TRADE, TREES, AND CONTINGENT TRADE AGREEMENTS

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Abstract

Can trade agreements motivate environmental conservation? I first present a model whereby the government in the South expands its production capacity (e.g., deforest) before trading with the North. After deriving negative relationships between tariff reductions and conservation, I show how all negative results are reversed if countries can negotiate a contingent trade agreement (CTA), where default tariffs vary with changes in the production capacity (or forest cover). A calibration suggests that growth and liberalization can cause Brazil's agricultural area to expand by 27%, but this expansion can be avoided if the EU and the US offer a CTA.

Key words: International trade, trade agreements, deforestation, environmental conservation.
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I. Introduction

Background. International trade and environmental concerns are often in conflict. Tens of thousands of protesters demonstrated in Seattle in 1999, and criticized trade negotiators for betraying environmental and social values.[1] They requested that trade should be limited rather than liberalized. Trade negotiations have mostly proceeded bilaterally in recent years, but the tension has not weakened.[2]

For instance, in June 2019, Brazil led the Mercosur trade bloc to conclude its largest trade agreement ever with the European Union. Two months later, Mercosur concluded an agreement with EFTA, and it continues to negotiate with other potential trading partners such as Canada, the US, and Asian countries. The trade agreements will change Brazil's economy. Up to now, Brazil has been relatively closed; 75 percent of the beef it produces, for example, is consumed domestically.[3] While the trade negotiations concluded, deforestation rates increased and the forest fires gained international media attention. Deforestation has continued to increase every year since that time.[4] Consequently, critics argue that the treaty with the EU should not be ratified in its current form, and it is opposed by countries such as France, Germany, Netherlands, Belgium, Ireland, Austria, and Luxembourg.[5]

The tension between trade and environmental concerns is not unjustified. Trade motivates countries to specialize in their comparative advantages and, for many countries in the South, this specialization leads to resource exploitation and agricultural expansion. Consistent with this logic, empirical investigations confirm that trade agreements do cause resource depletion, such as deforestation. Abman and Lundberg (2020), for instance, document that deforestation levels peak around the ratification date for regional trade agreements.[6]

The damages are immensely costly for the society: Franklin and Pindyck (2018:166) distinguish between the Amazon's direct value, indirect value (as a carbon stock), option value (because of its biodiversity), and existence value, and sum the valuations to almost USD40,000 per hectare. UN's most recent climate change report refers to deforestation 94 times and writes that: "Deforestation may have contributed to about one third of the warming" (IPCC, 2021:11-39).

To deal with the trade-environment conflict, trade agreements often include sustainability requirements. For instance, the EU adds so-called trade and sustainable development (TSD) chapters to its agreements. A major challenge is that threats and conditions might not be credible ex post. After the feared land-use-change has already taken place, it will be in everyone's interest to trade rather than to

[1] About the 1999 WTO negotiations in Seattle, *The New York Times* wrote (Oct. 13, 1999) that 50,000 demonstrators were expected and, underlying the protests, there "is a fundamental disagreement about the proper role of the trade organization."

[2] When the Transatlantic Trade and Investment Partnership (TTIP) agreement was negotiated, protesters said they expected 250,000 demonstrators to turn out in Germany because "TTIP threatens environmental and consumer protection" (*The Guardian*, Sept. 17, 2016).

[3] https://apps.fas.usda.gov/newgainapi/api/Report/DownloadReportByFileName?fileName=Livestock%20and%20 Products%20Annual_Brasilia_Brazil_08-15-2021.pdf

[4] See: http://terrabrasilis.dpi.inpe.br/app/dashboard/deforestation/biomes/legal_amazon/rates

[5] On March 23, 2021, *Financial Times* concluded that "the mood has turned sour and the prospects for ratification are fading." See: https://www.ft.com/content/e906b1b9-8749-467a-b445-36f2b0ee71de

[6] For earlier papers verifying the connection between trade liberalization and deforestation in the tropics, see, for example, the analysis by Barbier (2000), or evidence provided by Faria et al. (2016), or Pendrill et al. (2019).

impose costly sanctions. Thus, trade has generally failed the environment, according to the UN as well as the World Bank (2019).[7] Ferrari et al. (2021) confirm that the EU provisions have had little effect.

Consequently, "Member states and the European Parliament are looking for trade concessions to be made conditional on compliance with a wider range of sustainable development criteria," according to the *Financial Times*.[8]

More specifically, France and the Netherlands made a novel policy initiative in May 2020: In a so-called "non-paper," they first admit a "lack of progress in compliance" with the TSD chapters, before they propose that "Parties should introduce, where relevant, staged implementation of tariff reduction linked to the effective implementation of TSD provisions and clarify what conditions countries are expected to meet for these reductions, including the possibility of withdrawal of those specific tariff lines in the event of a breach of those provisions."[9]

The non-paper is brief and specifies neither exactly how one can achieve staged implementation and withdrawal of tariff lines, nor the extent to which such a design may motivate conservation.

This paper. The purpose of the present paper is to start an exploration of when and how a contingent trade agreement (CTA) can motivate environmental conservation rather than exploitation.

For this purpose, I first develop a model that intends to capture the negative interactions between free trade and conservation. In the model, the parties invest in production capacity before the market clears. In the business-as-usual (BAU) scenario, tariff levels are set noncooperatively after the investment stage. As in the standard literature, the tariff is set so as to improve the country's terms of trade.[10] The larger the investment, the larger is the equilibrium tariff.

The analysis builds on two complementary assumptions.

I. In the South (S), the investment decision is made by the government, taking into account how tariffs and prices will respond. This assumption is questionable when it comes to investments in production units in market economies, but reasonable when S exports agricultural products that necessitate land use change and deforestion. Burgess et al. (2019) have documented that governmental policies are determining deforestation rates in the Brazilian Amazon.

To illustrate the effect of this assumption, investments in production capacity are made by private price-taking actors in the North (N). In both countries, investments are reduced when the tariffs are expected to be positive in BAU. Compared to N, S limits investments for two additional reasons: A lower

[7] The UN report IPBES (2019, Ch 6:138) states: "the potential of WTO and other free trade agreements and WTO regulations to contribute to conservation and sustainability is criticized... While other regional or bilateral free trade agreements such as NAFTA include environmental provisions, these have mostly been implemented in a narrow way and have not resulted in significantly raised levels of environmental protection... At the global level, WTO has started to discuss environmental provisions as part of the Doha negotiations since 2001, but negotiations were not successful and ended in 2016." The World Bank (2019:8) states: "...the expansion of livestock production in Brazil could increase deforestation. Only if these adverse impacts are addressed through appropriate spatial and environmental policies will trade integration be a pathway to development."

[8] Sept. 21, 2020. The article is available here: https://www.ft.com/content/b508b3b1-999f-4528-a0d2-f1b37f0e0b87

[9] The non-paper is available here:
https://nl.ambafrance.org/Non-paper-from-the-Netherlands-and-France-on-trade-social-economic-effects-and

[10] Empirical studies verify that countries do set tariffs in order to improve their terms of trade: See Broda et al. (2008) or Bagwell and Staiger (2011).

production capacity raises the equilibrium price for S's product, and the lower capacity induces N to set a lower tariff. Therefore, I find that: (1) A free trade agreement (FTA) causes deforestation both because the tariff declines, and because S no longer needs to withold the investment in capacity to keep the tariff low, (2) larger gains from trade cause more deforestation, and (3) larger environmental damage reduces the value of the FTA.

A preferential trade agreement (PTA) allows the parties to negotiate and set fixed tariffs before investments adjust. If the environmental damage is large, it is optimal to have a higher tariff. The tariffs are distortionary, however, and every reasonable PTA leads to more deforestation than with BAU, I show.

II. Next, I assume that countries can negotiate tariffs that are contingent on the capacity level (or forest cover). After all, the forest cover is observable and verifiable. As discussed in Section VII, payments from central governments as well as from foreign donors have already been conditional on the forest cover.

Assumptions I and II complement each other: The contingency has *no* role to play when capacity investments are made by private actors that take prices and tariffs as given. This observation may explain why the usual trade agreements are not CTAs. But with public influence over the capacity level, as when the government monitors land-use change, S will pay attention to how the capacity will influence the terms of trade. To motivate conservation, S's terms of trade must be more attractive when the forest cover is large, and less attractive when the capacity to produce beef is large.

The CTA exploits the fact that there is more than one way of splitting the gains from trade: If the tariff in one country increases, then terms-of-trade effects imply that this country obtains more of the gains from trade, while the other loses. The CTA lets the point on the Pareto frontier be contingent on S's capacity (or forest cover). It is reasonable to require the tariffs to be Pareto optimal, i.e., credible and renegotiation proof, in that there should be no other tariff pair that both countries prefer ex post, no matter the forest cover that is realized. Therefore, there is a limit for how large the tariff can be before the parties want to renegotiate it, and thus there is a limit to what the CTA can achieve.

To achieve more, the CTA should make also S's tariff (and not only N's) contingent on S's capacity to produce. That is, S should be allowed to introduce positive tariffs on the goods imported from N as long as S's forest cover remains large.

My main finding is that the negative effect from traditional trade agreements on conservation, and several associated corollaries, are reversed when countries can negotiate a CTA: (1) More can be conserved under the CTA than with BAU, (2) larger gains from trade makes it possible to conserve more, and (3) larger environmental damage raises the value of this trade agreement.

Even in the absence of environmental damage, the CTA is strictly better than every FTA and PTA. With free trade, S invests less than private investors would. The CTA can motivate S to invest more by letting the terms of trade be more attractive to S when the capacity is large than when it is small.

In the extensions (and the Appendix), I allow for more than two products, more than two countries, and countries can be of different sizes. In all these cases, the CTA can motivate S to conserve or invest, depending on the environmental damage.

Although a serious calibration of the model is beyond the scope of this paper, I illustrate the possibility by matching the predictions of the model (with all generalizations) with the empirically reasonable 20% beef tariff, and the modest export fraction (25% of beef, 63% of soy) that we see in Brazil. The calibrated model indicates that Brazil exports to five major trading blocks, which is in line with other calibrated trade models (e.g., Ossa, 2011). The calibrated model predicts that trade liberalization can cause a 4.8% increase in the agricultural area, but that a CTA signed between Brazil and the EU can avoid the increase. If demand from three trading blocks (i.e., Asia) doubles, the agricultural area will increase by 27%, and the forest will diminish accordingly —with free trade. However, the increase is below 18%, 10%, or 1%, respectively, if the CTA is offered by the EU, the EU + US, or by 3 trading partners.

Literature. This paper brings together the large trade–environmental literature and a smaller literature on trade-specific investments.

The literature on trade and resource extraction goes back to Dasgupta et al. (1978), who studied depletion rates in open economies.[11] In the traditional literature on trade and the environment, countries may reduce environmental standards to become competitive (Markusen, 1975) or to specialize in their comparative advantages: The South may have a comparative advantage in environmentally damaging production because of policies (Pethig, 1976) or because of lower income levels (Copeland and Taylor, 1994).[12] If countries in the South struggle with an open-access problem, and are unable to control extraction rates, then trade can worsen the problem and cause depletion (Chichilnisky, 1994; Brander and Taylor, 1997 and 1998; Karp et al., 2001).[13]

To reduce the environmental damage, scholars have recommended trade sanctions (Barrett, 1997), border tax adjustments (Hoel, 1996; Elliott et al., 2010), costly international contracts (Horn et al., 2010), and climate clubs (Nordhaus, 2015). All these policies cause distortions that must be compared to the environmental benefits. They lead to leakages and, when the resource is non-renewable, to commitment problems: after the resource is exhausted, it is in everyone's interest to trade (Hsiao, 2022).

My basic model of trade and agreements draws on existing literature (see the surveys by Maggi, 2014; Bagwell and Staiger, 2016). For example, the tariffs are motivated by the terms-of-trade effects (as in Bagwell and Staiger, 2004 and 2011; Broda et al., 2008; Ludema and Mayda, 2013; Grossman, 2016), and I permit transfers at the negotiation stage (Aghion et al., 2007; Maggi and Ossa, 2020). I assume, however, that the environmental damage follows from up-front investments in capacity.

Because of the up-front investments, I connect with the literature on irreversible trade-specific in-

[11] Relatedly, Hillman and Van Long (1983) studied a country depleting a resource at the same time as it was importing extracted amounts from another country. If there is a (lower) risk of trade disruption, then the country conserves more (less) of its own resource. With a larger number of jurisdictions, depletion can be larger also because prices will be less sensitive to one's own supply (Markusen, 1981).

[12] For this reason, trade can increase global pollution if income differences are large (Copeland and Taylor, 1995).

[13] In other situations, trade can benefit the environment. In particular, trade can raise income levels, and because of the environmental Kuznets curve, the outcome can be a cleaner environment (Antweiler et al., 2001; Copeland and Taylor, 2004). In addition, trade can lead to technology upgrading (Bustos, 2011) which, in turn, can lead to structural transformations and a diminished reliance on resource exploitation (Bustos et al., 2016). My contribution to this literature is to show how, even when we abstract from these effects, trade agreements can be designed so as to motivate conservation.

vestments. When countries produce similar products, Krugman (1987) showed how protectionism can facilitate investments in the less competitive country. With different products, McLaren (1997) analyzed how investments influenced subsequent negotiations. McLaren and Bond and Park (2002) find, as I do, that the equilibrium tariff is larger if the investment level is larger. The importance of trade on capacity building is empirically supported.[14] Baier and Bergstrand (2007) find there to be a ten-year adjustment period from liberalization to increased trade. After the adjustment, trade doubles, in line with my results.

McLaren considered investments by firms, and Ossa (2011) the relocation of firms, but other scholars, such as Bond (2006), and Guriev and Klimenko (2015), study investments made by governments. In these papers, the authors refer to investments in infrastructure such as transportation facilities, ports, pipelines, or electricity grids. For these applications, it is natural that the authors abstract from potential environmental damage.

By permitting environmental damage, I combine this strand of literature with the trade–environmental one. When the capacity is a payoff-relevant stock, it becomes natural to consider tariffs that are contingent on the level of this stock. With this possibility, I characterize treaty designs that motivate environmental conservation more effectively than what seems to be possible in the earlier literature.

This possibility is highly policy relevant because using explicit transfers in return for conservation is often problematic.[15] The literature on issue linkages (surveyed by Maggi, 2016), as a type of transfer, typically considers structurally unrelated issues and assumes that the parties can commit (Abrego et al., 2001; Horstmann et al., 2005). Here, trade and the environment are structurally linked, because resource exploitation raises the gains from trade, and the parties cannot commit. The two issues are structurally linked also in the model by Copeland (2000), but he analyzes linkages between a trade agreement and an environmental agreement, while I consider only a trade agreement and how that, by itself, can influence conservation.

This characterization of the CTA adds to the literature on shallow vs. deep integration.[16] Even if there is no damage, free trade fails to implement the first best in my model, because S withholds capacity investment to influence terms of trade. In fact, the increase in capacity following trade liberalization is here comparable to the increase following privatization. When privatization is infeasible, I show that the CTA guarantees deeper and more efficient integration than can any traditional shallow trade agreement. With *local* environmental externalities, "deep agreements are very controversial," explain Maggi and Ossa (2020:1). They find that deep integration is not necessarily beneficial. With *international* externalities,

[14] For example, Juhasz (2018) shows that the 1803–1815 protectionist period in the French Empire influenced the production capacities in mechanized cotton spinning.

[15] Explicit compensations for conservation can, in some cases, be very effective (Souza-Rodrigues, 2019). However, IPBES (2019:54) reports that "the literature is currently mixed on the success rates of forest carbon projects in general and REDD+ has faced a number of challenges." The challenges with this approach include liquidity constraints (Jayachandran, 2013), contractual externalities (Harstad and Mideksa, 2017), and that they lead to corruption and a worse selection of political candidates (Brollo et al., 2013).

[16] The focus on terms of trade is normal in the shallow integration literature, but studies of deep integration also consider behind-the-border policies (Antras and Staiger, 2012), such as domestic regulation and product standards (Grossman et al. 2021), lobbies and process standards (Maggi and Ossa, 2020), or concentrate on principles such as reciprocity and nondiscrimination (Bagwell and Staiger, 1999) to prevent bilateral opportunism through "concession erosion" (Bagwell and Staiger, 2016).

however, the type of deep integration permitted by the CTA can motivate more conservation than what we can expect with shallow integration, I show.

Outline. After presenting the model, Section III derives the outcomes under BAU, FTA, and PTA. Section IV shows how the pessimistic findings from Section III are overturned with the CTA. The benchmark model is simple, but Section V illustrates how the case for the CTA is strengthened if there are many products, multiple trading partners, and countries can be of different sizes. Section VI makes a first step towards calibrating the model, and Section VII discusses the empirical relevance.

II. The Model

Demand. There are two countries, the North (N) and the South (S). Each country produces a unique good that is sold and consumed in both countries. Let $c_{ij} > 0$ measure country i's consumption level of country j's good, where $i,\, j \in \{N, S\}$.

A representative consumer enjoys the following consumption utility:

$$U_i = c_{i0} + \sum_{j \in \{N,S\}} u_{ij}\left(c_{ij}\right),\, i \in \{N, S\}. \tag{1}$$

Here, c_{i0} is the numeraire good that is freely traded.

I follow Maggi and Rodríguez-Clare (2007), or Bond and Park (2002), in assuming quadratic $u_{ij}\left(c_{ij}\right)$ with bliss point v_{ij}:

$$u_{ij}\left(c_{ij}\right) = -\frac{\left(v_{ij} - c_{ij}\right)^2}{2a_i}. \tag{2}$$

The Appendix, and Section V, allow for many goods and countries and heterogeneous country sizes.

Supply. Total consumption of i's product is limited by i's production capacity, X_i:

$$\sum_{j \in \{N,S\}} c_{ji} \leq X_i,\, \forall i \in \{N, S\}. \tag{3}$$

A key assumption in the model is that S's government decides on X_S before the good is traded by the consumers. This assumption is reasonable in certain important situations. When S produces beef, the amount is limited by S's amount of agricultural land, X_S, determined by S's policy regarding land use change, deforestation, and the monitoring of illegal logging.

In addition, we can permit a marginal production cost, $\kappa_S \geq 0$, when beef is produced on a unit of the land, the marginal cost of clearing the forest and converting it to agriculture can be $\varkappa_S$, and the marginal value of the lumber can be ν_S. S's decision on X_S will depend on the net marginal cost $k_S \equiv \kappa_S + \varkappa_S - \nu_S$. Assume $k_S \geq 0$, so that S will never clear land that is not used for beef production. This assumption implies that (3) will bind in equilibrium.

N may also face non-tariff measures or transport cost $t_{NS} \geq 0$ when importing a unit from S.[17]

[17] When prices can adjust, it's irrelevant whether the exporter or importer is responsible for paying the transport cost.

Analogously, the net marginal cost of increasing the capacity and produce X_N is $k_N \geq 0$, and S's marginal cost of transporting N's good is $t_{SN} \geq 0$. The single difference between the two countries is that while X_S is decided on by S's government, recognizing the effects on tariffs and prices, X_N is decided on by price-taking private investors.[18]

If capacity investments are irreversible, then we must also require $X_i \geq X_i^0$, where $X_i^0 \geq 0$ is the initial capacity. This inequality will not bind, and the level of X_i^0 will have no impact on the results, under the assumption that X_i^0 is weakly smaller than the BAU level.[19]

Externalities and payoffs. A representative consumer maximizes (1) subject to the budget constraint:

$$c_{i0} + p_i c_{ii} + (p_j + \tau_i + t_{ij}) c_{ij} \leq y_i, \forall \{i, j\} = \{N, S\}, \tag{4}$$

taking as given the export prices p_i and p_j, and i's tariff τ_i and income y_i (all measured relative to the price of the numeraire). If e_i is an exogenous endowment, the national income is:

$$y_i = e_i + \tau_i c_{ij} + (p_i - k_i) X_i, \ \forall \{i, j\} = \{N, S\}. \tag{5}$$

In (5), the second term is the country's tariff revenue, and the last term is the profit. In S, the profit is redistributed to the consumers. In N, the private sector invests to the point when $p_i = k_i$, i.e., there is zero profit, so (5) holds also in this case.

There can be externalities associated with production and/or capacity expansion. If S clears the forest to produce more beef, we lose biodiversity, carbon sinks, and the homes of indigenous tribes. The cost to S is internalized by S (this cost can be included in S's total capacity expansion cost, k_S). The expected environmental damage experienced by N is given by the function $d_N(X_S)$.[20]

Symmetrically, S may face the damage $d_S(X_N)$ when N invests or produces. For instance, N's production may contribute to climate change.[21] Each function $d_i(\cdot)$ is assumed to be weakly increasing and weakly convex. If it happens to be linear, then

$$d_i(X_j) = d_i' X_j, \tag{6}$$

for some damage $d_i' \geq 0$. If $d_i(X_j)$ is nonlinear, the marginal damage d_i' is a function of X_j.[22] The

[18] It is straightforward to allow both X_N and X_S to be decided on by governments, or by price-taking investors. The asymmetry, however, allows us to compare the two cases.

[19] That is, I will henceforth assume $X_i^0 \leq X_i^{BAU}$ $\forall i \in \{N, S\}$, where X_i^{BAU} is characterized by Proposition 2. Note that $X_S^0 < X_S^{BAU}$ can be reasonable if parameters change over time in a way that makes the new BAU level for X_S larger than in the past: see Section VI.

[20] With "expected damage", I permit a possible catastrophe if X_S exceeds threshold or tipping point $\widetilde{X}$, as in Maggi and Staiger (2021). They quote Pindyck (2020:22): "We don't know where a tipping point, if there is one, might lie." Thus, suppose $d_N(X_S) = \underline{d} X_S + h \Pr\left(X_S \geq \widetilde{X}\right)$, where h is the expected additional harm above threshold $\widetilde{X}$. If $\widetilde{X} \sim U\left[\mathrm{E}\widetilde{X} - 1/2\sigma, \mathrm{E}\widetilde{X} + 1/2\sigma\right]$, then $d_N(X_S) = \underline{d} X_S + \left(1/2 + \sigma\left(X_S - \mathrm{E}\widetilde{X}\right)\right) h$, so the marginal expected damage is $\underline{d} + \sigma h$.

[21] There may or may not be an externality from the private investments in X_N on country N. If there is, then we may either assume that N's government incentivizes the private investors to internalize the harm on N (for example, k_N may include the domestic Pigouvian CO_2 tax), or we can permit an externality that is not internalized. In the latter case, the analysis remains unchanged if $d_S(X_N)$ is interpreted as the global externality from X_N.

[22] If $d_i(\cdot)$ is quadratic, then the second-order derivative is constant ($d_i'' \geq 0$) and we can write:

$$d_i(X_j) = d_i' X_j + d_i'' X_j^2 / 2.$$

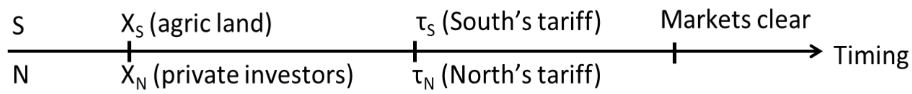

Figure 1: *The timing of the noncooperative game*

welfare in country i is the consumption utility (which includes the income from the tariff and profit), minus the damage:

$$U_i - d_i\left(X_j\right),\ \{i,j\} = \{N,S\}.$$

Timing. The timing of the noncooperative game is illustrated in Fig. 1. First, each country's capacity, X_i, is decided on. While S takes into account the effects on tariffs and prices when determining X_S, N's investors take prices and tariffs as given. Second, the government in every $i \in \{N,S\}$ decides on the tariff τ_i. Finally, price-taking consumers make their decisions and payoffs are realized. By solving the game by backward induction, we characterize the subgame-perfect equilibrium (SPE).

Extensions. Alternative assumptions on the timing will be discussed later. The next section considers free trade agreements and fixed tariffs negotiated at the beginning of the game. Section V allows for many products, multiple countries, and heterogeneous country sizes. The main results continue to hold with these generalizations, and each of them provides new insights.

