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OPTIMAL TIMING OF ENVIRONMENTAL POLICY; INTERACTION BETWEEN ENVIRONMENTAL TAXES AND INNOVATION EXTERNALITIES

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OPTIMAL TIMING OF ENVIRONMENTAL POLICY; INTERACTION BETWEEN ENVIRONMENTAL TAXES AND INNOVATION EXTERNALITIES

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ABSTRACT

This paper addresses the impact of endogenous technology through research and development (R&D) and learning by doing (LbD) on the timing of environmental policy. We develop two models, the first with technological change through R&D and the second with LbD. We study the interaction between environmental taxes and innovation externalities in a dynamic economy and prove policy equivalence between the second-best R&D and the LbD model. Our analysis shows that the difference found in the literature between optimal environmental policy in R&D and LbD models can partly be traced back to the set of policy instruments available, rather than being directly linked to the source of technological innovation. Arguments for early action in LbD models carry over to a second-best R&D setting. We show that environmental taxes should be high compared to the Pigouvian levels when an abatement industry is developing. We illustrate our analysis through numerical simulations on climate change policy.

JEL codes: H21, O30, Q42
Keywords: Environmental policy, technological change, research and development, learning by doing

1. INTRODUCTION

In the coming decades radical policy interventions are necessary to bring a halt to the continuing increase in the atmospheric greenhouse gas concentrations when the aim is to prevent a potentially dangerous anthropogenic interference with the global climate system, see, e.g., Stern Review (2007). Though most scientists agree on the need for some abatement in the coming decades, there is a debate on whether the major share of these efforts should be pursued from the beginning, or whether the largest share of abatement efforts should be delayed to the future. Three reasons stand out among advocates of delayed action. First, due to the discounting of future costs, saving our abatement efforts for the future will allow us to increase our efforts considerably at the same net present costs. Second, delaying emission reduction efforts will allow us to emit larger cumulative amounts of greenhouse gases, and thus to abate less in total, due to the natural depreciations of the atmospheric greenhouse gas concentrations. Third, delaying abatement efforts will allow us to benefit from cheaper abatement options that are available in the future, and also to develop these options through

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innovation. The first two arguments have taken firm ground in the literature, thanks to – among others – the analysis by Wigley et al. (1996).[2] The third argument, however, based on presumed technological advancements in abatement options, has raised a lively debate among economists studying technological change in relation to environmental policy.

There are arguments for accelerating abatement efforts rather than delaying them. Energy system analyses have clear empirical evidence for so-called experience curves suggesting that new low-carbon energy technologies, which will define the major long-term options for carbon dioxide emission reduction, need to accumulate experience for costs to come down sufficiently to make these technologies competitive.[3] Based on these experience curves, the more general argument is made that there is a need for up-front investment in abatement technologies to make them available at low prices, and thus, technological change would warrant early abatement action rather than a delay (Ha-Duong et al., 1997; Grübler and Messner, 1998; van der Zwaan et al., 2002; Kverndokk and Rosendahl, 2006). Models exploring the experience curves are typically referred to as learning by doing (LbD) models.[4] Many energy system models add another reason for a smooth transition towards clean energy supply, which is that diffusion of new technologies need the turnover of all existing vintages and therefore takes a considerable time (Knapp, 1999). A too rapid switch of the capital stock towards an entirely new technology is considered unrealistic (Gerlagh and van der Zwaan, 2004; Rivers and Jaccard, 2006).

Objections have been raised to these arguments. Though experience and diffusion curves have a strong empirical basis, many economists consider it a mechanistic view on technological development hiding the incentive-based structures that determine the level of research efforts by innovators. They prefer models with an explicit treatment of research and development (R&D) as the engine of innovation, and they have found that modelling innovations through R&D can lead to potentially very different outcomes on optimal timing of abatement policy. An important difference between LbD and R&D models is that the latter category of models does not assume from the outset that the technology needs to be used for its costs to fall. Thus, through R&D, future cheap abatement options may be made available without the need to use these abatement options while costs are still high. In an R&D model, it is then most efficient to focus mainly on R&D in the early stages of abatement policy, without employing the technologies, and to apply them only after the costs have sufficiently come down. Indeed, Goulder and Mathai (2000) found this pattern as an optimal environmental policy and they concluded that whereas LbD may warrant an advance of using abatement technologies compared to a situation without technological change, the presence of R&D unambiguously implies a delay in the use of such technologies.

The first objective of this paper is to test the robustness of Goulder and Mathai's finding in a second-best context, i.e., when we have several imperfections, but insufficient policy instruments available to correct them all. Caution is needed when results depend on first-best assumptions, since such a first-best innovation-

[2] They used these arguments to make the case that emission paths developed by the IPCC (1995) for ceiling atmospheric carbon dioxide concentrations tended to put too much effort up-front, while a delayed abatement response would be more cost-efficient.

[3] See Lieberman (1984) for an early contribution focused on the chemical industry, and Isoard and Soria (2001) for a recent empirical analysis for energy technologies.

[4] Manne and Richels (2004), however, find that LbD has almost no effect on the efficient timing of abatement.

abatement solution can be reached only when policy makers have a rich instrument set available. If R&D suffers from market imperfections, they need to be able to directly target environmental R&D, separately from abatement efforts. That is, policy makers need to have a tailored instrument available to bring environmental R&D efforts to their socially optimal level. In contrast, we assume that policy makers may use a common R&D instrument such as R&D subsidies over all sectors, and a generic environmental instrument such as environmental taxes to target environmental goals. Since environmental R&D makes up only a small portion of economy-wide R&D expenditures, we next consider the R&D subsidies as exogenous to the environmental policy problem, and consequently, the policy maker has to rely on one instrument, say the environmental tax, to steer both abatement levels and environment-specific R&D efforts. Since now the environmental tax affects both abatement efforts and innovation within the abatement sector, the functioning of the innovation market within the environmental sector, i.e., how the gap between private and social returns on R&D develops, becomes of crucial importance for determining the efficient level of the environmental tax. If the R&D distortion is largest during the phase of a fast increase in abatement efforts, this will have some impact on the efficient path of the environmental tax.

For our study we develop an R&D model in line with the endogenous growth literature and assume that R&D efforts are based on market-based incentives through patents. Patents protect the holders from others directly using their innovation in production, but at the same time, patents disclose the knowledge base underlying the innovation, which then can be used by rivals to develop substitute technologies. Also, patents have a finite lifetime and expire after a certain period. These properties can lead to intricate connections between R&D dynamics and environmental policy (cf. Encaoua and Ulph, 2004), and we need to see how they alter the first-best timing results.

We expect that the gap between an LbD model and a second-best R&D model with finite lifetime of patents will be considerably narrowed. Whereas in a first-best R&D model it is possible that innovators develop new technologies and continually improve these without the need to be used in production, in a second-best R&D model with finite patent lifetime, innovations will only occur when they are used in production before the patent's expiration date. This mechanism is similar to the mechanism in LbD models, where technology only advances if it is used. Thus, the representation of finite lifetime of patents in an R&D model will lead to the required use of abatement technologies in earlier periods so that innovators can earn back the costs of R&D.

The argument above makes clear that a finite patent lifetime creates an appropriation problem for innovators who cannot fully capture the social value of their innovations in the long future. Many R&D models incorporate the idea that innovators cannot appropriate the full value of their innovations – Nordhaus (2002), Popp (2004) and Gerlagh and Lise (2005) make precise assumptions on this. But whereas in the broad innovation literature the finite lifetime of patents is a common reason for this feature (for an early contribution, see Nordhaus, 1969), in the environmental economics literature, the time dimension of the appropriation problem is mostly neglected. If the appropriation gap would be a constant fraction of the social value (as assumed in these models), then a constant innovation subsidy would be sufficient to correct for this market failure. If, however, patents expire, innovations will be biased towards technologies that pay back within the patent's lifetime, while there is no incentive to develop and improve technologies whose value lies in the farther future. A generic R&D subsidy cannot correct for this timing dimension of the appropriation problem, and instead, a complementary environmental policy may be required for its correction.

This paper is organised in the following way. In Section 2 we develop a partial model for abatement and environmental quality, which for instance can be interpreted as climate change. The model has discrete time steps, and technological change is driven by the Romer (1987) type of endogenous growth through increasing varieties, based on the 'love of variety' concept (Dixit and Stiglitz, 1977). Subsequently, we develop an LbD model.

We analyse optimal environmental policies in Section 3, starting with a first-best setting as in Hartman and Kwon (2005) and Bramoulle and Olson (2005, cf Proposition 8). Then we consider the second-best setting, for which we analyse the development over time of efficient environmental taxes relative to Pigouvian taxes. As in Hart (2006), our timing analysis focuses on the transition paths for both R&D and LbD models, where the abatement sector is rapidly increasing in size, and slowly becomes mature characterised by a lower growth rate. Different from Goulder and Mathai (2000), the timing analysis is not based on a comparison of multiple scenarios, e.g. one with and another one without endogenous technological change.[5] Instead, we analyse the development over time of research subsidies and the gap between efficient environmental taxes and Pigouvian taxes in the first- and second-best setting. The relative gap between the two taxes tells us something about the relative stringency of environmental policy compared to the social cost of pollution, and we are particularly interested in its development over time.

Our focus on the gap between efficient environmental and Pigouvian taxes puts our analysis in a broad strand of literature. Much of this literature focused on tax interaction effects (c.f. Bovenberg and de Mooij, 1994) and it raised lively debates in policy circles when it explored the potential for so-called double dividends. In addition to tax interaction, reasons for a divergence between efficient environmental and Pigouvian taxes include trade effects (Hoel, 1996), scale effects in production (Liski, 2002), and, more recently, the processes underlying technological change. Rosendahl (2004) shows that in an LbD model, the environmental tax should be higher than a Pigouvian tax, with the largest gap for those countries and sectors that generate most of the learning. In a similar fashion, Golombek and Hoel (2005, Proposition 9) show that in an environmental treaty the optimal carbon price can exceed the Pigouvian level when abatement targets lead to innovation and international technology spillovers that are not internalised in domestic policies.[6] Our paper studies the dynamics of this gap between efficient and Pigouvian environmental taxes, in relation to endogenous technological change.

