Coalitions under Vertical Contracts: 
A Theory for Mergers, Collusion, and Conservation*

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Abstract

While traditional contract theory takes the institutional environment as exogenously given, this paper analyzes how the agents’ incentives to (de)centralize authority change when contracts are anticipated. In our model, induced institutional change will always harm the principal, and, under specified conditions, the outcome can be the reverse of what the principal seeks. The theory’s applications span from industrial organization, when collusion influences mergers, to environmental conservation agreements, when tropical countries strategically decide on whether to (de)centralize forest management. These decisions may change when conservation agreements are anticipated; the consequence can be increased deforestation overall, despite the principal’s payments for conservation.

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1 Introduction

1.1 Theory and results

The literature on contracts traditionally studies how a principal should optimally contract with agents, taking as given the institutional environment, such as the number of agents one can contract with (Bolton and Dewatripont, 2005). But in several situations, ranging from industrial organization to payments for environmental services, institutions may be endogenous, as when firms or districts can merge and centralize authority, or alternatively split and decentralize. The motivation for such institutional change can be influenced by the anticipation of contracts. The induced institutional changes are clearly important for the principal, but can they also harm the principal and undo the effects of the contracts themselves?

To shed light on these possibilities, this paper studies a simple setting with many agents, each taking an action. We allow for externalities between the agents that can be negative or positive, and that may work directly on the utility levels, or indirectly through a common market price, as with Cournot competition (so that the externalities are pecuniary). In these settings, a subset of the agents may have an incentive to merge and centralize authority, because they will then internalize the externalities on each other (e.g., to get market power). Alternatively, a subset of the agents may prefer decentralization because they want to influence the actions taken by the agents outside of this subset. When the agents also anticipate that a principal will offer contracts or payment schedules in return for particular actions, then the incentives to (de)centralize may change. If the incentives do change, we show that the induced institutional change will always harm the principal. Furthermore, we describe when the effects of the induced institutional change might indeed undo the effects of the contracts themselves. In these cases, the principal would clearly have been better off if she could have committed to abstain from offering contracts or payment schedules.

In the following, we will explain the theory’s relevance for both industrial organization and environmental policy such as conservation programs. To start with the former, it is well known that firms may prefer to merge to gain market power. On the other hand, if a set of firms merges to cut production, firms outside of the merger might decide to raise their production levels. When both effects are taken into account, a merger is profitable only when it includes most of the $n$ firms in the market. This result, by Salant et al. (1983), has later been extended in various directions (Gaudet and Salant, 1991; Kamien and Zang, 1990; Perry

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1Of these two effects, the first, i.e., the benefit from centralizing authority in the presence of externalities, has been recognized at least since the famous decentralization theorem of Oates (1972). However, the second effect was first detected in the literature on mergers in industrial organization, discussed below.
The condition derived by Salant et al. (1983: 191) is a special case of the condition we present below for the simplest benchmark case in which there is no principal or contracts (Proposition 0).

But these conditions can be influenced by the presence of a principal or third party, such as a common supplier. If the principal prefers lower quantities or a higher market price, she may offer payment schedules as a function of the quantities that the firms produce. The payments must be larger if the agents’ outside options are improved. Thus, to be granted more attractive payment schedules, the agents may want to merge or decentralize because contracts are anticipated. Our results show that such an induced institutional change will take the form of decentralization (if \( n \) is small) or centralization (when \( n \) is large). In either case, induced institutional change will always harm the principal and increase the total production level, despite the principal’s payments for lowering it, unless the principal is willing to offer sufficiently strong contracts.

Environmental policy often takes the form of conservation programs. Tropical deforestation is motivated in part by the profit from selling timber or agricultural products on a common market. Thus, Cournot competition has been used to better understand how deforestation in one district depends on deforestation elsewhere (Burgess et al., 2012). To motivate conservation, funds for Reducing Emissions from Deforestation and forest Degradation (REDD) are offered by the World Bank and by the United Nations thanks to several donor countries, such as Norway. Since these REDD agreements specify payment schedules that depend on the countries’ levels of deforestation, they fit particularly well with the contracts analyzed in this paper. Such contracts reduce deforestation if the institutions do not change, but we show that districts or countries may reform their institutions when REDD agreements can be anticipated. If this reform happens, the principal is worse off and the outcome can be more deforestation, despite the payments that are offered for conservation.

### 1.2 Literature

**Producers and collusions.** Firms’ incentive either to tacitly collude or to explicitly merge, to raise profit at the expense of social welfare, has been a central issue of research for a very long period of time. Whinston (2007) provides a comprehensive summary of the literature about the recent theoretical and empirical research focusing on the motivations for and the welfare effect of horizontal mergers and their consequences on prices. In an early study,

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2 Landeo and Spier (2009) show that, in experiments, even nonbinding communication among the agents (or, among the buyers, in their framework) may be sufficient for improving the offer made by the principal. They write (in the abstract): "Two-way non-binding pre-play communication among the buyers lowers the power of exclusive contracts and induces more generous contract terms from the seller."
Mankiw and Whinston (1986) show that whenever there is a market stealing effect, free entry leads to higher number of firms than what is socially desirable suggesting that mergers can be welfare enhancing. In a similar research, Horn and Wolinsky (1988) examine the incentives for merger between duopolies through the effect on input and output prices given the structure of competition in the upstream and downstream industries as well as the distribution of bargaining power between suppliers and buyers. Inderst and Wey (2003) address a similar problem to Horn and Wolinsky (1988) in a setting in which the final good market is not affected and show that mergers can still be be motivated by firms that intend to raise their bargaining power. Milliou and Petrakis (2007) extend this approach further by allowing vertical contracts. Inderst and Wey (2004) examine the incentives for mergers by analyzing the likelihood of its emergence as driven by the profits of both the insiders and the outsiders. Inderst and Shaffer (2007) explain retailers merger as a way to raise the buyer power and force suppliers of different product varieties compete for the merged retailer’s demand. Nishiwaki (2016) explains mergers as a way to reduce the cost of attrition war about which firm should scrap its capital stock first in a sunset industry and minimize the cost of scrapping capital in equilibrium.

Focusing on the empirical aspect of mergers, Hosken and Tenn (2015) describe antitrust analysis of horizontal mergers in U.S. retail markets and offer a very rich account of various merger cases such as Von’s grocery, Syufy Enterprises’, Stapels’, and Whole Foods’ cases. Ashenfelter and Hosken (2010) examine five different mergers and show that in four of these cases the mergers have raised market prices. Ashenfelter, Hosken, and Weinberg (2013) study the effect of a large manufacturing merger and show that the effect on market price is significant and high. Hosken, Olson, and Smith (2017) study the consequences of grocery mergers in fourteen regions including those in a highly concentrated and relatively unconcentrated markets and show that mergers raise prices in those highly concentrated markets.

Going deeper by examining the mergers and collusions in gasoline retailing, the recent research has made significant progress in empirical evaluation of the consequences of mergers.

Retailers under a supplier. Asker and Bar-Isaac (2014) study the effect of vertical contracts on manufacturers profits and show that such practices can reduce social welfare through incentivizing retailers to not accomodate entrant manufactureres. Similarly, Rey and Stiglitz (1995) study how manufactureres use vertical relations between manufactureres and retailers to influence the competition among manufacturers.