III. Trade, Tariffs, and Traditional Treaties

First best. With transferable utilities, social efficiency requires the sum of payoffs to be maximized. As a consequence, the marginal benefit, minus the total marginal cost, must equal zero for each country:

$$\frac{v_{ii}-c_{ii}}{a_i} - \left(k_i + d_j'\right) = \frac{v_{ji}-c_{ji}}{a_i} - \left(k_i + d_j' + t_{ji}\right) = 0 \quad \forall \{i,j\} = \{N,S\}. \tag{7}$$

If there were no damage, (7) would require:

$$c_{ii} = \overline{v}_{ii} \equiv v_{ii} - a_i k_i \text{ and } c_{ji} = \overline{v}_{ji} \equiv v_{ji} - a_i k_i - a_i t_{ji},$$

where the gains from trade is captured by $\overline{v}_{ji}$, decreasing in the transport cost (or non-tariff measure) t_{ji} and increasing in the value of lumber, for instance. With damage, (7) requires

$$c_{li} = \overline{v}_{li} - a_i d_j' \quad \forall \{i,j\} = \{N,S\},\ l \in \{N,S\}. \tag{8}$$

With the constraint $X_i \geq X_i^0$, the first-best is[23]

$$X_i^{FB}\left(d_j'\right) = \max\left\{X_i^0, \overline{v}_{ii} + \overline{v}_{ji} - 2a_i d_j'\right\}. \tag{9}$$

[23] Here, d_j' is a constant if the harm is linear. With nonlinear damage, d_j', on the right-hand side of (9), is a function of X_i. If the function is quadratic, with marginal damage $d_i' + d_i'' X_j$, then

$$X_i^{FB}(\cdot) = \max\left\{X_i^0, \overline{v}_{ii} + \overline{v}_{ji} - 2a_i \frac{\left(\overline{v}_{ii} + \overline{v}_{ji}\right) d_j'' + d_j'}{1 + 2a_i d_j''}\right\}.$$

The constraints $c_{ii} \geq 0$ and $c_{ji} \geq 0$ will never bind, one can show, if $\overline{v}_{ij} > 0\ \forall i, j \in \{N, S\}$, and $\overline{v}_{SS} > \overline{v}_{NS}/3$.[24]

A. *Business as Usual*

The market equilibrium. To derive the noncooperative SPE by backward induction, we first solve the consumers' problem. When consumers maximize (1), subject to (4), demand for i's good is:

$$\begin{aligned} c_{ii} &= v_{ii} - a_i p_i \text{ and} \\ c_{ji} &= v_{ji} - a_i \left(p_i + \tau_j + t_{ji}\right), \forall \{i, j\} = \{N, S\}. \end{aligned} \tag{10}$$

Note that the first-best (7) requires $\tau_j = 0$. A larger τ_j reduces the equilibrium price p_i, however. When (3) binds,

$$p_i = \frac{v_{ii} + v_{ji} - X_i}{2a_i} - \frac{\tau_j + t_{ji}}{2}. \tag{11}$$

Best-response tariffs. The lower import price implies that j's terms of trade improve when τ_j increases. Country j trades off this benefit with the distortions that follow when j's consumption falls because of the tariff. Country j's optimal tariff maximizes the consumer surplus plus the tariff revenues. The Appendix verifies that j's optimal τ_j is given by the following best response to X_i:

$$\tau_j^{BR}\left(X_j\right) = \frac{\overline{v}_{ji} - \overline{v}_{ii} + X_i}{3a_i}. \tag{12}$$

When (10)–(12) are combined, it is easy to check that when $c_{ji} > 0$, the r.h.s. of (12) is positive. Thus, the tariff increases in X_i. Intuitively, if X_i is large, j imports a lot, and it is more important for j to improve its terms of trade.

Equilibrium capacity. In country N, private price-taking and tariff-taking investors invest in X_N as long as $p_N \geq k_N$. Because p_N decreases in X_N, equilibrium X_N ensures that this inequality binds. With the definitions in (8), (11) implies that the market response to an expected tariff is:

$$X_N^{MR}\left(\tau_S\right) = \overline{v}_{NN} + \overline{v}_{SN} - a_N \tau_S. \tag{13}$$

In line with the first welfare theorem, the first best is implemented by a perfect market: $X_N^{MR}(0) = X_N^{FB}(0)$, when there is no tariff and no damage.

In (13), τ_S is actually the *expected* tariff, which N's investors take as given. It is intuitive that it is less profitable to invest in X_N if S's tariff is expected to be large.

Expectations are rational, so equilibrium pair (X_N, τ_S) satisfies both (12) and (13), as illustrated by the top-right intersection in Fig. 2.

EXAMPLE 1. All figures are drawn for $\overline{v}_{ji} = \overline{v}_{ii} = a_i = a_j = 1$, $i, j \in \{N, S\}$, implying $\tau_S^{BAU} = 1/2$.

[24] If $\overline{v}_{SS} < \overline{v}_{NS}/3$, S will only produce for the export market.

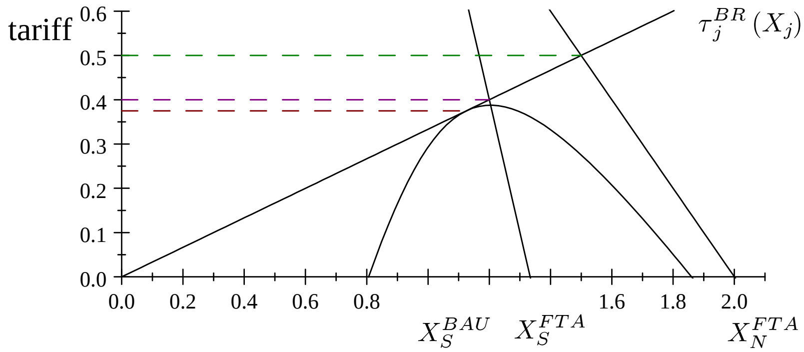

Figure 2: *S's indifference curve is tangent with N's best-response curve.*

More generally, combining (12) and (13), the BAU levels become:

$$X_N^{BAU} = \overline{v}_{NN} + \frac{\overline{v}_{SN}}{2} \text{ and } \tau_S^{BAU} = \frac{\overline{v}_{SN}}{2a_N}. \tag{14}$$

Equilibrium $\left(X_N^{BAU}, \tau_S^{BAU}\right)$ is also a Nash equilibrium if investors invest in X_N at the same time as (instead of before) τ_S is set. After all, N's investors do not attempt to influence τ_S.

When S decides on X_S, in contrast, S takes into account that a larger X_S increases τ_N and reduces p_S.

Even if we fixed τ_N, S would prefer to limit X_S in order to raise p_S. For a given τ_N, the Appendix shows that S's best response, X_S, to τ_N is:

$$X_S^{BR}(\tau_N) = \overline{v}_{SS} + \frac{\overline{v}_{NS} - a_S\tau_N}{3}. \tag{15}$$

Note that $X_S^{BR} < X_N^{MR}$, and X_S^{BR} is a steeper function than X_N^{MR} is, if the parameters are symmetric for the two countries. In Example 1, $X_S^{BR}(\tau_N)$ and $\tau_N^{BR}(X_S)$ cross at $\tau_N = 0.4$, as illustrated in Fig. 2. More generally, if the two policies were decided on simultaneously, the Nash equilibrium (X_S, τ_N) would be:

$$X_S^{NASH} = \overline{v}_{SS} + \frac{\overline{v}_{NS}}{5} \text{ and } \tau_N^{NASH} = \frac{2\overline{v}_{NS}}{5a_S}. \tag{16}$$

With the timing in Fig. 1, S will also take into account the effect on τ_N. By limiting X_S further, N will set a smaller τ_N. The smaller τ_N contributes to a higher p_S. In equilibrium, X_S is small, relative to X_N, both because S takes into account the effect on the price, and because S attempts to motivate the trading partner to reduce the tariff.

PROPOSITION 1: *The noncooperative SPE outcomes for X_N and τ_S are given by (14), while X_S and τ_N are:*

$$X_S^{BAU} = \overline{v}_{SS} + \frac{\overline{v}_{NS}}{8} \text{ and } \tau_N^{BAU} = \frac{3\overline{v}_{NS}}{8a_S}.$$

If there is no damage, each $X_i^{BAU} < X_i^{FB}(0)$ because $j \neq i$ cannot commit to $\tau_j = 0$. In addition, X_S^{BAU} is limited further because S internalizes the effect on the price (remember, $X_S^{BR}(\cdot) < X_S^{MR}(\cdot)$)

and the effect on the tariff (so, $X_S^{BAU} < X_S^{BR}\left(\tau_N^{BAU}\right)$). Thus, there are three reasons for X_S to be smaller than the first-best level if there is no damage.

Of course, if the marginal damage is sufficiently large, then $X_i^{FB} < X_i^{BAU}$. After all, equilibrium X_i^{BAU} does not depend on the damage.

B. *Free Trade*

An FTA is here defined as a commitment to zero tariffs. As observed after eq. (10), it is necessary with $\tau_j = 0$ to implement the first best, since marginal net benefits will otherwise differ for the two countries. In addition, with a FTA, the equilibrium capacities will be larger than in BAU. For both reasons, the total consumer surplus increases after liberalization.

PROPOSITION 2: *Compared to BAU, the FTA increases X_N, X_S, and consumer surpluses:*

$$\begin{aligned}
X_N^{FTA} &= \overline{v}_{NN} + \overline{v}_{SN} > \overline{v}_{NN} + \frac{\overline{v}_{SN}}{2} = X_N^{BAU},\\
X_S^{FTA} &= X_S^{BR}(0) = \overline{v}_{SS} + \frac{\overline{v}_{NS}}{3} > \overline{v}_{SS} + \frac{\overline{v}_{NS}}{8} = X_S^{BAU},\\
\left(U_N^{FTA} + U_S^{FTA}\right) - \left(U_N^{BAU} + U_S^{BAU}\right) &= \frac{1}{8}\frac{\overline{v}_{SN}^2}{a_N} + \frac{1}{8}\frac{133}{144}\frac{\overline{v}_{NS}^2}{a_S}.
\end{aligned}$$

It is easy to check that trade of the privately provided good doubles when we move from BAU to the FTA: c_{SN} increases from $\overline{v}_{SN}/2$ to $\overline{v}_{SN}$, while c_{NS} increases from $3\overline{v}_{NS}/8$ to $2\overline{v}_{NS}/3$.[25] These increases are in line with the empirical evidence from Baier and Bergstrand (2007), who show that trade doubles with the FTA, after a 10-year phase-in period (these years may be necessary to build the capacity).

Regarding the increases in capacity, consider, first, private investments in X_N. With a commitment to $\tau_S = 0$, N's investors expect higher demand, a higher price, and a larger return on a unit of capacity. As a result, the equilibrium capacity is $X_S^{FTA} = X_N^{MR}(0) > X_N^{BAU}$. As noted already, equilibrium X_N is first best if there is no tariff and no damage, so $X_S^{FTA} = X_N^{FB}(0)$. Consequently, if all investments were private, and there were no damage, the FTA would implement the first best.

Next, consider S's capacity. Even if $\tau_N = 0$, S limits X_S in order to improve its terms of trade: $X_S^{FTA} = X_S^{BR}(0) < X_S^{FB}(0)$. In fact, S has a stronger incentive to manipulate its terms of trade when S exports a lot, as when $\tau_N = 0$. From (13) and (15), we see that $X_i^{MR}(\cdot)$ is a steeper function than is $X_i^{BR}(\cdot)$. This comparison explains why the FTA leads to a larger increase in X_N than in X_S, if $\overline{v}_{NS} = \overline{v}_{SN}$.

COROLLARY 1: *The FTA has a greater effect on X_i when investments are private than when they are public.*

[25] These numbers are found by combining X_i and τ_j, from Propositions 1 and 2, with (10) and (11). S's import doubles, but N's import increases by only 78% because S has a stronger incentive to limit X_S (to raise p_S) when N imports more.

Nevertheless, the FTA leads to a larger X_S for two reasons. (i) When τ_N is no longer an increasing function of X_S, S no longer needs to limit X_S to keep τ_N from being raised to a level that is higher than τ_N^{BAU}. This effect corresponds to an increase from X_S^{BAU} to $X_S^{BR}\left(\tau_N^{BAU}\right)$. (ii) When τ_N is reduced, N demands more, the price (p_S) increases, and so does the return from making land available to agriculture. This effect corresponds to the increase from $X_S^{BR}\left(\tau_N^{BAU}\right)$ to $X_S^{BR}(0)$.

COROLLARY 2: *The FTA causes an increase in X_S both because (i) $\partial\tau_N/\partial X_S = 0$, and because (ii) $\tau_N = 0$.*

When there is no damage, the FTA is always valuable. Then, N's capacity is first best, and S's capacity is closer to the first best than with BAU. For the FTA to implement the first best in both markets, S must also privatize the investment decision. Interestingly, the increase in capacity following trade liberalization is comparable, in magnitude, to the effect of privatization.

Larger gains from trade, measured by $\overline{v}_{ji}$, lead to increases in X_i^{BAU}, X_i^{FTA}, and $X_i^{FTA} - X_i^{BAU}$.

COROLLARY 3: *If the gains from trade increase, X_S increases.*

Furthermore, a larger $\overline{v}_{ji}$ makes it more likely that the FTA will be signed, and then X_S increases also according to Corollary 2. It is easy to see that if the damages are linear and in line with (6), the value of the FTA is positive if and only if:

$$\frac{1}{8}\frac{\overline{v}_{SN}^2}{a_N} + \frac{1}{8}\frac{133}{144}\frac{\overline{v}_{NS}^2}{a_S} > \frac{1}{2}\overline{v}_{SN}d_S' + \frac{5}{24}\overline{v}_{NS}d_N'. \tag{17}$$

Hence, the FTA might not be valuable if the increase in X_i causes damage.

COROLLARY 4: *If the damage is larger, the value of the FTA is smaller.*

When the damage (d_N') from X_S is relatively large, it is N's value of the agreement that is low —not S's value.

COROLLARY 5: *If the damage facing N (d_N') is larger, N is worse off following liberalization, unless the transfer from S to N is larger.*

Without compensation, N loses from the larger damage and the additional capacity, and perhaps even from the FTA relative to BAU. If we assume that each party's bargaining surplus increases in the total bargaining surplus (as with the Nash bargaining solution), then S must compensate N for the damage if they agree on free trade. When side transfers are possible at the initial bargaining stage, the two countries sign the FTA if and only if (17) holds.

The corollaries are similar if the damage facing S is larger: Then, the value of the FTA decreases (as in Corollary 2), especially for S, so N will need to increase N's transfer to S (analogously to Corollary 4).

If the gains from trade (v_{NS} or v_{SN}) increase, the FTA is more likely to be signed, which increases the damage (as in Corollary 3). Thus, Corollaries 2–4 do not hinge on the assumption that X_S is decided on by the government rather than by private investors.

C. *Preferential Trade Agreements*

Free trade is always maximizing the sum of payoffs ex post, after the X_i's have been decided on. At the beginning of the game, however, it may well be that N and S benefit from committing to another pair of tariffs than $\tau_N = \tau_S = 0$. After all, fixing $\tau_i \in \left(0, \tau_i^{BAU}\right)$ will reduce the tariff distortions and the larger X_i will increase consumer utility compared to BAU, but X_i and the associated damage will not increase as much as with the FTA. Because consumer utility is concave in X_i, an interior solution for τ_i might maximize total welfare.

A typical PTA fixes the tariffs, but not necessarily at zero.

PROPOSITION 3: *If fixed tariffs are negotiated at the beginning of the game, the optimal and equilibrium levels are:*

$$\tau_S^* = \max\left\{\tau_S^{BAU}, d_S'\right\} \text{ and } \tau_N^* = \max\left\{\tau_N^{NASH}, \frac{3d_N' - \overline{v}_{NS}/a_S}{5}\right\}. \tag{18}$$

Given these tariffs, equilibrium capacity levels follow from (13) and (15).[26]

Interestingly, the price-taking investors in N should face a Pigouvian-like tariff, but the tariff in S should be *less* than the Pigouvian level. There are two reasons for this result.

First, S is voluntarily limiting X_S to improve its terms of trade. Thus, if d_N' is small, $X_S^{FTA} < X_S^{FB}$, and then (18) verifies that it would have been better to subsidize (rather than to tax) trade in S's good.

Second, the comparison of (13) and (15) shows that an increase in the tariff has a smaller effect when the capacity is decided on by the government instead of by private investors. As explained before Corollary 1, the intuition for this difference is that S benefits more from raising p_S if S produces a lot (as when τ_N is small). Consequently, the effect from τ_S on X_S will be relatively small compared to the ex post distortions from the tariff.

The essence of the corollaries continues to hold: A give tariff reduction influences X_N more than it reduces X_S (Corollary 1), but trade liberalization (i.e., a lower τ_N) does raise X_S (Corollary 2). And, because S's voluntary reduction in X_S (compared to $X_S^{FB}(0)$) is larger when S exports a lot, a larger $\overline{v}_{NS}$ reduces τ_N^*. It follows that larger gains from trade cause more deforestation (Corollary 3). More liberalization (i.e., a lower τ_N) is socially optimal, however, only if the damage is small (Corollary 4).

The tariffs facilitate the transfer from one country to the other, because each tariff influences the terms of trade. In particular, if N's marginal damage is larger, then Proposition 4 confirms that it becomes optimal to increase τ_N. This increase worsens S's terms of trade.

[26] With quadratic damage functions, with marginal damage $d_i' + d_i'' X_j$, we get:

$$\tau_S^* = \frac{d_S' + (\overline{v}_{NN} + \overline{v}_{SN})\, d_S''}{1 + a_N d_S''} \text{ and } \tau_N^* = \frac{3d_N' + (3\overline{v}_{SS} + \overline{v}_{NS})\, d_N'' - \overline{v}_{NS}/a_S}{5 + a_S d_N''}.$$

COROLLARY 6: *If the damage facing N (d'_N) is larger, S's terms of trade worsen.*

There are two justifications for max operators in (18). First, constraint $X_N \geq X_N^0$ implies that there is no point of setting τ_S so large that $X_N^{MR}(\tau_S) < X_N^0$, since a smaller tariff will reduce ex post distortions, without affecting X_N. Similarly, there is no point of setting τ_N so large that $X_S^{BR}(\tau_N) < X_S^0$. A larger X_S^0 reduces the possibility to conserve, and thus the benefit from a large τ_N.

Second, large tariffs may not be renegotiation proof. With the assumption $X_i^0 \leq X_i^{BAU}$, the renegotiation-proofness constraint binds before constraints $X_N^{MR}(\tau_S) \geq X_N^0$ and $X_S^{BR}(\tau_N) \geq X_S^0$.

REMARK 1: RENEGOTIATION-PROOFNESS. If the countries agreed on $\tau_S > \tau_S^{BAU}$, then, in equilibrium, $X_N = X_N^{MR}(\tau_S) < X_N^{BAU}$, implying that $\tau_S > \tau_S^{BR}(X_N)$. Consequently, both countries would strictly benefit by a reduction of τ_S. In other words, $\tau_S > \tau_S^{BAU}$ would not be renegotiation proof. Similarly, $\tau_N > \tau_N^{NASH}$ would not be renegotiation proof.[27] With the restrictions that $\tau_S \in \left[0, \tau_S^{BAU}\right]$ and $\tau_N \in \left[0, \tau_N^{NASH}\right]$, a PTA cannot induce $X_i \leq X_i^{BAU}$, $i \in \{N, S\}$.

Furthermore, a pair of tariffs is not renegotiation proof if $\tau_N \tau_S > 0$, because with $\tau_N \tau_S > 0$ it would be possible, ex post, to reduce both tariffs in a way that would make both countries better off. If only one of the tariffs is positive, and $\tau_i \leq \tau_i^{BR}(X_j)$, then the pair is Pareto optimal in that no other tariff pair can make both parties better off. In line with Bond and Park (2002:397), then: "no renegotiation takes place over the life of the agreement because the payoff of the two parties is always on the utility possibility frontier."[28]

IV. Contingent Trade Agreements

A. *Contingency and Credibility*

In general, traditional trade agreements cannot implement the first best. If there is no damage, X_N^{FTA} is first best but X_S^{FTA} is inefficiently small. With damage, the second-best tariff characterized by Proposition 3 trades off the deadweight loss from unequal marginal benefits with the effect on conservation.

In the trade agreements considered above, N and S committed to zero or fixed tariffs before capacity investments. In BAU, equilibrium tariffs were contingent on X_i. When X_i is verifiable, N and S may be able to consider *how* the tariffs should be contingent on X_i.

To be realistic, we must require the tariffs to be credible, or renegotiation proof, as discussed in Remark 1. If, at some X_i, $\tau_j^{CTA}(X_i) > \tau_j^{BR}(X_i)$, then both N and S benefit if j's tariff is reduced. Such a contingency would not be credible.

[27] If $\tau_N > \tau_N^{NASH}$, defined by (16), then $X_S = X_S^{MR}(\tau_N) < X_S^{NASH}$. With such a small X_S, both N and S benefit from reducing the tariff from τ_N to $\tau_N^{BR}\left(X_S^{MR}(\tau_N)\right)$.

[28] If the parties could renegotiate using side transfers, then every strictly positive tariff would be renegotiated (since it is distortionary ex post). This seems unrealistic, but even in this case, the effect of the initially set tariff on investments is exactly as in my analysis if country i has all the bargaining power when τ_i is renegotiated. This is in line with the assumption by Guriev and Klimenko (2015:1833), who write: "During each period, parties can renegotiate the previously concluded agreements. All the bargaining power belongs to the home country." If both countries have strictly positive bargaining power when τ_i is renegotiated, the effects of τ_i on X_j are quantitatively different, but the results will hold, qualitatively.

DEFINITION 1: *A contingent trade agreement (CTA) specifies* $\tau_j \in \left[0, \tau_j^{BR}(X_i)\right]$, *with* $\tau_N \tau_S = 0$, *for every* $X_S \geq X_S^0$, $X_N \geq X_N^0$, *and* $i, j \in \{N, S\}$.

The main point of this paper is to show that Corollaries 1–6 are all reversed when the countries can sign a CTA.

The reversal of Corollary 1 follows from Definition 1. The investors in N are assumed to be tariff- and price-takers. As before, N's equilibrium capacity will be characterized by $X_N^{MR}(\cdot)$, as a function of the expected and equilibrium tariff, and not of any hypothetical tariff at an out-of-equilibrium X_N. The socially optimal τ_S, given this market response, is as given by Proposition 3. The contingency has no role to play when investors are price-takers.

In contrast, when S decides on X_S, S takes into account how tariffs vary with X_S. Thus, S can be induced to select $X_S \neq X_S^{FTA}$, even if $\tau_N^{CTA}(X_S) = 0$, if the contingent tariffs are less attractive at other capacity levels.

COROLLARY 1^{CTA}: *The contingency has no effect on private investments, but can influence the government's capacity expansion.*

Consequently, there is no loss (of generality) from letting the CTA tariffs be contingent only on X_S.

B. *CTA with Free Trade*

The first best requires that there be no tariff on the equilibrium path. Thus, we start by characterizing what is feasible with an agreement, CTA^0, which is restricted in that trade must be free when X_S takes its equilibrium value, X_S^{CTA}.

Feasibility. The free-trade requirement is essentially equivalent to a requirement that only τ_N, and not τ_S, will be contingent on X_S. To see this, note, first, that $\tau_N = 0$ is both ex post efficient and it is preferred by S. It can only be harder, and socially less efficient, to implement X_S^{CTA} if $\tau_N^{CTA}\left(X_S^{CTA}\right) > 0$. Regarding $\tau_S = 0$, S generally prefers a larger τ_S after X_N has been decided on. If we require $\tau_S\left(X_S^{CTA}\right) = 0$, it can only be harder to implement X_S^{CTA} if we permit $\tau_S > 0$ when $X_S \neq X_S^{CTA}$. Thus, the best CTA^0, in this situation, allows τ_S to be independent of X_S. This independence implies that there is no linkage between the two markets. In fact, the analysis in this subsection is unchanged if τ_S is fixed at any other level, not necessarily zero. (In particular, the socially optimal non-contingent τ_S is characterized by Proposition 3.)