After the separate analyses of the R&D and LbD models, in Section 4 we compare the two models and present conditions under which the two models have

[5] As Pade and Greaker (2006) point out, the comparison made by Goulder and Mathai (2000) is problematic in the sense that their ETC scenario assumes technological change in addition to the benchmark (no-ETC) scenario. The scenario with ETC therefore has a more optimistic path of falling abatement costs compared to the scenario without ETC. Thus, the comparison between the two scenarios is mainly driven by the difference in technology paths, and is largely independent of the source of technological change, be it endogenous or exogenous. Though our set up is not directly comparable with Goulder and Mathai (2000), our broader context is comparable as both study the relation between sources of innovation (R&D vs. LbD) and timing of action.

[6] The analysis by Golombek and Hoel (2005) is in a game-theoretic context, and the result depends on the instrument used to define the treaty (compare Proposition 9 and 10).

identical efficient policy paths. That is, we establish conditions for policy equivalence between the second-best R&D model and the LbD model.

Finally, in Section 5 we carry out some numerical calculations to illustrate the analysis and to investigate its substance. Throughout the simulations, the model parameters are chosen to reflect the common climate change context. Section 6 concludes.

2. MODEL SET UP

We consider an economy where there are concerns for the environment due to stock pollution. This could for instance be climate change, caused by carbon dioxide emissions following from the combustion of fossil fuels. More generally, we assume a benchmark emission path and a demand for abatement of emissions because of environmental considerations.

Let θ_t and τ_t reflect the social marginal cost and the policy-induced market cost (e.g., environmental tax) of emission at time t, respectively. The social cost and the market cost may differ as the first reflects the pure pollution externality problem (and is often referred to as the Pigouvian tax), while the second is dependent on the policy measures applied and the structure of the economy. In the following, we are interested in studying the dynamic relations between θ_t and τ_t in first- and second-best settings, i.e., how does the market cost deviate from the social marginal cost under different assumptions about knowledge dynamics (R&D vs. LbD) and policy measures available.

The abatement sector in the R&D model

The model of research and development (R&D) is based on Romer's endogenous growth model (Romer, 1987, 1990; Barro and Sala-i-Martin, 1995). The model has an infinite horizon with discrete time steps, $t = 1,\ldots,\infty$. There is one representative abatement sector, which could either be interpreted as an alternative, emission-free resource sector (e.g. renewables or fossil fuels supplemented with carbon capturing and sequestration), or as abatement of emissions. There are H_t producers of abatement equipment at each point of time t, and an R&D sector producing new ideas or innovations. Technological progress takes the form of expansion in the number of abatement equipment varieties. The producers of the abatement equipment own patents and, therefore, receive monopoly profits. However, they have to buy the innovations from the R&D sector, where innovators are competitive and use research effort as an input. We assume that patents last for one period, and so innovations are public goods thereafter. Hence, there are positive spillovers to innovation from the previous-period stock of innovations (standing on shoulders). Also, we assume negative externalities from aggregate current research through crowding out of research effort. Thus, in this model there are three imperfections related to innovations; too little production of abatement equipment due to monopolistic competition, positive spillovers of the earlier period innovation stock on new innovations, and negative spillovers of total research effort on new innovations. Thus, the market outcome of innovations may exceed or fall short of the social optimal level.

Let E be emissions of the stock pollutant, Y is benchmark emissions without any environmental policies, while A is abatement. If we think of energy and CO_2 emissions, Y could be total energy demand, while A could either be renewable (CO_2-

free) energy that partly replaces fossil fuels in consumption and production, or a reduction in the emissions from fossil fuel combustion, e.g., through carbon capture and storage. Thus, total emissions are as follows:[7]

$$E_t = Y_t - A_t. \tag{1}$$

Production of abatement requires intermediate flow inputs Z_t, and the input x_i of abatement equipments, where subscript $i \in [0, H_t]$ refers to the variety, and H_t is the number of equipment varieties. H_t can be interpreted as the state of knowledge.

$$A_t = B\, Z_t^{\alpha} \left(\int_0^{H_t} x_{t,i}^{\beta} \mathrm{d}i\right)^{\gamma}. \tag{2}$$

B is a constant and $0<\beta<1$, $0<\alpha<1$, $0<\gamma<1$. Furthermore, we demand $\alpha+\beta\gamma\leq 1$, where a strict inequality implies that there is a fixed factor in production, e.g., due to site scarcity for renewables. The presence of a fixed factor implies that the value of output is strictly larger than the value of all variable inputs. In that case we can specify B as $B=cF^{1-\alpha-\beta\gamma}$, where F is the fixed factor and c is a constant, such that the total value of output is fully attributed to all inputs Z, x_i, and F.

The different abatement equipments are neither direct substitutes nor direct complements to other specific equipments. That is, the marginal product of each abatement equipment is independent of the quantity of any particular equipment, but depends on the total input of all other equipment varieties together. Since all varieties have the same production costs and decreasing marginal product, in equilibrium the same quantity will be employed of each equipment. Thus, assuming that the equipments can be measured in a common physical unit, we can write $x_i=X/H$, where X is the aggregate input of abatement equipment. The production identity then becomes:

$$A_t = B\, Z_t^{\alpha}\, X_t^{\beta\gamma}\, H_t^{(1-\beta)\gamma}. \tag{3}$$

It is clear that the abatement sector has increasing returns to scale when $\alpha+\gamma>1$, due to the technology H.[8] Now consider the case where abatement efforts have to increase over time continually to maintain a clean environment jointly with an increasing overall economic activity. For $\alpha+\gamma<1$, the abatement expenditures will have to increase more than proportionally with the abatement effort. For $\alpha+\gamma=1$, the costs of abatement rise in proportion with abatement levels. For $\alpha+\gamma>1$, the price of abatement decreases, and total expenditures increase less then the abatement effort.

Assume now that the public agent implements an emission tax τ_t, or more generally an environmental policy that induces a market cost of emission, τ_t. From (1) we see that this translates into a market price for abatement A_t. The abatement producer's optimisation problem is:

[7] The relation between emissions and benchmark emissions is specified as a linear function for convenience of notation. A more general function would give the same qualitative results. In the numerical simulations in Section 5, we use a CES aggregation.

[8] An interesting case arises when $\gamma=1-\alpha$. There are decreasing returns to scale for a given technological level H_t, e.g., due to a fixed factor. This can be understood as the short-term feature of the model. At the same time, there are constant returns to scale for endogenous level of knowledge. The technology effect precisely balances the fixed factor effect.

$$\text{Max } \tau_t A_t - Z_t - \int_0^{H_t} p_{t,i} x_{t,i} di, \tag{4}$$

subject to (2).

The price of Z is set to unity and the price of abatement equipement $x_{t,i}$ is equal to $p_{t,i.}$ Thus, the abatement producer maximises the value of abatement minus the abatement costs.

The first order conditions of this maximisation problem determine the abatement producer's demand for Z and x_i:

$$Z_t = \alpha \tau_t A_t \tag{5}$$

$$x_{t,i} = \{[\gamma\beta\tau_t B Z_t^{\alpha} (\int_0^{H_t} x_{t,k}^{\beta} dk)^{\gamma-1}]/p_{t,i}\}^{1/1-\beta} = \{[\gamma\beta\tau_t A_t (X_t^{\beta} H_t^{(1-\beta)})^{-1}]/p_{t,i}\}^{1/1-\beta}. \tag{6}$$

From (5) we see that the costs of Z should equal the share α of the production value, where α expresses the relative contribution of Z in production.

The demand for $x_{t,i}$ is given by (6). Alternatively, we can also express the demand for aggregated input of abatement equipment using $x_i = X/H$, and $p_{t,i} = p_t$:

$$p_t X_t = \beta\gamma\tau_t A_t. \tag{7}$$

Thus, the demand for abatement equipment is falling in the own price, but increasing in the environmental tax.

Production of abatement equipment in the R&D model

The producers of abatement equipment own patents and therefore act as monopolists. Their costs of producing intermediates $x_{t,i}$ are set to unity, and they maximise profits (or the value of the patent), $\pi_{t,i}$, taking into account the falling demand curves for abatement equipment. For a patent valid for one period, we get the following maximisation problem:

$$\text{Max } \pi_{t,i} = x_{t,i}(p_{t,i} - 1), \tag{8}$$

subject to (6).

The first order condition from maximising (8) with respect to $p_{t,i}$ determines the price of the abatement equipment:

$$p_{t,i} = p = 1/\beta. \tag{9}$$

From (7) and (9) we find the market equilibrium of X:

$$X_t = \beta^2 \gamma \tau_t A_t. \tag{10}$$

As all varieties are identical ($x_i = X/H$), and prices are equal across varieties, see (9), the value of a patent is also equal for all innovations, i.e., $\pi_{t,i} = \pi_t$. Using this in addition to (8), (9), (10) and $x_i = X/H$, we find the value of all patents:

$$\pi_t H_t = (1-\beta)\beta\gamma\tau_t A_t. \tag{11}$$

The innovation process in the R&D sector

The producers of abatement equipment buy patents from innovators that operate in a competitive market.[9] Innovators develop new varieties according to the following production function:

$$h_{t,j} = r_{t,j}\ (H_{t-1}/R_t)^{1-\psi}, \tag{12}$$

where $r_{t,j}$ is the research effort of innovator j, $h_{t,j}$ is the number of varieties produced by this innovator, and we assume $0<\psi<1$. R_t denotes aggregated research efforts by all innovators.

