THE INTERRELATION BETWEEN CAPITAL PRODUCTION AND CONSUMER-TAKING

I. STATEMENT OF PROBLEM

AT ONE of our informal meetings at the Campus Club of the University of Minnesota in the spring term of 1931, Professor Alvin H. Hansen and I happened to engage in a discussion regarding the nature of the interrelation between capital production and consumer-taking. Hansen took the position that, if we are in a phase of the business cycle where both capital production and consumer-taking are rising, then capital production must start declining as soon as the rate of increase of consumer-taking slackens. This being so, Professor Hansen argued further, the mere fact that the primary factors of production are limited must necessarily lead to a situation where capital production actually declines. In fact, if the primary factors of production are limited, the production of consumer goods (which we here may assume is the same as consumer-taking) cannot go on increasing to infinity. Therefore, the rate of increase of consumer-taking must sooner or later slacken; and, as soon as this happens, the production of capital goods must, as already mentioned, decline absolutely. In this way, Professor Hansen claimed, it should be possible to explain the turning-point of the business cycle. This argument was in conformity with the one he developed on page 113 in his book, Business Cycle Theory (1927).

I took the position that this argument is fundamentally wrong. I claimed that, even under the most simplified assumption, it is wrong to say that capital production must decline because the rate of increase in consumer-taking slackens. And it is, therefore, not true that the limited amount available of the primary factors must necessarily entail a situation where capital production actually declines. After some discussion, Professor Hansen agreed entirely with my argument, and he also pointed out to me that a number of other economists have made the same mistake, for instance, J. M. Clark and Wesley C. Mitchell.

1 I use the term “consumer-taking” to express the amount of consumer goods which is actually taken by the consumer per unit of time. Confusion is liable to arise, I think, from using the word “consumer demand” in this connection.


Clark’s interest in the relation between capital production and consumer-taking seems to have been aroused through Mitchell’s first statistical cycle-studies. Mitchell points out that the peak in consumer-taking follows after the peak in capital production and that it must, therefore, be in production rather than in consumption that we must seek for the key to the turning-point in general business. Clark points out that the time sequence in question does not prove that consumer-taking is a secondary phenomenon and capital production a primary phenomenon in the business cycle. In fact, the rate of increase in consumer-taking exerts an important influence on capital production, and this rate of increase is a time series that precedes the series of consumer-taking itself by about a quarter of a period. Consumer-taking is, therefore, just as much a primary phenomenon as capital production. So far, Clark’s argument is perfectly sound, I believe. His attempt to explain further the exact nature of the mechanism by which consumer-taking and capital production are interrelated, however, is wrong, so far as I can see. In his later paper, Mitchell accepted Clark’s point of view, and put up as the main thesis just that point in Clark’s analysis which is wrong.

In view of this situation, I have believed it worth while to write up a short explanation of my line of argument. If this analysis has any value at all, it is, to a large extent, due to the stimulating effect of the discussion with Professor Hansen.

2. THE RELATION BETWEEN CAPITAL PRODUCTION AND CONSUMER-TAKING REDUCED TO ITS LOWEST TERMS

Let $z$ be consumer-taking per unit of time, $w$ capital production per unit of time, and $W$ the capital stock that exists at any moment of time. All the three magnitudes $z$, $w$, and $W$ are, of course, functions of time. In practice they would be represented by time series.

Let us, for simplicity, make the following two assumptions:

A. Consumer-taking $z$ is the same as the production of the consumer good, and this again is at any time proportional to the existing capital stock $W$. In other words, we have

$$W = kz,$$

where $k$ is a constant independent of time.