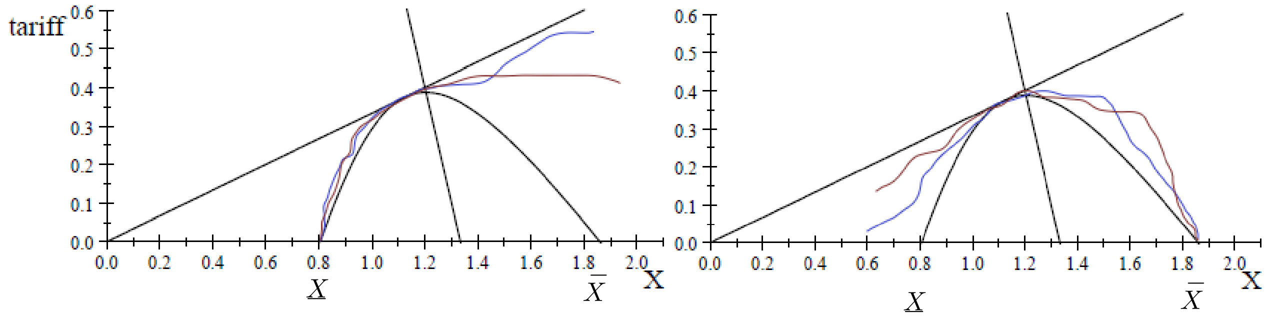

Figure 3: *Multiple credible tariff schedules can support conservation (a: left) or expansion (b: right).*

PROPOSITION 4: *With free trade in equilibrium, the CTA^0 can implement every $X_S \geq X_S^0$ if $X_S \in \left[\underline{X}, \overline{X}\right]$, where*

$$\begin{aligned}\underline{X} &= X_S^{FTA} - \overline{v}_{NS}\sqrt{10}/6 = \overline{v}_{SS} - \frac{\overline{v}_{NS}}{3}\left(\sqrt{\frac{5}{2}} - 1\right) \approx \overline{v}_{SS} - 0.19\overline{v}_{NS}, \\ \overline{X} &= X_S^{FTA} + \overline{v}_{NS}\sqrt{10}/6 = \overline{v}_{SS} + \frac{\overline{v}_{NS}}{3}\left(\sqrt{\frac{5}{2}} + 1\right) \approx \overline{v}_{SS} + 0.86\overline{v}_{NS}.\end{aligned}$$

To implement $X_S^{CTA} \geq X_S^0$, it must be that for every other $X_S \geq X_S^0$, $\tau_N^{CTA}(X_S)$ must be so large that S prefers the pair $\left(X_S^{CTA}, \tau_N = 0\right)$ to $\left(X_S, \tau_N^{CTA}(X_S)\right)$. To motivate S to conserve $X_S^{CTA} < X_S^{FTA}$, the tariff on S's export must be larger when X_S is large. Fig. 3(a) illustrates that multiple out-of-equilibrium tariff functions can implement $X_S = \underline{X}$. To motivate S to increase the production capacity, if that is socially optimal, the tariff must be larger as long as X_S remains small (Fig. 3(b)).

Because of the renegotiation constraint $\tau_N^{CTA}(X_S) \in \left[0, \tau_N^{BR}(X_S)\right]$, there is a lower and an upper boundary to which X_S's it is possible to implement. It is easy to check, however, that the feasibility set $\left[\underline{X}, \overline{X}\right]$ includes X_S^{BAU}, X_S^{FTA}, and every X_S implementable by the PTA in Section III.C. Thus, all these traditional trade agreements are dominated by some CTA^0.

The optimal and equilibrium CTA^0. Given the possibilities described by Proposition 4, it is straightforward to characterize an optimal agreement. If $X_S^{FB} \in \left[\underline{X}, \overline{X}\right]$, N and S will find it optimal to sign a CTA with $\tau_N^{CTA}\left(X_S^{FB}\right) = 0$. Consequently, the CTA implements the first-best allocation and production of X_S if $\underline{X} \leq X_S^{FB} \leq \overline{X}$. With (9), these inequalities can be written as follows if the damage is linear:[29]

$$\frac{0.07}{a_S} \leq d_N' \leq \frac{0.60}{a_S}.$$

[29]When these inequalities hold, CTA^0 implements the first best in *both* markets if there is no damage from N's good, i.e., if $h_S' = 0$. However, note that the first inequality fails when $d_N' = 0$. Consequently, $\overline{X}_S < X_S^{FB}(0)$, and the first best cannot be implemented by CTA^0 when there is no damage —in this simple model. With multiple products and cross contingency, discussed below, then it is possible for the CTA to implement $X_S^{FB}(0)$.

If the damage is so small that $X_S^{FB} > \overline{X}$, then the optimal and equilibrium CTA^0 implements $X_S = \overline{X}$ with free trade. If, instead, the damage is so large that $X_S^{FB} < \underline{X}$, the optimal and equilibrium CTA^0 implements $X_S = \underline{X}$ with free trade. In either case, the CTA^0 leads to larger payoffs than does the FTA —or any other PTA.

The rest of this subsection considers the case in which X_S is suboptimally large with BAU (i.e., $X_S^{FB} < X_S^{BAU}$). All corollaries are reversed:

COROLLARY 2^{CTA}: *The CTA^0 implements a smaller X_S than with BAU (i.e., $X_S^{CTA} < X_S^{BAU}$).*

Next, note that $\underline{X}$ decreases, while $\overline{X}$ increases, in $\overline{v}_{NS}$. Intuitively, if the gains from trade ($\overline{v}_{NS}$) increase, S has more to lose from a large τ_N. The potential loss makes S willing to select an X_S that is very different from X_S^{FTA}, if that is necessary to obtain the most attractive terms of trade. This willingness reverses the essence of Corollary 3.

COROLLARY 3^{CTA}: *If the gains from trade ($\overline{v}_{NS}$) increases, X_S^{CTA} decreases.*

Because the CTA^0 can motivate more conservation than BAU (in contrast to the FTA), the value of the CTA^0 is larger if the damage is large, so that conservation is more valuable. The insight of Corollary 4 is thus reversed:

COROLLARY 4^{CTA}: *If the damage (d_N') is larger, the value of the CTA^0, relative to BAU, is larger.*

When N's damage is large, it is N's value of the CTA^0 that is large —not S's value. Without compensation, S loses from the larger damage and the additional conservation, and perhaps even from the CTA^0 relative to BAU. This reverses Corollary 5:

COROLLARY 5^{CTA}: *If the damage (d_N') is larger, only N benefits more from the CTA, unless the transfer from N to S is larger.*

If we assume that each party's bargaining surplus increases in the total bargaining surplus (as with the Nash bargaining solution), then the transfer from N to S must increase when the damage d_N' increases.

C. *CTA with Tariffs in Equilibrium*

The CTA permits carrots as well as sticks. That is, S can be motivated to select a socially desirable X_S not only by the threat that S will otherwise face a larger tariff on its own product, as studied above, but also by the possibility to set a positive tariff on the goods imported from N. As discussed above, S's terms of trade are improved with $\tau_S > 0$. To take advantage of this carrot, or cross contingency, we will now establish and draw on a linkage between the two markets.

Feasibility. By definition, the CTA allows the positive τ_S to be conditioned on X_S. Thus, S can be induced to stay with $X_S^{CTA} < \underline{X}$, to be allowed $\tau_S\left(X_S^{CTA}\right) > 0$, if the tariff τ_S is smaller at alternative X_S's. In other words, the range of X_S's that can be supported by a CTA is larger when we permit a contingent $\tau_S > 0$ on the equilibrium path. As in Section III.C, it is not credible with $\tau_S > \tau_S^{BAU}$.

PROPOSITION 5: *With* $\tau_S \in \left[0, \tau_S^{BAU}\right]$ *in equilibrium, the CTA can implement every* $X_S \geq X_S^0$ *if* $X_S \in \left[\underline{X}(\tau_S), \overline{X}(\tau_S)\right]$, *where:*

$$
\begin{aligned}
\underline{X}(\tau_S) &= \overline{v}_{SS} - \frac{\overline{v}_{NS}}{3}\left(\sqrt{\frac{5}{2} + \frac{a_S}{\overline{v}_{NS}^2}\left(12\overline{v}_{SN} - 15a_N\tau_S\right)\tau_S} - 1\right), \\
\overline{X}(\tau_S) &= \overline{v}_{SS} + \frac{\overline{v}_{NS}}{3}\left(\sqrt{\frac{5}{2} + \frac{a_S}{\overline{v}_{NS}^2}\left(12\overline{v}_{SN} - 15a_N\tau_S\right)\tau_S} + 1\right).
\end{aligned}
$$

For every τ_S, N's capacity follows from $X_N = X_N^{MR}(\tau_S)$.

If the permitted τ_S increases marginally from 0, S's benefit from the CTA increases, and S accepts a larger range of X_S's. Consequently, $\underline{X}(\tau_S)$ decreases, and $\overline{X}(\tau_S)$ increases.

From Section III, we know that S does not prefer an arbitrarily large τ_S. It is easy to check that $\underline{X}(\tau_S)$ is minimal, and $\overline{X}(\tau_S)$ maximal, at

$$
\begin{aligned}
\tau_S^M &\equiv \arg\min_{\tau_S} \underline{X}(\tau_S) = \arg\max_{\tau_S} \overline{X}(\tau_S) = \frac{2}{5}\frac{\overline{v}_{SN}}{a_N} \Rightarrow \\
\underline{X}^M &\equiv \min_{\tau_S} \underline{X}(\tau_S) = \overline{v}_{SS} - \frac{\overline{v}_{NS}}{3}\left(\sqrt{\frac{5}{2} + \frac{12}{5}\frac{a_S}{a_N}\frac{\overline{v}_{SN}^2}{\overline{v}_{NS}^2}} - 1\right) \approx \overline{v}_{SS} - 0.40\overline{v}_{NS} \textit{ if } \overline{v}_{SN}^2/a_N \approx \overline{v}_{NS}^2/a_S, \\
\overline{X}^M &\equiv \max_{\tau_S} \overline{X}(\tau_S) = \overline{v}_{SS} + \frac{\overline{v}_{NS}}{3}\left(\sqrt{\frac{5}{2} + \frac{12}{5}\frac{a_S}{a_N}\frac{\overline{v}_{SN}^2}{\overline{v}_{NS}^2}} + 1\right) \approx \overline{v}_{SS} + 1.07\overline{v}_{NS} \textit{ if } \overline{v}_{SN}^2/a_N \approx \overline{v}_{NS}^2/a_S.
\end{aligned}
$$

If $\tau_S > \tau_S^M$, the tariff is not maximizing S's payoff from τ_S relative to $\tau_S = 0$, and thus the set of implementable X_S's is smaller when $\tau_S^{CTA} > \tau_S^M$ than when $\tau_S = \tau_S^M$. For Example 1, where $\tau_S^M = 0.4$ and $\tau_S^{BAU} = 0.5$, Fig. 4 illustrates how $\underline{X}(\tau_S)$ and $\overline{X}(\tau_S)$ vary with $\tau_S \in \left[0, \tau_S^{BAU}\right]$. The vertical axis measures $\tau_N - \tau_S$, so τ_S is measured on the negative vertical axis.[30]

The optimal and equilibrium CTA. When N and S can use side transfers when they negotiate the CTA, the equilibrium CTA will be the optimal one, which maximizes the sum of payoffs. Because a small τ_N is both minimizing ex post distortions, and preferred by S, the optimal CTA ensures that $\tau_N = 0$, in equilibrium. The remaining question regards the level of τ_S.

If $X_S^{FB} \in \left[\underline{X}(d'_S), \overline{X}(d'_S)\right]$, there is no trade-off. X_S^{FB} can be implemented with $\tau_S^* = d'_S$, which is optimal according to Proposition 3. Interestingly, this scenario is more likely with a large $d'_S \in \left(0, \tau_S^M\right)$. In this case, the large damage from N's production justifies a large tariff. With the (out-of-equilibrium) threat that this tariff will be reduced if $X_S \neq X_S^{FB}$, S becomes more willing to stick with X_S^{FB}.

[30] The τ_S that motivates the smallest X_S, τ_S^M, is less than τ_S^{BAU}. The intuition for why $\tau_S^M < \tau_S^{BAU}$ is that when τ_S is reduced below τ_S^{BAU}, X_N increases, and the larger X_N increases the cost for S if τ_S is lowered as a consequence of $X_S \neq X_S^{CTA}$.

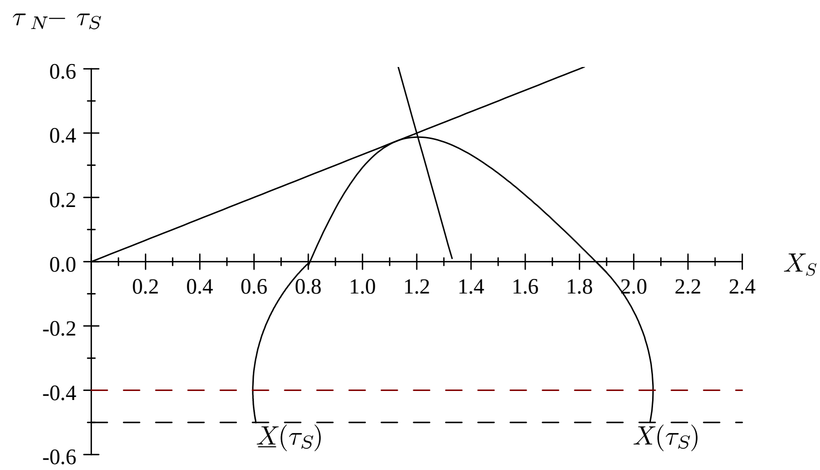

Figure 4: *For every $\tau_N - \tau_S$, the CTA can implement every X_S within S's indifference curve.*

If $X_S^{FB} \notin \left[\underline{X}\left(d'_S\right), \overline{X}\left(d'_S\right)\right]$, there is a trade-off. Even if the CTA *can* implement $\underline{X}^T$ with $\tau_S = \tau_S^M$, this is suboptimal, no matter how much damage N faces (unless $d'_S = \tau_S^M$). Instead, the optimal τ_S trades off the distortion in the market for N's good with the value of conserving X_S. If $\tau_S > \tau_S^*$ increases, the marginal distortion increases, but the marginal impact on $\underline{X}\left(\tau_S\right)$ declines. When $\tau_S \to \tau_S^M$, $\partial \underline{X}\left(\tau_S\right)/\partial \tau \to 0$. Consequently, the optimal τ_S^{CTA} is less than τ_S^M, if $\tau_S^* < \tau_S^M$.

PROPOSITION 6:
(i) If $X_S^{FB} \in \left[\underline{X}\left(\tau_S^*\right), \overline{X}\left(\tau_S^*\right)\right]$, it is optimal with $\tau_S^{CTA} = \tau_S^*$, and the CTA implements X_S^{FB}, $\tau_N = 0$, and $X_N^{MR}\left(\tau_S^*\right)$.
(ii) If $X_S^{FB} \notin \left[\underline{X}\left(\tau_S^*\right), \overline{X}\left(\tau_S^*\right)\right]$, the optimal τ_S^{CTA} is strictly between τ_S^* and τ_S^M.
(iii) Assume (a) $X_S^{FB} < \underline{X}\left(\tau_S^*\right)$ and (b) $d'_S < \tau_S^M$. With the optimal CTA, $X_S^{CTA} \in \left(X_S^{FB}, \underline{X}_S\right)$. If d'_N or d'_S increases, the optimal τ_S^{CTA} increases, and both X_N^{CTA} and X_S^{CTA} decrease.

The Appendix characterizes the optimal CTA without assumptions (a) and (b).

The two assumptions hold if the damage from S's capacity expansion is large, while the damage from N's production is small. In this case, the trade-off is the following. On the one hand, by increasing τ_S above τ_S^*, $\underline{X}\left(\tau_S\right)$ is reduced, and more can be conserved. On the other hand, the larger τ_S creates ex post distortions in the market for N's product, and it reduces the incentives to invest in X_N. The optimal τ_S^{CTA} trades off these two concerns. If d'_S increases, the *cost* of a given $\tau_S > d'_S$ is smaller, and it becomes socially optimal to increase τ_S. In other words: A larger damage associated with N's product makes it optimal to conserve more in country S if $X_S^{FB} < \underline{X}\left(d'_S\right)$. If, instead, d'_N increases, then, everything else equal, the *benefit* from raising τ_S is larger, while the cost of raising τ_S is unchanged. Again, it becomes socially optimal to increase τ_S. This comparative static implies that the essence of Corollary 6 is reversed.

COROLLARY 6^{CTA}: *Suppose (a) $X_S^{FB} < \underline{X}\left(\tau_S^*\right)$ and (b) $d'_S < \tau_S^M$. If the damage (d'_N or d'_S) increases, S's terms of trade improve.*

If $X_S^0 > \underline{X}^M$, then with a larger X_S^0, less can be conserved, and the optimal τ_S is smaller under assumptions (a) and (b). In other words, S's terms of trade worsen if X_S^0 increases. This contrasts our observation after Corollary 6, where τ_N could decline in X_S^0.

As mentioned, the Appendix characterizes the optimal CTA without requiring assumptions (a) and (b): See Proposition 6-A. The Appendix also allows for multiple goods, population sizes, countries, and CTA partners. The following section illustrates what the CTA can achieve with these generalizations.

V. Extensions

A. *Multiple Goods*

The scope for the CTA is strengthened if countries produce multiple types of goods. If each good is distinct, and satisfies a unique term in the quasilinear utility function, then each market can be modeled as in Section II, and analyzed as in Section III. The larger the number of goods is, the larger the gains from trade are.

With Section IV's CTA, S can be offered low tariffs on several goods contingent on the socially desirable conservation level. Even if S's other goods are privately provided, S is willing to limit the expansion of the agricultural sector (X_S) if that is necessary to avoid revenue losses in the private sector. To implement a socially desirable X_S^{CTA}, S will be offered zero tariff on all its products when $X_S = X_S^{CTA}$, but higher tariffs on all products if $X_S \neq X_S^{CTA}$.

Just as in Section IV.C, the CTA is further strengthened if we permit contingent positive tariffs on the goods that S import. The larger the number of goods that S imports from N is, the larger the potential loss for S is if S selects an $X_S \neq X_S^{CTA}$ that will lead to a reduction in the tariff levels on S's imported goods. Therefore, the feasibility set expands when there are many products.

PROPOSITION 7: *Suppose S exports q_S types of goods to N, while N exports q_N types of goods to S.*
(i) With free trade in equilibrium, the CTA^0 can implement every $X_S \geq X_S^0$ if $X_S \in \left[\underline{X}^{q_S}, \overline{X}^{q_S}\right]$, where $\underline{X}^{q_S}$ decreases, and $\overline{X}^{q_S}$ increases, in q_S. (Both thresholds are independent of q_N.)
(ii) With positive tariffs in equilibrium, the CTA can implement every $X_S \geq X_S^0$ if $X_S \in \left[\underline{X}^{q_S q_N}, \overline{X}^{q_S q_N}\right]$, where $\underline{X}^{q_S q_N}$ decreases, and $\overline{X}^{q_S q_N}$ increases, in q_S and in q_N.

B. *Multiple Consumers*

The proofs in the Appendix permit the mass of consumers to be m_N in N, and m_S in S. If we fix the ratio m_N/m_S, then capacity levels in BAU, the FTA, and the CTA's boundaries, $\underline{X}$ and $\overline{X}$, are all proportional to $m \equiv m_N + m_S$: When the population sizes double, these capacity levels double.

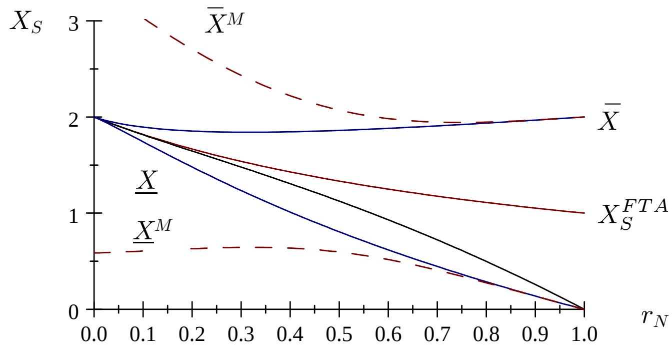

Figure 5: *More can be conserved with cross-contingency, especially if r_N is small.*

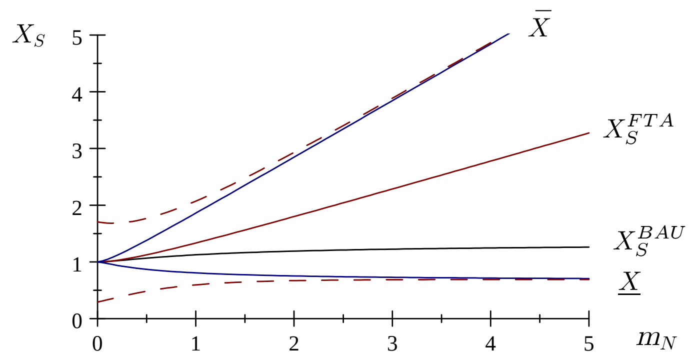

Figure 6: *More can be conserved when N is large.*

One way of learning about the effect of the relative sizes is to fix m, and consider changes in $r_N \equiv m_N/m \in (0,1)$. For Example 1, where $m = 2$, Fig. 5 illustrates that when r_N increases, then X_S^{FTA} decreases towards the monopoly quantity (which is 1). The intuition is that when S mainly produces for the international market, S becomes more willing to withhold X_S in order to raise the price. When $r_N \uparrow 1$, $\tau_N^{BAU} \uparrow \infty$, so $X_S^{BAU} \downarrow 0$, in line with the holdup problem.

When r_N is large, it is more expensive for S that τ_N is large, and S accepts a larger range of X_S's in return for $\tau_N = 0$. Therefore, $\underline{X}$ decreases, while $\overline{X}$ increases, in r_N.

The flip side of this logic is that when r_N is *small*, N's tariff is less important for how much the CTA can motivate S to conserve. That is, the "stick" $\tau_N > 0$ is less effective. In this case, it is instead more effective to use the "carrot" by allowing S to introduce a tariff on the import from N. This instrument is especially effective when r_N is small, because when S is the main market for N's product, S's tariff has a large influence on the equilibrium price on N's product.

This insight is confirmed in Fig. 6, where m_S is fixed while m_N increases along the horizontal axis. A larger m_N leads to a larger X_S^{BAU} and X_S^{FTA} because the total number of consumers increases.[31]

[31] Naturally, X_S^{FTA} approaches the monopoly quantity when m_N is large. If m_N is relatively small, S produces more

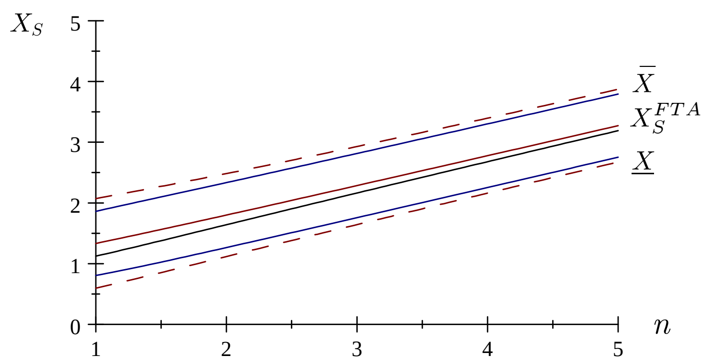

Figure 7: *CTA*^{0} *continues to motivate more conservation than with BAU, also if* n *is large.*

Nevertheless, $\underline{X}$ decreases in m_N. That is, the CTA succeeds in conserving more if N is large. The intuition is that, when N is large, it is more important for S to maintain $\tau_N = 0$.

C. *Multiple Countries*

The analysis in the Appendix allows for multiple countries. This is relevant in the EU–Mercosur context, because Mercosur exports to many countries. To include them, I henceforth assume that "our" northern country, N, is just one of n countries importing S's good. Each of them exports a unique good to all the others, just as modeled in Section II.

The Appendix allows the countries to include different masses of consumers. For the sake of illustration, assume here that all n countries are identical and with consumer mass 1, like S's consumer mass. If there were no damage, the first best would be $X_S = \overline{v}_{SS} + n\overline{v}_{NS}$. With free trade, equilibrium X_S^{FTA} is as in Fig. 6, if just m_N is replaced by n.

Appendix C allows all countries to set their tariffs strategically. Here, suppose that the $n-1$ other importers, except for our country, N, trade freely with S. In this case, the BAU outcome converges to the FTA outcome if n is large. The reason is that τ_N^{BAU} decreases in n, because N can influence the price less by its tariff when n is large.

With the lower equilibrium tariff, and with multiple buyers of beef, one may at first guess that a unilateral tariff is less effective in securing conservation (this is the finding by by Hsiao, 2022, for instance). Surprisingly, CTA^0 can motivate *more* conservation relative to BAU when n is large, even if CTA^0 is signed bilaterally between S and N only, and even if it implements free trade in equilibrium. Fig. 7 illustrates how X_S^{BAU}, X_S^{FTA}, $\underline{X}$, $\overline{X}$, $\underline{X}^M$, and $\overline{X}^M$ increase in n. A careful look at the figure (and the Appendix) discloses that when n increases, $X_S^{FTA} - X_S^{BAU}$ decreases (in line with the discussion above), but $X_S^{BAU} - \underline{X}$ increases, so the CTA has a larger effect (relative to BAU) when n is large.

than the monopoly quantity to reduce the domestic price.

