\right) - s_{SN}\left(X_N^{MR}\left(\tau_S^E\right), 0\right).$$

LEMMA 2: *We have*

$$\Delta_{SN}\left(\tau_S^E\right) = m_S r_S \bar{v}_{SN} \tau_S^E - \frac{m_S a_N}{2}\left(1 + r_S^2\right)\left(\tau_S^E\right)^2.$$

PROOF: To derive $s_{SN}\left(X_N, \tau_S\right)$, note that with expected τ_S^E, (26) gives:

$$X_N^{MR}\left(\tau_S^E\right) = \left(\bar{v}_{AN} - a_N r_S \tau_S^E\right) m.$$

When this $X_N^{MR}\left(\tau_S^E\right)$ is combined with (21) for $i =$ N, we get:

$$\bar{p}_N = r_S\left(\tau_S^E - \tau_S\right) \text{ and } \bar{p}_N + \tau_S = r_S \tau_S^E + \left(1 - r_S\right)\tau_S.$$

Thus, S's consumer surplus from N's product, plus S's tariff revenues, is:

$$s_{SN}\left(X_N^{MR}\left(\tau_S^E\right), \tau_S\right) = -m_S a_N \left(\bar{p}_N + \tau_S\right)^2 / 2 - m_S \bar{p}_N \left(\bar{v}_{SN} - a_N\left(\bar{p}_N + \tau_S\right)\right)$$
$$= -\frac{m_S a_N}{2}\left(r_S \tau_S^E + \left(1 - r_S\right)\tau_S\right)^2 - m_S r_S\left(\tau_S^E - \tau_S\right)\left[\bar{v}_{SN} - a_N\left(r_S \tau_S^E + \left(1 - r_S\right)\tau_S\right)\right].$$

With this, we can derive $\Delta_{SN}\left(\tau_S^E\right)$. It becomes:

$$-\frac{m_S a_N}{2}\left(r_S \tau_S^E + \left(1 - r_S\right)\tau_S^E\right)^2 - m_S r_S\left(\tau_S^E - \tau_S^E\right)\left[\bar{v}_{SN} - a_N\left(r_S \tau_S^E + \left(1 - r_S\right)\tau_S^E\right)\right]$$
$$+\frac{m_S a_N}{2}\left(r_S \tau_S^E + \left(1 - r_S\right)0\right)^2 + m_S r_S\left(\tau_S^E - 0\right)\left[\bar{v}_{SN} - a_N\left(r_S \tau_S^E + \left(1 - r_S\right)0\right)\right]$$
$$= m_S r_S \tau_S^E\left[\bar{v}_{SN} - a_N r_S \tau_S^E\right] - \frac{m_S a_N}{2}\left(\tau_S^E\right)^2\left(1 - r_S^2\right) \Leftrightarrow$$
$$\Delta_{SN}\left(\tau_S^E\right) = m_S r_S \tau_S^E \bar{v}_{SN} - \frac{m_S a_N}{2}\left(\tau_S^E\right)^2\left(1 + r_S^2\right), \tag{58}$$

which is positive if and only if

$$\tau_S^E \in \left[0, \bar{\tau}_S^E\right], \text{ where } \bar{\tau}_S^E \equiv \frac{2 r_S \bar{v}_{SN}/a_N}{1 + r_S^2} > \tau_S^{BAU},$$

according to (27). For every tariff that is ex post credible, $\Delta_{SN}\left(\tau_S^E\right) \geq 0$. ∥

The rest of the proof of Proposition 4 continues to hold if $s_{SS}^{BAU} - \Delta_{SN}\left(\tau_S^E\right)$ replaced s_{SS}^{BAU} in (53).

Note that $\Delta_{SN}\left(\tau_S^E\right)$ is maximized at:

$$
\begin{aligned}
\tau_S^M &= \frac{\overline{\tau}_S^E}{2} = \frac{r_S \overline{v}_{SN}/a_N}{1 + r_S^2} \Rightarrow \\
\overline{\Delta}_{SN} &\equiv \Delta_{SN}\left(\tau_S^M\right) = m_S \frac{r_S \overline{v}_{SN}/a_N}{1 + r_S^2}\left(\overline{v}_{SN} r_S - \frac{a_N}{2}\left(\frac{r_S \overline{v}_{SN}/a_N}{1 + r_S^2}\right)\left(1 + r_S^2\right)\right) \\
&= m_S \frac{\left(r_S \overline{v}_{SN}\right)^2/2a_N}{1 + r_S^2}.
\end{aligned}
\tag{59}
$$

At τ_S^M, (57) becomes:

$$
s_{SS}\left(X_S, 0\right) \geq s_{SS}^{BAU} - m_S \frac{\left(r_S \overline{v}_{SN}\right)^2/2a_N}{1 + r_S^2}.
$$

More generally: Given that $\underline{X}$ increases, and $\overline{X}$ decreases, in s_{SS}^{BAU}, and $\Delta_{SN}\left(\tau_S^E\right)$ increases in $\tau_S^E \in \left[0, \tau_S^M\right)$, when s_{SS}^{BAU} is replaced by $s_{SS}^{BAU} - \Delta_{SN}\left(\tau_S^E\right)$ it follows that $\underline{X}$ decreases, and $\overline{X}$ increases, in $\tau_S^E \in \left[0, \tau_S^M\right)$. Combined with (54), we get:

$$
\begin{aligned}
\underline{X}^M &= m\left[r_S \overline{v}_{SS} + \frac{\left(1 - r_S\right)^2}{2 - r_S}\overline{v}_{NS} - \frac{\left(1 - r_S\right)}{2 - r_S}\overline{v}_{NS}\sqrt{1 - \frac{2\left(2 - r_S\right)a_S}{\left(1 - r_S\right)^2 \overline{v}_{NS}^2 m}\left[s_{SS}^{BAU} - \overline{\Delta}_{SN}\right]}\right] \\
&= m\left[r_S \overline{v}_{SS} + \frac{\left(1 - r_S\right)^2}{2 - r_S}\overline{v}_{NS} - \frac{\left(1 - r_S\right)}{2 - r_S}\overline{v}_{NS}\sqrt{1 - \frac{\left(2 - r_S\right)r_S}{3 - 2r_S} + \frac{a_S}{a_N}\left(\frac{\overline{v}_{SN}}{\overline{v}_{NS}}\right)^2 \frac{2 - r_S}{\left(1 - r_S\right)^2}\frac{r_S^3}{1 + r_S^2}}\right]
\end{aligned}
\tag{60}
$$

With (A2), $r_S = 1/2$ and $m = 2$, so (58) becomes:

$$
\Delta_{SN}\left(\tau_S^E\right) = \left(\frac{\overline{v}_{SN}}{2} - \frac{5}{8}a_N \tau_S^E\right)\tau_S^E, \text{ so } \tau_S^M = \frac{2}{5}\frac{\overline{v}_{SN}}{a_N} \text{ and } \overline{\Delta}_{SN} = \frac{\overline{v}_{SN}^2}{10a_N}.
$$

Combined with (55), we now get:

$$
\begin{aligned}
\underline{X}\left(\tau_S^E\right) &= \overline{v}_{SS} + \frac{\overline{v}_{NS}}{3} - \frac{2}{3}\overline{v}_{NS}\sqrt{1 - \frac{6a_S}{\overline{v}_{NS}^2}\left[s_{SS}^{BAU} - \Delta_{SN}\left(\tau_S^E\right)\right]} \\
&= \overline{v}_{SS} - \frac{\overline{v}_{NS}}{3}\left(\sqrt{\frac{5}{2} + \frac{a_S}{\overline{v}_{NS}^2}\left(12\overline{v}_{SN} - 15a_N \tau_S^E\right)\tau_S^E} - 1\right).
\end{aligned}
$$

And:

$$
\underline{X}^M = \overline{v}_{SS} + \frac{\overline{v}_{NS}}{3} - \frac{2\overline{v}_{NS}}{3}\sqrt{\frac{10}{16} + \frac{6a_S}{\overline{v}_{NS}^2}\frac{\overline{v}_{SN}^2}{10a_N}} = \overline{v}_{SS} - \frac{\overline{v}_{NS}}{3}\left(\sqrt{\frac{5}{2} + \frac{12}{5}\frac{a_S}{a_N}\frac{\overline{v}_{SN}^2}{\overline{v}_{NS}^2}} - 1\right).
$$

The derivations of $\overline{X}\left(\tau_S^E\right)$ and $\overline{X}^M$ are analogous.

PROPOSITION A-6: *If $X_S^{FB} \in \left[\underline{X}\left(\tau_S^*\right), \overline{X}\left(\tau_S^*\right)\right]$, it is optimal with $\tau_S^{CTA} = \tau_S^*$, and the CTA implements X_S^{FB}, $\tau_N = 0$, and $X_N^{MR}\left(\tau_S^*\right)$. If $X_S^{FB} \notin \left[\underline{X}\left(\tau_S^*\right), \overline{X}\left(\tau_S^*\right)\right]$, there are five different cases to consider:*

(i) Suppose $X_S^{FB} > \overline{X}\left(\tau_S^*\right)$ and $\tau_S^* < \tau_S^M$. With the optimal CTA, $\tau_S^E \in \left(\tau_S^*, \tau_S^M\right)$ and $X_S \in \left(\underline{X}\left(\tau_S^*\right), X_S^{FB}\right)$. If either d_N' decreases or d_S' increases, the optimal τ_S increases, and X_N decreases while X_S increases.

(ii) Suppose $X_S^{FB} > \overline{X}\left(\tau_S^*\right)$ and $\tau_S^* > \tau_S^M$. With the optimal CTA, $\tau_S^E \in \left(\tau_S^M, \tau_S^*\right)$ and $X_S \in \left(\underline{X}\left(\tau_S^*\right), X_S^{FB}\right)$. If either d_N' or d_S' increases, the optimal τ_S increases, and both X_N and X_S decrease.

