Policies and Instruments for Self-Enforcing Agreements*

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Abstract

We study the optimal self-enforcing agreement based on price instruments and quantity mandates in a repeated game between countries, whose domestic firms invest in green technology before consumers emit. We find that technology must be regulated in addition to emissions, even in the absence of technological spillovers. Under a quantity agreement, emission is capped and countries must either overinvest in technology—to weaken the temptation to emit—or they must be punished unless they invest less—to maintain their willingness to retaliate on others. Under a price agreement, emission is taxed and investments subsidized. The price agreement dominates the quantity agreement because when firms are free to modify investment levels if another government defects, the punishment for defection is stronger.

Keywords: Climate change, prices vs. quantities, repeated games, self-enforcing agreements.

JEL: D86, F53, H87, Q54

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The fundamental challenge in achieving international cooperation is to motivate countries to cooperate rather than free ride. In the absence of a world government, countries may need to rely on reputation and repeated play. However, it is doubtful whether the prospect of future cooperation is sufficient to motivate cooperation in the present. A country will be tempted to increase its emissions or to reduce its abatement effort, especially when immediate retaliation is unfeasible. This will be more tempting for countries that rely on fossil fuels, but less so for countries that have invested in renewable energy. Thus, the technology levels will influence the temptation to emit.[1]

This observation leads to a number of important questions. What are the characteristics of the best agreement, if the first best cannot be attained? Should countries be required to invest more in renewable energy, so as to be less tempted to emit, or invest less, and therefore have the ability to credibly punish if others defect? Which policy implements the best self-enforcing agreement? Is a rigid quantity agreement better or worse than a flexible price agreement?

To answer these questions, Section 1 presents a game between countries. Each country is run by a benevolent government and has a large number of price-taking firms that choose how much to invest in renewables, before consumers choose fossil fuel consumption level. Under the quantity agreement, analyzed in Section 2, governments of all countries set emission quotas and investment mandates. Under the price agreement, analyzed in Section 3, governments specify emission taxes and investment subsidies. To isolate the role of technology, we abstract from technological spillovers. We assume that governments cannot commit, so that investment policies are set just before firms invest, while emission policies are set just before consumers emit. This extensive-form game is repeated indefinitely. International coordination on the best subgame-perfect equilibrium (SPE) is supported by a reversion to the "business-as-usual" Markov-perfect equilibrium (MPE).

Under a quantity agreement, there are two alternative types of distortions when the discount factor is too small to support the first best. Suppose, first, that technology is expensive relative to the cost of reducing consumption. In this case, the first-best investment level is low and the greatest temptation is to deviate from the cooperative emission quantity. To motivate compliance when the discount factor is low, countries must be allowed to emit more than the first-best level. In order to mitigate the necessity to increase the emission level, it is optimal to require countries to invest more in renewables, that is, more than they would find to be optimal *conditional* on the equilibrium emission level. By requiring overinvestment in order that countries will have an overabundance of renewable energy, the temptation to defect by emitting more is weakened, and the

[1]The importance of technology is recognized by practitioners. As stated by the IPCC (2014:1178): "*There is a distinct role for technology policy in climate change mitigation. This role is complementary to the role of policies aimed directly at reducing current GHG emissions.*"

necessary increase in the emission quantity (to motivate compliance) is smaller.

In contrast, if technology is inexpensive, or if it is costly to reduce consumption, then the first-best investment level is high and the temptation to defect is greatest at the investment stage. To reduce this temptation, countries must be required to invest less than the levels they would find to be optimal given the agreed-upon emission quantity. In principle, for a fixed emission quantity, an increase in investment leads to Pareto improvements. This, however, limits the possibility to punish, i.e., by emitting more, if another country defects, thus making it tempting to defect at the investment stage. To reduce this temptation, countries must be punished so that they will invest less.

A price agreement produces a different set of incentives. When the discount factor is large, the first best is implemented by a Pigouvian tax and no investment subsidy. If a country defected at the investment policy stage, then it will have to discourage firms from investing. When this is observed by firms in other countries, they will realize that cooperation is about to end, and as a result emissions will increase and the demand for technology will fall. Thus, firms in non-deviating countries will also invest less, and in turn emission levels in those countries will be higher at the subsequent emission policy stage. For this reason, the punishment is stronger and the temptation to defect is lower at the investment policy stage under a price agreement than under a quantity agreement.

This logic also explains why it is always more tempting under a price agreement to defect at the emission policy stage than at the investment policy stage. The harder it is to motivate compliance (i.e., the smaller is the discount factor), the lower the equilibrium emission tax will need to be. To maintain the incentive to invest in this situation, green technology investment must be subsidized, thus reducing the temptation to emit and making it possible to motivate compliance without a large increase in the permitted emission quantity.

Section 4 compares the payoffs sustained by the two designs and shows that a price agreement weakly Pareto dominates a quantity agreement. When technology is costly, the two designs are identical since the compliance constraint at the emission stage binds first under both types of agreement. In contrast, when technology is inexpensive, the price agreement is superior since the compliance constraint may bind at the investment stage under the quantity agreement, though not under the price agreement. Under a price agreement, firms in non-deviating countries invest less, and the subsequent punishment (in terms of higher emissions) will be stronger if a country defects. Under investment mandates, however, these firms will have to stick to that mandate. The comparison between the two designs therefore uncovers a novel reason to justify why flexible instruments can be preferred to rigid quantity mandates.

The importance of the sequential timing of climate policies in the optimal design of the agreement is discussed in Section 5. When climate policies are set simultaneously at the beginning of each period, technology investments do not influence compliance incentives with emission abatement, implying that there is not a strategic scope for technology to be internalized through investment regulation. In this case, a price agreement is welfare equivalent to a quantity agreement, but the equilibrium payoff is lower than under a sequential timing of climate policies.

Finally, Section 6 extends the baseline model by introducing imperfect transparency of domestic climate policies, and shows how different types of observational errors affect the design of the optimal agreement. In the presence of type II errors, a defection might go unnoticed. In this case, the temptation to defect at the investment policy stage can be the strongest also under a price agreement and investment in green technology may need to be punished through a tax when transparency is sufficiently low. In the presence of type I errors, countries might punish one another even if everyone has complied. In this case, firms can mistakenly react to what they believe are defections in other countries and induce unduly strong punishments, which lower the equilibrium payoff. Hence, flexible instruments can be detrimental for cooperation when transparency is imperfect.

Literature. The basic setup of the model follows Battaglini and Harstad (2016), which in turn draws on Harstad (2012; 2016). However, unlike the current model, they study MPEs and permit commitments to future contribution levels. Inspired by the literature on repeated games (see, Mailath and Samuelson, 2006), Barrett (1994; 2006), Dutta and Radner (2004; 2006), and Kerr, Lippert, and Lou (2020), among others, examine self-enforcing environmental agreements. However, they abstract from how technology can motivate compliance. This interaction has only been discussed previously by Harstad, Lancia, and Russo (2019) who restricted their attention to binary emission levels.[2] By allowing the emission levels to be continuous, the current analysis makes two important contributions.

First, we show that the best SPE does not necessarily require countries to overinvest in green technology (as in Harstad et al., 2019). When the technology is relatively inexpensive, one must instead punish countries for not reducing their investment in green technology. This links the study of self-enforcing environmental agreements to the industrial organization literature which looks at how firms can sustain collusion by overinvesting in capacity (see, Brock and Scheinkman, 1985, Benoit and Krishna, 1987, Davidson and Deneckere, 1990, and Compte, Jenny, and Rey, 2002). In that literature, firms overinvest in order to increase their ability to punish if another firm sells too much. This strategy is analogous to how countries may need to underinvest in green technology since that too enables the players to punish if someone defects. The strategies to underinvest (and thus increase the punishment) and to overinvest (and thus reduce one's own temptation) are

[2] In the relational contracting literature, Ramey and Watson (1997) and Halac (2015) show that technology investments can relax the compliance constraint on individual contributions to a public good; however, they focus on how an up-front investment by one party affects the hold-up problem.

mutually exclusive, but both are possible in our model, which unambiguously determines when one, rather than the other, is optimal.[3]

Second, the continuous emission level makes it meaningful to analyze the optimal price instrument. This analysis leads to the novel finding that investments must be regulated, even if there is no technological spillover, because technology influences the temptation to emit.[4] This result adds to the literature on whether two instruments are necessary in order to mitigate climate change (see, Acemoglu et al., 2012, and Golosov et al., 2014). Because a defection allows firms to react under a price agreement (by investing less), but not under a quantity agreement, we also derive a novel advantage of the flexible price instrument, relative to the rigid quantity mandate. Thus, the model contributes to the debate on the optimality of price regulation versus quantity regulation (see, Weitzman, 1974, and the subsequent literature), by studying the dilemma under a self-enforcing agreement.[5]

1. The Emission and Investment Game

1.1. *Countries and Payoffs*

Each country $i \in N \equiv \{1, ..., n\}$ is run by a benevolent government and has a large number of price-taking firms that invest in green technology—say, renewable energy—before they sell the energy they produce to consumers. At time $t \in \{1, 2, ...\}$, consumers in country i consume $y_{i,t}$ units of energy, where $r_{i,t}$ is from renewables and $g_{i,t}$ is from fossil fuels, such that $y_{i,t} = g_{i,t} + r_{i,t}$.

Variable $r_{i,t}$ can alternatively be interpreted as abatement technology, in which case $g_{i,t} = y_{i,t} - r_{i,t}$ is the actual emission level when $r_{i,t}$ units are abated. To simplify the analysis, we assume that $g_{i,t}$ and $r_{i,t}$ are perfect substitutes and there is no "brown" technology, assumptions that are relaxed in the Online Appendix.[6]

[3] Fudenberg and Tirole (1984) also separate two strategic forces: the incentive to invest in the capacity to compete, and the incentive to reduce the competitors' aggression; however, they focus on entry deterrence rather than collusion.

[4] The role of investment subsidies in the optimal environmental regulation is discussed by Fischer and Newell (2008) and Grimaud and Rouge (2008), among others.

[5] Hoel and Karp (2002) and Karp and Traeger (2020) analyze prices versus quantities in a dynamic setting with technology innovation and stock of pollution, but ignore strategic interactions between countries. Endres and Finus (2002) and Mideksa and Weitzman (2019) extend the prices versus quantities framework to a strategic setting, but neglet repeated interactions. Eichner and Pethig (2015) and Kornek and Marschinski (2018) compare prices versus quantities for environmental agreements, but consider a static coalition game and ignore the effects on compliance in international agreements.

[6] To simplify further, we do not require $g_{i,t}$ or $r_{i,t}$ to be positive. A negative $g_{i,t}$ may be feasible with carbon capture, for example. See, Harstad (2012) for how non-negative constraints can be taken into account.

The benefit from country i's energy consumption is concave and increasing in y_i up to a bliss point, $\overline{y}$:

$$B\left(y_{i,t}\right)=-\frac{b}{2}\left(\overline{y}-y_{i,t}\right)^2,$$

where $b>0$ reflects the cost of reducing energy consumption. The bliss point represents the ideal energy level if emitting is costless. Thus, a country would never produce more than $\overline{y}$ due to the implicit costs of generating or transporting the energy. It is straightforward to allow for heterogeneity in $\overline{y}$.

While the actual emission is privately beneficial, the environmental cost to each country is $c\sum_{i\in N} g_{i,t}$, where $c>0$ is the (present-discounted) marginal cost of emission. The cost of investing in green technology is:

$$K\left(r_{i,t}\right)=\frac{k}{2}r_{i,t}^2,$$

where $k>0$ is an investment cost parameter.

A government's objective is to maximize the present discounted value of its utility stream:

$$\sum_t \delta^t u\left(\mathbf{g}_t, r_{i,t}\right), \quad \text{with} \quad u\left(\mathbf{g}_t, r_{i,t}\right)\equiv B\left(g_{i,t}+r_{i,t}\right)-c\sum_{j\in N} g_{j,t}-K\left(r_{i,t}\right), \tag{1}$$

where $\delta\in(0,1)$ is the common discount factor and $\mathbf{g}_t\equiv\{g_{1,t},...,g_{n,t}\}\in\mathbb{R}^n$ is the vector of emission levels.

Stocks. One can easily permit emissions and technology to be stocks that accumulate over time, rather than outputs that depreciate from one period to the next. If the stock of greenhouse gases is accumulating, c can represent the present-discounted cost of an additional (long-lasting) unit of emission and the following analysis can be applied *mutatis mutandis*. The technology $r_{i,t}$ can also be a stock since the analysis is unchanged if the marginal cost of adding to the stock is (proportionally) larger when the existing stock is large. For details, see Battaglini and Harstad (2016) who consider the utility function (1) even though they focus on coalition formation and MPEs.

1.2. *Quantities and Prices*

In each period, private firms choose how much to invest in renewables, before consumers choose the energy consumption level. The sequential timing follows naturally when technology investment requires time to mature and become operational.[7] The

[7] The sequential timing of decisions taken by the private sector implies that there is a minimum length of time, $l\in(0,1)$, between the investment decision and the point at which the technology becomes operational. If the actual private cost of investment is, say, $\tilde{k}r^2/2$, then its present discounted value, evaluated at the time of the emission, is $kr^2/2$, with $k\equiv\delta^l\tilde{k}$. With this reformulation, we do not need to explicitly discount between the two stages within the same period.

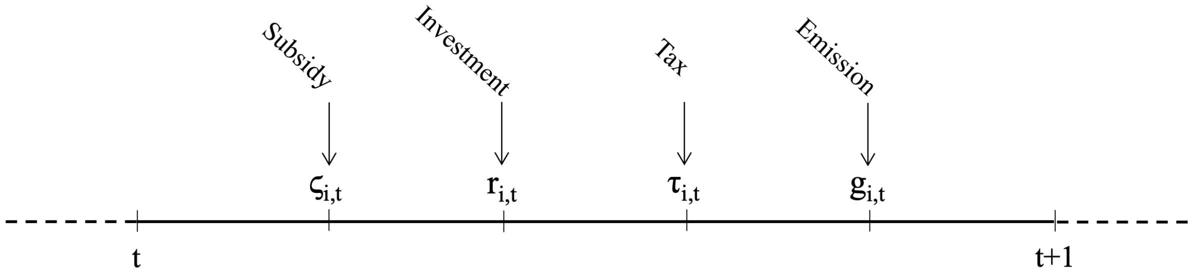

Figure 1: *Timing of the Game.*

private sector has no incentive to reduce emissions in the absence of a government intervention.

A government has the authority to regulate domestic emissions and technology investment using either *quantity* mandates or *price* instruments. In the case of the former, it sets emission and investment levels. In this case, the government is directly determining the variables of interest, and consumers and firms are not active players in the game. In the case of the latter, it specifies an emission tax, $\tau_{i,t} \in \mathbb{R}$, to be paid by consumers for each unit of fossil fuel that they consume, and an investment subsidy, $\varsigma_{i,t} \in \mathbb{R}$, to be paid to firms for each unit of investment in green technology that they make.

We assume that governments cannot commit to future policies. Thus, policies are set by governments in each period. Within a period, the investment subsidy is set just before the firms' investment in green technology, while the emission tax is set just before the consumption of fossil fuel.[8] The policies are implemented by all governments simultaneously and are observed by all. Hence, a crucial difference between the two types of regulations is that price instruments allow private firms to *flexibly* adjust investment to the anticipation of future policies, while quantity mandates do not. The sequential timing of climate policies and the private sector's responses are reported in Figure 1.