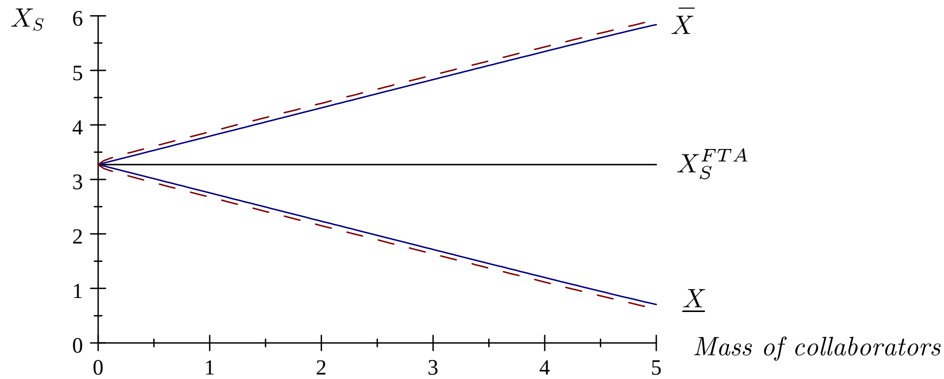

Figure 8: *The larger is the mass in countries offering the CTA, the more can be conserved.*

To understand the intuition for this result, consider, again, Fig. 2. When n is large, τ_N^{BR} is flatter, as a function of X_S, than when $n = 1$. Therefore, S's indifference curve is flatter at X_S^{BAU}, and thus S finds it inexpensive to reduce X_S from X_S^{BAU}. In other words, S is willing to reduce X_S by quite a lot, relative to X_S^{BAU}, in return for a decrease in N's tariff.

The cross-contingency discussed by Proposition 5 is less important when n is large, however. Because the tariffs are ex post distortionary (in that marginal utilities are unequal), it is generally less desirable to introduce a tariff at home than to eliminate the tariff abroad. This is especially true when n is large, because the distortions from the tariff are very large when there are many other importers that can purchase the good. Thus, the added value of cross-contingency (the red dashed lines) is smaller when n is large.

D. *Multiple Collaborators*

Naturally, the CTA can achieve more conservation if multiple countries collaborate in offering preliminary tariffs contingent on S's capacity to produce. The formulae derived in the Appendix allow the CTA-participant to include any mass of consumers. If two countries collaborate by jointly offering S low tariffs contingent on its capacity remaining low, the effect of the CTA^0 is equivalent to N having a larger mass, in this model. With cross contingency, S is permitted to introduce a positive tariff on two imported goods, contingent on X_S, when two beef importers collaborate on the CTA in addition to S.

Fig. 8 fixes the mass of S's foreign consumers at 5 (relative to S's mass of consumers). Then, $X_S^{FTA} = 3.27$. The (blue) solid curves show that $\underline{X}$ declines, and $\overline{X}$ increases, if the mass of consumers included in the CTA-participating importers increases. The (red) dashed lines illustrate $\underline{X}^M$ and $\overline{X}^M$. As discussed in the previous subsection, the additional effect from cross contingency is small when n is as large as 5, but the additional effect increases somewhat with the number of CTA-collaborators, because S can then introduce a positive tariff on a larger number of goods.

E. *Multiple Periods*

The results above continue to hold, qualitatively, if there are multiple periods. Regardless of whether we fix at the BAU, FTA, or PTA scenario, each X_i immediately adjusts to a stationary level, and there is no further capacity investment thereafter. In the working paper (Harstad, 2020), I consider a model with linear utility and an infinite number of periods. I find that the main results, emphasized above, continue to hold in that model.[32]

We can obtain a sharper characterization of the CTA tariff schedules when there are multiple periods. A tariff *strictly* above S's indifference curve for $X_S \in \left(\underline{X}, X_S^{BAU}\right)$ in Fig. 3 will *not* be Pareto optimal (and thus not renegotiation proof) when N realizes that unless this tariff is reduced to a point below S's indifference curve, S will continue to raise X_S in the next period. Thus, τ_N^{CTA} must increase in $X_S \in \left(\underline{X}, X_S^{BAU}\right)$ in line with S's indifference curve (see Fig. 3) for the CTA to be renegotiation proof in the dynamic game.

VI. Calibration

It is beyond the scope of this paper to provide a serious calibration and quantitative analysis. However, the formulae in the Appendix rest on few parameters, they allow for any number of countries, and they state predictions for equilibrium tariff levels and export ratios. When these predictions are matched with empirical observations, we can estimate the parameters in the model. With the parameters, we can proceed by deriving the quantitative effects of trade liberalization and of the CTA.

Calibration. To provide a vague idea of the promise for such an exercise, consider the agricultural export from Brazil. Brazil and the EU are major trading partners. Most of the EU's consumed soy is imported from Brazil, and 81% of the EU's beef import is from Mercosur (36% from Brazil).[33]

As a start, I begin by assuming that there are n identical importing blocks with equal population masses. Brazil's (S's) mass can be different. I will require that the equilibrium BAU tariff in each trading partner be 20%. After all, the tariff on high-quality beef from Brazil is 20% in Europe,[34] 24% in the US,[35] 30% in India,[36] and 15% in China[37] and Russia.[38] A tariff on 20% is also consistent with numbers from the World Bank, the WTO, and earlier calibrations.[39]

[32] The dynamic model generates some new insight, however: Suppose that N's harm is so large that the FTA has a negative sosial value, but that the parties are able to reconsider such an agreement in the future. Then, it can be desirable for S to increase X_S over time so that, eventually, the environmental harm is sunk, and N will agree on trade liberalization. See the working paper (Harstad, 2020) for details on this possibility.

[33] https://estadisticas.mercosur.int/

[34] https://trade.ec.europa.eu/access-to-markets/en/results?product=0201300031&origin=BR&destination=DE

[35] https://hts.usitc.gov/current

[36] https://www.exportgenius.in/india-import-duty/hscode-0201.30

[37] https://www.wto.org/english/res_e/statis_e/daily_update_e/tariff_profiles/CN_e.pdf

[38] https://www.fas.usda.gov/data/russia-russia-mulls-replacing-its-beef-trqs-tariff-2022

[39] See Table 3 and 4 in Ossa (2011). According to the World Bank, the weighted average tariff for food products from Brazil was 23.34 % in 2019:
https://wits.worldbank.org/CountryProfile/en/Country/WLD/StartYear/1988/EndYear/2019/TradeFlow/Import/Indicator/AHS-

Further, let's require that the BAU export fraction be 43%. This number is a weighted average of the soy export fraction (63%)[40] and the beef export fraction (25%),[41] where the weights reflect the values of the two sectors (the value of the soy production is 35% and the value of the beef production is 38% of Brazil's total agricultural production). This number is also similar to the total agricultural export fraction, according to the Brazilian government.[42] The transportation costs are very low,[43] so I will ignore them, but I take into account that existing non-tariff barriers on food seem to be about 20% (Cadot et al., 2018).

Appendix C shows that when the model is calibrated to match these numbers, then, approximately, $n \approx 5$ and $r_S \equiv m_S/m \approx 1/2$. Conversely, if $n = 5$ and $r_S = 1/2$, the BAU equilibrium predicts that the tariffs are 21% and S exports 45% of its own production.

The result that there are 5 major importing blocks, in addition to Brazil's market, is in line with Ossa (2011).[44] The result $r_S = 1/2$ seems high, but reflects the assumption that only S produces beef. In reality, much of the demand in the importing countries is saturated by other/domestic producers. In this light, it is not that odd if Brazil contains half of the consumers with demand not yet saturated by other producers.

Results. With these numbers, some of the model's predictions are shown in Table 1. That is, moving from BAU to free trade (with no non-tariff barriers) increases X_S by almost 5%. If one of the five importing blocks agrees on a CTA^0 with S, however, the entire increase can be avoided. (In fact, the CTA^0 can motivate more conservation than in BAU.)

X_S^{FTA}/X_S^{BAU}	$\underline{X}/X_S^{BAU}$
1.048	0.963

Table 1: With free trade, X_S increases by 4.8%, but the CTA can prevent the increase.

Doubled demand from Asia. From now, I refer to the above level of X_S^{BAU} as X_S^0, because I will consider a change in parameters. After all, the increase in X_S, following trade liberalization, does not capture how fast Brazil's exports have increased over the last few years.[45] Because of economic growth

WGHTD-AVRG/Partner/BRA/Product/all-groups

According to the WTO, the simple average tariff lines on agricultural products are 12.2% for China, 15.2% for EU, 17.1% for Japan, and 7.5% for the US:

https://www.wto.org/english/res_e/statis_e/daily_update_e/tariff_profiles/BR_E.pdf

However, the marginal tariff is higher: The WTO writes that the final bound is 35.4%: https://www.wto.org/english/thewto_e/countries_e/brazil_e.htm#statistics

[40] https://apps.fas.usda.gov/newgainapi/api/Report/DownloadReportByFileName?fileName=Oilseeds%20and%20 Products%20Update_Brasilia_Brazil_07-01-2021.pdf

[41] https://apps.fas.usda.gov/newgainapi/api/Report/DownloadReportByFileName?fileName=Livestock%20and%20 Products%20Annual_Brasilia_Brazil_08-15-2021.pdf

[42] In a typical month, such as in January, 2021, the agricultural export fraction was 41.76% according to https://www.gov.br/agricultura/pt-br/assuntos/politica-agricola/todas-publicacoes-de-politica-agricola/agrofoco/2021

[43] It is 3% on Brazilian pork: https://usdabrazil.org.br/wp-content/uploads/2021/11/Livestock-and-Products-Annual_Brasilia_Brazil_08-15-2021.pdf

[44] Ossa (2011:125) writes: "I focus on Brazil, China, the European Union, India, Japan, and the United States since these countries are typically considered to be the main players in GATT/WTO negotiations."

[45] Brazil's beef production is expected to increase sharply according to: https://www.ers.usda.gov/amber-waves/2019/july/brazil-once-again-becomes-the-world-s-largest-beef-exporter/

in Asia, and other factors, the mass of relevant consumers in the importing countries, relative to S's mass of consumers, has increased sharply.

To understand the effects of a similar development in the next few years, suppose that the mass of relevant consumers in each of three of the five importing blocks doubles in size. (The number of relevant consumers in the EU, the US, and in Brazil is held constant.) This doubling is not unreasonable: Beef export from Brazil to China more than doubled between 2018 and 2020.[46] As a consequence, Brazil's total beef exports increased from USD 5.3b in 2015 to 8.1b in 2020.[47]

I consider seven scenarios: (F) free trade, (1^0) the EU (which now has 1/13 of the relevant consumers) offers CTA^0, (1) the EU offers a CTA with cross-contingency, (2^0) the EU and the US (with 2/13 of the consumers) coordinate on CTA^0, (2) the EU and the US coordinate on a CTA with cross-contingency on the export from both the EU and the US, (3^0) the EU, the US, and one of the third importers (with doubled consumer mass) coordinate on CTA^0, and (3) same as with (3^0), but with cross contingency.

When I permit cross-contingency, I let S export a second (privately provided) good, in addition to beef, as discussed in Section V.A. For Brazil's export, manufacture is as important as agriculture.[48] Analogously, each beef importer exports two types of goods to the rest of the world. For simplicity, every good has the same value/characteristics as S's beef, except that they are privately provided.[49]

The effects on $\underline{X}$, as a function of the number of collaborators, relative to X_S^0, are derived in Appendix C and presented here:

(F)	(1^0)	(1)	(2^0)	(2)	(3^0)	(3)
X_S^{FTA}/X_S^0	$\underline{X}/X_S^0$	$\underline{X}^M/X_S^0$	$\underline{X}/X_S^0$	$\underline{X}^M/X_S^0$	$\underline{X}/X_S^0$	$\underline{X}^M/X_S^0$
1.27	1.18	1.14	1.10	1.03	0.93	0.82

Table 2: X_S can increase by 27%, or by only 10% if the EU and the US offer a CTA^0.

Conclusion. With free trade and larger demand from Asia, the model predicts that X_S will increase by 27%. The increase in X_S falls by 9–13 percentage points for each importer collaborating on the CTA, depending on the extent of cross-contingency permitted. Consequently, the EU's action alone can have a significant effect. If the EU acts in concert with the US, most of the expansion can be avoided.

Avoiding the expansion is important because the increase in X_S is associated with deforestation. Almost 60% of Brazil's area is forest: The forest area is 4.97m km^2, and the level of the agricultural area, X_S^0, is 2.37m km^2.[50] An increase in X_S by 27% means that the agricultural area will increase by 0.64m km^2, which amounts to 13% of the remaining forest area. The total value of Brazil's agricultural

[46] https://apps.fas.usda.gov/newgainapi/api/Report/DownloadReportByFileName?fileName=Livestock%20and%20 Products%20Annual_Brasilia_Brazil_08-15-2021.pdf

[47] https://apps.fas.usda.gov/newgainapi/api/Report/DownloadReportByFileName?fileName=Livestock%20and%20 Products%20Annual_Brasilia_Brazil_08-15-2021.pdf

[48] https://www.wto.org/english/thewto_e/countries_e/brazil_e.htm#statistics

[49] That is, after the growth in Asia, the EU includes 5/13 of the consumers demanding EU's products, while Brazil includes 1/13.

[50] See, respectively, https://data.worldbank.org/indicator/AG.LND.FRST.K2?locations=BR and https://data.oecd.org/agrland/agricultural-land.htm

production[51] (USD87.5b) is USD369 per hectare, about 0.9% of the conservation value (per hectare) estimated by Franklin and Pindyck (2018).

In practice, the loss of forest tends to be even larger than the increase in the agricultural area, because not all the former forest area continues to be productive for agriculture: Between 2010 and 2018, X_S increased by 0.51m km^2, but the decline in forest cover was more than twice that amount: 1.25m km^2.[52] If an increase in X_S leads to twice as much forest loss, we can expect that 26% of the remaining forest will disappear with growing demand and free trade. According to the results above, however, most (all) of this deforestation can be avoided if two northern countries offer the CTA^0 analyzed in this paper.

VII. Empirical Relevance

As explained in the Introduction, empirical evidence documents that the ratification of regional trade agreements is correlated with high deforestation levels. Deforestation has many causes, and a growing demand for beef, soy, and other agricultural products is likely to raise the pressure on tropical forests. The basic model in this paper is consistent with a negative relationship between trade agreements and natural resource conservation.

The purpose of this paper is not, however, to prove that this relationship necessarily must be negative. On the contrary, the purpose is to illustrate that, even in a situation in which the relationship can be negative, it is possible to design an alternative trade agreement that motivates conservation instead of depletion. A contingent trade agreement, where the default allocation of gains from trade is contingent on the forest cover, motivates more conservation if the gains from trade are large. These results are important because they show that although trade is often associated with resource depletion, such as deforestation, it *must* not be so. Clever agreements exploit the gains from trade and use the gains to motivate conservation rather than exploitation. This possibility should be kept in mind by scholars studying trade and environmental problems, but also by policymakers, public officials, and activists who struggle with how to balance trade and conservation.

The CTA is politically and empirically relevant. In fact, the formalization of the CTA in this paper is an interpretation of the proposal by France and the Netherlands in May 2020. In the non-paper, mentioned in the Introduction, these European countries admit that the EU's sustainability requirements have failed to motivate trading partners to implement sustainable policies, and that the EU should consider staged implementation where tariff reductions can be reversed if sustainable policies are not being implemented. The non-paper is brief, and non-technical, however. The present analysis provides a first exploration of how a contingent trade agreement might be implemented, and of how much conservation it might motivate. One conclusion is that the CTA can prevent much of the deforestation that would otherwise arise, even if Brazil has many export markets.

[51] https://data.worldbank.org/indicator/NV.AGR.TOTL.KD?locations=BR
[52] https://data.worldbank.org/indicator/AG.LND.FRST.K2?locations=BR

CTAs are viable in practice: verifiable measures of forest cover are already available, thanks to satellite monitoring.[53] In India, the regional forest cover has, since 2015, been part of the central government's allocation of tax revenue to its 29 states (Busch and Mukherjee, 2018). Angelsen et al. (2018:51) elaborate on this policy and conclude that: "This represents the first large-scale ecological fiscal transfers for forest cover, and could serve as a model for other countries." There are also reasons to believe that the contingency will succeed in motivating conservation. Abman et al. (2021:3) study the empirical effects of environmental provisions in regional trade agreements and document that "the inclusion of forest-related provisions has mitigated forest loss resulting from trade liberalization."

The US is also seeking ways of using trade agreements to motivate forest conservation. Nigel Purvis, the former US climate negotiator, admits that trade is "unintentionally creating a financial incentive for criminals to set fire to the Amazon and convert it into farmland." Nevertheless, "meaningful environmental provisions in trade agreements" could be the single most important way to curb deforestation, according to Bruce Babbitt. In January 2021, Mr. Babbitt, leading a group of US climate leaders, outlined and submitted an *Amazon Protection Plan* to the new Biden Administration. The heart of the proposal involves making the avoidance of deforestation central to trade agreements. They say the US government should be "working with Europe, Japan, China and other major economies to align international efforts and thereby spread globally the policies outlined above."[54]

[53] As IPBES (2019, Ch. 6:56) states: "The monitoring systems have been improved to the point of offering daily real-time data, constituting one of the most important tools for the fight against deforestation in Brazil."

[54] My sources here include the article in *The New York Times*, January 29, 2021, and the action plan itself:
https://www.nytimes.com/2021/01/29/climate/biden-amazon-deforestation.html
https://climateprincipals.org/wp-content/uploads/2021/01/Amazon-Protection-Plan-Final_Climate-Principals.pdf

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APPENDIX A: PROOFS

Notation. A generic country is indexed by i, j, or l. Generally, $j \neq i$.

Let $\overline{v}_{il} \equiv v_{il} - (k_l + t_{il})\, a_l$ measure the socially optimal c_{il}, all costs are taken into account, when t_{il} is the transport cost from l to i and there is no damage. When prices adjust, it does not matter whether the exporter or the importer is responsible for paying the transport cost. If we let the transport cost be paid by the importer, $\overline{p}_i \equiv p_i - k_i$ measures a producer's net profit, per unit of capacity (i.e., the price minus the net marginal production cost).

Generalization. The set of countries is $I = \{S, N, 1..., n-1\}$ .These countries do not produce the same goods that N and S produce.[55] A country i has a mass m_i of consumers, where $m \equiv \sum_{i \in I} m_i$ is the total mass, and $r_i \equiv m_i/m$ is the relative size of $i \in I$. For averages, write $\overline{v}_{Ai} \equiv \sum_{l \in I} r_l \overline{v}_{li}$. I will also use $\overline{v}_{-S} \equiv \sum_{l \in I \setminus S} r_l \overline{v}_{li}$, although I henceforth find it unnecessary to include I in the summation subscripts. In Sections II-IV, we impose:

ASSUMPTION A1:

$$n = 1 \text{ and } m_N = m_S = 1. \tag{A1}$$

With A1, $m = 2$ and $r_N = r_S = 1/2$.

With these definitions, the following lemma confirms that we can henceforth simplify the notation by ignoring the k_i's and the t_{il}'s in the proofs.

LEMMA 1: *Every individual utility (1) can be written as:*

$$\begin{aligned}
U_i &= \overline{U}_i + \kappa_i, \text{ where} \\
\overline{U}_i &\equiv -\sum_l \frac{(\overline{v}_{il} - c_{il})^2}{2a_l} - \sum_{l \neq i} \overline{p}_l c_{il} + \overline{p}_i \frac{\sum_{l \neq i} m_l c_{li}}{m_i}, \\
\overline{v}_{il} &\equiv v_{il} - (k_l + t_{il})\, a_l, \\
\overline{p}_i &\equiv p_i - k_i, \text{ and} \\
\kappa_i &\equiv e_i/m_i - \sum_l \left[v_{il} (k_l + t_{il}) - a_l (k_l + t_{il})^2 /2 \right],
\end{aligned}$$

where κ_i is a constant that is henceforth ignored.

Proof. With (2), a binding (3), (4), and (5), (1) is:

$$\begin{aligned}
U_i &= -\sum_l \frac{(v_{il} - c_{il})^2}{2a_l} + \frac{e_i}{m_i} - \sum_l p_l c_{il} + p_i \frac{\sum_l m_l c_{li}}{m_i} - k_i \frac{\sum_l m_l c_{li}}{m_i} - \sum_l t_{il} c_{il} \\
&= \frac{e_i}{m_i} - \sum_l \left[\frac{(v_{il} - c_{il})^2}{2a_l} + k_l c_{il} + t_{il} c_{il} \right] - \sum_l (p_l - k_l)\, c_{il} + (p_i - k_i) \frac{\sum_l m_l c_{li}}{m_i} \\
&= \frac{e_i}{m_i} - \sum_l \left[\frac{(v_{il} - c_{il})^2 + 2a_l (k_l + t_{il})\, c_{il}}{2a_l} \right] - \sum_l \overline{p}_l c_{il} + \overline{p}_i \frac{\sum_l m_l c_{li}}{m_i} \\
&= \frac{e_i}{m_i} - \sum_l \left[\frac{(v_{il} - a_l (k_l + t_{il}) - c_{il})^2 + 2a_l v_{il} (k_l + t_{il}) - [a_l (k_l + t_{il})]^2}{2a_l} \right] - \sum_l \overline{p}_l c_{il} + \overline{p}_i \frac{\sum_l m_l c_{li}}{m_i} \\
&= \kappa_i - \sum_l \frac{(\overline{v}_{il} - c_{il})^2}{2a_l} - \sum_{l \neq i} \overline{p}_l c_{il} + \overline{p}_i \frac{\sum_{l \neq i} m_l c_{li}}{m_i}.
\end{aligned}$$

[55] Because of the quasi-linear utility function, the market for the goods produced by these countries is characterized independently of the market for N's good and for S's good.

PROOF OF PROPOSITION 1:

As announced, in addition to country N and S, we begin by permitting $n-1$ other countries that are passive in that their tariffs are fixed and reflected by $\overline{v}_{li}$.[56] In Appendix C, all tariffs are set strategically.

The market equilibrium. Let $\{i,j\}=\{N,S\}$, while l can be any country. The consumption levels of i's product are as follows for each consumer in l (including i), and $j\neq i$:

$$c_{li}=v_{li}-a_i\left(p_i+t_{li}\right)=\overline{v}_{li}-a_i\overline{p}_i \text{ and } c_{ji}=v_{ji}-a_i\left(p_i+t_{ji}+\tau_j\right)=\overline{v}_{ji}-a_i\left(\overline{p}_i+\tau_j\right). \tag{19}$$

When (3) binds,

$$\overline{p}_i=\frac{\sum_l m_l\overline{v}_{li}-a_im_j\tau_j-X_i}{a_im}, \tag{20}$$

which can be written as (11) under A1.

