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Optimal Environmental Policy with Network Effects: Is Lock-in in Dirty Technologies Possible?

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No 13/13	Nils-Henrik M. von der Fehr *Transparency in Electricity Markets*
No 12/13	Nils Chr. Framstad *When Can Environmental Profile and Emissions Reduction Be Optimized Independently of the Pollutant Level*
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Optimal environmental policy with network effects: Is lock-in in dirty technologies possible?

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Abstract

Network externalities could be present for many low or zero emission technologies. One obvious example is alternative fuel cars, whose use value depends on the network of service stations.

The literature has only briefly looked at environmentally beneficial technologies. Yet, the general literature on network effects is mixed on whether governments need to intervene in order to correct for network externalities.

In this paper we study implications of network effects on environmental policy in a discrete time dynamic game. Firms sell a durable good. One type of durable is causing pollution when being used, while the other type is "clean". Consumers' utility increase in the number of other users of the same type of durable, which gives rise to the network effect.

We find that the optimal tax depends on the size of the clean network. If starting from a situation in which the dirty network dominates, the optimal tax may exceed the marginal environmental damage, thereby charging consumers for more than just their own emissions. Applying a Pigovian tax may, on the contrary, fail to introduce a socially beneficial clean network.

JEL-Classification: Q55, Q58, H23
Keywords: Network effects, lock-in, enviromnetal taxes

1 Introduction

The solution to an environmental problem often involves replacing an old dirty technology with a new clean technology. According to Barrett (1999) technological innovation was crucial for the success of the Montreal protocol protecting the ozone layer. When it comes to climate change, the technology options are not as evident. The question then arises; will a carbon tax implemented in the industrialized countries induce technological change such that in due time new carbon free technologies will overtake the markets?

According to several authors the answer could be no: The market entry of carbon free technologies is prevented by *lock-in* in fossil based technologies, and a carbon tax equal to the social cost of carbon may not lead to the necessary technological shift. Acemoglu et al. (2012), Chakravorty et al. (2011) and Greaker and Heggedal (2010) all argue that this may be the case for carbon free technologies.

Many mechanisms may lead to lock-in like situations. Acemoglu et al. (2012) describe a process of market driven directed research in which dirty technologies steadily improve and clean technologies are not developed further. Chakravorty et al. (2011) find that fossil fuel resource owners have an incentive to slow learning in the alternative emission free technology by increasing their own extraction. In this paper we will focus on *network externalities* as a potential source for technological lock-in.

Positive network externalities arise if one agent's adoption of a good (a) benefits other adopters of the good and (b) increases others' incentive to adopt it (Farrell and Klemperer (2007)). The literature so far has failed to agree on whether the market outcome will be efficient when network effects are present. Liebowitz and Margolis (1994), for instance, argue that in order for there to be inefficiencies, benefits of an unrealized outcome must exceed the costs, and this can be exploited by private agents with profit motives. Hence, they argue, inefficient outcomes due to network effects will rarely be observed.

In principle the argument of Liebowitz and Margolis should hold even if we have an environmental externality as long as the dirty technology faces a tax corresponding to the social cost of emissions. Yet, it is not obvious that there are such private agents who can exploit coordination failures; the market structure could vary from case to case. Define a *technology sponsor* to be a monopolist supplier of a network good. We investigate the case where both the green and dirty technology are sponsored, the case where only the clean technology is sponsored, and the case where non of the technologies are.

In this paper we pose the following research questions: I) Should environmental policy be adjusted when there are network effects?, II) Does the need for adjustment depend on the existence of sponsors? and III) May a failure to internalize the network externality lead to lock-in? First, we find that optimal environmental policies should take into account the network externality by making policy contingent on the size of the clean network. This holds for all the configurations of sponsors, and may be effectuated by setting an emission tax that departs from the Pigovian tax.

Finally, to answer the third question, we simulate a numerical version of the model. Our point of departure is the competition between fossil based and zero emission cars. Surprisingly, when only the zero emission technology has a sponsor, the market might be dominated by the inferior fossil based technology even if this technology is subject to a Pigovian tax. In this case the government can improve social welfare by subjecting the dirty technology to an emission tax far in excess of the social cost of emissions.

In our opinion network externalities could be present for many clean technologies. The literature distinguishes between *direct* and *indirect* network externalities. In the former case there is a direct benefit to existing consumers when a new consumer is recruited to the network, while in the latter case the benefit to existing consumers from a new consumer comes from increased supply of some complementary product Farrell and Klemperer (2007).