Taxes collected and subsidies paid by a government do not represent actual costs or revenues from the government's perspective. Their only effect is on the decisions to determine $g_{i,t}$ and $r_{i,t}$. After technology investments are made and carbon taxes are announced for period t, consumers choose the level of fossil fuel that maximizes the benefit of consumption net of taxes, i.e., $B(y_{i,t}) - \tau_{i,t} g_{i,t}$. It follows that:

$$B'(y_{i,t}) = \tau_{i,t} \Leftrightarrow g(\tau_{i,t}, r_{i,t}) = \overline{y} - \frac{\tau_{i,t}}{b} - r_{i,t}. \tag{2}$$

Naturally, the demand for fossil fuel is low when either the emission tax or the investment made in green technology is high.

[8]In Section 5, we show that the sequential timing of climate policies allows to sustain a higher equilibrium payoff than an alternative timing in which both climate policies are chosen simultaneously at the beginning of each period.

Firms choose the level of green technology that maximizes the benefit of production net of investment costs and subsidies, i.e., $B\left(y_{i,t}^{e}\right)-K\left(r_{i,t}\right)+\varsigma_{i,t}r_{i,t}$. At this stage, carbon taxes are yet to be announced and firms make investment decisions under the expectation that consumers' willingness to pay for renewable energy will be $\tau_{i,t}^{e}=B'\left(y_{i,t}^{e}\right)$. When firms can sell renewable energy to consumers, the marginal cost of investment in equilibrium equals the expected marginal benefit from consuming energy, plus the subsidy:

$$K'\left(r_{i,t}\right)=B'\left(y_{i,t}^{e}\right)+\varsigma_{i,t}\Leftrightarrow r\left(\tau_{i,t}^{e},\varsigma_{i,t}\right)=\frac{\tau_{i,t}^{e}+\varsigma_{i,t}}{k}. \tag{3}$$

A higher expected tax and a higher investment subsidy increase the firms' investment in green technology.[9]

1.3. *Benchmarks and Equilibria*

Business-as-Usual. Under business-as-usual (BAU), country i's investment and emission policies are set non-cooperatively to maximize $u\left(\mathbf{g}_t,r_{i,t}\right)$, as defined in (1). This outcome is equivalent to the unique MPE outcome of the repeated game and the unique SPE outcome of the stage game. At the emission stage, $u\left(\mathbf{g}_t,r_{i,t}\right)$ is maximized when i emits:

$$g_b\left(r_{i,t}\right)\equiv\overline{y}-\frac{c}{b}-r_{i,t}. \tag{4}$$

The larger $r_{i,t}$ is, the smaller will be $g_b\left(r_{i,t}\right)$, since the marginal benefit from emitting is smaller when i consumes more renewable energy.

At the investment stage, $u\left(\mathbf{g}_t,r_{i,t}\right)$ is maximized by the investment mandate:

$$r_b\left(g_{i,t}\right)\equiv\frac{b}{b+k}\left(\overline{y}-g_{i,t}\right).$$

Hence, the BAU investment and emission levels become:

$$r^b\equiv r_b\left(g^b\right)=\frac{c}{k}\quad\text{and}\quad g^b\equiv g_b\left(r^b\right)=\overline{y}-\frac{b+k}{bk}c. \tag{5}$$

According to (2) and (3), the government can also implement (5) by means of an emission tax and no investment subsidy:

$$\tau^b=c\quad\text{and}\quad\varsigma^b=0,$$

[9]Since there is a large number of price-taking firms, the impact of each firm's investment on the demand for fossil fuel is negligible and not internalized in the firms' decision problem. Optimal environmental regulation in a setting with strategic firms but without strategic interactions among countries is analyzed by Moledina et al. (2003) and Tarui and Polasky (2005).

since the firms anticipate an emission tax $\tau_{i,t}^e = \tau^b$ when they invest. The utility for each country is $u^b \equiv u\left(\mathbf{g}^b, r^b\right)$, where $\mathbf{g}^b \equiv \left\{g^b, ..., g^b\right\} \in \mathbb{R}^n$.

First Best. In the first-best case, countries coordinate on a fully enforceable agreement and policies are obtained by maximizing the sum of utilities of all countries. The first-best emission quota is:

$$g_*\left(r_{i,t}\right) \equiv \overline{y} - n\frac{c}{b} - r_{i,t},$$

and the first-best investment mandate is:

$$r_*\left(g_{i,t}\right) = r_b\left(g_{i,t}\right) = \frac{b}{b+k}\left(\overline{y} - g_{i,t}\right).$$

Hence, the first-best investment and emission levels become:

$$r^* \equiv r_*\left(g^*\right) = \frac{cn}{k} \quad \text{and} \quad g^* \equiv g_*\left(r^*\right) = \overline{y} - \frac{b+k}{bk}cn. \tag{6}$$

As in BAU, there is no need to regulate investment. Firms invest efficiently, conditional on the emission levels. Therefore, when using the price instrument, (2) and (3) imply that the government can implement (6) with a Pigouvian emission tax and no subsidy:

$$\tau^* = cn \quad \text{and} \quad \varsigma^* = 0,$$

since the emission tax anticipated by the firms is $\tau_{i,t}^e = \tau^*$. Clearly, $g^b > g^*$ and $r^b < r^*$ when $n > 1$. Because governments internalize the global emission cost in the first best, but not in BAU, the first-best utility is $u^* \equiv u\left(\mathbf{g}^*, r^*\right) > u^b$, where $\mathbf{g}^* \equiv \left\{g^*, ..., g^*\right\} \in \mathbb{R}^n$.

PROPOSITION 0. *In BAU, as well as in the first best, quantity mandates and price instruments are welfare equivalent and it is unnecessary to regulate investment.*

Equilibria. The stage game has the structure of a multilateral prisoner's dilemma since all countries are better-off in the first best than in BAU. The stage game is slightly more complex than in a standard prisoner's dilemma because each period is described by an extensive-form stage game with sequential decisions. Nevertheless, a folk theorem holds and there are many SPEs when the discount factor is large. Because countries can communicate and coordinate and they are symmetric, we henceforth characterize the symmetric SPE that maximizes (1), under the assumption that any deviation from the equilibrium path triggers a permanent reversion to BAU.[10] This assumption is made both because it enables us to illustrate the findings simply and pedagogically, and because

[10] We consider a linked enforcement strategy in which any deviation at either stage triggers a reversion to BAU. Unlinked enforcement—in which a deviation in one policy triggers a reversion to the BAU level of only that policy—is not sustainable in our context because countries would be always tempted to defect by setting investment at the efficient level, conditional on emissions. Hence, the mechanism here differs from that outlined in the literature of issue linkage in international agreements. See, Maggi (2014) for a review.

this punishment may be realistic when the countries can observe a deviation but not necessarily the identity of the deviator.[11]

2. Quantity Agreements

For an agreement to be self-enforcing, and supported by an SPE, it must be preferable to comply rather than defect. That is, the discounted payoff (1), when all countries play their equilibrium strategies, must be larger than the payoff from free riding one period before reverting to BAU. Given the extensive-form stage game, this requirement implies that the equilibrium must satisfy one "compliance constraint" at the investment stage and another at the emission stage.

Consider an equilibrium candidate in which every country emits g and invests r in every period. Each country's intertemporal value is a function of $(\mathbf{g}, r)$, with $\mathbf{g} \equiv \{g, ..., g\} \in \mathbb{R}^n$:

$$V(\mathbf{g}, r; \delta) = \frac{u(\mathbf{g}, r)}{1 - \delta}. \tag{7}$$

First, consider the temptation to defect at the investment stage. It is easy to verify that the most attractive defection is r^b, given by (5). Let $V^r(\mathbf{r}_{-i}; \delta)$ represent a country's intertemporal value when its government deviates by choosing r^b, while the governments of other countries comply with the equilibrium $\mathbf{r}_{-i} \equiv \{r, ..., r\} \in \mathbb{R}^{n-1}$. Since all other countries observe a deviation, they all revert to BAU at the subsequent emission stage. Thus, the defector will emit g^b, also given by (5), at the emission stage. This strategy leads to the continuation value $u^b/(1-\delta)$ for the deviator, plus the one-period gain thanks to the lower emission level in the non-deviating countries induced by the fact that they have invested $r \neq r^b$. As in (4), $g_b(r)$ is the non-cooperative emission level once the equilibrium investment level r is sunk. Thus,

$$V^r(\mathbf{r}_{-i}; \delta) = c(n-1)\left(g^b - g_b(r)\right) + \frac{u^b}{1-\delta}.$$

The compliance constraint at the investment stage is:

$$V(\mathbf{g}, r; \delta) \geq V^r(\mathbf{r}_{-i}; \delta). \tag{8}$$

Next, consider the temptation to free ride and defect at the emission stage. The intertemporal value from investing r and then emitting $g_b(r)$, while all other countries

[11] Mailath and Samuelson (2006) explain not only why punishments can be stronger with min-max strategies, but also why they might be weaker if the SPE must be renegotiation proof. A careful exploration of the set of SPEs under alternative assumptions is beyond the scope of the present paper.

emit $\mathbf{g}_{-i} \equiv \{g, ..., g\} \in \mathbb{R}^{n-1}$, is:

$$V^g\left(\{g_b(r), \mathbf{g}_{-i}\}, r; \delta\right) = -\frac{b}{2}\left(\overline{y} - [g_b(r) + r]\right)^2 - c\left[g_b(r) + (n-1)g\right] - \frac{k}{2}r^2 + \frac{\delta u^b}{1-\delta},$$

The compliance constraint at the emission stage is thus:

$$V(\mathbf{g}, r; \delta) \geq V^g\left(\{g_b(r), \mathbf{g}_{-i}\}, r; \delta\right). \tag{9}$$

DEFINITION 1. *The optimal quantity agreement is the vector pair* $(\mathbf{g}, \mathbf{r})$ *that maximizes the objective* (7) *s.t. the compliance constraints* (8) – (9).

If $\delta \to 1$, both (8) and (9) become $u(\mathbf{g}, r) \geq u^b$, which is trivially satisfied by any agreement that is better than BAU. Thus, when δ is sufficiently large, the first best can be sustained as the outcome of an SPE. When δ is smaller, long-term consequences become less important and it becomes tempting to defect at the investment or the emission stage. The following lemma establishes whether (8) or (9) binds first as δ falls. To formalize the condition, let $\delta_q^r(\mathbf{g}, \mathbf{r})$ and $\delta_q^g(\mathbf{g}, \mathbf{r})$ denote the levels of δ that solve $V(\mathbf{g}, r; \delta) = V^r(\mathbf{r}_{-i}; \delta)$ and $V(\mathbf{g}, r; \delta) = V^g\left(\{g_b(r), \mathbf{g}_{-i}\}, r; \delta\right)$, respectively, for any $(\mathbf{g}, \mathbf{r})$, and define $\overline{\delta}_q^g \equiv \delta_q^g(\mathbf{g}^*, \mathbf{r}^*)$ and $\overline{\delta}_q^r \equiv \delta_q^r(\mathbf{g}^*, \mathbf{r}^*)$, where $\mathbf{r}^* \equiv \{r^*, ..., r^*\} \in \mathbb{R}^n$.

LEMMA 1. *Since* $\overline{\delta}_q^r = \frac{b-k}{2b}$ *and* $\overline{\delta}_q^g = \frac{k}{b+2k}$, *then* $\overline{\delta}_q^r \leq \overline{\delta}_q^g$ *if and only if* $\frac{k}{b} \geq \frac{1}{2}$.

If $k/b \geq 1/2$, investments are costly and the cost-effective climate policy prioritizes a reduction in the consumption level. In this situation, it is more tempting to defect at the emission stage than at the investment stage, such that $\overline{\delta}_q^r \leq \overline{\delta}_q^g$. When $k/b < 1/2$, it is relatively costly to change the consumption level and the optimal investment levels are high. In this case, it is more tempting to defect at the investment stage than at the emission stage, such that $\overline{\delta}_q^r > \overline{\delta}_q^g$.

PROPOSITION 1. *The optimal quantity agreement is characterized by:*

i. *If* $\delta \geq \max\left\{\overline{\delta}_q^g, \overline{\delta}_q^r\right\}$, *all quantities are first best:*

$$g = g^* \quad \textit{and} \quad r = r^*.$$

ii. *Suppose* $k/b > 1/2$. *If* $\delta \in \left(\delta_q^r(\mathbf{g}, \mathbf{r}), \overline{\delta}_q^g\right)$, *the emission level is higher than in the first best, and the investment level is higher than is optimal, conditional on the emission level:*

$$g = g^* + \Lambda_q(\delta) > g_*(r) \quad \textit{and} \quad r = r^* = r_*(g) + \frac{b}{b+k}\Lambda_q(\delta), \textit{ where}$$

$$\Lambda_q(\delta) \equiv \frac{c(n-1)}{b}\left(1 - \delta - \sqrt{\frac{\delta(b+\delta k)}{k}}\right) > 0.$$

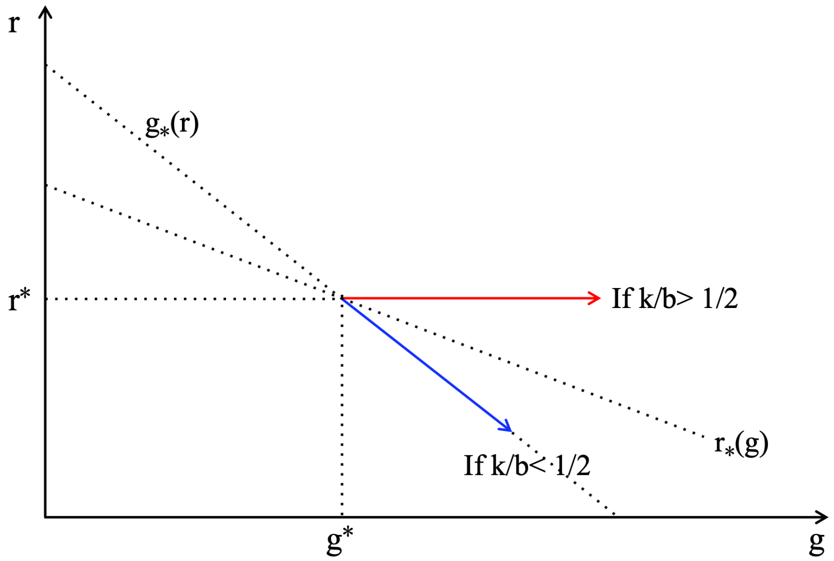

Figure 2: *As δ declines, the distortion follows the red arrow when $k/b > 1/2$ and the blue arrow when $k/b < 1/2$.*

iii. *Suppose $k/b < 1/2$. If $\delta \in \left(\delta_q^g(\mathbf{g},\mathbf{r}), \overline{\delta}_q^r\right)$, the emission level is higher than in the first best, and the investment level is lower than is optimal, conditional on the emission level:*

$$g = g_*(r) = g^* + \Xi_q(\delta) \qquad \text{and} \qquad r = r^* - \Xi_q(\delta) < r_*(g) < r^*, \text{ where}$$
$$\Xi_q(\delta) \equiv \frac{c(n-1)}{k}\left(1-\delta-\sqrt{\frac{k+\delta^2 b}{b}}\right) > 0.$$

iv. *If $\delta \leq \min\left\{\delta_q^g(\mathbf{g},\mathbf{r}), \delta_q^r(\mathbf{g},\mathbf{r})\right\}$, (8) and (9) both bind and together they determine $(\mathbf{g},\mathbf{r})$.*

Part (i) confirms that the first-best outcome is achievable when δ is large.