B. The depreciation per unit of time $u$, that is to say, the capital production that is needed for replacement purposes, is proportional to the existing capital stock. In other words, we have

$$u = hW,$$

where $h$ is a constant independent of time.
Now, the rate of change with respect to time of the capital stock is equal to
\[ \dot{W} = w - u. \tag{3} \]
By virtue of (1) we have, however,
\[ \dot{W} = k \dot{z}, \tag{4} \]
where \( \dot{z} \) is the rate of change of consumer-taking.
Inserting this into (3), and expressing \( u \) in terms of \( z \) by (2) and (1), we get
\[ k \dot{z} = w - h k z. \]
So that we finally have
\[ w = k (h z + \dot{z}). \tag{5} \]
The rate of change with respect to time of capital production is thus equal to
\[ \dot{w} = k (h \dot{z} + \ddot{z}). \tag{6} \]
Formula (5) indicates the two parts of which total capital production is made up. In the first place we have the part \( k h z \) that represents capital production for replacement purposes. This part is (under our simplified assumption) proportional to the size of consumer-taking. In the second place, we have the part \( k \dot{z} \) representing capital production for expansion purposes. This part is (under the present simplified assumption) proportional to the rate of change of consumer-taking.
Thus there are two forces that act upon total capital production. If consumer-taking is increasing, but at a constantly decreasing rate, the first of these two forces tends to increase, and the second tends to slow down capital production. Which one of the two forces shall have the upper hand depends on the manner in which the increase in consumer-taking slows down, and it depends also on the rate of depreciation. If the rate of depreciation is very small, the second force represents always the most important influence. In the limiting case where there is no depreciation at all, the second force (the rate of change of consumer-taking) is the dominating one, and, even if the rate of depreciation is not very small, there may be certain points of time where the second force is dominating, namely, those points of time where the change in consumer-taking is very slow. If the rate of depreciation is very high, the first force (the size of consumer-taking) is nearly...
always the dominating one. The necessary and sufficient condition that capital production shall be increasing (decreasing) at a given moment of time is, by (6), that the acceleration (the rate of change of the rate of change) of consumer-taking $\ddot{z}$, plus the product of the depreciation rate and the rate of change of consumer-taking, shall be positive (negative).

From the fact that we are in a situation where consumer-taking is increasing, but at a decreasing rate, there does not follow anything about the further development of the system. The system is, so far, quite indeterminate. In the reduced form (5) of the relationship, we have two variables but only one equation. It would be attempting the logically impossible if, from the conditions here considered, we should try to demonstrate that the system must after a while turn into depression. Such a demonstration is impossible even if we add the further condition that the ultimate productive resources of society are limited and that, therefore, there is a certain level beyond which the capital production cannot go. In fact, even if this is the case, and if we are in a situation where consumer-taking is increasing but at a decreasing rate, there are still many possibilities, for instance, the following:

I. Total capital production may immediately become constant, and consumer-taking continue to increase, however, at a constantly decreasing rate, and in such a way that consumer-taking approaches asymptotically to a certain level.

II. Total capital production may continue to increase and approach asymptotically to a certain level, and consumer-taking evolve as in the case of I above.

III. Consumer-taking may evolve as in the foregoing cases while total capital production approaches in dampened cycles to a stationary level.

In Table I below, I have given numerical examples illustrating the cases I and II. As figures for the first years I have taken those used by Professor Hansen. Professor Hansen assumes, as I have done here, that consumer-taking is proportional to the capital stock and that depreciation is proportional to capital stock. More precisely expressed, he starts with an index number of consumer-taking equal to $z_1 = 100$ in the year $t = 1$. In order to produce this amount of consumer good, he assumes that 100,000 looms are needed. If, in the second year, consumer-taking is increased to 110, there will be needed 10,000 looms for expansion purposes and 10,000 looms for replacement purposes, on the assumption that the depreciation each year is 10 per cent of the capital.

stock at the end of the preceding year. Total capital production the second year will, consequently, be 20,000 looms. From here on, I use my own figures. If consumer-taking in the third year is 119, there will be needed 9,000 looms for expansion and 11,000 looms for replacement, so that the total capital production still is 20,000. The rest of the figures in case I of the table are now self-explanatory. Although capital production is maintained unchanged, consumer-taking is increasing all

