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ON THE DYNAMICS OF GLOBAL ECONOMIC INEQUALITY

By

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1. *INTRODUCTORY REMARKS*

Economic progress for everybody, more equal distribution of goods and services and a more careful exploitation of world resources; these are some of the keynotes in current plans for global economic development. Confronted with such high ideals it may seem sour and petty to raise questions of practical feasibility or inner consistency.

It is, however, somewhat difficult to see how the danger of a catastrophe could be reduced by increasing the speed at which one is approaching it. The hopeful answers to this kind of worries seem to run along two different lines of thought.

On the one hand, there is the hope of positive feedback effects upon human attitudes, positive in the sense that material progress will remold human attitudes in a direction that will make plans for the good life in the future more and more easy to carry out. Thus, for example, if people are given the economic ability of bringing up more children, the effect would actually be a lower rate of population growth. If groups of suppressed people with a meager capacity for carrying out military activities could only have a better economic basis for doing so, they would contribute to world peace. If people in the industrially advanced nations could only be sure of steady economic growth for themselves, they would become more and more altruistic towards the rest of the world. And so on.

The other line of thought has to do with the world supply of resources. It is possible that the situation in this respect may become very precarious. So we have to conserve and economize. On the other hand, we cannot be sure of this. Such fears should, therefore, not stand in the way of current economic progress.

Some of the monstrosities that we have indicated are reflected in elementary models of economic growth. In such models the picture of the near future is often quite compatible with development that one can in fact expect. But the long-range aspects, the socalled asymptotic properties, serve more as a "reductio ad absurdum" than as a serious prediction. ("Infinitely" many people living of an "infinitely" large world product, etc.). Since we cannot have an impossible world, something else will happen. And it will. It is a reasonable axiom that actual history will be consistent.

Suppose we take it as an actual possibility that there are serious limits to world resources. Then a steady development of more for everybody, coupled with a more and more equal distribution of goods and services, cannot persist. For there is no evidence in sight that would support the belief in a stationary or falling world population under such conditions. Something will have to yield.

One idea, implicit in many hopeful views on the future of mankind, is that the greedy craving for economic growth will give way to an altrustic willingness to share and share alike. This could certainly lead to a rather dramatic final outcome where a maximum number of people, all equally near the starvation level, would drop dead simultaneously.

Another possibility, although not a very pleasant prospect, is that the willingness to share and to support those who are the weaker in the struggle for existence will be the residual factor. I do not say that this is necessarily a good prediction, but it may very well become so, if we keep on sweeping such ideas under the carpet as if they were only something belonging to ancient history.

In the following we shall try to explore this second possibility a little further, by means of a simple model framework. The distance between the broad ideas sketched above and what is covered by the dry model framework to be presented is admittedly still very great.

2. *SOME BASIC CONCEPTS*

Consider at time t $N(t)$ individuals, each characterized by a set of values $x_1,\ldots,x_m$ of m measurable properties, numbered $1,\ldots,m$. These characteristic properties may be resources (income), skill, physical strength, etc. For the vector $(x_1,\ldots,x_m)$ we shall use the shorter notation x. The individuals may produce offspring or change their own status. In such cases we shall denote the original or parent status of the m attributes by $(y_1,\ldots,y_m) = y$ and the new status of the m attributes by $(x_1,\ldots,x_m) = x$.

We define two concepts of frequency distributions, the <u>absolute</u> distribution $F_t(x)$ and the <u>relative</u> distribution $f_t(x)$. The two distributions are related by the definition

$$(2.1) \qquad F_t(x) = N(t)\, f_t(x) \quad .$$

We shall assume that $N(t)$ is so large at any time that the requirement of N(t) or subgroups of N(t) being an integer is of no material consequence.

The distributions F_t and f_t are the distributions of x as of time t. We shall assume that F_t and f_t can be written as follows

$$F_t(x) = F(x,t) \tag{2.2}$$

$$f_t(x) = f(x,t) \tag{2.3}$$

when the functional forms F and f are independent of the values of the arguments in the functions. Furthermore, we shall assume that the attributes are measured in such a way that an individual with a higher x_i is "stronger" than an individual with a lower x_i as far as attribute No. i is concerned. For the sake of simplicity we shall assume that all the attributes have non-negative values. Under these assumptions we may postulate

$$\int_{x=o}^{\infty} f(x,t)dx = 1 , \quad \text{for all } t \tag{2.4}$$

and, consequently,

$$\int_{x=o}^{\infty} \frac{\partial f}{\partial t} dx = 0 , \quad \text{for all } t . \tag{2.5}$$

As time passes, the distribution F(x,t) may change. This can be expressed as follows,

$$\frac{\partial F(x,t)}{\partial t} = \frac{dN}{dt}f + N\frac{\partial f}{\partial t} = F(x,t)\left(\frac{dN}{dt}\frac{1}{N} + \frac{\partial f}{\partial t}\frac{1}{f}\right) . \tag{2.6}$$

Here the first member in the right hand side brackets could be termed the <u>mass effect</u>, while the second could be termed the <u>distortion effect</u>, as far as the absolute frequency distribution is concerned.

