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Climate Contracts: A Game of Emissions, Investments, Negotiations, and Renegotiations

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The paper presents a dynamic game where players contribute to a public bad, invest in technologies, and write incomplete contracts. Despite the $n+1$ stocks in the model, the analysis is tractable and the symmetric Markov perfect equilibrium unique. If only the contribution levels are contractible, then investments are suboptimally small if the contract is short term or close to its expiration date. To encourage investments, the optimal contract is more ambitious if it is short term, and it is tougher to satisfy close to its expiration date and for players with small investment costs. If renegotiation is possible, such an incomplete contract implements the first-best. The framework helps to analyse emissions, investments, and international environmental agreements, and the results have important lessons for how to design a climate treaty.

Key words: Dynamic games, Incomplete contracts, Hold-up problems, Renegotiation design, Climate change, Environmental agreements

JEL Codes: Q54, D86, H87, F53

1. INTRODUCTION

This paper develops a dynamic model of private provision of public goods. The agents can also invest in substitute technologies, leading to $n+1$ stocks, but the analysis is nevertheless tractable. I derive and characterize a unique symmetric Markov perfect equilibrium (MPE) for the non-cooperative game as well as for situations where the agents can negotiate and contract on contribution levels.

The model is general and could fit various contexts, but the most important application might be climate change. Consistent with the model's assumptions, a country can reduce its emissions in multiple ways: a short-term solution is to simply consume less fossil fuel today, while a more long-term solution might be to invest in renewable energy sources or abatement technology. The Kyoto Protocol is a bargaining outcome limiting countries' emission levels, but it does not specify the extent to which a country should invest or simply reduce its consumption. At the same time, the Protocol is relatively short term since the commitments expire in 2012. This short duration may reflect the difficulties or costs of committing to the distant future.

With this motivation, I refer to the agents as countries, contributions as emissions, and the public bad as greenhouse gases. To explore the potential for a technological solution, I rule out technological spillovers and the associated free-riding problem. To isolate the interaction between the incentive to invest and negotiated emission quotas, I abstract from the difficulties of motivating participation and compliance.

Models with $n+1$ stocks and infinite time are often solved numerically.[1] To get analytical solutions, I make two simplifying assumptions. First, the investment cost is assumed to be linear. Second, the benefit of technology is additive and does not influence marginal emissions from consumption. With these assumptions, the continuation value turns out to be linear in *all* the $n+1$ stocks, making the analysis tractable. Furthermore, a unique symmetric MPE satisfies these conditions. This equilibrium is stationary and coincides with the unique symmetric subgame perfect equilibrium (SPE) if time were finite but approached infinity. These attractive equilibrium properties hold for every scenario studied in the paper.

In contrast to most of the literature, I do not focus on self-enforcing agreements sustained by trigger strategies. Instead, I derive the equilibrium outcome if contracts can be signed. For climate agreements, for example, countries may be able to commit at least to the near future since domestic stakeholders can hold the government accountable if it has ratified an international agreement. Rather than taking a stand on the countries' ability to commit, I derive the equilibrium contract as a function of this ability.

One extreme benchmark is the complete contracting environment. If all emissions and investments can be negotiated, the first-best is easily implemented.

At the other extreme, consider the non-cooperative outcome. Although the technology is private and investments are selfish, each country's technology stock is, in effect, a public good since its role is to substitute for the country's contribution to the public bad. If one country happens to pollute a lot, the other countries are induced to pollute less in the future since the problem is then more severe; they will also invest more in technology to be able to afford the necessary cuts in emissions. If a country happens to invest a lot in abatement technology, it can be expected to pollute less in the future. This induces the other countries to raise their emissions and reduce their own investments. Anticipating these effects, each country pollutes more and invests less than it would in an otherwise similar static model. This dynamic common-pool problem is thus particularly severe.

The real-world contracting environment probably lies between these two benchmarks: countries may be able to contract on certain variables but not on others. When analyzsing incomplete contracts, I let countries contract on emissions but not investments. There are several justifications for this choice: first, this assumption is in line with the literature on incomplete contracts (Segal and Whinston, 2010), making it possible to draw on that literature as well as to clarify my contribution to it. Second, the existing climate treaty (*i.e.* the Kyoto Protocol) does indeed specify levels of emission but not investments. Intuitively, if investment levels had been negotiated, it would be difficult for the enforcing party to distinguish actual investments from short-term abatement efforts.[2]

To begin, suppose countries commit to one period at a time. If there were no further periods, contracting on emission levels would be first-best since investments in technology are selfish. With multiple periods, however, the technology that survives to the next period is, in effect, a public good. The reason for this is that a hold-up problem arises when the countries negotiate emission levels: if one country has better technology and can cut its emissions fairly cheaply, then its opponents may ask it to bear the lion's share of the burden when collective emissions are reduced. Anticipating this, countries invest less when negotiations are coming up (as in Buchholtz and Konrad, 1994). On the one hand, with smaller investments, it is *ex post* optimal to allow for larger emission levels. On the other, since the countries are underinvesting, it is beneficial to encourage more investments and the parties can do this by negotiating a contract

1. See, *e.g.* the literature on industry dynamics, surveyed by Doraszelski and Pakes (2007).

2. Golombek and Hoelc (2006, p. 2) observe that "it would hardly be feasible for a country (or some international agency) to verify all aspects of other countries' R&D policies". Thus, this assumption is standard in the literature.

that is tough and allows few emissions. Thus, the best (and equilibrium) contract is tougher and stipulates *lower* emissions compared to the *ex post* optimum, particularly if the length of the contract is relatively short and the technology long-lasting.

If a country has contracted on the emission levels for several periods, then its technology is enjoyed privately during the agreement periods but thereafter becomes, essentially, a public good, thanks again to the hold-up problem. Thus, investments decline towards the end of the contract. In anticipation of this, and to further motivate investments, the optimal and equilibrium contract becomes tougher to satisfy over time. At the same time, the suboptimally low investments make it optimal to emit more *ex post*. Surprisingly, the two effects cancel: in every period, the equilibrium emission quotas are identical to the emission levels that would have been first-best if investments had been first-best.

However, none of these contracts are renegotiation-proof. Once the investments are sunk, countries have an incentive to negotiate *ex post* optimal emission levels rather than sticking to overambitious contracts. When renegotiation is possible and cannot be prevented, an investing country understands that in the end, it does not have to comply with an overambitious contract. Nevertheless, with renegotiation, all investments and emissions are *first-best*. Intuitively, emission levels are renegotiated to *ex post* optimal levels. Countries with poor technology find it particularly costly to comply with an initially ambitious agreement and will be quite desperate to renegotiate it. This gives them a weak bargaining position and a bad outcome. To avoid this fate, countries invest more in technology, particularly if the initial contract is very ambitious. To take advantage of this effect, the optimal contract is tougher if it has a relatively short duration or if it is close to its expiration date, just as in the case without renegotiation.

If investment costs are asymmetric, the first-best requires that only low-cost countries invest. This is also the non-cooperative equilibrium outcome, but non-investing countries pollute far too much since, as a consequence, the low-cost countries will invest more. The optimal incomplete contract specifies smaller emission quotas for low-cost countries in order to motivate them (and not high-cost countries) to invest. Paradoxically, countries with renewable energy sources end up consuming less energy than countries without.

The lessons for climate policies are important. The bad news is that the possibility of developing technology leads to a lot of free riding, even when technological spillovers are absent. The good news is that investments can be encouraged by negotiating emission quotas, but the treaty must be very tough, particularly towards its end and for countries with low investment costs. Efficiency is best achieved by long-lasting agreements that are renegotiated over time. This suggests that climate negotiators have something to learn from international trade policy negotiators since trade agreements are typically long-lasting, although they can expand or be renegotiated over time.

This paper reports on the relatively technical parts of a larger project on climate agreements. The current paper lays out the general theory and focuses on renegotiation, heterogeneity, and robustness. The companion paper (Harstad, 2011) rests on quadratic utility functions and goes further in analysing short-term agreements, when they are actually worse than business-as-usual, and it characterizes the optimal agreement length. The companion paper is also allowing for firms, imperfect property rights, technological trade, investment subsidies, as well as uncertainty and stochastic shocks.

The literature on climate and environmental agreements is better reviewed elsewhere (Kolstad and Toman, 2005; Andy and Stavins, 2009; Harstad, 2011). The paper's contribution to the earlier literature on dynamic games and incomplete contracts is clarified in the next section. The model is presented in Section 3. When solving the model in Section 4, I gradually increase the possibilities for negotiations and contracts by analysing the non-cooperative game, one-period contracts, multiperiod contracts, and contracts permitting renegotiation. Section 5 permits heterogeneous and convex investment costs and imposes non-negativity constraints on the levels of technology and emissions. Section 6 concludes and the Appendix contains all proofs.

2. CONTRIBUTIONS TO THE LITERATURE

2.1. *Dynamic games and the environment*

The private provision of public goods is often studied in differential games (or a difference game, if time is discrete) where each player's action influences the future stock or state parameter.[3] The result that dynamic common-pool problems are worse than their static versions is known from Fershtman and Nitzan (1991), who present a differential game where agents privately provide public goods in continuous time. Differential games are, however, often difficult to analyse. Many authors restrict attention to linear-quadratic functional forms.[4] While some papers arbitrarily select the linear MPE (*e.g.* Fershtman and Nitzan, 1991), there are typically multiple equilibria (Wirl, 1996; Tsutsui and Mino, 1990) and many scholars manage to construct more efficient non-linear MPEs.[5] In a model of climate change, Dutta and Radner (2009) explore various equilibria and analyse agreements enforced by the threat of reverting to the bad (linear) MPE. The cost of pollution is linear in their model, however, and this eliminates the dynamic effect emphasized below. Dutta and Radner (2004, 2006) also allow countries to invest in technologies that reduce the emission factor associated with their consumption. As in this paper, the cost of investment is linear. But since the cost of pollution is also assumed to be linear, their equilibrium is "bang-bang" where countries invest either zero or maximally in the first period and never thereafter.[6]

The first contribution of this paper is the development of a tractable model that can be used to analyse investments as well as emissions. Under the assumption that technology has a linear cost and an additive impact (unlike Dutta and Radner), I find that the continuation values must be linear in all the $n+1$ stocks, which implies a single symmetric MPE, sharp predictions, and a relatively simple analysis.[7] The second and most important contribution, made possible by the first, is the incorporation of incomplete contracts in dynamic games.[8]

3. Thus, such games are subclasses of stochastic games. For overviews, see Başar and Olsder (1999) or Dockner *et al.* (2000).

4. Fershtman and Nitzan (1991) is one example. For a comprehensive overview of linear-quadratic differential games, see Engwerda (2005).

5. See, *e.g.* Dockner and Long (1993), Dockner and Sorger (1996), Sorger (1998), and Marx and Matthews (2000).

6. Investments are also permitted by Hong and Karp (2010), studying the effect on participation, and Barrett (2006) and Hoel and de Zeeuw (2010), focusing on breakthrough technologies.

7. Potentially, this trick can also be applied when studying other economic problems. In the literature on industry dynamics (see the survey by Doraszelski and Pakes, 2007), *e.g.* analytical solutions are rare and numerical simulations typical. A firm typically overinvests in capacity to get a competitive advantage. While Reynolds (1987) restricts attention to the linear MPE in a linear-quadratic model, simple two-stage games are used by d'Aspremont and Jacquemin (1988) to discuss the benefits of cooperation and by Gatsios and Karp (1992) to show that firms may invest *more* if they anticipate future merger negotiations.

8. Few papers allow for policies or negotiation in stochastic games. Exceptions include Hoel (1993), studying a differential game with an emission tax; Yanase (2006), deriving the optimal contribution subsidy; Houba, Sneek and Vardy (2000), analysing negotiations over (fish) quotas lasting forever; and Sorger (2006), studying one-period agreements. Although Ploeg and de Zeeuw (1992) even allow for R&D, contracts are either complete or absent in all these papers. In Buchholz and Konrad (1994), legislators negotiate spending on "pork" and a long-lasting public good. The equilibrium public-good level is suboptimally but strategically low (as in this paper) to discourage future coalitions from wasting money on pork. However, their mechanism relies on majority rule, and the contract incompleteness is related to future policies rather than current investments.

2.2. *Contract theory*

By permitting contracts on emissions but not on investments, this paper is in line with the literature on incomplete contracts (*e.g.* Hart and Moore, 1988). It is well known that with only one period, selfish investments, and no uncertainty regarding the optimal quantity, the first-best is implemented simply by committing to the optimal quantity before the investment stage (see Proposition 5, Segal and Whinston, 2010). This is in line with my results if technology depreciates completely after each period or the countries can commit to the end of time. If these assumptions are violated, however, I find that investments are lower if the contract length is short and that investments decrease towards the end of a contract. To encourage more investments, the optimal and equilibrium contract is tougher to comply with if the contract is short term or close to its expiration date, particularly if the technology is long-lasting compared to the length of the agreement. These results have not been detected earlier, to the best of my knowledge.

