KEEPING BOTH EYES WIDE OPEN
The life of a competitive authority among sectoral regulators*

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This series is published by the Department of Economics in co-operation with The Frisch Centre for Economic Research.

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<table>
<thead>
<tr>
<th>No</th>
<th>Date</th>
<th>Authors</th>
<th>Title</th>
</tr>
</thead>
<tbody>
<tr>
<td>11</td>
<td>08</td>
<td>Kari Furu, Dag Morten Dalen, Marilena Locatelli and Steinar Strøm</td>
<td>Generic substitution</td>
</tr>
<tr>
<td>10</td>
<td>08</td>
<td>Reyer Gerlagh, Snorre Kverndokk and Knut Einar Rosendahl</td>
<td>Linking Environmental and Innovation Policy</td>
</tr>
<tr>
<td>9</td>
<td>08</td>
<td>Steinar Strøm and Jon Vislie</td>
<td>A Discrete-Choice Model Approach to Optimal Congestion Charge</td>
</tr>
<tr>
<td>8</td>
<td>08</td>
<td>M. I. Di Tomasso, Steinar Strøm and Erik Magnus Sæther</td>
<td>Nurses wanted.</td>
</tr>
<tr>
<td>7</td>
<td>08</td>
<td>Atle Seierstad</td>
<td>Existence of optimal nonanticipating controls in piecewise deterministic control problems</td>
</tr>
<tr>
<td>6</td>
<td>08</td>
<td>Halvor Mehlum and Karl Moene</td>
<td>King of the Hill</td>
</tr>
<tr>
<td>5</td>
<td>08</td>
<td>Zheng Song, Kjetil Storesletten and Fabrizio Zilibotti</td>
<td>Rotten Parents and Disciplined Children: A Politico-Economic Theory of Public Expenditure and Debt</td>
</tr>
<tr>
<td>4</td>
<td>08</td>
<td>Michael Hoel</td>
<td>Environmental taxes in an economy with distorting taxes and a heterogeneous population</td>
</tr>
<tr>
<td>3</td>
<td>08</td>
<td>Fedor Iskhakov</td>
<td>Dynamic programming model of health and retirement</td>
</tr>
<tr>
<td>2</td>
<td>08</td>
<td>Atle Seierstad and Sigve D. Stabrun</td>
<td>Discontinuous control systems</td>
</tr>
</tbody>
</table>

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KEEPING BOTH EYES WIDE OPEN:
The life of a competition authority among sectoral regulators*

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June 2, 2008

Abstract

Competition authorities must pay attention to many industries simultaneously. Sectoral regulators concentrate on their own industry. Often both types of authority may intervene in specific industries and there is an overlap of jurisdictions. We show how a competition authority’s resource allocation is affected by its relationships with sectoral regulators and their biases. If agencies collaborate (compete), the competition authority spends more effort on the industry with the more (less) consumer-biased sectoral regulator. The competition authority spends budget increases on the industry whose regulator reacts less to more effort. The socially optimal budget corrects for distortions due to regulatory bias, but only downwards.

Keywords: Competition authority, Sectoral regulators, Regulatory bias.
JEL: H11, L40, L51

*Corresponding author: shoernig@fe.unl.pt. We would like to thank Thomas Gehrig and audiences at EARIE 2007 (Valencia), ASSET 2007 (Padova) and the “Lisbon Discussion Group on Institutions and Public Policies”. We gratefully acknowledge financial support from INOVA.
1 Introduction

The evolution of deregulation in many industries implied a movement from monopoly to competition, with an intermediate phase where competition was gradually introduced but regulatory intensity could still increase before starting to fade out.\(^1\) As a part of this process, it has become common to create overlapping jurisdictions of sector-specific regulators and competition authorities. Examples of industries with this feature are telecommunications, energy, and media. For various aspects of this overlap, see Geradin and O’Donoghue (2005) on telecommunications, Harker and Waddams Price (2004) on energy, and Hope (2007) on media. For a general discussion of the merits of this overlap of jurisdictions, see, e.g., Fehr (2000).

Thus, in many countries there is significant interaction between the competition authority and various sector-specific regulators. Whereas the competition authority needs to spread its (limited) investigation resources across the whole economy, a sectoral regulator by definition has instructions to focus on a single sector or industry. The sectoral regulators vary potentially in terms of objectives and incentives. This will influence the intensity of activity by the competition authority over sectors in a non-trivial way.

In this paper, we aim at discussing this relationship between the competition authority and the various sectoral regulators.\(^2\) In particular, we build a model that makes it possible to discuss how the difference in scope among the economic authorities within a government affects their respective efforts in supervising industries, and in particular how it affects the competition authority’s allocation of its resources. We find that the outcome of the interaction between the economic authorities depends on the insti-

\(^1\)See a general discussion of this movement in Bergman et al. (1998).

\(^2\)We note in passing that our framework can also be used to study the interaction between a supranational agency and various national agencies or, in the US, between a federal agency and the corresponding state agencies. On the latter, see the study by Grace and Phillips (2007) on insurance regulation.
tutional arrangement and the sectoral regulators’ inclinations. We contrast two different institutional arrangements for multi-agency investigations, one competitive and one collaborative. We also let regulators differ in their biases - the relative weights they put on consumer surplus and industry profits when assessing industry performance.

The idea of biased regulators was first coherently formulated by Stigler (1971) and Peltzman (1976), who argue that high stakes and a small number of interested actors favour “tight” groups over larger ones, whose interest tend to be diffuse with small personal stakes. This argument implies that firms can more easily influence sectoral regulators than consumers, resulting in an industry bias with more weight on industry profit than on consumer surplus in the regulator’s objective. A different argument leading to the same phenomenon has been presented by Noll (1971), who stipulates that regulators act such as to minimize conflict with the entities with which they interact, in order not to have their decisions overturned. As a result, they may facilitate the latter’s participation in the process as compared to other actors, or may strategically make more favourable decisions. Martimort (1999a) explores the dynamics of situations where sectoral regulators become industry biased (captured) by firms over time. Recent accounts of this literature and related empirical evidence can be found in Dal Bó (2006), Yackee and Yackee (2006), and Holburn and Vanden Bergh (2006). What we take from this literature is that there are multiple factors deciding whether and how a regulatory agency becomes biased. These factors affect different regulators differently, so that we can expect to have sectoral regulators with varying biases across government, as industries differ in such aspects as the role of firms, regulation, and consumers’ rights institutions.