Direct network externalities are for instance likely to be present in the competition between advanced virtual meeting equipment and air travel. *Indirect* network externalities might be the case for both zero emission cars, and for carbon capture at powerplants and industries. Carbon capture requires the complementary pipeline transport service in which there are economies of scale. Thus, the more plants that adopt carbon capture, the lower the per-plant cost of carbon transportation to storage sites.

With respect to the car market, Nicholas and Ogden (2009) report from a survey, demonstrating a strong relationship between the willingness to pay for a hydrogen car and the availability of hydrogen filling stations. The same type of interdependency could also be the case for electric cars and the network of fast charging stations.

Possible indirect network externalities in the car market is also briefly explored in the environmental economics literature. Greaker and Heggedal (2010) builds an explicit model of the relationship between the market share of hydrogen cars and the density of hydrogen filling stations, and show that this could lead to multiple equilibria. In some equilibria fossil based cars dominate the market, although these equilibria are welfare inferior. However, unlike this paper, they do not include a potential sponsor in their analysis, and they only look at a static game.

Indirect network externalities in the transport market is also treated by Sartzetakis and Tsigaris (2005). Sartzetakis and Tsigaris (2005) do not model the network externality explicitly, but their game is dynamic as consumers arrive sequentially. However, prices follow exogenously given rules, and the government do not set taxes optimally. They therefore do not investigate optimal policy with technology sponsors.

This paper will extend the analysis in Cabral (2011) who studies a model with two sponsored networks competing in prices. Each period one consumer makes an adoption decision given the prices. His utility from the good depends on it's network size for each period he's alive. We introduce pollution from one of the networks, while the other network is clean. Moreover, we introduce a government who sets emission taxes to maximize social welfare.

In Cabral (2011) the two network technologies are equally good, while in our model the clean technology is superior from a social point of view, although identical in the eyes of the consumers. Further, the focus in Cabral is to characterize competition in a network industry, while we question whether the market achieves the correct mix between the networks, or whether the market is locked in to the inferior dirty technology. Lock-in has been a topic in the general literature on network externalities, which we will shortly review in the next subsection.

1.1 Literature on network externalities and lock-in

There is a body of literature looking at the lock-in phenomenon from a more general point of view. Farrell and Saloner (1985) also analyze a general model with network externalities. Firms choose whether to switch from an old to a new technology. The decisions of the firms are modeled as a multi stage game in which one firm starts and the other firms follow sequentially. Farrell and Saloner explore different versions of this game in which firms have either complete or incomplete information about other firms' pay-off functions. They define *excess inertia* to be a situation in which firms do not adopt a welfare dominant technology. This corresponds to how the literature defines a lock-in situation. In the Farrell and Saloner model, excess inertia cannot happen if firms have complete information.

In another paper Farrell and Saloner (1986) develop their ideas further, and introduce an installed base of users of the old technology. Due to the installed base, users of the old technology will adopt the new technology at a slow pace, depending on how fast the installed base depreciates. Early adopters of the new technology must then bear the cost of a small network while waiting for more consumers to adopt the new technology. This effect can lead to *excess inertia* even with complete information.

In our model early adopters of the clean durable must also bear the cost of a small network while waiting for more consumers to adopt the clean good. However, in our model the clean technology sponsor may speed up this process by offering the clean good at a low price. Katz and Shapiro (1986) introduced the concept of technology sponsors. With a sponsor they imply a private agent that has monopoly rights to a technology, and can claim a part of the future monopoly rents from this technology. They find that having a sponsor is crucial for the market development of a new technology. In particular, opposed to one of our results, they find that if the superior technology has a sponsor, it will dominate the market.

In Katz and Shapiro (1986) the two technologies only differ with respect to their network sizes, while in our model the technologies differ with respect to both network size and product characteristics. Since products are differentiated, firms may start to price high focusing only on the most eager customers when their network has reached a critical size. Thus, in our model an inferior technology my dominate the market even if only the superior technology has a sponsor.

Ochs and Park (2010) extend the analysis in Farrell and Saloner (1986), and find that as long as the timing of entry is endogenous (so that the most eager consumers move first) and entry decisions are irreversible (so that no network ever declines in size), then as the discount factor tends to one, any coordination problem found by Farrell and Saloner (1986) vanishes, and the equilibrium is efficient as the population grows large. On the other hand, in Ochs and Park (2010) none of the technologies have sponsors that can act stratigically. Moreover, durables wear out, and hence, consumers must choose network over again. Both these features are included in our model.

The paper proceeds as follows: In Section 2 we lay out the model, while in Section 3 we derive the main results. In Section 4 we simulate the model numerically, and in Section 5 we conclude.