Part (ii) describes the optimal agreement when δ is too small to support the first best and $k/b > 1/2$. In this case, the compliance constraint at the emission stage binds and emission quotas must be larger than in the first best, leading to the distortion measured by $\Lambda_q(\delta) > 0$. With regard to the investment mandate, there exist two countervailing effects. On the one hand, a larger emission quantity discourages technology investment, since technology is a substitute for fossil fuel. On the other hand, by requiring firms to invest more in technology, the increase in the emission level necessary to motivate compliance is limited. The two effects cancel each other out, and $r = r^*$ remains unchanged as δ declines.[12] Compared to the conditional efficient level, i.e., $r_*(g)$, which is decreasing in

[12]The two effects cancel each other out not because of the quadratic formulations, but because g and r are perfect substitutes in the utility function and the marginal cost of emission is constant.

g, it is evident that investments are distorted upward by the amount $(b/(b+k))\,\Lambda_q(\delta)$. If δ falls, the optimal quantity agreement changes, as illustrated by the horizontal (red) arrow in Figure 1.

When $k/b < 1/2$, as in part (iii), the declining (blue) arrow in Figure 1 illustrates how the optimal quantity agreement changes when δ falls. If b is large, or k is small, the cost-effective agreement prioritizes a reduction in technology investment. The high level of investment makes it more tempting to deviate at the investment stage than at the emission stage. If δ is smaller, the compliance constraint requires that the investment be reduced and, in turn, emission levels will increase (even though g remains optimal conditional on r). This distortion is measured by $\Xi_q(\delta) > 0$. The inequality $r < r_*(g)$ implies that countries and firms must be *punished* if they invest as much as they find to be optimal given the level of g. This punishment is surprising at first glance, since for a fixed level of g, it is a Pareto improvement to increase $r < r_*(g)$. Nevertheless, more investment cannot be allowed and must be punished by a reversion to BAU. Intuitively, if countries were permitted to invest the optimal amount, conditional on the agreed-upon g, then non-deviating countries would be unwilling to emit at a high level as a punishment following another country's deviation. Anticipating this unwillingness, countries would be tempted to defect at the investment stage.

COROLLARY 1. *When technology is inexpensive, each country invests less than it finds to be optimal given the emission level and is punished if it invests more.*

3. Price Agreements

We now consider the optimal agreement when governments set emission taxes and investment subsidies. At the emission policy stage, the compliance constraint is equivalent to (9) because, for any given technology level, there is a one-to-one mapping between the emission tax and the emission quantity. However, the compliance constraint at the investment policy stage differs from (8) and is equal to:

$$V(\mathbf{g}, r; \delta) \geq V^r(\widetilde{\mathbf{r}}_{-i}; \delta), \tag{10}$$

where $\mathbf{r}_{-i}$ is replaced by $\widetilde{\mathbf{r}}_{-i} \equiv \{\widetilde{r}, ..., \widetilde{r}\} \in \mathbb{R}^{n-1}$ and $\widetilde{r} \equiv (\tau^b + \varsigma)/k$—which is obtained from (3) when $\tau^e = \tau^b$—is the investment level by firms in a non-deviating country after the firms have witnessed that another country has deviated from the equilibrium investment subsidy. After such a deviation, firms rationally anticipate that emissions are about to increase and that consumers' willingness to pay for renewable energy will shift from τ to τ^b. They will therefore accordingly adjust their investment level to $\widetilde{r}$ rather than to $r = (\tau + \varsigma)/k$. Thus, the subsequent emission level in a non-deviating country will be $g_b(\widetilde{r}) = g^b - \varsigma/k$, such that:

$$V^r\left(\boldsymbol{\varsigma}_{-\boldsymbol{i}}; \delta\right) = c\left(n-1\right)\frac{\varsigma}{k} + \frac{u^b}{1-\delta}. \tag{11}$$

When $\tau^b < \tau$, then $\widetilde{r} < r$, and $g_b\left(\widetilde{r}\right) > g_b\left(r\right)$. Thus, if a country defects at the investment policy stage, the subsequent emission level in a non-deviating country is larger than it would have been under a quantity agreement. There is therefore less temptation to defect at the investment policy stage under a price agreement than under a quantity agreement, since the punishment for defection is stronger in the former case.

DEFINITION 2. *The optimal price agreement is the vector pair* $(\boldsymbol{\tau}, \boldsymbol{\varsigma})$*, with* $\boldsymbol{\tau} \equiv \{\tau, ..., \tau\} \in \mathbb{R}^n$ *and* $\boldsymbol{\varsigma} \equiv \{\varsigma, ..., \varsigma\} \in \mathbb{R}^n$*, that maximizes the objective* (7) *s.t. the private sector's response functions* $(2)-(3)$ *and compliance constraints* $(9)-(10)$*.*

If $\delta \to 1$, both compliance constraints become $u\left(\mathbf{g}, r\right) \geq u^b$, which is trivially satisfied by any agreement that is better than BAU. If δ declines below some threshold, $\overline{\delta}_p^g$, it will eventually be tempting to defect at the emission policy stage, since τ^* is not optimal from a national perspective. The threshold $\overline{\delta}_p^g$ is obtained by solving $V\left(\boldsymbol{\tau}^*, \boldsymbol{\varsigma}^*; \delta\right) = V^g\left(\boldsymbol{\tau}^*, \boldsymbol{\varsigma}^*; \delta\right)$. Since the compliance constraint at the emission policy stage under a price agreement is equivalent to that under a quantity agreement, so are the critical thresholds, i.e., $\overline{\delta}_p^g = \overline{\delta}_q^g$. In addition, let $\delta_p^r(\boldsymbol{\tau}, \boldsymbol{\varsigma})$ denote the threshold level of δ that solves $V\left(\boldsymbol{\tau}, \boldsymbol{\varsigma}; \delta\right) = V^r\left(\boldsymbol{\varsigma}_{-\boldsymbol{i}}; \delta\right)$ for any $(\boldsymbol{\tau}, \boldsymbol{\varsigma})$. When $\varsigma = 0$, (7), (10), and (11) imply that the compliance constraint at the investment policy stage reduces to $u\left(\mathbf{g}, r\right) \geq u^b$. Since $\varsigma^* = 0$, we obtain Lemma 2.

LEMMA 2. *If* $\delta > \overline{\delta}_p^g$, $(\boldsymbol{\tau}, \boldsymbol{\varsigma}) = (\boldsymbol{\tau}^*, \boldsymbol{\varsigma}^*)$ *and* $(9)-(10)$ *do not bind. As* δ *falls below* $\overline{\delta}_p^g$*,* (9) *binds before* (10)*.*

Lemma 2 shows that, unlike a quantity agreement, the compliance constraint at the investment policy stage never binds first under a price agreement, even when technology is inexpensive. The result is quite intuitive. Starting from the first best, the investment subsidy is zero. If a country attempts to further discourage its firms from investing, then investment falls in all countries and the deviator ends up with the BAU utility level.

PROPOSITION 2. *The optimal price agreement is characterized by:*

i. *If* $\delta \geq \overline{\delta}_p^g$*, the emission tax is Pigouvian and investments are unregulated:*

$$\tau = \tau^* \quad \textit{and} \quad \varsigma = \varsigma^*.$$

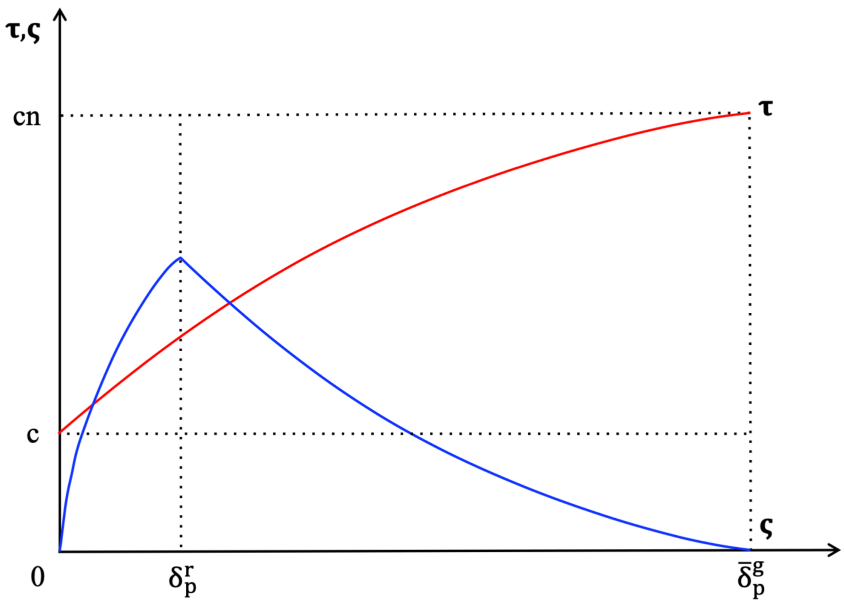

Figure 3: *The blue line represents the optimal investment subsidy while the red line represents the optimal emission tax, both as functions of the discount factor.*

ii. *If* $\delta \in \left(\delta_p^r(\boldsymbol{\tau}, \boldsymbol{\varsigma}), \bar{\delta}_p^g\right)$, *the emission tax is smaller than the Pigouvian level, and investments are subsidized:*

$$\tau = \tau^* - \Lambda_p(\delta) \quad \text{and} \quad \varsigma = \Lambda_p(\delta), \text{ where } \Lambda_p(\delta) = b\Lambda_q(\delta);$$

iii. *If* $\delta \leq \delta_p^r(\boldsymbol{\tau}, \boldsymbol{\varsigma})$, (8) *and* (9) *both bind and together they determine* $(\boldsymbol{\tau}, \boldsymbol{\varsigma})$.

Part (i) confirms that if δ is large, it is possible to implement the first-best climate policy.

Part (ii) shows that if δ is sufficiently small, then (9) binds and governments are tempted to defect by lowering the domestic emission tax. To mitigate this temptation, the required emission tax must be reduced. To maintain the incentive to invest, it becomes then necessary to subsidize investment. As before, a higher investment level reduces the temptation to defect at the emission policy stage. Consequently, the increase in the equilibrium emission level necessary to motivate compliance is smaller than if investments were unregulated.

Figure 2 illustrates the optimal price agreement as a function of δ. When the compliance constraint at the emission policy stage starts to bind, the optimal emission tax is reduced below the Pigouvian level, but the optimal investment subsidy increases. If δ is even smaller, then part (iii) states that the compliance constraint at the investment policy stage may also bind. In this case, the optimal subsidy is reduced. In fact, $\varsigma \to 0$

when $\delta \to 0$, in the optimal price agreement.[13]

4. Welfare Comparison

We have characterized the optimal climate policies for a self-enforcing agreement when governments have the authority to regulate the domestic private sector using either quantity mandates or price instruments. In both cases, technology, as well as emissions, must be regulated when the discount factor is sufficiently small in order to secure the self-enforceability of the optimal agreement. An important corollary of the analysis is that the equilibrium payoff under the price agreement is either equal to or higher than that under the quantity agreement. When $k/b > 1/2$, the equilibrium levels g and r under the price agreement are identical to those under the quantity agreement because the compliance constraint at the emission stage is the relevant one under both types of agreement. When $k/b < 1/2$, however, welfare is higher under the price agreement because it is less tempting to deviate from the optimal investment subsidy than from the investment mandate. Under a price agreement, firms in the non-deviating countries react to another country's deviation at the investment policy stage by reducing their own investment. Subsequently, emission levels will be higher, thus reducing the benefit from defecting. In contrast, investment choices under a quantity agreement are rigid and unresponsive to another country's recent deviation at the investment stage. Specifically, when $k/b < 1/2$, such that $\overline{\delta}_q^g < \overline{\delta}_q^r$, a price agreement can implement the first best for any $\delta \in \left[\overline{\delta}_q^g, \overline{\delta}_q^r\right]$, while a quantity agreement cannot. The welfare loss induced by quantities, relative to prices, increases as δ declines.

COROLLARY 2. *The price agreement weakly Pareto dominates the quantity agreement because, under the former, firms react by investing less if a country defects, while under the latter, they cannot.*

Corollary 2 provides a novel insight about the divergence between price instruments and quantity mandates in terms of welfare. Starting from Weitzman (1974), the price-vs-quantity literature has examined the performance of the two types of regulation when an abatement cost shock occurs after the authority has committed to an instrument. We abstract from such a shock and isolate the influence of climate policies on the compliance constraints. The price instrument allows firms to adjust the investment after a deviation,

[13] In the Online Appendix, we introduce the possibility of investing in a "brown" technology in addition to green technology. We show that the optimal self-enforcing climate policy involves a subsidy for green technology and a tax on brown technology. Furthermore, the distortion from the first-best climate policy is greater when the discount rate is lower and is influenced by the elasticity of substitution between the two types of technologies in a non-linear way.

whereas the quantity mandate does not. The investment adjustment turns out to be valuable because it relaxes the compliance constraints necessary to maintain the cooperative outcome.

5. Timing and Commitment

The model relies on the sequential timing of climate policies. In this section, we demonstrate that when governments set emission and investment policies simultaneously, a lower equilibrium payoff is attained. Hence, permitting a sequential timing of policy decisions is necessary in order to support the highest welfare.

To see this, consider a setting in which all governments decide upon climate policies at the beginning of each period and commit to them for the duration of the period. In this case, a government who is tempted to deviate from the agreement will always do so from both emission and investment policies. The compliance constraint is then unique and given by:

$$V(\mathbf{g}, r; \delta) \geq c(n-1)\left(g^b - g\right) + \frac{1}{1-\delta}u^b. \tag{12}$$

Incentives underlying (12) are similar to those underlying (8). That is, if the agreed-upon emission level g is lower, then the one-period gain from deviation is larger and a country's temptation to defect is stronger.

PROPOSITION 3. *When governments commit to both emission and investment policies at the beginning of each period:*

i. *The optimal price agreement is characterized by:*

If $\delta > 1/2$, $\tau = \tau^*$ *and* $\varsigma = 0$, *and if* $\delta \in [0, 1/2]$,

$$\tau = \tau^* - \tilde{\Lambda}(\delta) \quad \text{and} \quad \varsigma = 0, \text{ where } \tilde{\Lambda}(\delta) \equiv (1-2\delta)\, c\,(n-1)\,.$$

ii. *The optimal quantity agreement is characterized by:*

If $\delta > 1/2$, $g = g^*$ *and* $r = r^*$, *and if* $\delta \in [0, 1/2]$,

$$g = g^* + \frac{b+k}{bk}\tilde{\Lambda}(\delta) \quad \text{and} \quad r = r_*(g) = r^* - \frac{1}{k}\tilde{\Lambda}(\delta)\,.$$

iii. *For any* $\delta \in [0, 1)$, *the price agreement and the quantity agreement are welfare equivalent.*

Parts (i) and (ii) highlight an important implication of the model. If governments announce both climate policies before the private sector's decisions, then it is optimal to *not* regulate investment decisions, and let firms decide on r themselves. In this case, technology investments do not influence a country's incentives to comply with emission abatement. Hence, there is not a strategic scope for technology and in turn there is no need of investment regulation.