As seen from the production function in (12), and as explained above, there is a positive externality through a spillover from the previous period knowledge stock through H_{t-1}, and a negative externality through crowding out of current research via R_t.[10] We also see that both externalities are higher the lower the value of ψ.

The innovators maximise profit with respect to research effort, where the price of the innovation equals the monopoly profit of equipment producers, or equivalently the value of the patent.

$$\text{Max}\ \pi_t h_{t,j} - r_{t,j}, \tag{13}$$

subject to (12).

The price of research effort is set equal to one. First order conditions give that the unit cost of research (i.e., one) is equal to the value of the patent, π, multiplied by the productivity of r.

Due to the zero-profit condition, in equilibrium the value of all patents is equal to the value of all research effort:

$$\pi_t H_t = R_t. \tag{14}$$

Substitution of (14) in (11) and aggregation of (12) give the following two conditions for research effort and knowledge dynamics in the economy:

$$R_t = (1-\beta)\beta\gamma\tau_t A_t \tag{15}$$
$$H_t = R_t^{\psi} H_{t-1}^{1-\psi}. \tag{16}$$

Market equilibrium in the R&D model

The five equations (3), (5), (10), (15) and (16) define a market equilibrium through the variables A_t, Z_t, X_t, R_t, H_t, for a given environmental tax policy τ_t.

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[9] Alternatively we could assume that the innovators are producing the abatement equipments, such that they own the patents and get the monopoly rent. This would not change the arguments or conclusions of the analysis.

[10] Encaoua and Ulph (2004) distinguish between knowledge and technology information flows. *Knowledge flow* or knowledge diffusion is equal to ςH_{t-1}, which means that a fraction $0<\varsigma<1$ of previous knowledge is public information at time t. The *technology flow* is the technology spillover according to which a technology can be imitated by others, such that a patent does not offer a perfect protection to its holder. In our model this would mean that χH_t will be private property of the patent holders, where $0<\chi<1$, while $(1-\chi)H_t$ can be copied by others. In our model we assume that both ς and χ are set to unity.

PROPOSITION 1. *For given initial state of knowledge, H_0, and tax policy defined by $\tau_t>0$, a unique equilibrium path exists if* $[1-\alpha-\beta\gamma-\psi\gamma(1-\beta)]>0$.

Proof. Substitution of the four equations (5), (10), (15) and (16) into (3) gives:

$$A_t^{1-\alpha-\beta\gamma-\psi\gamma(1-\beta)}=B\alpha^{\alpha}\gamma^{\beta\gamma+\psi\gamma(1-\beta)}\beta^{2\beta\gamma+\psi\gamma(1-\beta)}(1-\beta)^{\psi\gamma(1-\beta)}\tau_t^{\alpha+\beta\gamma+\psi\gamma(1-\beta)}H_{t-1}^{(1-\psi)(1-\beta)\gamma}. \quad (17)$$

If $[1-\alpha-\beta\gamma-\psi\gamma(1-\beta)]>0$, the left hand side is increasing in A_t from zero without bound, and the right-hand side is constant at time t. Thus, for given H_{t-1} there is a unique A_t that solves the equation. H_t is then defined by (15) and (16). By forward induction, this defines a unique path. Q.E.D.

Note that the unique path essentially requires that there are decreasing returns to scale within a period t (i.e., with H_{t-1} fixed, but not H_t).

Model with LbD

We now present a learning by doing (LbD) model that is comparable to the R&D model. However, in this model, technological progress takes form of expansion in knowledge following passively from experience with abatement. Thus, there is no separate innovation activity in the model. As in the R&D model, there is a positive spillover from the knowledge stock in the previous period, such that knowledge is a public good after one period. Because of this externality, the social optimal abatement level is higher than abatement in the market equilibrium.

Market equilibrium in the LbD model

As before, the model has an infinite horizon with discrete time steps, $t=1,\ldots,\infty$. Abatement requires intermediate inputs Z_t for which a competitive market exists, and H_t is the state of technology or knowledge.

$$A_t = BZ_t^{\alpha} H_t^{\gamma}, \quad (18)$$

with $0<\gamma<1$, $0<\alpha<1$, and B is a constant. The condition $\alpha<1$ implies that there is a fixed factor in production. Compared to the abatement production in the R&D model (see equation (2)), abatement in the LbD model is produced without specific abatement equipments.

As before, the public agent implements an emission tax τ_t. The representative abatement producer is competitive and maximises (with the price of Z_t set to unity):

$$\text{Max } \tau_t A_t - Z_t, \quad (19)$$

subject to (18).

From profit maximisation we derive the first order condition

$$Z_t = \alpha\tau_t A_t. \quad (20)$$

Note that the first order condition for the intermediate input is the same as in the R&D model, see (5).

Under LbD, knowledge dynamics are given by

$$H_t = A_t^{\psi} H_{t-1}^{1-\psi}. \quad (21)$$

The three equations (18), (20), and (21) define a market equilibrium through the variables A_t, Z_t, H_t.

PROPOSITION 2. *For given initial state of knowledge*, H_0, *and a tax path* $\tau_t > 0$, *a unique equilibrium path exists if* $1-\alpha-\psi\gamma > 0$.

Proof. Substitution of (20) and (21) in (18) gives

$$A_t^{1-\alpha-\psi\gamma} = \mathrm{B}\alpha^{\alpha}\tau_t^{\alpha} H_{t-1}^{(1-\psi)\gamma}. \quad (22)$$

The left-hand-side is increasing in A_t from zero without bound for $1-\alpha-\psi\gamma > 0$, the right-hand-side is constant at time t. Thus, there is a unique solution A_t to the equation. For given H_{t-1}, this solution determines A_t, and H_t through (21), such that the entire equilibrium path can be determined by forward induction. Q.E.D.

3. EFFICIENT POLICY IMPLEMENTATIONS

First-best policy in the R&D model

The social planner aims at minimising the present value of abatement costs plus the damage from the stock pollutant. This can for instance be interpreted as the damage from the concentration of carbon in the atmosphere, i.e., the carbon stock. The minimisation problem becomes (where $\delta < 1$ is the social discount factor):

$$\text{Min } \Sigma_1^{\infty} \delta^{t-1}[Z_t + R_t + X_t + D(S_t)], \quad (23)$$

subject to (1), (3), (16) and stock accumulation dynamics

$$S_t = (1-\varepsilon)S_{t-1} + E_t . \quad (24)$$

The social abatement costs are the sum of the costs of Z, R and X, which all have price equal to 1.

$D(S)$ is the damage cost function, where damage depends on the stock of emissions, S. We assume that $D(S_0) \geq 0$, $D'(S_t) > 0$ and $D''(S_t) > 0$, and that the stock depreciates by the rate $\varepsilon < 1$.

The first order conditions from this minimisation problem are:

$$Z_t = \alpha\theta_t A_t \quad (25)$$
$$X_t = \beta\gamma\theta_t A_t \quad (26)$$
$$R_t = \psi\eta_t H_t \quad (27)$$
$$\eta_t H_t = \delta(1-\psi)\eta_{t+1}H_{t+1} + (1-\beta)\gamma\theta_t A_t \quad (28)$$
$$\theta_t = D'(S_t) + \delta(1-\varepsilon)\theta_{t+1}. \quad (29)$$

Note that $\theta_t = -\lambda_t \geq 0$, where λ_t is the dual variable for equation (1), and, hence, the current value shadow price of emissions. As mentioned before, θ_t is often referred to as the Pigouvian tax. Note also that θ_t is equal to the social price (or marginal value) of abatement in this model, as E_t and A_t are perfect substitutes, and since A_t has no effects on knowledge (as it has in the LbD model). $\eta_t \geq 0$ is the dual variable of equation (16) and, therefore, the current value shadow price of knowledge.

The first order conditions for Z and X defined by (25) and (26), are similar to the corresponding conditions for the market equilibrium given by (5) and (7), with the exception that market prices are replaced by the corresponding social prices.

As seen from (27), the value of research should equal the share ψ of the social value of knowledge. ψ expresses the relative contribution of R in producing knowledge. Equation (28) shows that the shadow price of knowledge is in general positive, but equal to 0 if there is no abatement throughout the time horizon.

According to (29), the social cost of emissions at time t, θ_t, is the present value of the damages caused by one unit of emission emitted at time t. It follows from a comparison of (5) and (25) that in the first-best policy, θ_t is equal to the optimal emission tax τ_t at time t.

As there are three types of imperfections in the model; pollution, imperfect competition in the market for abatement equipment, and positive and negative externalities of research effort, we would need three policy instruments to implement the social optimum: A Pigouvian tax on emissions, a subsidy to producers of abatement equipment, and a subsidy or tax on research effort.

PROPOSITION 3. *Through a tax on emissions equal to the Pigouvian tax, $\tau_t=\theta_t$, a subsidy on abatement equipment equal to $s_{x,t} = 1-\beta$, and a subsidy/tax on R&D effort equal to $s_{r,t}=1-(1-\beta)\gamma\theta_t A_t/\psi\eta_t H_t$, the first best outcome can be implemented.*

Proof: We introduce three policy instruments to implement the first-best outcome; an emission tax, τ_t, a subsidy on abatement equipment, $s_{x,t}$, and a subsidy/tax on research, $s_{r,t}$. We can then write the market conditions corresponding to (25), (26) and (27) as

$$Z_t = \alpha\tau_t A_t \tag{30}$$
$$(1-s_{x,t})p_t X_t = \beta\gamma\tau_t A_t \tag{31}$$
$$(1-s_{r,t})R_t = (p_t-1)\,\beta\gamma\tau_t A_t/(1-s_{x,t})p_t. \tag{32}$$

First, equation (30) is equal to the market condition defined in (5). Second, replacing p_t in equation (7) with $(1-s_{x,t})p_t$, gives the demand for X_t expressed by (31). Finally, (32) is derived in the same way as equation (11) and (15), apart from that we use (31) instead of equation (10). The price innovators pay for r_t is now set to $(1-s_{r,t})$ instead of unity.