The literature on repeated moral hazard is also somewhat related to the present paper. This literature, as surveyed by Chiappori *et al.* (1994), derives a principal's optimal contract when motivating an agent to exert effort. If the optimal long-term contract happens to be "memoryless", then it is also implementable by short-term or spot contracts (see their Proposition 2). This is consistent with my results if the technology depreciates completely after each period.[9] In other dynamic settings, hold-up problems may be solved if the parties can invest while negotiating and agreements can be made only once (Che and Sakovics, 2004) or if there are multiple equilibria in the continuation game (Evans, 2008). Neither requirement is met in this paper, however.

When considering renegotiation, the paper is closer to the existing literature. Chung (1991) and Aghion *et al.* (1994) have shown how the initial contract can provide incentives by affecting the bargaining position associated with particular investments. If investments are selfish, Edlin and Reichelstein (1996) find that the first-best is achieved by initially contracting on the expected optimal quantity and thereafter allowing for renegotiation. When introducing direct investment externalities, Segal and Whinston (2002, Figure 3) show that the larger the externality, the larger the optimal default quantity compared to the expected *ex post* first-best quantity. Renegotiation is then necessary to ensure *ex post* efficiency. My model needs neither uncertainty nor externalities for renegotiation to be beneficial: with an infinite time horizon, I find that the optimal contract is very aggressive if its length is relatively short and the technology long-lasting, and renegotiation is necessary to ensure *ex post* efficiency. One country's investment affects everyone's continuation value when the agreement expires, creating an intertemporal externality. The dynamic model with selfish investments is thus different from its static counterpart but similar to one with positive direct externalities.

Guriev and Kvasov (2005) present a dynamic moral hazard problem emphasizing the termination time. Their contract is renegotiated at every point in time, to keep the remaining time horizon constant. Contribution levels are not negotiated, but contracting on time is quite similar to contracting on quantity: to increase investments, Guriev and Kvasov let the contract length increase, while Segal and Whinston (2002) let the contracted quantity increase. In this paper, agents can contract on quantity (of emissions) as well as on time, which permits the study of how the two interact. I also allow an arbitrary number of agents, in contrast to the buyer–seller situations in most papers. It is still realistic to assume that unanimity is required in the negotiations, even though a smaller majority requirement would have strengthened the incentives to invest (Harstad, 2005).

9. In general, however, Chiappori *et al.* show that the optimal long-term contract is not implementable by spot contracts because of the benefit from intertemporal consumption smoothing. This benefit does not exist in the present paper (since there is no uncertainty and since linear investment costs make agents risk neutral). On the other hand, while Chiappori *et al.* assume that current effort does not influence future outcomes, I let investments cumulate to a long-lived technology stock. This prevents spot contracts from implementing the first-best in the present paper.

3. THE MODEL

3.1. *Description of the game*

This section presents a game where a set N of $n = |N| > 1$ agents contributes over time to a public bad while they also invest in technology. The public bad is represented by the stock G. Let $1 - q_G \in [0,1]$ measure the fraction of G that "depreciates" from one period to the next. The stock G may nevertheless increase, depending on each agent's contribution level, g_i:

$$G = q_G G_- + \sum_N g_i. \tag{3.1}$$

Parameter G_- represents the level of the public bad left from the previous period; subscripts for periods are thus skipped. To fix ideas and illustrate the importance of the results, I will henceforth refer to the agents as countries, the public bad as greenhouse gases, and contributions as emissions.

Each country $i \in N$ benefits privately from emitting g_i. In order to reduce the necessity of contributing to the public bad, i may invest in a private substitute technology. For example, $i \in N$ can consume fossil fuel (measured by g_i) or renewable energy. Let the technology stock R_i measure the capacity of i's renewable energy sources (*e.g.* the windmills). The total consumption of i is then

$$y_i = g_i + R_i. \tag{3.2}$$

As an alternative interpretation of R_i, it may measure country i's "abatement technology", *i.e.* the amount by which i can reduce (or clean) its potential emissions at no cost. If energy production, measured by y_i, is otherwise polluting, the actual emission level of country i is given by $g_i = y_i - R_i$, which again implies (3.2).

The stock R_i might also depreciate over time, at the rate $1 - q_R \in [0,1]$. If $R_{i,-}$ represents i's technology stock in the previous period, while r_i measures i's recent investments in this stock, then i's current technology level is given by

$$R_i = q_R R_{i,-} + r_i.$$

The investment stages and the consumption stages alternate over time. A period is defined as the shortest time in which investments and thereafter consumptions take place (as indicated by Figure 1).

The per-unit investment cost is $K > 0$. Let the benefit of consumption be given by the increasing and concave function $B(y_i)$, while the individual disutility from the public bad is represented by the increasing and convex function $C(G)$. The one-period utility can then be written as

$$u_i = B(y_i) - C(G) - K r_i.$$

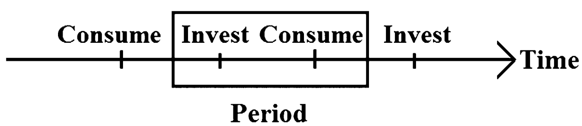

FIGURE 1
The definition of a period

Country i's objective is to maximize the present discounted value of future utilities,

$$V_{i,t} \equiv \sum_{\tau=t}^{\infty} u_{i,\tau}\delta^{\tau-t},$$

where δ is the common discount factor and $V_{i,t}$ is i's continuation value as measured at the start of period t. As mentioned, subscripts denoting period t are typically skipped when this is not confusing.

To allow for alternative applications and to simplify the analysis, note that I have *not required* g_i or r_i to be positive. Even for climate policies, a negative g_i may be technically feasible if carbon capture is an option, although this may be costly, as captured by a large $B'(\cdot)$ for such a small g_i. Similarly, r_i may be negative if the abatement technology and the associated infrastructure can be put to other uses. Nevertheless, non-negativity constraints are imposed in Section 5.2, where I discuss conditions under which the equilibria below generalize. Section 5.1 allows investment costs to be heterogeneous, Section 5.3 lets them be convex (rather than linear) functions, and Section 5.4 introduces uncertainty.

3.2. *The equilibrium concept*

As in most stochastic games, attention is restricted to Markov perfect equilibria where strategies are conditioned on the physical stocks only. As do Maskin and Tirole (2001), I look for the coarsest set of such strategies. Maskin and Tirole (2001, pp. 192–193) defend MPEs since they are "often quite successful in eliminating or reducing a large multiplicity of equilibria" and they "prescribe the simplest form of behavior that is consistent with rationality" while capturing the fact that "bygones are bygones more completely than does the concept of subgame-perfect equilibrium".

Because the investment cost is linear in the model above, multiple MPEs exist, distinguished by how the sum of investment is allocated. Since the model is otherwise symmetric, in the non-cooperative game I will restrict attention to symmetric MPEs where identical countries invest identical amounts. The asymmetric equilibria would, in any case, not survive if the investment cost functions were slightly but strictly convex, rather than linear (see Section 5).

If the countries are negotiating a contract, I assume that the bargaining outcome is efficient and symmetric *if* the pay-off-relevant variables are symmetric across countries. These assumptions are weak and satisfied in several situations. For example, we could rely on cooperative solution concepts, such as the Nash bargaining solution (with or without side transfers). Alternatively, consider a non-cooperative bargaining game where one country can make a take-it-or-leave-it offer to the others, and side transfers are feasible. If every country has the same chance of being recognized as the proposal-maker, the equilibrium contract is exactly as described below.[10]

With this equilibrium concept, there turns out to be a unique equilibrium for each of the contracting environments investigated below; furthermore, it coincides with the unique symmetric SPE if time were finite but approached infinity. This result is desirable; in fact, Fudenberg and Tirole (1991, p. 533) have suggested that "one might require infinite-horizon MPE to be limits of finite-horizon MPE".

10. For the negotiation game with side transfers, there exists multiple asymmetric MPEs since it is irrelevant if i is induced to invest more than j as long as i is compensated for this. But the pay-offs must be the same across these equilibria since the pay-offs in the bargaining outcome are pinnned down by the threat of reverting to the symmetric non-cooperative MPE.

All countries participate in the contract in equilibrium since there is no stage at which they can commit to not negotiating with the others.

4. ANALYSIS

Six contracting environments are analysed in the following subsections.

Contract	Complete	None	Incomplete	+ Renegotiations
One period	1	2	3	5
Multiperiod			4	6

Naturally, different contracting environments generate different outcomes. Nevertheless, all the contracting environments share a number of attractive features. Rather than repeating these features for each case, let me start by emphasizing them since they constitute the main methodological contribution of the paper.

Theorem. *For each contracting environment* (1–6)*, it is true that*

(1) *The individual continuation value V is a function of only G_- and $R_- \equiv \sum_N R_{i,-}$.*
(2) *Continuation values are linear in the stocks and satisfy*

$$\frac{\partial V(G_-, R_-)}{\partial G_-} = -q_G(1-\delta q_R)\frac{K}{n};$$
$$\frac{\partial V(G_-, R_-)}{\partial R_-} = q_R\frac{K}{n}.$$

(3) *There is a unique symmetric MPE.*
(4) *This MPE coincides with the unique symmetric SPE if time is finite, $t \in \{1, \dots, \bar{t}\}$, and $t < \bar{t} - 1$.*

Part (1) is easily illustrated. In principle, the continuation value V_i is a function of the $n+1$ stocks G and $\mathbf{R} \equiv [R_1, R_2, \dots, R_n]$. However, note that choosing g_i is equivalent to choosing y_i, once the R_is are sunk. Thanks to the assumption that technology and emissions are perfect substitutes, by combining (3.1) and (3.2), we get

$$G = q_G G_- + \sum_N y_i - R.$$

This way, the R_is are eliminated from the model: they are *pay-off irrelevant* as long as R is given, and i's Markov perfect strategy for y_i is therefore not conditioned on them.[11] Hence, a country's continuation value V_i is a function of G_- and R_-, and not of $R_{i,-} - R_{j,-}$, and we can write that value as $V(G_-, R_-)$, without the subscript i.

The remaining parts of the theorem are best understood after the analysis of each scenario and by following the proofs in the Appendix. For example, in every scenario, the value of another unit of technology is that each of the n countries can invest q_R/n units less in the following period: this explains the level $\partial V/\partial R_-$ and why this is constant across scenarios. The fact that V is linear in *all* stocks hinges on the linear investment costs, revealing the power of this assumption.

11. This follows from the definition by Maskin and Tirole (2001, p. 202), where Markov strategies are measurable with respect to the coarsest partition of histories consistent with rationality.

The linearity of V makes the model tractable and permits only one equilibrium. To simplify notation, equations often rely on the conventional definitions $V_G \equiv \partial V / \partial G_-$, $V_R \equiv \partial V / \partial R_-$, $B' \equiv dB(y_i)/dy_i$, $C' \equiv dC(G)/dG$, etc.

4.1. *The first-best outcome*

For future reference, I start by describing the first-best outcome, *i.e.* the outcome if every $g_{i,t}$ and $r_{i,t}$ were contractible or chosen by a planner maximizing the sum of utilities $\sum_N V_{i,t}$.

Proposition 1.

(1) *The first-best consumption $y_i = g_i + R_i$ is independent of R_i, given R:*

$$B'(g_i^*(\mathbf{R}) + R_i) - n(C' - \delta V_G) = 0. \tag{4.1}$$

(2) *The first-best investment ensures that R_i satisfies*

$$B'(g_i^*(\mathbf{R}^*) + R_i^*) = (1 - \delta q_R)K. \tag{4.2}$$

(3) *The first-best pollution level G satisfies*

$$C'(G^*) = (1 - \delta q_G)(1 - \delta q_R)K/n. \tag{4.3}$$

Part (1) simply states that, at the emission stage, the (private) marginal benefit of consumption should equal the (social) cost of pollution. Intuitively, this requires identical consumption levels, $y_i^* = y_j^*, \forall i, j \in N^2$, regardless of the differences in technologies. Technological differences do imply, however, that the *ex post* first-best *emission* level should be smaller for i if R_i is large:

$$g_i^*(\mathbf{R}) \equiv y_i^* - R_i.$$

Part (2) states that the first-best investment level equalizes the marginal benefit to the marginal cost, recognizing that more investments today reduce the need to invest in the next period. By substituting (4.2) in (4.1), we find the first-best pollution level, stated in part (3). Interestingly, note that both y_i^* and G^* are independent of the past stocks: G^* is pinned down by (4.3) and, together with G_-, this pins down $\sum_N g_i$. Since y_i is pinned down by (4.2), $R_i = y_i - g_i$ follows as a residual, and so does $r_i = R_i - q_R R_{i,-}$.

4.2. *The non-cooperative equilibrium*

Suppose now that countries act non-cooperatively at every decision node. At the emission stage, a marginally larger g_i gives country i the benefit B', but i's increased cost of present pollution is C' and the cost for the future is δV_G, where V is now i's continuation value in the non-cooperative environment. Of course, each country internalizes only $1/n$ of the total harm.