In particular, some of the research has focused on horizontal mergers under vertical relations and the consequences of divorcement legislations in gasoline retailing empirically (Hastings, 2004; Taylor et al., 2010; and Hastings 2010). Houde (2012) takes the spatial dif-
ferentiation and vertical mergers in a vertically integrated retailers of gasoline in Quebec City and shows that it leads to sizable price increase. Silvia and Taylor (2013) argue that the 2004 acquisition of El Paso’s by Sunoco’s 2004, and New Jersey refinery and Valero’s 2005 acquisition of Premcor’s Delaware refinery has created a significant consolidation of the refinery sector in the Northeast U.S. and report that prices have increased after the mergers. Simpson and Taylor (2008) study the merger between the Marathon Ashland Petroleum and Ultramar Diamond Shamrock and find no evidence that the merger of these gasoline retailers has raised the price for consumers. However, Taylor and Hosken (2007) show that the wholesale prices have increased in response to the merger despite the retail prices fail to increase.

**Regulators and resource extraction.** Our main extension of traditional contract theory is thus to endogenize the institutions and to allow agents to (de)centralize before the principal can commit to the contracts. We build on Segal (1999), who study contracts in the presence of externalities. Che and Spier (2008) study the effects of the principal’s "divide and conquer" strategy, Segal (2003) analyze prohibitions on such strategies, and Genicot and Ray (2006) allow agents to coordinate (but not centralize). None of these papers endogenize the institutional structure, although such an endogeneity is clearly realistic in many contexts. Districts and countries are periodically reforming their institutions by either centralizing or decentralizing authority. Regarding forest management, the trend over the last few decades has been reforms toward more decentralized management, and such decentralization has often been successful in reducing deforestation; see Agrawal et al. (2008), Irawan and Tacconi (2009), and Berkes (2010). In other words, if the anticipation of REDD agreements leads to centralization instead, the effect on deforestation can be reduced or even reversed. And, in fact, several studies are indeed concerned about the possibility that REDD agreements can lead to centralization. Phelps et al. (2010: 312) summarize some findings by stating that "when presented with strong incentives, central governments have at times reversed forest policy decentralization." Ribot et al. (2006) has a detailed evaluation of donor-induced decentralization in forest management in many countries and of how governments introduced different institutional changes that recentralized forest management in six different countries: Senegal, Uganda, Nepal, Indonesia, Bolivia, and Nicaragua. Larson and Soto (2008) offer similar arguments. Larson and Soto (2008) also argue that central governments often maintain control of forest resources even when countries formally decide to decentralize forest management.

Formally, the model we present in Section 2.1 is one that is also analyzed in our companion paper (Harstad and Mideksa, 2016). This model is a simple extension of the traditional model of Cournot competition with linear demand, where we also permit a direct externality or spillover from one agent’s production level to the payoff of another. This spillover can be
negative, as when production leads to pollution or biodiversity loss. But the spillover can also
be positive since increased production reduces the price, and the lower price reduces the profit
from illegally logging forest. When the profit of illegal logging is small, the expenditures on
monitoring and enforcement, to protect the forest, are also smaller. In this way, our model
continues to be relevant if deforestation is driven by illegal logging and high enforcement costs.

Harstad and Mideksa (2017) apply this model to study the effects of (de)centralization on
deforestation, and we there focus on the optimal conservation contracts. The approach in that
paper, however, follows approaches in most of the literature by assuming that the institutions
are exogenously given. In the present paper, we instead study the districts’ incentives to
(de)centralize, and how these incentives are influenced by the anticipation of contracts.\footnote{Since we here focus on the harmful effects of induced institutional change, we contribute to the literature on explanations of deforestation, see, for example, Burgess et al. (2012); Amacher et al. (2012); Amacher et al. (2007); Mendelsohn (1994); Angelsen and Kaimowitz (1999) and the references therein. Furthermore, our focus on institutional change is different than the papers on REDD that emphasize moral hazard (Gjertsen et al., 2016), private information (Chiroleu-Assouline et al., 2012; Mason, 2015; Mason and Plantinga, 2013), or observability (Delacote and Simonet, 2013).}

The next section first presents the simple model, before we state a benchmark result
describing when a reform is beneficial for a subset of agents if neither any contract nor the
principal is present. Given this benchmark, Section 3 introduces the principal and the principal’s contracts, and describes when the benchmark result is overturned. Section 3 also shows
that the induced institutional change will always harm the principal, and describes when the
overall effect is that production increases, despite the effects of the contracts in isolation. We
conclude after discussing some of the key assumptions. The Appendix contains all proofs.

2 Benchmark: Institutions without Contracts

2.1 A Simple Model

This section presents a simple and stylized model before we discuss three interpretations of it.
There are \( n \) agents, each taking action \( x_i \geq 0, i \in N \equiv \{1, ..., n\} \). Agent \( i \) ("he") produces \( x_i \)
units at marginal cost \( v_i \) and sells this in the common market at price \( p \). The common market
means that there are pecuniary externalities between the firms.

In addition, we can permit nonpecuniary externalities, or spillovers, and these are reason-
able in all the applications discussed in the next subsection, as we will explain. Thus, we allow
for both standard (non-pecuniary) externalities and pecuniary externalities, working through
the market price. In sum, the model is basically a classic Cournot game, extended to allow
for non-pecuniary externalities and contracts. Agent $i$’s payoff is given by:

$$ u_i(x_i, x_{-i}) = wp_i x_i - v_i x_i - s_i x + t_i, \text{ where} $$

$$ p = \bar{p} - ax, \text{ and } x \equiv \sum_{i \in N} x_i. \quad (1) $$

Parameter $w$ measures the weight associated with the gross revenue ($px_i$) relative to direct transfer from contracting ($t_i$). The direct transfer to $i$ might come from the "principal" in our model. The role of the principal will be explained in a later section, whereas the interpretation of the principal ("she", or agent "0") is given in the next subsection. In short, the principal’s objective function depends on $x$ as well as the transfers:

$$ u_0 = -d_M x_M - d_Z x_Z - t, $$

where $x_M \equiv \sum_{i \in M} x_i$, $x_Z \equiv \sum_{i \in Z} x_i$, while $M$ and $Z$ are two disjoint groups of agents ($M \cup Z = N$). We will permit the principal to be able to contract with $M$ but not with $Z$ (which we, in turn, permit but not require to be empty). Thus, $t \equiv \sum_{i \in N} t_i = \sum_{i \in M} t_i$.

### 2.2 Interpretations and Applications

(a) Producers and collusions

The agents can be producers of a uniform good. Moreover, we can suppose that, in addition to the marginal cost $v_i$, there might be fixed production costs measured by an amount of the final good, $k_i$, implying that only $x_i - k_i$ can be sold by $i$ on the market at the price $p$. Even if there are no direct spillovers between the firms, $u_i$ can with this be written as:

$$ u_i(x_i, x_{-i}) = wp_i (x_i - k_i) - v_i x_i + t_i $$

$$ = wp_i - w (\bar{p} - ax) k_i - v_i x_i + t_i $$

$$ = wp_i - v_i x_i - s_i x + t_i, $$

minus the constant $wp_i k_i$ if just

$$ s_i \equiv -wak_i $$

In this application, one interpretation of the principal is that she is one of the firms that benefits from a higher price. Suppose "agent 0" is the "collusion-maker" that pays the other firms to produce less in order to raise the price. If, for simplicity, agent 0 has a fixed production
capacity, $x_0$, then her objective function is:

$$u_0 = px_0 - t = (\bar{p} - ax)x_0 - t$$
$$= -d_M x_M - d_Z x_Z - t$$

(3)

plus the constant $\bar{p}x_0$, if

$$d_M = d_Z = ax_0.$$ 

For this interpretation of the model, the two $d$’s are the same.