A benevolent and utilitarian social planner would maximize the unweighted sum of consumer surplus and total profits in an industry. We call an agency with such an objective unbiased. If an agency puts a higher weight on indus-
try profits than on consumer surplus, it is said to be *industry biased*. If, on the contrary, higher weight is put on consumer surplus, it is *consumer biased*. In order to concentrate on the effects of bias, we will discuss the case of two regulated industries that are identical in all respects except possibly the biases of their respective sectoral regulators. In our analysis, for simplicity we assume an unbiased competition authority. A recent literature, summarized by Farrell and Katz (2006), gives arguments for a consumer-biased competition policy, and there seems also to be agreement on current competition policy in the US and the EU being consumer biased. It turns out, however, that the bias of the competition authority is irrelevant in the simple model we put forth.

The competition authority being placed in the middle, interacting with each of them, means that the sectoral regulators interact, indirectly and through the competition authority. This notion of multiple regulators interacting indirectly through the competition authority complements earlier analyses where multiple regulators interact indirectly through a common regulated industry; see, *e.g.*, Martimort (1996). From this literature, we note in particular the notion that biases among regulators may be an efficient way to deal with the regulators’ lack of commitment; see Dewatripont and Tirole (1996). Moreover, Martimort (1999b) argues that multiple regulators mitigate the commitment problem by making renegotiations between regulators and firms harder. In order to focus on the relationship between the sectoral regulators and the competition agency, we assume a very simplistic view of firm behaviour, treating it as exogenous.

Our analysis complements and follows up on the paper by Barros and Hoernig (2008). They discuss the interaction between two regulators of the same industry - such as a competition authority and a sectoral regulator - and find that results depend crucially on whether the outcome of the combined regulatory efforts is based on collaboration or competition between
the two agencies. Collaboration between agencies, defined precisely in the next section, has the feature that the regulators’ efforts are strategic complements, in the sense of Bulow et al. (1985): The more effort one agency puts in, the higher is the marginal benefit for the other of raising its own effort. The opposite holds when the agencies compete: Now the efforts of the two regulators are strategic substitutes.

In focusing on the limited resources of a competition authority, we are related to Martin (2000), who considered a competition authority watching several unregulated industries. Contrary to his emphasis on how the relative performance of an industry affects the resources spent on it, however, we play down the role of characteristics of the industries under study in order to focus on the role of the sectoral regulators and their relationships with the competition authority in the determination of the latter’s resource allocation.

There is a parallel to our model of the interaction between agencies in the work of Sah and Stiglitz (e.g., 1985) on the quality of decisions in organizations where decision-makers are subject to human fallibility, which is a probability of making mistakes. In their work, this probability is fixed, while in Gehrig (2004) and Barros and Hoernig (2008), it is endogenous. The setting of Gehrig’s paper is different from ours and that of Barros and Hoernig (2008), in that he focuses on comparing independent and collaborative R&D by symmetric firms.

Let us now describe our main results. First of all, if decision regimes are the same in both industries (what we call a pure regime), then sectoral regulators’ efforts are indirectly strategic substitutes through their interactions with the competition authority. But if sectoral agencies collaborate with the competition authority in one industry and compete in the other (a mixed regime), they are indirectly strategic complements.

If, in each of the two industries, the competition authority and the sectoral regulator collaborate, it will spend more on the industry with the more
consumer-biased regulator. This holds even if the competition authority is not itself consumer biased or industry biased - what matters is the relative positioning of the two sectoral regulators. In fact, this result holds even if both sectoral regulators are consumer-biased, as long as one of them is more so than the other. This means that, even if we consider the competition agency as an unbiased maximizer of social welfare, the way its decisions are related to the sectoral regulators may magnify the biased behavior of these regulators rather than mitigate it.

If, in each of the two industries, the competition authority and the sectoral regulator compete, we obtain the opposite result: The competition authority will spend more on the industry with the more industry-friendly regulator, thus reducing the effect of the bias. We also show that in a mixed regime either industry may be favoured, depending on how high detection probabilities become.

Thus, the allocation of resources by a competition authority to investigate regulated sectors depends on the combination of biases of the sectoral regulators and the type of formal decision-making authority across authorities, and its intervention may counteract or magnify the effect of the sectoral regulators’ biases. A different question that we also consider is where the competition authority will spend additional resources when they become available. We find that it will spend more on the industry whose sectoral regulator reacts less to additional effort by the competition authority, independently of the decision regime. It is even possible that the competition authority will spend less than before on one of the industries.

We also consider how much money the competition authority should have at its disposal. If sectoral regulators are unbiased, then the socially optimal budget of the competition authority corresponds to the one that exactly finances the effort the competition authority would choose if it cared about the cost of public funds. Yet the optimal budget of the competition au-
authority will be binding if sectoral regulatory bias magnifies changes in the competition authority’s efforts. That is, in order to avoid larger distortions in sectoral regulators’ efforts due to their bias, the competition authority’s spending may have to be reduced.

Our results are robust to a number of extensions which are not reported in the text: the existence of unregulated sectors (which call in some of the resources of the competition authority); firms determining endogenously the probability of occurring anti-competitive behavior; activities by companies aiming at making it harder to gather evidence about their behavior; and information exchange across institutions.\footnote{The proof that our main results hold under these conditions is available upon request from the authors.}

The paper is organized as follows. In the next section, we present the model. In Section 3, we collect some preliminary results. The main analysis is done in Section 4, and implications for the competition authority’s budget are given in Section 5. Finally, we offer some concluding remarks in Section 6.

\section{The basic set-up}

Consider an economy consisting of two identical industries. The welfare gained from the activities in each industry consists of consumer surplus, $S$, and (total industry) profits, $\Pi$, so that welfare per industry is $W := S + \Pi$. The government seeks to maximize total welfare in the economy, and for this it uses a competition authority, $CA$, monitoring both industries, and two sectoral regulators, $SR_1$ and $SR_2$, monitoring one industry each. Since our interest lies with the interactions between agencies, we ignore the agency problem between the government and its competition authority and sectoral regulators.
While we assume that the two industries are identical, the sectoral regulators are not. They are allowed to differ with respect to the weights they put on consumer surplus and profits, respectively. In particular, gross of own resources spent, regulator $SR_i$ maximizes

$$U_i := S + \lambda_i \Pi, \quad i = 1, 2,$$

where $\lambda_i > 0$. Regulator $i$ is said to be unbiased if $\lambda_i = 1$, consumer-biased if $\lambda_i \in (0, 1)$, and industry-biased if $\lambda_i > 1$. We assume the competition authority to be unbiased, thus maximizing $W = S + \Pi$.5