Proposition 2 (emission). (1) *The equilibrium consumption y_i^{no} is independent of R_i and suboptimally large, given R:*

$$B'(y_i^{\text{no}}) = C' - \delta V_G. \tag{4.4}$$

(2) *Country i pollutes less but $j \neq i$ pollutes more if R_i is larger, fixing R_j, $\forall j \neq i$:*

$$\partial g_i^{\text{no}} / \partial R_i = -\frac{C''(n-1) - B''}{nC'' - B''} < 0,$$

$$\partial g_j^{\text{no}} / \partial R_i = \frac{C''}{nC'' - B''} > 0, \quad \forall j \neq i.$$

Part (1) verifies that all countries choose the same y_i^{no}, regardless of the R_is. While perhaps surprising at first, the intuition is straightforward. Since (3.2) assumes that technology has an additive impact on consumption, the returns to technology are always inframarginal and the level of technology does not change the *marginal* cost of consumption. Emissions increase one by one in consumption, no matter what the level of technology is.[12]

Since $C\left(q_G G_- + \sum_N y_i - R\right)$ is convex, (4.4) implies that a larger R must increase every y_i. This means that if R_i increases but R_j is constant, then $g_j = y_j - R_j$ must increase. Furthermore, substituting (3.2) in (4.4) implies that if R_i increases, g_i must decrease, as shown in part (2). In words, a country that has better technology pollutes less but—because of this—other countries pollute more.

These results mean that a country's technology stock is, in effect, a public good. A larger R_i raises every country's consumption and, in addition, reduces every investment in the following period. Since each country captures only $1/n$ of the benefits, it invests less than optimally.

Proposition 3 (investments).

(1) *Equilibrium investments are suboptimally low and ensure that*

$$B'(R_i^{\mathrm{no}} + g_i^{\mathrm{no}}) = (1 - \delta q_R/n)\left[\frac{nC'' - B''}{C'' - B''}\right]K, \tag{4.5}$$

assuming that the following second-order condition holds:

$$C'' \geq (n-1)\left[\frac{B'''(C'')^2/B'' - C'''B''}{(C'' - B'')(nC'' - B'')}\right]B'. \tag{4.6}$$

(2) *Every country invests more if R_- is small and G_- large:*

$$\partial r_i^{\mathrm{no}}/\partial R_- = -q_R/n,$$
$$\partial r_i^{\mathrm{no}}/\partial G_- = q_G/n.$$

(3) *The resulting pollution level is suboptimally large and given by*

$$C'(G^{\mathrm{no}}) = (1-\delta q_G)(1-\delta q_R)K/n + \left[1 + \frac{(n-\delta q_R)C''}{C'' - B''}\right](1 - 1/n)K. \tag{4.7}$$

Compared to the first-best (4.2), the right-hand side of (4.5) is larger. This implies that country i's consumption, $R_i^{\mathrm{no}} + g_i$, is *smaller* than at the first-best. Since Proposition 2 states that consumption levels are suboptimally large, given $\mathbf{R}$, it follows that investments must be suboptimally small: $g_i^{\mathrm{no}}(\mathbf{R}^{\mathrm{no}}) + R_i^{\mathrm{no}} < g_i^*(\mathbf{R}^*) + R_i^*$ and $g_i^*(\mathbf{R}^{\mathrm{no}}) > g_i^*(\mathbf{R}^{\mathrm{no}})$ implies $\mathbf{R}^{\mathrm{no}} < \mathbf{R}^*$. Furthermore, the effect of suboptimally small investments dominate the suboptimally large emissions, leading to lower equilibrium consumption than at the first-best.[13]

12. Obviously, this would not necessarily be true if I instead had focused on technologies that reduce the emission *factor* of *each* produced unit (as do Dutta and Radner, 2004). The additive form (3.2) is chosen not only because it simplifies the analysis tremendously but also because the resulting crowding-out effects might be reasonable in reality.

13. Note the qualifying second-order condition, which is violated if C''' is positive and large and/or if B''' is very negative. In both situations, an increase in R raises y_j at a decreasing rate. Thus, the discouragement effect when i invests (*i.e.* the fact that other countries pollute more) is reduced when i invests more, and this implies that an interior solution for r_i may not exist. However, the second-order condition holds if C''' and $-B'''$ are small relative to C'' and $-B''$, and it always holds for quadratic functions (implying $C''' = B''' = 0$).

Part (2) first states that, if the technology stock is small today, every country invests more in the next period. Naturally, this contributes to explaining the underinvestment result. Second, if the pollution level is large, all countries invest more. This contributes to explaining the suboptimally large emissions, as described by Proposition 2. Part (3) states that, when all these effects are taken into account, the resulting pollution level is larger than the first-best level.

4.3. *One-period incomplete contracts*

From now on, I let the countries negotiate and contract on their contributions to the public bad. In each period, the timing is the following. First, the countries negotiate a vector of emission levels. Thereafter, each country invests and, finally, every country complies with the contract. The bargaining outcome is assumed to be efficient and symmetric *if* the game itself is symmetric.

The bargaining game is indeed symmetric, even if technology levels differ across the countries. Just as before, the $R_{i,-}$s are eliminated from the model and the continuation value is a function of only G_- and $R_- \equiv \sum_N R_{i,-}$. Moreover, the linear investment cost implies that the continuation value must be linear in both stocks, pinning down a unique equilibrium.

When investing, country i prefers a larger stock of technology if its quota, g_i^{co}, is small since, otherwise, its consumption level would be very low. Consequently, r_i^{co} decreases in g_i^{co}.

Proposition 4 (investments).

(1) *Investments are larger if the contract is tough:* $\partial r_i/\partial g_i^{\text{co}} = -1$.
(2) *For any quota* g_i^{co}, *i's investment is suboptimally low if* $\delta q_R > 0$ *and ensures that*

$$B'(g_i^{\text{co}} + R_i^{\text{co}}) = (1 - \delta q_R/n)K. \tag{4.8}$$

In contrast to the non-cooperative game, a country's technology stock is no longer a "public good": once the emission levels are pinned down, i's investment increases y_i but not y_j, $j \neq i$. However, the technology that survives to the next period, $q_R R_i$, does become a public good since, for a fixed R, the continuation value at the start of every period is independent of R_i. Intuitively, if the agreement does not last forever, a country anticipates that having good technology will worsen its bargaining position in the future, once a new agreement is negotiated. At that stage, good technology leads to a lower quota $g_{i,+}$ since the other countries can hold up i when it is cheap for i to reduce its emissions.[14] In fact, the future consumption $y_{i,+}$ will be the same across countries, regardless of any difference in technology. This discourages a country from investing now, particularly if the current agreement is relatively short (δ large), the technology likely to survive (q_R large), and the number of countries (n) large.

Thus, if n, δ, and q_R are large, it is beneficial to encourage more investments. On the one hand, this can be achieved by a *small* g_i^{co}. On the other hand, the *ex post* optimal g_i^{co} is *larger* when equilibrium investments are low. The optimal quota must trade off these concerns. As shown in the Appendix, the equilibrium and optimal quotas are identical to the emission levels in the first-best![15] However, since the equilibrium investments are less than the optimal ones, the g_i^{co}s are suboptimally low *ex post*.

14. Alternatively, if no agreement is expected in the future, a large $R_{i,+}$ reduces $g_{i,+}$ and increases $g_{j,+}$, as stated in Proposition 2.

15. Intuitively, since the investment cost is linear, consumption is constant and independent of g_i^{co}. Increasing the quota thus has only two effects. On the one hand, investments decrease one by one in g_i^{co} (since $\partial R_i/\partial g_i^{\text{co}} = -1$) and the marginal reduction in investment cost is constant, thanks again to the linear investment cost. On the other hand, the marginal cost of pollution is $C'(G)$. Therefore, setting the optimal quota generates exactly the same trade-off as in the first-best case, and the optimal G is therefore identical.

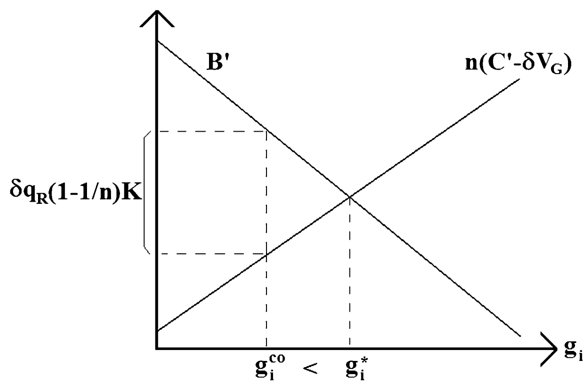

FIGURE 2

The shorter the agreement, the lower is the contracted emission level relative to the *ex post* optimum

Proposition 5 (emission).

(1) *The contracted emission levels are equal to the levels at the first-best:* $g_i^{\text{co}} = g_i^*(\mathbf{R}^*) \Leftrightarrow G^{\text{co}} = G^*$.

(2) *Since* $R_i^{\text{co}} < R_i^*$ *if* $\delta q_R > 0$, $g_i^{\text{co}} < g_i^*(\mathbf{R}^{\text{co}})$ *and the emission levels are lower than what is ex post optimal:*

$$B'(g_i^{\text{co}} + R_i^{\text{co}}) - n(C' - \delta V_G) = \delta q_R(1 - 1/n)K > 0. \tag{4.9}$$

Not only is the shadow value of polluting, B', larger than in the non-cooperative case, but it is even larger than it would be in the first-best, (4.1). For a fixed investment level, optimally g_i^{co} should have satisfied $B' - n(C' - \delta V_G) = 0$ rather than (4.9). Only then would marginal costs and benefits be equalized. Relative to this *ex post* optimal level, the quotas satisfying (4.9) must be *lower* since $B' - n(C' - \delta V_G)$ decreases in g_i^{co}. If n, q_R, and δ are large, the additional term $\delta q_R(1 - 1/n)K$ is large, and g_i^{co} must decline. This makes the contract more demanding or *tougher* to satisfy at the emission stage compared to what is *ex post* optimal. In equilibrium, the quotas play a dual role: they control current emissions and mitigate the underinvestment in technology. The optimal emission quotas are therefore particularly stringent if the contract is short term, the technology long-lasting, and the number of countries large, as illustrated in Figure 2.

On the other hand, if $\delta q_R = 0$, the right-hand side of (4.9) is zero, meaning that the commitments under the best long-term agreement also maximize the expected utility *ex post*. In this case, the countries are not concerned with how current technologies affect future bargaining power either because the existing agreement is lasting forever ($\delta = 0$) or because the technology will not survive the length of the contract ($q_R = 0$). Investments are first-best and there is no need to distort the g_i^{co}s downwards.

4.4. *Multiperiod contracts*

Assume now that at the beginning of period 1, the countries negotiate the quotas for every period $t \in \{1, 2, \ldots, T\}$. The time horizon T may be limited by the countries' ability to commit to future promises.

At the start of period 1, the pay-off-relevant stocks are G_- and R_- only. Once again, this simplifies the analysis. There is a unique MPE, the continuation value at the start of period 1 (and $T+1$) is linear in the stocks and has the same slopes as before.

When investing in period $t \in \{1,2,\ldots,T\}$, countries take the quotas $g_{i,t}^{mc}$s as given, and the continuation value in period $T+1$ is $V(G_T, R_T)$, just as before. At the last investment stage, the problem is the same as in Section 4.3 and a country invests until (4.8) holds. Investments are then suboptimally low since the technology that survives to the next period is basically a public good.

However, by investing more in period $T-1$, a country can invest less in period T. The net investment cost for a country is then $(1-\delta q_R)K$, which leads to first-best investments. The same logic applies to every previous period since, when the quotas for the following period are fixed, technology is a private rather than a public good. Investments fall only towards the end of the commitment period, right before a new round of negotiations and hold-up problems is anticipated.[16]

Proposition 6 (investments). *Investments are first-best at $t < T$ but suboptimally low in the last period for any set of quotas:*

$$B'(g_{i,t}^{mc}+R_{i,t}^{mc}) = (1-\delta q_R)K, \quad t < T, \tag{4.10}$$
$$B'(g_{i,t}^{mc}+R_{i,t}^{mc}) = (1-\delta q_R/n)K, \quad t = T.$$

All this is anticipated when the countries negotiate the quotas. As shown in the Appendix, the optimal and equilibrium quotas must satisfy (4.3) for every $t \leq T$: the equilibrium pollution level is similar to the first-best level, for every period!

In the beginning of the agreement, when $t < T$, the $g_{i,t}^{mc}$s are *ex post* optimal as well since the investments are first-best. In the last period, however, investments decline and the contracted emission levels are lower than the *ex post* optimal levels. In other words, the optimal contract becomes tougher to satisfy towards its end.

Proposition 7 (emission).

(1) *The negotiated quotas equal the first-best emission levels in every period: $g_{i,t}^{\text{co}} = g_i^*(\mathbf{R}^*) \Leftrightarrow G_t^{mc} = G^*$.*

(2) *Thus, quotas are ex post first-best ($g_{i,t}^{mc} = g_i^*(\mathbf{R}_t^{mc})$) for $t < T$, but for $t = T$, since $R_{i,T}^{mc} < R_{i,T}^*$, the quotas are suboptimally low ex post and satisfying (4.9). Consequently, the contract becomes tougher to satisfy toward the end.*

4.5. *Renegotiation*

The contracts above are not renegotiation-proof since they specify emission levels that are less than what is optimal *ex post*, after the investments are sunk. The countries may thus be tempted to renegotiate the treaty. This section derives equilibria when renegotiation is costless.