Part (iii) states that quantity and price agreements yield the same equilibrium outcome, even when the discount factor is small, since governments rely only on emission policies.[14] This observation also explains why a sequential timing of climate policies achieves a higher welfare than a simultaneous timing, since under the former, governments can use two instruments, while under the latter, they can use only one.

COROLLARY 3. *The optimal self-enforcing agreement is associated with a higher payoff when governments decide on investment and emission policies sequentially rather than simultaneously.*

6. Transparency and Flexibility

The baseline model assumes no uncertainty. In that context, a deviation at the investment stage induces a prompt reaction by firms, which correctly anticipate the breakdown of the agreement already from the current period. In this section, we introduce imperfect transparency of domestic climate policies to show how observational errors affect the design of the optimal agreement.

To show this simply, we assume that a country's domestic policy can sometimes be misperceived by other countries because a shock occurs after climate policy decisions have been made at each stage. These shocks generate two types of errors: a *type I error* occurring with probability λ, due to a country incorrectly perceiving that another has defected from the agreement, even if it has not; a *type II error* occurring with probability η, due to the fact that when a country is defecting, the defection goes undetected.[15]

For the sake of comparison, we focus on the optimal self-enforcing agreements in which any observed deviation triggers a reversion to BAU. To show how the lack of transparency

[14] The results are related to those in Tarui and Polasky (2005), who also find that price instruments and quantity mandates accomplish the same outcome in a regulatory regime based on rules (in which the authority is the initial mover), but they generate different outcomes under a discretionary regulation (in which emission regulation takes place after investment).

[15] We assume that shocks occurring at each stage are independently drawn from the same distribution and do not depend on the type of policy employed. Furthermore, while the private sector perfectly observes its own country's climate policy, it misperceives those of other countries to the same extent as its country's government. This stark signal technology is similar to that used in Ederington (2003) and Maggi (1999) and permits a simple analysis, which is however robust to a more general signal structure, such as the one in Green and Porter (1984).

modifies the optimal agreement, we separately analyze the case of type I error and the case of type II error. Clearly, the overall effect can be viewed as a combination of the two.

6.1. *Type II Errors*

We first consider the case in which $\lambda = 0$ and $\eta > 0$, that is, punishments are *not* triggered when all countries comply, while countries may comply by mistake after a deviation. Hence, when punishment is triggered, its optimal duration is $T = \infty$, which implies that the intertemporal value of cooperation remains (7).

The compliance constraint at the emission stage is as (9) apart from the fact that the continuation value following a defection is now $(1-\eta)u^b/(1-\delta) + \eta V(\mathbf{g}, r; \delta)$, since a country's defection from g goes undetected with probability η.

At the investment stage, if a country deviates from r, it will also deviate from g since any deviation is punished through a reversion to BAU. Hence, the compliance constraint at the investment stage can be written as:

$$V(\mathbf{g}, r; \delta) \geq c(n-1)\left(g^b - g_\eta(g, r)\right) + u^b + \delta\left((1-\eta^2)\frac{u^b}{1-\delta} + \eta^2 V(\mathbf{g}, r; \delta)\right), \quad (13)$$

where η^2 is the joint probability that a country's deviation from both climate policies goes undetected. After a country defects at the investment stage, each non-deviating country emits:

$$g_\eta(g, r) = \eta g + (1-\eta)\, g_b(r), \quad (14)$$

since it complies with g with probability η. Under a price agreement, r in (14) is replaced by $\widetilde{r} = (\tau^b + \varsigma)/k$, that is, the level of investment carried out by firms in non-deviating countries when they anticipate that $\tau^e = \tau^b$.

As in the case without uncertainty, the first best can be sustained when δ is higher than a threshold level obtained from solving the binding compliance constraint at either the emission or investment stage when $g = g^*$ and $r = r^*$ under each type of agreement.

LEMMA 3. *The compliance constraint binds at the* s-stage, $s \in \{g, r\}$*, for policy instrument* $\iota \in \{p, q\}$*, when* $\delta < \overline{\delta}_\iota^s$*, where:*

$$\overline{\delta}_q^g = \frac{k}{(1-\eta)\, b + (2-\eta)\, k} \quad \text{and} \quad \overline{\delta}_q^r = \frac{b - (1-2\eta)\, k}{(2-\eta^2)\, b + \eta\,(2-\eta)\, k},$$

and

$$\overline{\delta}_p^g = \overline{\delta}_q^g \quad \text{and} \quad \overline{\delta}_p^r = \frac{2\eta - 1}{\eta\,(2-\eta)}.$$

When $\eta = 0$, the threshold levels of δ in Lemma 3 coincide with those reported in the baseline model. When $\eta > 0$, however, countries may now be more tempted to deviate at the investment stage than at the emission stage under *both* a quantity and a price agreement: that is, $\overline{\delta}_q^r > \overline{\delta}_q^g$ when $\eta > \eta_q \equiv 2 - b/k$ (which requires $k/b < 1/2$) and $\overline{\delta}_p^r > \overline{\delta}_p^g$ when $\eta > \eta_p \equiv (b + 2k) / (2b + k)$. Less transparency of domestic policies induces non-deviating countries to respond less promptly to a deviation that occurs at the investment stage. From (14), we see that $g_\eta(\cdot) < g_b(\cdot)$ any time that $g < g_b(\cdot)$, which increases the gain from deviation at the investment stage. Nevertheless, Lemma 3 shows that $\max\{\overline{\delta}_q^g, \overline{\delta}_q^r\} \geq \max\{\overline{\delta}_p^g, \overline{\delta}_p^r\}$ for any η. Hence, even under uncertainty, a price agreement sustains the first best for a wider range of δ than a quantity agreement and therefore is Pareto dominant. This result hinges on the fact that the response in emissions of non-deviating countries is still stronger when using price instruments than quantity mandates since $g_b(\widetilde{r}) > g_b(r)$ and therefore, for a given g, $g_\eta(g, \widetilde{r}) > g_\eta(g, r)$. Even if welfare implications are similar to those in the baseline case, policy implications may be different, as shown in the following proposition.

PROPOSITION 4. *When* $\delta < \max\{\overline{\delta}_\iota^g, \overline{\delta}_\iota^r\}$ *for each* $\iota \in \{p, q\}$*:*

The optimal price agreement is characterized by:

P.i. *Suppose* $\eta < \eta_p$. *If* $\delta \in (\delta_p^r(\boldsymbol{\tau}, \boldsymbol{\varsigma}), \overline{\delta}_p^g]$,

$$\begin{aligned} \tau &= \tau^* - \Lambda_p^\eta(\delta) \quad \text{and} \quad \varsigma = \Lambda_p^\eta(\delta), \text{ where} \\ \Lambda_p^\eta(\delta) &= \frac{c(n-1)}{1-\eta\delta}\left(1 - \delta - \sqrt{\frac{\delta(1-\eta)((1-\eta\delta)b + (1-\eta)\delta k)}{k}}\right); \end{aligned}$$

P.ii. *Suppose* $\eta > \eta_p$. *If* $\delta \in (\delta_p^g(\boldsymbol{\tau}, \boldsymbol{\varsigma}), \overline{\delta}_p^r]$,

$$\begin{aligned} \tau &= \tau^* - \Xi_p^\eta(\delta) \quad \text{and} \quad \varsigma = -\frac{(1-\eta)(b+k)}{\eta k - (1-\eta)b}\Xi_p^\eta(\delta), \text{ where} \\ \Xi_p^\eta(\delta) &= \frac{c(n-1)(\eta k - (1-\eta)b)}{(1-\eta^2\delta)k}\left(1 - \delta - (1-\eta)\sqrt{\frac{(1+\eta\delta)^2 k + (1-\delta)^2 b}{\eta^2 k + (1-\eta)^2 b}}\right). \end{aligned}$$

The optimal quantity agreement is characterized by:

Q.i. *Suppose* $\eta < \eta_q$ *(which requires* $k/b > 1/2$*). If* $\delta \in (\delta_q^r(\mathbf{g}, \mathbf{r}), \overline{\delta}_q^g]$,

$$g = g^* + \Lambda_q^\eta(\delta) \quad \text{and} \quad r = r^*, \text{ where } \Lambda_q^\eta(\delta) = \frac{1}{b}\Lambda_p^\eta(\delta);$$

Q.ii. *Suppose $\eta > \eta_q$ (which requires $k/b < 1/2$). If $\delta \in (\delta_q^g(\mathbf{g},\mathbf{r}), \overline{\delta}_q^r]$,*

$$\begin{aligned} g &= g^* + \Xi_q^\eta(\delta) \quad \textit{and} \quad r = r^* - \frac{b}{b+\eta k}\Xi_q^\eta(\delta), \textit{ where} \\ \Xi_q^\eta(\delta) &= \frac{c(n-1)(b+\eta k)}{(1-\eta^2\delta)\,bk}\left(1-\delta-\sqrt{\frac{((1-\eta^2)\,\delta)^2\,b + ((1-\eta)\,(1+\delta\eta))^2 k}{b+\eta^2 k}}\right). \end{aligned}$$

Parts (P.i) and (Q.i) characterize the optimal agreement when transparency is sufficiently high and therefore the compliance constraint at the emission stage binds first under both types of agreement. As in the baseline case, the optimal price and quantity agreements coincide, that is, they implement the same levels of emissions and investments. A lower δ implies that emissions must be increased above g^* and investment must be raised above the conditional efficient level, $r_*(g)$.

Parts (P.ii) and (Q.ii) characterize the optimal agreement when transparency is sufficiently low. In this case, the constraint at the investment stage binds first under both types of agreement. Compliance with the agreement then requires punishing countries that invest too much in green technology in order to dampen the temptation to defect at the investment stage. In a price agreement, investments in green technology must therefore be taxed rather than subsidized.

COROLLARY 4. *When η is large, the optimal price agreement requires a tax on investment in green technology, rather than a subsidy.*

6.2. *Type I Errors*

We now consider the case in which $\lambda > 0$ and $\eta = 0$, that is, punishments *can* be triggered by mistake even if all country comply, while countries never comply after a deviation. The possibility of undue punishments implies that the duration of punishment, T, should be as short as is necessary to motivate compliance. Hence, a country's intertemporal value of cooperation differs from (7) and becomes:

$$V(\mathbf{g},\mathbf{r};\delta) = u(\mathbf{g},r) - c(n-1)\left[g_\lambda(g,r) - g\right] + \delta\left((1-(1-\lambda)^2)V^P(\mathbf{g},\mathbf{r};\delta) + (1-\lambda)^2 V(\mathbf{g},\mathbf{r};\delta)\right),$$

where $(1-\lambda)^2$ is the joint probability of observing a country complying with both climate policies, and the discounted value of punishment is:

$$V^P(\mathbf{g},\mathbf{r};\delta) = \frac{1-\delta^T}{1-\delta}u^b + \delta^T V(\mathbf{g},\mathbf{r};\delta).$$

Even if no deviation actually occurred at the investment stage, a country can incorrectly perceives that another country has defected with probability λ and in response emits $g_b(r)$. Hence, each non-deviating country emits:

$$g_\lambda(g, r) = (1-\lambda) g + \lambda g_b(r), \tag{15}$$

where r in (15) is replaced by $\widetilde{r} = (\tau^b + \varsigma)/k$ under a price agreement. The compliance constraint at the emission stage can then be written as:

$$V(\mathbf{g}, \mathbf{r}; \delta) \geq -\frac{b}{2}(\overline{y} - [g_b(r) + r])^2 - c[g_b(r) + (n-1) g_\lambda(g, r)] - \frac{k}{2}r^2 + \delta V^P(\mathbf{g}, \mathbf{r}; \delta),$$

while the compliance constraint at the investment stage is as (8) in which the continuation value $u^b/(1-\delta)$ is replaced by $u^b + \delta V^P(\mathbf{g}, \mathbf{r}; \delta)$.

The presence of type I errors implies that the first best cannot be achieved, since retaliation by non-deviating countries takes place along the equilibrium path. Furthermore, the absence of type II errors implies that any deviation at the investment stage is immediately detected and punished by non-deviating countries. Therefore, it is never more tempted to deviate at the investment stage than at the emission stage in a price agreement, although—when uncertainty is sufficiently high, i.e., $\lambda > \lambda_p^g$—the compliance constraints at both stages can simultaneously bind.

In a quantity agreement, the temptation to defect is stronger at the emission stage when $\lambda < \lambda_q^g \equiv 1 - (b/2k)^{1/4}$ (which requires $k/b > 1/2$), while it is stronger at the investment stage when $\lambda < \lambda_q^r \equiv 1 - (2k/b)^{1/2}$ (which requires $k/b < 1/2$). It follows that both constraints simultaneously bind when $\lambda > \max\{\lambda_q^g, \lambda_q^r\}$.

All binding constraints can be slackened by increasing T, that is, when δ decreases, T can be increased in order to restore the incentives to comply. We can then determine the threshold levels of δ, above which the most cooperative agreement can be sustained by the prospect of a punishment of finite length and below which $T = \infty$.

LEMMA 4. *The compliance constraint binds at the* s-stage, $s \in \{g, r\}$, *for policy instrument* $\iota \in \{p, q\}$, *and the punishment duration is* $T = \infty$, *when* $\delta < \bar{\delta}_\iota^s$, *where:*

$$\bar{\delta}_q^g = \frac{(1-\lambda)^4 k}{b + 2(1-\lambda)^4 k} \quad \text{and} \quad \bar{\delta}_q^r = \frac{(1-\lambda)^2 b - k}{2(1-\lambda)^2 b},$$

and

$$\bar{\delta}_p^g = \frac{((1-\lambda)(\lambda b - (1-\lambda) k))^2}{bk + 2((1-\lambda)(\lambda b - (1-\lambda) k))^2}.$$

Clearly, such thresholds include those in the baseline model as a special case when $\lambda = 0$. The following proposition characterizes the optimal self-enforcing agreements when T is finite (since $\delta > \bar{\delta}^s_\iota$) and the compliance constraints do not simultaneously bind (since λ is small enough).[16]

PROPOSITION 5. *When* $\delta > \max\{\overline{\delta}^g_\iota, \overline{\delta}^r_\iota\}$ *for each* $\iota \in \{p, q\}$*:*

The optimal price agreement is characterized by:

P. *Suppose* $\lambda < \lambda^g_p$*. If* $\delta \in [\overline{\delta}^g_p, 1)$*,*

$$\tau = \tau^* - \frac{c\,(n-1)\,((1-\lambda)^2\lambda b + (1-(1-\lambda)^3)\,k)}{k} \quad \textit{and} \quad \varsigma = \tau^* - \tau, \ \textit{with}$$

$$T = (\log(\delta))^{-1} \log\left(\frac{\delta bk - (1-2\delta)\,((1-\lambda)\,(\lambda b - (1-\lambda)\,k))^2}{bk + ((1-\lambda)\,(\lambda b - (1-\lambda)\,k))^2}\right) - 1.$$

The optimal quantity agreement is characterized by:

Q.i. *Suppose* $\lambda < \lambda^g_q$ *(which requires* $k/b > 1/2$*). If* $\delta \in [\overline{\delta}^g_q, 1)$*,*

$$g = g^* + \frac{c\,(n-1)\,(1-(1-\lambda)^3)}{b} \quad \textit{and} \quad r = r^*, \ \textit{with}$$

$$T = (\log(\delta))^{-1} \log\left(\frac{\delta b - (1-2\delta)\,(1-\lambda)^4 k}{b + (1-\lambda)^4 k}\right) - 1;$$

Q.ii. *Suppose* $\lambda < \lambda^r_q$ *(which requires* $k/b < 1/2$*). If* $\delta \in [\overline{\delta}^r_q, 1)$*,*

$$g = g^* + \frac{c(n-1)\,(\lambda k + (1-(1-\lambda)^2)\,b)}{bk} \quad \textit{and} \quad r = r^* - \frac{c(n-1)\,(1-(1-\lambda)^2)}{k}, \ \textit{with}$$

$$T = (\log(\delta))^{-1} \log\left(\frac{k - (1-\lambda)^2\,(1-2\delta)b}{(1-\lambda)^2\,b + k}\right) - 1.$$

Policy predictions are similar to those in the baseline case. Investment must be regulated in addition to emissions by means of a subsidy (part (P)) or a quantity mandate that induces overinvestment in green technology when $k/b > 1/2$ (part (Q.i)) or under-investment when $k/b < 1/2$ (part (Q.ii)).