TABLE I

<table>
<thead>
<tr>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4) Capital Production Needed for Expansion (Number of Looms) (20000(1 + \frac{1}{110})) (3 Zeros Omitted)</th>
<th>(5) Capital Production Needed for Replacement (Number of Looms) (20000(1 + \frac{1}{110})) (3 Zeros Omitted)</th>
<th>(6) Total Capital Production (Sum of col. 4 and col. 5) (\frac{100}{110}) (3 Zeros Omitted)</th>
<th>(7) Stock of Capital at End of Year (W_t) (3 Zeros Omitted)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Year</td>
<td>Number of Consumer-taking (z_t)</td>
<td>Rate of Increase of Consumer-taking (z_t = z_{t-1} - z_{t-1} - 1)</td>
<td>10</td>
<td>10</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>1</td>
<td>100</td>
<td>110</td>
<td>9</td>
<td>8.1</td>
<td>8.1</td>
<td>12.71</td>
</tr>
<tr>
<td>3</td>
<td>119</td>
<td>10</td>
<td>9.5</td>
<td>9.5</td>
<td>11.9</td>
<td>20</td>
</tr>
<tr>
<td>5</td>
<td>134.39</td>
<td>8.57375</td>
<td>8.57375</td>
<td>8.57375</td>
<td>21.42625</td>
<td>30</td>
</tr>
<tr>
<td>0</td>
<td>200</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>300</td>
</tr>
</tbody>
</table>

Case I

<table>
<thead>
<tr>
<th>(t=1)</th>
<th>(100)</th>
<th>(10)</th>
<th>(10)</th>
<th>(10)</th>
<th>(10)</th>
<th>(10)</th>
<th>(10)</th>
<th>(10)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>110</td>
<td>10</td>
<td>10</td>
<td>10</td>
<td>10</td>
<td>10</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>3</td>
<td>119</td>
<td>9.5</td>
<td>9.5</td>
<td>9.5</td>
<td>9.5</td>
<td>9.5</td>
<td>9.5</td>
<td>9.5</td>
</tr>
<tr>
<td>5</td>
<td>137.09875</td>
<td>8.57375</td>
<td>8.57375</td>
<td>8.57375</td>
<td>8.57375</td>
<td>8.57375</td>
<td>8.57375</td>
<td>8.57375</td>
</tr>
<tr>
<td>6</td>
<td>300</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Case II

<table>
<thead>
<tr>
<th>(t=1)</th>
<th>(100)</th>
<th>(10)</th>
<th>(10)</th>
<th>(10)</th>
<th>(10)</th>
<th>(10)</th>
<th>(10)</th>
<th>(10)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>110</td>
<td>10</td>
<td>10</td>
<td>10</td>
<td>10</td>
<td>10</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>3</td>
<td>119.5</td>
<td>9.5</td>
<td>9.5</td>
<td>9.5</td>
<td>9.5</td>
<td>9.5</td>
<td>9.5</td>
<td>9.5</td>
</tr>
<tr>
<td>5</td>
<td>137.09875</td>
<td>8.57375</td>
<td>8.57375</td>
<td>8.57375</td>
<td>8.57375</td>
<td>8.57375</td>
<td>8.57375</td>
<td>8.57375</td>
</tr>
<tr>
<td>6</td>
<td>300</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

the time and approaches the asymptotic level 200. This asymptotic level is determined as

\[
100 + 10(1 + 0.9 + 0.9^2 + \ldots) = 100 + \frac{10}{0.1} = 200.
\]

Total capital stock is also increasing toward an asymptotic level. In case II of the table the increase in consumer-taking is selected in such a manner that also capital production partakes in the evolution toward the stationary situation. The stationary level in consumer-taking is here equal to

\[
100 + 10(1 + 0.95 + 0.95^2 + \ldots) = 100 + \frac{10}{0.05} = 300.
\]
A great number of other examples could be given. The only condition that must be fulfilled in order that we shall have a monotonic evolution toward a stationary level similar to the one exemplified in Table I is that

\[(1 - h)\hat{z}_{t-1} \leq \hat{z}_t \leq \hat{z}_{t-1}\]

and that the series \(\hat{z}_3 + \hat{z}_4 + \ldots\) is convergent. Otherwise, the numbers \(\hat{z}_t\) may be selected quite arbitrarily.