Starting with one-period contracts, the timing in each period is the following. First, the countries negotiate the initial commitments, the g_i^{de}s, referred to as "the default". If these negotiations fail, it is natural to assume that the threat point is no agreement. Thereafter, the countries invest. Before carrying out their commitments, the countries get together to renegotiate the g_i^{de}s.

16. The sudden fall in investments from period $T-1$ to period T is due to the linear investment cost. If the investment cost were instead a convex function, then one should expect investments to gradually decline when approaching the expiration date.

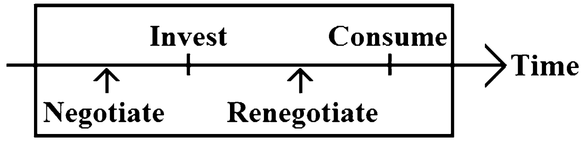

FIGURE 3
The timing when renegotiation is possible

Relative to the threat point g_i^{de}, the bargaining surplus is assumed to be split equally in expectation (this is standard in the literature, see Segal and Whinston 2010). This bargaining outcome is implemented by the Nash bargaining solution if side transfers are available or, alternatively, by randomly letting one country make a take-it-or-leave-it offer regarding quantities and transfers.

Renegotiation ensures that emission levels are *ex post* optimal, in contrast to the contracts discussed above. When investing, a country anticipates that it will not, in the end, have to comply with an overambitious contract. Will this jeopardize the incentives to invest?

The answer is no and the explanation as follows: when renegotiating a very ambitious agreement, countries that have invested little are desperate to reach a new agreement that would replace the tough initial commitments. In other words, they have a high willingness to pay when renegotiating the default outcome. Since the bargaining surplus is split evenly (in expectation) in equilibrium, the other countries take advantage of this willingness to pay and require side transfers in return. The fear of such a poor bargaining position motivates all countries to invest more, particularly if the default emission levels are small.

Proposition 8 (investments). *Country i's investment level r_i decreases in the initial quota g_i^{de}.*

This is anticipated when negotiating the initial agreement. The more ambitious this agreement is, the more the countries invest. This is desirable if the countries are otherwise tempted to underinvest. Thus, the agreement should be more ambitious if δ and q_R are large. Since the investments are influenced by the initial agreement, the g_i^{de}s can always be set such that the investments are first-best. In any case, the emission levels remain optimal, thanks to renegotiation. In sum, renegotiation provides a second instrument when the countries attempt to mitigate two common pool problems, and this permits the first-best.

Proposition 9 (emission).

(1) *The initial contract is tougher than what is ex post optimal if $\delta q_R > 0$ and is given by*

$$\begin{aligned} &B'(g_i^{\mathrm{de}} + R_i^*) = K \\ \Rightarrow\ &g_i^{\mathrm{de}} = g_i^*(\mathbf{R}) - [B'^{-1}([1-\delta q_R]K) - B'^{-1}(K)] < g_i^*(\mathbf{R}). \end{aligned} \tag{4.11}$$

(2) *In equilibrium, all investments and emissions are first-best.*

If the technology depreciates completely after each period ($q_R = 0$) or if the players are impatient or there is no further period ($\delta = 0$), then $g_i^{\mathrm{de}} = g_i^*(\mathbf{R})$. Otherwise, the quota g_i^{de} decreases if the length of the agreement is short while the technology is long-lasting since countries fear that more technology today will hurt their bargaining position in the near future. They thus invest less than what is optimal, unless the agreement is more ambitious. This

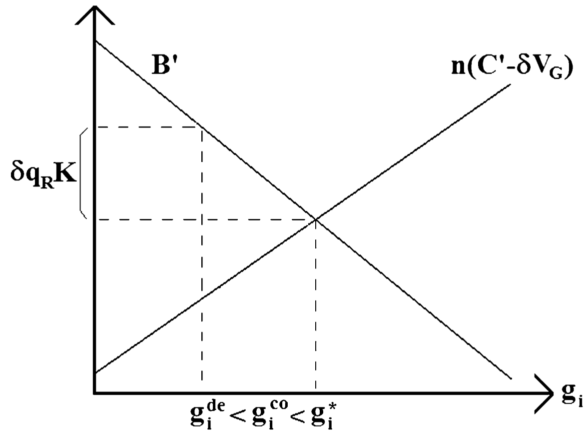

FIGURE 4
The shorter the agreement, the lower is the contracted emission level relative to the *ex post* optimum

result, illustrated in Figure 4, confirms the comparative static for the case without renegotiation (Proposition 5).[17]

Compared to the optimal contract without renegotiation, given by (4.9), the initial agreement should be tougher when renegotiation is possible: $g_i^{\text{de}} < g_i^{\text{co}}$. Intuitively, without renegotiation the contract balances the concern for investments (by reducing g_i^{co}) and for *ex post* efficiency (where g_i^{co} should be larger). The latter concern is irrelevant when renegotiation ensures *ex post* optimality, so the initial contract can be tougher—indeed, so tough that investments are first-best.

Corollary 1. *The initial contract under renegotiation* (4.11) *is tougher and specifies lower emission levels than the equilibrium contract when renegotiation is not possible* (4.9).

4.6. *Multiperiod contracts and renegotiation*

If the countries can negotiate and commit to a T-period agreement, we know from Section 4.4 that investments (and consumption) are first-best in every period—except for the last. Thus, the contracted quantities are also *ex post* optimal, and there is no need to renegotiate them. It is only in the last period that the quantities are lower than what is optimal *ex post*, and only then is there an incentive to renegotiate the contract.

Thus, when renegotiation is possible, for every period but the last the optimal and equilibrium initial contract specifies the *ex post* optimal quotas, and these are also equal to the first-best quantities since investments are optimal for $t < T$. The initial contract for the last period, $t = T$, is given by (4.11), just as in the one-period contract with renegotiation. As in Section 4.4, the initial contract becomes tougher to satisfy towards its end since the initial quotas are smaller for the last period than for the earlier periods.[18]

Proposition 10. *The equilibrium initial contract is given by* $g_{i,t}^{\text{de}} = g_i^*(\mathbf{R}^*)$ *for* $t < T$*, but for* $t = T$*, the contract is tougher and given by* (4.11)*. This contract implements the first-best and is renegotiated only after the investment stage in the last period.*

17. In practice, there might be a limit to how tough the agreement can be if it must be self-enforcing or the enforcement capacity is small. Furthermore, one may require $g_i^{\text{de}} \geq 0$, as discussed in Section 5.2. With such constraints, multiperiod agreements may be necessary.

18. To see this, just compare the last-period contract (4.11) to (4.10).

In principle, there cannot be a unique equilibrium when contracts last multiple periods and renegotiation is possible. Instead of renegotiating only in the last period, the countries do equally well by, *e.g.* renegotiating in every period. However, Segal and Whinston (2010, p. 31) use the term *renegotiation* only when renegotiation changes the equilibrium pay-offs. Furthermore, with an arbitrarily small cost of renegotiating contracts, the countries would always prefer to renegotiate as little and as late as possible. This would imply a unique prediction: the outcome described by Proposition 10.

Interestingly, the equilibrium outcome is observationally equivalent to a time-inconsistency problem where the countries make ambitious plans for the future, while repeatedly backing down from promises made in the past. But rather than reflecting a time-inconsistency problem, this actually leads to the first-best.

Corollary 2. *In equilibrium, the countries repeatedly promise to pollute little in the future but when the future arrives, they renege on these promises. This procedure implements the first-best.*

5. GENERALIZATIONS AND EXTENSIONS

5.1. *Heterogeneous investment costs*

This section relaxes the assumption on identical investment costs. Instead, let country $i \in N$ face the linear investment cost K_i. Define $K \equiv \min_i K_i$, $\underline{N} \equiv \{i \mid K_i = K\}$, and $\underline{n} \equiv |\underline{N}|$. Thus, $\underline{n}$ is the number of countries that face the smallest cost, K. I will assume that K is so small that it is optimal with $R_i > 0$ for at least some country.[19] Each country can invest as much as it wants to, subject to the constraint that its technology cannot be negative: $r_i \geq -q_R R_{i,-}$.

The first-best outcome is simple: the relevant investment cost for the planner is K, as before. Thus, only the low-cost countries should invest, while countries with higher investment costs should not.[20] Further, the first-best g_i^* requires that marginal utilities are equalized. In sum, Proposition 1 holds as before.

The equilibria are a bit more complicated. When countries are heterogeneous, it matters a great deal whether they can use side transfers at the negotiation stage; it is assumed here that they can and that the bargaining surplus is split evenly, in expectation.[21] I will continue to restrict attention to "symmetric" equilibria where identical countries invest the same amount. But since K_i may vary across all countries, this condition does not have much bite. However, for each contracting environment, it turns out that exactly one symmetric MPE exists where only low-cost countries invest. Furthermore, this equilibrium is identical to the unique symmetric SPE if time $t \in \{1, 2, \ldots, \bar{t}\}$ were finite and $t < \bar{t} - 1$.

Proposition 11. *For each contracting environment, an MPE exists which coincides to the unique symmetric SPE if $t < \bar{t} - 1 < \infty$. In this equilibrium, the following is true:*

19. The requirement on K is strongest in the non-cooperative case, since investments are then smallest. An implicit condition for the requirement on K is given by Assumption A1 in the proof of Proposition 11. When contracts are possible, the requirement on K is weaker and as in Proposition 12.

20. Of course, this hinges on the linear investment cost (see Section 5.3)

21. This would be the case with the Nash bargaining solution or if a randomly drawn country makes a take-it-or-leave-it offer.

(1) *Only $i \in \underline{N}$ invest and the continuation values satisfy*

$$\frac{\partial V_i(G,R)}{\partial G} = -\frac{q_G K(1-\delta q_R)}{\underline{n}} \quad \textit{and} \quad \frac{\partial V_i(G,R)}{\partial R} = \frac{q_R K}{\underline{n}} \quad \textit{if } i \in \underline{N}; \tag{5.1}$$
$$\frac{\partial V_i(G,R)}{\partial G} = 0 \quad \textit{and} \quad \frac{\partial V_i(G,R)}{\partial R} = 0 \quad \textit{if } i \in N \backslash \underline{N}.$$

(2) *With no contract, low-cost countries consume less:*

$$B'(y_i^{\text{no}}) = C' - \delta \frac{\partial V_i(G,R)}{\partial G} \quad \forall i \in N, \tag{5.2}$$

assuming a decreasing $B'(y_i)/(-B''(y_i))$ and the following second-order condition:

$$C'' \geq B'(y_i)\left[\frac{C'''/B'' - B'''(C'')^2/(B'')^3}{1-C''/B''(y_i)} - \frac{\sum_N [C'''/B'' - (C'')^2 B'''/(B'')^3]}{1-\sum_N C''/B''}\right]. \tag{5.3}$$

(3) *With one-period contracts, $G = G^*$ and quotas are ex post optimal, $g_i^{\text{co}} = g_i^*(\boldsymbol{R}^{\text{co}})$, for $i \in N\backslash\underline{N}$. Low-cost countries consume less and pollute suboptimally little ex post:*

$$B'(g_i^{\text{co}} + R_i^{\text{co}}) - \sum_N (C' - \delta \partial V_j/\partial G) = \delta q_R (1 - 1/\underline{n}) K > 0, \quad i \in \underline{N}. \tag{5.4}$$

(4) *With multiperiod contracts, at $t < T$, $G = G^*$ and emissions are ex post optimal, $g_{i,t}^{mc} = g_{i,t}^*(\boldsymbol{R}^{mc}) = g_{i,t}^*(\boldsymbol{R}^*)$, for all $i \in N$. At $t = T$, the outcome is as in part* (3).

(5) *With one-period contracts and renegotiation, the first-best is implemented, the initial contract is $g_i^{\text{de}} = g_i^*$ for $i \in N\backslash\underline{N}$, while for $i \in \underline{N}$, g_i^{de} is given by*

$$B'(g_i^{\text{de}} + R_i^*) = \left(1 - \delta q_R \frac{n-\underline{n}}{\underline{n}(n-1)}\right) K \tag{5.5}$$
$$\Rightarrow g_i^{\text{de}} = g_i^*(\boldsymbol{R}^*) - \left[B'^{-1}([1-\delta q_R]K) - B'^{-1}\left(\left[1 - \delta q_R \frac{n-\underline{n}}{\underline{n}(n-1)}\right] K\right)\right] < g_i^*(\boldsymbol{R}^*).$$

(6) *With multiperiod contracts and renegotiation, the first-best is implemented and the initial contract is $g_{i,t}^{\text{de}} = g_i^*(\boldsymbol{R}^*)$, $\forall i \in N$ for $t < T$. At $t = T$, the contract is as in part* (5).