Nevertheless, welfare implications may be different. A quota agreement *can* Pareto dominate a price agreement when the compliance constraint at the emission stage is the hardest to satisfy under both types of regulations, i.e., when $k/b > 1/2$.[17] By inspecting

[16] The characterization of the optimal self-enforcing agreement when T is infinite is provided in the proof of Proposition 5.

[17] Clearly, when countries must be punished for overinvestment under a quantity agreement—because they are more tempted to deviate at the investment stage, which requires $k/b < 1/2$—while the same countries would not have been punished under a price agreement, price instruments are superior.

parts (P) and (Q.i.), we see that—although investment is at the first-best level under both types of agreement—a price agreement permits countries to emit more than a quantity agreement. To understand why, suppose that emissions are required to be less than the implemented level g, say $g' < g$, but investment is forced to remain at the first-best level. To implement this allocation, the emission tax should be set higher, say at $\tau' > \tau$, and the investment subsidy lower, say at $\varsigma' < \varsigma$, since technology is expensive in the case of $k/b > 1/2$. Given these policies, firms in non-deviating countries adjust their investment to a level of $\widetilde{r}' = (\tau^b + \varsigma')/k < \widetilde{r} = (\tau^b + \varsigma)/k$ if they mistakenly perceive that a country has defected at the investment policy stage. This lower level of investment allows to punish deviations by means of higher emissions, i.e., $g_b(\widetilde{r}') > g_b(\widetilde{r})$, even if no deviation has occurred. Overall, each country faces a larger welfare loss because of unduly strong punishments at the emission policy stage following mistakenly observed deviation at the investment policy stage.[18]

COROLLARY 5. *When λ is small and technology is expensive, the quantity agreement Pareto dominates the price agreement because, under the former, firms cannot mistakenly react if all countries cooperate, while under the latter, they can.*

The computed welfare gain associated with a quantity agreement relative to a price agreement is $c^2(n-1)^2(1-\lambda)^2\lambda(2k(1-\lambda)-b\lambda)/2k^2$, a level that tends to zero as λ approaches zero, which reconciles with the result on the welfare equivalence between the two types of agreement in the absence of uncertainty when technology is expensive.

The superiority of price instruments over quantity mandates stated in Corollary 2 relies on the observation that allowing firms to flexibly adjust their investment decisions to an anticipated reversion to BAU—following a country's actual defection—permits countries to use a stronger punishment in a price agreement than in a quantity agreement and, in turn, to obtain more cooperation. The same logic applies in the case where transparency is imperfect. However, since uncertainty implies that the strong punishment can be triggered when it should not be, the welfare implication can be the opposite of that in the case without uncertainty.

7. Conclusions

International agreements must be self-enforcing. The necessity to motivate compliance influences the optimal design of the agreement. When compliance is challenging, it is crucial to strengthen the treaty to ensure that compliance is more attractive than is

[18] Since more emissions are allowed under a price agreement than under a quantity agreement, the temptation to defect from the agreement is weaker and the optimal duration of punishment is shorter in the former case compared to the latter. Nevertheless, this effect is of second order relative to the change in emissions and, as shown in Corollary 5, a quantity agreement dominates a price agreement.

defection. This implies that, under a quantity agreement, firms must overinvest in green technology if investments are costly, but underinvest if they are inexpensive. Under a price agreement, investments should be subsidized. The optimal price agreement weakly Pareto dominates the quantity agreement because it frees firms to react following another country's actual defection so the punishment for defection is stronger.

These results are policy relevant for the effectiveness of international climate treaties. There is a long-standing debate on the optimality of price instruments versus quantity mandates (discussed in our literature review). The current paper provides a novel insight in favor of price instruments when the temptation to defect is explicitly considered. This advantage can be reversed if the information and transparency are not sufficiently precise. We thus highlight a trade-off between flexibility and transparency when countries negotiate a self-enforcing agreement. This type of analysis has so far been missing from the literature.

To guarantee analytic tractability, the baseline model is kept simple. Because the model is simple, it can be used as a workhorse model and future research can and should generalize it in several directions. In this paper, we have already shown that the model can be extended to allow for observational errors and imperfect substitutability between renewables and dirty technology. So far, however, we have abstracted from fossil-fuel extraction and trade, adaptation to climate change, and renegotiation. Future research should generalize the model in these directions to obtain a deeper understanding of the optimal agreement design.

Proofs of Lemmas 1-3 are in the text. The Proof of Lemma 4 is reported in the Proof of Proposition 5.

PROOF OF PROPOSITION 1: Part (i) follows from Lemma 1. Part (ii) holds when $k/b > 1/2$ because in this case, $\overline{\delta}_q^g > \overline{\delta}_q^r$. For smaller δ that are close to $\overline{\delta}_q^g$, the optimization problem becomes:

$$V_q(\delta) \equiv \max_{\mathbf{g},r} V(\mathbf{g}, r; \delta) = \max_{g,r} \frac{1}{1-\delta}\left(-\frac{b}{2}(\overline{y} - [g+r])^2 - cng - \frac{k}{2}r^2\right), \tag{A.1}$$

subject to

$$\begin{aligned} V(\mathbf{g}, r; \delta) \geq & V^g(\{g_b(r), \mathbf{g}_{-i}\}, r; \delta) \\ & = -\frac{b}{2}(\overline{y} - [g_b(r) + r])^2 - c[g_b(r) + (n-1)g] - \frac{k}{2}r^2 + \frac{\delta u^b}{1-\delta}, \end{aligned} \tag{A.2}$$

where $g_b(r) = \overline{y} - c/b - r$. Solving for r, we obtain $r = r^*$. Substituting the equilibrium r into $V(\mathbf{g}, r; \delta) = V^g(\{g_b(r), \mathbf{g}_{-i}\}, r; \delta)$ holding with equality and solving for g, yields:

$$g = g^* + \Lambda_q(\delta), \text{ where } \Lambda_q(\delta) \equiv \frac{c(n-1)}{b}\left(1 - \delta - \sqrt{\frac{\delta(b+\delta k)}{k}}\right)$$

and $\partial \Lambda_q(\delta)/\partial\delta < 0$. Hence, $r = r^* = r_*(g) + (b/(b+k))\Lambda_q(\delta)$, where $r_*(g) = (b/(b+k))(\overline{y} - g)$. Part (iii) holds when $k/b < 1/2$ because in this case, $\overline{\delta}_q^g < \overline{\delta}_q^r$. For smaller δ that are close enough to $\overline{\delta}_q^r$, the optimization problem consists in maximizing (A.1), subject to

$$V(\mathbf{g}, r; \delta) \geq V^r(\mathbf{r}_{-i}; \delta) = c(n-1)\left(g^b - g_b(r)\right) + \frac{u^b}{1-\delta}, \tag{A.3}$$

where $g^b = \overline{y} - c/b - r^b$. Solving for r, we obtain $r = \overline{y} - cn/b - g$. Substituting the equilibrium r into $V(\mathbf{g}, r; \delta) = V^r(\mathbf{r}_{-i}; \delta)$ and solving for g, yields:

$$g = g^* + \Xi_q(\delta), \text{ where } \Xi_q(\delta) = \frac{c(n-1)}{k}\left(1 - \delta - \sqrt{\frac{k + \delta^2 b}{b}}\right)$$

and $\partial \Xi_q(\delta)/\partial\delta < 0$. Hence, $r = r^* - \Xi_q(\delta)$. Part (iv) requires (A.2) and (A.3) to be simultaneously satisfied when $\delta \leq \min\left\{\delta_q^g(\mathbf{g}, \mathbf{r}), \delta_q^r(\mathbf{g}, \mathbf{r})\right\}$, where $\delta_q^r(\mathbf{g}, \mathbf{r})$ is obtained by solving $V(\mathbf{g}, r; \delta) = V^r(\mathbf{r}_{-i}; \delta)$ with respect to δ for $(\mathbf{g}, \mathbf{r})$ determined in part (ii) and $\delta_q^g(\mathbf{g}, \mathbf{r})$ is obtained by solving $V(\mathbf{g}, r; \delta) = V^g(\{g_b(r), \mathbf{g}_{-i}\}, r; \delta)$ with respect to δ for $(\mathbf{g}, \mathbf{r})$ determined in part (iii). As $\delta \to 0$, the optimal quantities approach $g = g^b$ and $r = r^b$.

PROOF OF PROPOSITION 2: Part (i) follows from Lemma 2. To demonstrate part (ii), we solve the following optimization problem, for smaller δ that are close to $\overline{\delta}_p^g$:

$$\begin{aligned} V_p(\delta) &\equiv \max_{\boldsymbol{\tau},\boldsymbol{\varsigma}} V(\boldsymbol{\tau},\boldsymbol{\varsigma};\delta) \\ &= \max_{\tau,\varsigma} \frac{1}{1-\delta}\left(-\frac{b}{2}\left(\overline{y}-\left[g(\tau,\varsigma)+r(\tau,\varsigma)\right]\right)^2 - cng(\tau,\varsigma) - \frac{k}{2}\left(r(\tau,\varsigma)\right)^2\right), \end{aligned}$$

subject to $g(\tau,\varsigma)=\overline{y}-(b\varsigma+(b+k)\tau)/bk$, $r(\tau,\varsigma)=(\varsigma+\tau)/k$ and

$$\begin{aligned} V(\boldsymbol{\tau},\boldsymbol{\varsigma};\delta) \geq& V^g(\boldsymbol{\tau},\boldsymbol{\varsigma};\delta) \qquad \text{(A.4)} \\ &= -\frac{b}{2}\left(\overline{y}-\left[g_b(\tau,\varsigma)+r(\tau,\varsigma)\right]\right)^2 - c\left[g_b(\tau,\varsigma)+(n-1)g(\tau,\varsigma)\right] - \frac{k}{2}\left(r(\tau,\varsigma)\right)^2 + \frac{\delta u^b}{1-\delta}, \end{aligned}$$

where $g_b(\tau,\varsigma)=\overline{y}-c/b-r(\tau,\varsigma)$. Solving for ς, we obtain $\varsigma=\tau^*-\tau$. Substituting the equilibrium ς into $V(\boldsymbol{\tau},\boldsymbol{\varsigma};\delta)=V^g(\boldsymbol{\tau},\boldsymbol{\varsigma};\delta)$ and solving for τ, yields:

$$\tau=\tau^*-\Lambda_p(\delta)\,, \text{ where } \Lambda_p(\delta)=c(n-1)\left(1-\delta-\sqrt{\frac{\delta(b+\delta k)}{k}}\right)$$

and $\partial\Lambda_p(\delta)/\partial\delta<0$. Hence, $\varsigma=\Lambda_p(\delta)$. Part (iii) requires (A.4) and the compliance constraint at the investment policy stage,

$$V(\boldsymbol{\tau},\boldsymbol{\varsigma};\delta)\geq V^r(\boldsymbol{\varsigma}_{-\boldsymbol{i}};\delta)=\frac{c(n-1)}{k}\left(\varsigma-\varsigma^b\right)+\frac{u^b}{1-\delta},$$

to be simultaneously satisfied when $\delta<\delta_p^r(\boldsymbol{\tau},\boldsymbol{\varsigma})$, where $\delta_p^r(\boldsymbol{\tau},\boldsymbol{\varsigma})$ is obtained by solving $V(\boldsymbol{\tau},\boldsymbol{\varsigma};\delta)=V^r(\boldsymbol{\varsigma}_{-\boldsymbol{i}};\delta)$ with respect to δ for $(\boldsymbol{\tau},\boldsymbol{\varsigma})$ determined in part (ii). As $\delta\to 0$, the optimal prices approach $\tau=\tau^b$ and $\varsigma=\varsigma^b$.

PROOF OF COROLLARY 2: Let $\mathcal{L}_\iota(\delta)\equiv u^*/(1-\delta)-V_\iota(\delta)$ for $\iota\in\{q,p\}$, where $V_\iota(\delta)$ is reported in the proof of Propositions 1 and 2. Under a price agreement, we have:

$$\mathcal{L}_p(\delta)=\begin{cases} 0 & \text{if } \delta\in[\bar{\delta}_p^g,1) \\ (1/2b)\,\Lambda_p^2(\delta) & \text{if } \delta\in[\delta_p^r(\boldsymbol{\tau},\boldsymbol{\varsigma}),\bar{\delta}_p^g) \end{cases}.$$

Under a quantity agreement, we have:

$$\mathcal{L}_q(\delta)=\begin{cases} \begin{array}{ll} 0 & \text{if } \delta\in[\bar{\delta}_q^g,1) \\ (b/2)\,\Lambda_q^2(\delta) & \text{if } \delta\in[\delta_q^r(\mathbf{g},\mathbf{r}),\bar{\delta}_q^g) \end{array}, & \text{for } k/b>1/2 \\ \\ \begin{array}{ll} 0 & \text{if } \delta\in[\bar{\delta}_q^r,1) \\ (b/2)\,\Xi_q^2(\delta) & \text{if } \delta\in[\delta_q^g(\mathbf{g},\mathbf{r}),\bar{\delta}_q^r) \end{array}, & \text{for } k/b<1/2 \end{cases},$$

where $\Lambda_q(\delta) = (1/b)\Lambda_p(\delta)$. It follows that $\mathcal{L}_p(\delta) = \mathcal{L}_q(\delta)$ when $k/b > 1/2$ and $\mathcal{L}_p(\delta) < \mathcal{L}_q(\delta)$ when $k/b < 1/2$, since $\bar{\delta}_q^r > \bar{\delta}_p^g$ and $\Xi_q(\delta) > \Lambda_q(\delta)$ for $\delta \in [\max\{\delta_p^r(\boldsymbol{\tau}, \boldsymbol{\varsigma}), \delta_q^g(\mathbf{g}, \mathbf{r})\}, \bar{\delta}_p^g]$.