(iii) Suppose $X_S^{FB} < \underline{X}(\tau_S^*)$ and $\tau_S^* < \tau_S^M$. With the optimal CTA, $\tau_S^E \in (\tau_S^*, \tau_S^M)$ and $X_S \in (X_S^{FB}, \underline{X}_S)$. If either d_N' or d_S' increases, the optimal τ_S increases, and both X_N and X_S decrease.

(iv) Suppose $X_S^{FB} < \underline{X}(\tau_S^*)$ and $\tau_S^* > \tau_S^M$. With the optimal CTA, $\tau_S^E \in (\tau_S^M, \tau_S^*)$ and $X_S \in (X_S^{FB}, \underline{X}_S)$. If either d_N' decreases or d_S' increases, the optimal τ_S increases, and X_N decreases while X_S increases.

(v) If $\tau_S^* = \tau_S^M$, the optimal CTA ensures that $\tau_S^E = \tau_S^*$ and $X_S = \underline{X}(\tau_S^*)$ if $X_S^{FB} < \underline{X}(\tau_S^*)$, while $X_S = \overline{X}(\tau_S^*)$ if $X_S^{FB} > \overline{X}(\tau_S^*)$.

PROOF: Generalizing (57), we have

$$s_{SS}(X_S, 0) \geq s_{SS}^{BAU} - \Delta_{SN}(\tau_S^E),$$

and combined with (53) and (58), we find that the smallest implementable X_S is a function of the tariff S that is permitted by the CTA:

$$\underline{X}(\tau_S^E) = m\left[r_S \overline{v}_{SS} + \frac{1 - r_S}{2 - r_S}\overline{v}_{-S} - \frac{\overline{v}_{-S}}{2 - r_S}\sqrt{(1 - r_S)^2 + (2 - r_S)\left(r_S - \frac{2ma_S}{(mv_{-S})^2}\left(s_{SS}^{BAU} - \Delta_{SN}(\tau_S^E)\right)\right)}\right].$$

Note that $\underline{X}(\tau_S^E)$ is decreasing and convex in $\Delta_{SN}(\tau_S^E)$. The derivative of $\underline{X}(\tau_S^E)$ w.r.t. τ_S^E is:

$$\frac{\partial \underline{X}(\tau_S^E)}{\partial \tau_S^E} = m\frac{\overline{v}_{-S}}{2 - r_S}\frac{-2ma_S(2 - r_S)/(mv_{-S})^2}{2\sqrt{(1 - r_S)^2 + (2 - r_S)\left(r_S - \frac{2ma_S}{(mv_{-S})^2}\left(s_{SS}^{BAU} - \Delta(\tau_S^E)\right)\right)}}\frac{\partial \Delta(\tau_S^E)}{\partial \tau_S^E}$$

$$= \frac{-a_S}{\overline{v}_{-S}\sqrt{(1 - r_S)^2 + (2 - r_S)\left(r_S - \frac{2ma_S}{(mv_{-S})^2}\left(s_{SS}^{BAU} - \Delta(\tau_S^E)\right)\right)}}\frac{\partial \Delta(\tau_S^E)}{\partial \tau_S^E}.$$

From (58), which is concave, we see

$$\frac{\partial \Delta(\tau_S^E)}{\partial \tau_S^E} = mr_S\left[r_S\left(\overline{v}_{SN} - 2a_N r_S \tau_S^E\right) - a_N r_N(r_N + 2r_S)\tau_S^E\right],$$

which is positive for small τ_S^E, but decreases in τ_S^E. Combining the two equations above,

$$\frac{\partial \underline{X}(\tau_S^E)}{\partial \tau_S^E} = \frac{-a_S mr_S\left[r_S\left(\overline{v}_{SN} - 2a_N r_S \tau_S^E\right) - a_N r_N(r_N + 2r_S)\tau_S^E\right]}{\overline{v}_{-S}\sqrt{(1 - r_S)^2 + (2 - r_S)\left(r_S - \frac{2ma_S}{(mv_{-S})^2}\left(s_{SS}^{BAU} - \Delta(\tau_S^E)\right)\right)}}.$$

Similarly, we have:

$$\frac{\partial \overline{X}(\tau_S^E)}{\partial \tau_S^E} = -\frac{\partial \underline{X}(\tau_S^E)}{\partial \tau_S^E} = \frac{a_S mr_S\left[r_S\left(\overline{v}_{SN} - 2a_N r_S \tau_S^E\right) - a_N r_N(r_N + 2r_S)\tau_S^E\right]}{\overline{v}_{-S}\sqrt{(1 - r_S)^2 + (2 - r_S)\left(r_S - \frac{2ma_S}{(mv_{-S})^2}\left(s_{SS}^{BAU} - \Delta(\tau_S^E)\right)\right)}}.$$

When $X_S^{FB} \in [\underline{X}(\tau_S^*), \overline{X}(\tau_S^*)]$, X_S^{FB} is implemented with $\tau_N = 0$ and $\tau_S = \tau_S^*$ on the equilibrium path.

If, instead, $\overline{X}(\tau_S^*) < X_S^{FB}$, the best CTA ensures $X_S = \overline{X}(\tau_S^E)$ for the τ_S^E maximizing the sum of payoffs. When $X_S^{FB} < \underline{X}(\tau_S^*)$, the best CTA ensures $X_S = \underline{X}(\tau_S^E)$ for the τ_S^E maximizing the sum of payoffs. S's tariff influences four parts of the total payoffs:

$$s_S(X_S, 0) - d_N(X_S) + s_N\left(X_N^{BR}(\tau_S^E), \tau_S^E\right) - d_S\left(X_N^{BR}(\tau_S^E)\right).$$

The f.o.c. can be written as:

$$\left[d'_N(X_S) - \frac{\partial s_S(X_S,0)}{\partial X_S}\right]\left(-\frac{\partial X_S}{\partial \tau_S^E}\right) + \frac{\partial s_N(\tau_S^E)}{\partial \tau_S^E} + d'_S\left(X_N^{BR}(\tau_S^E)\right)\left(-\frac{\partial X_N^{BR}(\tau_S^E)}{\partial \tau_S^E}\right) = 0, \qquad (61)$$

where each of the three terms is decreasing in τ_S^{E}.[52] The f.o.c. is thus sufficient, and it pins down τ_S^E to be strictly between τ_S^M (which makes the first term equal to zero) and τ_S^* (which makes the second two terms equal to zero). With (21), (45), and (49), (61) can be written as:

$$\left[d'_N(X_S) - \frac{m_N\bar{v}_{NS} + m_S\bar{v}_{SS} - X_S}{ma_A}\right]\left(-\frac{\partial X_S}{\partial \tau_S^E}\right) + \left[d'_S\left(X_N^{BR}(\tau_S^E)\right)(a_N m_S) - m_S a_N \tau_S^E\right] = 0. \quad (62)$$

Because s.o.c. holds, the left-hand side (l.h.s.) of (61), and of (62), decreases in τ_S^E. Because it also increases in d'_S, it follows that if d'_S increases, then τ_S^E must increase for (62) to continue to hold, and then $X_N = X_N^{BR}(\tau_S)$ decreases. For other comparative statics, we must distinguish four possibilities.

(i) Suppose $\tau_S^* \equiv d'_S < \tau_S^M$ and $X_S^{FB} > \overline{X}(\tau_S^M)$. Then, (62) holds when $\tau_S \in (\tau_S^*, \tau_S^M)$, and $X_S \in (\overline{X}(\tau_S^M), X_S^{FB})$, so that the first bracket is negative, $-\partial X_S/\partial \tau_S^E = -\partial \overline{X}(\tau_S^E)/\partial \tau_S^E < 0$, and the second bracket is negative as before. A larger τ_S will then increase $X_S = \overline{X}(\tau_S^E)$. A larger d'_N reduces the first term, so τ_S must decrease, which incrases X_N and reduces $X_S = \overline{X}(\tau_S^E)$.

(ii) Suppose $\tau_S^* > \tau_S^M$ and $X_S^{FB} > \overline{X}(\tau_S^*)$. Then, (62) holds when $\tau_S \in (\tau_S^M, \tau_S^*)$, implying $X_S \in (\overline{X}(\tau_S^M), X_S^{FB})$, so that the first bracket is negative, but $-\partial X_S/\partial \tau_S^E = -\partial \overline{X}(\tau_S^E)/\partial \tau_S^E > 0$, while the second bracket is positive. A larger τ_S^E will then reduce $X_S = \overline{X}(\tau_S^E)$. A larger d'_N increases the first bracket, the l.h.s. increases, so τ_S^E must increase, which decreases both X_N and $X_S = \underline{X}(\tau_S^E)$.

(iii) Suppose $\tau_S^* < \tau_S^M$ and $X_S^{FB} < \underline{X}(\tau_S^M)$. Then, (62) holds when $\tau_S \in (\tau_S^*, \tau_S^M)$, implying $X_S \in (X_S^{FB}, \underline{X}^M)$, so that the first bracket is positive, $-\partial \underline{X}(\tau_S^E)/\partial \tau_S^E > 0$, and the second bracket is negative. A larger τ_S will then reduce $X_S = \underline{X}(\tau_S^E)$. A larger d'_N increases the first term, so τ_S must increase, which reduces X_N and $X_S = \underline{X}(\tau_S^E)$.

(iv) Suppose $\tau_S^* > \tau_S^M$ and $X_S^{FB} < \underline{X}(\tau_S^*)$. Then, (62) holds when $\tau_S \in (\tau_S^M, \tau_S^*)$, implying $X_S \in (X_S^{FB}, \underline{X}(\tau_S^M))$, so that the first bracket is positive, but $-\partial \underline{X}(\tau_S^E)/\partial \tau_S^E < 0$, while the second bracket is positive. A larger τ_S will then increase $X_S = \underline{X}(\tau_S^E)$. A larger d'_N increases the first bracket, the l.h.s. decreases, so τ_S must decrease, which increases X_N and decreases $X_S = \underline{X}(\tau_S^E)$.