Part 1 states that, for each contracting environment, only low-cost countries invest. Intuitively, the marginal value of adding more technology is similar for all countries, but the marginal cost is always smaller for $i \in \underline{N}$. If the stock of inherited technology is small and the pollution large, the need to invest is larger and the low-cost countries' continuation value smaller. Non-investing countries, in contrast, face a continuation value that depends on neither past technology nor past pollution. With this asymmetry, the results from Section 4 generalize in interesting ways.

Part 2 shows that, in the non-cooperative setting, the first-order condition for emission is as in Proposition 2. But only the investing countries internalize the larger future investment cost if pollution increases today. Given the difference in continuation values, the investing countries try harder to reduce emissions than do non-investing countries. Consequently, and perhaps paradoxically, they end up consuming less than countries that do not have green technology at all. The impact of the stocks on future investments is divided by the number of investing countries, $\underline{n}$. Thus, if $\underline{n}$ decreases, the free-riding problem between the investors decreases, and one may conjecture that the investors will invest more and pollute less. This conjecture is indeed true if $B(\cdot)$ and $C(\cdot)$ are quadratic functions, as shown in the Appendix.[22]

Part 3 says that, with optimal one-period contracts, only low-cost countries are encouraged to invest. This necessitates small quotas for low-cost countries, even though this is suboptimal *ex post*. The optimal contract is particularly tough for investing countries if $\underline{n}$ is large, since every $i \in \underline{N}$ understands that it benefits only $1/\underline{n}$ from the technology that survives the current contract. If $\underline{n} = 1$, however, $i = \underline{N}$ invests optimally and there is no need to reduce i's quota below the *ex post* optimal level. Countries with higher investment costs are not encouraged to invest and they consume and pollute the *ex post* optimal amount.

Multiperiod contracts share the same logic, part 4 finds. As in Section 4.4, every country invests optimally in the periods before the last and it is not necessary to have a suboptimally tough contract for these periods.

Parts 5 and 6 affirm that, with renegotiation, the emission levels are *ex post* optimal, as before. The equilibrium (and optimal) initial default quotas are set so as to motivate first-best investments. Since only low-cost countries invest in the first-best, the default quota must be tougher than the *ex post* optimal quota only for the low-cost countries. Furthermore, the initial demand on these countries should be particularly tough if their number, $\underline{n}$, is large, just as in the case without renegotiation.[23]

5.2. *Requiring non-negative emissions and technologies*

The main body of this paper does not consider any non-negativity constraints on g_i or r_i. This may be reasonable, as argued in Section 3.2, but the main motivation for ignoring non-negativity constraints is simplicity: at the start of Section 3, the $n+1$ stocks were reduced to only 2 after arguing that the R_is were pay-off-irrelevant, given R. With non-negativity constraints, however, this argument is invalid and one can no longer prove that the continuation value is necessarily a function of only G and R. Nevertheless, it is still possible to show that the equilibria described above continue to be an MPE, under specific conditions.

As in the previous subsection, technology stocks are required to be non-negative but countries are allowed to put their technology to other uses: $r_i \geq -q_R R_{i,-}$ (the alternative constraint $r_i \geq 0$ is briefly discussed below). In addition, emission levels are required to be positive: $g_i \geq 0$. To save space, attention is restricted to the four scenarios with incomplete contracts.

Proposition 12. *Consider the constraints $R_i \geq 0$ and $g_i \geq 0 \Rightarrow G \geq 0$. For each contracting environment, the equilibrium described in Section 4 continues to be an MPE under certain conditions:*

(1) *If there is no renegotiation, the condition is*

$$K \leq \frac{C'(q_G G_- + nB'^{-1}(K - \delta V_R))}{1 - \delta q_R/n - \delta q_G(1 - \delta q_R)}. \tag{5.6}$$

(2) *For contracts that can be renegotiated, the condition is $K \leq B'(R_i^*)$.*

22. For the non-cooperative case, I also need to assume a decreasing $B'/(-B'')$ to guarantee that only low-cost countries invest. The reason is that while B' measures the benefit of consuming more, the actual increase in consumption induced by a larger R decreases in $-B''$. This assumption is sufficient but not necessary and it always holds for quadratic $B(\cdot)$, *e.g.* The second-order condition is a generalization of the one in Proposition 3.

23. In principle, multiple contracts implement the first-best when renegotiation is possible, so the equilibrium is technically not unique, for the same reasons as in Section 4.6. In particular, any initial g_i^{de} so large that $i \in N \backslash \underline{N}$ does not invest can implement the first-best.

Part 1 is explained as follows. Remember that for one-period agreements, R_i decreased in g_i^{co}. Thus, the constraint $R_i \geq 0$ binds when the negotiated g_i^{co} is sufficiently generous, *i.e.* if $g_i^{\mathrm{co}} \geq \widehat{g}_i^{\mathrm{co}}$ for some threshold $\widehat{g}_i^{\mathrm{co}}$. Thus, when the quotas are negotiated, it is anticipated that the benefit of reducing g_i^{co} below the *ex post* optimal level (*i.e.* that i invests more) ceases to exist when $g_i^{\mathrm{co}} > \widehat{g}_i^{\mathrm{co}}$. Above this threshold, the optimal g_i^{co}s are larger and set so as to be *ex post* optimal, without the need of motivating investments, and R is then zero. Hence, when the quota g_i^{co} is to be chosen, the second-order condition may not hold at the threshold $\widehat{g}_i^{\mathrm{co}}$. Permitting $g_i^{\mathrm{co}} > \widehat{g}_i^{\mathrm{co}}$ is nevertheless not an equilibrium if investments are sufficiently cheap and pollution costly. This is the intuition behind condition (5.6), derived in the Appendix. For multiperiod agreements, the incentive to invest is lowest in the last period, and the condition ensuring that $R_i > 0$ in the last period, (5.6), is sufficient to ensure that $R_i > 0$ and $g_i > 0$ in the earlier periods as well.

Part 2 is even simpler. When renegotiation is possible, the trick in Section 4.5 was to negotiate a tough default agreement. The agreement had to be particularly tough (in that g_i^{de} was small) for K large and $B'(\cdot)$ small. When we require that $g_i^{\mathrm{de}} \geq 0$, however, there is a limit to how tough the agreement can be. The condition for the first-best to be implemented by some $g_i^{\mathrm{de}} \geq 0$ is simply $K \leq B'(R_i^*)$, as shown in the Appendix. With more than one period, the first-best was implemented in the earlier periods simply by initially committing to the *ex post* optimal emission levels. The default requirement $g_{i,t}^{\mathrm{de}}$ is therefore at the lowest in the last period, leading to the same condition.[24]

Things are more complicated if the constraint $r_i \geq -q_R R_{i,-}$ is replaced by $r_i \geq 0$. The most interesting consequence of such a constraint might be for the multiperiod agreement. Since Proposition 6 predicts a (possibly large) fall in R_i from period $T-1$ to period T, the constraint $r_{i,T} \geq 0$ may then bind. If it does, it may be optimal to also invest zero in period $T-1$ and perhaps also in period $T-2$. Thus, as we approach the final period, investments may fall to zero for several of the last periods.[25]

5.3. *Nonlinear investment costs*

The framework above rests on the assumption that investment costs are linear. This is certainly a strong assumption: realistically, investment costs are given by a convex function (if the cheapest investments are made first) or a concave function (with returns to scale).

Let i's cost of investing r_i units be given by an increasing and convex function, $k(r_i)$. The reasoning that reduced the number of pay-off-relevant stocks from $n+1$ to 2 continues to hold. However, it is no longer possible to derive constant derivatives for the continuation value. Thus, the linear cost assumption is crucial for the simplicity above.

While a general analysis would be difficult, the steady state at the first-best is still characterized by (4.1) together with the analogous equation for (4.2):

$$B'(g_i^* + R_i^*) = (1 - \delta q_R) k'(r_s^*),$$

where $r_s^* \equiv (1-q_R)R^*/n$ and R_i^* is the first-best level of technology. Furthermore, the most optimistic result from the above analysis continues to hold: the first-best is still implementable

24. With multiperiod contracts and renegotiation, the first-best can be implemented even if $K > B'(R_i^*)$, although such a contract is different from the one described by Proposition 10. For example, a sequence of quotas equal to the *ex post* optimal level induces first-best investments as long as the countries renegotiate (and postpone) the expiration date no later than in period $T-1$.

25. This would be more likely if n is large while δ and q_R are large, *i.e.* if the period length is short. While it is easy to derive such investment levels as functions of the quotas, it is not straightforward to characterize the equilibrium (or optimal) quotas when $r_i \geq 0$ may bind.

by one-period contracts if renegotiation is permitted, and the necessary initial contract is similar to the one described by Proposition 9.

Proposition 13. *Suppose the investment–cost function is increasing and convex, countries can negotiate one-period contracts, and these can be renegotiated.*

(1) *Country i's investment level r_i decreases in the initial quota g_i^{de}.*
(2) *In steady state, where $r = (1-q_R)R/n$, the initial contract is given by*

$$B'(g_i^{\text{de}} + R_i^*) = k'(r_s^*)$$

$$\Rightarrow g_i^{\text{de}} = g_i^*(\boldsymbol{R}^*) - [B'^{-1}([1-\delta q_R]k'(r_s^*)) - B'^{-1}(k'(r_s^*))] < g_i^*(\mathbf{R}^*) \quad \text{if } \delta q_R > 0.$$

(3) *In equilibrium, all investments and emissions are first-best.*

A convex investment cost can in principle help to further generalize the results above. In particular, we no longer need to restrict attention to *symmetric* MPEs in the equilibria above: the existence of asymmetric equilibria (where some countries invest more than others) hinge on the linear investment cost; for a convex investment cost function, these would cease to exist and the equilibria emphasized above are unique. Regarding the non-negativity constraints, the constraint $r_i \geq 0$ would never bind if $k'(0) = 0$. Thus, for a multiperiod contract (without renegotiation), we should expect that $r_{i,t}$ would gradually decrease over time as t approaches T. If investment costs were heterogeneous but convex functions, it would be optimal to let every country invest and not only the lowest cost countries as in Section 5.1. Deriving the optimal contract in this situation is certainly both important and interesting, suggesting a promising path for future research.

6. CONCLUSIONS

This paper presents a dynamic game in which n agents contribute to a public bad while also investing in substitute technologies. If the impact and the cost of investments are assumed to be linear, the symmetric MPE is unique and the analysis tractable. While the unique equilibrium rules out self-enforcing agreements, the framework can be employed to analyse incomplete contracts in a dynamic setting.

The model's assumptions fit well with the context of climate change and the results have important consequences for how to design a treaty. For those hoping for a technological solution, the *bad news* is that there will be as much free-riding on green investments as on emissions. Furthermore, countries have an incentive to invest little today in order to induce other countries to pollute less and invest more tomorrow. Finally, countries with large investment costs prefer to pollute a lot since, as a consequence, the technology leaders will simply invest more. In sum, green technology has the characteristic of a public good even when abstracting from technological spillovers and even if countries can sign contracts on emission levels.

The *good news* are, first, that investments are larger if the negotiated quotas are small. Second, the investments increase and can be socially optimal if the agreement is long-lasting. Third, the more long-lasting the agreement is, the less ambitious it needs to be, although it should be tougher to satisfy towards its end and for technology-leading countries. Fourth, permitting renegotiation does not reduce incentives to invest but rather enables the countries to design an agreement that motivates efficient investments as well as emissions.

A comparison to trade agreements is illuminating. Currently, the commitments made under the Kyoto Protocol expire in 2012 and the threat point for present negotiations is no agreement at all. In contrast, when the Doha-round trade negotiations broke down, the default outcome was not the non-cooperative equilibrium but the *existing* set of long-term trade agreements. Trade negotiations can thus be viewed as attempts to renegotiate existing treaties. The procedure used for negotiating trade agreements is more efficient than the one currently used for climate, according to the above analysis.

This paper has abstracted from firms, technological trade, intellectual property rights, and uncertainty (all analysed in Harstad, 2011). I have also ignored the problems of nonparticipants (discussed in Harstad, 2012). Instead, the paper has isolated the two-way interaction between investments in green technology and the design of international climate agreements. As a result, the analysis has detected and explored challenges that arise *even if* we abstract from domestic politics, technological spillovers, private information, monitoring, compliance, coalition formation, and the possibility of opting out of the agreement. While the effects discussed in this paper are likely to persist, allowing for such complications will certainly generate several new results and thereby enhance our understanding of the best agreement design. Relaxing these assumptions is thus the natural next step.

APPENDIX A

Let each i's interim continuation value (after the investment stage) be represented by $W(\cdot)$. Note that, just like $V(\cdot)$, W is a function of only G_- and R and thus is independent of i or $R_i - R_j$, $(i,j) \in N^2$.

Proof of Proposition 1. I start with the following lemma.

Lemma A1. $V_R^* = q_R K/n$.