PROOF OF PROPOSITION 3: To demonstrate part (i), we derive the optimal price agreement. The threshold level of δ, above which the first best can be sustained, is $\bar{\delta} = 1/2$, which is obtained by solving $V\left(\boldsymbol{\tau}^*, \boldsymbol{\varsigma}^*; \bar{\delta}\right) = \hat{V}\left(\boldsymbol{\tau}^*_{-\boldsymbol{i}}, \boldsymbol{\varsigma}^*_{-\boldsymbol{i}}; \bar{\delta}\right)$, where $V(\boldsymbol{\tau}, \boldsymbol{\varsigma}; \delta) = (1/(1-\delta))\left(-(b/2)\left(\overline{y} - (g(\tau,\varsigma) + r(\tau,\varsigma))\right)^2 - cng(\tau,\varsigma) - (k/2)\, r(\tau,\varsigma)^2\right)$ and $\hat{V}(\boldsymbol{\tau}_{-\boldsymbol{i}}, \boldsymbol{\varsigma}_{-\boldsymbol{i}}; \delta) = c(n-1)\left(g^b - g(\tau,\varsigma)\right) + u^b/(1-\delta)$, subject to $g(\tau,\varsigma) = \overline{y} - (b\varsigma + (b+k)\tau)/bk$ and $r(\tau,\varsigma) = (\varsigma + \tau)/k$. For $\delta < 1/2$, the maximization problem becomes:

$$\tilde{V}(\delta) = \max_{\boldsymbol{\tau}, \boldsymbol{\varsigma}} V(\boldsymbol{\tau}, \boldsymbol{\varsigma}; \delta),$$

subject to:

$$V(\boldsymbol{\tau}, \boldsymbol{\varsigma}; \delta) \geq \hat{V}(\boldsymbol{\tau}_{-\boldsymbol{i}}, \boldsymbol{\varsigma}_{-\boldsymbol{i}}; \delta). \tag{A.5}$$

Solving for ς, we obtain $\varsigma = 0$. Substituting the equilibrium ς into (A.5) yields:

$$\tau = \tau^* - \tilde{\Lambda}(\delta), \text{ where } \tilde{\Lambda}(\delta) \equiv (1 - 2\delta)\, c(n-1).$$

To demonstrate part (ii), we derive the optimal quantity agreement by solving the maximization problem described in part (i) with respect to g and r. It is immediate to verify that $g = g^*$ and $r = r^*$ when $\delta > 1/2$, and

$$g = g^* + \frac{b+k}{bk}\tilde{\Lambda}(\delta) \;\text{ and }\; r = r^* - \frac{1}{k}\tilde{\Lambda}(\delta)$$

when $\delta \in [0, 1/2]$. Finally, to demonstrate part (iii), notice that the equilibrium taxes and subsidies reported in part (i) implement emissions and investment levels equal to those in part (ii). Hence, a price agreement and a quantity agreement achieve the same payoff for any $\delta \in [0, 1)$.

PROOF OF COROLLARY 3: Let $\tilde{\mathcal{L}}(\delta) \equiv u^*/(1-\delta) - \tilde{V}(\delta)$, where $\tilde{V}(\delta)$ is reported in the proof of Proposition 3:

$$\tilde{\mathcal{L}}(\delta) = \begin{cases} 0 & \text{if } \delta \in [1/2, 1) \\ (((b+k)/b)\tilde{\Lambda}(\delta) + cn)\tilde{\Lambda}(\delta)/2k & \text{if } \delta \in [0, 1/2) \end{cases},$$

Under sequential climate policy decisions, the first best can be sustained for a wider range of δ than under simultaneous climate policy decisions, since $\max\left\{\bar{\delta}_q^g, \bar{\delta}_q^r\right\} < 1/2$. Furthermore, $\tilde{\mathcal{L}}(\delta) \geq \mathcal{L}_q(\delta) \geq \mathcal{L}_p(\delta)$, where $\mathcal{L}_q(\delta)$ and $\mathcal{L}_p(\delta)$ are reported in the proof of Corollary 2. Suppose that $\tilde{\mathcal{L}}(\delta) < \mathcal{L}_q(\delta)$. This implies that $\delta > (1/2)\sqrt{b/(b+k)}$ when $k/b < 1/2$ and $\delta > (b + k + \sqrt{k(b+k)})/4(b+k)$ when $k/b > 1/2$, which cannot hold since in the former case $\delta < \bar{\delta}_q^r < (1/2)\sqrt{b/(b+k)}$, while in the latter case $\delta < \bar{\delta}_q^g <$

$(b+k+\sqrt{k(b+k)})/4(b+k)$.

PROOF OF PROPOSITION 4: First, consider a price agreement when $\delta < \max\left\{\overline{\delta}_p^r, \overline{\delta}_p^g\right\}$. Part (P.i) holds when $\eta < \eta_p \equiv (b+2k)/(2b+k)$. In this case, $\overline{\delta}_p^g > \overline{\delta}_p^r$. For smaller δ that are close enough to $\overline{\delta}_p^g$, the optimization problem is:

$$\begin{aligned} V_p^{\eta}(\delta) &\equiv \max_{\boldsymbol{\tau},\boldsymbol{\varsigma}} V(\boldsymbol{\tau},\boldsymbol{\varsigma};\delta) \qquad \text{(A.6)} \\ &= \max_{\tau,\varsigma} \frac{1}{1-\delta}\left(-\frac{b}{2}\left(\overline{y}-\left[g(\tau,\varsigma)+r(\tau,\varsigma)\right]\right)^2 - cng(\tau,\varsigma) - \frac{k}{2}r(\tau,\varsigma)^2\right), \end{aligned}$$

subject to $g(\tau,\varsigma) = \overline{y} - (b\varsigma + (b+k)\tau)/bk$ and $r(\tau,\varsigma) = (\varsigma+\tau)/k$, and

$$\begin{aligned} V(\boldsymbol{\tau},\boldsymbol{\varsigma};\delta) &\geq V^g(\boldsymbol{\tau},\boldsymbol{\varsigma};\delta) \\ &= \frac{1}{1-\delta\eta}\left(-\frac{b}{2}\left(\overline{y}-\left[g_b(\tau,\varsigma)+r(\tau,\varsigma)\right]\right)^2 - c\left[(n-1)g(\tau,\varsigma)+g_b(\tau,\varsigma)\right] - \frac{k}{2}(\tau,\varsigma)^2\right), \\ &\quad + \frac{1}{1-\delta\eta}\frac{\delta(1-\eta)u^b}{1-\delta} \end{aligned}$$

where $g_b(\tau,\varsigma) = \overline{y} - r(\tau,\varsigma) - c/b$. Solving for ς, we obtain $\varsigma = \tau^* - \tau$. Replacing the equilibrium ς into $V(\boldsymbol{\tau},\boldsymbol{\varsigma};\delta) = V^g(\boldsymbol{\tau},\boldsymbol{\varsigma};\delta)$ and solving for τ yields:

$$\tau = \tau^* - \Lambda_p^{\eta}(\delta,\eta), \text{ where } \Lambda_p^{\eta}(\delta,\eta) = \frac{c(n-1)}{1-\eta\delta}\left(1-\delta-\sqrt{\frac{\delta(1-\eta)\left((1-\eta\delta)b+(1-\eta)\delta k\right)}{k}}\right),$$

$\partial\Lambda_p^{\eta}(\delta,\eta)/\partial\delta < 0$ and $\partial\Lambda_p^{\eta}(\delta,\eta)/\partial\eta > 0$. Hence, $\varsigma = \Lambda_p^{\eta}(\delta,\eta)$. Part (P.ii) holds when $\eta > \eta_p$. In this case, $\overline{\delta}_p^g < \overline{\delta}_p^r$. For smaller δ that are close to $\overline{\delta}_p^r$, the optimization problem consists in maximizing (A.6), subject to:

$$V(\boldsymbol{\tau},\boldsymbol{\varsigma};\delta) \geq V^r(\boldsymbol{\tau}_{-i},\boldsymbol{\varsigma}_{-i};\delta) = \frac{c(n-1)}{1-\delta\eta^2}\left(g^b - g_{\eta}(g,\widetilde{r})\right) + \frac{u^b}{1-\delta},$$

where $g_{\eta}(g,\widetilde{r}) = \eta g(\tau,\varsigma) + (1-\eta)g_b(\widetilde{r})$, $g_b(\widetilde{r}) = \overline{y} - \widetilde{r} - c/b$, and $\widetilde{r} = (c+\varsigma)/k$. Solving for ς, we obtain $\varsigma = -((1-\eta)(k+b)/(\eta k - (1-\eta)b))(\tau^* - \tau)$. Replacing the equilibrium ς into $V(\boldsymbol{\tau},\boldsymbol{\varsigma};\delta) = V^r(\boldsymbol{\tau}_{-i},\boldsymbol{\varsigma}_{-i};\delta)$ and solving for τ yields:

$$\begin{aligned} \tau &= \tau^* - \Xi_p^{\eta}(\delta,\eta), \text{ where} \\ \Xi_p^{\eta}(\delta,\eta) &= \frac{c(n-1)(\eta k-(1-\eta)b)}{(1-\eta^2\delta)k}\left(1-\delta-(1-\eta)\sqrt{\frac{(1+\delta\eta)^2k+(1-\delta)^2b}{\eta^2k+(1-\eta)^2b}}\right), \end{aligned}$$

$\partial\Xi_p^{\eta}(\delta,\eta)/\partial\delta < 0$ and $\partial\Xi_p^{\eta}(\delta,\eta)/\partial\eta > 0$. Hence,

$$\varsigma = -\frac{(1-\eta)(k+b)}{\eta k-(1-\eta)b}\Xi_p^{\eta}(\delta,\eta).$$

Next, consider a quantity agreement when $\delta < \max\left\{\overline{\delta}^r_q, \overline{\delta}^g_q\right\}$. Part (Q.i) holds when $\eta < \eta_q \equiv 2 - b/k$. In this case, $\overline{\delta}^g_q > \overline{\delta}^r_q$. For smaller δ that are close to $\overline{\delta}^g_q$, the optimization problem becomes:

$$V^{\eta}_q(\delta) \equiv \max_{\mathbf{g},r} V(\mathbf{g}, r; \delta) = \max_{g,r} \frac{1}{1-\delta}\left(-\frac{b}{2}(\overline{y} - [g + r])^2 - cng - \frac{k}{2}r^2\right), \qquad \text{(A.7)}$$

subject to:

$$\begin{aligned} V(\mathbf{g}, r; \delta) &\geq V^g(\{g_b(r), \mathbf{g}_{-i}\}, r; \delta) \\ &= \frac{1}{1-\delta\eta}\left(-\frac{b}{2}(\overline{y} - [g_b(r) + r])^2 - c[g_b(r) + (n-1)g] - \frac{k}{2}r^2 + \frac{\delta(1-\eta)u^b}{1-\delta}\right), \end{aligned}$$

where $g_b(r) = \overline{y} - r - c/b$. Solving for r, we obtain $r = r^*$. Substituting the equilibrium r into $V(\mathbf{g}, r; \delta) = V^g(\{g_b(r), \mathbf{g}_{-i}\}, r; \delta)$ and solving for g, we obtain:

$$g = g^* + \Lambda^{\eta}_q(\delta, \eta), \text{ where } \Lambda^{\eta}_q(\delta, \eta) = \frac{1}{b}\Lambda^{\eta}_p(\delta, \eta),$$

$\partial\Lambda^{\eta}_q(\delta, \eta)/\partial\delta < 0$ and $\partial\Lambda^{\eta}_q(\delta, \eta)/\partial\eta > 0$. Part (Q.ii) holds when $\eta > \eta_q$. In this case, $\overline{\delta}^g_q < \overline{\delta}^r_q$. For smaller δ that are close to $\overline{\delta}^r_q$, the optimization problem consists in maximizing (A.7), subject to:

$$V(\mathbf{g}, r; \delta) \geq V^r(\mathbf{g}_{-i}, \mathbf{r}_{-i}; \delta) = \frac{c(n-1)}{1-\delta\eta^2}\left(g^b - g_{\eta}(g, r)\right) + \frac{u^b}{1-\delta},$$

where $g_{\eta}(g, r) = \eta g + (1-\eta) g_b(r)$ and $g_b(r) = \overline{y} - r - c/b$. Solving for r, we obtain $r = (b/(b + \eta k))(\overline{y} - g) - (1-\eta) cn/(b + \eta k)$. Substituting the equilibrium r into $V(\mathbf{g}, r; \delta) = V^r(\mathbf{g}_{-i}, \mathbf{r}_{-i}; \delta)$ and solving for g yields:

$$\begin{aligned} g &= g^* + \Xi^{\eta}_q(\delta, \eta), \text{ where} \\ \Xi^{\eta}_q(\delta, \eta) &= \frac{c(n-1)(b+\eta k)}{(1-\eta^2\delta)\, bk}\left(1 - \delta - \sqrt{\frac{\left((1-\eta^2)\delta\right)^2 b + ((1-\eta)(1+\delta\eta))^2 k}{b + \eta^2 k}}\right), \end{aligned}$$

$\partial\Xi^{\eta}_q(\delta, \eta)/\partial\delta < 0$ and $\partial\Xi^{\eta}_q(\delta, \eta)/\partial\eta > 0$. Hence,

$$r = r^* - \frac{b}{b + \eta k}\Xi^{\eta}_q(\delta, \eta).$$

PROOF OF PROPOSITION 5: First, consider a price agreement. To demonstrate part (P), assume that the compliance constraint at the investment policy stage is slack, while the compliance constraint at the emission policy stage binds. In this case, the maximization problem is:

$$\begin{aligned} V_p^{\lambda}(\delta) &\equiv \max_{\boldsymbol{\tau},\boldsymbol{\varsigma}} V(\boldsymbol{\tau},\boldsymbol{\varsigma};\delta) \\ &= \max_{\tau,\varsigma} \frac{1}{1-\delta} \begin{pmatrix} -\frac{b}{2}(\overline{y}-[g(\tau,\varsigma)+r(\tau,\varsigma)])^2 - cng(\tau,\varsigma) - \frac{k}{2}r(\tau,\varsigma)^2 \\ -c(n-1)[g_\lambda(g,\widetilde{r})-g(\tau,\varsigma)] - \frac{1-(1-\lambda)^2}{(1-\lambda)^2}\psi_g(\tau,\varsigma) \end{pmatrix}, \end{aligned}$$

where $\psi_g(\tau,\varsigma) \equiv (c-b(\overline{y}-[g(\tau,\varsigma)+r(\tau,\varsigma)]))^2/2b$, $g(\tau,\varsigma) = \overline{y}-(b\varsigma+(b+k)\tau)/bk$, $r(\tau,\varsigma) = (\varsigma+\tau)/k$, $g_\lambda(g,\widetilde{r}) = (1-\lambda)g(\tau,\varsigma)+\lambda(\overline{y}-\widetilde{r}-c/b)$, and $\widetilde{r} = (c+\varsigma)/k$. Solving for τ and ς, we obtain:

$$\tau = \tau^* - \frac{c(n-1)((1-\lambda)^2\lambda b + (1-(1-\lambda)^3)k)}{k} \quad \text{and} \quad \varsigma = \tau^* - \tau, \tag{A.8}$$

which implements the allocation:

$$g = g^* + c(n-1)\left(\frac{\lambda(1-\lambda)^2}{k} + \frac{1-(1-\lambda)^3}{b}\right) \quad \text{and} \quad r = r^*.$$