(b) Retailers under a supplier

Instead of assuming that agent 0 is one of the producers, we can interpret her as the supplier of a crucial input. In particular, suppose that $v_i$ is $i$’s marginal production cost in the outside market in which $i$ finds an alternative supplier, and suppose that the supplier more efficiently can provide an equally valuable input at the lower marginal cost $\tilde{v}_i < v_i$. The supplier can ‘cash in’ on the difference in production costs if she contracts efficiently with the retailers $i \in M$, as we will assume below (see the later contracting section). Then, given any (equilibrium) set of transfers, the supplier’s objective function will be:

$$u_0 = \sum_{i \in M} (v_i - \tilde{v}_i) x_i - t$$
$$= -d_M x_M - d_Z x_Z - t,$$

if the efficiency gain $(v_i - \tilde{v}_i)$ is the same for every $i \in M$ and when we define

$$d_M = -(v_i - \tilde{v}_i) \quad \text{while} \quad d_Z = 0$$

Thus, for the supplier-retailers interpretation of the model, $d_M$ is negative while $d_Z$ is zero.

(c) Regulators and resource extraction

The producers can cause negative externalities. For example, suppose the producers are different districts logging tropical forests. In this case, the direct spillovers make a lot of sense. Alternatively, we can simplify to the modelling framework of Harstad and Mideksa (2017), where we allow each district to be endowed with a forest size $X_i$. The fraction of the forest that is not logged, $X_i - x_i$, must be protected, implying that the expected penalty on each unit of the forest must outweigh the profit from illegal logging, as measured by the market price for timber, $p$. If the marginal cost of raising the expected penalty (because one needs to monitor more intensively) is $c > 0$ per unit of the forest, then a district’s payoff can be
written as:

\[ u_i = bpx_i - cp(X_i - x_i) - v_ix_i + t_i \]
\[ = (b + c)px_i - c(p - ax)X_i - v_ix_i + t_i \]
\[ = wpx_i - s_ix - v_ix_i + t_i, \]

minus the constant \( cpX_i \), if we define

\[ s_i \equiv -caX_i \text{ and } w \equiv b + c. \]

In this case, the spillover is negative since more deforestation in one district reduces the cost of enforcing the laws in another district. It is also natural that \( s_i \) is heterogeneous when the forest size varies among the districts.

For this application, it makes sense that a donor (such as the World Bank, an NGO, or a national government such as Norway) has the utility function directly assumed above:

\[ u_0 = -d_Mx_M - d_Zx_Z - t. \]

If the principal can contract with \( i \in M \) only, then it is natural that \( d_M = d_Z \) if the environmental damage from deforestation is the same regardless of whether deforestation takes place in \( M \) or in \( Z \). However, it might also be natural that \( d_M > d_Z \), if the higher damage from deforestation in \( M \) is the reason for why the principal contracts with exactly these districts, and not as well as with the others.

Relatedly, we might suppose that the agents are countries producing fossil fuels while the principal is a climate coalition paying countries to extract less fossil fuel (as in Harstad, 2012). If the climate coalition contracts with coal-producers only, and if we approximate by assuming that coal and natural gas are perfect substitutes in the market for energy, then it is reasonable that the model above holds with \( d_M > d_Z \).

The purpose of this discussion is to illustrate that even though we have a very simple and stylized model, as presented in the previous subsection, it is sufficiently flexible to capture a wide range of applications. We will return to the various applications below, after we have derived the model’s results.
2.3 Equilibrium Production

In choosing their production activities, producers take the number of players as fixed. For a given number of producers, the equilibrium activities is given by:

\[ x_i = \frac{w\bar{p} + \sum_{j \in N} (v_j + s_j) - (n + 1) (v_i + s_i)}{wa(n + 1)}, \text{ and} \]

\[ x = \frac{n w\bar{p} - \sum_{j \in N} (v_j + s_j)}{wa(n + 1)}. \]

All else held the same, the equilibrium amount of activity increases when the market size \( \bar{p} \) is increases or a producer’s effective marginal cost \( (v_i + s_i) \) becomes lower than the average effective marginal cost of the remaining producers in the market. Similarly, the equilibrium amount of activity increases when the number players producing \( x_i \) becomes smaller.

The envelope theorem implies that a player’s decision to reduce activity \( x_i \) has an interesting implication the welfare of other players:

\[ \frac{\partial u_i(x_i, x_{-i})}{\partial (-x_j)} = awx_i + s_i \]

\[ = \frac{e_i}{n + 1}, \text{ where} \]

\[ e \equiv w\bar{p} + n \left( \frac{\sum_{j \in N} v_j}{n} - v_i \right) - v_i + s, \]

when we substitute for the equilibrium \( x_i \). The object \( e_i \) is a pecuniary externality from a player’s reduction of activity \( x_i \) on the welfare of players other than player \( i \).

The externality \( e_i \) is large if the weight on profit \( w \) is large, or if the market size \( \bar{p} \) is large. In such cases, the revenue from producing and selling \( x_i \) is important to player \( i \), and \( i \) benefits when the others keep the supply low since low supply raises the market price and thus profit beomes larger. If a producer’s \( v_i \) is greater or equal to the average \( \sum_{j \in N} v_j/n \), the producer finds it valuable to reduce its own supply, thus it produces less. In this case, it becomes less important to obtain a high price for the relatively small quantity that \( i \) has produced. However, when when a producer’s \( v_i \) is less than average marginal cost \( \frac{1}{n+1} \sum_{j \in N} v_j \), the producer finds it valuable to raise its own supply, and thus it produces more. In this case, it becomes more important to obtain a lower price for a relatively higher quantity it has produced.

In addition, the direct non-pecuniary externality from \( s \) enters in the equation for \( e_i \). However, \( e_i \) depends on the aggregate \( s \equiv \sum_{i \in N} s_i \), instead of depending simply on individual \( s_i \). This is because, the direct effect on \( e_i \) of individual \( s_i \) in \( awx_i + s_i \) neutralized by the indirect effect on \( e_i \) of individual \( s_i \) through \( awx_i \). That is, a higher \( s_i \) reduces the equilibrium choice.
of \( x_i \), and this in turn reduces the benefit of \( i \) when \( j \) reduces the supply \( x_j \). Simultaneously, when \( s_j \) is higher, player \( j \) reduces its production of \( x_j \), and then \( i \) finds it optimal to increase \( x_i \). The larger \( x_i \) makes it more beneficial for \( i \) that \( j \) reduces supply. The total payoff externality is positive \( s > 0 \) if agents are "strong" and production is motivated by the profit, while it is negative \( s < 0 \) if instead agents are weak and production is illegal and costly to prevent. Identical externality across all players \( e_i = e \) arises when \( v_i = v \) and in this case, a larger \( s \) implies a larger \( e \), so \( x_i \) is sales-driven if \( e \) is large, but that it is illegal or driven by the high enforcement costs when \( e \) takes the opposite sign.