2 Model primitives

Following the original model of Cabral, we will have discrete timing with two competing networks and a fixed number N of consumers. The networks will be indexed by $k = c$ for clean and d for dirty. For each network there is an access price the consumer has to pay to join the network. These prices are set by the firms, and can be thought of as prices for some durable goods that grant the consumer access to the network in question. Denote these prices p_c and p_d, respectively.

The government will set two different taxes, one tax t on the purchase of the dirty durable, and one tax τ on the use of the dirty good. We will study markov-perfect equilibria (MPEs). The setup will be time homogeneous, hence we suppress all time subscripts. The only payoff relevant variables will be the network sizes, denoted n_c and n_d. We assume that the market is fully covered, so that all consumers own a good. Since the total number of consumers is fixed at N, we only need to keep the clean network size n_c as a state variable.

2.1 The consumers

At the beginning of each period, there are $N - 1$ consumers present in the market. One consumer arrives, and is confronted with the prices and taxes. Subject to these, he has to choose which network he wants to enter. After he makes his choice, there is an intermediate stage, the aftermarket stage, in which the durable goods are being put to use. At this stage all consumers each enjoy some aftermarket benefits $\lambda(n_k)$, common to all consumers and weakly increasing in the network size n_k.[1] At the end of the period, with uniform probability, one random consumer is chosen to exit the market.

Due to this random exit, an entering consumer neither knows for how many periods he will enjoy the aftermarket benefits nor how large the network

[1]For instance, the function $\lambda(n_k)$ can be seen as the reduced form of the explicit network model in Greaker and Heggedal (2010).

is going to be in the future. We therefore introduce the function $u_k(n_k)$ which is the expected present value (EPV) of entering network k at size n_k. That is, it is the expected discounted sum of the aftermarket benefits $\lambda(n_k)$ over all the future periods the consumer expects to be in the market.

In addition to the aftermarket benefits that are common to all consumers, each consumer draws two idiosyncratic, private utility components at birth. The components, $\{\zeta_c, \zeta_d\} \in \mathbb{R}^2$, determine the technology-specific utility he enjoys from joining either of the networks. The total net benefit B_k is then given by:

$$B_k = \begin{cases} \zeta_c + u_c(n_c + 1) - p_c(n_c), & \text{if clean network} \\ \zeta_d + u_d(n_d + 1) - p_d(n_d) - t(n_d), & \text{if dirty network.} \end{cases}$$

We assume that the values of ζ_k are sufficiently high such that the consumer always chooses one of the networks. Since the market is then completely covered, we can restrict our attention to the distribution of the difference between the two utility parameters $\xi_c \equiv \zeta_c - \zeta_d$. As we assume that the ζ_k are $i.i.d$, ξ_c has expected value equal to zero.

The consumer who is indifferent between the two networks will have: $B_c = B_d$, or $\xi_c = x(n_c)$ where the latter is given by:

$$x(n_c) = p_c(n_c) - p_d(n_d) - t(n_d) - u_c(n_c + 1) + u_d(n_d + 1). \tag{1}$$

That is, $x(n_c)$ indicates the position along the real line of the consumer who is indifferent between the two goods when the clean network has size n_c, and prices and taxes are as given. Now, assuming that ξ_c is normally distributed with cdf $\Phi(\cdot)$ and density $\phi(\cdot)$, we derive the probability that a newborn consumer chooses the clean network:

$$\begin{aligned} q_c(n_c) &= Pr\left[\xi_c \geq x(n_c)\right] = 1 - Pr\left[\xi_c < x(n_c)\right] \\ &= 1 - \Phi\left[x(n_c)\right], \end{aligned} \tag{2}$$

and the probability of choosing the polluting network is:

$$\begin{aligned} q_d(n_d) &= Pr\left[\xi_c < x(n_c)\right] \\ &= \Phi\left[x(n_c)\right]. \end{aligned} \tag{3}$$