Setting the environmental tax equal to the Pigouvian tax, i.e., $\tau_t=\theta_t$, implements the optimal use of Z_t, see (25) and (30).

To find the optimal subsidy rate on abatement equipment, $s_{x,t}$, we first replace θ_t for τ_t, which gives the following demand for X_t:

$$(1-s_{x,t})p_t X_t = \beta\gamma\theta_t A_t. \tag{33}$$

From (9) we know that $p=1/\beta$. Thus, $s_{x,t}=1-\beta$ implements the optimal use of X, cf. (26).

Finally, to find the optimal subsidy/tax on research, $s_{r,t}$, we insert $p=1/\beta$ from (9), $\tau_t=\theta_t$ and $s_{x,t}=1-\beta$ in equation (32). The market outcome of R then changes to:

$$(1-s_{r,t})R_t = (1-\beta)\gamma\theta_t A_t. \tag{34}$$

Inserting the first-best level of R from (27) gives after some calculation:

$$s_{r,t} = 1 - (1-\beta)\gamma\theta_t A_t/\psi\eta_t H_t. \tag{35}$$

Q.E.D.

The optimal level of $s_{r,t}$ in equation (35) may be positive or negative. This is because research effort has both positive and negative external effects.

The development of the research subsidy/tax, $s_{r,t}$, will depend on the development of the ratio $\theta_t A_t/\eta_t H_t$, i.e., the social value of abatement relative to the social value of knowledge, see equation (35). Note that the social value of abatement is proportional to the abatement expenditure (i.e., Z_t+X_t+R_t), as $\theta_t=\tau_t$. To see how this ratio develops over time, we need some definitions. The abatement expenditure growth factor is defined as $\varphi_t=\tau_{t+1}A_{t+1}/\tau_t A_t$. In a mature abatement sector, this growth factor is constant. For an infant industry, growth will exceed the matured growth level. When the sector is becoming mature, expenditure growth will gradually fall from its infant level to its mature level. We define the abatement sector to be *maturing* when $\varphi_t\geq\varphi_{t+1}$, and *constantly maturing* when this inequality applies for all $t\geq0$. We can now state and prove:

PROPOSITION 4. *In the R&D model, for a constantly maturing abatement sector, the efficient R&D subsidy/tax $s_{r,t}$ will fall over time.*

Proof: Given (35), it suffices to prove that $\eta_t H_t/\tau_t A_t$ decreases over time. Notice that $\theta_t=\tau_t$. Writing out equation (28) for the entire horizon, we have

$$\eta_t H_t/\theta_t A_t = (1-\beta)\gamma\ \{1+\delta(1-\psi)\varphi_t + [\delta(1-\psi)]^2\varphi_t\,\varphi_{t+1} + \ldots\}. \tag{36}$$

It is obvious that when φ_t is decreasing in t, then when we compare the equation for $\eta_t H_t/\theta_t A_t$ and $\eta_{t+1}H_{t+1}/\theta_{t+1}A_{t+1}$, in the latter equation, each of the terms on the right-hand side will be smaller, and thus, $\eta_{t+1}H_{t+1}/\theta_{t+1}A_{t+1}\leq\eta_t H_t/\theta_t A_t$. Q.E.D.

Second-best policy in the R&D model

Even if the social optimum in principle may be implemented using the appropriate number of policy instruments, it may be hard to target R&D at the firm level (as long as R&D effort is not completely undertaken in the public sector). For instance, R&D is not specified as a separate activity or sector in most national accounts. Consequently, it is difficult to use instruments such as a subsidy to producers of abatement equipment and a subsidy/tax on research effort. Based on this, we specify a second-best optimum, where the social planner has only one policy instrument available, namely the environmental tax.

The second-best optimisation problem of the social planner is, therefore, the minimisation problem (23) subject to (1), (3), (16), and (24), but also subject to the market equilibrium for Z, R and X given by equations (5), (10) and (15). The social planner now sets the value of τ_t that minimises social costs subject to the functioning of the environmental stock, the technology stock, and the different markets.

We can solve this social optimisation problem by substitution. In combination with (5), equations (10) and (15) give

$$X_t = (\beta^2\gamma/\alpha)Z_t \tag{37}$$
$$R_t = ((1-\beta)\beta\gamma/\alpha)Z_t. \tag{38}$$

Substitution of (37) and (38) in (23), (3), and (16) give

$$Min\ \Sigma_1^\infty\ \delta^{t-1}[wZ_t + D(S_t)], \tag{39}$$

subject to (1), (24), and

$$A_t = CZ_t^{\alpha+\beta\gamma}H_t^{(1-\beta)\gamma} \tag{40}$$
$$H_t = K\ Z_t^{\psi}H_{t-1}^{1-\psi}, \tag{41}$$

where w=$1+\beta^2\gamma/\alpha+(1-\beta)\beta\gamma/\alpha=1+\beta\gamma/\alpha>0$, C=$B(\beta^2\gamma/\alpha)^{\beta\gamma}>0$ and K=$((1-\beta)\beta\gamma/\alpha)^{\psi}>0$.

As before, let θ_t be the Pigouvian tax, so that $\lambda_t=-\theta_t\leq 0$ is the dual variable for equation (1). Let η_t be the dual variable for equation (41). The first order condition for Z_t and the optimal level of H_t are given by

$$wZ_t = (\alpha+\beta\gamma)\theta_t A_t + \psi\eta_t H_t \tag{42}$$
$$\eta_t H_t = \delta(1-\psi)\eta_{t+1}H_{t+1} + (1-\beta)\gamma\theta_t A_t. \tag{43}$$

In addition, equation (29) carries over from the first-best solution. While equation (43) is equal to the corresponding equation (28) in the first-best solution, the first order condition for Z is different due to the restrictions on the use of policy instruments (compare (42) with (25)).

From (5) and (42) and inserting for w, we derive

$$\tau_t/\theta_t = 1 + [\psi/(\alpha + \beta\gamma)]\ \eta_t H_t/\theta_t A_t. \tag{44}$$

This formula calculates the efficient second-best environmental tax relative to the Pigouvian tax on basis of the constant parameters α, β, γ, ψ, w, and the ratio of the value of knowledge over the value of abatement, $\eta_t H_t/\theta_t A_t$. As we see from (44), $\tau_t/\theta_t>1$, which means that the efficient environmental tax will be higher than the Pigouvian tax. This is stated in the following proposition:

PROPOSITION 5. *In the second-best R&D model, the efficient environmental tax, τ_t, will always be higher than the Pigouvian tax, θ_t, as long as abatement is positive.*

Will τ_t/θ_t rise or fall over time? As seen from (44), this depends on the development in the ratio of the value of knowledge over the value of abatement, i.e., $\eta_t H_t/\theta_t A_t$. This means that the development in τ_t/θ_t follows a similar path as the development in the optimal subsidy/tax on research, see (35). Thus, without the possibility to target research effort, the difference between the efficient emission tax and the Pigouvian tax should mimic the development in the optimal research subsidy/tax. This gives the following proposition.

PROPOSITION 6. *In the second-best R&D model, for a constantly maturing abatement sector, the relative difference between the efficient environmental tax, τ_t, and the Pigouvian tax, θ_t, will fall over time.*

Proof. From (5) and (42), we calculate a slight deviation from (44),

$$\theta_t/\tau_t = 1 - [\psi/(\alpha + \beta\gamma)]\eta_t H_t/\tau_t A_t]. \tag{45}$$

It suffices to prove that $\eta_t H_t/\tau_t A_t$ decreases over time, which follows the same argument as the proof of Proposition 4. Q.E.D.

Optimal policy in the LbD model

The social planner aims at minimising the present value of abatement costs plus the damage from the stock pollution, where $\delta<1$ is the social discount factor:

$$\text{Min } \Sigma_1^\infty \delta^{t-1}[Z_t + D(S_t)], \tag{46}$$

subject to (1), (18), (21) and (24). The first order conditions from this minimisation problem are:

$$Z_t = \alpha\theta_t A_t + \alpha\psi\eta_t H_t \tag{47}$$
$$\eta_t H_t = [\delta(1-\psi)/(1-\psi\gamma)]\eta_{t+1}H_{t+1} + [\gamma/(1-\psi\gamma)]\theta_t A_t \tag{48}$$
$$\theta_t = D'(S_t) + \delta(1-\varepsilon)\theta_{t+1}. \tag{49}$$

As before, $\theta_t \geq 0$ is the Pigouvian tax or the current value shadow cost of emissions, and $\eta_t \geq 0$ is the current value shadow price on knowledge. The dynamics of the Pigouvian tax is the same in the two different models, as the dynamics of the pollutant is the same. Also, even if the value of knowledge does not have the same dynamics in the two models, it follows a similar pattern, see (28) and (48).

While research effort creates knowledge in the R&D model, the use of the input Z creates learning and, therefore, knowledge in the LbD model. As opposed to the first order condition for Z in the R&D model, see (25), we see that in the LbD model, the value of knowledge has an impact on the optimal use of Z.

There are two imperfections in the LbD model: pollution and spillovers from knowledge. As knowledge follows from abatement, and abatement affects pollution, only one policy instrument is needed to implement the social optimum, i.e., a tax on emissions (or alternatively a subsidy on abatement). Therefore, the first-best solution and the second-best solution (with only one instrument) will be equivalent in the LbD model. The optimal policy is to choose the tax level τ_t that minimises the present value of abatement costs plus damage costs. This tax will in general be different from the Pigouvian tax as the latter only reflects the shadow cost of emissions.

As the abatement firm sets the optimal level of abatement according to (20), the optimal environmental tax level τ_t follows from equalising equations (20) and (47). Thus, we find:

$$\tau_t = \theta_t + \psi\eta_t H_t/A_t, \tag{50}$$

where $\eta_t H_t$ follows the development described by (48).