It is thus evident that the mere fact that there is a decline in the rate of increase in consumer-taking does by no means entail a decline in capital production. The whole situation depends on the manner in which the rate of increase declines. And it is, of course, perfectly possible to indicate such a time-shape of consumer-taking that will actually bring capital production to turn downward. By assuming a given time-shape of consumer-taking we can even by (5) deduce exactly the corresponding time-shape of capital production—but we cannot deduce both time-shapes from the single condition (5). We may, for instance, take for granted that consumer-taking will evolve in cycles, and then deduce the corresponding time-shape of capital production. This time-shape will then also turn out to be cyclical. And if we assume a fair degree of regularity in the cycles in consumer-taking, the cycles in capital production will also be fairly regular. It is even possible to indicate by how much one of the curves will lag behind the other. One interesting question in this connection is, for instance, this: Will the turning-points in capital production come much later than the points of fastest increase (or decrease) in consumer-taking? That will depend on the depreciation rate. And it will also depend on the length of the cycle. The nature of this dependency is the following:

Instead of expressing the depreciation rate per year, we shall express it per the length of time included in a cycle. If \(h\) is the depreciation rate per year, and \(\rho\) is the length of the cycle, then the depreciation rate per cycle will be equal to

\[H = \rho h.\]  \hspace{1cm} (7)

The depreciation rate \(H\) we may perhaps call the *normalized depreciation* rate. Furthermore, the time interval by which the turning-points in capital production lag behind the points of fastest change in consumer-taking we shall measure as a fraction \(c\) of the length of the cycle. The lag measured in absolute units will consequently be \(cp\). The number \(c\) we may call the *lag fraction.*
The interesting thing is that if the cycles are fairly regular, the lag here considered will always be less than a quarter of a period. And the magnitude of the lag fraction will depend only on the normalised depreciation rate. For small depreciation rates (i.e., \( H \) not larger than about unity), we have approximately

\[
c = \frac{H}{2\pi^2}.
\]  

(8)

A more correct formula, holding good also for larger magnitudes of \( H \), is

\[
\tan 2\alpha = \frac{H}{2\pi},
\]

(9)

\[1 \leq c \leq \frac{1}{4}.
\]

Table II gives an expression of how \( c \) changes with \( H \). It is remarkable that for depreciation rates of the order of magnitude which we would expect to find in most actual cases the lag is a very small one, only amounting to a small percentage of the cycle length. It should, of course, be remembered that this result is obtained by assuming that consumer-taking evolves in cycles.

**TABLE II**

<table>
<thead>
<tr>
<th>Depreciation Rate Counted per Cycle Length</th>
<th>Fraction of the Cycle length by Which the Turning-points in Capital Production Lags behind the Point of Fastest Change in Consumer-taking</th>
</tr>
</thead>
<tbody>
<tr>
<td>( (H) )</td>
<td>( (c) )</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>0.025</td>
</tr>
<tr>
<td>2</td>
<td>0.049</td>
</tr>
<tr>
<td>3</td>
<td>0.071</td>
</tr>
<tr>
<td>4</td>
<td>0.090</td>
</tr>
<tr>
<td>5</td>
<td>0.107</td>
</tr>
<tr>
<td>10</td>
<td>0.160</td>
</tr>
<tr>
<td>100</td>
<td>0.241</td>
</tr>
<tr>
<td>( \infty )</td>
<td>0.250</td>
</tr>
</tbody>
</table>