Inserting back (4) and (5) into (1) and using (6) implies that a player’s payoff in equilibrium given by:

\[
u_i = \frac{1}{aw} \left( \frac{e}{n+1} \right)^2 \frac{wp - v}{aw} s_i.
\]

### 2.4 Equilibrium Institutions

The externality in (6) implies that agents can be better off when they centralize and set the activity levels taking the externalities into account. If \( e > 0 \), a reduction in the production level of a firm raises other agents’ payoffs, and it becomes optimal to conserve more. However, if \( e < 0 \), as when agents are weak, a reduction in the production level of a firm reduces other agents’ payoffs and it becomes optimal to conserve less. Regardless of whether the total externality is positive or negative, centralization is always optimal for all the agents combined.\(^4\)

Since \( e = \sum_{i \in N} s_i + wp - v \), it may seem intuitive that \( e \) also changes \( n \) and when agents centralize. This is incorrect, at least, in our applications. For the application in (c), the \( \sum_{i \in N} s_i \) is the total resource (e.g., forest) size, which is fixed whether agents choose to merge, collude, or decentralize further. For applications (a) and (b), consider the following. A traditional argument for mergers is that then the firms will benefit from cost reductions by omitting one of the fixed costs in productions. In order to isolate the effects on mergers/collusions from the contracts and the market competition, we abstract from the possibility to save on the fixed costs. In effect, this means that for applications (a) and (b), we assume that the units must pay the fixed costs \( (k_i \text{'s}) \) even if the firms merge. With this, the sum of the fixed costs \( s \) do not change when \( n \) changes (i.e., when some firms merge/collude). Consequently, \( s \) and \( e \) are unchanged in \( n \).

\(^4\)From the previous equation, we have:

\[
\sum_{i \in N} u_i = \frac{n}{aw} \left( \frac{e}{n+1} \right)^2 + \frac{v - wp}{aw} s,
\]

which decreases in \( n \) for every \( e \neq 0 \).
To study institutional change, consider a subset of agents \( L \subseteq N \), who are able to centralize authority. Centralization to a single authority would imply that the number of decision-makers in \( L \) is reduced from \( l \equiv |L| \) to only 1. If this reform increases the equilibrium level of \( \sum_{i \in L} u_i \), we will say that \( L \) prefers centralization. Otherwise, we will say that \( L \) prefers decentralization.

Centralization means that the externalities on the other agents in the merger are taken into account, but also that agents in \( N \setminus L \) will react to the reform. Thus, the size of \( L \) relative to \( N \) will turn out to be crucial for the effect of (de)centralization. We will say that \( L \), considering the centralization, is "large" (relative to \( N \)) if:

\[
\xi_L \equiv 1 - \frac{l}{n+1} - \frac{1}{n-l+2} < 0. \tag{7}
\]

For the set of agents \( L \) to be large, it is necessary that \( L \) contains a majority of the decision-making agents before centralization \( (l > (n+1)/2) \). If \( L \) is not large, we say that \( L \) is "small."

**Proposition 0.** Suppose \( L \subseteq N \) is a subset of agents.

(i) If \( L \) is large, \( L \) always prefers centralization.

(ii) If \( L \) is small, \( L \) always prefers decentralization.

To focus on intuition, all the proofs are kept in the appendix.\(^5\) If \( L = N \), we know that the sum of payoffs is highest under centralization, since externalities make decentralization inefficient. It is thus intuitive that if \( L \) is large, \( L \) will prefer centralization. Part (i) of Proposition 0 is therefore exactly as one would expect. Part (ii) states that the opposite happens if the subset \( L \) is small. Suppose \( e > 0 \). If \( L \) decentralizes, then \( L \) produces more when \( e > 0 \) and, in response, the remaining players in \( N \setminus L \) produce less, which is beneficial to \( L \) since \( e > 0 \). Similarly, suppose \( e < 0 \). If \( L \) decentralizes, \( L \) produces less when \( e < 0 \) and, in response, the remaining players in \( N \setminus L \) produce more, which again is beneficial to \( L \) when \( e < 0 \). In both cases, the reaction of the other agents in \( N \setminus L \) to \( L \)'s further decentralization is beneficial to \( L \). When \( N \setminus L \) is a large set of agents, the responses are very important and thus \( L \) prefers decentralization in order to trigger the response from \( N \setminus L \), despite the fact that decentralization also implies that \( L \) will internalize fewer of the externalities on each other.

\(^5\)With equation (??), Proposition 0 can be shown to generalize equation (3') in Salant et al. (1983), p. 191, where the firms' incentive to merge is analyzed.
3 Contracts and Institutions

We now consider a principal or "donor" who benefits from reductions in activities $x_i$. We assume that the principal benefits from agents’ reduced production with whom she can contract with and agents’ production with whom she cannot contract with. The payoff is $u_D = -d_M x_M - d_Z x_Z$, where $d_M > 0$ measures the marginal damage the principal experiences from the agents’ production with whom she can contract with, and $d_Z \geq 0$ measures the marginal damage the principal experiences from the agents’ production with whom she cannot contract with. Let the set $M \subseteq N$ be the subset of agents that the principal is able to contract with.

The principal has a quasi-linear utility function for the payoff, $u_0$. The transfer to $i \in M$ will be in exchange for reduced production activity. In particular, we assume that the principal commits to a set of transfer functions before the agents decide on the action $x_i$. The transfer function to $i$ can be a general function $t_i(x)$, where $x = (x_1, ..., x_n)$ is the vector of actual production levels that are simultaneously chosen by the agents.

If a set of functions $t_i(x)$ leads to some equilibrium vector $x^*$, then $x^*$ will also be an equilibrium if the principal simply offers the fixed payment $\tilde{t}_i = t_i(x^*)$ if $x = x^*$, and zero otherwise. We assume also that every agent has a utility function that is additive in the transfer $t_i$. The problem for the principal is then simply to maximize $u_D = \sum_{i \in N} t_i$ subject to the following $m \equiv |M|$ incentive constraints:

$$u_i(x_i^*, x_{-i}^*) + t_i \geq \max_{\tilde{x}_i \geq 0} u_i(\tilde{x}_i, x_{-i}^*).$$

An agent $i \in N \setminus M$ is simply selecting quantities as a best response to the other production levels, just as in the previous section.

If we solve the Principal’s problem subject to the $(IC_i)$’s and derive the equilibrium contracts, we get:

$$t_i^* = \frac{1}{aw} \left( \frac{d}{n+1} \right)^2, \text{ where}$$

$$d \equiv d_M + (n - m) d_Z,$$

---

6We ignore the possibility that the donor may value the consumer surplus since exports, and not domestic consumption, are the main drivers of tropical deforestation (DeFries et al., 2010; Rudel, 2007; Rudel et al., 2009). Moreover, the donor should not be interpreted as a benevolent planner, but rather as an NGO or a country offering REDD contracts, such as Norway. Furthermore, allowing the principal to internalize the consumer surplus would make the analysis somewhat messier, without changing the results qualitatively.
\[ x^*_i = \frac{1}{aw} \left[ \frac{e}{n+1} - \frac{2}{n+1} \frac{n+1-m}{n+1} d - s_i \right], \text{ implying} \quad (8) \]

\[ \sum_{i \in M} x^*_i = \frac{1}{aw} \left[ \frac{me}{n+1} - \frac{2m}{n+1} \frac{n-m+1}{n+1} d - \sum_{i \in M} s_i \right]. \]

Naturally, when \( d_M \) or \( d_Z \) is larger, the induced cuts in the \( x^*_i \) from \( i \in M \) are larger, and the transfers to induce agents accept the contract must be larger as well. When \( d_M = d_Z \) and the principal is a collusion maker or is a donor contracting for conservation of resources, the transfer increases with increasing \( d_M \) and decreases with the share of the number of agents \( \frac{m}{n+1} \) the principal is contracting with. All else being the same, the transfer she makes to agents increases with the number agents that she cannot contract with.