From (50) we can derive the following relationship between the environmental tax and the Pigouvian tax:

$$\tau_t/\theta_t = 1 + \psi\eta_t H_t/\theta_t A_t. \tag{51}$$

As in the second-best R&D model, see (44), the environmental tax will always be higher than the Pigouvian tax as long as abatement is positive. This result is in accordance with Rosendahl (2004), who finds that the optimal tax rate should be higher than the shadow cost of emissions when there are spillover effects from learning by doing. We then have the following proposition:

PROPOSITION 7. *In the LbD model, the efficient environmental tax, τ_t, will always be higher than the Pigouvian tax, θ_t, as long as abatement is positive.*

Comparing (44) with (51), we see that the development of the environmental tax relative to the Pigouvian tax follows a similar pattern in the LbD model as in the R&D model. This leads us to the following proposition:

PROPOSITION 8. *In the LbD model, for a constantly maturing abatement sector, the relative difference between the efficient environmental tax, τ_t, and the Pigouvian tax, θ_t, will fall over time.*

Proof. The proof follows exactly the same line of argument as Propositions 4 and 6.

4. MODEL EQUIVALENCE

As mentioned in the introduction, we expected the gap between an LbD model and a second-best R&D model with finite patent's lifetime to be considerably narrowed. This is partly confirmed by Propositions 6 and 8, which show that the R&D model share similarities with the LbD model in the second-best optimum. To study this further, we would like to see if equivalence between the two models can be shown to hold more generally. If so, this would mean that the choice of knowledge dynamics would not matter for policy implications, given the second-best setting.

To study the possible equivalence between the two models, we need to define equivalence. Assume now that the social welfare program for the second-best R&D model defined by (39), and the similar program for the LbD model defined by (46), both give well defined paths for the abatement effort, A, efficient environmental tax, τ, and the Pigouvian tax, θ. We define the two models to be equivalent if they give the same outcomes of these three variables for the same environmental preferences captured by $D(S_t)$.

First, we would like the models to produce the same market equilibrium, i.e., the same abatement level for a given tax on emissions. Second, we would like the second-best social optimum to be the same, which means that the efficient environmental tax should be the same in both models. The second condition is equivalent to the condition that the relative difference between the efficient tax and the Pigouvian tax should be equal in the two models. The reason is that the Pigouvian tax is equal across models as long as the abatement paths are equal (cf. (29) and (49)). Thus, if $(\tau_t)_t$ is the efficient tax for one model, it produces the same abatement path and hence the same Pigouvian tax path for both models. Therefore, if the ratio $(\tau_t/\theta_t)_t$ is the same, $(\tau_t)_t$ is the efficient tax for the other model as well.

Based on these requirements, we can define equivalence in the second-best setting if the R&D model and the LbD model

(i) produce the same abatement path, $(A_t)_t$, resulting from an arbitrary tax path, $(\tau_t)_t$ (equivalence of the market equilibrium).

(ii) give the same ratio between the efficient and the Pigouvian tax $(\tau_t/\theta_t)_t$ for any abatement path, $(A_t)_t$ (equivalence of the social optimum).

Thus equivalence means that for any second-best R&D model, an LbD model can be made that has exactly the same dynamic response function in the market equilibrium, and produces exactly the same socially optimal tax and abatement paths (and vice versa).

To continue, we need to set up both models on the same format. Consider, therefore, the reduced form specifications of the market equilibrium for the second-best R&D model and the LbD model, where the subscript R denotes the R&D model and L denotes the LbD model.

Based on (5), (40), and (41), the reduced form specification for the R&D model is:

$$A_{R,t} = B_R Z_{R,t}^{\alpha_R} H_{R,t}^{\gamma_R} \tag{52}$$

$$Z_{R,t} = G_R \tau_t A_{R,t} \tag{53}$$

$$H_{R,t} = K_R Z_{R,t}^{\psi_R} H_{R,t-1}^{1-\psi_R}, \tag{54}$$

where $B_R=B\ (\beta^2\gamma/\alpha)^{\beta\gamma}$, $G_R=\alpha$, $K_R=((1-\beta)\beta\gamma/\alpha)^{\psi}$, $\alpha_R=\alpha+\beta\gamma$, $\gamma_R=(1-\beta)\gamma$ and $\psi_R=\psi$.

In the same way, we specify the reduced form of the LbD model based on (18), (20) and (21):

$$A_{L,t} = B_L Z_{L,t}^{\alpha_L} H_{L,t}^{\gamma_L} \tag{55}$$

$$Z_{L,t} = G_L \tau_t A_{L,t} \tag{56}$$

$$H_{L,t} = A_{L,t}^{\psi_L} H_{L,t-1}^{1-\psi_L}, \tag{57}$$

where $B_L=B$, $G_L=\alpha$, $\alpha_L=\alpha$, $\gamma_L=\gamma$ and $\psi_L=\psi$.

Both reduced form versions of the models have three parameters, α, γ and ψ, which, as we will show, completely determine the dynamic behaviour of the model and the optimality conditions.

To see if the reduced form models are equivalent, we first introduce three intuitive requirements, which we use to derive relationships between the three parameters in the two models. Then we use these relationships to prove equivalence between the models.

The three requirements are that the short-term scale elasticities, the long-term scale elasticities, and the discount factor of the value of knowledge should be equal across models. Let us denote by μ the short-term returns to scale of production, i.e., $\mu=(\mathrm{d}A_t/\mathrm{d}Z_t)/(A_t/Z_t)$ with fixed H_{t-1}. Let v denote the long-term (steady state) returns to scale of production, i.e., $v=(\mathrm{d}A/\mathrm{d}Z)/(A/Z)$, with $H=Z$ for the R&D model[11] and $H=A$ for the LbD model[12]. Let ρ be the discount factor of the value of knowledge, i.e., the factor before $\eta_{t+1}H_{t+1}$ in (43) and (48). We find that:

$$\mu_R = \alpha_R + \gamma_R\psi_R \tag{58}$$
$$v_R = \alpha_R + \gamma_R \tag{59}$$
$$\rho_R = \delta(1-\psi_R) \tag{60}$$

$$\mu_L = \alpha_L/(1-\gamma_L\psi_L) \tag{61}$$
$$v_L = \alpha_L/(1-\gamma_L) \tag{62}$$
$$\rho_L = \delta(1-\psi_L)/(1-\gamma_L\psi_L). \tag{63}$$

Now, we can prove full dynamic equivalence of the market equilibrium between the two models, cf. (i) in the definition of equivalence above.

PROPOSITION 9. *The market equilibrium: When the second-best R&D model and the LbD model have the same characteristics, $\mu_R=\mu_L$, $v_R=v_L$ and $\rho_R=\rho_L$, and have parameters B_R, B_L, G_R, G_L and K_R,* (*in notation of* (52)-(57)) *and initial knowledge stocks $H_R=H_R^*$ and $H_L=H_L^*$ that support the same steady state, $\tau_R^*=\tau_L^*$ and $A_R^*=A_L^*$, then the two models have exactly the same dynamic behaviour. Formally, for given exogenous tax path $(\tau_t)_t$, both models produce the same equilibrium abatement path $(A_t)_t$.*

Proof. See the Appendix.

Whereas Proposition 9 states the dynamic equivalence of the market equilibrium between the second-best R&D model and the LbD model, we also have to prove that both models generate the same optimal policy, cf. (ii) in the definition of equivalence above. The following proposition states the equivalence of the social optimum:

PROPOSITION 10. *Social optimum: When the second-best R&D model and the LbD model have the same characteristics, $\mu_R=\mu_L$, $v_R=v_L$ and $\rho_R=\rho_L$, and follow the same abatement path $(A_t)_t$ for a given tax path $(\tau_t)_t$, then both models have the same ratio between the efficient and the Pigouvian tax $(\tau_t/\theta_t)_t$. Thus, the R&D and LbD models produce exactly identical optimal tax paths.*

Proof. See the Appendix.

[11] This follows from inserting $H_t=H_{t-1}$ in equation (54).
[12] This follows from inserting $H_t=H_{t-1}$ in equation (57).

The two propositions together make the proposition of full equivalence between the second-best R&D and the LbD model:

PROPOSITION 11. *When the second-best R&D model and the LbD model have the same characteristics, $\mu_R=\mu_L$, $v_R=v_L$ and $\rho_R=\rho_L$, and have parameters and initial stock levels that support the same steady state, the R&D and LbD models are equivalent if only one policy instrument, i.e., a tax on emissions, is available.*

Proof. The proof follows directly from Proposition 9 and10. Q.E.D.

5. SIMULATIONS

In this section we want to illustrate the theory by developing and simulating a numerical model that mimics a transition from a fossil fuel based to a carbon free energy system. The speed of transition is determined by technological progress, driven by policies and market forces. The numerical model gives insight into development over time of the relationship between the optimal environmental tax and the Pigouvian tax in a situation where research policies are not available, cf. Proposition 8. We will also use the model to get confirmation on equivalence between the LbD model and the second-best R&D model, cf. Proposition 11.