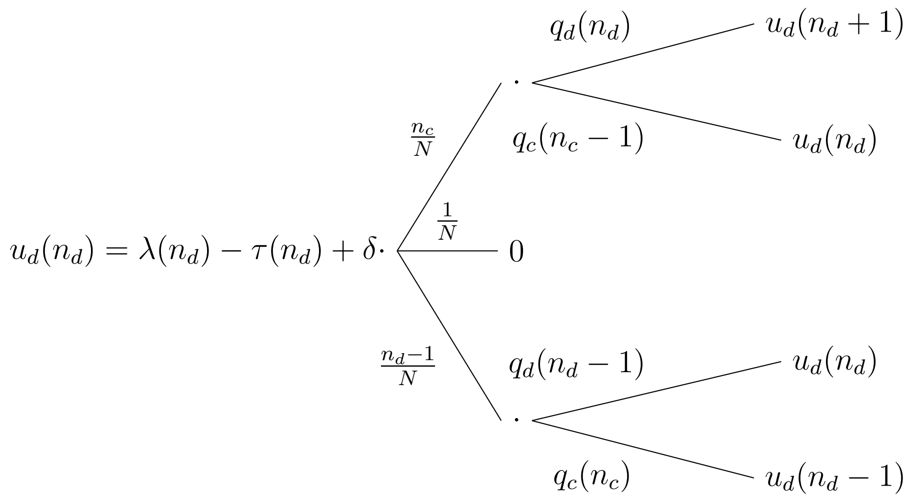
The taxes levied on the dirty network introduces asymmetries, such that $q_c(a) \neq q_d(a)$ in equilibrium, but the probabilities are related through $q_c(a) + q_d(N-1-a) \equiv 1$. From these expressions, we can see that the probability that firm k makes the next sale is, ceteris paribus, continuously and monotonically decreasing in p_k.

Given a sequence of taxes and prices, we now have the law of motion for the network shares. Given that every consumer has the same probability of being chosen to leave the market, the EPV of future network benefits does not depend on how long a consumer has been present. We can therefore define $u_k(n_k)$ recursively in the following way (first for the dirty network):

$$\begin{aligned} u_d(n_d) = \lambda(n_d) - \tau(n_d) + \frac{1}{N} \cdot 0 + \delta \frac{n_c}{N} q_d(n_d) u(n_d+1) \qquad (4) \\ + \delta \left[\frac{n_c}{N} q_c(n_c-1) + \frac{n_d-1}{N} q_d(n_d-1) \right] u_d(n_d) + \delta \frac{n_d-1}{N} q_c(n_c) u(n_d-1) \end{aligned}$$

Each period you enjoy the aftermarket benefit as a function of the market share, and consumers in the dirty network also pay a tax $\tau(\cdot)$ every period for the use of their good. At the end of each period, there is a probability $1/N$ that you are the one who dies, after which you get zero by assumption. If you are not chosen to exit, there are three possibilities: your network increases, decreases or remains at the same size. There is only one possible way your network can increase in size: with a probability of n_c/N someone in the clean network exits, and with probability $q_d(n_d)$ the arriving consumer opts for the dirty network, and the network size increases one step. There are two events that may reproduce the current state the next period; that is when one of the networks experience exit and the arriving consumer chooses to join that same network. And finally your network may decrease by one step if someone other than you dies, and the next consumer chooses the clean network. See Figure 1 for a visualization of (4).

Figure 1 "Expected utility of entering the dirty network"



For a consumer present in the clean network, we get the following value:

$$\begin{aligned} u_c(n_c) = \lambda(n_c) + \frac{1}{N} \cdot 0 + \delta \frac{n_d}{N} q_c(n_c) u(n_c+1) + \qquad (5) \\ \delta \left[\frac{n_d}{N} q_d(n_d-1) + \frac{n_c-1}{N} q_c(n_c-1) \right] u_c(n_c) + \delta \frac{n_c-1}{N} q_d(n_d) u(n_c-1) \end{aligned}$$

Note that there is no use tax $\tau(\cdot)$ in (5).

To gain some intuition on these expressions, we can consider the case with a constant use tax and zero network benefits e.g. $\lambda(\cdot) = 0$. Equation (4) then collapses to $u_d = -\tau(1 - \delta\frac{N-1}{N})^{-1}$, i.e. the expected net present value of the future outlays on the use tax, while (5) collapses to 0. Note that the discount factor is augmented with the factor $\frac{N-1}{N}$, that is the probability that the consumer will stay alive. Further, with constant access prices, the marginal consumer is given by: $x_c = p_c - p_d - t - \tau(1 - \delta\frac{N-1}{N})^{-1}$. Hence, only consumers with $\zeta_c - \zeta_d < p_c - p_d - t - \tau(1 - \delta\frac{N-1}{N})^{-1}$ will choose the dirty network.

2.2 Firms

Firms derive revenue equal to the entry price p_k every time a new consumer enters their technology.[2] Costs are normalized to zero, and hence revenue is equal to profits. Remember that the utility a consumer gets from a technology depends on the number of consumers already using the technology. Hence, expected revenue for a given p_k will depend positively on the size of the network.