It would not be difficult to make the present analysis more realistic by taking account of the fact that capital production takes time. In Aftalion’s manner we could distinguish between capital goods ordered and capital goods delivered. Furthermore, we could consider the depreciation rate and the ratio between consumer-taking and capital stock as changing with time. But all this would not materially alter that feature of the relationship in which we have here been interested.
3. THE MISTAKE IN CLARK'S, MITCHELL'S, AND HANSEN'S ANALYSES

It is now easy to pin down exactly in what the mistake in Clark's, Mitchell's, and Hansen's analyses consists. Let us take Clark's argument first.

First of all, we notice that Clark's argument is essentially based on the same two assumptions that I adopted in Section 2, namely, that consumer-taking is proportional to the existing capital stock and that depreciation is also proportional to the existing capital stock (see, in particular, pages 391–92 in The Economics of Overhead Costs). Basing his argument on these assumptions, Clark maintains that capital production must necessarily decline as soon as the rate of increase in consumer-taking slackens: "The makers of capital equipment are bound, in the nature of the case, to suffer an absolute decline in the demand for their products . . . whenever ultimate demand slackens its rate of growth."5 This is not correct, and apparently the error is that Clark overlooks the fact that the amount of capital production at any given point of time depends on two things: capital production for expansion purposes and capital production for replacement purposes. It is true that at certain points in the discussion he makes some allusion to the difference between these two things; but when it comes to a discussion of the consequences which can be drawn from the nature of the relationship between capital production and consumer-taking, he argues definitely as if total capital production should consist only of capital production for expansion purposes. As soon as we take into account both forces that act on capital production, and, particularly, when we look upon the matter in the light of the rôle played by the depreciation rate as explained in Section 2, it becomes clear that we cannot make any such statement as the one quoted above.

It seems to me that the mistake on this point is of a rather vital sort. Of course, if Clark's object had been merely to point out the fact that there exists a certain mechanism connecting consumer-taking and capital production, the correction here discussed may perhaps be looked upon as a minor one. But Clark's object is much more than this. It is obvious from his whole discussion that he wants to do something more than just draw attention to the existence of a mechanism connecting consumer-taking and capital production. He also wants to deduce a certain consequence from the particular nature of this mechanism, namely, the consequence that the limited amount of the primary factors of

production (or any other cause that prevents consumer-taking from increasing to infinity) must lead necessarily to a situation where capital production declines absolutely. Indeed, it is this consequence of the mechanism connecting consumer-taking and capital production which obviously is his main thesis (see page 392). And this consequence can only be deduced from Clark's incorrect description of the mechanism. It does not follow from the nature of the mechanism when this nature is correctly stated.

It is very characteristic of the whole situation that it was just on this incorrect part of Clark's analysis that Hansen based his argument in our discussion at the Campus Club meeting. In Business Cycle Theory, page 113, he formulates the same idea in the proposition that "as soon as the rate of increase in consumer demand begins to decline, an absolute decline in the demand for fixed capital ensues."

No less significant is it that Mitchell, when he took up Clark's point of view, put the same erroneous proposition in the foreground. Mitchell says: "But their business [i.e., the business of the equipment trades] falls off again . . . . provided the increase in the physical quantity of product slackens before it stops." As will be seen from the numerical examples in Table I, it is by no means necessary that capital production must decline because the rate of increase in production slackens.

Both Hansen and Mitchell use numerical examples to support their thesis; and as they stand, these examples may seem convincing, but in reality they do not prove what they were intended to prove. They only show that, on the simplified assumption adopted, the rate of increase in consumer-taking may slacken in such a particular manner that a decrease in capital production follows. The numerical figures in their examples happened to turn out that way. But the examples do by no means show that a decrease in capital production must necessarily follow.

Oslo, Norway