Calibration

We set out to calibrate a model that reproduces key characteristics of the climate change debate in a stylised manner. The starting point is a business as usual scenario of a LbD model with the following characteristics:

(i) Global emissions of CO_2 are 6 Gigatons carbon per year in the base year 2000.
(ii) Fossil fuel production costs grow (exogenously) from €200 per ton carbon in 2000 to €600 in 2200. €200 per ton carbon corresponds approximately to the average international market price of fossil fuels in 2004 and 2005 (BP, 2006). The rising unit costs over time reflect the exhaustion of easy-to-recover reserves.
(iii) CO_2-free energy amounts to 0.5 per cent of fossil energy in the base year. This is the share of commercial non-hydro, non-bio renewables in global energy supply (see IEA, 2005). Moreover, the annual growth in CO_2-free energy in 2000 is set to 4.5 per cent, which is consistent with actual growth rates in the 1990's for those renewables (cf. IEA, 2002, p. 27).
(iv) The long-term returns to scale in CO_2-free energy (v) is 1.2. This is consistent with an initial learning rate of 15-20 per cent, which is often seen in studies of CO_2-free energy (e.g., IEA, 2000).[13]
(v) CO_2-free energy constitutes 50 per cent of total energy use in 2250 in a benchmark BaU scenario. This benchmark scenario assumes that the spillover effects from learning are internalised, but not the damages from CO_2 emissions.
(vi) In the benchmark scenario, the marginal damages of CO_2 emissions (the equivalent of the Pigouvian tax on CO_2 emissions if it were levied) in 2000 are

[13] In an initial steady state, we have $Z/A = A^{(1/\alpha)-1}B^{-(1/\alpha)}H^{\gamma/\alpha}=C\cdot A^{(1/\alpha)-1+\gamma/\alpha}$, where C is a constant, and we have assumed fixed growth rate in abatement. The learning rate is then given by $1-2^{(1-\alpha-\gamma)/\alpha}$, which varies between 0.13 and 0.24 when α varies between 0.5 and 1 (and v=1.2).

€100 per ton carbon, or €28 per ton CO_2. As a comparison, the price of allowances in the EU's Emission Trading Scheme has hovered between €7 and €30 per ton CO_2 since the scheme was initiated in 2005. On the other hand, the Stern Review (2007) suggests that the social cost of carbon today is around \$85 per ton CO_2, if the world continues on the BaU path, and \$25-30 if the concentration of CO_2-equivalents is stabilised between 450-550 ppm CO_2e.

Production of energy is modelled slightly differently in the simulation model compared to the theoretical model (cf. equation (1)):

$$Y = \left(E^{(\sigma-1)/\sigma} + A^{(\sigma-1)/\sigma}\right)^{\sigma/(\sigma-1)}. \tag{64}$$

This CES-aggregate is used to give the simulation model more realism. It means that CO_2-free energy is an imperfect substitute to fossil fuels, so that the prices of fossil and CO_2-free energy may differ (whereas in the theoretical analyses abatement is a perfect substitute for emissions with identical price). Still, we assume that both E and A are measured in the same units. In energy system analysis, this would typically be in primary energy equivalents (EJ), but for convenience of our presentation, we present energy in Gigaton carbon equivalents, using the average carbon content of fossil fuels for conversion. The substitution parameter σ is set to 2, which implies that the price of A is 14 times higher than the price of E initially. Note that with $\sigma=\infty$, equation (64) reduces to equation (1).

In addition to the calibration data, the following assumptions are made. The length of the simulation periods, and thus the lifetime of patents, is set to ten years.[14] Future costs and benefits are discounted at a rate of 5 per cent per year. This is a compromise between typical market rates and social discount rates used in e.g. Stern Review (2007). Concentration of CO_2 in the atmosphere above the pre-industrial level decays by 1 per cent annually.[15] Energy use per capita grows by 1 per cent per year, whereas population grows by 1.2 per cent initially, but levels off at around 11 billion people during the first century. Marginal damage costs grow with economic growth per capita, for which we take 2 per cent per year. The assumptions about growth in population, economy and energy use seem to be in between the A1 and the A2 scenarios put forward by the IPCC's Special Report on Emission Scenarios (IPCC, 2000).

Table 1 shows the remaining (endogenous) model parameters in the LbD model that agree with the calibration requirements above. The table also shows the equivalent parameters in the second-best R&D model, which are calculated based on the equations in Section 4.

[14] Ten years may seem a bit short for the lifetime of patents. Note, however, that the main results regarding the ratio of efficient tax over Pigouvian tax (cf. Figure 6) are quite similar when we e.g. double the length of the simulation periods.

[15] This is of course a simplification of the carbon cycle, i.e., the interaction between CO_2 in the atmosphere and CO_2 in the land and in the ocean (see e.g. IPCC, 2001, Chapter 3.5). In particular, it may overestimate the decay of CO_2 when the concentration level gets higher.

TABLE 1. *Parameters in LbD model and equivalent second-best R&D model*

	LbD model	R&D model
α	0.85	0.69
β	-	0.3
γ	0.30	0.51
ψ	0.19	0.14

Scenarios

We run four alternative scenarios, see Table 2. All scenarios have the same stock levels in 2000, and environmental policy is introduced in 2010 in all scenarios except *S0* (the BaU scenario). *S1* and *S2* denote the first- and second-best R&D scenarios, whereas the *S3* scenario is based on a cost minimisation instead of a cost-benefit optimisation. That is, the first-best R&D model is used to minimise:

$$Min\ \Sigma_0^\infty\ \delta^t[Z_t + R_t + X_t], \tag{65}$$

with the additional constraint:

$$\Sigma_0^\infty\ \delta^t D(S_t) \leq \Sigma_0^\infty\ \delta^t D(S_t^*), \tag{66}$$

where S_t^* is the concentration level in the *S2* solution. The purpose of introducing this scenario is to examine the timing of abatement within a first- and second-best R&D model, where the discounted environmental damage costs are equal.

TABLE 2. *Model scenarios*

	Scenarios
S0	Business as Usual (BaU)
S1	First-best R&D
S2	Second-best R&D (=LbD)
S3	First-best R&D with same damage as in *S2*

Numerical results

First of all, the simulations clearly confirm Proposition 11, i.e., that the second-best R&D model and the LbD model are equivalent. The models produce the same optimal tax and abatement paths. Thus, the difference between the R&D and the LbD model can be interpreted along the lines of differences in access to policy instruments, at least within our model framework. Our conclusions about the *S2* scenario therefore relate to both the second-best R&D model and the LbD model.

Figures 1 and 2 show the development of fossil (E_t) and CO_2-free (A_t) energy over the next two centuries, measured in Gigaton carbon per year (on a logarithmic scale). In the policy scenarios, we see from Figure 1 that fossil energy reaches a top in the middle of this century, and falls below CO_2-free energy just before 2100 (only shown for *S2*). From Figure 2 we note that all policy scenarios produce very similar energy paths. Differences between first-best and second-best scenarios are small compared to overall policy effects. CO_2-free energy rises more rapidly in the first-best scenario (*S1*) than in the second-best scenario (*S2*), despite a higher environmental tax in the latter scenario (see below). The reason for this is that a first-best policy can stimulate abatement more cost-effectively through appropriate taxes and subsidies, compared to a second-best policy that only can stimulate CO_2-free energy through taxes on fossil fuels. Costs of abatement are thus lower in scenario *S1*, compared to *S2*.

In the cost-effective scenario (*S3*), CO_2-free energy is initially used slightly less than in the second-best policy scenario (*S2*), but catches up around 2040. The smaller market share in scenario *S3* in the beginning is due to the fact that innovation for and deployment of CO_2-free energy can be targeted separately by appropriate subsidies or taxes. Thus, with all policy instruments available, R&D is shifted upfront, whereas abatement is delayed.

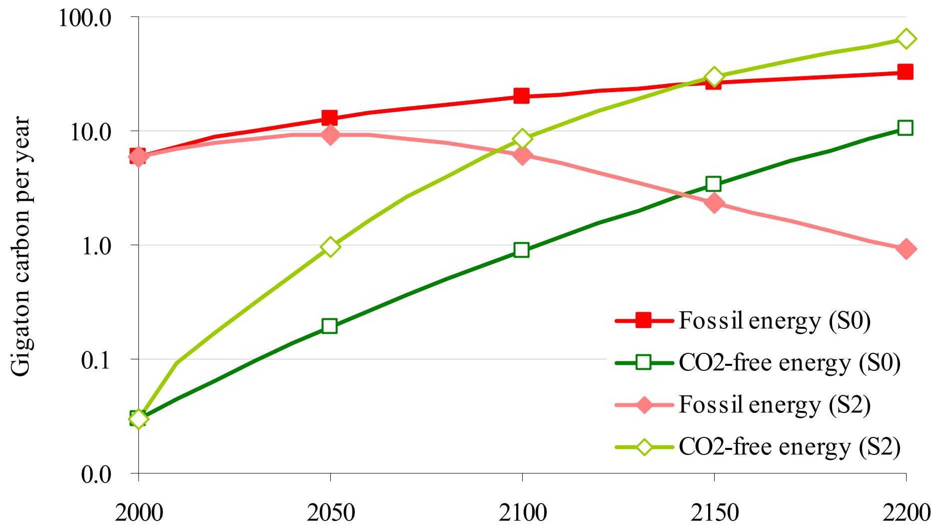

FIGURE 1. *Fossil and CO_2-free energy in the S0 and S2 scenarios*

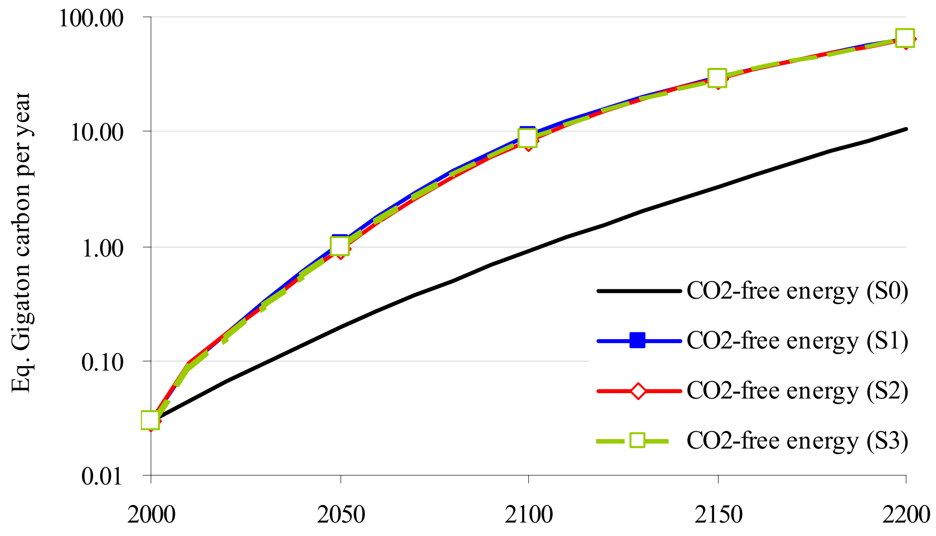

FIGURE 2. *CO_2 free energy in the different scenarios*

The timing issue is better seen in Figure 3, which shows the annual growth rate in CO_2-free energy expenditures (i.e., growth in Z_t+X_t). We notice that in all scenarios the growth rate falls, that is, the abatement sector is maturing as defined above Proposition 4. The transition from an infant industry into a matured industry is most pronounced in the policy scenarios. Also, we notice that growth rates in the scenarios

S1 and *S3* virtually coincide. Obviously, climate change policy increases abatement growth substantially over the first century, but eventually, the CO_2-free energy sector matures around the middle of the next century, as it takes over the energy market almost completely. From that time onwards, CO_2-free energy expenditures grow at the same rate as total energy use, i.e., by 1 per cent per year. When comparing the first-best and second-best scenarios, we find that expenditures start at a lower level in the first-best R&D model (*S3*), and grow slightly faster throughout the simulation period compared to the second-best R&D model (*S2*).