Our picture of regulatory activity is one of monitoring specific industries in order to detect violations of competition law and/or regulatory dispositions: Each industry is in a state which is unobservable to both the sectoral regulator and the competition authority. There are two possible states, which we call “Violation” and “No violation” of the rules of competition. The probability that an industry is in the violation state is $\pi$, which is exogenous and the same for both industries.6

The two agencies involved in an industry expend effort to find out about the true state of nature in that industry. If it is established that an industry is in the state of violation, “remedies” will be imposed. There are thus three pairs of consumer surplus and profits relevant for the discussion: $(S_V, \Pi_V)$ when the industry is in violation, but this fact is not established and there are no remedies imposed; $(S_R, \Pi_R)$ when the industry is in violation, this

---

4Indeed, we assume that both industries are identical precisely in order to concentrate on the effect of bias. A relative increase of $\Delta S$ and/or reduction of $\Delta \Pi$ in one market would simply make the CA spend more in this market, similar to Martin (2000). Differences in regulatory regimes may also give rise to variations in capital intensity, due to the Averch-Johnson effect [Averch and Johnson (1962)]. We neglect this effect here.

5This is admittedly a very benign view of competition authorities. But as it will turn out, introducing a bias here does not affect our results. Our way of modelling a biased regulator goes back at least to Baron and Myerson (1982).

6This is not essential to the main results. We have considered an extension with endogenous probabilities of violation chosen by firms, and results are not qualitatively different.
is discovered, and remedies are imposed; and \((S_N, \Pi_N)\) when there is no violation. Naturally, we assume \(\Pi_R < \Pi_V\), and \(S_R + \Pi_R > S_V + \Pi_V\). The latter inequality can be rewritten as:

\[
\Delta_S := S_R - S_V > \Delta_\Pi := \Pi_V - \Pi_R > 0. \tag{1}
\]

In other words, remedies have a positive effect on consumer surplus and a negative effect on profits, where the former effect is larger than the latter.

With this formulation, we allow for false negatives, so-called Type II errors, in that the industry may be in violation without the agencies finding out. We disregard, however, the possibility of false positives, or Type I errors: The agencies do not claim a violation and impose remedies when in fact there is no violation. We find this a reasonable assumption, as the presence of courts and the possibilities to appeal regulatory and antitrust decisions tend to minimize Type I errors.\(^7\)

We focus on cases where each agency prefers some monitoring over no monitoring; in other words, it prefers the remedies outcome to the violation outcome. For sectoral regulator \(i\) this means that

\[
U_{iR} := S_R + \lambda_i \Pi_R \geq U_{iV} := S_V + \lambda_i \Pi_V,
\]

or

\[
\Delta_i := U_{iR} - U_{iV} = \Delta_S - \lambda_i \Delta_\Pi \geq 0. \tag{2}
\]

The term \(\Delta_i\) describes regulator \(i\)'s gain in utility from detecting a violation. If \(\lambda_1 \geq \lambda_2\) then we have \(\Delta_1 \leq \Delta_2\). For the competition authority,

\[
W_R := S_R + \Pi_R > W_V := S_V + \Pi_V,
\]

and

\[
\Delta := W_R - W_V = \Delta_S - \Delta_\Pi > 0,
\]

\(^7\)See Barros and Hoernig (2008) for a discussion of Type I errors in a one-industry setting.
where the inequality follows from (1).

The condition in (2) holds always for consumer-biased regulators. However, it does not hold for regulators that are sufficiently industry-biased. Imposing the condition implies, in effect, putting limits on how industry-biased a regulator can be. We assume in the following that (2) holds for both sectoral regulators.

The probability that the agencies monitoring an industry are successful in finding a violation (meaning sufficient evidence to hold up the case in court, if necessary) depends on the institutional relationship between the authorities (the decision regime), and on the resources the agencies spend on monitoring.

The competition authority has a given budget at its disposal. Let $M$ denote total resources available at the competition authority, who is to decide how to split this among the two industries, with $e_{10} = e_0$ spent on industry 1 and $e_{20} = M - e_0$ on industry 2. We assume that $M$ is low enough so that the CA’s budget constraint is binding. At the same time, sectoral regulator $i$ decides to put in resources $e_i$ in monitoring its industry, $i = 1, 2$. These resources include any shadow cost of public funds, so that the CA and SRs take these into account when choosing how much to spend. We will analyze the Nash equilibrium of this game.

The timing of decisions deserves some further discussion. Any of the authorities may actually start an investigation. There is no particular reason why one should move earlier than the other. The start of an investigation is triggered by some information received or suspicion raised at any of the authorities. As long as, after the official start of an investigation, the other authority becomes aware of it and is able to start its own investigation before the other one is concluded, we can consider this as simultaneous moves by authorities.

Thus, the probabilities that monitoring is successful in finding violations
in the two industries are

\[ P_1(e_0, e_1) \text{ and } P_2(M - e_0, e_2). \]

We assume each of the two functions to be twice continuously differentiable, increasing and strictly concave in each argument, with \( P_1(0, 0) = 0 \): without any effort nothing can be proved.

Following Barros and Hoernig (2008), we focus on two distinct institutional relationships between the competition authority and the sectoral regulators, while we will allow the relationships to differ across industries. One arrangement corresponds to “collaboration”, or joint decisions, between the two regulators. It implies that a violation is successfully identified and remedies imposed if and only if both the two agencies have succeeded in finding evidence of violation. Under this heading, we include situations where a mandatory opinion from the other authority is required for a final decision to be reached. Thus only when both authorities gather enough evidence to support action there is a case against the firms. Conditional on a violation taking place in industry 1, the competition authority finds enough evidence for action with probability \( p(e_0) \). The sector-specific regulator correspondingly obtains sufficient information with probability \( p(e_1) \). Here, \( p(e) \) is a function that maps resources of an agency into a probability of success, with \( p' > 0, \ p'' < 0, \ p(0) = 0, \) and \( \lim_{e \to \infty} p(e) \leq 1 \). Our description of collaboration implies that we can write the probability \( P_1 \) of the two agencies operating in industry 1 detecting a violation as

\[ P_1^J = p(e_0) p(e_1). \]  

Similarly, if there is collaboration in industry 2, we can write \( P_2 \) as

\[ P_2^J = p(M - e_0) p(e_2). \]  

The other arrangement is called “competition”, or independent decisions, and implies that a violation is successfully identified and remedies imposed
if at least one of the two agencies involved has succeeded in finding evidence of violation. So if there is a “competition” decision regime in industry 1, we can write

\[ P_1^I = 1 - [1 - p(e_0)] [1 - p(e_1)], \quad (5) \]

while for industry 2 the expression is

\[ P_2^I = 1 - [1 - p(M - e_0)] [1 - p(e_2)]. \quad (6) \]

The case of “collaboration” resembles what Sah and Stiglitz (1985) call hierarchy, whereas “competition” corresponds to what they call polyarchy.