(v) In the knife-edge case in which $\tau_S^* = \tau_S^M$, then no other tariff than $\tau_S^* = \tau_S^M$ can increase $\overline{X}(\tau_S^E)$ or decrease $\underline{X}(\tau_S^E)$.

---

[52] It is easy to verify that both the second and the third term decreases in t_S^E. For the first term, one must check that it decreases in t_S^E of each of the four cases (i)-(iv), discussed below. I have omitted the explicit discussion of these checks for brevity, but will provide it upon requests.

The purpose of this appendix is to show that the results in Section 3 do not hinge on the particular instruments considered in the main text.

As proven below, if government i regulates domestic investment or production with a tax, set before the investment stage, the equilibrium tax is $\tau_i^I = \bar{v}_{ij}/4a_i$, resulting in $X_i^I = \bar{v}_{ii} + \bar{v}_{ji}/8$, exactly as in (17). Thus, such a policy can replace the assumption that X_S is set directly, and the comparison between S and N can be interpreted as a comparison between governments that do, and do not, set production taxes.

If, instead, a government sets an export tariff τ_i^E before the investment stage, the equilibrium export tariff is $\tau_i^E = \bar{v}_{ji}/2a_i$, implying $X_i^E > X_i^I$. In this case, i does not need to limit X_i so much to improve its terms of trade. However, it is easy to check that X_i^I is still smaller than X_i would be with no regulation and free trade. The intuition is simply that $\tau_i^E > 0$ reduces the equilibrium quantity that is consumed.

When both instruments can be combined, it is optimal with the export tariff $\tau_i^E = \frac{44}{79}\bar{v}_{ji}/a_i$ combined with a tax on investment (or production) $\tau_i^I = -\frac{5}{79}\frac{\bar{v}_{ji}}{a_i} < 0$, which is negative (and thus a subsidy) since i's government seeks to raise international prices but not domestic prices. This combination leads to the capacity $X_i^{IE} = \bar{v}_{ii} + \frac{25}{79}\bar{v}_{ji} > X_i^E$. The additional instrument reduces the need to limit X_i. By comparison, X_i^{IE} is still smaller than X_i would be under free trade and no regulation. Thus, in all these cases, (i) governmental regulation leads to less investments, (ii) letting a government determine the capacity leads to less investments, and, (iii) free trade leads to capacity increases.

PROPOSITION A-7:

(i) If country i regulates X_i with an investment tax or, equivalently, with a production tax, set before the investment stage, then:

$$\tau_i^I = \frac{1}{4}\frac{\bar{v}_{ji}}{a_i}, \text{ implying } X_i^I = \bar{v}_{ii} + \frac{\bar{v}_{ji}}{8}.$$

(ii) If country i instead commits to an export tariff before the investment stage, then:

$$\tau_i^E = \frac{44}{79}\frac{\bar{v}_{ji}}{a_i}, \text{ implying } X_i^E := \bar{v}_{ii} + \frac{\bar{v}_{ji}}{4} > X_i^I.$$

(iii) If country i can commit to both an investment tax (or production tax) and an export tariff, then:

$$\tau_i^I = -\frac{5}{79}\frac{\bar{v}_{ji}}{a_i} \text{ and } \tau_i^E = \frac{44}{79}\frac{\bar{v}_{ji}}{a_i}, \text{ implying } X_i^{IE} = \bar{v}_{ii} + \frac{25}{79}\bar{v}_{ji} > X_i^E.$$

(iv) If parameters are identical for N and S, then:

$$X_S^{BAU} = X_i^I < X_i^E < X_i^{IE} < X_N^{FTA}.$$

PROOF: Instead of referring to the country setting the taxes as i, I refer to it as S. The proposition is only proven for $n = 1$ and $m_N = m_S = 1$.

If the physical investment cost is k_{S0} and transport cost to N is t_{N0}, then, if we define $k_S := k_{S0} + \tau^I_S$, $t_N := t_{N0} + \tau^E_S$, and $\bar{v}_{ij0} := v_{ij} - a_j (k_j + t_{ij})$, all consumers act as above. (I only use these (re)definitions in this proof.)

Just as X_N is characterized by (42),

$$X_S = m_S \left(v_{SS} - a_S k_{S0} - a_S \tau^I_S \right) + r_S m_N \left(v_{NS} - a_S k_{S0} - a_S t_{N0} - a_S \tau^E_S - a_S \tau^I_S \right). \tag{63}$$

From (41),

$$\bar{p}_S + \tau_N = \frac{2 + r_N}{1 + 2r_N} \frac{r_N \bar{v}_{NS}}{a_S}, \quad \text{so}$$

$$c_{NS} = \bar{v}_{NS} - a_S \left(\bar{p}_S + \tau_N \right) = \bar{v}_{NS} - a_S \bar{p}_S - \frac{1 + r_N}{1 + 2r_N} r_N \bar{v}_{NS}. \tag{64}$$

Revenues. The taxes τ^I_S and τ^E_i give revenues $\tau^I_S X_S + \tau^E_i m_N c_{NS}$, spent by S on the numeraire good. When investors in S ensure that $\bar{p}_S := p_S - a_S \left(k_{S0} + \tau^I_S \right) = 0$, the revenues to S from τ^E_S and τ^I_S are (when we use (63), (64), and $\bar{v}_{ij0} := v_{ij} - a_j (k_j + t_{ij})$):

$$\tau^I_S \left(m_S \left(v_{SS} - a_S \left(k_{S0} + \tau^I_S \right) \right) + r_S m_N \left(\bar{v}_{NS0} - a_S \tau^I_S - a_S \tau^E_S \right) \right)$$
$$+ \tau^E_S m_N \left[v_{NS} - a_S \left(k_{S0} + \tau^I_S + t_{N0} + \tau^E_S \right) \right] \left(1 - \frac{1 + r_N}{1 + 2r_N} r_N \right)$$
$$= \tau^I_S m_S \left(\bar{v}_{SS0} - a_S \tau^I_S \right) + \left(\bar{v}_{NS0} - a_S \tau^I_S - a_S \tau^E_S \right) m_N \left[r_S \tau^I_S + \left(1 - \frac{1 + r_N}{1 + 2r_N} r_N \right) \tau^E_S \right].$$

Consumer Surplus. From Lemma 1, the consumer surplus in S is $U_S = \overline{U}_S + \kappa_S$. With $\bar{p}_S = 0$, (29) implies $s_{SS} = 0$, so $U_S = s_{SN} + \kappa_S$. Here, s_{SN} is independent of S's policy regarding S's good, but κ_S is:

$$\kappa_S = e_S/m_S - \left[v_{SN} (k_N + t_S) - a_N (k_N + t_S)^2 /2 \right] - v_{SS} \left(k_{S0} + \tau^I_S \right) + a_S \left(k_{S0} + \tau^I_S \right)^2 /2$$
$$= e_S/m_S - \left[v_{SN} (k_N + t_S) - a_N (k_N + t_S)^2 /2 \right] - v_{SS} k_{S0} + a_S (k_{S0})^2 /2$$
$$- \tau^I_S \left(v_{SS} - a_S k_{S0} \right) + a_S \left(\tau^I_S \right)^2 /2.$$

Only the last two terms depend on S's policy. When these two terms are added to the revenues from S's policy, we find that S's objective when choosing τ^I_S and τ^E_S is to maximize:

$$-m_S a_S \frac{\left(\tau^I_S \right)^2}{2} + \left(\bar{v}_{NS0} - a_S \tau^I_S - a_S \tau^E_S \right) m_N \left[r_S \tau^I_S + \left(1 - \frac{1 + r_N}{1 + 2r_N} r_N \right) \tau^E_S \right].$$

Optimal Policy. The f.o.c. w.r.t. τ^I_S is:

$$-m_S a_S \tau^I_S - a_S m_N \left[r_S \tau^I_S + \left(1 - \frac{1 + r_N}{1 + 2r_N} r_N \right) \tau^E_S \right] + \quad r_S m_N \left(\bar{v}_{NS0} - a_S \tau^I_S - a_S \tau^E_S \right) = 0 \Leftrightarrow$$

$$- \left(1 - \frac{1 + r_N}{1 + 2r_N} r_N \right) \tau^E_S + r_S \left(\frac{\bar{v}_{NS0}}{a_S} - \tau^E_S \right) = \left(\frac{r_S}{r_N} + 2r_S \right) \tau^I_S \Leftrightarrow$$

$$r_S \frac{\bar{v}_{NS0}}{a_S} - \left(r_S + 1 - \frac{1 + r_N}{1 + 2r_N} r_N \right) \tau^E_S = \left(\frac{r_S}{r_N} + 2r_S \right) \tau^I_S.$$

With $m_N = m_S = 1$,

$$\tau^I_S = \bar{v}_{NS0}/4a_S - \frac{1}{2} \left(\frac{1}{8} + 1 \right) \tau^E_S.$$

(i) With $\tau_S^E = 0$ and $m_N = m_S = 1$,

$$\tau_S^I = \bar{v}_{NS0}/4a_S, \text{ so}$$

$$
\begin{aligned}
X_S^I &= m_S\left(\bar{v}_{SS0} - a_S\tau_S^I\right) + r_S m_N\left(\bar{v}_{NS0} - a_S\tau_S^I - a_S\tau_S^E\right) \\
&= \bar{v}_{SS0} - \frac{\bar{v}_{NS0}}{4} + \frac{1}{2}\left(\bar{v}_{NS0} - \frac{\bar{v}_{NS0}}{4}\right) = \bar{v}_{SS0} + \frac{1}{8}\bar{v}_{NS0} = X_S^{BAU}.
\end{aligned}
$$

This equation confirms the intuition that setting an investment tax is equivalent to setting X_S, when it comes to the outcome for X_S.