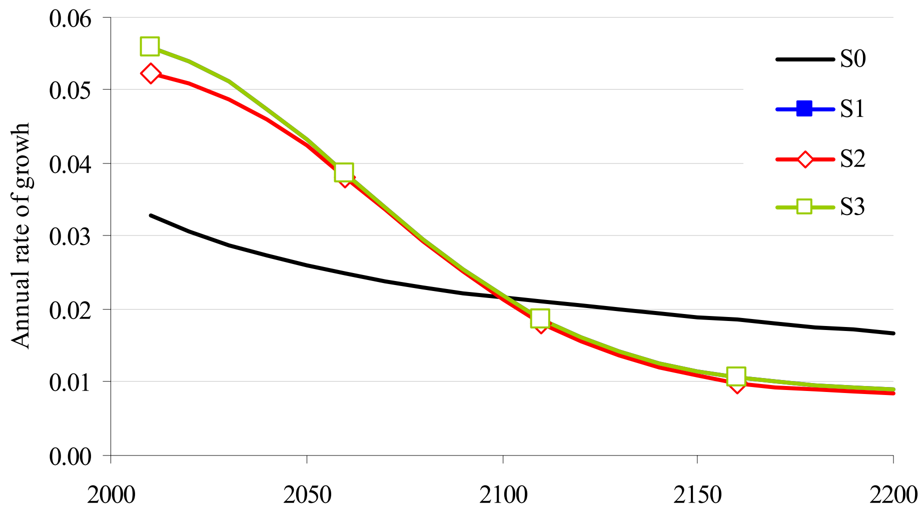

FIGURE 3. *Growth in CO_2-free energy expenditures in the different scenarios*

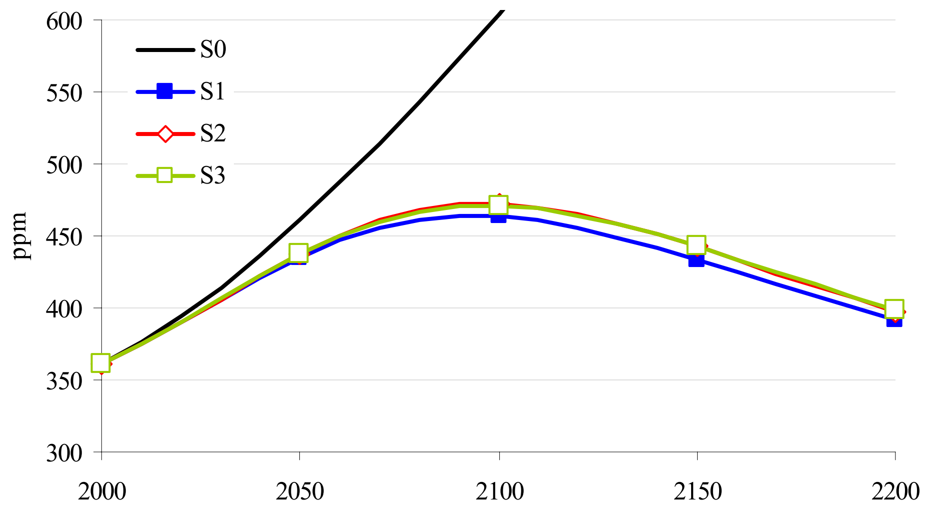

FIGURE 4. *Concentration level of CO_2 in the different scenarios*

Though we apply only a simple one-box resource model, still it can produce qualitative insights in the concentration level of CO_2 in the atmosphere (S_t). The concentration peaks around 2100 in the policy scenarios at a stock level equivalent to about 475 ppmv. Under the first-best R&D policy, *S1*, more action is taken and thus the concentration level peaks at a slightly lower level than with a second-best R&D

policy *S2*. The two scenarios with equal net present value of damages, *S2* and *S3*, are very similar.

In Figure 5 we show how the Pigouvian tax (θ_t) and the efficient tax (τ) develop in the three policy scenarios. In the first-best R&D scenario (*S1*), these two taxes are equal (cf. Proposition 3). In the second-best R&D scenario (*S2*) they are generally not (cf. equation (44)), and in our numerical simulations the efficient tax is well above the Pigouvian tax. The figure further shows that the Pigouvian tax is higher in the second-best scenario (*S2*) and the cost-effective scenario (*S3*) than in the first-best scenario (*S1*), which reflects the higher concentration level, and thus higher marginal environmental damages, in these scenarios. Consequently, in the cost-effectiveness scenario (*S3*), the efficient tax lies below the Pigouvian tax, as less abatement is needed compared to the first-best (*S1*) (cost-benefit) scenario. Note that in the cost-effective scenario, abatement levels are less than optimal (given the environmental damage function), as the objective is to minimise abatement costs for a fixed present value of future environmental damages (based on *S2*). Remember that environmental damages are higher in *S2* than in the first-best outcome *S1*. That is why marginal abatement costs (τ_t) are below marginal damage costs (θ_t) in this scenario.

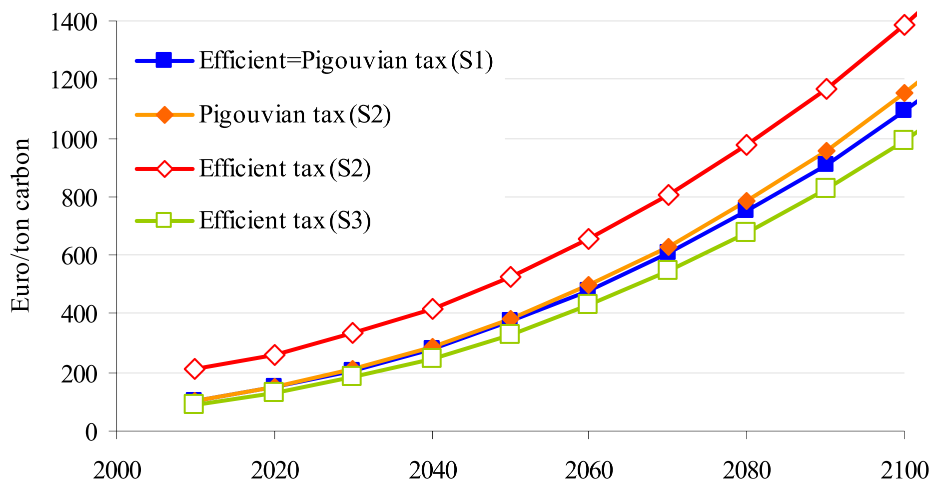

FIGURE 5. *Efficient tax and Pigouvian tax in the different scenarios*

The ratio between the efficient and the Pigouvian taxes in the second-best scenario *S2* is displayed in Figure 6, showing that the ratio exceeds unity and is falling monotonically over time. This confirms Proposition 5 and Proposition 6. The latter proposition states that the relative difference between the efficient and the Pigouvian tax will fall over time in the case with a maturing abatement sector (see Figure 3). We notice that the initial tax in this scenario exceeds the Pigouvian tax by factor 2.

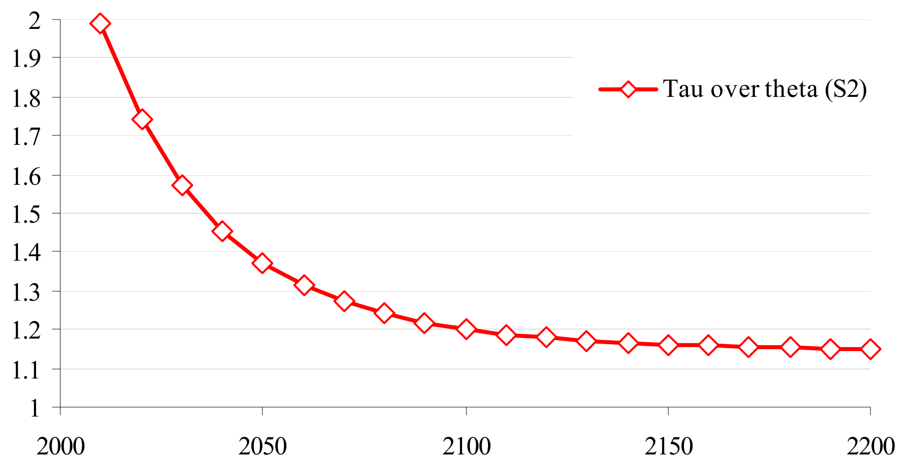

FIGURE 6. *Ratio of Efficient tax over Pigouvian tax in second-best R&D scenario*

6. CONCLUSION

In the climate change literature a pressing question is whether currently it is sufficient to stimulate the development of clean technologies for future use (technology push), or alternatively, that we need to start emission abatement sooner rather than later. Some take the technology push perspective even one step further, and assume that the foresight of a future need for abatement is sufficient to lead private firms to develop clean technologies. Within this optimistic perspective, it is unwarranted to start with abatement activities too hastily, as these early abatement efforts are unnecessarily costly compared to the cheaper options that will become available in the future. In the literature on technology development and climate change, the proponents of delayed and early action have often been divided along the lines of users of R&D models versus users of LbD models. Our analysis suggests that this distinction in perspective arising from the two types of models is not justified (Proposition 11).