Equilibrium tariff. Anticipating (19) and (20), j sets τ_j to maximize j's surplus from the market for i's good, s_{ji}:

$$s_{ji}=-m_ja_i\left(\overline{p}_i+\tau_j\right)^2/2-m_j\overline{p}_i\left(\overline{v}_{ji}-a_i\left(\overline{p}_i+\tau_j\right)\right). \tag{21}$$

The first-order condition (f.o.c.) with respect to (w.r.t.) τ_j is (note that the second-order condition (s.o.c.) holds):

$$\begin{aligned}
-m_ja_i\left(\overline{p}_i+\tau_j\right)+a_im_j\overline{p}_i-m_j\left[a_i\left(\overline{p}_i+\tau_j\right)+\left(\overline{v}_{ji}-a_i\left(2\overline{p}_i+\tau_j\right)\right)\right]\frac{\partial\overline{p}_i}{\partial\tau_j} &= 0\Leftrightarrow\\
-a_i\left(\overline{p}_i+\tau_j\right)+a_i\overline{p}_i-\left[a_i\left(\overline{p}_i+\tau_j\right)+\left(\overline{v}_{ji}-a_i\left(2\overline{p}_i+\tau_j\right)\right)\right]\left(-\frac{m_j}{m}\right) &= 0\Leftrightarrow\\
-a_i\tau_j+\left(\overline{v}_{ji}-a_i\overline{p}_i\right)r_j &= 0\Leftrightarrow\\
-\tau_j+\left(\frac{\overline{v}_{ji}}{a_i}-\frac{\sum_l m_l\overline{v}_{li}-a_im_j\tau_j-X_i}{a_im}\right)r_j &= 0\Leftrightarrow \qquad (22)\\
\tau_j=\tau_j^{BR}\left(X_i\right)\equiv\frac{r_j}{a_i}\frac{\overline{v}_{ji}-\overline{v}_{Ai}+X_i/m}{1-r_j^2}. & \qquad (23)
\end{aligned}$$

When we combine (20) and (23), $\overline{p}_i$ becomes :

$$\overline{p}_i=\frac{\overline{v}_{Ai}}{a_i}-\frac{X_i}{a_im}-\frac{r_j^2}{a_i}\frac{\overline{v}_{ji}-\overline{v}_{Ai}+X_i/m}{1-r_j^2}=\frac{\overline{v}_{Ai}-r_j^2\overline{v}_{ji}-X_i/m}{a_i\left(1-r_j^2\right)}. \tag{24}$$

Equilibrium capacity. When investors are price-takers, and expect tariff τ_i, the market response is that investments increase as long as $\overline{p}_i\geq 0$. From (20):

$$X_i^{MR}\left(\tau_j\right)=\left(\overline{v}_{Ai}-a_ir_j\tau_j\right)m. \tag{25}$$

Expectations are rational, so the combination of (23) and (25) characterizes the BAU equilibrium in a country with private investments. For i =N, this gives:

$$\begin{aligned}
\tau_S^{BAU} &= \tau_S^{BR}\left(X_N^{BAU}\right)\equiv\frac{r_S}{a_N}\frac{\overline{v}_{SN}-\overline{v}_{AN}+\left(\overline{v}_{AN}-a_Nr_S\tau_S\right)}{1-r_S^2}\Leftrightarrow\\
\tau_S^{BAU} &= \frac{r_S\overline{v}_{SN}}{a_N}\text{ and } X_N^{BAU}=\left(\overline{v}_{AN}-r_S^2\overline{v}_{SN}\right)m. \qquad (26)
\end{aligned}$$

By assumption, S internalizes the effect on the price. S selects X_S to maximize S's surplus from the market for S's good, s_{SS}, which is the sum of S's consumer surplus from c_{SS} and S's profit:

$$\begin{aligned}
s_{SS} &= m_S\left[-\frac{\left(\overline{v}_{SS}-c_{SS}\right)^2}{2a_S}-\overline{p}_Sc_{SS}\right]+\overline{p}_SX_S \qquad (27)\\
&= m_Sa_S\overline{p}_S^2/2+\overline{p}_S\left(X_S-m_S\overline{v}_{SS}\right). \qquad (28)
\end{aligned}$$

[56] That is, if country $l\notin\{N,S\}$ has tariff τ_{li} on good i, l's demand is $\overline{v}_{li}-a_i\overline{p}_i$ if $\overline{v}_{il}\equiv v_{il}-\left(k_l+t_{il}+\tau_{li}\right)a_l$.

The f.o.c. of s_{SS} w.r.t. X_S is, given (20):

$$\overline{p}_S + (m_S a_S \overline{p}_S + X_S - m_S \overline{v}_{SS}) \left(\frac{\partial \overline{p}_S}{\partial X_S} \right) = 0 \Leftrightarrow \tag{29}$$

$$\overline{p}_S + (m_S a_S \overline{p}_S + X_S - m_S \overline{v}_{SS}) \left(-\frac{1}{m a_S} - \frac{m_N}{m} \frac{\partial \tau_N}{\partial X_S} \right) = 0. \tag{30}$$

If τ_N was fixed, $\partial \tau_N / \partial X_S = 0$, and (30) would simplify to:

$$\begin{aligned}
\overline{p}_S - \frac{m_S}{m}\overline{p}_S - \frac{X_S}{m a_S} + \frac{m_S \overline{v}_{SS}}{m a_S} &= 0 \Leftrightarrow \\
\frac{\sum_l m_l \overline{v}_{lS} - a_S m_N \tau_N - X_S}{m a_S} (1 - r_S) - \frac{X_S}{m a_S} + \frac{m_S \overline{v}_{SS}}{m a_S} &= 0 \Leftrightarrow \\
(2 - r_S)\, m_S \overline{v}_{SS} + (1 - r_S) \left(\sum_{l \neq i} m_l \overline{v}_{lS} - a_S m_N \tau_N \right) &= (2 - r_S)\, X_S \Leftrightarrow \\
m_S \overline{v}_{SS} + \frac{1 - r_S}{2 - r_S} m \left(\overline{v}_{-S} - a_S r_N \tau_N \right) &= X_S.
\end{aligned}$$

So, S's best response to a fixed τ_N would be:

$$X_S^{BR}(\tau_N) = m_S \overline{v}_{SS} + \frac{1 - r_S}{2 - r_S} m \left(\overline{v}_{-S} - a_S r_N \tau_N \right), \text{ and, with zero tariff:} \tag{31}$$

$$X_S^{FTA} \equiv X_S^{BR}(0) = m_S \overline{v}_{SS} + \frac{1 - r_S}{2 - r_S} m \overline{v}_{-S}. \tag{32}$$

N's tariff is endogenous with BAU. From (23), $\partial \tau_N / \partial X_S = r_N / m a_S \left(1 - r_N^2\right)$. Thus, (30) becomes:

$$\begin{aligned}
\overline{p}_S + (m_S a_S \overline{p}_S + X_S - m_S \overline{v}_{SS}) \left(-\frac{1}{m a_S} - \frac{m_N}{m} \frac{r_N}{m a_S (1 - r_N^2)} \right) &= 0 \Leftrightarrow \\
\left(1 - r_S - r_S \frac{r_N^2}{1 - r_N^2} \right) \overline{p}_S - \left(\frac{X_S - m_S \overline{v}_{SS}}{m a_S} \right) \left(1 + \frac{r_N^2}{1 - r_N^2} \right) &= 0 \Leftrightarrow \\
m a_S \left(1 - r_N^2 - r_S \right) \overline{p}_S - (X_S - m_S \overline{v}_{SS}) &= 0 \Leftrightarrow \\
m a_S \left(1 - r_N^2 - r_S \right) \left(\frac{\overline{v}_{AS} - r_N^2 \overline{v}_{NS} - X_S / m}{a_S (1 - r_N^2)} \right) - (X_S - m_S \overline{v}_{SS}) &= 0 \Leftrightarrow \\
\left(1 - r_N^2 - r_S \right) \left(\frac{\sum_l m_l \overline{v}_{lS} - m r_N^2 \overline{v}_{NS} - X_S}{1 - r_N^2} \right) - (X_S - m_S \overline{v}_{SS}) &= 0 \Leftrightarrow
\end{aligned} \tag{33}$$

$$\begin{aligned}
\left(1 - r_N^2 - r_S \right) \frac{\sum_{l \neq S} m_l \overline{v}_{lS} - m r_N^2 \overline{v}_{NS}}{1 - r_N^2} &= (X_S - m_S \overline{v}_{SS}) \left(1 + \frac{1 - r_N^2 - r_S}{1 - r_N^2} \right) \Leftrightarrow \\
\left(1 - r_N^2 - r_S \right) \left(\overline{v}_{-S} - r_N^2 \overline{v}_{NS} \right) m &= (X_S - m_S \overline{v}_{SS}) \left(2 - 2 r_N^2 - r_S \right) \Leftrightarrow
\end{aligned}$$

$$X_S^{BAU} = m_S \overline{v}_{SS} + \frac{1 - r_N^2 - r_S}{2 - 2 r_N^2 - r_S} \left(\overline{v}_{-S} - r_N^2 \overline{v}_{NS} \right) m. \tag{34}$$

S's payoffs. Combining (24) and (34), we can write:

$$\begin{aligned}
\overline{p}_S &= \frac{\overline{v}_{AS} - r_N^2 \overline{v}_{NS}}{a_S (1 - r_N^2)} - \frac{1 / m a_S}{1 - r_N^2} \left[m_S \overline{v}_{SS} + \frac{1 - r_N^2 - r_S}{2 - 2 r_N^2 - r_S} \left(\overline{v}_{-S} - r_N^2 \overline{v}_{NS} \right) m \right] \\
&= \frac{\overline{v}_{AS} - r_N^2 \overline{v}_{NS}}{a_S (1 - r_N^2)} - \frac{1 / a_S}{1 - r_N^2} \left[r_S \overline{v}_{SS} + \frac{1 - r_N^2 - r_S}{2 - 2 r_N^2 - r_S} \left(\overline{v}_{AS} - r_S \overline{v}_{SS} - r_N^2 \overline{v}_{NS} \right) \right] \\
&= \frac{\overline{v}_{AS} - r_N^2 \overline{v}_{NS} - r_S \overline{v}_{SS}}{a_S (2 - 2 r_N^2 - r_S)}.
\end{aligned} \tag{35}$$

And, from above, where (33) established,

$$X_S - m_S\overline{v}_{SS} = \left(1 - r_N^2 - r_S\right) m a_S \overline{p}_S,$$

we get from (28) and (35):

$$\begin{aligned} s_{SS}^{BAU} &= m_S a_S \overline{p}_S^2/2 + \left(1 - r_N^2 - r_S\right) m a_S \overline{p}_S^2 = \frac{m a_S}{2}\left(2 - 2r_N^2 - r_S\right)\left(\frac{\overline{v}_{AS} - r_N^2\overline{v}_{NS} - r_S\overline{v}_{SS}}{a_S\left(2 - 2r_N^2 - r_S\right)}\right)^2 \\ &= \frac{m}{2a_S}\frac{\left(\overline{v}_{AS} - r_N^2\overline{v}_{NS} - r_S\overline{v}_{SS}\right)^2}{2 - 2r_N^2 - r_S} = \frac{m}{2a_S}\frac{\left(\overline{v}_{-S} - r_N^2\overline{v}_{NS}\right)^2}{2 - 2r_N^2 - r_S}. \end{aligned} \tag{36}$$

When all foreign countries share the value $\overline{v}_{NS}$, s_{SS}^{BAU} becomes

$$\frac{m v_{NS}^2}{2a_S}\frac{\left(1 - r_S - r_N^2\right)^2}{2 - r_S - 2r_N^2}.$$

Equilibrium when $n = 1$. When $n = 1$,

$$s_{SS}^{BAU} = \frac{m\overline{v}_{NS}^2}{2a_S}\frac{\left(r_N - r_N^2\right)^2}{2 - 2r_N^2 - (1 - r_N)} = \frac{m\overline{v}_{NS}^2}{2a_S}\frac{r_N^2\left(1 - r_N\right)^2}{\left(1 - r_N\right)\left(1 + 2r_N\right)} = \frac{m\overline{v}_{NS}^2}{2a_S}\frac{r_N^2\left(1 - r_N\right)}{1 + 2r_N}, \tag{37}$$

and with A1:

$$s_{SS}^{BAU} = \frac{\overline{v}_{NS}^2}{16a_S}. \tag{38}$$

S's product. With (34), and $r_S = 1 - r_N$,

$$X_S^{BAU} = m_S\overline{v}_{SS} + \frac{1 - r_N^2 - (1 - r_N)}{2 - 2r_N^2 - (1 - r_N)}\left(r_N - r_N^2\right) m\overline{v}_{NS} = m_S\overline{v}_{SS} + \frac{r_N\left(1 - r_N\right)}{1 + 2r_N} m_N\overline{v}_{NS}.$$

The tariff (23) becomes

$$\begin{aligned} \tau_N^{BAU} &= \tau_N^{BR}\left(X_S^{BAU}\right) = \frac{r_N}{a_S}\frac{\overline{v}_{NS} - \overline{v}_{AS} + X_S/m}{1 - r_N^2} = \frac{r_N}{a_S}\frac{\overline{v}_{NS} - r_N\overline{v}_{NS} + \left(1 - r_N\right) r_N^2\overline{v}_{NS}/\left(1 + 2r_N\right)}{1 - r_N^2} \\ &= \frac{r_N}{a_S}\frac{\left(1 - r_N\right)\left(1 + 2r_N\right)\overline{v}_{NS} + \left(1 - r_N\right) r_N^2\overline{v}_{NS}}{\left(1 - r_N\right)\left(1 + r_N\right)\left(1 + 2r_N\right)} = \frac{1 + r_N}{1 + 2r_N}\frac{r_N\overline{v}_{NS}}{a_S}. \end{aligned}$$

From (35),

$$\begin{aligned} \overline{p}_S &= \frac{r_N\overline{v}_{NS} + \left(1 - r_N\right)\overline{v}_{SS} - r_N^2\overline{v}_{NS} - \left(1 - r_N\right)\overline{v}_{SS}}{a_S\left(2 - 2r_N^2 - \left(1 - r_N\right)\right)} = \frac{r_N\overline{v}_{NS}/a_S}{1 + 2r_N}, \text{ so} \qquad (39) \\ \overline{p}_S + \tau_N &= \frac{2 + r_N}{1 + 2r_N}\frac{r_N\overline{v}_{NS}}{a_S}. \qquad (40) \end{aligned}$$

N's product. (23) and (25) both holds when

$$\begin{aligned} \tau_S &= \tau_S^{BR}\left(X_N\right) \equiv r_S\frac{\left(\overline{v}_{SN} - \overline{v}_{AN}\right)/a_N + X_N/m a_N}{1 - r_S^2} \text{ and} \\ X_N &= X_N^{MR}\left(\tau_S\right) = m v_{AN} - a_N m_S\tau_S, \text{ so} \end{aligned}$$

$$\begin{aligned} X_N^{BAU} &= v_{AN} m - a_N m_S\left[\frac{r_S}{a_N}\frac{\overline{v}_{SN} - \overline{v}_{AN} + X_N^{BAU}/m}{1 - r_S^2}\right] \Leftrightarrow \\ X_N^{BAU} &= \left(1 - r_S^2\right) m v_{AN} - m_S r_S\left(\overline{v}_{SN} - \overline{v}_{AN}\right) = m v_{AN} - m r_S^2\overline{v}_{SN} \\ &= m\left(r_N\overline{v}_{NN} + r_S\overline{v}_{SN}\right) - m r_S^2\overline{v}_{SN} = m_N\overline{v}_{NN} + r_N m_S\overline{v}_{SN}, \text{ and} \end{aligned}$$

$$\tau_S^{BAU} = \frac{r_S}{a_N}\frac{\overline{v}_{SN} - \overline{v}_{AN} + X_N^{BAU}/m}{1-r_S^2}$$

$$= \frac{r_S}{a_N}\frac{(\overline{v}_{SN} - r_N\overline{v}_{NN} - r_S\overline{v}_{SN}) + (r_N\overline{v}_{NN} + r_S\overline{v}_{SN}) - r_S^2\overline{v}_{SN}}{1-r_S^2} = \frac{r_S\overline{v}_{SN}}{a_N}. \tag{41}$$

With A1, the equations simplify to Proposition 1.

PROOF OF PROPOSITION 2:

N's product. In equilibrium, $\overline{p}_N = 0$, and the payment for the product is simply a transfer from one country to the other. Thus, for every given τ_S, the total BAU surplus associated with N's product follows from (1), (2), and (19). When $n = 1$,

$$s_N(\tau_S) \equiv s_{NN} + s_{SN} = -\frac{m_N a_N}{2}\overline{p}_N^2 - \frac{m_S a_N}{2}(\overline{p}_N + \tau_S)^2 = -\frac{m_S a_N}{2}\tau_S^2, \tag{42}$$

when $\overline{p}_N = 0$. From (41), $\tau_S = \frac{r_S\overline{v}_{SN}}{a_N}$, so

$$s_N^{BAU} = -\frac{m_S a_N}{2}\left(\frac{r_S\overline{v}_{SN}}{a_N}\right)^2 = -\frac{m r_S^3\overline{v}_{SN}^2}{2a_N}.$$

Under the FTA, $\tau_S = 0$, and $s_N(\tau_S) = 0$, so:

$$s_N^{FTA} - s_N^{BAU} = \frac{m r_S^3\overline{v}_{SN}^2}{2a_N}, \text{ which is } \frac{\overline{v}_{SN}^2}{8a_N} \text{ under A1.}$$

S's product. From (21) and (27), (39), and (40), we find the total BAU surplus associated with S's product. When $n = 1$,

$$s_S^{BAU} \equiv s_{NS} + s_{SS} = -m_N a_S(\overline{p}_S + \tau_N)^2/2 - m_S a_S\overline{p}_S^2/2 \tag{43}$$

$$= -\frac{m_N a_S}{2}\left(\frac{2+r_N}{1+2r_N}\frac{r_N\overline{v}_{NS}}{a_S}\right)^2 - \frac{m_S a_S}{2}\left(\frac{r_N\overline{v}_{NS}/a_S}{1+2r_N}\right)^2$$

$$= -\frac{m}{2a_S}\left(\frac{r_N\overline{v}_{NS}}{1+2r_N}\right)^2\left(r_N(2+r_N)^2 + r_S\right) = -\frac{m}{2a_S}\left(\frac{r_N\overline{v}_{NS}}{1+2r_N}\right)^2\left(1+3r_N+4r_N^2+r_N^3\right).$$

Next, consider the surplus for any fixed τ_N. When $n = 1$, (31) becomes:

$$X_S^{BR}(\tau_N) = \frac{m(\overline{v}_{AS}(1-r_S) + r_S\overline{v}_{SS}) - a_S(1-r_S)m_N\tau_N}{2-r_S} = m_S\overline{v}_{SS} + \frac{m_N r_N\overline{v}_{NS}}{1+r_N} - \frac{a_S r_N m_N\tau_N}{1+r_N}.$$

From (20), we have:

$$\overline{p}_S = \frac{m_N\overline{v}_{NS} - a_S m_N\tau_N - (X_S - m_S\overline{v}_{SS})}{m a_S}$$

$$= \frac{m_N\overline{v}_{NS} - a_S m_N\tau_N - \left(\frac{m_N r_N\overline{v}_{NS}}{1+r_N} - \frac{a_S r_N m_N\tau_N}{1+r_N}\right)}{m a_S}$$

$$= \frac{m_N\overline{v}_{NS} - a_S m_N\tau_N}{(1+r_N)m a_S} = \frac{r_N}{1+r_N}\left(\frac{\overline{v}_{NS}}{a_S} - \tau_N\right) \tag{44}$$

$$\overline{p}_S + \tau_N = \frac{r_N\overline{v}_{NS}/a_S + \tau_N}{1+r_N}. \tag{45}$$

Substituting (44) and (45) into (43), we get:

$$s_S(\tau_N) = -\frac{m_N a_S}{2}\left(\frac{r_N\overline{v}_{NS}/a_S + \tau_N}{1+r_N}\right)^2 - \frac{m_S a_S}{2}\left(\frac{r_N}{1+r_N}\left(\frac{\overline{v}_{NS}}{a_S} - \tau_N\right)\right)^2$$

$$= -\frac{m a_S}{2}\left(\frac{r_N\overline{v}_{NS}/a_S}{1+r_N}\right)^2 - \frac{(m_N - m_S r_N)r_N\overline{v}_{NS}\tau_N}{(1+r_N)^2} - \frac{a_S(m_N + m_S r_N^2)}{2}\left(\frac{\tau_N}{1+r_N}\right)^2$$

$$= -\frac{m a_S}{2}\left(\frac{r_N\overline{v}_{NS}/a_S}{1+r_N}\right)^2 - \frac{m_N r_N^2\overline{v}_{NS}\tau_N}{(1+r_N)^2} - \frac{a_S m_N(1+r_S r_N)}{2}\left(\frac{\tau_N}{1+r_N}\right)^2. \tag{46}$$

With free trade,

$$s_S^{FTA} \equiv s_S(0) = -\frac{ma_S}{2}\left(\frac{r_N \overline{v}_{NS}/a_S}{1+r_N}\right)^2.$$

By comparison, the gains from liberalizing trade for S's good is:

$$\begin{aligned} s_S^{FTA} - s_S^{BAU} &= \frac{m}{2a_S}\left(\frac{r_N \overline{v}_{NS}}{1+2r_N}\right)^2 \left(r_N (2+r_N)^2 + (1-r_N)\right) - \frac{ma_S}{2}\left(\frac{r_N \overline{v}_{NS}/a_S}{1+r_N}\right)^2 \\ &= \frac{m (r_N \overline{v}_{NS})^2}{2a_S}\left(\frac{r_N (2+r_N)^2 + 1 - r_N}{(1+2r_N)^2} - \frac{1}{(1+r_N)^2}\right). \end{aligned}$$

If $r_N = 1/2$,

$$s_S^{FTA} - s_S^{BAU} = \frac{1}{8}\frac{133}{144}\frac{\overline{v}_{NS}^2}{a_S} = \frac{133}{1152}\frac{\overline{v}_{NS}^2}{a_S}.$$

PROOF OF PROPOSITION 3:

When the damage is taken into account, the ex ante socially optimal fixed τ_S solves:

$$\max_{\tau_S} s_N(\tau_S) - d_S\left(X_N^{MR}(\tau_S)\right),$$

where $s_N(\tau_S)$ is given by (42) when $n = 1$. With (25), the f.o.c. is (note that s.o.c. holds):

$$-m_S a_N \tau_S + d_S'(X_N)(a_N m_S) = 0 \Leftrightarrow \tau_S = d_S'(X_N). \tag{47}$$

Similarly, the optimal τ_N solves:

$$\max_{\tau_N} s_S(\tau_N) - d_N\left(X_S^{BR}(\tau_N)\right),$$

and with (31) and (46), the f.o.c. is (again, s.o.c. holds):

$$\begin{aligned} -\frac{m_N r_N^2 \overline{v}_{NS}}{(1+r_N)^2} - \frac{a_S m_N (1+r_S r_N)}{(1+r_N)^2}\tau_N - d_N'(X_S)\left(\frac{\partial X_S^{BR}}{\partial \tau_N}\right) &= 0 \Leftrightarrow \\ -\frac{m_N r_N^2 \overline{v}_{NS}}{(1+r_N)^2} - \frac{a_S m_N (1+r_S r_N)}{(1+r_N)^2}\tau_N - d_N'(X_S)\left(-\frac{a_S r_N m_N}{1+r_N}\right) &= 0 \Leftrightarrow \\ -m_N r_N^2 \overline{v}_{NS} + (1+r_N) a_S r_N m_N d_N'(X_S) &= a_S m_N (1+r_S r_N)\tau_N \Leftrightarrow \\ r_N \frac{(1+r_N) d_N'(X_S) - r_N \overline{v}_{NS}/a_S}{1+r_S r_N} &= \tau_N. \end{aligned}$$

PROOF OF PROPOSITION 4:

Without cross-contingency, it suffices to consider S's surplus from S's product, as a function of X_S and τ_N. When (20) is substituted in (27), and we define $Z \equiv X_S - m_S \overline{v}_{SS}$ and $Y_\tau \equiv \sum_{i \neq S} m_i \overline{v}_{iS} - a_S m_N \tau_N$, s_{SS} can be written as:

$$\begin{aligned} s_{SS}(X_S, \tau_N) &= \frac{m_S a_S}{2}\left(\frac{m v_{AS} - a_S m_N \tau_N - X_S}{m a_S}\right)^2 + \left(\frac{m v_{AS} - a_S m_N \tau_N - X_S}{m a_S}\right)(X_S - m_S \overline{v}_{SS}) \\ &= \frac{m_S a_S}{2}\left(\frac{Y_\tau - Z}{m a_S}\right)^2 + \frac{Y_\tau - Z}{m a_S} Z = \frac{1}{2 m a_S}\left(r_S (Y_\tau - Z)^2 + 2(Y_\tau - Z) Z\right). \end{aligned} \tag{48}$$

To implement X_S^{CTA}, we must have

$$s_{SS}\left(X_S^{CTA}, \tau_N^{CTA}\left(X_S^{CTA}\right)\right) \geq s_{SS}\left(X_S', \tau_N^{CTA}(X_S')\right) \quad \forall X_S' \geq X_S^0. \tag{49}$$