(ii) The f.o.c. w.r.t. τ_S^E is:

$$-a_S\left[r_S\tau_S^I + \left(1 - \frac{1+r_N}{1+2r_N}r_N\right)\tau_S^E\right] + \left(1 - \frac{1+r_N}{1+2r_N}r_N\right)\left(\bar{v}_{NS0} - a_S\tau_S^I - a_S\tau_S^E\right) = 0 \Leftrightarrow$$

$$-a_S r_S\tau_S^I + \left(1 - \frac{1+r_N}{1+2r_N}r_N\right)\left(\bar{v}_{NS0} - a_S\tau_S^I\right) = 2a_S\tau_S^E\left(1 - \frac{1+r_N}{1+2r_N}r_N\right) \Leftrightarrow$$

$$\frac{\bar{v}_{NS0}}{a_S} - \tau_S^I\left(\frac{r_S}{1 - \frac{1+r_N}{1+2r_N}r_N} + 1\right) = 2\tau_S^E.$$

If $m_N = m_S = 1$, $\frac{\bar{v}_{NS0}}{a_S} - \frac{9}{5}\tau_S^I = 2\tau_S^E$. If also $\tau_S^I = 0$,

$$
\begin{aligned}
\tau_S^E &= \bar{v}_{NS0}/2a_S, \text{ and} \\
X_S^E &= \left(m_S\bar{v}_{SS0} + r_S m_N\left(\bar{v}_{NS0} - \bar{v}_{NS0}/2\right)\right) = \bar{v}_{SS0} + \bar{v}_{NS0}/4 > X_S^{BAU}.
\end{aligned}
$$

(iii) When both τ_S^I and τ_S^E can be used, then we can combine the two f.o.c.'s to get:

$$
\begin{aligned}
\frac{\bar{v}_{NS0}}{4a_S} - \frac{1}{2}\left(\frac{1}{8} + 1\right)\frac{1}{2}\left(\frac{\bar{v}_{NS0}}{a_S} - \tau_S^I\frac{9}{5}\right) &= \tau_S^I \Leftrightarrow \\
\frac{\bar{v}_{NS0}}{4a_S} - \frac{1}{2}\frac{9}{16}\frac{\bar{v}_{NS0}}{a_S} + \tau_S^I\frac{1}{2}\frac{9}{5}\frac{9}{16} &= \tau_S^I \Leftrightarrow \\
-\left(\frac{9}{8} - 1\right)\frac{\bar{v}_{NS0}}{4a_S} &= \left(1 - \frac{1}{2}\frac{81}{80}\right)\tau_S^I \Leftrightarrow \\
-\frac{\bar{v}_{NS0}}{32a_S} &= \left(\frac{160 - 81}{160}\right)\tau_S^I \Leftrightarrow \\
\tau_S^I &= -\frac{5}{79}\frac{\bar{v}_{NS0}}{a_S}, \text{ and}
\end{aligned}
$$

$$\tau_S^E = \frac{1}{2}\left(\frac{\bar{v}_{NS0}}{a_S} - \frac{9}{5}\tau_S^I\right) = \frac{1}{2}\left(1 + \frac{9}{5}\frac{5}{79}\right)\frac{\bar{v}_{NS0}}{a_S} = \frac{44}{79}\frac{\bar{v}_{NS0}}{a_S}.$$

Thus,

$$
\begin{aligned}
X_S^{IE} &= \bar{v}_{SS0} - a_S\tau_S^I + \frac{1}{2}\left(\bar{v}_{NS0} - a_S\tau_S^I - a_S\tau_S^E\right) \\
&= \bar{v}_{SS0} + \frac{5}{79}\bar{v}_{NS0} + \frac{1}{2}\left(\bar{v}_{NS0} + \frac{5}{79}\bar{v}_{NS0} - \frac{44}{79}\bar{v}_{NS0}\right) \\
&= \bar{v}_{SS0} + \frac{1}{2}\left(1 - \frac{29}{79}\right)\bar{v}_{NS0} = \bar{v}_{SS0} + \frac{25}{79}\bar{v}_{NS0} > X_S^E.
\end{aligned}
$$

In addition to the agricultural capacity X_S, invested in by S's government, suppose private investors in S invest to produce X_J in sector $J \in \{1, ..., q_S\}$. Similarly, private investors in N invest to produce X_K in sector $K \in \{1, ..., q_N\}$. For each J, the tariff in N is measured by τ_{NJ}, and for each K, the tariff in S is τ_{SK}.

PROPOSITION A-8: *Suppose S exports q_S types of goods to N, while N exports q_N types of goods to S.*

(i) With free trade in equilibrium, the CTA^0 can implement every $X_S \geq X_S^0$ if $X_S \in \left[\underline{X}^{q_S}, \overline{X}^{q_S} \right]$, where $\underline{X}^{q_S}$ decreases, and $\overline{X}^{q_S}$ increases, in q_S. (Both thresholds are independent of q_N.)

(ii) With positive tariffs in equilibrium, the CTA can implement every $X_S \geq X_S^0$ if $X_S \in \left[\underline{X}^{q_S q_N}, \overline{X}^{q_S q_N} \right]$, where $\underline{X}^{q_S q_N}$ decreases, and $\overline{X}^{q_S q_N}$ increases, in q_S and in q_N.

PROOF: The surplus for S from sector J is denoted by $s_{SJ}(X_J, \tau_{NJ})$. (As before, this surplus equals the country's consumer surplus and the producers' revenues minus the cost of investing and producing.) Given the market response to the tariffs, equilibrium capacity is $X_J^E = X_J^{MR}(\tau_{NJ}^E)$, where $\tau_{NJ}^E = \tau_{NJ}^{CTA}(X_S^{CTA})$ is the equilibrium and expected tariff when X_S takes its equilibrium value under the CTA. Define X_K^E in the equivalent way.

Under the CTA, the tariffs can be functions (with superscripts CTA) of X_S. For the CTA to implement X_S^{CTA}, the following incentive constraint is analogous to (51):

$$s_{SS}\left(X_S^{CTA}, \tau_{NJ}^E\right) + \sum_J s_{SJ}\left(X_J^E, \tau_{NJ}^E\right) + \sum_K s_{SK}\left(X_K^E, \tau_{SK}^E\right) \geq \qquad (65)$$

$$s_{SS}\left(X_S', \tau_{NS}^{CTA}(X_S')\right) + \sum_J s_{SJ}\left(X_J^E, \tau_{NJ}^{CTA}(X_S')\right) + \sum_K s_{SK}\left(X_K^E, \tau_{SK}^{CTA}(X_S')\right) \; \forall X_S' \geq X_S^0.$$

In equilibrium, investors expect that S selects X_S^{CTA} when they invest in X_J or X_K.

As before, four observations help to simplify (65): (1) For the CTA to be credible, $\tau_{NJ} \in \left[0, \tau_{NJ}^{BR}(X_J)\right]$ and $\tau_{SK} \in \left[0, \tau_{SK}^{BR}(X_K)\right]$, just as before. As in the proof of Proposition 4, $\tau_{NJ}^E = 0$ is both ex post efficient and it helps to satisfy (65). (2) Inequality (65) can hold if and only if it holds at $\tau_{NJ}^{CTA}(X_S) = \tau_{NJ}^{BR}(X_J)$, when $X_S \neq X_S^{CTA}$, because, ex post, S likes this large tariff the least, among all the credible tariffs $\tau_{NJ} \in \left[0, \tau_{NJ}^{BR}(X_J)\right]$. (3) As in the proof of Proposition 5, a deviation from X_S^{CTA} is punished the most if it leads to $\tau_{SK} = 0$. (4) As before, S's best deviation X_S is X_S^{BAU} (i.e., (66) is hardest to satisfy when $X_S = X_S^{BAU}$). With (1)-(4), (65) simplifies to:

$$s_{SS}\left(X_S^{CTA}, 0\right) \geq s_{SS}^{BAU} - \sum_J \Delta_{SJ} - \sum_K \Delta_{SK}\left(\tau_{SK}^E\right) \qquad (66)$$

$$= s_{SS}\left(X_S', \tau_{NS}^{CTA}(X_S')\right) - \sum_J \Delta_{SJ} - \sum_K \Delta_{SK}\left(\tau_{SK}^E\right) \; \forall X_S' \geq X_S^0, \text{ where}$$

$$\Delta_{SJ} \equiv s_{SJ}\left(X_J^{MR}(0), 0\right) - s_{SJ}\left(X_J^{MR}(0), \tau_{NJ}^{BR}\left(X_J^{MR}(0)\right)\right) \text{ and}$$

$$\Delta_{SK}\left(\tau_{SK}^E\right) \equiv s_{SK}\left(X_K^{MR}\left(\tau_{SK}^E\right), \tau_{SK}^E\right) - s_{SK}\left(X_K^{MR}\left(\tau_{SK}^E\right), 0\right).$$

LEMMA 3:

$$(a) \qquad \Delta_{SJ} \equiv \left(\frac{r_N^2}{a_J}\frac{\overline{v}_{NJ}}{1-r_N^2}\right)\left(\sum_{i\backslash S} m_i\overline{v}_{iJ} - \frac{m_S}{2}\left(\frac{r_N^2\overline{v}_{NJ}}{1-r_N^2}\right)\right),$$

$$(b) \quad \Delta_{SK}\left(\tau_{SK}^E\right) \equiv m_S r_S \overline{v}_{SN}\tau_S^E - \frac{m_S a_N}{2}\left(1+r_S^2\right)\left(\tau_S^E\right)^2, \quad so$$

$$\overline{\Delta}_{SK} \equiv \max_{\tau_{SK}^E}\Delta_{SK}\left(\tau_{SK}^E\right) = m_S\frac{\left(r_S\overline{v}_{SK}\right)^2/2a_K}{1+r_S^2}.$$