If the principal is a supplier of crucial input so that \( d_M < 0 \), and \( d_Z = 0 \), she benefits from a higher activity by retailers in the set \( M \). To this end, she reduces the transfer she makes to each agent. Agents that the donor cannot contract with react to the contract by adjusting their activities, and their total activity becomes:

\[ \sum_{i \in Z} x_i = \frac{1}{aw} \left[ \frac{(n-m)e}{n+1} + \frac{2m}{n+1} \frac{n-m}{n+1} d - \sum_{i \in Z} s_i \right]. \]

When the principal is a collusion maker or is a donor contracting for conservation of resources, and \( d > 0 \), the principal offers a contract to its agents that ensures reductions in production activity. The reduction raises the market price, which in turn gives the right incentive for the remaining producers to raise their activities. However, if the principal is a supplier of crucial input so that \( d < 0 \), she benefits from a higher activity by retailers in the set \( M \) and her contract takes the incentive into account. When the production activity by the contracting parties increases, the market price decreases. A reduced price, in turn, incentivizes the remaining agents to reduce their production activity.

Together, in response to the contract, the total activity becomes:

\[ x^* = \frac{1}{aw} \left[ \frac{ne}{n+1} - \frac{2m}{n+1} \frac{d}{n+1} - s \right] , \text{ and} \]

the total payoff for an agent accepting the contract becomes:

\[ u_i = \frac{1}{aw} \left[ \left( \frac{e + d - 2 \frac{n-m+1}{n+1} d}{n+1} \right)^2 + \frac{v - w^0 s_i}{aw} \right]. \]

Thus, the principal is a collusion maker or is a donor contracting for conservation of resources, an agent facing a contract is better off when the principal is contracted with the majority of agents (i.e., when \( \frac{m}{n+1} > \frac{1}{2} \)) and is worse-off when the principal is contracted with a minority of
agents. However, if the principal is a supplier of crucial input, then an agent that she contracts with is better off, in equilibrium, when the contract is accepted by minority of agents.

### 3.1 Induced Institutional Change

In the setting without any principal or contracts, Proposition 0 stated that large coalitions will centralize, while small coalitions will decentralize. The presence of the principal and the anticipation of contracts may change these results. Suppose now that a subset of agents $L \subseteq M$ can centralize and reduce the number of authorities in $L$ by the number $l - 1$ and centralize to a single authority. We find it natural to assume that if $D$ can contract with the $l$ agents in $L$ if there is no reform, $D$ can also contract with the centralized decision-maker in $L$ if the reform takes place.

The reform decision of $L$ is made before $D$ has committed to the actual contracts with the agents.\(^8\) Thus, the equilibrium contracts offered by $D$ will be influenced by any reform made by $L$. When $L$ anticipates these effects, Proposition 0 may be overturned, implying that the presence of $D$ and the anticipation of $D$'s contracts lead to induced institutional change.

**Proposition 1.** With $D$ present in the game, Proposition 0 may be overturned.

(i) If $L$ is large, $L$ prefers decentralization if and only if $e/d \in [\hat{\epsilon}_L, \tau_L]$, where $\tau_L > \hat{\epsilon}_L > \varepsilon_M > 2\varepsilon_M < 0$.

(ii) If $L$ is small, $L$ prefers centralization if and only if $e/d \in [\tau_L, \hat{\epsilon}_L]$, where $\tau_L < \hat{\epsilon}_L < \varepsilon_M$.

The thresholds are given by:

\[
\hat{\epsilon}_L \equiv 1 - \frac{2m}{n+1} + \frac{\frac{2m}{n+1} - \frac{2(m+1-l)}{n-l+2}}{1 + \frac{\sqrt{l(n-l+2)}}{n+1}}, \quad (9)
\]

\[
\tau_L \equiv 1 - \frac{2m}{n+1} + \frac{\frac{2m}{n+1} - \frac{2(m+1-l)}{n-l+2}}{1 - \frac{\sqrt{l(n-l+2)}}{n+1}}, \quad (10)
\]

\[
\varepsilon_M \equiv 1 - \frac{m}{n+1} - \frac{m-l+1}{n-l+2}. \quad (11)
\]

Part (i) shows that a large $L$ may prefer to decentralize authority in the presence of the principal. However, this happens only for a set of $e$’s that are so large that $e/d > \varepsilon_M$. To understand this result, consider the right-hand side of the incentive constraint (IC), measuring the outside option. The larger the outside options are, the larger the transfers from $D$ will be. The right-hand side of (IC) is $i$’s outside option by deviating alone, keeping the other\(^8\) Equivalently, we allow $D$ to revise the contracts after the reform. This is reasonable, since institutional reforms take time, while conservation contracts, for example, can be rapidly adjusted.

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\(^8\)Equivalently, we allow $D$ to revise the contracts after the reform. This is reasonable, since institutional reforms take time, while conservation contracts, for example, can be rapidly adjusted.
agents’ actions fixed. If $L$ merges, the new outside option is instead to choose the $x_i$’s so as to maximize $\sum_{i\in L} u_i$. This outside option is more attractive if the other agents in $L$ benefit when one $x_i$ increases. This holds if the externality from conservation, as measured by $e$, is small and negative. If instead this externality is large and positive, the utility sum under such a joint deviation from the contract is smaller than the sum of outside options if each $i \in L$ deviates alone. For this reason, a merger improves the outside options (and thus the transfers from $D$) if $e$ is small but not if $e$ is large.

The effect on the outside option is important if $d$ is large, so that the contracts will be substantial. If instead $d$ is small, the effect on the (small) transfers is not important enough to overturn the importance of internalizing the externality, so then Proposition 0 will continue to hold. Formally, if $d \to 0$, $|e/d| \to \infty$, so it cannot be that $e/d \in [\bar{e}_L, \bar{e}_L]$ or $e/d \in [\underline{e}_L, \underline{e}_L]$, as is required to overturn Proposition 0.

Part (ii) similarly shows that a small set $L$ may prefer centralization in the presence of $D$, even if $L$ would always have preferred decentralization when $D$ were absent. This happens only when $e$ is so small that $e/d < \xi_M$. Under this condition, the externality from producing more is larger, and therefore the sum of the outside options is larger if one considers a joint deviation, as when the agents have centralized authority.

The intuition for the result can also be explained by considering the utilities in the contracts relative to outside options. If the externality from conservation is large ($e/d > \xi_M$), then centralization would have allowed $D$ to offer less payments, since centralization would have implied that the positive externalities would have been internalized. In this case, decentralization is preferred, even if $L$ is large. If instead the externality from conservation is negative or small ($e/d < \xi_M$), then centralization would imply that $D$ would find it necessary to offer more in transfers to overcome the hesitation of the central authority who takes the negative effects into account. In this case, the agents prefer centralization, perhaps even for the case in which $L$ is small.