Three observations help us to simplify (49). (1) For the CTA to be credible, $\tau_N^{CTA}(X_S) \in \left[0, \tau_N^{BR}(X_S)\right]$. To implement X_S^{CTA}, $\tau_N^{CTA}\left(X_S^{CTA}\right) = 0$ is both ex post efficient, and it helps to satisfy (49), because S prefers the lowest tariff among the credible alternatives $\tau_N^{CTA}(X_S) \in \left[0, \tau_N^{BR}(X_S)\right]$. (2) The r.h.s. of (49) is smallest when $\tau_N^{CTA}(X'_S) \le \tau_N^{BR}(X'_S)$ binds. (3) When S considers a deviation X'_S accompanied with $\tau_N^{CTA}(X'_S) = \tau_N^{BR}(X'_S)$, the proof of Proposition 1 established that S prefers X_S^{BAU}, inducing $s_{SS}^{BAU} = s_{SS}\left(X_S^{BAU}, \tau_N^{BR}\left(X_S^{BAU}\right)\right)$, which is characterized already: see (36). With (1)-(3), (48), and $Y_0 \equiv mv_{-S}$, (49) can be simplified to:

$$\begin{aligned} \frac{1}{2ma_S}\left(r_S\left(Y_0 - Z\right)^2 + 2\left(Y_0 - Z\right)Z\right) &\ge s_{SS}^{BAU} \Leftrightarrow \\ Z^2\left(2 - r_S\right) - 2ZY_0\left(1 - r_S\right) - r_S Y_0^2 + 2ma_S s_{SS}^{BAU} &\le 0, \end{aligned} \tag{50}$$

which binds when:

$$Z = \frac{Y_0\left(1 - r_S\right)}{2 - r_S} \pm \frac{1}{2\left(2 - r_S\right)}\sqrt{\left[2Y_0\left(1 - r_S\right)\right]^2 - 4\left(2 - r_S\right)\left(2ma_S s_{SS}^{BAU} - r_S Y_0^2\right)}.$$

With $Y_0 \equiv mv_{-S}$, $X_S = m_S\overline{v}_{SS} + Z$, and (36), (50) require:

$$\begin{aligned} X_S &\in \left[\underline{X}, \overline{X}\right], \text{ where} \\ \underline{X} &\equiv m_S\overline{v}_{SS} + \frac{Y_0\left(1 - r_S\right)}{2 - r_S} - \frac{1/2}{2 - r_S}\sqrt{\left[2Y_0\left(1 - r_S\right)\right]^2 - 4\left(2 - r_S\right)\left(2ma_S s_{SS}^{BAU} - r_S Y_0^2\right)} \\ &= m\left[r_S\overline{v}_{SS} + \frac{1 - r_S}{2 - r_S}\overline{v}_{-S} - \frac{\overline{v}_{-S}}{2 - r_S}\sqrt{1 - \frac{2\left(2 - r_S\right)a_S}{m\overline{v}_{-S}^2}s_{SS}^{BAU}}\right] \\ &= m\left[r_S\overline{v}_{SS} + \frac{1 - r_S}{2 - r_S}\overline{v}_{-S} - \frac{\overline{v}_{-S}}{2 - r_S}\sqrt{1 - \frac{2\left(2 - r_S\right)a_S}{m\overline{v}_{-S}^2}\left[\frac{m}{2a_S}\frac{\left(\overline{v}_{-S} - r_N^2\overline{v}_{NS}\right)^2}{2 - 2r_N^2 - r_S}\right]}\right], \\ \overline{X} &\equiv m\left[r_S\overline{v}_{SS} + \frac{1 - r_S}{2 - r_S}\overline{v}_{-S} + \frac{\overline{v}_{-S}}{2 - r_S}\sqrt{1 - \frac{2\left(2 - r_S\right)a_S}{m\overline{v}_{-S}^2}s_{SS}^{BAU}}\right] \\ &= m\left[r_S\overline{v}_{SS} + \frac{1 - r_S}{2 - r_S}\overline{v}_{-S} + \frac{\overline{v}_{-S}}{2 - r_S}\sqrt{1 - \frac{2\left(2 - r_S\right)a_S}{m\overline{v}_{-S}^2}\left[\frac{m}{2a_S}\frac{\left(\overline{v}_{-S} - r_N^2\overline{v}_{NS}\right)^2}{2 - 2r_N^2 - r_S}\right]}\right]. \end{aligned} \tag{51}$$

If $n = 1$, then $r_N = 1 - r_S$ and $\overline{v}_{-S} = \left(1 - r_S\right)\overline{v}_{NS}$. With (37), (51) becomes:

$$\begin{aligned} \underline{X} &= m\left[r_S\overline{v}_{SS} + \frac{\left(1 - r_S\right)^2}{2 - r_S}\overline{v}_{NS} - \frac{\left(1 - r_S\right)}{2 - r_S}\overline{v}_{NS}\sqrt{1 - \frac{2\left(2 - r_S\right)a_S}{\left(1 - r_S\right)^2\overline{v}_{NS}^2 m}\left[s_{SS}^{BAU}\right]}\right] \\ &= m\left[r_S\overline{v}_{SS} + \frac{\left(1 - r_S\right)^2}{2 - r_S}\overline{v}_{NS} - \frac{\left(1 - r_S\right)}{2 - r_S}\overline{v}_{NS}\sqrt{1 - \frac{2\left(2 - r_S\right)a_S}{\left(1 - r_S\right)^2\overline{v}_{NS}^2 m}\left[\frac{m\overline{v}_{NS}^2}{2a_S}\frac{r_N^2\left(1 - r_N\right)}{1 + 2r_N}\right]}\right] \\ &= m\left[r_S\overline{v}_{SS} + \frac{\left(1 - r_S\right)^2}{2 - r_S}\overline{v}_{NS} - \frac{\left(1 - r_S\right)}{2 - r_S}\overline{v}_{NS}\sqrt{1 - \frac{\left(2 - r_S\right)r_S}{3 - 2r_S}}\right]. \end{aligned} \tag{52}$$

If also $m = 2$ and $r_S = 1/2$, as under A1, then, with (38):

$$\begin{aligned} \underline{X} &= \overline{v}_{SS} + \frac{\overline{v}_{NS}}{3} - \frac{2}{3}\overline{v}_{NS}\sqrt{1 - \frac{6a_S}{\overline{v}_{NS}^2}\left[s_{SS}^{BAU}\right]} \\ &= \overline{v}_{SS} + \frac{\overline{v}_{NS}}{3} - \frac{2}{3}\overline{v}_{NS}\sqrt{1 - \frac{6a_S}{\overline{v}_{NS}^2}\left[\frac{\overline{v}_{NS}^2}{16a_S}\right]} = \overline{v}_{SS} + \frac{\overline{v}_{NS}}{3} - \frac{\overline{v}_{NS}}{6}\sqrt{10} \approx \overline{v}_{SS} - 0.19\overline{v}_{NS}. \end{aligned} \tag{53}$$

Similarly,

$$\overline{X} = \overline{v}_{SS} + \overline{v}_{NS}\left(\frac{1}{3} + \frac{1}{6}\sqrt{10}\right) \approx \overline{v}_{SS} + 0.86\overline{v}_{NS}.$$

PROOF OF PROPOSITION 5:

With $\tau_S^{CTA}\left(X_S^{CTA}\right)$, $X_N = X_N^{MR}\left(\tau_S^E\right)$, where $\tau_S^E = \tau_S^{CTA}\left(X_S^{CTA}\right)$ is the equilibrium and expected tariff in S when the CTA implements X_S^{CTA}.

For S to prefer X_S^{CTA}, we must have:

$$\begin{aligned} s_{SS}\left(X_S^{CTA}, \tau_N^{CTA}\left(X_S^{CTA}\right)\right) + s_{SN}\left(X_N^{MR}\left(\tau_S^E\right), \tau_S^E\right) &\geq \\ s_{SS}\left(X_S', \tau_N\left(X_S'\right)\right) + s_{SN}\left(X_N^{MR}\left(\tau_S^E\right), \tau_S^{CTA}\left(X_S'\right)\right) \quad \forall X_S' &\geq X_S^0. \end{aligned} \tag{54}$$

Four observations help to simplify (54): (1) For the CTA to be credible, $\tau_N^{CTA}(X_S) \in \left[0, \tau_N^{BR}(X_S)\right]$ and $\tau_S^{CTA}(X_S) \in \left[0, \tau_S^{BR}(X_N)\right]$. (2) The right-hand side (r.h.s.) of (54) is most likely to hold when $\tau_N^{CTA}(X_S) \leq \tau_N^{BR}(X_S)$ binds. (3) The r.h.s. of (54) is most likely to hold when $\tau_S^{CTA}(X_S) = 0$. (4) The most attractive deviation for S is X_S^{BAU}. Given (1)-(4), (54) simplifies to:

$$\begin{aligned} s_{SS}(X_S, 0) &\geq s_{SS}^{BAU} - \Delta_{SN}\left(\tau_S^E\right) \text{ , where} \\ \Delta_{SN}\left(\tau_S^E\right) &\equiv s_{SN}\left(X_N^{MR}\left(\tau_S^E\right), \tau_S^E\right) - s_{SN}\left(X_N^{MR}\left(\tau_S^E\right), 0\right). \end{aligned} \tag{55}$$

LEMMA 2: *We have*

$$\Delta_{SN}\left(\tau_S^E\right) = m_S r_S \overline{v}_{SN} \tau_S^E - \frac{m_S a_N}{2}\left(1 + r_S^2\right)\left(\tau_S^E\right)^2.$$

Proof. To derive $s_{SN}(X_N, \tau_S)$, note that with expected τ_S^E, (25) gives:

$$X_N^{MR}\left(\tau_S^E\right) = \left(\overline{v}_{AN} - a_N r_S \tau_S^E\right) m.$$

When this $X_N^{MR}\left(\tau_S^E\right)$ is combined with (20) for i =N, we get:

$$\overline{p}_N = r_S\left(\tau_S^E - \tau_S\right) \text{ and } \overline{p}_N + \tau_S = r_S \tau_S^E + (1 - r_S)\tau_S.$$

Thus, S's consumer surplus from N's product, plus S's tariff revenues, is:

$$\begin{aligned} & s_{SN}\left(X_N^{MR}\left(\tau_S^E\right), \tau_S\right) = -m_S a_N \left(\overline{p}_N + \tau_S\right)^2/2 - m_S \overline{p}_N\left(\overline{v}_{SN} - a_N\left(\overline{p}_N + \tau_S\right)\right) \\ = & -\frac{m_S a_N}{2}\left(r_S \tau_S^E + (1 - r_S)\tau_S\right)^2 - m_S r_S\left(\tau_S^E - \tau_S\right)\left[\overline{v}_{SN} - a_N\left(r_S \tau_S^E + (1 - r_S)\tau_S\right)\right]. \end{aligned}$$

With this, we can derive $\Delta_{SN}\left(\tau_S^E\right)$. It becomes:

$$\begin{aligned} & -\frac{m_S a_N}{2}\left(r_S \tau_S^E + (1 - r_S)\tau_S^E\right)^2 - m_S r_S\left(\tau_S^E - \tau_S^E\right)\left[\overline{v}_{SN} - a_N\left(r_S \tau_S^E + (1 - r_S)\tau_S^E\right)\right] \\ & + \frac{m_S a_N}{2}\left(r_S \tau_S^E + (1 - r_S)0\right)^2 + m_S r_S\left(\tau_S^E - 0\right)\left[\overline{v}_{SN} - a_N\left(r_S \tau_S^E + (1 - r_S)0\right)\right] \\ = & \ m_S r_S \tau_S^E\left[\overline{v}_{SN} - a_N r_S \tau_S^E\right] - \frac{m_S a_N}{2}\left(\tau_S^E\right)^2\left(1 - r_S^2\right) \Leftrightarrow \end{aligned}$$

$$\Delta_{SN}\left(\tau_S^E\right) = m_S r_S \tau_S^E \overline{v}_{SN} - \frac{m_S a_N}{2}\left(\tau_S^E\right)^2\left(1 + r_S^2\right), \tag{56}$$

which is positive if and only if

$$\tau_S^E \in \left[0, \overline{\tau}_S^E\right], \text{ where } \overline{\tau}_S^E \equiv \frac{2 r_S \overline{v}_{SN}/a_N}{1 + r_S^2} > \tau_S^{BAU},$$

according to (26). For every tariff that is ex post credible, $\Delta_{SN}\left(\tau_S^E\right) \geq 0$. ‖

The rest of the proof of Proposition 4 continues to hold if $s_{SS}^{BAU} - \Delta_{SN}\left(\tau_S^E\right)$ replaced s_{SS}^{BAU} in (51).

Note that $\Delta_{SN}\left(\tau_S^E\right)$ is maximized at:

$$\begin{aligned} \tau_S^M &= \frac{\overline{\tau}_S^E}{2} = \frac{r_S \overline{v}_{SN}/a_N}{1 + r_S^2} \Rightarrow \\ \overline{\Delta}_{SN} &\equiv \Delta_{SN}\left(\tau_S^M\right) = m_S \frac{r_S \overline{v}_{SN}/a_N}{1 + r_S^2}\left(\overline{v}_{SN} r_S - \frac{a_N}{2}\left(\frac{r_S \overline{v}_{SN}/a_N}{1 + r_S^2}\right)\left(1 + r_S^2\right)\right) \\ &= m_S \frac{\left(r_S \overline{v}_{SN}\right)^2/2a_N}{1 + r_S^2}. \end{aligned} \tag{57}$$

At τ_S^M, (55) becomes:

$$s_{SS}(X_S,0) \geq s_{SS}^{BAU} - m_S \frac{(r_S\overline{v}_{SN})^2/2a_N}{1+r_S^2}.$$

More generally: Given that $\underline{X}$ increases, and $\overline{X}$ decreases, in s_{SS}^{BAU}, and $\Delta_{SN}(\tau_S^E)$ increases in $\tau_S^E \in [0,\tau_S^M)$, when s_{SS}^{BAU} is replaced by $s_{SS}^{BAU} - \Delta_{SN}(\tau_S^E)$ it follows that $\underline{X}$ decreases, and $\overline{X}$ increases, in $\tau_S^E \in [0,\tau_S^M)$. Combined with (52), we get:

$$\begin{aligned}
\underline{X}^M &= m\left[r_S\overline{v}_{SS} + \frac{(1-r_S)^2}{2-r_S}\overline{v}_{NS} - \frac{(1-r_S)}{2-r_S}\overline{v}_{NS}\sqrt{1-\frac{2(2-r_S)a_S}{(1-r_S)^2\overline{v}_{NS}^2 m}\left[s_{SS}^{BAU}-\overline{\Delta}_{SN}\right]}\right] \\
&= m\left[r_S\overline{v}_{SS} + \frac{(1-r_S)^2}{2-r_S}\overline{v}_{NS} - \frac{(1-r_S)}{2-r_S}\overline{v}_{NS}\sqrt{1-\frac{(2-r_S)r_S}{3-2r_S}+\left(\frac{2(2-r_S)a_S}{(1-r_S)^2\overline{v}_{NS}^2 m}\right)\left(m_S\frac{(r_S\overline{v}_{SN})^2}{2a_N(1+r_S^2)}\right)}\right] \\
&= m\left[r_S\overline{v}_{SS} + \frac{(1-r_S)^2}{2-r_S}\overline{v}_{NS} - \frac{(1-r_S)}{2-r_S}\overline{v}_{NS}\sqrt{1-\frac{(2-r_S)r_S}{3-2r_S}+\frac{a_S}{a_N}\left(\frac{\overline{v}_{SN}}{\overline{v}_{NS}}\right)^2\frac{2-r_S}{(1-r_S)^2}\frac{r_S^3}{1+r_S^2}}\right]. \qquad (58)
\end{aligned}$$

With A1, $r_S = 1/2$ and $m = 2$, so (56) becomes:

$$\Delta_{SN}(\tau_S^E) = \left(\frac{\overline{v}_{SN}}{2} - \frac{5}{8}a_N\tau_S^E\right)\tau_S^E, \text{ so } \tau_S^M = \frac{2}{5}\frac{\overline{v}_{SN}}{a_N} \text{ and } \overline{\Delta}_{SN} = \frac{\overline{v}_{SN}^2}{10a_N}.$$

Combined with (53), we now get:

$$\begin{aligned}
\underline{X}(\tau_S^E) &= \overline{v}_{SS} + \frac{\overline{v}_{NS}}{3} - \frac{2}{3}\overline{v}_{NS}\sqrt{1-\frac{6a_S}{\overline{v}_{NS}^2}\left[s_{SS}^{BAU}-\Delta_{SN}(\tau_S^E)\right]} \\
&= \overline{v}_{SS} + \frac{\overline{v}_{NS}}{3} - \frac{2}{3}\overline{v}_{NS}\sqrt{1-\frac{6a_S}{\overline{v}_{NS}^2}\left[\frac{\overline{v}_{NS}^2}{16a_S}-\Delta_{SN}(\tau_S^E)\right]} \\
&= \overline{v}_{SS} + \frac{\overline{v}_{NS}}{3} - \frac{2\overline{v}_{NS}}{3}\sqrt{\frac{10}{16}+\frac{6a_S}{\overline{v}_{NS}^2}\left(\frac{\overline{v}_{SN}}{2}-\frac{5}{8}a_N\tau_S^E\right)\tau_S^E} \\
&= \overline{v}_{SS} - \frac{\overline{v}_{NS}}{3}\left(\sqrt{\frac{5}{2}+\frac{a_S}{\overline{v}_{NS}^2}\left(12\overline{v}_{SN}-15a_N\tau_S^E\right)\tau_S^E}-1\right).
\end{aligned}$$

And:

$$\underline{X}^M = \overline{v}_{SS} + \frac{\overline{v}_{NS}}{3} - \frac{2\overline{v}_{NS}}{3}\sqrt{\frac{10}{16}+\frac{6a_S}{\overline{v}_{NS}^2}\frac{\overline{v}_{SN}^2}{10a_N}} = \overline{v}_{SS} - \frac{\overline{v}_{NS}}{3}\left(\sqrt{\frac{5}{2}+\frac{12}{5}\frac{a_S}{a_N}\frac{\overline{v}_{SN}^2}{\overline{v}_{NS}^2}}-1\right).$$

The derivations of $\overline{X}(\tau_S^E)$ and $\overline{X}^M$ are analogous.

PROPOSITION 6-A: *If $X_S^{FB} \in [\underline{X}(\tau_S^*), \overline{X}(\tau_S^*)]$, it is optimal with $\tau_S^{CTA} = \tau_S^*$, and the CTA implements X_S^{FB}, $\tau_N = 0$, and $X_N^{MR}(\tau_S^*)$. If $X_S^{FB} \notin [\underline{X}(\tau_S^*), \overline{X}(\tau_S^*)]$, there are five different cases to consider:*
(i) Suppose $X_S^{FB} > \overline{X}(\tau_S^*)$ and $\tau_S^* < \tau_S^M$. With the optimal CTA, $\tau_S^E \in (\tau_S^*, \tau_S^M)$ and $X_S \in (\underline{X}(\tau_S^*), X_S^{FB})$. If either d_N' decreases or d_S' increases, the optimal τ_S increases, and X_N decreases while X_S increases.
(ii) Suppose $X_S^{FB} > \overline{X}(\tau_S^*)$ and $\tau_S^* > \tau_S^M$. With the optimal CTA, $\tau_S^E \in (\tau_S^M, \tau_S^*)$ and $X_S \in (\underline{X}(\tau_S^*), X_S^{FB})$. If either d_N' or d_S' increases, the optimal τ_S increases, and both X_N and X_S decrease.
(iii) Suppose $X_S^{FB} < \underline{X}(\tau_S^*)$ and $\tau_S^* < \tau_S^M$. With the optimal CTA, $\tau_S^E \in (\tau_S^*, \tau_S^M)$ and $X_S \in (X_S^{FB}, \underline{X}_S)$. If either d_N' or d_S' increases, the optimal τ_S increases, and both X_N and X_S decrease.
(iv) Suppose $X_S^{FB} < \underline{X}(\tau_S^*)$ and $\tau_S^* > \tau_S^M$. With the optimal CTA, $\tau_S^E \in (\tau_S^M, \tau_S^*)$ and $X_S \in (X_S^{FB}, \underline{X}_S)$. If either d_N' decreases or d_S' increases, the optimal τ_S increases, and X_N decreases while X_S increases.
(v) If $\tau_S^* = \tau_S^M$, the optimal CTA ensures that $\tau_S^E = \tau_S^*$ and $X_S = \underline{X}(\tau_S^*)$ if $X_S^{FB} < \underline{X}(\tau_S^*)$, while $X_S = \overline{X}(\tau_S^*)$ if $X_S^{FB} > \overline{X}(\tau_S^*)$.

Proof. Generalizing (55), we have

$$s_{SS}(X_S, 0) \geq s_{SS}^{BAU} - \Delta_{SN}\left(\tau_S^E\right),$$

and combined with (51) and (56), we find that the smallest implementable X_S is a function of the tariff S that is permitted by the CTA:

$$\underline{X}\left(\tau_S^E\right) = m\left[r_S\overline{v}_{SS} + \frac{1-r_S}{2-r_S}\overline{v}_{-S} - \frac{\overline{v}_{-S}}{2-r_S}\sqrt{(1-r_S)^2 + (2-r_S)\left(r_S - \frac{2ma_S}{(mv_{-S})^2}\left(s_{SS}^{BAU} - \Delta_{SN}\left(\tau_S^E\right)\right)\right)}\right].$$

Note that $\underline{X}\left(\tau_S^E\right)$ is decreasing and convex in $\Delta_{SN}\left(\tau_S^E\right)$. The derivative of $\underline{X}\left(\tau_S^E\right)$ w.r.t. τ_S^E is:

$$\begin{aligned}
\frac{\partial\underline{X}\left(\tau_S^E\right)}{\partial\tau_S^E} &= m\frac{\overline{v}_{-S}}{2-r_S}\frac{-2ma_S(2-r_S)/(mv_{-S})^2}{2\sqrt{(1-r_S)^2+(2-r_S)\left(r_S-\frac{2ma_S}{(mv_{-S})^2}\left(s_{SS}^{BAU}-\Delta\left(\tau_S^E\right)\right)\right)}}\frac{\partial\Delta\left(\tau_S^E\right)}{\partial\tau_S^E} \\
&= \frac{-a_S}{\overline{v}_{-S}\sqrt{(1-r_S)^2+(2-r_S)\left(r_S-\frac{2ma_S}{(mv_{-S})^2}\left(s_{SS}^{BAU}-\Delta\left(\tau_S^E\right)\right)\right)}}\frac{\partial\Delta\left(\tau_S^E\right)}{\partial\tau_S^E}.
\end{aligned}$$

From (56), which is concave, we see

$$\frac{\partial\Delta\left(\tau_S^E\right)}{\partial\tau_S^E} = mr_S\left[r_S\left(\overline{v}_{SN} - 2a_Nr_S\tau_S^E\right) - a_Nr_N(r_N+2r_S)\tau_S^E\right],$$

which is positive for small τ_S^E, but decreases in τ_S^E. Combining the two equations above,

$$\frac{\partial\underline{X}\left(\tau_S^E\right)}{\partial\tau_S^E} = \frac{-a_Smr_S\left[r_S\left(\overline{v}_{SN}-2a_Nr_S\tau_S^E\right)-a_Nr_N(r_N+2r_S)\tau_S^E\right]}{\overline{v}_{-S}\sqrt{(1-r_S)^2+(2-r_S)\left(r_S-\frac{2ma_S}{(mv_{-S})^2}\left(s_{SS}^{BAU}-\Delta\left(\tau_S^E\right)\right)\right)}}.$$

Similarly, we have:

$$\frac{\partial\overline{X}\left(\tau_S^E\right)}{\partial\tau_S^E} = -\frac{\partial\underline{X}\left(\tau_S^E\right)}{\partial\tau_S^E} = \frac{a_Smr_S\left[r_S\left(\overline{v}_{SN}-2a_Nr_S\tau_S^E\right)-a_Nr_N(r_N+2r_S)\tau_S^E\right]}{\overline{v}_{-S}\sqrt{(1-r_S)^2+(2-r_S)\left(r_S-\frac{2ma_S}{(mv_{-S})^2}\left(s_{SS}^{BAU}-\Delta\left(\tau_S^E\right)\right)\right)}}.$$

When $X_S^{FB} \in \left[\underline{X}(\tau_S^*), \overline{X}(\tau_S^*)\right]$, X_S^{FB} is implemented with $\tau_N = 0$ and $\tau_S = \tau_S^*$ on the equilibrium path.