With $n = 2$, (a) becomes:

$$\Delta_{SJ} = m_N\frac{\overline{v}_{NJ}^2}{2a_J}\left(\frac{1}{1/r_N^2 - 1}\right)\left(\frac{2+r_N}{1+r_N}\right).$$

In Example 1,

$$\Delta_{SJ} = \frac{5}{18} \text{ and } \overline{\Delta}_{SK} = \frac{1}{10}.$$

PROOF OF LEMMA 3: (a) To derive country S's surplus from S's sector J, $s_{SJ}\left(X_J, \tau_{NJ}\right)$, note that, analogously to (21), we have:

$$\overline{p}_J = \frac{\sum m_i\overline{v}_{iJ} - a_J m_N\tau_{NJ} - X_J}{ma_J} \text{ and } \overline{p}_J + \tau_{NJ} = \frac{\sum m_i\overline{v}_{iJ} + a_J m_S\tau_{NJ} - X_J}{ma_J}.$$

With $X_J^E = X_J^{MR}(0) = \sum m_i\overline{v}_{iJ}$, we find that $s_{SJ}\left(X_J^E, \tau_{NJ}\right)$ is:

$$-m_S a_J\left(\overline{p}_J\right)^2/2 + \overline{p}_J\left(\sum_{i\backslash S} m_i\left(\overline{v}_{iJ} - a_J\overline{p}_J\right) - m_N a_J\tau_{NJ}\right)$$

$$= -\frac{m_S a_J}{2}\left(r_N\tau_{NJ}\right)^2 - \left(r_N\tau_{NJ}\right)\left(\sum_{i\backslash S} m_i\overline{v}_{iJ} + \sum_{i\backslash S} m_i a_J r_N\tau_{NJ} - m_N a_J\tau_{NJ}\right)$$

$$= -a_J\left(\frac{m_S}{2} + \sum_{i\backslash S} m_i - m\right)\left(r_N\tau_{NJ}\right)^2 - r_N\tau_{NJ}\sum_{i\backslash S} m_i\overline{v}_{iJ} = \frac{a_J m_S}{2}\left(r_N\tau_{NJ}\right)^2 - r_N\tau_{NJ}\sum_{i\backslash S} m_i\overline{v}_{iJ}.$$

which decreases in τ_{NJ} as long as $\tau_{NJ} \in [0, \widehat{\tau}_{NJ}]$, where $\widehat{\tau}_{NJ} \equiv \sum_{i\backslash S} r_i\overline{v}_{iJ}/a_J r_S r_N$. I will now show that, when the CTA is renegotiation proof, in that $\tau_{NJ} \in [0, \tau_{NJ}^{BR}(X_J)]$, then $\tau_{NJ} < \widehat{\tau}_{NJ}$. For a renegotiation proof CTA, when τ_{NJ} can be a function of X_S, $\tau_{NJ} \leq \tau_{NJ}^{BR}(X_J)$, given that $X_J = X_J^{MR}(0)$ when no tariff is expected in N in equilibrium. From (24),

$$\tau_{NJ}^{BR}\left(X_J^{MR}(0)\right) \equiv \frac{r_N}{a_J}\frac{\overline{v}_{NJ} - \overline{v}_{AJ} + X_J^{MR}(0)/m}{1-r_N^2} = \frac{r_N}{a_J}\frac{\overline{v}_{NJ}}{1-r_N^2},$$

which is smaller than $\widehat{\tau}_{NJ}$. This confirms that renegotiation-proof Δ_{SJ} is maximized at $\tau_{NJ}^{BR}\left(X_J^{BR}(0)\right)$. With this (out-of-equilibrium) tariff in N, $s_{SJ}\left(X_J^E, \tau_{NJ}\right)$ is:

$$\frac{m_S a_J}{2}\left(r_N\frac{r_N\overline{v}_{NJ}/a_J}{1-r_N^2}\right)^2 - r_N\left(\frac{r_N}{a_J}\frac{\overline{v}_{NJ}}{1-r_N^2}\right)\sum_{i\backslash S} m_i\overline{v}_{iJ} = \frac{m_S a_J}{2}\left(r_N\frac{r_N\overline{v}_{NJ}/a_J}{1-r_N^2}\right)^2$$

$$-r_N\left(\frac{r_N}{a_J}\frac{\overline{v}_{NJ}}{1-r_N^2}\right)\sum_{i\backslash S} m_i\overline{v}_{iJ} = -\left(\frac{r_N^2}{a_J}\frac{\overline{v}_{NJ}}{1-r_N^2}\right)\left(\sum_{i\backslash S} m_i\overline{v}_{iJ} - \frac{m_S}{2}\left(\frac{r_N^2\overline{v}_{NJ}}{1-r_N^2}\right)\right).$$

The proof of (a) is completed by noting that $s_{SJ}\left(X_J^E, 0\right) = 0$.

(b) The proof of Proposition 5 holds for every product produced by N. So, $\Delta_{SK}\left(\tau_{SK}^E\right)$ follows from Lemma 2. $\|$

The reasoning in the proof of Proposition 4 continues to hold if just s_{SS}^{BAU} is replaced by $s_{SS}^{BAU} - \sum_J \Delta_{SJ} - \sum_K \Delta_{SK}\left(\tau_{SK}^E\right)$. Given that $\underline{X}$ increases, and $\overline{X}$ decreases, in s_{SS}^{BAU}, when s_{SS}^{BAU} is replaced by $s_{SS}^{BAU} - \sum_J \Delta_{SJ} - \sum_K \Delta_{SK}\left(\tau_{SK}^E\right)$ it follows that $\underline{X}$ decreases, and $\overline{X}$ increases, when either q_S or q_N increases (if the τ_{SK}^E's stay unchanged)

(i) With free trade in equilibrium, $\tau_{SK} = 0$ and $\Delta_{SK}\left(\tau_{SK}\right) = 0$. Note that $\Delta_{SK} = 0$ also if τ_{SK} is fixed (not contingent on X_S) at any other level than 0.

The reasoning in the proof of Proposition 4 continues to hold if just s_{SS}^{BAU} is replaced by $s_{SS}^{BAU} - \sum_J \Delta_{SJ}$. Given that $\underline{X}$ increases, and $\overline{X}$ decreases, in s_{SS}^{BAU}, when s_{SS}^{BAU} is replaced by $s_{SS}^{BAU} - \sum_J \Delta_{SJ}$ it follows that $\underline{X}$ decreases, and $\overline{X}$ increases, when q_S increases.

(ii) As in (59),

$$\overline{\Delta}_{SK} \equiv \max_{\tau_{SK}^E} \Delta_{SK}\left(\tau_{SK}^E\right) = m_S \frac{\left(r_S \overline{v}_{SK}\right)^2 / 2a_K}{1 + r_S^2}.$$

In this case, the reasoning in the proof of Proposition 4 continues to hold if just s_{SS}^{BAU} is replaced by $s_{SS}^{BAU} - \sum_J \Delta_{SJ} - \sum_K \overline{\Delta}_{SK}$. Given that $\underline{X}$ increases, and $\overline{X}$ decreases, in s_{SS}^{BAU}, when s_{SS}^{BAU} is replaced by $s_{SS}^{BAU} - \sum_J \Delta_{SJ} - \sum_K \overline{\Delta}_{SK}$ it follows that $\underline{X}$ decreases, and $\overline{X}$ increases, when either q_S or q_N increases.

Multiple Consumers. When $n = 1$, $m = 2$, $\overline{v}_{SS} = \overline{v}_{NS} = a_S = 1$, and $\frac{a_S}{a_N}\left(\frac{\overline{v}_{SS}}{\overline{v}_{NS}}\right) = 1$, (32), (35), (54), and (60) become:

$$X_S^{FTA} = 2\left((1 - r_N) + \frac{r_N}{1 + r_N}r_N\right),$$

$$X_S^{BAU} = 2\left((1 - r_N) + \frac{r_N(1 - r_N)}{1 + 2r_N}r_N\right),$$

$$\underline{X} = 2\left((1 - r_N) + \frac{r_N^2}{1 + r_N} - \frac{r_N}{1 + r_N}\sqrt{1 - \frac{(1 + r_N)(1 - r_N)}{1 + 2r_N}}\right),$$

$$\underline{X}^M = 2\left((1 - r_N) + \frac{r_N^2}{1 + r_N} - \frac{r_N}{1 + r_N}\sqrt{1 - \frac{(1 + r_N)(1 - r_N)}{1 + 2r_N} + \frac{1 + r_N}{(r_N)^2}\frac{(1 - r_N)^3}{1 + (1 - r_N)^2}}\right),$$

and similarly:

$$\overline{X} = 2\left((1 - r_N) + \frac{r_N^2}{1 + r_N} + \frac{r_N}{1 + r_N}\sqrt{1 - \frac{(1 + r_N)(1 - r_N)}{1 + 2r_N}}\right),$$

$$\overline{X}^M = 2\left((1 - r_N) + \frac{r_N^2}{1 + r_N} + \frac{r_N}{1 + r_N}\sqrt{1 - \frac{(1 + r_N)(1 - r_N)}{1 + 2r_N} + \frac{1 + r_N}{(r_N)^2}\frac{(1 - r_N)^3}{1 + (1 - r_N)^2}}\right).$$