### 3.2 The Consequence of Induced Institutional Change

Induced institutional change refers to the possibility that a large $L$ decentralizes because $D$ is present in the game, or that a small $L$ centralizes because $D$ is present. When $D$’s presence leads to institutional change, the reforming agents in $L$ are motivated by the prospects of larger transfers from $D$. It may thus not surprise that the induced institutional change always reduces the payoff to $D$. In fact, it is easy to show that centralization among a set of agents $L \subseteq M$ always increases $D$’s equilibrium payoff if and only if $e/d > \xi_M$. Intuitively, a
large \( e \) means that a central authority will accept conservation for a lower transfer, when the externalities are positive. However, when \( e/d > \xi_M \), Proposition 1(i) states that induced institutional change takes the form of decentralization, thus harming \( D \). If instead \( e/d < \xi_M \), so that \( D \) prefers decentralization, Proposition 1(ii) shows that induced institutional change takes the form of centralization. In either case, induced institutional change harms \( D \).

**Proposition 2.** *Induced institutional change will always make \( D \) worse off.*

Induced institutional change will also influence the equilibrium \( x^* \), expressed by equation (8). This equation can be used to show that if \( L \subseteq M \) centralizes, \( x^* \) declines if and only if \( e/d > 2\xi_M \). It is clearly intuitive that centralization leads to a smaller \( x^* \) if and only if the externality from conservation is positive. A comparison to Proposition 1 leads to the following result.

**Proposition 3.**

(i) If \( L \) is large, induced institutional change always increases \( x^* \).

(ii) If \( L \) and \( M \) are small, induced institutional change always increases \( x^* \).

Thus, \( D \)’s presence is likely to motivate institutional change that leads to a larger \( x \), exactly the reverse of what \( D \) attempts to achieve. In part (ii), when \( L \) is small, then a small \( M \) is sufficient but not necessary for induced institutional change to increase \( x^* \), as shown in the Appendix.

### 3.3 Counterproductive Contracts

Given the above findings, one may question whether the larger extraction conservation levels following reform can outweigh the direct effect of the contracts themselves. If so, the very presence of \( D \) leads to reform and so much more production that more would have been conserved if \( D \), as well as \( D \)’s contracts, had been absent. In this case, \( D \)’s presence is clearly doing more harm than good and \( D \) would have preferred to commit to abstaining from offering contracts, if such a commitment were feasible.

**Proposition 4.** *Contracts can be counterproductive.*

(i) Suppose \( L \) is large. If \( e/d \in [\tilde{\varepsilon}_L, \tau_L] \), the presence of \( D \) induces decentralization and, despite the contracts, \( x^* \) increases when \( e/d > \tilde{\varepsilon}_L \), where:

\[
\tilde{\varepsilon}_L = \frac{2m}{l-1} \frac{n-l+2}{n+1} > 0.
\]
(ii) Suppose L is small. If $e/d \in [\tilde{\epsilon}_L, \epsilon_L]$, the presence of $D$ induces centralization and, when also $e/d < -\tilde{\epsilon}_L$, $x^*$ increases, despite the contracts.

The results are intuitive. When $L$ is large and $e > 0$, contracts are counterproductive only when $d$ is so large that the anticipation of the contracts leads to induced institutional change, but while $d$ simultaneously is so small that $e/d > \tilde{\epsilon}_L$. For a larger $d$, the contracts are stronger, and their effects will outweigh the isolated effect of the induced institutional change. A similar argument holds when $L$ is small and $e < 0$.

To summarize and illustrate the result, suppose $l = m = n$ and that the agents consider to merge ($l = n$). Such centralization would implement the socially efficient first-best outcome, since no third party would be affected by the contracts. However, the thresholds become:

\[
\begin{align*}
\hat{\epsilon}_L &= -\frac{n - 1}{n + 1} \frac{2\sqrt{n}}{n + 1 + 2\sqrt{n}}, \\
\bar{\epsilon}_L &= \frac{n - 1}{n + 1} \frac{2\sqrt{n}}{n + 1 - 2\sqrt{n}}, \\
\xi_M &= -\frac{1}{2} \frac{n - 1}{2n + 1}, \\
\tilde{\epsilon}_L &\equiv \frac{4n}{(n + 1)(n - 1)},
\end{align*}
\]

and it is easy to check that:

\[2\xi_M < \xi_M < \tilde{\epsilon}_L < 0 < \bar{\epsilon}_L < \epsilon_L.\]

Figure 1 illustrates the various settings for the case in which $n = 2$. Decentralized contracts are preferred by $D$ in the shaded area, where $e/d < \xi_M = -1/6 \approx -0.17$, even though decentralization increases production when $e/d > 2\xi_M = -1/3$ (where the shaded area has downward-sloping lines). The agents, however, prefer decentralization only when they are "stronger" and $e/d \in (\tilde{\epsilon}_L, \epsilon_L) \approx (-0.16, 5.5)$ (i.e., in the dotted area). Furthermore, note that $\bar{\epsilon}_L = 8/3 \approx 2.7 \in (\tilde{\epsilon}_L, \epsilon_L)$. Thus, for every $e/d \in (2.7, 5.5)$, which corresponds to the area that is both colored and dotted, the presence of $D$ motivates the agents to decentralize and the accompanying increase in $x^*$ outweighs the effect of the contracts.

4 Assumptions, Limitations, and Future Research

To keep the paper short and to illustrate the insight in a pedagogical way, we have simplified the model and thus imposed a number of strong assumptions. In this section, we discuss some of these assumptions, and suggest a path for future research.
Figure 1: *Even if centralization leads to the first best, the principal prefers decentralized contracts when e and d are small (shaded area), while the agents prefer decentralization when e and d are large (dotted area). In the colored dotted area, the regime change raises production by more than the contracts reduce it. The lines are drawn for an example with two agents.*

Regarding the simple game between the agents, we followed the early literature on industrial organization by assuming a linear demand function. This approach resulted in a linear-quadratic utility function for every agent, and simplified the expressions of the solutions. The subsequent literature on industrial organization has relaxed the linear-demand assumption, but the essence of the results regarding mergers has been shown to hold. The best illustration of this robustness is the paper by Gaudet and Salant (1991), already mentioned above, which considers when a subset of firms would benefit from marginally reducing their production levels. Gaudet and Salant (1991: 658) find that "a marginal contraction is strictly beneficial (strictly harmful) if and only if the number of firms in the designated subset exceeds the "adjusted" number of firms outside it by strictly more (strictly less) than one. The adjustment factor is unity when cost and demand functions are linear but, more generally, depends on the convexity of the cost and demand curves."

Regarding our results in Section 3, these results are driven by the consideration of the agents’ outside options, so we are confident that the results continue to hold qualitatively with nonlinear demand. (The proofs in our companion paper, Harstad and Mideksa (2017), do allow for nonlinear demand.)

A limitation that has larger effects on the results is that we have considered the incentive to centralize only among a single set of agents $L \subseteq M$. This assumption ought to be relaxed in two ways. If we permit the reforming set to be among the set of agents which $D$ cannot contract with, $L \subseteq N\setminus M$, then it may not necessarily be that an induced institutional reform by $L$ is always harms $D$ or is always likely to increase $x^*$. An earlier version of our paper
did consider (de)centralization among agents $L \subseteq N \setminus M$, so these results are available upon request.