Proof. The $1/n$ continuation value at the start of each period can be written as

$$V(G_-, R_-) = \max_R W(G_-, R) - (R - q_R R_-)K/n. \tag{A.1}$$

The lemma follows from the envelope theorem. ‖

(1) When the objective is to maximize the sum of utilities, the first-order condition at the emission stage is

$$B'(y_i) - nC'\left(q_G G_- + \sum_N y_i - R\right) + n\delta V_G\left(q_G G_- + \sum_N y_i - R, R\right) = 0. \tag{A.2}$$

Thus, $y_i \equiv y$ is the same for every i. The second-order condition is

$$B''(y_i) - nC''\left(q_G G_- + \sum_N y_i - R\right) + n\delta V_{GG}\left(q_G G_- + \sum_N y_i - R, R\right) \leq 0,$$

which holds when $V_{GG} = 0$, as implied by the following lemma.

Lemma A2. $V_G^* = -(1-\delta q_R) q_G K/n$.

Proof. From Lemma A1, $V_{GR} = 0$ and V_G is independent of R. Therefore, (A.2) implies that $y_i = y$ is a function only of $\xi^* \equiv q_G G_- - R$, as is $G = q_G G_- - R + \sum_N y_i$, which can be written as the function $G = \chi^*(\xi^*)$. Furthermore, we can write $B(y_i) - C(G) \equiv \gamma^*(\xi^*)$, where $\gamma^*(\cdot)$ is a function of $\xi^* \equiv q_G G_- - R$. Since $R = q_G G_- - \xi^*$, the continuation value at the start of each period can be written as

$$V(G_-, R_-) = \max_{\xi^*} \gamma^*(\xi^*) + \delta V(\chi^*(\xi^*), q_G G_- - \xi^*) - (q_G G_- - \xi^* - q_R R_-)K/n$$

$$\Rightarrow \partial V/\partial G_- = \delta q_G V_R - q_G K/n,$$

by the envelope theorem. Substituting for V_R concludes the proof. ‖

Part (3) is proven before part (2). The interim continuation value can be written as

$$W(G_-, R) = \max_y B(y) - C(q_G G_- + ny - R) + \delta V(q_G G_- + ny - R, R)$$
$$\Rightarrow \partial W/\partial R = C' - \delta V_G + \delta V_R = C' + \delta(1-\delta q_R) K q_G/n + \delta q_R K/n,$$

by the envelope theorem. From (A.1), the first-order condition for the investment level is

$$\partial W/\partial R - K/n = 0 \Rightarrow C' = (1-\delta q_R)(1-\delta q_G)K/n. \tag{A.3}$$

The second-order condition is $-C'' \leq 0$, which is always satisfied.

(2) Note that (A.3) can be written as $C' - \delta V_G = K/n - \delta V_R$. From (A.2), we have $C' - \delta V_G = B'(y_i)/n$. Combined, $B'(y_i) = K - n\delta V_R = (1-\delta q_R)K$.

Proof of Proposition 2. I start with the following lemma.

Lemma A3. $V_R^{\text{no}} = q_R K/n$.

Proof. In the symmetric equilibrium, i anticipates that every $j \neq i$ invests the same amount, $(R^{\text{no}} - q_R R_-)/n$, where R^{no} is the equilibrium R. At the start of each period, i maximizes its interim continuation value w.r.t. R, anticipating that $r_i = R - q_R R_- - \sum_{j \neq i}(R^{\text{no}} - q_R R_-)(n-1)/n$. Hence, i's continuation value at the start of the period can be written as

$$V(G_-, R_-) = \max_R W(G_-, R) - [R - q_R R_- - (R^{\text{no}} - q_R R_-)(1-1/n)]K.$$

Clearly, the equilibrium R (and thus R^{no}) is independent of R_-. Applying the envelope theorem concludes the proof. ‖

(1) At the emission stage, i's first-order condition for y_i is

$$B'(y_i) - C'\left(q_G G_- - R + \sum_N y_i\right) + \delta V_G\left(q_G G_- - R + \sum_N y_i, R\right) = 0, \tag{A.4}$$

implying that all y_is are identical. The second-order condition is

$$B'' - C'' + \delta V_{GG}(\cdot) \leq 0,$$

which holds in equilibrium since $V_{GG} = 0$, as implied by the following lemma.

Lemma A4. $V_G^{\text{no}} = -q_G(1-\delta q_R)K/n$.

From Lemma A3, $V_{GR} = 0$ and V_G is independent of R. Therefore, (A.4) implies that y_i is a function of only $\xi^{\text{no}} \equiv q_G G_- - R$, as is $G = q_G G_- - R + \sum_N y_i$, which can be written as the function $G = \chi^{\text{no}}(\xi^{\text{no}})$. Furthermore, we can write $B(y_i) - C(G) \equiv \gamma^{\text{no}}(\xi^{\text{no}})$, where $\gamma^{\text{no}}(\cdot)$ is a function of $\xi^{\text{no}} \equiv q_G G_- - R$.

Let $\overline{\xi}^{\text{no}}$ be the equilibrium ξ^{no}. In equilibrium, j invests $r_j = (q_G G_- - \overline{\xi}^{\text{no}} - q_R R_-)/n$. Anticipating this, i's maximization problem when choosing r_i is equivalent to choosing ξ^{no}, recognizing $r_i = R - q_R R_- - \sum_{j \neq i} r_j = q_G G_- - \xi^{\text{no}} - q_R R_- - (q_G G_- - \overline{\xi}^{\text{no}} - q_R R_-)(n-1)/n$. At the start of each period, i's continuation value can be written as

$$V(G_-, R_-) = \max_{\xi^{\text{no}}} \gamma(\xi^{\text{no}}) + \delta V(\chi(\xi^{\text{no}}), q_G G_- - \xi^{\text{no}})$$
$$- [q_G G_- - \xi^{\text{no}} - q_R R_- - (q_G G_- - \overline{\xi}^{\text{no}} - q_R R_-)(1-1/n)]K.$$

Since V_R is a constant, it follows that the equilibrium ξ^{no} (which equals $\overline{\xi}^{\text{no}}$) is a constant, independent of the stocks. By the envelope theorem, it follows that

$$V_G = \delta V_R q_G - q_G K/n = -q_G(1-\delta q_R)K/n. \qquad \|$$

(2) Differentiating (A.4) w.r.t. R_i or, equivalently, w.r.t. $R = R_i + \sum_{j \neq i} R_j$ gives

$$B'' \frac{dy_i}{dR} - C'' \frac{n dy_i - dR}{dR} = 0 \Rightarrow \frac{dy_i^{no}}{dR} = \frac{C''}{nC'' - B''} \tag{A.5}$$

$$\Rightarrow \frac{dg_i}{dR_j} = \frac{C''}{nC'' - B''} > 0, \quad \text{if } j \neq i, \text{ while}$$

$$\frac{dg_i}{dR_i} = \frac{C''}{nC'' - B''} - 1 = -\frac{C''(n-1) - B''}{nC'' - B''} < 0.$$

Proof of Proposition 3.

(1) The first-order condition w.r.t. r_i $\left(\text{or, equivalently, w.r.t. } R = R_i + \sum_{j \neq i} R_j\right)$ is

$$B'(y_i) \frac{dy_i}{dR} - [C'(G) - \delta V_G] \sum_{j \in N} \frac{dg_j}{dR} + \delta V_R = K \Rightarrow (4.5), \tag{A.6}$$

when substituting for (A.4) and (A.5). The right-hand side of (4.5) is larger than the first-best (4.2), so $y_i^{no} < y_i^*$ when the second-order condition holds. Since $g_i^{no}(\mathbf{R}^{no}) + R_i^{no} = y_i^{no} < y_i^* = g_i^*(\mathbf{R}^*) + R_i^*$ while $g_i^{no}(\mathbf{R}^*) > g_i^*(\mathbf{R}^*)$ from Proposition 2, we have $g_i^{no}(\mathbf{R}^{no}) + R_i^{no} < g_i^{no}(\mathbf{R}^*) + R_i^*$. This implies that $R_i^{no} < R_i^*$ because $\partial(g_i^{no}(\mathbf{R}) + R_i)/\partial R = \partial y_i^{no}/\partial R > 0$ from (A.5).

Lemma A5. *The second-order condition at the investment stage holds if*

$$C'' \geq (n-1) B'(y_i) \left[\frac{-B'''(C'')^2/(B'') - C''' B''}{(C'' - B'')(nC'' - B'')} \right].$$

Proof. The second-order condition of (A.6) is

$$B''(y_i) \left(\frac{dy_i}{dR} \right)^2 + B'(y_i) \frac{d^2 y_i}{(dR)^2} - C''(G) \left[n \frac{dy_i}{dR} - 1 \right]^2 - [C'(G) - \delta V_G] \left[n \frac{d^2 y_i}{(dR)^2} \right] \leq 0$$

$$\Rightarrow \frac{B'' C'' (C'' - B'')}{(nC'' - B'')^2} + (B' - n[C' - \delta V_G]) \left[\frac{d^2 y_i}{(dR)^2} \right] \leq 0, \tag{A.7}$$

we substitute with (A.5). By differentiating (A.5), we get

$$0 = \left(\frac{dy_i}{dR} \right)^2 B''' + B'' \frac{d^2 y_i}{(dR)^2} - \left(\frac{n dy_i - dR}{dR} \right)^2 C''' - C'' n \frac{d^2 y_i}{(dR)^2}$$

$$\Rightarrow \frac{d^2 y_i}{(dR)^2} = \frac{(C'')^2 B''' - (B'')^2 C'''}{(nC'' - B'')^3}.$$

Substituted in (A.7),

$$\frac{B'' C'' (C'' - B'')}{(nC'' - B'')^2} - B'(n-1) \left[\frac{(C'')^2 B''' - (B'')^2 C'''}{(B'' - nC'')^3} \right] \leq 0 \Rightarrow (4.6). \quad \|$$

(2) From the proof of Lemma A4, we learned that, for the equilibrium R, $R = q_G G_- - \xi^{no}$, where ξ^{no} was a constant. Since $r = (R - q_R R_-)/n$, $\partial r/\partial G_- = q_G/n$ and $\partial r_i/\partial R_- = -q_R/n$.

(3) By substituting (4.5) in (A.4), we can write

$$C' = B' + \delta V_G = (1 - \delta q_R/n) K \frac{nC'' - B''}{C'' - B''} + \delta V_G,$$

which, with some algebra, can be rewritten as (4.7).

Proof of Proposition 4.

(1) When the level g_i^{co} is already committed to, the first-order condition for i's investment is

$$0 = B'(g_i^{\text{co}} + R_i) + \delta V_R - K \tag{A.8}$$

$$\Rightarrow y_i^{\text{co}} = B'^{-1}(K - \delta V_R), \quad R_i^{\text{co}} = B'^{-1}(K - \delta V_R) - g_i^{\text{co}}, \tag{A.9}$$

$$r_i^{\text{co}} = B'^{-1}(K - \delta V_R) - g_i^{\text{co}} - q_R R_{i,-}. \tag{A.10}$$

Substituting $V_R^{\text{co}} = q_R K/n$ (confirmed by Lemma A6) into (A.8) and (A.10) gives part (1). Since V_R^{co} is constant, the second-order condition is $B'' \le 0$, which holds by assumption. A comparison to the first-best completes the proof of part (2).

Proof of Proposition 5. First, note that if the negotiations fail, the default outcome is the non-cooperative outcome, giving everyone the same utility. Since the r_is follow from the g_is by (A.10), negotiating the g_is is equivalent to negotiating the *equilibrium* r_is. Since all countries have identical preferences w.r.t. the r_is (and their default utility is the same), symmetry requires that r_i be the same for every i. Thus, preferences are aligned, and efficiency requires utility to be maximized w.r.t. the g_is or, equivalently, to the r_is, or to G. Anticipating (A.9)–(A.10), we can write the continuation value at the start of the period as

$$V^{\text{co}}(G_-, R_-) = \max_G B(B'^{-1}(K - \delta V_R)) - C(G) - Kr_i + \delta V, \quad \text{where} \tag{A.11}$$
$$r_i = [R - q_R R_-]/n \quad \text{and}$$
$$R = \sum_{j \in N} (y_j - g_j) = nB'^{-1}(K - \delta V_R) - (G - q_G G_-).$$

Lemma A6. *(i)* $V_R^{\text{co}} = q_R K/n$. *(ii)* $V_G^{\text{co}} = -q_G K(1 - \delta q_R)/n$.

Proof. Part (i) follows from applying the envelope theorem to (A.11). Part (ii) follows from applying the envelope theorem to (A.11) and substituting for V_R^{co}. ‖

(1) The first-order condition from (A.11) is

$$0 = -C' + K/n + \delta V_G - \delta V_R \Rightarrow C' = (1 - \delta q_G)(1 - \delta q_R)K/n. \tag{A.12}$$

The second-order condition is $-C'' \le 0$, which holds by assumption. By comparing to (4.3), $G^{\text{co}} = G^*$, which, given G_-, is equivalent to $\sum_N g_i^{\text{co}} = \sum_N g_i^*(\mathbf{R}^*)$. This, in turn, is equivalent to $g_i^{\text{co}} = g_i^*(\mathbf{R}^*)$ since both the r_is and the y_is are the same across the is for both scenarios.

(2) Combining (A.12) and (A.8) gives (4.9).