If, instead, $m_S = 1$, $m = 1 + m_N$, then $r_N = \frac{m_N}{m_N + 1}$ and:

$$X_S^{FTA} = 1 + \frac{\frac{m_N}{m_N + 1}}{1 + \frac{m_N}{m_N + 1}}m_N,$$

$$X_S^{BAU} = 1 + \frac{\frac{m_N}{m_N + 1}\left(1 - \frac{m_N}{m_N + 1}\right)}{1 + 2\frac{m_N}{m_N + 1}}m_N,$$

$$\underline{X} = 1 + \frac{m_N\frac{m_N}{m_N + 1}}{1 + \frac{m_N}{m_N + 1}} - \frac{m_N}{1 + \frac{m_N}{m_N + 1}}\sqrt{1 - \frac{\left(1 + \frac{m_N}{m_N + 1}\right)\left(1 - \frac{m_N}{m_N + 1}\right)}{1 + 2\frac{m_N}{m_N + 1}}},$$

$$\underline{X}^M = 1 + \frac{m_N\frac{m_N}{m_N + 1}}{1 + \frac{m_N}{m_N + 1}} - \frac{m_N}{1 + \frac{m_N}{m_N + 1}}\sqrt{1 - \frac{\left(1 + \frac{m_N}{m_N + 1}\right)\left(1 - \frac{m_N}{m_N + 1}\right)}{1 + 2\frac{m_N}{m_N + 1}} + \frac{1 + \frac{m_N}{m_N + 1}}{\left(\frac{m_N}{m_N + 1}\right)^2}\frac{\left(1 - \frac{m_N}{m_N + 1}\right)^3}{1 + \left(1 - \frac{m_N}{m_N + 1}\right)^2}},$$

$$\overline{X} = 1 + \frac{m_N\frac{m_N}{m_N + 1}}{1 + \frac{m_N}{m_N + 1}} + \frac{m_N}{1 + \frac{m_N}{m_N + 1}}\sqrt{1 - \frac{\left(1 + \frac{m_N}{m_N + 1}\right)\left(1 - \frac{m_N}{m_N + 1}\right)}{1 + 2\frac{m_N}{m_N + 1}}},$$

$$\overline{X}^M = 2\left((1 - r_N) + \frac{r_N^2}{1 + r_N} + \frac{r_N}{1 + r_N}\sqrt{1 - \frac{(1 + r_N)(1 - r_N)}{1 + 2r_N} + \frac{1 + r_N}{(r_N)^2}\frac{(1 - r_N)^3}{1 + (1 - r_N)^2}}\right).$$

Multiple Countries. When $r_S = r_N = 1/(n+1)$, and $\frac{a_S}{a_N}\left(\frac{\overline{v}_{SS}}{\overline{v}_{NS}}\right) = 1$, (32), (35), (53), and (59) give:

$$X_S^{FTA} = 1 + \frac{1 - \frac{1}{n+1}}{2 - \left(\frac{1}{n+1}\right)}n, \ X_S^{BAU} = 1 + \frac{1 - \left(\frac{1}{n+1}\right)^2 - \left(\frac{1}{n+1}\right)}{2 - 2\left(\frac{1}{n+1}\right)^2 - \left(\frac{1}{n+1}\right)}\left(n - \left(\frac{1}{n+1}\right)\right),$$

$$\underline{X} = 1 + \frac{1 - \left(\frac{1}{n+1}\right)}{2 - \left(\frac{1}{n+1}\right)}n - \frac{n}{2 - \left(\frac{1}{n+1}\right)}\sqrt{1 - \frac{2\left(2 - \left(\frac{1}{n+1}\right)\right)}{n\left(\frac{n}{n+1}\right)}\left(\frac{n+1}{2}\frac{\left(\left(\frac{n}{n+1}\right) - \left(\frac{1}{n+1}\right)^2\right)^2}{2 - 2\left(\frac{1}{n+1}\right)^2 - \left(\frac{1}{n+1}\right)}\right)},$$

$$\underline{X}^M = 1 + \frac{1 - \left(\frac{1}{n+1}\right)}{2 - \left(\frac{1}{n+1}\right)}n - \frac{n}{2 - \left(\frac{1}{n+1}\right)}\sqrt{1 - \frac{2\left(2 - \left(\frac{1}{n+1}\right)\right)}{n\left(\frac{n}{n+1}\right)}\left(\frac{n+1}{2}\frac{\left(\left(\frac{n}{n+1}\right) - \left(\frac{1}{n+1}\right)^2\right)^2}{2 - 2\left(\frac{1}{n+1}\right)^2 - \left(\frac{1}{n+1}\right)} - \frac{\left(\left(\frac{1}{n+1}\right)\right)^2\frac{1}{2}}{1 + \left(\frac{1}{n+1}\right)^2}\right)},$$

$$\overline{X} = 1 + \frac{1 - \left(\frac{1}{n+1}\right)}{2 - \left(\frac{1}{n+1}\right)}n + \frac{n}{2 - \left(\frac{1}{n+1}\right)}\sqrt{1 - \frac{2\left(2 - \left(\frac{1}{n+1}\right)\right)}{n\left(\frac{n}{n+1}\right)}\left(\frac{n+1}{2}\frac{\left(\left(\frac{n}{n+1}\right) - \left(\frac{1}{n+1}\right)^2\right)^2}{2 - 2\left(\frac{1}{n+1}\right)^2 - \left(\frac{1}{n+1}\right)}\right)},$$

$$\overline{X}^M = 1 + \frac{1 - \left(\frac{1}{n+1}\right)}{2 - \left(\frac{1}{n+1}\right)}n + \frac{n}{2 - \left(\frac{1}{n+1}\right)}\sqrt{1 - \frac{2\left(2 - \left(\frac{1}{n+1}\right)\right)}{n\left(\frac{n}{n+1}\right)}\left(\frac{n+1}{2}\frac{\left(\left(\frac{n}{n+1}\right) - \left(\frac{1}{n+1}\right)^2\right)^2}{2 - 2\left(\frac{1}{n+1}\right)^2 - \left(\frac{1}{n+1}\right)} - \frac{\left(\left(\frac{1}{n+1}\right)\right)^2\frac{1}{2}}{1 + \left(\frac{1}{n+1}\right)^2}\right)}.$$

Multiple Collaborators. With $m_S = 1$ and $m = 6$, we can vary the mass of consumers included by the CTA-collaborators (in addition to S) from 0 to 5. That is, we use $m = 6$, $r_S = 1/6$, $m_N = m_C$, and $r_N = m_C/6$, in (33), (53), and (59) to get:

$$X_S^{FTA} = 1 + \frac{1 - \frac{1}{6}}{2 - \frac{1}{6}}5,$$

$$\underline{X} = 1 + \frac{1 - \frac{1}{6}}{2 - \frac{1}{6}}5 - \frac{5}{2 - \frac{1}{6}}\sqrt{1 - \frac{2\left(2 - \frac{1}{6}\right)}{6\left(1 - \frac{1}{6}\right)^2}\left(\frac{6}{2}\frac{\left(\left(1 - \frac{1}{6}\right) - \left(\frac{m_C}{6}\right)^2\right)^2}{2 - 2\left(\frac{m_C}{6}\right)^2 - \frac{1}{6}}\right)},$$

$$\underline{X}^M = 1 + \frac{1 - \frac{1}{6}}{2 - \frac{1}{6}}5 - \frac{5}{2 - \frac{1}{6}}\sqrt{1 - \frac{2\left(2 - \frac{1}{6}\right)}{6\left(1 - \frac{1}{6}\right)^2}\left(\frac{6}{2}\frac{\left(\left(1 - \frac{1}{6}\right) - \left(\frac{m_C}{6}\right)^2\right)^2}{2 - 2\left(\frac{m_C}{6}\right)^2 - \frac{1}{6}} - m_C\frac{\left(\frac{1}{6}\right)^2\frac{1}{2}}{1 + \left(\frac{1}{6}\right)^2}\right)},$$

$$\overline{X} = 1 + \frac{1 - \frac{1}{6}}{2 - \frac{1}{6}}5 + \frac{5}{2 - \frac{1}{6}}\sqrt{1 - \frac{2\left(2 - \frac{1}{6}\right)}{6\left(1 - \frac{1}{6}\right)^2}\left(\frac{6}{2}\frac{\left(\left(1 - \frac{1}{6}\right) - \left(\frac{m_C}{6}\right)^2\right)^2}{2 - 2\left(\frac{m_C}{6}\right)^2 - \frac{1}{6}}\right)},$$

$$\overline{X}^M = 1 + \frac{1 - \frac{1}{6}}{2 - \frac{1}{6}}5 + \frac{5}{2 - \frac{1}{6}}\sqrt{1 - \frac{2\left(2 - \frac{1}{6}\right)}{6\left(1 - \frac{1}{6}\right)^2}\left(\frac{6}{2}\frac{\left(\left(1 - \frac{1}{6}\right) - \left(\frac{m_C}{6}\right)^2\right)^2}{2 - 2\left(\frac{m_C}{6}\right)^2 - \frac{1}{6}} - m_C\frac{\left(\frac{1}{6}\right)^2\frac{1}{2}}{1 + \left(\frac{1}{6}\right)^2}\right)}.$$