Future research should also consider the setting in which many subsets of agents can reform simultaneously. For example, the districts in $N$ might be divided amongst different countries, and each country may decide on whether to centralize or decentralize. In this setting, the countries’ actions will be strategic complements, since when one subset $L \subseteq M$ centralizes, the total number of decision makers declines, and this decline makes it more likely that also another subset $L' \subseteq M$ will also prefer to centralize. This complementarity would lead to multiple equilibria, and one may search for the best equilibrium and for how the principal may help the agents to coordinate on such an equilibrium.

Empirical research should take the testable predictions above to the data. Proposition 1 predicts that when $D$ is anticipated to be a player in the game, then we may have two types of induced institutional change. First, we predict that a relatively large country may decide to decentralize, but this happens only when the externality $e$ is relatively large, as when districts are "strong" and the enforcement cost $c$, discussed in Section 2.1, is small. In contrast, a relatively small country may prefer to centralize authority, but only when $e$ is smaller or negative, as when districts are "weak" in that the enforcement cost $c$ is larger. These predictions are testable, and data concerning the effects of conservation contracts will increasingly be available when such contracts become more common.

If one can identify cases with induced institutional change, Proposition 3 further predicts that the isolated effects on $x^*$ may be positive (in that $x^*$ will increase). This prediction may be harder to test, because the increase in $x^*$ is relative to the counter-factual level in which the induced institutional change did not take place. Only when the contracts are so weak (and $d$ is so small) that Proposition 4 is relevant should we be able to easily test this prediction, since then $x^*$ is predicted to increase after the contracts have appeared in the game, accompanied by institutional change, relative to the situation before the contracts and before agents’ could reform in the anticipation of the contracts. This possibility is predicted by Proposition 4 only when the externality $e$ and the country $L$ are both quite large (and then induced institutional change takes the form of decentralization) or when $e$ and $L$ are both small (and then centralization is the predicted reform).

5 Conclusions

In the traditional contract theory literature, where the institutional environment is perceived as being exogenous, contracts and payment schedules will have an effect that is to some
extent in line with the principal’s objective. If institutions are endogenized, however, the anticipation of the contracts may lead to induced institutional change that can harm the principal and result in an outcome that is the opposite of what the principal seeks. This paper analyzes these effects in a simple model that can be applied to settings ranging from industrial organization to environmental policy such as conservation programs. As an example of the mechanics analyzed above, Sandbrook et al. (2010: 332) find that REDD conservation programs are "likely to create incentives for forest managers to return to past centralized models of forest conservation...Such governance arrangements have often been ineffective at sustaining forests...."

In our view, our theoretical results should not be interpreted as saying that these effects necessarily are going to be important in the real world. On the contrary, since we specify exact conditions under which the effects will be present, our ambition is to contribute to a better understanding of when contracts are likely to influence institutions, and of when the consequence may be dramatic for the principal and the outcome of interest. For example, it is when the principal’s preference is relatively weak, but still sufficiently strong to induce institutional change, that the situation may arise in which the contracts will backfire. By recognizing when these effects may reasonably be expected to arise, one can take steps to avoid them. One policy solution is for the principal to invest in developing a reputation for always contracting with the appropriate jurisdictional level. Similarly, if institutions change in order to elicit more payments from the principal, the principal would benefit from establishing a reputation that penalizes such behavior. These possibilities are in line with how Wunder (2010: 336) respond to the concern of Sandbrook et al.: "Conditionality is the key conceptual safeguard: if inefficient governments waste rents centrally without avoiding deforestation then international REDD transfers must be stopped."

References


658-665.


6 Appendix: Proofs (to be revised)

Proof of Proposition 0. This result follows from Proposition 1 in the special case where \( m \to 0 \) and \( d \to 0 \). Thus, the proof is omitted here.

Proof of Proposition 1. We first derive the equilibrium payoff for a single agent. The equilibrium contracts, as presented by (8), are easy to derive from the principal’s maximization problem. Combined with (1), we can derive an agent’s total payoff \( u_i + \tau_i \):

\[
\frac{1}{aw} \left[ \left( e - \frac{2}{n+1}d \right)^2 + \left( v + \frac{2}{n+1}d \right) s_i \right] + \frac{(2n+1)d^2}{4aw} + \frac{2}{n+1}d e - \frac{(n+1)s_i}{aw(n+1)} - \frac{2}{n+1}d \frac{2d}{aw(n+1)^2},
\]

where \( q \equiv n - m \). With some algebra, this payoff can be rewritten as:

\[
u_i = \frac{1}{aw} \left[ \left( \frac{(e + d)(n+1) - 2d(q+1)}{(n+1)^2} \right)^2 + vs_i \right].
\]

Consider now a set \( L \) of \( l = \mid L \mid \) agents, taking as given \( n - l \). The sum of \( L \)’s payoffs is:

\[
\sum_{i \in L} \frac{1}{aw} \left( \frac{(e + d)(n+1) - 2d(q+1)}{(n+1)^2} \right)^2 + v \sum_{i \in L} s_i,
\]

where the last term, \( \sum_{i \in L} s_i \), stays unchanged if the districts reform. But if the set \( L \) centralizes, the new numbers are reduced to \( 1 < l \), \( m' = m - l + 1 \), and \( n' = n - l + 1 \), while \( q = n - m \) stays the same. This change increases total welfare for the set \( L \) if and only if:

\[
\frac{l}{aw} \left( \frac{(e + d)(n+1) - 2d(q+1)}{(n+1)^2} \right)^2 < \frac{1}{aw} \left( \frac{(e + d)(n'+1) - 2d(q+1)}{(n'+1)^2} \right)^2 \iff
\]

\[
l \left( \frac{n'+1}{n+1} \right)^2 < \left( \frac{e/d + 1 - \frac{2(q+1)}{(n'+1)}}{e/d + 1 - \frac{2(q+1)}{(n+1)}} \right)^2 \iff
\]

\[
\sqrt{l} \left( \frac{n'+1}{n+1} \right) < \left| 1 - \frac{2m}{n+1} - \frac{2m'}{n'+1} \right|, \quad (12)
\]

Lemma 1. The left-hand side of (12) is smaller than 1 if and only if \( L \) is large.
Proof. Note that

\[
\sqrt{l} \left( \frac{n' + 1}{n + 1} \right) < 1 \iff \frac{1}{l} \left( \frac{n + 1}{n' + 1} \right)^2 > 1 \iff 
\]

\[
\frac{1}{l} \frac{(n + 1)^2}{(n + 1)^2 - 2(l - 1)(n + 1) + (l - 1)^2} > 1 \iff 
\]

\[
1 - \frac{(l - 1)}{l} > 1 - \frac{2(l - 1)(n + 1) - (l - 1)^2}{(n + 1)^2} \iff 
\]

\[
\frac{l}{1 + n} > \frac{n + 1}{2(n + 1) - (l - 1)}. \quad (14)
\]

In addition, (13) is equivalent to

\[
\frac{2(n' + 1)(l - 1) + (l - 1)^2}{(n' + 1)^2} > \frac{(l - 1)}{l} \iff 
\]

\[
\frac{1}{n' + 1} > \frac{n' + 1}{2(n' + 1) + (l - 1)}. \quad (15)
\]

Since (14) and (15) are equivalent, we can also sum them and thus write:

\[
\frac{1}{1 + n'} + \frac{l}{n + 1} > \frac{n' + 1}{2(n' + 1) + (l - 1)} + \frac{n + 1}{2(n' + 1) - (l - 1)} = \frac{n' + 1}{n + n' + 2} + \frac{n + 1}{n + n' + 2} = 1. \quad \|
\]

To further evaluate (12), note that the expression within the absolute sign is negative if and only if:

\[
1 - \frac{2m}{n + 1} < e/d < 1 - \frac{2m'}{n' + 1}. \quad (16)
\]

(i) Suppose, first, that \( L \) is large, so that the l.h.s. of (12) is smaller than 1. Proposition 0 is overned if and only if (12) fails.