If, instead, $\overline{X}(\tau_S^*) < X_S^{FB}$, the best CTA ensures $X_S = \overline{X}\left(\tau_S^E\right)$ for the τ_S^E maximizing the sum of payoffs. When $X_S^{FB} < \underline{X}(\tau_S^*)$, the best CTA ensures $X_S = \underline{X}\left(\tau_S^E\right)$ for the τ_S^E maximizing the sum of payoffs. S's tariff influences four parts of the total payoffs:

$$s_S(X_S,0) - d_N(X_S) + s_N\left(X_N^{BR}\left(\tau_S^E\right), \tau_S^E\right) - d_S\left(X_N^{BR}\left(\tau_S^E\right)\right).$$

The f.o.c. can be written as:

$$\left[d_N'(X_S) - \frac{\partial s_S(X_S,0)}{\partial X_S}\right]\left(-\frac{\partial X_S}{\partial\tau_S^E}\right) + \frac{\partial s_N\left(\tau_S^E\right)}{\partial\tau_S^E} + d_S'\left(X_N^{BR}\left(\tau_S^E\right)\right)\left(-\frac{\partial X_N^{BR}\left(\tau_S^E\right)}{\partial\tau_S^E}\right) = 0, \tag{59}$$

where each of the three terms is decreasing in τ_S^E.[57] The f.o.c. is thus sufficient, and it pins down τ_S^E to be strictly between τ_S^M (which makes the first term equal to zero) and τ_S^* (which makes the second two terms equal to zero). With (20), (43), and (47), (59) can be written as:

$$\left[d_N'(X_S) - \frac{m_N\overline{v}_{NS} + m_S\overline{v}_{SS} - X_S}{ma_A}\right]\left(-\frac{\partial X_S}{\partial\tau_S^E}\right) + \left[d_S'\left(X_N^{BR}\left(\tau_S^E\right)\right)(a_Nm_S) - m_Sa_N\tau_S^E\right] = 0. \tag{60}$$

[57] It is easy to verify that both the second and the third term decreases in t_S^E. For the first term, one must check that it decreases in t_S^E of each of the four cases (i)-(iv), discussed below. I have omitted the explicit discussion of these checks for brevity, but will provide it upon requests.

Because s.o.c. holds, the left-hand side (l.h.s.) of (59), and of (60), decreases in τ_S^E. Because it also increases in d_S', it follows that if d_S' increases, then τ_S^E must increase for (60) to continue to hold, and then $X_N = X_N^{BR}(\tau_S)$ decreases. For other comparative statics, we must distinguish four possibilities.

(i) Suppose $\tau_S^* \equiv d_S' < \tau_S^M$ and $X_S^{FB} > \overline{X}(\tau_S^M)$. Then, (60) holds when $\tau_S \in (\tau_S^*, \tau_S^M)$, and $X_S \in (\overline{X}(\tau_S^M), X_S^{FB})$, so that the first bracket is negative, $-\partial X_S/\partial \tau_S^E = -\partial \overline{X}(\tau_S^E)/\partial \tau_S^E < 0$, and the second bracket is negative as before. A larger τ_S will then increase $X_S = \overline{X}(\tau_S^E)$. A larger d_N' reduces the first term, so τ_S must decrease, which incrases X_N and reduces $X_S = \overline{X}(\tau_S^E)$.

(ii) Suppose $\tau_S^* > \tau_S^M$ and $X_S^{FB} > \overline{X}(\tau_S^*)$. Then, (60) holds when $\tau_S \in (\tau_S^M, \tau_S^*)$, implying $X_S \in (\overline{X}(\tau_S^M), X_S^{FB})$, so that the first bracket is negative, but $-\partial X_S/\partial \tau_S^E = -\partial \overline{X}(\tau_S^E)/\partial \tau_S^E > 0$, while the second bracket is positive. A larger τ_S^E will then reduce $X_S = \overline{X}(\tau_S^E)$. A larger d_N' increases the first bracket, the l.h.s. increases, so τ_S^E must increase, which decreases both X_N and $X_S = \underline{X}(\tau_S^E)$.

(iii) Suppose $\tau_S^* < \tau_S^M$ and $X_S^{FB} < \underline{X}(\tau_S^M)$. Then, (60) holds when $\tau_S \in (\tau_S^*, \tau_S^M)$, implying $X_S \in (X_S^{FB}, \underline{X}^M)$, so that the first bracket is positive, $-\partial \underline{X}(\tau_S^E)/\partial \tau_S^E > 0$, and the second bracket is negative. A larger τ_S will then reduce $X_S = \underline{X}(\tau_S^E)$. A larger d_N' increases the first term, so τ_S must increase, which reduces X_N and $X_S = \underline{X}(\tau_S^E)$.

(iv) Suppose $\tau_S^* > \tau_S^M$ and $X_S^{FB} < \underline{X}(\tau_S^*)$. Then, (60) holds when $\tau_S \in (\tau_S^M, \tau_S^*)$, implying $X_S \in (X_S^{FB}, \underline{X}(\tau_S^M))$, so that the first bracket is positive, but $-\partial \underline{X}(\tau_S^E)/\partial \tau_S^E < 0$, while the second bracket is positive. A larger τ_S will then increase $X_S = \underline{X}(\tau_S^E)$. A larger d_N' increases the first bracket, the l.h.s. decreases, so τ_S must decrease, which increases X_N and decreases $X_S = \underline{X}(\tau_S^E)$.

(v) In the knife-edge case in which $\tau_S^* = \tau_S^M$, then no other tariff than $\tau_S^* = \tau_S^M$ can increase $\overline{X}(\tau_S^E)$ or decrease $\underline{X}(\tau_S^E)$.

PROOF OF PROPOSITION 7:

In addition to the agricultural capacity X_S, invested in by S's government, suppose private investors in S invest to produce X_J in sector $J \in \{1, ..., q_S\}$. Similarly, private investors in N invest to produce X_K in sector $K \in \{1, ..., q_N\}$. For each J, the tariff in N is measured by τ_{NJ}, and for each K, the tariff in S is τ_{SK}.

The surplus for S from sector J is denoted by $s_{SJ}(X_J, \tau_{NJ})$. (As before, this surplus equals the country's consumer surplus and the producers' revenues minus the cost of investing and producing.) Given the market response to the tariffs, equilibrium capacity is $X_J^E = X_J^{MR}(\tau_{NJ}^E)$, where $\tau_{NJ}^E = \tau_{NJ}^{CTA}(X_S^{CTA})$ is the equilibrium and expected tariff when X_S takes its equilibrium value under the CTA. Define X_K^E in the equivalent way.

Under the CTA, the tariffs can be functions (with superscripts CTA) of X_S. For the CTA to implement X_S^{CTA}, the following incentive constraint is analogous to (49):

$$\begin{aligned} s_{SS}(X_S^{CTA}, \tau_{NJ}^E) + \sum_J s_{SJ}(X_J^E, \tau_{NJ}^E) + \sum_K s_{SK}(X_K^E, \tau_{SK}^E) &\geq \quad (61) \\ s_{SS}(X_S', \tau_{NS}^{CTA}(X_S')) + \sum_J s_{SJ}(X_J^E, \tau_{NJ}^{CTA}(X_S')) + \sum_K s_{SK}(X_K^E, \tau_{SK}^{CTA}(X_S'))\ \forall X_S' &\geq X_S^0. \end{aligned}$$

In equilibrium, investors expect that S selects X_S^{CTA} when they invest in X_J or X_K.

As before, four observations help to simplify (61): (1) For the CTA to be credible, $\tau_{NJ} \in [0, \tau_{NJ}^{BR}(X_J)]$ and $\tau_{SK} \in [0, \tau_{SK}^{BR}(X_K)]$, just as before. As in the proof of Proposition 4, $\tau_{NJ}^E = 0$ is both ex post efficient and it helps to satisfy (61). (2) Inequality (61) can hold if and only if it holds at $\tau_{NJ}^{CTA}(X_S) = \tau_{NJ}^{BR}(X_J)$, when $X_S \neq X_S^{CTA}$, because, ex post, S likes this large tariff the least, among all the credible tariffs $\tau_{NJ} \in [0, \tau_{NJ}^{BR}(X_J)]$. (3) As in the proof of Proposition 5, a deviation from X_S^{CTA} is punished the most if it leads to $\tau_{SK} = 0$. (4) As before, S's best deviation X_S is X_S^{BAU} (i.e., (62) is hardest to satisfy when $X_S = X_S^{BAU}$). With (1)-(4), (61) simplifies to:

$$
\begin{aligned}
s_{SS}\left(X_S^{CTA},0\right) &\geq s_{SS}^{BAU}-\sum_J \Delta_{SJ}-\sum_K \Delta_{SK}\left(\tau_{SK}^E\right) \qquad (62)\\
&= s_{SS}\left(X_S',\tau_{NS}^{CTA}\left(X_S'\right)\right)-\sum_J \Delta_{SJ}-\sum_K \Delta_{SK}\left(\tau_{SK}^E\right)\ \forall X_S'\geq X_S^0, \text{ where}\\
\Delta_{SJ} &\equiv s_{SJ}\left(X_J^{MR}(0),0\right)-s_{SJ}\left(X_J^{MR}(0),\tau_{NJ}^{BR}\left(X_J^{MR}(0)\right)\right) \text{ and}\\
\Delta_{SK}\left(\tau_{SK}^E\right) &\equiv s_{SK}\left(X_K^{MR}\left(\tau_{SK}^E\right),\tau_{SK}^E\right)-s_{SK}\left(X_K^{MR}\left(\tau_{SK}^E\right),0\right).
\end{aligned}
$$

LEMMA 3:

$$
\begin{aligned}
\text{(a)} \qquad \Delta_{SJ} &\equiv \left(\frac{r_N^2}{a_J}\frac{\overline{v}_{NJ}}{1-r_N^2}\right)\left(\sum_{i\backslash S} m_i\overline{v}_{iJ}-\frac{m_S}{2}\left(\frac{r_N^2\overline{v}_{NJ}}{1-r_N^2}\right)\right),\\
\text{(b)} \quad \Delta_{SK}\left(\tau_{SK}^E\right) &\equiv m_S r_S\overline{v}_{SN}\tau_S^E-\frac{m_S a_N}{2}\left(1+r_S^2\right)\left(\tau_S^E\right)^2, \textit{ so}\\
\overline{\Delta}_{SK} &\equiv \max_{\tau_{SK}^E}\Delta_{SK}\left(\tau_{SK}^E\right)=m_S\frac{\left(r_S\overline{v}_{SK}\right)^2/2a_K}{1+r_S^2}.
\end{aligned}
$$

With $n=2$, (a) becomes:

$$
\Delta_{SJ}=m_N\frac{\overline{v}_{NJ}^2}{2a_J}\left(\frac{1}{1/r_N^2-1}\right)\left(\frac{2+r_N}{1+r_N}\right).
$$

In Example 1,

$$
\Delta_{SJ}=\frac{5}{18} \text{ and } \overline{\Delta}_{SK}=\frac{1}{10}.
$$

Proof. (a) To derive country S's surplus from S's sector J, $s_{SJ}\left(X_J,\tau_{NJ}\right)$, note that, analogously to (20), we have:

$$
\overline{p}_J=\frac{\sum m_i\overline{v}_{iJ}-a_J m_N\tau_{NJ}-X_J}{ma_J} \text{ and } \overline{p}_J+\tau_{NJ}=\frac{\sum m_i\overline{v}_{iJ}+a_J m_S\tau_{NJ}-X_J}{ma_J}.
$$

With $X_J^E=X_J^{MR}(0)=\sum m_i\overline{v}_{iJ}$, we find that $s_{SJ}\left(X_J^E,\tau_{NJ}\right)$ is:

$$
\begin{aligned}
&-m_S a_J\left(\overline{p}_J\right)^2/2+\overline{p}_J\left(\sum_{i\backslash S} m_i\left(\overline{v}_{iJ}-a_J\overline{p}_J\right)-m_N a_J\tau_{NJ}\right)\\
=\ & -\frac{m_S a_J}{2}\left(\frac{\sum m_i\overline{v}_{iJ}-a_J m_N\tau_{NJ}-X_J^E}{ma_J}\right)^2\\
&+\left(\frac{\sum m_i\overline{v}_{iJ}-a_J m_N\tau_{NJ}-X_J^E}{ma_J}\right)\left(\sum_{i\backslash S} m_i\left(\overline{v}_{iJ}-\frac{\sum m_i\overline{v}_{iJ}-a_J m_N\tau_{NJ}-X_J^E}{m}\right)-m_N a_J\tau_{NJ}\right)\\
=\ & -\frac{m_S a_J}{2}\left(r_N\tau_{NJ}\right)^2-\left(r_N\tau_{NJ}\right)\left(\sum_{i\backslash S} m_i\overline{v}_{iJ}+\sum_{i\backslash S} m_i a_J r_N\tau_{NJ}-m_N a_J\tau_{NJ}\right)\\
=\ & -a_J\left(\frac{m_S}{2}+\sum_{i\backslash S} m_i-m\right)\left(r_N\tau_{NJ}\right)^2-r_N\tau_{NJ}\sum_{i\backslash S} m_i\overline{v}_{iJ}=\frac{a_J m_S}{2}\left(r_N\tau_{NJ}\right)^2-r_N\tau_{NJ}\sum_{i\backslash S} m_i\overline{v}_{iJ}.
\end{aligned}
$$

which decreases in τ_{NJ} as long as $\tau_{NJ}\in[0,\widehat{\tau}_{NJ}]$, where $\widehat{\tau}_{NJ}\equiv\sum_{i\backslash S} r_i\overline{v}_{iJ}/a_J r_S r_N$. I will now show that, when the CTA is renegotiation proof, in that $\tau_{NJ}\in\left[0,\tau_{NJ}^{BR}\left(X_J\right)\right]$, then $\tau_{NJ}<\widehat{\tau}_{NJ}$. For a renegotiation proof CTA, when τ_{NJ} can be a function of X_S, $\tau_{NJ}\leq\tau_{NJ}^{BR}\left(X_J\right)$, given that $X_J=X_J^{MR}(0)$ when no tariff is expected in N in equilibrium. From (23),

$$
\tau_{NJ}^{BR}\left(X_J^{MR}(0)\right)\equiv\frac{r_N}{a_J}\frac{\overline{v}_{NJ}-\overline{v}_{AJ}+X_J^{MR}(0)/m}{1-r_N^2}=\frac{r_N}{a_J}\frac{\overline{v}_{NJ}}{1-r_N^2},
$$

which is smaller than $\widehat{\tau}_{NJ}$. This confirms that renegotiation-proof Δ_{SJ} is maximized at $\tau_{NJ}^{BR}\left(X_J^{BR}(0)\right)$. With this (out-of-equilibrium) tariff in N, $s_{SJ}\left(X_J^E, \tau_{NJ}\right)$ is:

$$\begin{aligned} & \frac{m_S a_J}{2}\left(r_N \frac{r_N \overline{v}_{NJ}/a_J}{1-r_N^2}\right)^2 - r_N\left(\frac{r_N}{a_J}\frac{\overline{v}_{NJ}}{1-r_N^2}\right)\sum_{i\backslash S} m_i \overline{v}_{iJ} \\ = \quad & \frac{m_S a_J}{2}\left(r_N \frac{r_N \overline{v}_{NJ}/a_J}{1-r_N^2}\right)^2 - r_N\left(\frac{r_N}{a_J}\frac{\overline{v}_{NJ}}{1-r_N^2}\right)\sum_{i\backslash S} m_i \overline{v}_{iJ} \\ = \quad & -\left(\frac{r_N^2}{a_J}\frac{\overline{v}_{NJ}}{1-r_N^2}\right)\left(\sum_{i\backslash S} m_i \overline{v}_{iJ} - \frac{m_S}{2}\left(\frac{r_N^2 \overline{v}_{NJ}}{1-r_N^2}\right)\right). \end{aligned}$$

The proof of (a) is completed by noting that $s_{SJ}\left(X_J^E, 0\right) = 0$.

(b) The proof of Proposition 5 holds for every product produced by N. So, $\Delta_{SK}\left(\tau_{SK}^E\right)$ follows from Lemma 2. ‖

The reasoning in the proof of Proposition 4 continues to hold if just s_{SS}^{BAU} is replaced by $s_{SS}^{BAU} - \sum_J \Delta_{SJ} - \sum_K \Delta_{SK}\left(\tau_{SK}^E\right)$. Given that $\underline{X}$ increases, and $\overline{X}$ decreases, in s_{SS}^{BAU}, when s_{SS}^{BAU} is replaced by $s_{SS}^{BAU} - \sum_J \Delta_{SJ} - \sum_K \Delta_{SK}\left(\tau_{SK}^E\right)$ it follows that $\underline{X}$ decreases, and $\overline{X}$ increases, when either q_S or q_N increases (if the τ_{SK}^E's stay unchanged)

(i) With free trade in equilibrium, $\tau_{SK} = 0$ and $\Delta_{SK}\left(\tau_{SK}\right) = 0$. Note that $\Delta_{SK} = 0$ also if τ_{SK} is fixed (not contingent on X_S) at any other level than 0.

The reasoning in the proof of Proposition 4 continues to hold if just s_{SS}^{BAU} is replaced by $s_{SS}^{BAU} - \sum_J \Delta_{SJ}$. Given that $\underline{X}$ increases, and $\overline{X}$ decreases, in s_{SS}^{BAU}, when s_{SS}^{BAU} is replaced by $s_{SS}^{BAU} - \sum_J \Delta_{SJ}$ it follows that $\underline{X}$ decreases, and $\overline{X}$ increases, when q_S increases.

(ii) As in (57),

$$\overline{\Delta}_{SK} \equiv \max_{\tau_{SK}^E} \Delta_{SK}\left(\tau_{SK}^E\right) = m_S \frac{\left(r_S \overline{v}_{SK}\right)^2 / 2a_K}{1 + r_S^2}.$$

In this case, the reasoning in the proof of Proposition 4 continues to hold if just s_{SS}^{BAU} is replaced by $s_{SS}^{BAU} - \sum_J \Delta_{SJ} - \sum_K \overline{\Delta}_{SK}$. Given that $\underline{X}$ increases, and $\overline{X}$ decreases, in s_{SS}^{BAU}, when s_{SS}^{BAU} is replaced by $s_{SS}^{BAU} - \sum_J \Delta_{SJ} - \sum_K \overline{\Delta}_{SK}$ it follows that $\underline{X}$ decreases, and $\overline{X}$ increases, when either q_S or q_N increases.

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APPENDIX B (ONLINE ONLY): EQUATIONS FOR THE FIGURES

Multiple consumers. When $n = 1$, $m = 2$, $\overline{v}_{SS} = \overline{v}_{NS} = a_S = 1$, and $\frac{a_S}{a_N}\left(\frac{\overline{v}_{SS}}{\overline{v}_{NS}}\right) = 1$, (31), (34), (52), and (58) become:

$$
\begin{aligned}
X_S^{FTA} &= 2\left((1-r_N)+\frac{r_N}{1+r_N}r_N\right),\\
X_S^{BAU} &= 2\left((1-r_N)+\frac{r_N(1-r_N)}{1+2r_N}r_N\right),\\
\underline{X} &= 2\left((1-r_N)+\frac{r_N^2}{1+r_N}-\frac{r_N}{1+r_N}\sqrt{1-\frac{(1+r_N)(1-r_N)}{1+2r_N}}\right),\\
\underline{X}^M &= 2\left((1-r_N)+\frac{r_N^2}{1+r_N}-\frac{r_N}{1+r_N}\sqrt{1-\frac{(1+r_N)(1-r_N)}{1+2r_N}+\frac{1+r_N}{(r_N)^2}\frac{(1-r_N)^3}{1+(1-r_N)^2}}\right),
\end{aligned}
$$

and similarly:

$$
\begin{aligned}
\overline{X} &= 2\left((1-r_N)+\frac{r_N^2}{1+r_N}+\frac{r_N}{1+r_N}\sqrt{1-\frac{(1+r_N)(1-r_N)}{1+2r_N}}\right),\\
\overline{X}^M &= 2\left((1-r_N)+\frac{r_N^2}{1+r_N}+\frac{r_N}{1+r_N}\sqrt{1-\frac{(1+r_N)(1-r_N)}{1+2r_N}+\frac{1+r_N}{(r_N)^2}\frac{(1-r_N)^3}{1+(1-r_N)^2}}\right).
\end{aligned}
$$

If, instead, $m_S = 1$, $m = 1 + m_N$, then $r_N = \frac{m_N}{m_N+1}$ and:

$$
\begin{aligned}
X_S^{FTA} &= 1+\frac{\frac{m_N}{m_N+1}}{1+\frac{m_N}{m_N+1}}m_N,\\
X_S^{BAU} &= 1+\frac{\frac{m_N}{m_N+1}\left(1-\frac{m_N}{m_N+1}\right)}{1+2\frac{m_N}{m_N+1}}m_N,\\
\underline{X} &= 1+\frac{m_N\frac{m_N}{m_N+1}}{1+\frac{m_N}{m_N+1}}-\frac{m_N}{1+\frac{m_N}{m_N+1}}\sqrt{1-\frac{\left(1+\frac{m_N}{m_N+1}\right)\left(1-\frac{m_N}{m_N+1}\right)}{1+2\frac{m_N}{m_N+1}}},\\
\underline{X}^M &= 1+\frac{m_N\frac{m_N}{m_N+1}}{1+\frac{m_N}{m_N+1}}-\frac{m_N}{1+\frac{m_N}{m_N+1}}\sqrt{1-\frac{\left(1+\frac{m_N}{m_N+1}\right)\left(1-\frac{m_N}{m_N+1}\right)}{1+2\frac{m_N}{m_N+1}}+\frac{1+\frac{m_N}{m_N+1}}{\left(\frac{m_N}{m_N+1}\right)^2}\frac{\left(1-\frac{m_N}{m_N+1}\right)^3}{1+\left(1-\frac{m_N}{m_N+1}\right)^2}},\\
\overline{X} &= 1+\frac{m_N\frac{m_N}{m_N+1}}{1+\frac{m_N}{m_N+1}}+\frac{m_N}{1+\frac{m_N}{m_N+1}}\sqrt{1-\frac{\left(1+\frac{m_N}{m_N+1}\right)\left(1-\frac{m_N}{m_N+1}\right)}{1+2\frac{m_N}{m_N+1}}},\\
\overline{X}^M &= 2\left((1-r_N)+\frac{r_N^2}{1+r_N}+\frac{r_N}{1+r_N}\sqrt{1-\frac{(1+r_N)(1-r_N)}{1+2r_N}+\frac{1+r_N}{(r_N)^2}\frac{(1-r_N)^3}{1+(1-r_N)^2}}\right).
\end{aligned}
$$

Multiple countries. When $r_S = r_N = 1/(n+1)$, and $\frac{a_S}{a_N}\left(\frac{\overline{v}_{SS}}{\overline{v}_{NS}}\right) = 1$, (31), (34), (51), and (57) give:

$$X_S^{FTA} = 1 + \frac{1-\frac{1}{n+1}}{2-\left(\frac{1}{n+1}\right)}n,\ X_S^{BAU} = 1 + \frac{1-\left(\frac{1}{n+1}\right)^2-\left(\frac{1}{n+1}\right)}{2-2\left(\frac{1}{n+1}\right)^2-\left(\frac{1}{n+1}\right)}\left(n-\left(\frac{1}{n+1}\right)\right),$$

$$\underline{X} = 1+\frac{1-\left(\frac{1}{n+1}\right)}{2-\left(\frac{1}{n+1}\right)}n - \frac{n}{2-\left(\frac{1}{n+1}\right)}\sqrt{1-\frac{2\left(2-\left(\frac{1}{n+1}\right)\right)}{n\left(\frac{n}{n+1}\right)}\left(\frac{n+1}{2}\frac{\left(\left(\frac{n}{n+1}\right)-\left(\frac{1}{n+1}\right)^2\right)^2}{2-2\left(\frac{1}{n+1}\right)^2-\left(\frac{1}{n+1}\right)}\right)},$$

$$\underline{X}^M = 1+\frac{1-\left(\frac{1}{n+1}\right)}{2-\left(\frac{1}{n+1}\right)}n - \frac{n}{2-\left(\frac{1}{n+1}\right)}\sqrt{1-\frac{2\left(2-\left(\frac{1}{n+1}\right)\right)}{n\left(\frac{n}{n+1}\right)}\left(\frac{n+1}{2}\frac{\left(\left(\frac{n}{n+1}\right)-\left(\frac{1}{n+1}\right)^2\right)^2}{2-2\left(\frac{1}{n+1}\right)^2-\left(\frac{1}{n+1}\right)}-\frac{\left(\left(\frac{1}{n+1}\right)\right)^2\frac{1}{2}}{1+\left(\frac{1}{n+1}\right)^2}\right)},$$

$$\overline{X} = 1+\frac{1-\left(\frac{1}{n+1}\right)}{2-\left(\frac{1}{n+1}\right)}n + \frac{n}{2-\left(\frac{1}{n+1}\right)}\sqrt{1-\frac{2\left(2-\left(\frac{1}{n+1}\right)\right)}{n\left(\frac{n}{n+1}\right)}\left(\frac{n+1}{2}\frac{\left(\left(\frac{n}{n+1}\right)-\left(\frac{1}{n+1}\right)^2\right)^2}{2-2\left(\frac{1}{n+1}\right)^2-\left(\frac{1}{n+1}\right)}\right)},$$

$$\overline{X}^M = 1+\frac{1-\left(\frac{1}{n+1}\right)}{2-\left(\frac{1}{n+1}\right)}n + \frac{n}{2-\left(\frac{1}{n+1}\right)}\sqrt{1-\frac{2\left(2-\left(\frac{1}{n+1}\right)\right)}{n\left(\frac{n}{n+1}\right)}\left(\frac{n+1}{2}\frac{\left(\left(\frac{n}{n+1}\right)-\left(\frac{1}{n+1}\right)^2\right)^2}{2-2\left(\frac{1}{n+1}\right)^2-\left(\frac{1}{n+1}\right)}-\frac{\left(\left(\frac{1}{n+1}\right)\right)^2\frac{1}{2}}{1+\left(\frac{1}{n+1}\right)^2}\right)}.$$