BAU with Many Strategic Importers. I first derive the BAU equilibrium when S faces n equal-sized and identical strategic importers. Set $r \equiv r_S$, so $r_j = r_N \equiv (1-r)/n$ and $\overline{v}_{ji} = \overline{v}_{Ni}$ for $j \neq$ S. Then, the tariff in each of them is equal, and (23) becomes:

$$-\tau_j + \left(\frac{\overline{v}_{ji}}{a_i} - \frac{\sum_l m_l \overline{v}_{li} - \sum_l a_i m_j \tau_j - X_i}{a_i m} \right) r_j = 0 \Rightarrow$$

$$\tau = \tau_j = \frac{(1-r)/n}{1 - (1-r)^2/n} \left(\frac{\overline{v}_{ji}}{a_i} - \frac{\sum_l m_l \overline{v}_{li} - X_i}{a_i m} \right) = \frac{(1-r)/n}{1 - (1-r)^2/n} \left(\frac{r \overline{v}_{Ni}}{a_i} + \frac{Z}{a_i m} \right), \tag{67}$$

where $Z \equiv X_S - m_S \overline{v}_{SS}$. With this, the price from (21) becomes:

$$
\begin{aligned}
\overline{p}_i &= \frac{\overline{v}_{Ai}}{a_i} - \frac{X_i}{a_i m} - \frac{(1-r)^2/n}{1 - (1-r)^2/n} \left(\frac{r \overline{v}_{Ni}}{a_i} + \frac{Z}{a_i m} \right) \\
&= (1-r) \frac{\overline{v}_{Ni}}{a_i} - \frac{Z/m}{a_i} - \frac{(1-r)^2/n}{1 - (1-r)^2/n} \left(\frac{r \overline{v}_{Ni}}{a_i} + \frac{Z}{a_i m} \right) \\
&= \frac{\overline{v}_{Ni}}{a_i} (1-r) \left(1 - \frac{r(1-r)/n}{1 - (1-r)^2/n} \right) - \frac{Z/m}{a_i} \left(1 + \frac{(1-r)^2/n}{1 - (1-r)^2/n} \right) \\
&= \frac{\overline{v}_{Ni}}{a_i} (1-r) \left(\frac{1 - (1-r)/n}{1 - (1-r)^2/n} \right) - \frac{Z/m}{a_i} \left(\frac{1}{1 - (1-r)^2/n} \right) \Leftrightarrow
\end{aligned}
$$

$$\overline{p}_i = \frac{\overline{v}_{Ni}}{a_i} \left(1 - \frac{r}{1 - (1-r)^2/n} \right) - \frac{Z/m}{a_i} \left(\frac{1}{1 - (1-r)^2/n} \right), \quad \text{so} \tag{68}$$

$$\frac{\partial \overline{p}_S}{\partial X} = -\frac{1}{am} \left(\frac{1}{1 - (1-r)^2/n} \right).$$

With this, the f.o.c. for X_S, in (30), becomes:

$$
\begin{aligned}
\overline{p}_S - (m_S a_S \overline{p}_S + X_S - m_S \overline{v}_{SS}) \frac{1}{am} \left(\frac{1}{1 - (1-r)^2/n} \right) &= 0 \Leftrightarrow \\
\overline{p}_S \left(1 - \frac{r}{1 - (1-r)^2/n} \right) - \frac{Z}{am} \left(\frac{1}{1 - (1-r)^2/n} \right) &= 0 \Leftrightarrow \\
\frac{\overline{v}_{Ni}}{a_i} \left(1 - \frac{r}{1 - (1-r)^2/n} \right)^2 - \frac{Z}{am} \left(\frac{1}{1 - (1-r)^2/n} \right) \left(2 - \frac{r}{1 - (1-r)^2/n} \right) &= 0 \Leftrightarrow
\end{aligned}
$$

$$Z = Z^{BAU} \equiv m \overline{v}_{Ni} \frac{\left[1 - (1-r)^2/n - r \right]^2}{2 - 2(1-r)^2/n - r}. \tag{69}$$

With this, (68) becomes

$$\overline{p}_i = \frac{\overline{v}_{Ni}}{a_i} \left(1 - \frac{r}{1 - (1-r)^2/n} \right) - \frac{\overline{v}_{Ni}}{a_i} \left(\frac{1}{1 - (1-r)^2/n} \right) \frac{\left[1 - (1-r)^2/n - r \right]^2}{2 - 2(1-r)^2/n - r} = \frac{\overline{v}_{Ni}}{a_i} \xi, \quad \text{where}$$

$$\xi \equiv 1 - \frac{r}{1 - (1-r)^2/n} - \left(\frac{1}{1 - (1-r)^2/n} \right) \frac{\left(1 - (1-r)^2/n - r \right)^2}{2 - 2(1-r)^2/n - r}.$$

The tariff (67) and thus τ/p become:

$$\tau = \frac{(1-r)/n}{1-(1-r)^2/n}\left(\frac{r\bar{v}_{Ni}}{a_i}+\frac{1}{a_i m}m\bar{v}_{Ni}\frac{\left[1-(1-r)^2/n-r\right]^2}{2-2(1-r)^2/n-r}\right)$$

$$= \frac{\bar{v}_{Ni}}{a_i}\frac{(1-r)/n}{1-(1-r)^2/n}\left(r+\frac{\left(1-(1-r)^2/n-r\right)^2}{2-2(1-r)^2/n-r}\right),$$

$$\frac{\tau}{p} = \frac{1}{\xi}\frac{(1-r)/n}{1-(1-r)^2/n}\left(r+\frac{\left(1-(1-r)^2/n-r\right)^2}{2-2(1-r)^2/n-r}\right).$$

The fraction consumed domestically, relative to X_S, becomes:

$$f \equiv m_S\frac{\bar{v}_{SS}-a_i\bar{p}_i}{X_S} = mr\frac{\bar{v}_{SS}-\bar{v}_{Ni}\xi}{m_S\bar{v}_{SS}+m\bar{v}_{Ni}\frac{[1-(1-r)^2/n-r]^2}{2-2(1-r)^2/n-r}} = r\frac{1-\omega\xi}{r+\omega\frac{\left(1-(1-r)^2/n-r\right)^2}{2-2(1-r)^2/n-r}},$$

where $\omega \equiv \bar{v}_{Ni}/\bar{v}_{SS}$. A non-tariff barrier (NTB) has the same effect as a transport cost, if we, for simplicity, treat it as being exogenous. With the 20% NTB on food from Cadot et al. (2018), but otherwise equal preferences, we may write:

$$\bar{v}_{Ni} = \bar{v}_{SS}-0.2a_i\bar{p}_i = \bar{v}_{SS}-0.2\bar{v}_{Ni}\xi, \text{ and } \bar{v}_{Ni}/\bar{v}_{SS} = \omega = \frac{1}{1+\frac{20}{100}\xi}.$$

Insisting that $T = \frac{\tau}{p} = 0.2$ and $f \equiv m_S\frac{\bar{v}_{SS}-a_i\bar{p}_i}{X_S} = 0.57$, as argued for in the text, we get:

$$\frac{1}{g}\frac{(1-r)\frac{1}{n}}{1-(1-r)^2\frac{1}{n}}\left(r+\frac{\left(1-(1-r)^2\frac{1}{n}-r\right)^2}{2-2(1-r)^2\frac{1}{n}-r}\right) = \frac{2}{10}$$

$$r\left(\frac{1-\left(\frac{1}{1+\frac{20}{100}g}\right)g}{r+\left(\frac{1}{1+\frac{20}{100}g}\right)\frac{\left(1-(1-r)^2\frac{1}{n}-r\right)^2}{2-2(1-r)^2\frac{1}{n}-r}}\right) = \frac{57}{100}$$

$$1-\frac{r}{1-(1-r)^2\frac{1}{n}}-\left(\frac{1}{1-(1-r)^2\frac{1}{n}}\right)\frac{\left(1-(1-r)^2\frac{1}{n}-r\right)^2}{2-2(1-r)^2\frac{1}{n}-r} = g$$

The solution is approximately $n = 5$ and $r = 1/2$. Vice versa, with $n = 5$ and $r = 1/5$ the predicted T and

f are:

$$\frac{1}{g}\left(\frac{(1-r)\frac{1}{n}}{1-(1-r)^2\frac{1}{n}}\right)\left(r+\left(\frac{\left(1-(1-r)^2\frac{1}{n}-r\right)^2}{2-2(1-r)^2\frac{1}{n}-r}\right)\right) = T$$

$$\left(\frac{1-\left(\frac{1}{1+\frac{20}{100}g}\right)g}{r+\left(\frac{1}{1+\frac{20}{100}g}\right)\left(\frac{\left(1-(1-r)^2\frac{1}{n}-r\right)^2}{2-2(1-r)^2\frac{1}{n}-r}\right)}\right)r = f$$

$$1-\left(\frac{r}{1-(1-r)^2\frac{1}{n}}\right)-\left(\frac{1}{1-(1-r)^2\frac{1}{n}}\right)\left(\frac{\left(1-(1-r)^2\frac{1}{n}-r\right)^2}{2-2(1-r)^2\frac{1}{n}-r}\right) = g$$

$$n = 5$$

$$r = \frac{1}{2}$$

The solution is $T = 0.21$ and $f = 0.55$. Thus, from now on, let $n = 5$ and $r = 1/2$.

Predictions on FTA. With (33) and (69), the ratio free-trade vs. BAU becomes $F \equiv X_S^{FTA}/X_S^{BAU}$:

$$F = \frac{m_S\bar{v}_{SS}+\frac{1-r_S}{2-r_S}m\bar{v}_{-S}}{m_S\bar{v}_{SS}+m\bar{v}_{Ni}\frac{[1-(1-r)^2/n-r]^2}{2-2(1-r)^2/n-r}} = \frac{r+\frac{(1-r)^2}{2-r}\omega^{FTA}}{r+\left(\frac{1}{1+\frac{20}{100}g}\right)\left(\frac{\left(1-(1-r)^2\frac{1}{n}-r\right)^2}{2-2(1-r)^2\frac{1}{n}-r}\right)}.$$

If the NTB is removed, then $\omega^{FTA} = 1$, so

$$\frac{r+\frac{(1-r)^2}{2-r}\left(\frac{1}{1+\frac{0}{100}g}\right)}{r+\left(\frac{1}{1+\frac{20}{100}g}\right)\left(\frac{\left(1-(1-r)^2\frac{1}{n}-r\right)^2}{2-2(1-r)^2\frac{1}{n}-r}\right)} = F$$

$$1-\left(\frac{r}{1-(1-r)^2\frac{1}{n}}\right)-\left(\frac{1}{1-(1-r)^2\frac{1}{n}}\right)\left(\frac{\left(1-(1-r)^2\frac{1}{n}-r\right)^2}{2-2(1-r)^2\frac{1}{n}-r}\right) = g$$

$$n = 5$$

$$r = \frac{1}{2}$$

The solution is $F = 1.048$. In this case, X_S increases with 4.8% compared to X_S^{BAU}.