(i-a) Suppose (16) fails. Then, (12) fails if and only if:

\[
1 - \frac{2m}{n + 1} < e/d < 1 - \frac{2m'}{n' + 1} + \frac{2m}{n + 1} + \frac{2m'}{n' + 1} < 1 - \sqrt{l \left( \frac{n' + 1}{n + 1} \right)}. 
\]

So, in this case, (12) fails if and only if \( e/d \) is such that:

\[
1 - \frac{2m'}{n' + 1} < e/d < 1 - \frac{2m}{n + 1} + \frac{2m}{n + 1} - \frac{2m'}{n' + 1} \quad \|(17)
\]

\[
= 1 - \frac{2m'}{n' + 1} + \left( \frac{2m}{n + 1} - \frac{2m'}{n' + 1} \right) \left( \frac{\sqrt{l \left( \frac{n' + 1}{n + 1} \right)}}{1 - \sqrt{l \left( \frac{n' + 1}{n + 1} \right)}} \right) > 1 - \frac{2m'}{n' + 1}. \quad (18)
\]
(i-b) If (16) holds, then (12) can be written as:

\[ \sqrt{\frac{n'}{n+1}} < \frac{2m}{\sqrt(n'+1)} - 1 \iff \]

\[ 1 - \frac{2m}{n+1} < e/d < 1 - \frac{2m}{\sqrt(n'+1)} + 1 = \hat{\epsilon}_L \]

\[ \epsilon_M = 1 - \frac{m'}{n'+1} - \frac{m}{n+1} < \epsilon_L < \epsilon_M. \]

(ii) Suppose, next, that \( L \) is small. Proposition 0 is overturned if (12) holds, requiring:

(ii-a) If (16) fails, then (12) holds if and only if:

\[ \sqrt{\frac{n'}{n+1}} - 1 < \frac{2m}{n+1} < \frac{m'}{n'+1} - \frac{m}{n+1} \]

\[ \hat{\epsilon}_L = 1 - \frac{2m}{n+1} < e/d < 1 - \frac{2m}{n+1} \]

(ii-b) If (16) holds, then (12) holds if and only if:

\[ \sqrt{\frac{n'}{n+1}} < \frac{2m}{\sqrt(n'+1)} - 1 \iff \]

\[ 1 - \frac{2m}{n+1} < e/d < 1 - \frac{2m}{\sqrt(n'+1)} + 1 = \hat{\epsilon}_L \]

\[ \epsilon_L < \epsilon_L < \epsilon_M = 1 - \frac{m'}{n'+1} - \frac{m}{n+1}, \text{ when } \sqrt{\frac{n'+1}{n+1}} > 1. \]

(ii-c) Combined, when \( L \) is small, (12) holds if and only if:

\[ 1 - \frac{2m}{n+1} < e/d < 1 - \frac{2m}{n+1} + \frac{2m}{\sqrt(n'+1)} \]

Note that Proposition 0 is confirmed in the following result for the special case where \( d \to 0 \Rightarrow |e/d| \to \infty. \ Q.E.D.

**Proof of Proposition 2.** Given (8), it is straightforward to derive the principal’s equilibrium payoff and confirm that \( D \) is better off if \( L \) centralizes if and only if \( e/d > \epsilon_M \).

When \( L \) is large, \( L \) prefers decentralization only for the set of \( e/d \) in which \( e/d \in [\epsilon_L, \hat{\epsilon}_L] \), but it is easy to check that \( \epsilon_M < \epsilon_L \), so \( D \) always prefer that \( L \) centralizes in these cases.
When $L$ is small, $L$ prefers centralization only for the situation in which $e/d < \xi_L$, but, when $L$ is small, it is easy to check that $\hat{\xi}_L < \xi_M$, so $D$ always prefer decentralization when a small $L$ turns to centralization because $d > 0$. Q.E.D.

Proof of Proposition 3. Since $x^*$ as expressed by (8) is a function of $m, n, d$ and $e = s + w\bar{p} - v$, it is easy to consider $m'$ and $n'$ and confirm that if $L$ centralizes, then $x^*$ increases if and only if $e/d > 2\hat{\xi}_L$.

(i) When $L$ is large, $2\xi_M < \xi_M < 0$. By comparing to the conditions in Proposition 1, it is clear that whenever a large $L$ prefers to decentralize because $d > 0$, then $e/d > 2\xi_M$, so decentralization increases $x^*$.

(ii) In general, we cannot rank $2\xi_M$ and $\hat{\xi}_L$. However, if $L = M$ and $L$ is small, then $\xi_L = \xi_M > 0$, so $2\xi_M > \xi_M > \hat{\xi}_L$, where the last inequality follows from Proposition 1. Consequently, whenever $L = M$ is small and $L$ prefers centralization because $d > 0$, then $L$’s centralization will increase $x^*$. More generally, we have that:

$$\frac{2m}{n+1} + \frac{2m'}{n'+1} - 2 < \left(1 - \frac{2m'}{n'+1}\right)\left(\sqrt{1 +\left(\frac{n'+1}{n+1}\right)} - 1\right),$$

which always holds when $M$ is small. Q.E.D.

Proof of Proposition 4.

(i) Suppose $L$ is large. If $D$ is not present, $L$ prefers to centralize, and $x^*$ follows from (8) if we set $d = 0$ and $n = n - l + 1$:

$$x^0_{l-1} = \frac{(n - l + 1)w\bar{p} - s - (n - l + 1)v}{aw(n - l + 2)}.$$

If the presence of $D$ discourages $L \subseteq M$ from centralizing, then $x^*$ is as stated by (8). By comparing $x^0_{l-1}$ and $x^*$, we can after a few steps conclude that $x^* > x^0_{l-1}$ if and only if:

$$\frac{e}{d} > \hat{\xi}_L \iff \frac{2m(n - l + 2)}{(n + 1)(l - 1)}.$$

(ii) If $L$ is small, $L$ prefers to abstain from centralization when $D$ is absent, and $x^0$ would be:

$$x^0 = \frac{nw\bar{p} - s - nv}{aw(n + 1)}.$$

If, however, $L$ prefers to centralize because of the presence of $D$ and $d > 0$, then $x^*$ is replaced by:

$$x^*_{l-1} = \frac{(n - l + 1)w\bar{p} - s - (n - l + 1)v}{aw(n - l + 2)} - \frac{2(m - l + 1)d}{aw(n - l + 2)^2}.$$

If we compare $x^0$ and $x^*_{l-1}$, a few steps gives that $x^*_{l-1} > x^0$ if and only if:

$$\frac{e}{d} < -\frac{2m(n - l + 2)}{(n + 1)(l - 1)} \iff -\hat{\xi}_L.$$

Q.E.D.