Combining $V^P(\boldsymbol{\tau},\boldsymbol{\varsigma};\delta) = \left(\left(1-\delta^T\right)/(1-\delta)\right)u^b + \delta^T V(\boldsymbol{\tau},\boldsymbol{\varsigma};\delta)$ and $V(\boldsymbol{\tau},\boldsymbol{\varsigma};\delta) - V^P(\boldsymbol{\tau},\boldsymbol{\varsigma};\delta) = (1/(\delta(1-\lambda)^2))\psi_g(\tau,\varsigma)$ and using (A.8), we obtain:

$$T = [\log(\delta)]^{-1}\log(\Gamma_p^g(\delta)) - 1, \tag{A.9}$$

where $\Gamma_p^g(\delta) \equiv ((1-\delta)bk)/(bk+(1-\lambda)^2(\lambda b-(1-\lambda)k)^2)+2\delta-1$. By solving $\Gamma_p^g(\delta) = 0$ with respect to δ yields:

$$\bar{\delta}_p^g = \frac{((1-\lambda)(\lambda b-(1-\lambda)k))^2}{bk+2((1-\lambda)(\lambda b-(1-\lambda)k))^2}.$$

We next show the condition under which compliance constraint at the investment policy stage is satisfied when climate policies are (A.8) and the length of punishment is (A.9), by verifying that:

$$V(\boldsymbol{\tau},\boldsymbol{\varsigma};\delta) - V^P(\boldsymbol{\tau},\boldsymbol{\varsigma};\delta) \geq \frac{\psi_r(\tau,\varsigma)}{\delta(1-\lambda)^2}, \tag{A.10}$$

where $\psi_r(\tau,\varsigma) \equiv c(n-1)\left(g^b - g_b(\widetilde{r})\right) - \left(u(g,r)-u^b\right)$, $g_b(\widetilde{r}) = \overline{y}-\widetilde{r}-c/b$, and $\widetilde{r} = (c+\varsigma)/k$. Inequality (A.10) is satisfied when $\lambda \leq \lambda_p^g$, where λ_p^g is implicitly determined from (A.10). It follows that both compliance constraints at the emission and investment policy stages simultaneously bind when $\lambda > \lambda_p^g$. When $\lambda < \lambda_p^g$, the compliance constraint at the emission policy stage is binding. For $\delta \leq \bar{\delta}_p^g$, $T = \infty$ and the optimal agreement is obtained by $\max_{\boldsymbol{\tau},\boldsymbol{\varsigma}} V(\boldsymbol{\tau},\boldsymbol{\varsigma};\delta)$, where $V(\boldsymbol{\tau},\boldsymbol{\varsigma};\delta)$ is given by:

$$
\begin{aligned}
&-\frac{1}{1-\delta(1-\lambda)^2}\left(\frac{b}{2}\left(\overline{y}-\left[g(\tau,\varsigma)+r(\tau,\varsigma)\right]\right)^2+c\left[g(\tau,\varsigma)+(n-1)\,g_\lambda(g,\widetilde{r})\right]+\frac{k}{2}r(\tau,\varsigma)^2\right)\\
&+\frac{1-(1-\lambda)^2}{1-\delta(1-\lambda)^2}\frac{\delta u^b}{1-\delta},
\end{aligned}
$$

subject to $g(\tau,\varsigma)=\overline{y}-(b\varsigma+(b+k)\tau)/bk$ and $r(\tau,\varsigma)=(\varsigma+\tau)/k$, and

$$
\begin{aligned}
V(\boldsymbol{\tau},\boldsymbol{\varsigma};\delta) &\geq V^g(\boldsymbol{\tau},\boldsymbol{\varsigma};\delta)\\
&= -\left(\frac{b}{2}\left(\overline{y}-\left[g_b(\tau,\varsigma)+r(\tau,\varsigma)\right]\right)^2+c\left[g_b(\tau,\varsigma)+(n-1)\,g_\lambda(g,\widetilde{r})\right]+\frac{k}{2}r(\tau,\varsigma)^2\right)+\frac{\delta u^b}{1-\delta}.
\end{aligned}
$$

where $g_b(\tau,\varsigma)=\overline{y}-c/b-r(\tau,\varsigma)$, $g_\lambda(g,\widetilde{r})=(1-\lambda)\,g(\tau,\varsigma)+\lambda g_b(\widetilde{r})$, $g_b(\widetilde{r})=\overline{y}-\widetilde{r}-c/b$, and $\widetilde{r}=(c+\varsigma)/k$. Solving for ς, we obtain $\varsigma=\tau^*-\tau$. Substituting the equilibrium ς into $V(\boldsymbol{\tau},\boldsymbol{\varsigma};\delta)=V^g(\boldsymbol{\tau},\boldsymbol{\varsigma};\delta)$ and solving for τ, we obtain:

$$
\begin{aligned}
\tau &= \tau^g-\frac{(1-\delta)c(n-1)(1-\lambda)^2(\lambda b-(1-\lambda)k)}{k}\\
&\quad -\frac{(1-\lambda)\,c(n-1)}{k}\sqrt{\delta(\delta(1-\lambda)^2\left((1-\lambda)^2k^2+\lambda^2b^2\right)+bk(1-2\lambda(1-\lambda)^3\delta))}
\end{aligned}
$$

where $\tau^g=\tau^*-c(n-1)\left((1-\lambda)^2\lambda b+(1-(1-\lambda)^3)k\right)/k$.

Next, consider a quantity agreement. To show part (Q.i), assume that the compliance constraint at the investment policy stage is slack, while the compliance constraint at the emission policy stage binds. In this case, the maximization problem is:

$$
V_q^\lambda(\delta)\equiv\max_{\mathbf{g},\mathbf{r}}V(\mathbf{g},\mathbf{r};\delta)=\max_{g,r}\frac{1}{1-\delta}\left(\begin{array}{c}-\frac{b}{2}\left(\overline{y}-[g+r]\right)^2-cng-\frac{k}{2}r^2\\ -c(n-1)\left[g_\lambda(g,r)-g\right]-\frac{1-(1-\lambda)^2}{(1-\lambda)^2}\psi_g(g,r)\end{array}\right),
$$

where $\psi_g(g,r)\equiv\left(c-b\left(\overline{y}-(g+r)\right)\right)^2/2b$ and $g_\lambda(g,r)=(1-\lambda)\,g+\lambda\left(\overline{y}-r-c/b\right)$. Solving for g and r, we obtain:

$$
g=g^*+\frac{c(n-1)\left(1-(1-\lambda)^3\right)}{b}\quad\text{and}\quad r=r^*. \tag{A.11}
$$

Combining $V^P(\mathbf{g},\mathbf{r};\delta)=\left(\left(1-\delta^T\right)/(1-\delta)\right)u^b+\delta^T V(\mathbf{g},\mathbf{r};\delta)$ and $V(\mathbf{g},\mathbf{r};\delta)-V^P(\mathbf{g},\mathbf{r};\delta)=(1/(\delta(1-\lambda)^2))\psi_g(g,r)$ and using (A.11), we obtain:

$$
T=[\log(\delta)]^{-1}\log\left(\Gamma_q^g(\delta)\right)-1, \tag{A.12}
$$

where $\Gamma_q^g(\delta)\equiv(\delta b-(1-2\delta)(1-\lambda)^4k)/(b+(1-\lambda)^4k)$. By solving $\Gamma_q^g(\delta)$ with respect to δ, we obtain:

$$\bar{\delta}_q^g = \frac{(1-\lambda)^4 k}{b + 2(1-\lambda)^4 k}.$$

We next show the condition under which compliance constraint at the investment policy stage is satisfied when climate policies are (A.11) and the length of punishment is (A.12). We verify that:

$$V(\mathbf{g}, \mathbf{r}; \delta) - V^P(\mathbf{g}, \mathbf{r}; \delta) > \frac{\psi_r(g,r)}{\delta(1-\lambda)^2}, \tag{A.13}$$

where $\psi_r(g,r) \equiv c(n-1)\left(g^b - g_b(r)\right) - \left(u(g,r) - u^b\right)$ and $g_b(r) = \overline{y} - r - c/b$. Inequality (A.13) is satisfied when $\lambda < \lambda_q^g \equiv 1 - (b/2k)^{1/4}$, which requires $k/b > 1/2$. To show part (Q.ii), assume that the compliance constraint at the emission policy stage is slack, while the compliance constraint at the investment policy stage binds. In this case, the maximization problem is:

$$V_q^\lambda(\delta) \equiv \max_{\mathbf{g},\mathbf{r}} V(\mathbf{g}, \mathbf{r}; \delta) = \max_{g,r} \frac{1}{1-\delta} \begin{pmatrix} -\frac{b}{2}(\overline{y} - [g+r])^2 - cng - \frac{k}{2}r^2 \\ -c(n-1)[g_\lambda(r) - g] - \frac{p}{q_r - p}\psi_r(g,r) \end{pmatrix},$$

and it is equal to:

$$g = g^* + \frac{c(n-1)((1-(1-\lambda)^2)b + \lambda k)}{bk} \quad \text{and} \quad r = r^* - \frac{c(n-1)(1-(1-\lambda)^2)}{k}. \tag{A.14}$$

Using (A.14), the optimal T is obtained by solving $V(\mathbf{g}, \mathbf{r}; \delta) - (1/(1-\delta))\,u^b = (1/(\delta(1-\delta^T)(1-\lambda)^2))\psi_r(g,r)$, which is equal to:

$$T = [\log(\delta)]^{-1} \log\left(\Gamma_q^r(\delta)\right) - 1, \tag{A.15}$$

where $\Gamma_q^r(\delta) \equiv (k - (1-\lambda)^2(1-2\delta)b)/((1-\lambda)^2 b + k)$. Solving $\Gamma_q^r(\delta)$ with respect to δ, we obtain:

$$\bar{\delta}_q^r = \frac{(1-\lambda)^2 b - k}{2(1-\lambda)^2 b}.$$

We next show under which condition compliance constraint at the emission policy stage is satisfied when climate policies are by (A.14) and the length of punishment is (A.15). We verify that:

$$V(\mathbf{g}, \mathbf{r}; \delta) - V^P(\mathbf{g}, \mathbf{r}; \delta) > \frac{\psi_g(g,r)}{\delta(1-\lambda)^2}. \tag{A.16}$$

Condition (A.16) is satisfied when $\lambda < \lambda_q^r \equiv 1 - (2k/b)^{1/2}$, which requires $k/b < 1/2$. It follows that both compliance constraint at the emission and investment policy stages simultaneously bind when $\lambda > \max\left\{\lambda_q^g, \lambda_q^r\right\}$. When $\lambda < \lambda_q^g$, the compliance constraint at the emission policy stage is binding. For $\delta \leq \bar{\delta}_q^g$, $T = \infty$ and the optimal agreement is obtained by $\max_{\mathbf{g},\mathbf{r}} V(\mathbf{g},\mathbf{r};\delta)$, where $V(\mathbf{g},\mathbf{r};\delta)$ is given by:

$$-\frac{1}{1-\delta(1-\lambda)^2}\left(\frac{b}{2}(\overline{y}-[g+r])^2+c\left[g+(n-1)\,g_\lambda(g,r)\right]+\frac{k}{2}r^2\right)+\frac{1-(1-\lambda)^2}{1-\delta(1-\lambda)^2}\frac{\delta u^b}{1-\delta},$$

subject to:

$$\begin{aligned} V(\mathbf{g},\mathbf{r};\delta) &\geq V^g(\mathbf{g},\mathbf{r};\delta) \\ &= -\left(\frac{b}{2}(\overline{y}-[g_b(r)+r])^2+c\left[g_b(r)+(n-1)\,g_\lambda(g,r)\right]+\frac{k}{2}r^2\right)+\frac{\delta u^b}{1-\delta}. \end{aligned}$$

where $g_\lambda(g,r)=(1-\lambda)\,g+\lambda(\overline{y}-r-c/b)$ and $g_b(r)=\overline{y}-r-c/b$. Solving for r, we obtain $r=r^*$. Substituting the equilibrium r into $V(\mathbf{g},\mathbf{r};\delta)=V^g(\mathbf{g},\mathbf{r};\delta)$ and solving for g, we obtain:

$$g=g^g+\frac{c(n-1)(1-\lambda)}{b}\left((1-\delta)(1-\lambda)^2-\sqrt{\frac{\delta(b+\delta k(1-\lambda)^4)}{k}}\right),$$

where $g^g=g^*+c\,(n-1)\,(1-(1-\lambda)^3)\,/b$. When $\lambda<\lambda_q^r$, the compliance constraint at the investment policy stage is binding. For $\delta\leq\bar{\delta}_q^r$, $T=\infty$ and the optimal agreement is obtained by $\max_{\mathbf{g},\mathbf{r}} V(\mathbf{g},\mathbf{r};\delta)$, subject to:

$$\begin{aligned} V(\mathbf{g},\mathbf{r};\delta) &\geq V^r(\mathbf{r}_{-i};\delta) \\ &= -\left(\frac{b}{2}\left(\overline{y}-\left[g^b+r^b\right]\right)^2+c\left[g^b+(n-1)\,g_b(r)\right]+\frac{k}{2}\left(r^b\right)^2\right)+\frac{\delta u^b}{1-\delta}, \end{aligned}$$

Solving for r, we obtain:

$$r=(\overline{y}-g)-\frac{c\,(\lambda+(1-\lambda)\,n)}{b}.$$

Substituting the equilibrium r into $V(\mathbf{g},\mathbf{r};\delta)=V^r(\mathbf{r}_{-i};\delta)$ and solving for g, we obtain:

$$g=g^r+\frac{c(n-1)(1-\lambda)}{k}\left((1-\delta)(1-\lambda)-\sqrt{\frac{(\delta^2(1-\lambda)^2b+k)}{b}}\right),$$

where $g^r=g^*+c(n-1)((1-(1-\lambda)^2)b+\lambda k)/bk$. Hence,

$$r=r^r-\frac{c(n-1)(1-\lambda)}{k}\left((1-\delta)(1-\lambda)-\sqrt{\frac{(\delta^2(1-\lambda)^2b+k)}{b}}\right).$$

where $r^r=r^*-c(n-1)(1-(1-\lambda)^2)/k$.

PROOF OF COROLLARY 5: When $\lambda<\lambda_q^g$, which requires $k/b>1/2$, we obtain

$V_q^\lambda(\delta) - V_p^\lambda(\delta) = c^2(n-1)^2(1-\lambda)^2\lambda(2k(1-\lambda) - b\lambda)/2k^2 > 0$, where the values $V_q^\lambda(\delta)$ and $V_p^\lambda(\delta)$ are reported in the proof of Proposition 5.