### 3 Equilibrium and Interactions

Consider first industry 1, monitored by CA and SR\(_1\). Given the decision \(e_0\) by CA, \(SR_1\)’s optimum effort is the solution to

\[
\max_{e_1 \geq 0} E[U_1] - e_1 = P_1(e_0, e_1) (\pi U_{1R} + (1 - \pi) U_{1N})
+ (1 - P_1(e_0, e_1)) (\pi U_{1V} + (1 - \pi) U_{1N}) - e_1.
\]

We can rewrite the objective function as

\[
E[U_1] - e_1 = P_1(e_0, e_1) \pi \Delta_1 - e_1 + \{\pi U_{1V} + (1 - \pi) U_{1N}\}.
\]

The term in curly brackets is constant, and so we can restate the problem of \(SR_1\) as

\[
\max_{e_1 \geq 0} P_1(e_0, e_1) \pi \Delta_1 - e_1.
\]

We can now express the best response of \(SR_1\), \(\hat{e}_1(e_0)\), as follows: If \(\frac{\partial}{\partial e_1} P_1(e_0, 0) < \frac{1}{\pi \Delta_1}\), then \(\hat{e}_1(e_0) = 0\). If \(\frac{\partial}{\partial e_1} P_1(e_0, 0) \geq \frac{1}{\pi \Delta_1}\), then \(\hat{e}_1(e_0)\) solves

\[
\frac{\partial}{\partial e_1} P_1(e_0, \hat{e}_1(e_0)) = \frac{1}{\pi \Delta_1}.
\]

(7)
Since $P_1$ is strictly concave in $e_1$, the sufficient second-order condition holds and there is a unique best response.

Similarly, we find the best response of $SR_2$ in industry 2: If $\frac{\partial}{\partial e_2} P_2 (M - e_0, 0) < \frac{1}{\pi \Delta_2}$, then $\hat{e}_2 (e_0) = 0$. If $\frac{\partial}{\partial e_2} P_2 (M - e_0, 0) \geq \frac{1}{\pi \Delta_2}$, then $\hat{e}_2 (e_0)$ solves

$$\frac{\partial}{\partial e_2} P_2 (M - e_0, \hat{e}_2 (e_0)) = \frac{1}{\pi \Delta_2}. \quad (8)$$

Denote the slopes of the best responses of $SR_1$ and $SR_2$ as $r_1$ and $r_2$, respectively. They are positive (negative) if the SRs’ efforts are strategic complements (substitutes) with those of the competition authority, which happens iff $\partial^2 P_i / \partial e_i \partial e_i$ is positive (negative).

The problem of the competition authority, who operates in both industries, is:

$$\max_{0 \leq e_0 \leq M} E [W_1 + W_2] - M = [P_1 (e_0, e_1) + P_2 (M - e_0, e_2)] (\pi W_R + (1 - \pi) W_N)$$
$$+ [\{1 - P_1 (e_0, e_1)\} + (1 - P_2 (M - e_0, e_2))] (\pi W_V + (1 - \pi) W_N) - M,$$

where $W_i$ denotes welfare in industry $i$, $i \in \{1, 2\}$. We can rewrite this objective function as

$$E [W_1 + W_2] - M =$$

$$[P_1 (e_0, e_1) + P_2 (M - e_0, e_2)] \pi \Delta + \{2\pi W_V + 2 (1 - \pi) W_N - M\},$$

where, again, the term in curly brackets is a constant. Note that, since total resources are fixed, the competition authority’s total cost is fixed, which is different from the problem of a sectoral regulator. This has the further consequence that the term $\pi \Delta$ here is simply a multiplier; thus, as already noted, results are not dependent on whether or not the competition authority is unbiased. Thus its problem simplifies to

$$\max_{0 \leq e_0 \leq M} P_1 (e_0, e_1) + P_2 (M - e_0, e_2).$$
The best response \( \hat{e}_0 (e_1, e_2) \) solves

\[
\frac{\partial}{\partial e_{10}} P_1 (\hat{e}_0 (e_1, e_2), e_1) = \frac{\partial}{\partial e_{20}} P_2 (M - \hat{e}_0 (e_1, e_2), e_2).
\] (9)

The efforts \( (e_0^*, e_1^*, e_2^*) \) constitute an interior Nash equilibrium if and only if they satisfy the conditions (7), (8), and (9). We assume that there is a unique interior Nash equilibrium.\(^8\)

Denote by \( S_i \) the slope of CA’s best response with respect to \( e_i \), given \( e_j \). In Appendix A we show that the interior Nash equilibrium is stable iff

\[
\alpha := 1 - r_1 S_1 - r_2 S_2 > 0.
\]

In the following we assume that this condition holds.

Furthermore, since the sign of \( S_i \) is equal to that of \( \partial^2 P_i / \partial e_{i0} \partial e_i \), \( S_i \) has the same sign as \( r_i \) for both \( i = 1, 2 \), so that \( 0 < r_i S_i < 1 \) in a stable equilibrium. Thus, an SR’s and the CA’s efforts are either strategic substitutes or complements to each other. This begs the question of how SRs’ efforts relate to each other, i.e. how they indirectly interact through the CA. The following Lemma describes what happens to the equilibrium in market \( j \) if \( SR_i \) changes its behavior, which is at the heart of our results presented below:

**Lemma 1** Indirectly, the efforts of \( SR_1 \) and \( SR_2 \) are strategic substitutes if \( r_1 r_2 > 0 \), and strategic complements if \( r_1 r_2 < 0 \).

**Proof:** Wlog let \( i = 1 \). For given \( e_1 \), consider partial equilibrium efforts \( e_0 = \hat{e}_0 (e_1, e_2) \) and \( e_2 = \hat{e}_2 (M - e_0) \). Totally differentiating both equations we obtain \( de_0 = S_1 de_1 + S_2 de_2 \) and \( de_2 = -r_2 de_0 \). Solving this for \( de_0 \) and \( de_2 \) leads to

\[
\frac{de_0}{de_1} = \frac{S_1}{1 - S_2 r_2}, \quad \frac{de_2}{de_1} = -\frac{r_2 S_1}{1 - S_2 r_2}.
\]

\(^8\)Under collaboration, there is also always a degenerate boundary Nash equilibrium where nobody expends any effort and the CA does not touch its budget.
The latter has the same sign as $-r_1 r_2$ since in a stable equilibrium $r_2 S_2 < 1$.