In the following, two <u>conditional</u> distributions will be of importance.

The first is the conditional distribution of children with regard to the m attributes $(x_1,\ldots,x_m)$, <u>given</u> that they came from parents with attributes $(y_1,\ldots,y_m)$. This conditional frequency distribution we shall denote by

$$g(x/y,t), \quad \int_{x=o}^{\infty} g(x/y,t)dx = 1 . \tag{2.7}$$

The reason for time t appearing in this distribution is that the conditional frequencies may change over time.

The second conditional frequency distribution has to do with migrations. We may imagine that an individual having properties $(y_1,\ldots,y_m) = y$ moves into a group having properties $(x_1,\ldots,x_m) = x$. The conditional distribution of x will be denoted by

$$h(x/y,t), \quad \int_{x=o}^{\infty} h(x/y,t)dx = 1 \tag{2.8}$$

with similar interpretation as for g above.

The two conditional distributions defined above could be said to represent <u>diffusion</u> processes.

In most statistical applications of frequency - or probability distributions the "identity" of the individual is in a way immaterial. The only thing that counts is <u>how many</u> of them have the characteristic x. If we have two frequency distributions $F_{t_1}(x)$ and $F_{t_2}(x), t_2 > t_1$ and if they are identical in form, it is still possible that a lot of things could have happened to the individuals between t_1 and t_2. Two individuals could have changed places on the x-scale or, more generally, the whole population could have been re-shuffled in various ways. Such possibilities may be of importance in the dynamics of inequality.

3. *AN EQUATION FOR THE EVOLUTION OF INEQUALITIES* *)

Consider the distributions $F(x,t)$ and $F(x,t+\Delta t)$. They may be different for the following reasons:

(a) The individuals at time t may have produced children who have been scattered over the whole range of attributes at time $(t+\Delta t)$.

(b) Some individuals in status x at time t may have died.

(c) Some individuals in status x at time t may have "migrated" into a different status at time $t+\Delta t$.

We shall assume that our model is global in the sense that there is no net migration to or from the "outside". Using a continuous formulation, we are then led to the following dynamic process,

(3.1)
$$\frac{\partial F(x,t)}{\partial t} = \int_{y=o}^{\infty} g(x/y,t)\ \alpha(y,t)\ F(y,t)dy - \beta(x,t)\ F(x,t) + \int_{y=o}^{\infty} h(x/y,t)\ \gamma(y,t)\ F(y,t)dy - \gamma(x,t)\ F(x,t)$$

where $\alpha(x,t)$ is the birth rate in group x, while $\beta(x,t)$ is the death rate in group x, and $\gamma(x,t)$ the rate of total emmigration from group x, per unit of time.

Using (2.1) and (2.4) - (2.8) we get the following equations for the mass effect and the distortion effect, respectively

(3.2)
$$\frac{\frac{dN}{dt}}{N} = \bar{\alpha}(t) - \bar{\beta}(t),$$

where $\bar{\alpha}(t)$ is the average birth rate and $\bar{\beta}(t)$ the average death rate, and (before dividing both sides by $f(x,t)$)

*) I owe thanks to my colleagues Arne Strøm and Knut Sydsæter for helpful mathematical advice. Needless to say, they have no responsibility for defects in the final product.

(3.3)
$$\frac{\partial f(x,t)}{\partial t} = \int_{y=o}^{\infty} (g(x/y,t)\ \alpha(y,t) - \bar{\alpha}(t)\ f(x,t))\ f(y,t)dy - (\beta(x,t) - \bar{\beta}(t))\ f(x,t) + \int_{y=o}^{\infty} (h(x/y,t)\ \gamma(y,t) - \gamma(x,t)f(x,t))f(y,t)dy\ .$$

If we integrate both sides of (3.3) with respect to x, we see that the left hand side and the right hand side both become zero, as should be the case.

The analytical problem in connection with equations (3.2) and (3.3) is the following: Given the functions g, h, α, β, γ, what are the functions $N(t)$ and $f(x,t)$? The first problem, the form of $N(t)$, is simple, while the second problem, the form of $f(x,t)$, may be complicated. (It should be noticed, however, that the two problems are interrelated.)

4. *FURTHER ANALYSIS OF SPECIAL CASES*

Consider the particularly simple case where the conditional distributions g and h do not depend on y. Let these two simplified distributions be denoted by $g^*(x,t)$ and $h^*(x,t)$ respectively. The equation (3.3) then takes the form

(4.1)
$$\begin{aligned}\frac{\partial f(x,t)}{\partial t} &= (g^*(x,t) - f(x,t))\bar{\alpha}(t) - (\beta(x,t) - \bar{\beta}(t))f(x,t) + h^*(x,t)\ \bar{\gamma}(t) - \gamma(x,t)\ f(x,t) \\ &= -(\beta(x,t) + \gamma(x,t) + \bar{\alpha}(t) - \bar{\beta}(t))f(x,t) + g^*(x,t)\ \bar{\alpha}(t) + h^*(x,t)\ \bar{\gamma}(t)\ .\end{aligned}$$