If the public authority can directly steer the development of environmental technology, either through public environmental R&D or through targeted private environmental R&D, then it is efficient to spend much of the initial effort on this technological development. In both cases it is to be noted that in the phase of an emerging environmental problem, substantial public funds are to be directed to developing environmentally friendly technologies, either through public R&D or through high subsidies on private R&D (Proposition 4).

However, if the public authority cannot directly determine the development of an environmentally friendly technology, then efficiency considerations suggest that the clean technology should be extra stimulated through an increased demand for its produced goods. The technology pull policy should be relatively strong during the emerging phase of the environmental problem, when the abatement technologies still have to mature. Notably, this result is found in both the R&D and the LbD model (Proposition 6, Proposition 8). The major feature responsible for this equivalence between the R&D and the LbD models is an assumed finite lifetime of patents in the R&D model, a credible assumption we think.

As a final comment, we notice that the theoretical analysis we carried out has been fairly general, so that our findings may imply more generally that infant industries should be stimulated to a larger degree than mature industries. This topic may be worked out in future research.

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APPENDIX

For the proofs of Proposition 9 and Proposition 10, we have to do some preliminary work. Assume that $\mu_R=\mu_L=\mu$, $v_R=v_L=v$ and $\rho_R=\rho_L=\rho$. We can then invert the equations (58)-(63), which enables us to calculate the parameters α_R, γ_R, ψ_R and α_L, γ_L, ψ_L as functions of μ, v and ρ:

$$\alpha_R = v(1-\delta/\rho)+\mu\delta/\rho \tag{67}$$
$$\gamma_R = (v-\mu)\delta/\rho \tag{68}$$
$$\psi_R = 1-\delta/\rho \tag{69}$$
$$\alpha_L = v(1-\delta/\rho)+\mu\delta/\rho \tag{70}$$
$$\gamma_L = (1-\mu/v)\delta/\rho \tag{71}$$
$$\psi_L = (1-\delta/\rho)v/\mu. \tag{72}$$

It follows that when the two models have the same characteristics μ, v and ρ, then the following relations hold between the two sets of model parameters:

$$\alpha_R = \alpha_L \tag{73}$$
$$\gamma_R = v\ \gamma_L \tag{74}$$
$$\psi_R = (\mu/v)\ \psi_L \text{ or } (1-\psi_R) = (\mu/\alpha_L)(1-\psi_L). \tag{75}$$

Proof of Proposition 9: Let us denote by a tilde on top of a variable the log-difference compared to the steady state, e.g., $\tilde{\tau}_t=\ln(\tau_t)-\ln(\tau^*)$. Since (52)-(57) hold in steady state, we can now log-linearize them:

$$\tilde{A}_{R,t} = \alpha_R\ \tilde{Z}_{R,t} + \gamma_R \tilde{H}_{R,t} \tag{76}$$
$$\tilde{Z}_{R,t} = \tilde{\tau}_t + \tilde{A}_{R,t} \tag{77}$$
$$\tilde{H}_{R,t} = \psi_R\ \tilde{Z}_{R,t} + (1-\psi_R)\tilde{H}_{R,t-1}. \tag{78}$$

Similarly, for the LbD model, we take the log-linearization

$$\tilde{A}_{L,t} = \alpha_L \tilde{Z}_{L,t} + \gamma_L \tilde{H}_{L,t} \quad (79)$$
$$\tilde{Z}_{L,t} = \tilde{\tau}_t + \tilde{A}_{L,t} \quad (80)$$
$$\tilde{H}_{L,t} = \psi_L \tilde{A}_{L,t} + (1-\psi_L)\tilde{H}_{L,t-1}. \quad (81)$$

For the R&D model, substituting $\tilde{Z}_{R,t}$ out of (76)-(78), we derive

$$\tilde{A}_{R,t} = \alpha_R \tilde{\tau}_t + \alpha_R \tilde{A}_{R,t} + \gamma_R \tilde{H}_{R,t} \quad (82)$$
$$\tilde{H}_{R,t} = \psi_R \tilde{\tau}_t + \psi_R \tilde{A}_{R,t} + (1-\psi_R)\tilde{H}_{R,t-1}. \quad (83)$$

Substituting (83) in (82) and using (58) we derive

$$\tilde{A}_{R,t} = [\mu/(1-\mu)]\tilde{\tau}_t + [\gamma_R(1-\psi_R)/(1-\mu)]\tilde{H}_{R,\,t-1}. \quad (84)$$

Further, deriving $\tilde{\tau}_t$ from (82), substituting in (83) and using (58), we find

$$\tilde{H}_{R,t} = [\psi_R/\mu]\tilde{A}_{R,t} + (1-\psi_R)[1-\gamma_R\psi_R/\mu]\tilde{H}_{R,\,t-1}. \quad (85)$$

And similarly, for the LbD model we derive

$$\tilde{A}_{L,t} = [\mu/(1-\mu)]\tilde{\tau}_t + [\gamma_L(1-\psi_L)\mu/\alpha_L(1-\mu)]\tilde{H}_{L,t-1} \quad (86)$$
$$\tilde{H}_{L,t} = \psi_L\tilde{A}_{L,t} + (1-\psi_L)\tilde{H}_{L,\,t-1}. \quad (87)$$

Evaluating the equations for the first period t=1, we have $\tilde{H}_{R,0}$=$\tilde{H}_{L,0}$=0 as we assume an initial steady state, and thus $\tilde{A}_{R,1}$=$\tilde{A}_{L1}$ from (84) and (86). Also, from (85),(87) and (75), we find $\tilde{H}_{R,1}/\tilde{H}_{L,1}=\psi_R/\mu\psi_L=1/\nu$. By forward induction, we can show that $\tilde{A}_{R,t}$=$\tilde{A}_{L,t}$ and $\tilde{H}_{R,t}/\tilde{H}_{L,t}$=1/$\nu$ for all t. Assume that the equalities hold for t (and they do for t=1). Then, for t+1 it follows from (73), (74) and (75) that

$$\gamma_R(1-\psi_R)/(1-\mu) = \nu[\gamma_L(1-\psi_L)\mu/\alpha_L(1-\mu)]. \quad (88)$$

If we substitute this equality in (84) and (86), we find

$$\tilde{A}_{R,t+1} = \tilde{A}_{L,t+1}.$$

Furthermore, it follows from (58) and (75) that

$$(1-\psi_R)[1-\gamma_R\psi_R/\mu] = (1-\psi_L), \quad (89)$$

which, after substitution in (85) and (87), yields $\tilde{H}_{R,t+1}/\tilde{H}_{L,t+1}=1/\nu$.

To conclude, for all t we have established that $\tilde{A}_{R,t}=\tilde{A}_{L,t}$ and consequently, $A_{R,t}=A_{L,t}$ as we assume that $A_R^*=A_L^*$. Q.E.D.

Proof of Proposition 10: The optimal policy is determined by (43) and (44) for the R&D model and by (48) and (51) for the LbD model. Rewriting (43) and (44), using (60) and the parameter adjustments immediately below equation (54), we find for the R&D model:

$$\eta_t H_t = \rho_R \eta_{t+1} H_{t+1} + \gamma_R \theta_t A_t \tag{90}$$

$$\tau_t/\theta_t = 1 + \psi_R/\alpha_R\ \eta_t H_t/\theta_t A_t. \tag{91}$$

In the same way we can rewrite the optimal policy conditions for the LbD model:

$$\eta_t H_t = \rho_L \eta_{t+1} H_{t+1} + [\gamma_L/(1-\psi_L\gamma_L)]\theta_t A_t \tag{92}$$

$$\tau_t/\theta_t = 1 + \psi_L \eta_t H_t/\theta_t A_t. \tag{93}$$

Now, using (90), (91) can be rewritten as

$$\tau_t/\theta_t = 1 + [\psi_R\gamma_R/\alpha_R](1+\rho_R\theta_{t+1}A_{t+1}/\theta_t A_t+(\rho_R)^2\theta_{t+2}A_{t+2}/\theta_t A_t+\ldots). \tag{94}$$

Also, using (92), (93) can be rewritten as

$$\tau_t/\theta_t=1+[\psi_L\gamma_L/(1-\psi_L\gamma_L)](1+\rho_L\theta_{t+1}A_{t+1}/\theta_t A_t+(\rho_L)^2\theta_{t+2}A_{t+2}/\theta_t A_t+..). \tag{95}$$

Assume that τ_t is the efficient tax in one of the models. Then we know from Proposition 9 that it produces the same abatement path A_t, and hence the same Pigouvian tax θ_t in both models. Furthermore, we have by assumption $\rho_R=\rho_L$. Therefore, in order to prove that τ_t is the efficient tax in the other model as well, we only need to prove that

$$\psi_R\gamma_R/\alpha_R = \psi_L\gamma_L/(1-\psi_L\gamma_L). \tag{96}$$

Using (58) for the left-hand side and (61) for the right-hand side, (96) can be rewritten as

$$\mu_R/\alpha_R-1 = \mu_L/\alpha_L-1, \tag{97}$$

which holds by assumption, see (73). Thus, we have established that the paths for the efficient tax τ_t are identical for the R&D and LbD models. Q.E.D.

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