Proof of Proposition 6. The investment level in the last period is given by (A.10) for the same reasons as in Proposition 4. Anticipating the equilibrium $R_{i,T}$ (and $R_{j,T}$), i can invest q_R units fewer in period T for each unit invested in period $T-1$. Thus, in period $T-1$, the first-order condition for i's investment is

$$0 = B'(g_{i,T-1}^{mc} + R_{i,T-1}) - K + \delta q_R K$$
$$\Rightarrow \; R_{i,t} = B'^{-1}((1 - \delta q_R)K) - g_{i,t}^{mc}, \quad t = T-1. \tag{A.13}$$

The same argument applies to every period $t \in \{1, \ldots, T-1\}$, and the investment level is given by the analogous equation for each period $t < T$. The second-order condition, $B'' \le 0$, holds by assumption. A comparison to the first-best completes the proof.

Proof of Proposition 7. At the start of $t = 1$, countries negotiate emission levels for every period $t \in \{1, \ldots, T\}$. In equilibrium, all countries enjoy the same default utilities. Just as before, they will therefore negotiate the quotas such that the equilibrium investment will be the same for all is. Using (A.13) for each period $t < T$, this implies

$$\begin{aligned} R_t &= \sum_N (B'^{-1}(K(1-\delta q_R)) - g_{i,t}) = nB'^{-1}(K(1-\delta q_R)) - (G_t - q_G G_{t-1}) \\ \Rightarrow r_{i,t} &= (R_t - q_R R_{t-1})/n, \quad \forall t \\ &= B'^{-1}(K(1-\delta q_R)) - (G_t - q_G G_{t-1})/n \\ &\quad - q_R B'^{-1}(K(1-\delta q_R)) + q_R (G_{t-1} - q_G G_{t-2})/n, \quad t \in \{2, \ldots, T-1\}. \end{aligned} \tag{A.14}$$

At the start of the first period, i's continuation value can be written as

$$V = \max_{\{G_t\}_t} \frac{1-\delta^T}{1-\delta} B(B'^{-1}(K(1-\delta q_R))) - \sum_{t=1}^{T} \delta^{t-1}[K r_{i,t} + C(G_t)] + \delta^T V(G_T, R_T), \tag{A.15}$$

where $r_{i,t}$ is given by (A.14) and R_T is given by (A.9).

Lemma A7. *(i)* $V_R = q_R K/n$. *(ii)* $V_G = -q_G K(1-\delta q_R)/n$.

Proof. Both parts follow from (A.15) by applying the envelope theorem. ‖

(1) The first-order condition of (A.15) w.r.t. any G_t, $t \in \{1, \ldots, T\}$, gives (4.3). As in the proof of Proposition 5, $G_t^{mc} = G^* \Leftrightarrow g_{i,t}^{mc} = g_i^*(\mathbf{R}^*)$. The second-order condition, $-C'' < 0$, always holds.
(2) Since $\mathbf{R}_t^{mc} = \mathbf{R}^*$ for $t < T$, $g_{i,t}^{mc} = g_i^*(\mathbf{R}_t^{mc})$. In the last period, however, investments are as described by Proposition 4 and the equilibrium quotas are suboptimally low, just as described by Proposition 5.

Proof of Proposition 8. In the default outcome (*i.e.* if the renegotiations fail), i's interim utility is

$$W_i^{\mathrm{de}} = B(g_i^{\mathrm{de}} + R_i) - C\left(q_G G_- + \sum_N g_j^{\mathrm{de}}\right) + \delta V.$$

Since i also expects $1/n$ of the renegotiation surplus, i's expected utility at the start of the period is

$$W_i^{\mathrm{de}} + \frac{1}{n}\sum_N (W_j^{\mathrm{re}} - W_j^{\mathrm{de}}) - K r_i, \tag{A.16}$$

where W_j^{re} is j's utility after renegotiation. Maximizing (A.16) w.r.t. r_i gives the first-order condition:

$$K = [B'(g_i^{\mathrm{de}} + R_i) + \delta V_R]\left(1 - \frac{1}{n}\right) + \frac{1}{n}\partial\left(\sum_N W_j^{\mathrm{re}}\right) \Big/ \partial R - \sum_{j \in N\setminus i} \frac{\delta V_R}{n}. \tag{A.17}$$

Note that V_R drops out. The term $\sum_N W_j^{\mathrm{re}}$ can be written as

$$\sum_N W_j^{\mathrm{re}} = \max_{\{g_1^{\mathrm{re}}, \ldots, g_n^{\mathrm{re}}\}} \sum_N B(g_j^{\mathrm{re}} + R_j) - C\left(q_G G_- + \sum_N g_j^{\mathrm{re}}\right) + \delta V.$$

Since the right-hand side is concave in R_i, the second-order condition of (A.17) holds. Furthermore, $\partial\left(\sum_N W_j^{\mathrm{re}}\right)/\partial R$ is not a function of g_i^{de} (as can be seen when applying the envelope theorem). Thus, if g_i^{de} increases, the right-hand side of (A.17) decreases and, to restore equality, R_i must decrease.

Proof of Proposition 9. Part (2) is proven before part (1): Renegotiation implies that the g_i^{re}s are first-best, conditional on the stocks. Furthermore, the R_is decrease in the g_i^{de}s, as shown in the previous proof. Finally, negotiating the initial quotas ensures that these are set so as to motivate the first-best R_is, if possible.

(1) First-best investments are ensured if $\partial\left(\sum W_i^{\mathrm{re}}\right)/\partial R = K$. With this, (A.17) can be rewritten as:

$$B'(g_i^{\mathrm{de}} + R_i^*) = K \Rightarrow g_i^{\mathrm{de}} = B'^{-1}(K) - R_i^*.$$

Since, for the default outcome, the continuation value is the same for every country and $\sum_N V_R$ and $\sum_N V_G$ are the same as in the first-best, implemented here, the bargaining surplus at the start of the period is independent of the stocks. Thus, V_R and V_G are the same as in the non-cooperative situation.

Proof of Proposition 10. From Propositions 6–7, we know the first-best is not possible if $T < \infty$ without renegotiation. Renegotiation must take place at least in the last period. Assume first that renegotiation is not expected in period $t < T$. It is then easy to see that $R_{i,t}$ will be first-best for the same reason as in Proposition 6, conditional on, but regardless of, the quotas $g_{i,t}^{\text{de}}$s. For $t = T$, investments are first-best if $g_{i,T}^{\text{de}} = g_i^{\text{de}}$, given by Proposition 9, for the same reasons as given there.

Regarding the emission levels, renegotiation ensures that $g_{i,T} = g_i^*(\mathbf{R})$, while, for $t < T$, $g_{i,t} = g_i^*(\mathbf{R})$ is ensured simply by setting $g_{i,t}^{\text{de}} = g_i^*$.

Assume instead that $g_{i,t}^{\text{de}}$, $t < T$, are also expected to be renegotiated in period t. If $g_{i,t}^{\text{de}} = g_i^*(\mathbf{R}^*)$, a reasoning similar to that for Proposition 9 shows that investments are first-best. Thus, the renegotiated $g_{i,t}$s will be identical to the initial contract, $g_{i,t}^{\text{de}} = g_i^*$, and such renegotiation at $t < T$ changes neither emission levels nor incentives.

Proof of Theorem.

(1) The first part is proved in the text.

(2)–(3) For each contracting environment, the above proofs derived necessary conditions for a symmetric MPE. This pinned down V_G, V_R, and a unique equilibrium.

(4) Let time $t \in \{1, \ldots, \bar{t}\}$ be finite and $V^t(G_{t-1}, R_{t-1})$ denote the continuation value for period t. Consider the proofs above: for each scenario, except for the multiperiod agreement scenario, the continuation value was derived in two steps. First, it was first proven that $\partial V^t/\partial R_{t-1} = q_R K/n$, regardless of V^{t+1}. This certainly also holds for the last period, if time is finite. Second, it was proven that $\partial V^t/\partial G_{t-1} = -q_G K(1-\delta q_R)/n$ if one could substitute for $\partial V^{t+1}/\partial R_t = q_R K/n$ for the next period. Since $\partial V^t/\partial R_{t-1} = q_R K/n$ in every period $t \in \{1, \ldots, \bar{t}\}$, $\partial V^t/\partial G_{t-1} = -q_G K(1-\delta q_R)/n$ in every period $t \le \bar{t} - 1$. Thus, the continuation value is as above for every period $t \le \bar{t} - 1$ and, anticipating this, the equilibrium play must be as described above for all periods $t \le \bar{t} - 2$. For multiperiod contracts, it was proven that, before the negotiation stage, $V_R = q_R K/n$ and $V_G = -q_G K(1-\delta q_R)/n$, independently of the continuation value following the expiration of these contracts.

Proof of Proposition 11.

(1) Follows from the proofs of (2)–(6).

(2) Although continuation values depend on i, the proofs are analogous to those above: at the emission stage, the first-order condition is (5.2). If $V_{i,G} \equiv \partial V_i/\partial G_-$ is a constant (verified below), then differentiating (5.2) gives

$$\frac{dy_i}{dR} = -\frac{C''/B''(y_i)}{1-\sum_N C''/B''(y_j)}. \tag{A.18}$$

Substituting this into the first-order condition w.r.t. $R_i \ge 0$ gives

$$\frac{B'(y_i)}{-B_i''}\left(\frac{C''}{1-\sum_N C''/B''(y_j)}\right) + \left[\frac{C'(G)-\delta\partial V_{i,t+1}/\partial G}{1-\sum_N C''/B''(y_j)}\right] \le K_i - \delta\frac{\partial V_{i,t+1}}{\partial R} \tag{A.19}$$

$$\Rightarrow B'(y_i) \le \left(K_i - \delta\frac{\partial V_{i,t+1}}{\partial R}\right)\left(\frac{1-C''/B''(y_i)}{1-\sum_N C''/B''(y_j)}\right), \tag{A.20}$$

with equality if $R_i > 0$.

Lemma A8. *The second-order condition holds if and only if (5.3) is satisfied.*

Proof. The second-order condition of (A.6) is

$$B''(y_i)\left(\frac{dy_i}{dR}\right)^2 + B'(y_i)\frac{d^2y_i}{(dR)^2} - C''(G)\left(\sum_N \frac{dy_i}{dR} - 1\right)^2 - [C'(G)-\delta V_G]\left[\sum_N \frac{d^2y_i}{(dR)^2}\right] \le 0. \tag{A.21}$$

By differentiating (A.18), we get

$$\sum \frac{d^2 y_i}{dR^2} = \frac{\sum_N [C'''/B''(y_j) - (C'')^2 B'''(y_j)/(B''(y_j))^3]}{(1-\sum_N C''/B''(y_j))^3},$$

$$\frac{d^2 y_i}{dR^2} = \frac{C'''/B''(y_i) - (C'')^2 B'''(y_i)/(B''(y_i))^3}{(1-\sum_N C''/B''(y_j))^2} + \frac{C''}{B''(y_i)}\left(\frac{\sum_N [C'''/B''(y_j) - (C'')^2 B'''(y_j)/(B''(y_j))^3]}{(1-\sum_N C''/B''(y_j))^3}\right).$$

Substituting these equations and (A.18) into (A.21) gives (5.3) after some algebra. ‖

As for the other scenarios, I assume that *some* K_i is small enough to invest a positive amount.

Assumption A1. (A.20) would be violated for $R = 0$ and $\min_i K_i$, *i.e.*

$$K \le B'(g_i)\left(\frac{1-\sum_N C''/B''(g_j)}{1-C''/B''(g_i)}\right) + \delta \frac{\partial V_{i,t+1}}{\partial R},$$

where the g_js are determined by (5.2) when $R = 0$.

In the last period, $\partial V_{i,t+1}/\partial G = \partial V_{i,t+1}/\partial R = 0$, (5.2) implies that y_i is the same for every i, and (A.20) can bind only for the smallest K_i. Then, as in the earlier proofs, $q_G G_- - R$ will be a constant, in equilibrium, and the equilibrium R will be independent of R_-. If every $i \in \underline{N}$ invests the same amount, then, for the same reasons as before, it holds for $i \in \underline{N}$ that

$$\partial r_i/\partial R_- = -q_R/\underline{n}, \quad \partial V_i/\partial R_- = q_R K/\underline{n} \quad \text{and} \quad \partial V_i/\partial G_- = -q_G K/\underline{n} + q_G \delta \partial V_{i,t+1}/\partial R. \tag{A.22}$$

On the other hand, $i \in N \backslash \underline{N}$ anticipates that $q_G G_- - R$, and thus y_i and G will be independent of R_- and G_-, implying that $\partial V_i/\partial R_- = \partial V_i/\partial G_- = 0$ for $i \in N \backslash \underline{N}$.