Multiple collaborators. With $m_S = 1$ and $m = 6$, we can vary the mass of consumers included by the CTA-collaborators (in addition to S) from 0 to 5. That is, we use $m = 6$, $r_S = 1/6$, $m_N = m_C$, and $r_N = m_C/6$, in (32), (51), and (57) to get:

$$X_S^{FTA} = 1+\frac{1-\frac{1}{6}}{2-\frac{1}{6}}5,$$

$$\underline{X} = 1+\frac{1-\frac{1}{6}}{2-\frac{1}{6}}5-\frac{5}{2-\frac{1}{6}}\sqrt{1-\frac{2\left(2-\frac{1}{6}\right)}{6\left(1-\frac{1}{6}\right)^2}\left(\frac{6}{2}\frac{\left(\left(1-\frac{1}{6}\right)-\left(\frac{m_C}{6}\right)^2\right)^2}{2-2\left(\frac{m_C}{6}\right)^2-\frac{1}{6}}\right)},$$

$$\underline{X}^M = 1+\frac{1-\frac{1}{6}}{2-\frac{1}{6}}5-\frac{5}{2-\frac{1}{6}}\sqrt{1-\frac{2\left(2-\frac{1}{6}\right)}{6\left(1-\frac{1}{6}\right)^2}\left(\frac{6}{2}\frac{\left(\left(1-\frac{1}{6}\right)-\left(\frac{m_C}{6}\right)^2\right)^2}{2-2\left(\frac{m_C}{6}\right)^2-\frac{1}{6}}-m_C\frac{\left(\frac{1}{6}\right)^2\frac{1}{2}}{1+\left(\frac{1}{6}\right)^2}\right)},$$

$$\overline{X} = 1+\frac{1-\frac{1}{6}}{2-\frac{1}{6}}5+\frac{5}{2-\frac{1}{6}}\sqrt{1-\frac{2\left(2-\frac{1}{6}\right)}{6\left(1-\frac{1}{6}\right)^2}\left(\frac{6}{2}\frac{\left(\left(1-\frac{1}{6}\right)-\left(\frac{m_C}{6}\right)^2\right)^2}{2-2\left(\frac{m_C}{6}\right)^2-\frac{1}{6}}\right)},$$

$$\overline{X}^M = 1+\frac{1-\frac{1}{6}}{2-\frac{1}{6}}5+\frac{5}{2-\frac{1}{6}}\sqrt{1-\frac{2\left(2-\frac{1}{6}\right)}{6\left(1-\frac{1}{6}\right)^2}\left(\frac{6}{2}\frac{\left(\left(1-\frac{1}{6}\right)-\left(\frac{m_C}{6}\right)^2\right)^2}{2-2\left(\frac{m_C}{6}\right)^2-\frac{1}{6}}-m_C\frac{\left(\frac{1}{6}\right)^2\frac{1}{2}}{1+\left(\frac{1}{6}\right)^2}\right)}.$$

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APPENDIX C (ONLINE ONLY): EQUATIONS AND CALIBRATIONS

BAU with many strategic importers. I first derive the BAU equilbrium when S faces n equal-sized and identical strategic importers. Set $r \equiv r_S$, so $r_j = r_N \equiv (1-r)/n$ and $\overline{v}_{ji} = \overline{v}_{Ni}$ for $j \neq$ S. Then, the tariff in each of them is equal, and (22) becomes:

$$-\tau_j + \left(\frac{\overline{v}_{ji}}{a_i} - \frac{\sum_l m_l \overline{v}_{li} - \sum_l a_i m_j \tau_j - X_i}{a_i m}\right) r_j = 0 \Rightarrow$$

$$\tau = \tau_j = \frac{(1-r)/n}{1-(1-r)^2/n}\left(\frac{\overline{v}_{ji}}{a_i} - \frac{\sum_l m_l \overline{v}_{li} - X_i}{a_i m}\right) = \frac{(1-r)/n}{1-(1-r)^2/n}\left(\frac{r\overline{v}_{Ni}}{a_i} + \frac{Z}{a_i m}\right), \tag{63}$$

where $Z \equiv X_S - m_S \overline{v}_{SS}$. With this, the price from (20) becomes:

$$\begin{aligned}
\overline{p}_i &= \frac{\overline{v}_{Ai}}{a_i} - \frac{X_i}{a_i m} - \frac{(1-r)^2/n}{1-(1-r)^2/n}\left(\frac{r\overline{v}_{Ni}}{a_i} + \frac{Z}{a_i m}\right) \\
&= (1-r)\frac{\overline{v}_{Ni}}{a_i} - \frac{Z/m}{a_i} - \frac{(1-r)^2/n}{1-(1-r)^2/n}\left(\frac{r\overline{v}_{Ni}}{a_i} + \frac{Z}{a_i m}\right) \\
&= \frac{\overline{v}_{Ni}}{a_i}(1-r)\left(1 - \frac{r(1-r)/n}{1-(1-r)^2/n}\right) - \frac{Z/m}{a_i}\left(1 + \frac{(1-r)^2/n}{1-(1-r)^2/n}\right) \\
&= \frac{\overline{v}_{Ni}}{a_i}(1-r)\left(\frac{1-(1-r)/n}{1-(1-r)^2/n}\right) - \frac{Z/m}{a_i}\left(\frac{1}{1-(1-r)^2/n}\right) \Leftrightarrow
\end{aligned}$$

$$\overline{p}_i = \frac{\overline{v}_{Ni}}{a_i}\left(1 - \frac{r}{1-(1-r)^2/n}\right) - \frac{Z/m}{a_i}\left(\frac{1}{1-(1-r)^2/n}\right), \text{ so} \tag{64}$$

$$\frac{\partial \overline{p}_S}{\partial X} = -\frac{1}{am}\left(\frac{1}{1-(1-r)^2/n}\right).$$

With this, the f.o.c. for X_S, in (29), becomes:

$$\begin{aligned}
\overline{p}_S - (m_S a_S \overline{p}_S + X_S - m_S \overline{v}_{SS})\frac{1}{am}\left(\frac{1}{1-(1-r)^2/n}\right) &= 0 \Leftrightarrow \\
\overline{p}_S\left(1 - \frac{r}{1-(1-r)^2/n}\right) - \frac{Z}{am}\left(\frac{1}{1-(1-r)^2/n}\right) &= 0 \Leftrightarrow \\
\frac{\overline{v}_{Ni}}{a_i}\left(1 - \frac{r}{1-(1-r)^2/n}\right)^2 - \frac{Z}{am}\left(\frac{1}{1-(1-r)^2/n}\right)\left(2 - \frac{r}{1-(1-r)^2/n}\right) &= 0 \Leftrightarrow
\end{aligned}$$

$$Z = Z^{BAU} \equiv m\overline{v}_{Ni}\frac{\left[1-(1-r)^2/n - r\right]^2}{2-2(1-r)^2/n - r}. \tag{65}$$

With this, (64) becomes

$$\begin{aligned}
\overline{p}_i &= \frac{\overline{v}_{Ni}}{a_i}\left(1 - \frac{r}{1-(1-r)^2/n}\right) - \frac{\overline{v}_{Ni}}{a_i}\left(\frac{1}{1-(1-r)^2/n}\right)\frac{\left[1-(1-r)^2/n - r\right]^2}{2-2(1-r)^2/n - r} = \frac{\overline{v}_{Ni}}{a_i}\xi, \text{ where} \\
\xi &\equiv 1 - \frac{r}{1-(1-r)^2/n} - \left(\frac{1}{1-(1-r)^2/n}\right)\frac{\left(1-(1-r)^2/n - r\right)^2}{2-2(1-r)^2/n - r}.
\end{aligned}$$

The tariff (63) and thus τ/p become:

$$\begin{aligned}
\tau &= \frac{(1-r)/n}{1-(1-r)^2/n}\left(\frac{r\overline{v}_{Ni}}{a_i}+\frac{1}{a_i m}m\overline{v}_{Ni}\frac{\left[1-(1-r)^2/n-r\right]^2}{2-2(1-r)^2/n-r}\right)=\frac{\overline{v}_{Ni}}{a_i}\frac{(1-r)/n}{1-(1-r)^2/n}\left(r+\frac{\left(1-(1-r)^2/n-r\right)^2}{2-2(1-r)^2/n-r}\right),\\
\frac{\tau}{p} &= \frac{1}{\xi}\frac{(1-r)/n}{1-(1-r)^2/n}\left(r+\frac{\left(1-(1-r)^2/n-r\right)^2}{2-2(1-r)^2/n-r}\right).
\end{aligned}$$

The fraction consumed domestically, relative to X_S, becomes:

$$f\equiv m_S\frac{\overline{v}_{SS}-a_i\overline{p}_i}{X_S}=mr\frac{\overline{v}_{SS}-\overline{v}_{Ni}\xi}{m_S\overline{v}_{SS}+m\overline{v}_{Ni}\frac{\left[1-(1-r)^2/n-r\right]^2}{2-2(1-r)^2/n-r}}=r\frac{1-\omega\xi}{r+\omega\frac{\left(1-(1-r)^2/n-r\right)^2}{2-2(1-r)^2/n-r}},$$

where $\omega\equiv\overline{v}_{Ni}/\overline{v}_{SS}$. The non-tariff barrier (NTB) has the same effect as a transport cost. With 20% NTB, but otherwise equal preferences, we may write:

$$\overline{v}_{Ni}=\overline{v}_{SS}-0.2a_i\overline{p}_i=\overline{v}_{SS}-0.2\overline{v}_{Ni}\xi,\ \text{and}\ \overline{v}_{Ni}/\overline{v}_{SS}=\omega=\frac{1}{1+\frac{20}{100}\xi}.$$

Insisting that $T=\frac{\tau}{p}=0.2$ and $f\equiv m_S\frac{\overline{v}_{SS}-a_i\overline{p}_i}{X_S}=0.57$, as argued for in the text, we get:

$$\begin{aligned}
\frac{1}{g}\frac{(1-r)\frac{1}{n}}{1-(1-r)^2\frac{1}{n}}\left(r+\frac{\left(1-(1-r)^2\frac{1}{n}-r\right)^2}{2-2(1-r)^2\frac{1}{n}-r}\right) &= \frac{2}{10}\\
r\left(\frac{1-\left(\frac{1}{1+\frac{20}{100}g}\right)g}{r+\left(\frac{1}{1+\frac{20}{100}g}\right)\frac{\left(1-(1-r)^2\frac{1}{n}-r\right)^2}{2-2(1-r)^2\frac{1}{n}-r}}\right) &= \frac{57}{100}\\
1-\frac{r}{1-(1-r)^2\frac{1}{n}}-\left(\frac{1}{1-(1-r)^2\frac{1}{n}}\right)\frac{\left(1-(1-r)^2\frac{1}{n}-r\right)^2}{2-2(1-r)^2\frac{1}{n}-r} &= g
\end{aligned}$$

The solution is approximately $n=5$ and $r=1/2$. Vice versa, with $n=5$ and $r=1/5$ the predicted T and f are:

$$\begin{aligned}
\frac{1}{g}\left(\frac{(1-r)\frac{1}{n}}{1-(1-r)^2\frac{1}{n}}\right)\left(r+\left(\frac{\left(1-(1-r)^2\frac{1}{n}-r\right)^2}{2-2(1-r)^2\frac{1}{n}-r}\right)\right) &= T\\
\left(\frac{1-\left(\frac{1}{1+\frac{20}{100}g}\right)g}{r+\left(\frac{1}{1+\frac{20}{100}g}\right)\left(\frac{\left(1-(1-r)^2\frac{1}{n}-r\right)^2}{2-2(1-r)^2\frac{1}{n}-r}\right)}\right)r &= f\\
1-\left(\frac{r}{1-(1-r)^2\frac{1}{n}}\right)-\left(\frac{1}{1-(1-r)^2\frac{1}{n}}\right)\left(\frac{\left(1-(1-r)^2\frac{1}{n}-r\right)^2}{2-2(1-r)^2\frac{1}{n}-r}\right) &= g\\
n &= 5\\
r &= \frac{1}{2}
\end{aligned}$$

The solution is $T=0.21$ and $f=0.55$. Thus, from now on, let $n=5$ and $r=1/2$.

Predictions on FTA. With (32) and (65), the ratio free-trade vs. BAU becomes $F \equiv X_S^{FTA}/X_S^{BAU}$:

$$F = \frac{m_S \overline{v}_{SS} + \frac{1-r_S}{2-r_S} m\overline{v}_{-S}}{m_S \overline{v}_{SS} + m\overline{v}_{Ni} \frac{\left[1-(1-r)^2/n-r\right]^2}{2-2(1-r)^2/n-r}} = \frac{r + \frac{(1-r)^2}{2-r}\omega^{FTA}}{r + \left(\frac{1}{1+\frac{20}{100}g}\right)\left(\frac{\left(1-(1-r)^2\frac{1}{n}-r\right)^2}{2-2(1-r)^2\frac{1}{n}-r}\right)}.$$

If the NTB is removed, then $\omega^{FTA} = 1$, so

$$\begin{aligned}
\frac{r + \frac{(1-r)^2}{2-r}\left(\frac{1}{1+\frac{0}{100}g}\right)}{r + \left(\frac{1}{1+\frac{20}{100}g}\right)\left(\frac{\left(1-(1-r)^2\frac{1}{n}-r\right)^2}{2-2(1-r)^2\frac{1}{n}-r}\right)} &= F \\
1 - \left(\frac{r}{1-(1-r)^2\frac{1}{n}}\right) - \left(\frac{1}{1-(1-r)^2\frac{1}{n}}\right)\left(\frac{\left(1-(1-r)^2\frac{1}{n}-r\right)^2}{2-2(1-r)^2\frac{1}{n}-r}\right) &= g \\
n &= 5 \\
r &= \frac{1}{2}
\end{aligned}$$

The solution is $F = 1.048$. In this case, X_S increases with 4.8% compared to X_S^{BAU}.

If the foreign market doubles (in each of three importing countries), then population grows by 13/10, and S's relative size shrinks from $r = 5/10$ to $a = 5/13$, so:

$$\begin{aligned}
\frac{m\left(a + \frac{(1-a)^2}{2-a}\right)}{r + \left(\frac{1}{1+\frac{20}{100}g}\right)\left(\frac{\left(1-(1-r)^2\frac{1}{n}-r\right)^2}{2-2(1-r)^2\frac{1}{n}-r}\right)} &= F \\
1 - \left(\frac{r}{1-(1-r)^2\frac{1}{n}}\right) - \left(\frac{1}{1-(1-r)^2\frac{1}{n}}\right)\left(\frac{\left(1-(1-r)^2\frac{1}{n}-r\right)^2}{2-2(1-r)^2\frac{1}{n}-r}\right) &= g \\
n &= 5 \\
r &= \frac{1}{2} \\
a &= \frac{5}{13} \\
m &= \frac{13}{10}
\end{aligned}$$

The solution is $F = 1.265$. So, X_S increases with almost 27% compared to X_S^0.

Predictions on CTA^0. With one importer signing the CTA^0, $\underline{X}$ follows from (51) where

$$s_{SS}^{BAU} = \frac{m}{2a_S}\frac{\left(\overline{v}_{-S} - r_N^2\overline{v}_{NS}\right)^2}{2-2r_N^2-r_S},$$

so $\underline{X}$ becomes, in our case,

$$\underline{X} = m\overline{v}_{SS}\left[r + \frac{(1-r)^2}{2-r} - \frac{1-r}{2-r}\sqrt{1 - \frac{2(2-r)a_S}{m}\left[\frac{m}{2a_S}\frac{\left(1-r_N^2/(1-r)\right)^2}{2-2r_N^2-r}\right]}\right]. \tag{66}$$

With (65), we get that with $C = \underline{X}/X_S^{BAU}$,

$$\begin{aligned}
\frac{r + \frac{(1-r)^2}{2-r} - \frac{1-r}{2-r}\sqrt{1 - \frac{2-r}{2-2\left(\frac{1-r}{n}\right)^2 - r}\left(1 - \frac{\left(\frac{1-r}{n}\right)^2}{1-r}\right)^2}}{r + \left(\frac{1}{1+\frac{20}{100}g}\right)\left(\frac{\left(1-(1-r)^2\frac{1}{n}-r\right)^2}{2-2(1-r)^2\frac{1}{n}-r}\right)} &= C \\
1 - \left(\frac{r}{1-(1-r)^2\frac{1}{n}}\right) - \left(\frac{1}{1-(1-r)^2\frac{1}{n}}\right)\left(\frac{\left(1-(1-r)^2\frac{1}{n}-r\right)^2}{2-2(1-r)^2\frac{1}{n}-r}\right) &= g \\
n &= 5 \\
r &= \frac{1}{2}
\end{aligned}$$

The solution is $C = 0.962$. So, with CTA^0, X_S can be reduced by 3.8% compared to BAU.

Now, suppose that 3 of the 5 importing blocks double in mass. Then, m grows by 13/10, and, in (66), $r_N = 1/13$ and $r = 5/13$. Compared to the original BAU (before doubling and liberalization), $\underline{X}/X_S^0$ is:

$$\begin{aligned}
m\frac{a + \frac{(1-a)^2}{2-a} - \frac{1-a}{2-a}\sqrt{1 - \frac{2-a}{2-2\left(\frac{1}{13}\right)^2 - a}\left(1 - \frac{\left(\frac{1}{13}\right)^2}{1-a}\right)^2}}{r + \left(\frac{1}{1+\frac{20}{100}g}\right)\left(\frac{\left(1-(1-r)^2\frac{1}{n}-r\right)^2}{2-2(1-r)^2\frac{1}{n}-r}\right)} &= C \\
1 - \left(\frac{r}{1-(1-r)^2\frac{1}{n}}\right) - \left(\frac{1}{1-(1-r)^2\frac{1}{n}}\right)\left(\frac{\left(1-(1-r)^2\frac{1}{n}-r\right)^2}{2-2(1-r)^2\frac{1}{n}-r}\right) &= g \\
n &= 5 \\
r &= \frac{1}{2} \\
a &= \frac{5}{13} \\
m &= \frac{13}{10}
\end{aligned}$$

The solution is $C = 1.1806$. With two beef importers participating in the CTA, and a doubling of the consumers in the other three importers, $r_N = 2/13$ and $r = 5/13$ in (66). With $C = \underline{X}/X_S^0$,

$$\begin{aligned}
m\frac{a + \frac{(1-a)^2}{2-a} - \frac{(1-a)}{2-a}\sqrt{1 - \frac{2-a}{2-2\left(\frac{2}{13}\right)^2 - a}\left(1 - \frac{\left(\frac{2}{13}\right)^2}{1-a}\right)^2}}{r + \left(\frac{1}{1+\frac{20}{100}g}\right)\left(\frac{\left(1-(1-r)^2\frac{1}{n}-r\right)^2}{2-2(1-r)^2\frac{1}{n}-r}\right)} &= C \\
1 - \left(\frac{r}{1-(1-r)^2\frac{1}{n}}\right) - \left(\frac{1}{1-(1-r)^2\frac{1}{n}}\right)\left(\frac{\left(1-(1-r)^2\frac{1}{n}-r\right)^2}{2-2(1-r)^2\frac{1}{n}-r}\right) &= g \\
n &= 5 \\
r &= \frac{1}{2} \\
a &= \frac{5}{13} \\
m &= \frac{13}{10}
\end{aligned}$$

The solution is $C = 1.0957$. The predictions with three beef importers collaborating on CTA^0 can be derived analogously.

Predictions with cross contingency. With contingent tariffs on all q_S privately provided goods exported by S (in addition to S's beef), and all q_N privately provided goods exported by N, then, with Δ_{SJ} and $\overline{\Delta}_{SK}$ defined by Lemma 3, $\underline{X}$ follows from (51) when s_{SS}^{BAU} is replaced by:

$$\frac{m}{2a_S}\frac{\left(\overline{v}_{-S}-r_N^2\overline{v}_{NS}\right)^2}{2-2r_N^2-r_S}-\sum_J\Delta_{SJ}-\sum_K\overline{\Delta}_{SK},$$

so $\underline{X}$ becomes, in this case,

$$\underline{X}^M=m\overline{v}_{NS}\left[r+\frac{(1-r)^2}{2-r}-\frac{1-r}{2-r}\sqrt{1-\frac{2(2-r)a_S}{m}\left[\frac{m}{2a_S}\frac{\left(1-r_N^2/(1-r)\right)^2}{2-2r_N^2-r}-\frac{1}{\overline{v}_{NS}^2}\sum_J\Delta_{SJ}-\frac{1}{\overline{v}_{NS}^2}\sum_K\overline{\Delta}_{SK}\right]}\right].$$

When $\overline{v}_{NJ}=\overline{v}_{iJ}$, Lemma 3 gives:

$$\Delta_{SJ}\equiv\frac{m\overline{v}_{NJ}^2}{a_J}\left(\frac{r_N^2}{1-r_N^2}\right)\left((1-r)-\frac{(1-r)}{2}\left(\frac{r_N^2}{1-r_N^2}\right)\right)\text{ and }\overline{\Delta}_{SK}=\frac{m_S\overline{v}_{SK}^2}{a_K}\frac{r_S^2/2}{1+r_S^2},$$

but for good K, exported from N to S, we should set $r_S=1/13$ because S includes only this fraction of the consumer mass for N's product. With that, and if $\overline{v}_{iK}^2/a_K$ is equal for all goods and countries, then:

$$\begin{aligned}\underline{X}^M&=m\overline{v}_{NS}\left[r+\frac{(1-r)^2}{2-r}-\frac{1-r}{2-r}\sqrt{1-(2-r)\left(\frac{\left(1-r_N^2/(1-r)\right)^2}{2-2r_N^2-r}-\Lambda\right)}\right]\text{, where}\\ \Lambda&=q_S\left(\frac{r_N^2}{1-r_N^2}\right)\left((1-r)-\frac{(1-r)}{2}\left(\frac{r_N^2}{1-r_N^2}\right)\right)+q_N\frac{1}{2}\frac{(1/13)^2}{1+(1/13)^2}.\end{aligned}$$

So $\underline{X}^M$, divided by the original X_S^0, becomes as follows:

$$\begin{aligned}m\frac{a+\frac{(1-a)^2}{2-a}-\frac{(1-a)}{2-a}\sqrt{1-(2-a)\left(\frac{\left(1-r_N^2\frac{1}{(1-a)}\right)^2}{2-2r_N^2-a}-\Lambda\right)}}{r+\left(\frac{1}{1+\frac{20}{100}g}\right)\left(\frac{\left(1-(1-r)^2\frac{1}{n}-r\right)^2}{2-2(1-r)^2\frac{1}{n}-r}\right)}&=C\\ 1-\left(\frac{r}{1-(1-r)^2\frac{1}{n}}\right)-\left(\frac{1}{1-(1-r)^2\frac{1}{n}}\right)\left(\frac{\left(1-(1-r)^2\frac{1}{n}-r\right)^2}{2-2(1-r)^2\frac{1}{n}-r}\right)&=g\\ q_S\left(\frac{r_N^2}{1-r_N^2}\right)\left((1-a)-\frac{(1-a)}{2}\left(\frac{r_N^2}{1-r_N^2}\right)\right)+q_N\frac{1}{2}\frac{\left(\frac{1}{13}\right)^2}{1+\left(\frac{1}{13}\right)}&=\Lambda\\ n&=5\\ r&=\frac{1}{2}\\ a&=\frac{5}{13}\\ m&=\frac{13}{10}\\ q_S&=1\\ q_N&=2\\ r_N&=\frac{1}{13}\end{aligned}$$

The solution is $C=1.1383$. The final numbers in Table 2 are derived by setting $r_N=2/13$ and $q_N=4$ when two beef-importers collaborate on the CTA, and by setting $r_N=4/13$ and $q_N=8$ when also one of the other beef-importers (which doubles in size) collaborates on the CTA.