ECON4140 Mathematics 3 – remarks on a 2023–05–12 exam paper

This paper got an «A». It does have its imperfections. At the precision level of this course this year (giving some slack for the pitfalls) and the five problems could be A–B–A–B–A (and no. 5 was only five percent worth), which suggests a very narrow A – but it was so clearly the best-in-show that the grading committee wasn't in much doubt.

These remarks might sometimes suggest to write an answer slightly different even when I consider it full score. Those suggestions are not only for A-level papers:

- Yes if you aspire for A or B, it surely should be clear what you conclude upon.
- But: for weaker papers – or weaker *parts of* a paper – documenting correct work might save a grade just as much as for a near-top paper. When things do go wrong, there is value in whatever shows you can apply a correct method (and how far). Jumping straight to wrong conclusion might be scored to F – pointing out how you attempted at something correct, and partly carried it out before messing up, can save some points which in the end can tilt a grade.

Examples from this paper:

- 2(a): calculations inaccuracies, but method is right.
- 4(b): generally right method, and enough written out to show that it is not too far from complete.
- 3(b): balancing on uncomfortably sparse. An inaccuracy could have been costly.

But on the other hand, see the remark to Problem 5.

Problem 1 (linear algebra) is «A» level, though (a) had a (very common) shortcoming.

(a) When $R = -5$, the rank is < 2, and that means it is 0 or 1. Logically, one needs to point out the rank is not zero in order to conclude that it is 1. Only null matrices have rank $= 0$, but does that go without saying? Not all graders would buy that, but it is an omission of the kind that is easier to forgive when a paper handles all linear algebra so well.

(b) Full score, but it wouldn't *hurt* to complete the first line with « $= 2\vec{u}$ ».

(c) Of course one may wonder why the candidate writes «(if $0 \in \mathbb{R}$)», and maybe there is some confusion here connected to the fact that the null *vector* is not any eigenvector, but it is hard to penalize this.

(d) Full score. Readers should take «must be symmetric:» to mean «must be symmetrized:» – see what the paper then *does,* that is indeed an an essential element in solving this problem, and it was deliberate to pose the form in terms of a non-symmetric matrix.

Problem 2 (differential and difference equations and systems) might be «B» level, but it is mildly surprising that an otherwise excellent paper makes so many different mistakes. Half of the elements are «fully accomplished» though, and the rest are more than halfway there, that makes for a «B».

(a) There are calculations inaccuracies here. The third-to-last line is correct, but then the penultimate line should read $\frac{1}{2}\ddot{x} - \frac{1+R}{2}\dot{x} + (R+5)x = -R^2 \qquad | \cdot 2$
That error is carried over to the last line, where all for sudden the «$(R+5)$» becomes «4» before both errors are undone in the last step.

Unfortunately we often see answers where the candidate jumps from something obviously wrong to the answer that was given. Sometimes it is clearly a glitch not coming from any theoretical shortcoming, sometimes it is much worse (like, many will just routinely write the correct conclusion with or without it having any connection to their previous line). In this case, the *method* is correct, and the mistakes – be they in calculations or in copying from drafts – don't simplify away anything. That justifies a high (but certainly not full) score.

(b)
- First bullet item: The paper misses the «2023», i.e. the shift by -4046. Even if one doesn't know how shifts can enter into a «solving» procedure, one can answer part (b) by *verifying.* We have a constant particular solution $Q = -1/4$, and then for the homogeneous: we can verify that $\sin(2(t-2023))$ and $\cos(2(t-2023))$ are solutions, and they are non-proportional functions. Given that, it should be known that one can just «write them up with constants in front and plus sign in between».
- Second bullet item is full score. Note the word «fit»: candidate shows the knowledge that those constants are not arbitrary integration constants, they need to be fit to the right-hand side in question.

(c) There are two elements to this. One is to find the constant particular solution, and to know that it is not the same as for the differential equation. Despite the problem text emphasizing «(the *inhomogeneous* equation!)», this paper skips that completely, thus getting the wrong answer in the end.

The other element is to solve a linear equation system for the constants. The paper accomplishes that.

(d) Full score. This question tests whether the candidates know the criteria, and that they are different for difference equations and differential equations. The paper reaches the contradiction for the former and verifies the latter.

Problem 3 is «A» level. Some remarks still:

(a) The paper could be better on clarifying that it isn't really the concavity of $x \mapsto H$, but of the maximized Hamiltonian. The reason that it falls out as easy as this, is that everything with u is constant in x.

(Well ahem, it *does* keep the «$\frac{5}{4}$» that my grading guideline document forgot about.)

(b) This sparse text is closing in on «dangerous»: it isn't so in general that when there is no tomorrow, one can disregard it and maximize running utility. It hinges upon there being no terminal condition, and the free end leads to $p(2024) = 0$. I would have wanted a little bit more of text. Had one written a simple «$H(x, u, 0) = \frac{5}{4}x^2 - u^2$» it would have been clear that the condition is actually used.

(c) Splendid. The paper used the information from problem 2 to deliver a clear and concise answer.

Problem 4 might have been a bit unfortunate – functions with split definition seem to be awkwardly unfamiliar – and so good score was given despite not getting done with (b).

(a) Not fully happy about the notation («max» over what you say?), but it looks correct apart from that.

(b) At first glance this doesn't look too good. But, considering what the problem set out to test:

- Proof by induction is syllabus in its own right, and a meritable part of the problem is to pose an induction proof – it could be half the score, and especially when the split definition function turned out to cause all sorts of trouble for everyone. (Disregard that the paper writes «$N = 1$» for the base case, when it is the final time – because the latter is what is actually established.)
- The last few lines are actually very close to establishing the case $x \leq 2$.

The lack of thoroughness in analyzing all the cases and writing correct notations puts it at the «B» level; but not worse, since the general approach is sound and for part (b) – counting up what has actually been achieved – the paper actually gets most bases covered.

Problem 5 is correct – *but ...*

Here there is ink better spent elsewhere. The paper is on the sparse side on several other items, and here one could have stopped after the text «line segment containing the two points», and if more is wanted: why not add «meaning, infinitely many.» And stop there.

In fact the end of that page is close to doing *harm,* depending on graders' discretion: The paper suggests that an interval in n-space isn't convex. Now an «interval» would then probably mean a line segment, or possibly a rectangle – those are indeed convex. So for someone who has caught what the problem is about ... don't talk nonsense when you have solved it!