The intuition of this result is the following: For example, if efforts are strategic complements in both industries, then a rise in SR 1’s effort leads the CA to spend more on this industry and less on the other, which in turns triggers a reduction in SR 2’s effort. All other cases follow the same logic.

4 Where to look?

We can now address the question of where does the competition authority invests more and how its decision relates to the biases of sectoral regulators and to the decision regime adopted.

In this section we consider how much, in relative terms, the CA will spend on one or the other industry, and how this will depend on the decision regimes and the SRs biases.

Suppose first that the government agencies are in a collaborative mode, in the sense that there are “collaboration” decision regimes in both industries. Based on eqs. (3), (4), (7), (8) and (9), Nash equilibrium efforts are given by the following set of equations:

\begin{align}
    p' (e_0) p (e_1) &= p' (M - e_0) p (e_2); \\
    p' (e_1) p (e_0) &= \frac{1}{\pi \Delta_1}; \\
    p' (e_2) p (M - e_0) &= \frac{1}{\pi \Delta_2}.
\end{align}

We find the following result:

**Proposition 1** If decision regime in both industries is “collaboration”, then (i) the more consumer-biased regulator exerts more effort than the other; and
(ii) the competition authority exerts more effort in the industry with the more consumer-biased regulator.

Proof. Assume that regulator 2 is the more consumer-biased, i.e. \( \Delta_1 \leq \Delta_2 \). Suppose that \( e_0 = M/2 \). It follows from (11), (12), and our assumptions on \( p \) that \( e_1 < e_2 \). Making use of this in (10), we find that \( e_0 < M/2 \). Going back to (11) and (12), we find that this change only strengthens our previous result: \( e_1 < e_2 \) also when \( e_0 < M/2 \).

Basically, this follows from the result of Barros and Hoernig (2008) that efforts of the two agencies involved with an industry are strategic complements in the “collaboration” case. Because of strategic complementarity, the competition authority gets, on the margin, more mileage out of effort spent on the industry with the more consumer-biased sectoral regulator, because this regulator is more inclined to spend effort itself than the other regulator is. Thus, under the “collaboration” decision regime, the effect of the bias of the sectoral regulator is amplified by the intervention of the competition authority.

Suppose next that there are “competition” decision regimes in both industries. The equilibrium efforts now follow from (5), (6), (7), (8) and (9), and are given by the following set of equations:

\[
\begin{align*}
p'(e_0) [1 - p(e_1)] &= p'(M - e_0) [1 - p(e_2)]; \\
p'(e_1) [1 - p(e_0)] &= \frac{1}{\pi \Delta_1}; \\
p'(e_2) [1 - p(M - e_0)] &= \frac{1}{\pi \Delta_2}.
\end{align*}
\]

According to Barros and Hoernig (2008), efforts of the two agencies involved in an industry are strategic substitutes in the “competition” case. This turns around the previous result on the competition authority’s resource allocation. Now, the competition authority gets more out of its effort.
when spending it on the industry whose sectoral regulator is less interested in putting in own effort. We have:

**Proposition 2** If the decision regime in both industries is “competition”, then

(i) the more consumer-biased regulator spends more effort than the other;

and

(ii) the competition authority spends more effort on the industry with the **less** consumer-biased regulator.

**Proof.** (immediate) ■

Between the cases of the “collaboration” and “competition” decision regimes, which are *pure* regimes, there is a *mixed* regime as a third possibility, in which the relationship between the competition authority and one of the sectoral regulators is “collaboration” and the other one is “competition”. Suppose, say, that regulator 1 is the one with “collaboration” decision regime with the competition authority. Equilibrium efforts are now determined by the three equations (11), (15), and

\[ p'(e_0) p(e_1) = p'(M - e_0)(1 - p(e_2)). \]  

(16)

As we will show through an example, without further assumptions no generic ordering of efforts by regulators or by the competition authority across regimes or according to consumer bias can be established. In Appendix B we show that spending can still be compared if the individual probabilities of detection are either all very low or all very high.

Now, consider the example. Let industry 1 be the one with the “collaboration” decision regime. Assume \( p(x) = 1 - \exp(-x) \), \( M = 3 \), and \( \pi \Delta_1 = 10 \), and compute the equilibrium effort values for a range of \( \Delta_2 \) values, defined by \( \Delta_2 = \gamma \Delta_1 \), with \( \gamma \) ranging from 0.5 to 2.0. Figure 1 reports equilibrium efforts. We observe that efforts from sectoral regulators have
no general ordering. They change relative positions, in our example, with higher effort in the industry where interaction with the sectoral regulator is set as “collaboration” for $\gamma < 1.441$, and lower effort otherwise. Also, the competition authority devotes more resources, $e_0 > M/2$, to the sector where “collaboration” decision regime is set for $\gamma > 0.515$, and less otherwise.

![Figure 1: Equilibrium efforts $e_0$ (blue), $e_1$ (red), $e_2$ (green).](image)

We can now highlight an interesting difference between the mixed and pure regimes. Even though the two sectoral regulators are not in direct interaction, they are indirectly related to each other through their pairwise interactions with the competition authority, as already noted in Lemma 1 above. The nature of this indirect interaction turns out to depend on whether we are in a pure or mixed regime. In a pure regime, both SRs’ best responses have either positive or negative slopes, thus $r_1 r_2 > 0$. In a mixed regime we have $r_1 r_2 < 0$, and it follows directly from Lemma 1 that:

**Proposition 3** In a pure regime, the efforts of the sectoral regulators are strategic substitutes, while in a mixed regime, they are strategic complements.
The intuition for this result is that in any regime an increase in CA’s effort in one industry implies a reduction of its effort in the other industry. In a pure (mixed) regime this means that the SRs’ incentives to supply more or less effort are opposite (identical).

Let us now see how equilibrium efforts react to a change in bias by one of the agencies, say $SR_1$. The results depend on the type of interaction in industry 1 and whether we have a pure or mixed regime:

**Proposition 4** If a sectoral regulator becomes more consumer biased, then:

(i) his own effort increases;

(ii) the competition authority increases its effort in this industry under the “collaboration” decision regime and decreases it under the “competition” decision regime;

(iii) the other sectoral regulator decreases its effort in a pure regime, and increases it in a mixed regime.