If the foreign market doubles (in each of three importing countries), then population grows by 13/10,

and S's relative size shrinks from $r = 5/10$ to $a = 5/13$, so:

$$\frac{m\left(a + \frac{(1-a)^2}{2-a}\right)}{r + \left(\frac{1}{1+\frac{20}{100}g}\right)\left(\frac{\left(1-(1-r)^2\frac{1}{n}-r\right)^2}{2-2(1-r)^2\frac{1}{n}-r}\right)} = F$$

$$1 - \left(\frac{r}{1-(1-r)^2\frac{1}{n}}\right) - \left(\frac{1}{1-(1-r)^2\frac{1}{n}}\right)\left(\frac{\left(1-(1-r)^2\frac{1}{n}-r\right)^2}{2-2(1-r)^2\frac{1}{n}-r}\right) = g$$

$$n = 5$$
$$r = \frac{1}{2}$$
$$a = \frac{5}{13}$$
$$m = \frac{13}{10}$$

The solution is $F = 1.265$. So, X_S increases with almost 27% compared to X_S^0.

Predictions on CTA^0. With one importer signing the CTA^0, $\underline{X}$ follows from (53) where

$$s_{SS}^{BAU} = \frac{m}{2a_S}\frac{\left(\overline{v}_{-S} - r_N^2\overline{v}_{NS}\right)^2}{2 - 2r_N^2 - r_S},$$

so $\underline{X}$ becomes, in our case,

$$\underline{X} = m\overline{v}_{SS}\left[r + \frac{(1-r)^2}{2-r} - \frac{1-r}{2-r}\sqrt{1 - \frac{2(2-r)a_S}{m}\left[\frac{m}{2a_S}\frac{(1-r_N^2/(1-r))^2}{2-2r_N^2-r}\right]}\right]. \tag{70}$$

With (69), we get that with $C = \underline{X}/X_S^{BAU}$,

$$\frac{r + \frac{(1-r)^2}{2-r} - \frac{1-r}{2-r}\sqrt{1 - \frac{2-r}{2-2\left(\frac{1-r}{n}\right)^2-r}\left(1 - \frac{\left(\frac{1-r}{n}\right)^2}{1-r}\right)^2}}{r + \left(\frac{1}{1+\frac{20}{100}g}\right)\left(\frac{\left(1-(1-r)^2\frac{1}{n}-r\right)^2}{2-2(1-r)^2\frac{1}{n}-r}\right)} = C$$

$$1 - \left(\frac{r}{1-(1-r)^2\frac{1}{n}}\right) - \left(\frac{1}{1-(1-r)^2\frac{1}{n}}\right)\left(\frac{\left(1-(1-r)^2\frac{1}{n}-r\right)^2}{2-2(1-r)^2\frac{1}{n}-r}\right) = g$$

$$n = 5$$
$$r = \frac{1}{2}$$

The solution is $C = 0.962$. So, with CTA^0, X_S can be reduced by 3.8% compared to BAU.

Now, suppose that 3 of the 5 importing blocks double in mass. Then, m grows by $13/10$, and, in (70),

$r_N = 1/13$ and $r = 5/13$. Compared to the original BAU (before doubling and liberalization), $\underline{X}/X_S^0$ is:

$$m\frac{a + \frac{(1-a)^2}{2-a} - \frac{1-a}{2-a}\sqrt{1 - \frac{2-a}{2-2\left(\frac{1}{13}\right)^2-a}\left(1 - \frac{\left(\frac{1}{13}\right)^2}{1-a}\right)^2}}{r + \left(\frac{1}{1+\frac{20}{100}g}\right)\left(\frac{\left(1-(1-r)^2\frac{1}{n}-r\right)^2}{2-2(1-r)^2\frac{1}{n}-r}\right)} = C$$

$$1 - \left(\frac{r}{1-(1-r)^2\frac{1}{n}}\right) - \left(\frac{1}{1-(1-r)^2\frac{1}{n}}\right)\left(\frac{\left(1-(1-r)^2\frac{1}{n}-r\right)^2}{2-2(1-r)^2\frac{1}{n}-r}\right) = g$$

$$n = 5$$
$$r = \frac{1}{2}$$
$$a = \frac{5}{13}$$
$$m = \frac{13}{10}$$

The solution is $C = 1.1806$. With two beef importers participating in the CTA, and a doubling of the consumers in the other three importers, $r_N = 2/13$ and $r = 5/13$ in (70). With $C = \underline{X}/X_S^0$,

$$m\frac{a + \frac{(1-a)^2}{2-a} - \frac{(1-a)}{2-a}\sqrt{1 - \frac{2-a}{2-2\left(\frac{2}{13}\right)^2-a}\left(1 - \frac{\left(\frac{2}{13}\right)^2}{1-a}\right)^2}}{r + \left(\frac{1}{1+\frac{20}{100}g}\right)\left(\frac{\left(1-(1-r)^2\frac{1}{n}-r\right)^2}{2-2(1-r)^2\frac{1}{n}-r}\right)} = C$$

$$1 - \left(\frac{r}{1-(1-r)^2\frac{1}{n}}\right) - \left(\frac{1}{1-(1-r)^2\frac{1}{n}}\right)\left(\frac{\left(1-(1-r)^2\frac{1}{n}-r\right)^2}{2-2(1-r)^2\frac{1}{n}-r}\right) = g$$

$$n = 5$$
$$r = \frac{1}{2}$$
$$a = \frac{5}{13}$$
$$m = \frac{13}{10}$$

The solution is $C = 1.0957$. Predictions with three collaborating beef importers can be derived analogously.

Predictions with Cross Contingency. With contingent tariffs on all q_S privately provided goods exported by S (in addition to S's beef), and all q_N privately provided goods exported by N, then, with Δ_{SJ} and $\overline{\Delta}_{SK}$ defined by Lemma 3, $\underline{X}$ follows from (53) when s_{SS}^{BAU} is replaced by:

$$\frac{m}{2a_S}\frac{\left(\overline{v}_{-S} - r_N^2\overline{v}_{NS}\right)^2}{2 - 2r_N^2 - r_S} - \sum_J \Delta_{SJ} - \sum_K \overline{\Delta}_{SK}, \text{ so } \underline{X} \text{ becomes}$$

$$\underline{X}^M = m\overline{v}_{NS}\left[r + \frac{(1-r)^2}{2-r} - \frac{1-r}{2-r}\sqrt{1 - \frac{2(2-r)a_S}{m}\left[\frac{m}{2a_S}\frac{\left(1-r_N^2/(1-r)\right)^2}{2-2r_N^2-r} - \frac{1}{\overline{v}_{NS}^2}\sum_J \Delta_{SJ} - \frac{1}{\overline{v}_{NS}^2}\sum_K \overline{\Delta}_{SK}\right]}\right].$$

When $\overline{v}_{NJ} = \overline{v}_{iJ}$, Lemma 3 gives:

$$\Delta_{SJ} \equiv \frac{m\overline{v}_{NJ}^2}{a_J}\left(\frac{r_N^2}{1-r_N^2}\right)\left((1-r) - \frac{(1-r)}{2}\left(\frac{r_N^2}{1-r_N^2}\right)\right) \text{ and } \overline{\Delta}_{SK} = \frac{m_S\overline{v}_{SK}^2}{a_K}\frac{r_S^2/2}{1+r_S^2},$$

but for good K, exported from N to S, we should set $r_S = 1/13$ because S includes only this fraction of the consumer mass for N's product. With that, and if $\bar{v}_{iK}^2/a_K$ is equal for all goods and countries, then:

$$\underline{X}^M = m\bar{v}_{NS}\left[r + \frac{(1-r)^2}{2-r} - \frac{1-r}{2-r}\sqrt{1 - (2-r)\left(\frac{(1-r_N^2/(1-r))^2}{2-2r_N^2-r} - \Lambda\right)}\right], \text{ where}$$

$$\Lambda = q_S\left(\frac{r_N^2}{1-r_N^2}\right)\left((1-r) - \frac{(1-r)}{2}\left(\frac{r_N^2}{1-r_N^2}\right)\right) + q_N\frac{1}{2}\frac{(1/13)^2}{1+(1/13)^2}.$$

So $\underline{X}^M$, divided by the original X_S^0, becomes as follows:

$$m\frac{a + \frac{(1-a)^2}{2-a} - \frac{(1-a)}{2-a}\sqrt{1 - (2-a)\left(\frac{\left(1-r_N^2\frac{1}{(1-a)}\right)^2}{2-2r_N^2-a} - \Lambda\right)}}{r + \left(\frac{1}{1+\frac{20}{100}g}\right)\left(\frac{\left(1-(1-r)^2\frac{1}{n}-r\right)^2}{2-2(1-r)^2\frac{1}{n}-r}\right)} = C$$

$$1 - \left(\frac{r}{1-(1-r)^2\frac{1}{n}}\right) - \left(\frac{1}{1-(1-r)^2\frac{1}{n}}\right)\left(\frac{\left(1-(1-r)^2\frac{1}{n}-r\right)^2}{2-2(1-r)^2\frac{1}{n}-r}\right) = g$$

$$q_S\left(\frac{r_N^2}{1-r_N^2}\right)\left((1-a) - \frac{(1-a)}{2}\left(\frac{r_N^2}{1-r_N^2}\right)\right) + q_N\frac{1}{2}\frac{\left(\frac{1}{13}\right)^2}{1+\left(\frac{1}{13}\right)} = \Lambda$$

$$n = 5, r = \frac{1}{2}, a = \frac{5}{13}, m = \frac{13}{10}, q_S = 1, q_N = 2, r_N = \frac{1}{13}.$$

The solution is $C = 1.1383$. The final numbers in Table 2 are derived by setting $r_N = 2/13$ and $q_N = 4$ when two beef-importers collaborate on the CTA, and by setting $r_N = 4/13$ and $q_N = 8$ when also one of the other beef-importers (which doubles in size) collaborates on the CTA.