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ONLINE APPENDIX: MULTIPLE TECHNOLOGIES

In the baseline model, country can invest only in green technology, which reduces the marginal value of emitting. In reality, energy can be produced from various sources, some of which are brown, such as drilling technology, which is beneficial in the extraction and consumption of fossil fuel and therefore complementary to emitting.[19] We can then expand the baseline model by permitting countries to invest in a technology portfolio, which includes clean and renewable technology r_{Ci} and brown technology r_{Bi}. The benefit function of country i is modified as follows:

$$B(g_i, r_{Bi}, r_{Ci}) = -\frac{b}{2}\left(\overline{y} - [g_i + r_{Ci}]\right)^2 - \frac{q}{2}\left(g_i - r_{Bi}\right)^2 + z r_{Ci} r_{Bi},$$

with $q > 0$ and $z \in (\underline{z}, \overline{z})$ measures the elasticity of substitution between the two types of technologies. The term $-(q/2)(g_i - r_{Bi})^2$ represents the cost of extracting fossil fuel beyond the capacity level r_{Bi}. The capacity to provide fossil fuel is brown since it reduces the cost of providing g_i and thus increases the net benefit from consuming fossil fuel. Brown technologies and clean technologies can be interdependent. The parameter z is positive when energy produced from, for example, solar or wind power is complemented by traditional fossil fuel sources in order to ensure a constant flow of electricity. In contrast, the case of substitute technologies arises when $z \leq 0$. We allow the cost of investment $(k_\sigma/2)\, r_{\sigma i}^2$ for each $\sigma \in \{C, B\}$, where $k_\sigma > 0$, to vary across technologies.

LEMMA O.1: *When $n > 1$, we have $g^* < g^b$, $r_B^* > (\leq)\, r_B^b$ for $z > (\leq)\, (b + k_C)q/b$ and $r_C^* > (\leq)\, r_C^b$ for $z < (\geq)\, (q + k_B)b/q$.*

Clearly, the first-best level of emissions is lower than the BAU level, since the global emission cost is internalized only in the first best. The first-best levels of investment in technologies, however, can be higher or lower than the BAU levels depending on the type of technology and on the degree of interdependence between types. If technologies are substitutes, namely $z \leq 0$, then there will be more investment in clean technologies and less in brown ones in the first-best case relative to the case in which neither emissions nor investment are internationally negotiated. If technologies are complements, namely $z > 0$, then investment in both types of technologies must be either larger (when $z > (b+k_C)q/b$) or smaller (when $z \geq (q + k_B)b/q$) in the first best than in BAU.[20]

We have shown that global welfare is higher in a price agreement than in a quantity agreement in the absence of uncertainty. Clearly, the result also holds true in the case

[19] In Harstad, Lancia, and Russo (2019), we studied how technologies of different types affect incentives to comply with emissions when g is a binary variable, and thus we abstracted from climate policy implications.

[20] The former occurs when $q/b < \sqrt{(q + k_B)/(b + k_C)}$, that is, when the use of clean technologies is more effective than the use of brown technologies, while the latter occurs in the opposite case.

of multiple technologies. Therefore, we focus here on the characterization of the optimal self-enforcing agreements when national governments use price instruments. As shown in the paper, it is optimal to implement a Pigouvian emission tax equal to $\tau^* = cn$ and investment subsidies for brown and clean technologies equal to $\varsigma_B^* = \varsigma_C^* = 0$, when δ is sufficiently large. As δ becomes smaller, emissions must be allowed to increase in order to provide incentives for governments to comply with the agreement by reducing the emission tax. However, the dampening of the emission tax is partially mitigated by the introduction of subsidies for investment in clean technologies and by taxes imposed on firms investing in dirty technologies. The fiscal distortion generated by the friction of limited enforcement is quantified in the following proposition.

PROPOSITION O.1: *If* $\delta \in (\delta_p^r(\boldsymbol{\tau}, \boldsymbol{\varsigma_B}, \boldsymbol{\varsigma_C}), \overline{\delta}_p^g]$*, the optimal price agreement is characterized by:*

$$\tau^* - \tau = \Lambda_m(\delta, z) > 0,$$

and

$$\varsigma_B - \varsigma_B^* = -(q/(b+q))\Lambda_m(\delta, z) \;\; \text{and} \;\; \varsigma_C - \varsigma_C^* = (b/(b+q))\Lambda_m(\delta, z),$$

where $\partial\Lambda_m(\delta, z)/\partial\delta < 0$ *and*

$$\frac{\partial\Lambda_m(\delta, z)}{\partial z} \begin{cases} \geq 0 & if \quad z \leq \min\left\{(b+k_C)\frac{q}{b}, (q+k_B)\frac{b}{q}\right\} \\ < 0 & if \quad z > \min\left\{(b+k_C)\frac{q}{b}, (q+k_B)\frac{b}{q}\right\} \end{cases}.$$

Proposition O.1 provides new insights by generalizing the results of Proposition 2 in the paper to an environment in which countries invest in a technology portfolio. Tax distortions from first-best policies, denoted by $\Lambda_m(\delta, z)$, are greater when the discount rate is lower. Such distortions are also affected by the elasticity of substitution between brown technologies and clean technologies in a non-linear way. In particular, $\Lambda_m(\delta, z)$ is the largest when $z = \min\{(b+k_C)q/b, (q+k_B)b/q\}$, since either $r_B^* = r_B^b$ or $r_C^* = r_C^b$. Intuitively, when $r_\sigma^* = r_\sigma^b$ for some σ, it is optimal to implement policies so that the domestic private sector responds by investing in technology σ at the first-best level, i.e., $r_\sigma(\tau, \varsigma_B, \varsigma_C) = r_\sigma^*$, which restricts the ability of the two complementary investment policies, ς_B and ς_C, to curb fiscal distortions.[21]

MATHEMATICAL ONLINE APPENDIX

[21] Acemoglu et al. (2016) develop a growth model in which dirty and clean technologies compete in each of many product lines. They find that a shift toward clean technology is possible only when the two energy technologies are not complementary. In contrast, we find that global emissions can be reduced also when technologies are complementary by means of an optimal combination of taxes on dirty technology and subsidies for clean technology, whose respective amounts depend on the degree of complementarity between the two types of technologies.

PROOF OF LEMMA O.1:

The first-best levels of emission and investment in brown and clean technologies are obtained by solving:

$$\max_{g,r_B,r_C} -\frac{b}{2}\left(\overline{y}-[g+r_C]\right)^2-\frac{q}{2}\left(g-r_B\right)^2+zr_Cr_B-cng-\frac{k_B}{2}r_B^2-\frac{k_C}{2}r_C^2,$$

and are equal to:

$$\begin{aligned} g^* &= \left(\left((q+k_B)k_C+(q-z)\,z\right)\overline{y}b+cn\left(z^2-(q+k_B)(b+k_C)\right)\right)/\varrho, \\ r_B^* &= \left((z+k_C)\,\overline{y}bq-cn\left((b+k_C)q-bz\right)\right)/\varrho, \text{ and} \\ r_C^* &= \left((z+k_B)\,\overline{y}bq+cn\left((q+k_B)b-qz\right)\right)/\varrho, \end{aligned}$$

where $\varrho \equiv (b+q)\left(k_Bk_C-z^2\right)+bq\left(2z+k_B+k_C\right)>0$ for $z\in(\underline{z},\overline{z})$ with

$$\begin{aligned} \underline{z} &\equiv \frac{bq-\sqrt{\left(bq+(b+q)\,k_B\right)\left(bq+(b+q)\,k_C\right)}}{b+q} \text{ and} \\ \overline{z} &\equiv \frac{bq+\sqrt{\left(bq+(b+q)\,k_B\right)\left(bq+(b+q)\,k_C\right)}}{b+q}. \end{aligned}$$

Since $z<\overline{z}$, then $g^b>g^*$. Furthermore, $r_B^*(n)>(\leq)\,r_B^*(1)=r_B^b$ when $z>(\leq)\,(b+k_C)q/b$ and $r_C^*(n)>(\leq)\,r_C^*(1)=r_C^b$ when $z<(\geq)\,(q+k_B)b/q$.

PROOF OF PROPOSITION O.1:

The optimal self-enforcing price agreement is obtained by solving $\max_{\boldsymbol{\chi}} V(\boldsymbol{\chi};\delta)$, where $\boldsymbol{\chi}\equiv(\boldsymbol{\tau},\boldsymbol{\varsigma_B},\boldsymbol{\varsigma_C})$ and $V(\boldsymbol{\chi};\delta)$ is equal to:

$$-\frac{1}{1-\delta}\left(\frac{b}{2}\left(\overline{y}-[g+r_C(\chi)]\right)^2+\frac{q}{2}\left(g-r_B(\chi)\right)^2-zr_C(\chi)\,r_B(\chi)+cng(\chi)+\sum_\sigma\frac{k_\sigma}{2}r_\sigma^2(\chi)\right)$$

subject to:

$$\begin{aligned} V(\boldsymbol{\chi};\delta) \;\geq\; & V^g(\boldsymbol{\chi};\delta)=-\frac{b}{2}\left(\overline{y}-[g_b(\cdot)+r_C(\chi)]\right)^2-\frac{q}{2}\left(g_b(\cdot)-r_B(\chi)\right)^2+zr_B(\chi)\,r_C(\chi) \\ & -c(n-1)\,g(\chi)-cg_b(\cdot)-\sum_\sigma\frac{k_\sigma}{2}r_\sigma^2(\chi)+\frac{\delta}{1-\delta}u^b, \end{aligned}$$

where $g_b(\cdot)=(b\overline{y}-br_C+qr_B-c)\,/\,(q+b)$, and

$$V(\boldsymbol{\chi};\delta)\geq V^r(\boldsymbol{\chi_{-i}};\delta)=c(n-1)\left(g^b-g_b(\cdot)\right)+\frac{u^b}{1-\delta}, \text{ where} \tag{O.1}$$

$$u^b=-\frac{b}{2}\left(\overline{y}-\left[g^b+r_C^b\right]\right)^2-\frac{q}{2}\left(g^b-r_B^b\right)^2+zr_B^br_C^b-cng^b-\sum_\sigma\frac{k_\sigma}{2}\left(r_\sigma^b\right)^2(\chi),$$

$g^b = g^*$, $r_B^b = r_B^*$ and $r_C^b = r_C^*$ for $n = 1$. The optimal private responses $g(\chi)$, $r_B(\chi)$ and $r_C(\chi)$ are obtained by solving:

$$b(\overline{y} - (g + r_C)) - q(g - r_B) - \tau = 0, \tag{O.2}$$

$$q(g - r_B) + zr_C - k_B r_B + \varsigma_B = 0, \tag{O.3}$$

$$b(\overline{y} - (g + r_C)) + zr_B - k_C r_C + \varsigma_C = 0. \tag{O.4}$$

The function $g_b(\cdot) = (b\overline{y} - b\widetilde{r}_C + q\widetilde{r}_B - c)/(q + b)$ in (O.1) is the optimal domestic level of emission conditioned on investment $\widetilde{r}_B$ and $\widetilde{r}_C$, which are obtained by solving (O.2)-(O.4) when $\tau = \tau^b$. Replacing g^b and $g_b(\cdot)$ into (O.1), we obtain:

$$V(\boldsymbol{\chi}; \delta) \geq V^r(\boldsymbol{\chi_{-i}}; \delta) = \frac{c(n-1)}{\varrho}(b((-q+z)\varsigma_B + (q+k_B)\varsigma_C) - q(k_C\varsigma_B + z\varsigma_C)) + \frac{u^b}{1-\delta}.$$

Since $\varsigma_B^* = \varsigma_C^* = 0$, we have that $V^r(\boldsymbol{\chi}_{-i}^*; \delta) = u^b/(1-\delta)$ with $\boldsymbol{\chi}^* \equiv (\boldsymbol{\tau}^*, \boldsymbol{\varsigma_B^*}, \boldsymbol{\varsigma_C^*})$, which implies that $V(\boldsymbol{\chi}^*; \delta) > V^r(\boldsymbol{\chi}_{-i}^*; \delta)$ is always satisfied. Let $\delta_p^g(\boldsymbol{\chi})$ denote the level of δ that solves $V(\boldsymbol{\chi}; \delta) = V^g(\boldsymbol{\chi}; \delta)$ and $\overline{\delta}_p^g \equiv \delta_p^g(\boldsymbol{\chi}^*)$, which is equal to:

$$\overline{\delta}_p^g = \frac{(b+q)k_B k_C + bq(k_B + k_C) + 2bqz - (b+q)z^2}{b^2(q+k_B) + q((2k_B+q)k_C - 2z^2) + b((2k_C+q)q + 2((q+k_C)k_B - (z-q)z))}.$$

If $\delta > \overline{\delta}_p^g$, then neither the compliance constraint at the emission policy stage nor the compliance constraint at the investment policy stage binds when taxes and subsidies are set to χ^*. Hence, the optimal self-enforcing price agreement is characterized by $\tau = \tau^*$ and $\varsigma_B = \varsigma_C = 0$. If $\delta \leq \overline{\delta}_p^g$, then the compliance constraint at the emission policy stage binds when $\chi = \chi^*$. Since $\partial V^g(\boldsymbol{\chi}; \delta)/\partial\varsigma_B = \partial V(\boldsymbol{\chi}; \delta)/\partial\varsigma_B$ and $\partial V^g(\boldsymbol{\chi}; \delta)/\partial\varsigma_C = \partial V(\boldsymbol{\chi}; \delta)/\partial\varsigma_C$, the equilibrium policies $\varsigma_B(\tau)$ and $\varsigma_C(\tau)$ are obtained by solving $\partial V(\boldsymbol{\chi}; \delta)/\partial\varsigma_B = 0$ and $\partial V(\boldsymbol{\chi}; \delta)/\partial\varsigma_C = 0$ and are equal to $\varsigma_B(\tau) = (q/(b+q))(\tau - cn)$ and $\varsigma_C(\tau) = (b/(b+q))(cn - \tau)$. Substituting $\varsigma_B(\tau)$ and $\varsigma_C(\tau)$ into $V(\boldsymbol{\chi}; \delta) = V^g(\boldsymbol{\chi}; \delta)$ and solving for τ, we obtain:

$$\begin{aligned}
\tau &= \tau^* - \Lambda_m(\delta, z), \text{ where} \\
\Lambda_m(\delta, z) &= (n-1)c\left((1-\delta) + \sqrt{\frac{\delta q(k_C(q+k_B\delta) - \delta z^2)}{(b+q)k_B k_C + qb(k_B+k_C) + 2bqz - (b+q)z^2}}\right) \\
&\quad - (n-1)c\sqrt{\frac{\delta((b^2(q+k_B) + bq(q-2z)) + b\delta(qk_C + k_B(q+k_C) - (z-2q)z))}{(b+q)k_B k_C + qb(k_B+k_C) + 2bqz - (b+q)z^2}},
\end{aligned}$$

$\Lambda_m(\delta, z) > 0$, $\partial\Lambda_m(\delta, z)/\partial\delta < 0$ and

$$\frac{\partial \Lambda_m(\delta, z)}{\partial z} \begin{cases} \geq 0 & if \quad z \leq \min\left\{(b+k_C)\frac{q}{b}, (q+k_B)\frac{b}{q}\right\} \\ < 0 & if \quad z > \min\left\{(b+k_C)\frac{q}{b}, (q+k_B)\frac{b}{q}\right\} \end{cases} .$$

The level $\delta^r(\boldsymbol{\tau}, \boldsymbol{\varsigma_B}, \boldsymbol{\varsigma_C})$ is obtained by solving $V(\boldsymbol{\chi}; \delta) = V^r(\boldsymbol{\chi_{-i}}; \delta)$ with respect to δ for equilibrium levels $(\boldsymbol{\tau}, \boldsymbol{\varsigma_B}, \boldsymbol{\varsigma_C})$.

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