For any given x this is a simple first order linear differential equation, the solution of which is

$$(4.2)\qquad f(x,t) = \phi(x)\, e^{-\int_0^t A(x,\tau)d\tau} + \int_0^t B(x,\tau)\, e^{-\int_\tau^t A(x,s)ds}\, d\tau$$

where

$$(4.3)\qquad A(x,t) = \beta(x,t) + \gamma(x,t) + \bar{\alpha}(t) - \bar{\beta}(t)$$

and

$$(4.4)\qquad B(x,t) = g^*(x,t)\, \bar{\alpha}(t) + h^*(x,t)\, \bar{\gamma}(t) \quad .$$

The arbitrary function $\phi(x)$ represents the initial distribution $f(x,o)$. The solution is not quite explicit, however, as the quantities $\bar{\alpha}(t)$, $\bar{\beta}(t)$ and $\bar{\gamma}(t)$ actually depend on the form $f(x,t)$. This problem can be solved by using the definition of the expected value of a function of x.

A further drastic specialization is of some interest, at least for illustrative purposes. Consider the case where there is a) <u>no migration</u> and b) where the children have the <u>same distribution as their parents</u>, i.e. $g^*(x,t) = f(x,t)$. In that case the equation (4.1) reduces to

$$(4.1.a)\qquad \frac{\partial f(x,t)}{\partial t} = -\,(\beta(x,t) - \bar{\beta}(t))f(x,t) \ ,$$

and we obtain

$$(4.5)\qquad f(x,t) = \phi(x)e^{-\int_0^t (\beta(x,\tau) - \bar{\beta}(\tau))d\tau} \quad .$$

We shall consider this last very simple case in order to say something explicit about the <u>mass effect</u> of evolution. And let us now think of x not as a <u>vector</u> but as a <u>single characteristic</u>, e.g. income or the total supply of goods available to the individual.

Let $\bar{x}(t)$ be the <u>average</u> x in the distribution $f(x,t)$. And suppose that $\beta(x,t)$ can be written more explicitly in the form

$$(4.6)\qquad \beta(x,t) = b(x)\, \frac{\bar{x}(t)\, N(t)}{A}$$

where A is the total of resources available to all the N(t) individuals. The fraction to the right in (4.6) can then be interpreted as the total pressure upon resources at time t, while b(x) characterizes the dependence of the death rate upon the status x. We may think of b(x) as a function that <u>falls</u> when x increases.

Consider the mass effect relation (3.2). It now takes the form

$$(4.7)\qquad \frac{dN}{dt} = \bar{\alpha}(t)\, N(t) - \bar{b}(t)\, \frac{\bar{x}(t)}{A}\, N(t)^2 \ ,$$

where $\bar{b}(t)$ is the average of b(x) at time t.

It is seen that this is the equation of some sort of <u>generalized logistic</u> curve.

From the equation (4.5) it is possible to say something about $\bar{b}(t)$ and $\bar{x}(t)$. In fact, it is intuitively obvious that if b(x) and x are negatively "correlated", then $\bar{b}(t)$ will fall and $\bar{x}(t)$ will rise as functions of time. It then appears that the "braking element" represented by the last term to the right in (4.7) can develop in many different ways. One possibility is, for example, that $\bar{b}(t)$ continues to decrease, while $\bar{x}(t)$ increases and as far as N(t) is concerned it may reach a maximum and then <u>start falling</u>. This may have some interest in view of the much-discussed possibility that N(t) may increase to the point of a catastrophe.

The tentative conclusions drawn from the simple model above may be somewhat one-sided. There are several other possibilities. Let me mention a few of them.

We have not said much about the function $\alpha(x,t)$. As far as empirical evidence is concerned this function is certainly complicated, both with regard to the effect of x and with regard to the effect of time. It is, therefore, much too simple to assume that $\bar{\alpha}(t)$ should remain relatively constant. It is very difficult to have any founded opinion regarding the evolution of this parameter.

Next, there is the question of the function $\beta(x,t)$ and, therefore, the development of $\bar{\beta}(t)$. In the formula (4.6) we have assumed that $\beta(x,t)$ has a component, $b(x)$. This may be too simple. It is possible that the ability to survive depends also on the number of people struggling for survival. When this struggle gets really tough, it is possible that the small and many could somehow eat the bigger and more demanding ones out of existence. (Cf. what happens in a fishing lake when it becomes overpopulated. Then it is not the big ones that take over, but rather the small and many who can somehow survive on very little. This may indicate that the number of individuals is not necessarily the best measure of population pressure.)

In the formula (4.6) we have assumed that total resources are constant. This assumption may overlook the fact that resources may be a relative concept, depending on the human ability to utilize the resources. Thus, for example, the parameter A could be positively correlated with $\bar{x}$, which would mean that the strong would not only get more, but that they would also show increasing ability to produce goods and services on which to subsist.

Considering some of these possibilities and many others not mentioned, it is in fact possible that evolution does not necessarily have to end in some kind of steady state condition. Wavelike motions, perhaps with a very long period, are quite feasible.