Finally, take *any* period $t < \bar{t}$ where $V_{i,t+1,R} \equiv \partial V_{i,t+1}/\partial R = q_R K/n$ and $\partial V_{i,t+1}/\partial G > \partial V_{j,t+1}/\partial G$, $i \in \underline{N}$, $j \in N \backslash \underline{N}$. Together with (5.2), this implies that, in period t, y_i is larger for $i \in N \backslash \underline{N}$. The first-order condition w.r.t. R_i can bind only for the smallest K_i if $B'(y_i)/(-B''(y_i))$ is decreasing in y_i, as can be seen from (A.19). Thus, $j \in N \backslash \underline{N}$ ensures that $R_j = 0$, while $R_i > 0$ for $i \in \underline{N}$ under Assumption A1. As before, $q_G G_- - R$ is going to be a constant and (A.22) holds, implying that $\partial V_i/\partial G_- = -q_G K(1-\delta q_R)/n$, while for $i \in N \backslash \underline{N}$, $\partial V_i/\partial R_- = \partial V_i/\partial G_- = 0$. This argument is certainly valid for $t = \bar{t} - 1$, and the argument will then also hold for period $t-1$. The proof is completed by induction.

If B and C are quadratic functions, B'' and C'' are constant, $B'(y_i)/(-B''(y_i))$ does indeed decrease in y_i, and (A.20) binds for $K_i = K$. Combined with (5.2), we get

$$C'(G) - \delta V_{i,t+1,G} = \left(K - \delta \frac{\partial V_i}{\partial R}\right) \frac{1-\sum_N C''/B''(y_j)}{1-C''/B''(y_i)}, \quad i \in \underline{N}. \tag{A.23}$$

If $\underline{n}$ increases, $V_{i,t+1,G}$ increases and $V_{i,t+1,R}$ decreases, implying that G must also increase. Then, however, (5.2) shows that $y_i = g_i$ decreases for $i \in N \backslash \underline{N}$. Consequently, g_i must increase in $\underline{n}$ for $i \in \underline{N}$. Furthermore, substituting (A.23) in (A.19) implies that $y_i = g_i + R_i$ must decrease in $\underline{n}$ for $i \in \underline{N}$. It follows that R_i must decrease in $\underline{n}$ for $i \in \underline{N}$. ‖

(3) Given g_i^{co}, the first-order condition w.r.t. $R_i \ge 0$ is

$$B'(g_i^{\text{co}} + R_i) + \delta V_{i,R} - K_i \le 0 \tag{A.24}$$

$$\Rightarrow R_i = \max\{0, B'^{-1}(K_i - \delta V_{i,R}) - g_i^{\text{co}}\}. \tag{A.25}$$

The second-order condition holds if $V_{i,R}$ is a constant (confirmed by the next lemma). Anticipating (A.25), the problem at the negotiation stage is

$$\max_{\{g_i^{\text{co}}\}_i} \sum_N \left[B(g_i^{\text{co}} + R_i) - (R_i - q_R R_{i,-})K_i - C\left(q_G G_- + \sum g_i^{\text{co}}\right) + \delta V_i(G, R)\right].$$

When substituting for R_i, it is clear that $R_i > 0$ (implying that $y_i = B'^{-1}(K_i - \delta V_{i,R})$) is optimal only if $K_i = K$. Thus, the first-order condition w.r.t. g_i^{co} is

$$B'(g_i^{co}) - \sum_N (C' - \delta V_{i,G}) = 0 \quad \text{if } i \in N \setminus \underline{N}; \tag{A.26}$$

$$K - \sum_N (C' - \delta V_{i,G} + \delta V_{i,R}) = 0 \quad \text{if } i \in \underline{N}. \tag{A.27}$$

The second-order conditions hold if V_i is linear. Thus, $g_i^{co} = g_i^*$ for $i \in N \setminus \underline{N}$, while combining a binding (A.24) for $i \in \underline{N}$ with (A.27) gives (5.4). The proof is completed by substituting for $V_{i,G}$ and $V_{i,R}$ from the following lemma.

Lemma A9. *For any $t < \bar{t}$, (5.1) holds.*

Proof. For the same reasons as before, $\sum_N V_{i,R} = q_R K$. Then, (A.27) pins down G. Given G, (A.26) pins down g_i^{co}, $i \in N \setminus \underline{N}$. Consequently, (A.25) and $\sum_{\underline{N}} g_i^{co} = G - q_G G_- - \sum_{N \setminus \underline{N}} g_i^{co}$ together imply that $\partial R / \partial G_- = q_G$ and that $\sum_N V_{i,G} = -q_G K + \delta q_G \sum_N V_{i,R} = -q_G K(1 - \delta q_R)$ for $t < \bar{t}$. Hence, the sum of the continuation values' derivatives are the same as in the case with no contract. This implies that the surplus from cooperation is independent of the stocks! The bargaining outcome gives i its continuation value in the case of no agreement plus $1/n$ of the (constant) total surplus. Thus, $V_{i,G}$ and $V_{i,R}$ are the same as in the case of no agreement (in other words, any necessary side transfers are independent of the stocks). ‖

(4) The proof is similar to the one above and is thus omitted.
(5) Following the proof of Proposition 8, the first-order condition w.r.t. $R_i \geq 0$ becomes

$$K_i \geq [B'(g_i^{de} + R_i) + \delta V_{i,R}](1 - 1/n) + \frac{1}{n}\partial \left(\sum_N W_j^{re} \right) / \partial R - \sum_{j \in N \setminus i} \frac{1}{n} \delta V_{j,R}, \tag{A.28}$$

with equality if $R_i > 0$. Since B is concave, the second-order condition holds as in the proof of Proposition 8. The first-best requires that (A.28) binds only if $K_i = K$ and $\partial \left(\sum_N W_i^{re} \right) / \partial R = K$. Furthermore, at the first-best, $V_{i,R}$ is as in the non-cooperative case for the same reason as in the proof of Lemma A9. With this fact, (A.28) can be written as (5.5). For $i \in N \setminus \underline{N}$, the first-best investment level (zero) is ensured by setting $g_i^{de} = g_i^*$. All this holds for any period $t < \bar{t}$.
(6) The proof is analogous to the one above and is thus omitted.

Proof of Proposition 12.

(1) The proof is by induction: I show that *if* the equilibrium is as described in Section 3 (with the same V_G and V_R) for period $t+1$, then this can be true for period t as well. I will start with one-period contracts.

Lemma A10. $R_i > 0,\ \forall i$, *if* $g_i^{co} < \widehat{g}_i \equiv B'^{-1}(K - \delta V_R)$.

Proof. For a given contract in period t, the first-order condition for $r_i \geq -R_i$ is

$$B'(g_i^{co} + R_i) - K + \delta V_R \leq 0,$$

with equality if $R_i > 0$, but $R_i = 0$ if $B'(g_i^{co}) - K + \delta V_R \leq 0 \Rightarrow g_i^{co} \geq B'^{-1}(K - \delta V_R)$. In either case, the second-order conditions hold since $B(\cdot)$ is concave. ‖

Lemma A11. *In equilibrium, $g_i^{co} < \widehat{g}_i$ if (5.6) holds.*

Proof. At the negotiation stage, it is anticipated that $g_i^{co} \geq \widehat{g}_i$ implies $R_i = 0$. In a symmetric equilibrium, $g = g_i^{co}$ solves

$$\max_{g \geq 0} \left\{ \begin{array}{c} B(B'^{-1}(K - \delta V_R)) - C(q_G G_- + ng) - K[B'^{-1}(K - \delta V_R) - g - q_R R_-/n] + \delta V \\ \text{if } g \leq B'^{-1}(K - \delta V_R), \\ B(g_{i,t}) - C(q_G G_- + ng) + \delta V \text{ if } g > B'^{-1}(K - \delta V_R) \end{array} \right\}. \tag{A.29}$$

If $g < B'^{-1}(K - \delta V_R)$, then $R_i > 0$, and increasing g reduces R_i (exactly as in the proof of Proposition 4). If $g \geq B'^{-1}(K - \delta V_R)$, $R_i = 0$, and increasing g increases $B(g)$. When $g_i^{co} < \widehat{g}_i$, the second-order condition holds since $-C$

is concave; when $g_i^{co} > \widehat{g}_i$, the second-order condition holds since B is concave. However, the utility function in (A.29) has a kink at $g = B'^{-1}(K-\delta V_R)$ and may not be single-peaked. Single-peakedness is ensured if, at $g = B'^{-1}(K-\delta V_R)$, an increase in g reduces $B(g) - C(q_G G_- + ng) + \delta V$. This is guaranteed if

$$B'(B'^{-1}(K-\delta V_R)) + n\delta V_G \leq nC'(q_G G_- + ng) \Rightarrow$$
$$K[1-\delta q_R/n - \delta q_G(1-\delta q_R)] \leq C'(q_G G_- + nB'^{-1}(K-\delta V_R)). \quad \|$$

Lemma A12. *In equilibrium, the constraint $g_i \geq 0$ does not bind.*

Proof. When (5.6) holds, the first-order condition w.r.t. g is, from (A.29),

$$-C'(q_G G_- + ng) + K + \delta V_G - \delta V_R \leq 0,$$

with equality if $g > 0$ (the second-order condition holds since $-C'' < 0$). Thus, $g \geq 0$ does not bind if

$$-C'(q_G G_-) + K + \delta V_G - \delta V_R \geq 0 \Rightarrow C'(q_G G_-) \leq (1-\delta q_G)(1-\delta q_R)K/n.$$

For the equilibrium in Proposition 4, we know that $C'(G_-) = (1-\delta q_G)(1-\delta q_R)K/n$. Since $q_G \leq 1$ and C is convex, $C'(q_G G_-) \leq C'(G_-) = (1-\delta q_G)(1-\delta q_R)K/n$. $\|$

Together, Lemmas A1–A12 show that if (5.6) holds, the equilibrium described by Propositions 3–4 continues to be an equilibrium also when the constraints $g_i \geq 0$ and $r_i \geq -R_i$ are imposed. For the same reasons as in Section 4.3, V_G and V_R are as stated by the Theorem.

For multi-period contracts, the proof is similar. To see this, note that the last period is exactly as in part (1). Compared to the last period, for every period $t < T$ investments are larger and the condition for $R_{i,t} > 0$ weaker (and it always holds when (5.6) holds). $\|$

(2) Consider first one-period contracts with renegotiation. At the renegotiation stage, countries renegotiate every $g_i \geq 0$ and these will be equal to the first-best, as before. At the investment stage, the first-order condition is

$$[B'(g_i^{de} + R_i) + \delta V_R](1-1/n) + \frac{1}{n}\partial\left(\sum_N W_j^{re}\right)\Big/\partial R - \sum_{j\in N\setminus i}\frac{1}{n}\delta V_R - K \leq 0,$$

with equality if $R_i > 0$. Define $\overline{g}_i^{de} \equiv \sup(g_i^{de}: R_i > 0)$. If $g_i^{de} < \overline{g}_i^{de}$, $R_i > 0$ and the second-order condition is as before.

At the initial negotiation stage, if g_i^{de}, as described by Proposition 9, is such that $g_i^{de} \in [0, \overline{g}_i^{de}]$, then g_i^{de} implements the first-best and these quotas represent an equilibrium contract. Note that $g_i^{de} \leq \overline{g}_i^{de}$ is indeed satisfied since $g_i^{de} > \overline{g}_i^{de}$ implies $R_i = 0$, while g_i^{de} induces first-best investments if R_i is unconstrained, and $R_i^* > 0$ by assumption. However, the g_i^{de} implementing the first-best is positive only if $B'(g_i^{de} + R_i^*) = K$ for *some* $g_i^{de} \geq 0$, which requires that $B'(R_i^*) \geq K$. The proof is analogous if there are multiple periods.

Proof of Proposition 13.

(1) As in the proof of Proposition 8, the first-order condition w.r.t. r_i is

$$k'(r_i) = [B'(g_i^{de} + R_i) + \delta V_R]\left(1 - \frac{1}{n}\right) + \frac{1}{n}\frac{\partial \sum_N W_j^{re}}{\partial R} - \sum_{j\in N\setminus i}\frac{\delta V_R}{n}. \quad \text{(A.30)}$$

Again, V_R drops out. The term $\sum_N W_j^{re}$ can be written as

$$\sum_N W_j^{re} = \max_{\{g_1^{re},\ldots,g_n^{re}\}} \sum_N B(g_j^{re} + R_j) - C\left(q_G G_- + \sum_N g_j^{re}\right) + \delta V. \quad \text{(A.31)}$$

When the first-best is implemented, $V(G,R) = \max_{R_+} W(G,R_+) - k((R_+ - q_R R)/n)$ which is concave in R (as can be seen from the envelope theorem). Thus, (A.31) is concave in R_j, implying that the second-order condition of (A.30) holds. Furthermore, $\partial\left(\sum_N W_j^{re}\right)/\partial R$ is not a function of g_i^{de}. Hence, if g_i^{de} increases, the right-hand side of (A.30) decreases and, to restore equality, R_i must decrease.

(2)–(3) Renegotiation implies that the g_i^{re}s are first-best, conditional on the stocks. Negotiating the initial quotas implies that these are set such as to induce the first-best R_is, if possible. First-best investments are ensured if $\partial\left(\sum W_i^{\text{re}}\right)/\partial R = k'(r_i^*)$. With this equality, (A.30) can be written as $B'(g_i^{\text{de}} + R_i^*) = k'(r_i^*)$. A comparison to the steady-state first-best, $B'(g_i^* + R_i^*) = (1 - \delta q_R)k'(r_i^*)$, concludes the proof.

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