**Proof.** Assume that $\Delta_1$ increases. From Proposition A-1 in Appendix A, we have

\[
\frac{de_0^*}{d\Delta_1} = Kr_1, \quad \frac{de_1^*}{d\Delta_1} = K \frac{r_1}{S_1} (1 - r_2 S_2), \quad \frac{de_2^*}{d\Delta_1} = -Kr_1 r_2,
\]

for some $K > 0$. We have $de_1^*/d\Delta_1 > 0$ because $r_2 S_2 < 1$, while the other signs are obvious. $\blacksquare$

The proposition shows that a more consumer-biased regulator in one industry may be enough to trigger more activity by other regulators. Yet, it may also crowd out activity by the other regulators. This depends precisely on whether we have a pure or a mixed regime, with the competition authority mediating this effect.
5 How much?

While in the previous section we have considered how a CA will spend a given budget, we will now see what happens if additional money becomes available, and at what level the optimal budget should be.

Let us first consider the first question: Assume the CA receives an additional Euro, which industry should it spend more on? Or should it take away effort from one industry and hand over more than a Euro’s worth of effort to the other industry? The following Proposition states how budget increases will be shared.

Let \( A_i = -\partial^2 P_i / \partial e_{i0}^2 > 0 \), \( A = A_1 + A_2 \), and \( \alpha_i = \frac{A_i}{A} (1 - r_is_i) \), where \( s_i \) is the slope of the CA’s best response in a game without the other industry. We have \( 1 > \alpha_1 + \alpha_2 = \alpha > 0 \) by the stability condition. The condition of \( \alpha_i \) being small, which appears below, basically requires efforts to have low sensitivity to each other, which in equilibrium means that the competition authority can spend more in the analysis of that industry without triggering a large reaction of the sectoral regulator.

**Proposition 5** If the CA’s budget increases it spends more of the increase on the industry where a smaller reaction by the sectoral regulator will occur (i.e., the industry with the lower \( \alpha_i \)).

**Proof.** Assume that \( M \) increases. From Proposition A-1 in Appendix A, we have \( de_0^*/dM = \alpha_2/\alpha \). Thus \( de_0^*/dM > 1/2 \) if \( \alpha_1 < \alpha_2 \) and vice-versa.

Does this mean that the CA spends additional money always on both industries? No, because the above result does not depend on the \( \alpha_i \)s being both positive. More precisely, while they cannot be both negative, which would violate stability, it is still possible that one is negative while the other is positive. In this case, for example if \( \alpha_2 < 0 \), the money spent in industry 1 actually decreases after an increase in the budget. In order to see that
negative $\alpha_i$ is not ruled out by the stability condition, note that $\alpha_i > 0$ is equivalent to (see details in Appendix A)

$$r_i s_i < \frac{\partial^2 P_i / \partial e_i^2}{\partial^2 P_1 / \partial e_{10}^2 + \partial^2 P_2 / \partial e_{20}^2}.$$  

The sum of these conditions for $i = 1, 2$ implies stability, but stability does not imply that both conditions hold.

As to the intuition behind this result, note that the sign of $\alpha_i$ depends on whether $r_i s_i$ is smaller than 1 or not. The former would be the case in a stable equilibrium (if it occurred at these effort levels) in the two-regulator game. Thus, a negative $\alpha_i$ corresponds to an unstable situation in industry $i$. Additional effort by the CA in this market will lead to a disproportionate response by the respective sectoral regulator, the anticipation of which makes the CA withdraw resources from the other market.$^9$

Let us now consider the size of the optimal budget. The two questions to be answered are: Should the budget constraint be binding? and, How do sectoral regulators’ biases affect the optimal budget?

A social planner maximizing welfare has an objective function very similar to that of the CA. While he has the same aim of detecting violations of the law, he will take into account the spending of all three authorities. Total welfare (neglecting constants) is:

$$W = (P_1 (e_0^*, e_1^*) + P_2 (M - e_0^*, e_2^*)) \pi \Delta - e_1^* - e_2^* - M. \tag{17}$$

We assume that the social planner only has powers to define the size of the CA’s budget, but cannot interfere with any agency’s effort. This implies that $(e_0^*, e_1^*, e_2^*)$ are the Nash equilibrium efforts in the game played out by the authorities, and are therefore functions of $M$.

$^9$In the case of joint decisions and $p(e) = Ae^{1/n}$ with $n > 2$, the $\alpha_i$s are positive, and so here the CA will spend more on both industries.
Proposition 6 (i) If both regulators are unbiased, then the optimal budget does not constrain the competition authority.

(ii) Assume $SR_2$ is unbiased. The CA’s optimal budget is reduced if the bias of $SR_1$ makes its own effort increase more (or decrease less) in the size of the budget of the competition authority.

The social planner maximizes $W$ over $M$, where $(e_0^*, e_1^*, e_2^*)$ are defined by the first-order conditions (7), (8) and (9). The first-order condition for an interior maximum is the following:

\[
\frac{dW}{dM} = \left( \frac{\partial P_2}{\partial e_20} \pi \Delta - 1 \right) + \left( \frac{\partial P_1}{\partial e_{10}} - \frac{\partial P_2}{\partial e_{20}} \right) \pi \Delta \frac{de_0^*}{dM} \\
+ \left( \frac{\partial P_1}{\partial e_1} \pi \Delta - 1 \right) \frac{de_1^*}{dM} + \left( \frac{\partial P_2}{\partial e_2} \pi \Delta - 1 \right) \frac{de_2^*}{dM} = 0
\]

By (9) we have $\frac{\partial P_1}{\partial e_{10}} = \frac{\partial P_2}{\partial e_{20}}$, and by (7) and (8) $\frac{\partial P_i}{\partial e_i} \pi \Delta_i = 1$. Plugging these in we obtain

\[
\frac{dW}{dM} = \left( \frac{\partial P_2}{\partial e_20} \pi \Delta - 1 \right) + \frac{\Delta - \Delta_1}{\Delta_1} \frac{de_1^*}{dM} + \frac{\Delta - \Delta_2}{\Delta_2} \frac{de_2^*}{dM} = 0.
\]

This condition defines the optimal budget $M^*$ implicitly. If both sectoral regulators are unbiased then the optimal budget is given by the conditions

\[
\frac{\partial P_1}{\partial e_0} \pi \Delta = \frac{\partial P_2}{\partial e_{20}} \pi \Delta = 1.
\]

These are exactly the conditions that would describe the CA’s choice of effort if it were not subject to a budget constraint, but were to take into account its expenses.

In order to see how the optimal budget changes with the bias of a regulatory agency, assume that $\Delta_1$ changes. Thus we want to find $dM^*/d\Delta_1$, which has the same sign as $\partial^2W/\partial\Delta_1\partial M$. It turns out to be simpler to use the following route: First compute, using the three first-order conditions and $\Delta_2 = \Delta$,

\[
\frac{\partial W}{\partial \Delta_1} = 0 + \frac{\Delta - \Delta_1}{\Delta_1} \frac{\partial e_1^*}{\partial \Delta_2} + 0,
\]
then immediately
\[
\frac{\partial^2 W}{\partial \Delta_1 \partial M} = \Delta - \Delta_1 \frac{\partial^2 e^*_1}{\partial \Delta_1 \partial M}.
\]
Clearly at \( \Delta_1 = \Delta \) we have \( \frac{\partial^2 W}{\partial \Delta_1 \partial M} = 0 \) and thus \( \frac{dM^*}{d\Delta_1} = 0 \). If \( \frac{\partial^2 e_1^*}{\partial \Delta_1 \partial M} > 0 \) (bias makes effort of the sectoral regulator to increase more or to decrease less with an increase in the competition authority’s budget) then \( \frac{dM^*}{d\Delta_1} > (\leq) 0 \) if \( \Delta_1 < (\geq) \Delta \), which implies \( M^* \) has a local maximum at \( \Delta_1 = \Delta \). The opposite logic (local minimum if \( \frac{\partial^2 e_1^*}{\partial \Delta_1 \partial M} < 0 \)) does not apply if the social planner cannot force the CA to spend its budget on these two industries.

Part (i) of Proposition 6 states that if the sectoral regulators are unbiased then the competition authority should be able to exert effort without restrictions, whether there is a pure or a mixed regime. That is, the social planner should provide the competition authority with enough funds to cover the costs for the effort levels the competition authority decides to make. Note, though, that this result depends on the assumption that the CA would care about spending additional resources in the absence of a budget constraint.

Part (ii) of Proposition 6 says that if a sectoral regulator’s bias magnifies the effect of a change in the budget on the sectoral regulator’s effort, then the budget should be reduced to correct for this effect. This holds for both consumer and industry bias.

In Appendix A we show that \( \partial e_1^*/\partial M = r_1 \alpha_2/\alpha \), so this effect has two components:
\[
\frac{\partial^2 e_1^*}{\partial \Delta_1 \partial M} = \frac{\partial}{\partial \Delta_1} \left( \frac{\alpha_2}{\alpha} \right) r_1 + \frac{\alpha_2}{\alpha} \frac{\partial r_1}{\partial \Delta_1}.
\]
The first term describes the effect of the change in the distribution of additional money in the budget, while the second term shows how the SR’s reaction to higher CA effort changes.\(^{10}\)

\(^{10}\)Note that in the discussion we make the implicit assumption that funding of the sector-specific regulators cannot be directly controlled by the social planner (or, at least, cannot be done such as to indirectly determine the efforts made by the sectoral regulators).
A related question is whether the optimal CA’s budget should be larger in the “collaboration” or in the “competition” decision regime. With unbiased regulators we can give a quick answer. It is easy to see that both the CA and the sectoral regulators’ choosing the same level of effort \( m \) such that 
\[
\frac{\partial P}{\partial m} (m,m) = \frac{1}{\pi \Delta}
\]
defines the optimal budget. The above condition for a maximum becomes, for joint and independent decisions, respectively,
\[
\begin{align*}
p' \left( \frac{M^J}{2} \right) p \left( \frac{M^J}{2} \right) &= \frac{1}{\pi \Delta} \\
p' \left( \frac{M^I}{2} \right) \left( 1 - p \left( \frac{M^I}{2} \right) \right) &= \frac{1}{\pi \Delta}.
\end{align*}
\]
If \( p \left( \frac{M^J}{2} \right) > 1/2 \), then \( p' \left( \frac{M^J}{2} \right) < p' \left( \frac{M^I}{2} \right) \) and \( M^J > M^I \), while for \( p \left( \frac{M^J}{2} \right) < 1/2 \) the opposite holds. Only for \( p \left( \frac{M^J}{2} \right) = 1/2 \) will both optimal budgets be the same.

6 Concluding remarks

The overlap of jurisdictions between different economic authorities creates interactions, which should not be overlooked. How the legal framework establishes the formal relationships between different authorities is not neutral from an economic point of view, as they affect their incentives to intervene. The main contribution of our analysis lies in pointing out the mediation role of strategic effects, in the choice of effort from sectoral regulators, performed by a competition authority. Not only authorities in direct contact in the same market interact, but there are also interactions across industries, mediated by the competition authority. Thus the design of regulatory agencies and their relationships with each other should not neglect the resulting strategic interactions.

These results raise a question for future research: if a social planner wants to induce a competition authority to behave as if it was more consumer oriented, would it be appropriate to use the decision regime in overlapping
jurisdictions as an instrument to credibly commit the competition authority to such a behavior? Answering this question to full extent is beyond the scope of this paper, but our results suggest that the answer depends on the bias of the sectoral regulator, and thus decision regimes are not robust commitment devices.

Appendix A: Stability and Comparative Statics

Let \( \alpha := 1 - r_1S_1 - r_2S_2 \).

**Lemma A-1** A Nash equilibrium is stable if and only if \( \alpha > 0 \).

With \( A_i = -\frac{\partial^2 P_i}{\partial e_i^2} > 0 \), \( A = A_1 + A_2 \), \( B_i = -\frac{\partial^2 P_i}{\partial e_i^2} > 0 \), and \( c_i = \frac{\partial^2 P_i}{\partial e_i^2} \), we can write \( r_i = \frac{c_i}{A_i} \) and \( S_i = \frac{c_i}{A} \). The Hessian of the system (7), (8), and (9) is

\[
\Phi = \begin{bmatrix}
-A & c_1 & -c_2 \\
c_1 & -B_1 & 0 \\
-c_2 & 0 & -B_2
\end{bmatrix}.
\]

The equilibrium is stable if \( \Phi \) is negative definite, which is true if the following conditions hold: \(-A < 0, -B_i < 0 \) for \( i = 1, 2 \), and \( B_1B_2 > 0 \), which are true by concavity; \( AB_i - c_i^2 > 0 \) for \( i = 1, 2 \), which is equivalent to \( r_iS_i < 1, i = 1, 2 \); and \( -AB_1B_2 + c_1^2B_2 + c_2^2B_1 < 0 \), which is equivalent to the stronger condition \( r_1S_1 + r_2S_2 < 1 \).

**Proposition A-1** Let \( \alpha > 0 \), so that the Nash equilibrium is stable.

1. If CA’s budget \( M \) increases, then \( \frac{\partial e_0^*}{\partial M} = \frac{\alpha_2}{\alpha_1}, \frac{\partial e_1^*}{\partial M} = r_1\frac{\partial e_0^*}{\partial M} \) and \( \frac{\partial e_2^*}{\partial M} = r_2\left(1 - \frac{\partial e_0^*}{\partial M}\right) \).

2. If SR\( _i \) becomes more consumer friendly, i.e. \( \Delta_i \) increases, then \( \frac{\partial e_i^*}{\partial \Delta_i} > 0 \), the sign of \( \frac{\partial e_i^*}{\partial \Delta_i} \) equals that of \( r_i \), and \( \frac{\partial e_i^*}{\partial \Delta_i} = -r_j \frac{\partial e_j^*}{\partial \Delta_i} \).
3. If the probability of a violation \( \pi \) increases, then \( \frac{d\pi}{d\pi} > 0 \) if \( \frac{\pi_i}{\Delta_i} > \frac{\pi_j}{\Delta_j} \), and \( \frac{d\pi}{d\pi} < 0 \) if \( \frac{\beta_i}{\Delta_i} < \frac{\pi_j}{\Delta_j} \), where \( \beta_i = \frac{\pi_i}{S_i}(1 - \pi_jS_j) > 0 \).

The negative inverse of \( \Phi \) is

\[
-\Phi^{-1} = \frac{1}{AB_1B_2 - c_1^2B_2 - c_2^2B_1} \begin{bmatrix}
  B_1B_2 & c_1B_2 & -c_2B_1 \\
  c_1B_2 & B_2A - c_2^2 & -c_1c_2 \\
  -c_2B_1 & -c_1c_2 & B_1A - c_1^2
\end{bmatrix}
\]

\[
= \frac{1}{A\alpha} \begin{bmatrix}
  1 & r_1 & -r_2 \\
  r_1 & \beta_1 & -r_1r_2 \\
  -r_2 & -r_1r_2 & \beta_2
\end{bmatrix},
\]

with \( \beta_i = \frac{\pi_i}{S_i}(1 - \pi_jS_j) > 0 \), for \( i, j = 1, 2, i \neq j \).

The effect of a change in the size of the budget \( M \) is:

\[
\begin{bmatrix}
  \frac{de}{d\Delta_1}
  \frac{de}{d\Delta_2}
  \frac{de}{d\Delta}
\end{bmatrix} = -\Phi^{-1} \begin{bmatrix}
  A_2 \\
  0 \\
  c_2
\end{bmatrix} = \begin{bmatrix}
  \frac{\alpha^2}{\alpha}r_1 \\
  \frac{\alpha^2}{\alpha}r_2
\end{bmatrix}.
\]

If, say, \( \Delta_1 \) increases, then

\[
\begin{bmatrix}
  \frac{de}{d\Delta_1} \\
  \frac{de}{d\Delta_2} \\
  \frac{de}{d\Delta}
\end{bmatrix} = -\Phi^{-1} \begin{bmatrix}
  0 \\
  \frac{1}{\pi\Delta} \\
  0
\end{bmatrix} = \begin{bmatrix}
  \frac{r_1}{A\alpha\pi\Delta_1} \\
  \frac{\beta_1}{A\alpha\pi\Delta_1} \\
  -\frac{r_1r_2}{A\alpha\pi\Delta_1}
\end{bmatrix}
\]

Thus, \( \frac{de}{d\Delta_1} > 0 \), and the sign of \( \frac{de}{d\Delta_1} \) is equal to that of \( r_1 \).

Appendix B: Additional Result for the Mixed Regime

As shown above, with a mixed regime no straight results are available as to in which market the CA spends more effort. If we allow for some specific assumptions about which range of values the individual probabilities of detection can take, some of which break with the general assumptions made in the text, then we can provide conclusive results.
Proposition B-1 Consider a mixed regime, where in one sector the relationship between the competition authority and the sectoral regulator is characterized by joint decisions and in the other sector by independent decisions.

a) If $p(x) < 1/2, \forall x$, then:

i) If the more consumer-biased regulator is in the industry with independent decisions, then he spends more effort than the other. If he is in the industry with joint decisions, he may spend more or less effort than the other.

ii) The competition authority spends more effort in the industry with independent decisions.

b) If $p(0) > 1/2$, then:

i) If the more consumer-biased regulator is in the industry with joint decisions, then he spends more effort than the other. If he is in the industry with independent decisions, he may spend more or less effort than the other.

ii) The competition authority devotes more effort to the market with joint decisions.

Proof. Denote the sector with joint decisions sector $J$ and the other sector $I$. The set of first-order conditions defining the (interior) equilibrium values are:

\[ p'(e_J)p(e_{J0}) = \frac{1}{\pi \Delta_J} \]  \hspace{1cm} (18)

\[ p'(e_I)(1 - p(e_{I0})) = \frac{1}{\pi \Delta_I} \]  \hspace{1cm} (19)

\[ p'(e_{I0})p(e_J) = p'(e_{I0})(1 - p(e_I)) \]  \hspace{1cm} (20)
Part a): Assume \( p(x) \leq 1/2, \forall x \). Then \( p(e_J) < 1 - p(e_I), \forall e_I, e_J \). From (20),
\[ p'(e_{J0}) > p'(e_{I0}), \] implying \( e_{J0} < e_{I0} \). In this case, the competition authority always does more effort in the sector under ID. If \( \Delta_J \leq \Delta_I \) (\( SR_I \) is more consumer-biased), then, from (18) and (19), \( p'(e_I) < p'(e_J) \) and \( e_I > e_J \). However, for \( \Delta_J > \Delta_I \) (\( SR_J \) is more consumer-biased), it is only when \( \Delta_J \) is sufficiently large that \( e_J > e_I \).

Part b): Assume \( p(0) \geq 1/2 \). Then \( p(e_J) > 1 - p(e_I), \) for \( e_J > 0, e_I > 0 \). Now, from (20), \( p'(e_{J0}) < p'(e_{I0}) \) and \( e_{J0} > e_{I0} \). The competition authority devotes more effort to the market where joint decisions prevail. If \( \Delta_J \geq \Delta_I \) (\( SR_J \) is more consumer-biased), then, from (18) and (19), \( p'(e_I) > p'(e_J) \) and \( e_J > e_I \). However, for \( \Delta_I > \Delta_J \) (\( SR_I \) is more consumer-biased), it is only when \( \Delta_I \) is sufficiently large that \( e_I > e_J \). ■
References