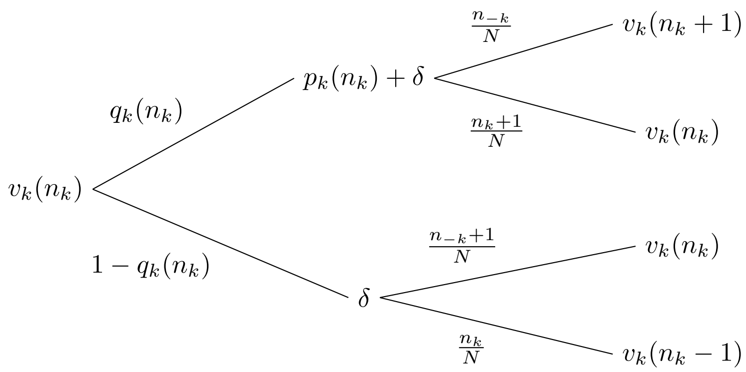
The value functions of the firms are evaluated before the firms set the price and the arriving consumer makes his choice. The total number of consumers who *currently* are in the market is therefore $N-1$. For a network of technology $k = c, d$ we have:

$$\begin{aligned} v_k(n_k) = q_k(n_k) & \left(p_k(n_k) + \delta\frac{n_{-k}}{N} v_k(n_k+1) + \delta\frac{n_k+1}{N} v_k(n_k) \right) \\ & + (1 - q_k(n_k)) \left(\delta\frac{n_{-k}+1}{N} v_k(n_k) + \delta\frac{n_k}{N} v_k(n_k - 1) \right) \end{aligned} \tag{6}$$

where $n_k + n_{-k} = N - 1 \Rightarrow n_{-k} = N - 1 - n_k$. The first line above is the event that the newborn consumer chooses network k when it's size is n_k. In that case network k sells a unit at value $p_k(n_k)$ and it's network size increases to $n_k + 1$. In the next period there are two possibilities; either the other network has experienced exit (with probability n_{-k}/N), or someone in network k has exited (with probability $(n_k+1)/N$). The network size at the beginning of the next period is updated accordingly. The second line is the event that the arriving consumer chooses the other network. In that case there is a higher probability that the other network experiences an exit, and vice versa. It is visualized in Figure 2.

Figure 2 "Expected firm value"

[2] In Cabral (2011) firms also enjoy aftermarket benefits depending on the size of their networks. For simplicity, we disregard these here. This can for instance be the case if the complimentary services are supplied from a sector separate from the two technology owners.



As mentioned, we consider three different market configurations, all compatible with (6): I) both technologies are sponsored, II) only the clean technology is sponsored, while the dirty technology is supplied by several firms, and III) both technologies are supplied by more firms.[3]

2.3 The government

Environmental damages from the polluting network accrues according to $d * n_d$, where d is a parameter and n_d is the number of consumers present in the polluting network today. This is a reasonable representation of environmental costs as long as a) the emissions from the network in question is only a part of the total emissions, and b) the use intensity is exogenous to the agents once they have joined the dirty network.

We equip the government with two instruments: a purchase tax $t(n_d)$ levied at the time of purchase, and a flow tax $\tau(n_d)$ levied each period on all consumers present in the polluting network and thus affecting the the expected present value of entering network d.

In addition to the environmental damage function, the public welfare function is assumed to be utilitarian, it is the unweighted sum of profits and consumer utility. We are thus lead to the following value function evaluated before the consumer chooses a network:

[3] The first configuration corresponds to the set up used by Cabral. In the transport market application the durables could be either a fossil fuel car or a hydrogen (electric) car which both provide a tranportation service that depends on the density of refueling stations. Further, the inventor owning the patent on the premium fuelcell (rechargeable battery) can through her pricing of the patent set the access price for the clean network. For the dirty network, we can either assume that current car companies act as a cartel using their pricing to keep consumers in the dirty technology, or we may have that only the green network has a sponsor.

$$
\begin{aligned}
g(n_c) = & \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad (7)\\
& q_c(n_c) \cdot \Big\{ \mathbb{E}\left[\zeta_c | \xi_c > x(n_c)\right] + (n_c+1)\lambda(n_c+1) - p_c(n_c) \\
& + n_d \left[\lambda(n_d) - \tau(n_d)\right] - d n_d + \tau(n_d) n_d + p_c(n_c) \\
& + \delta_G \left[\frac{n_c+1}{N} g(n_c) + \frac{n_d}{N} g(n_c+1) \right] \Big\} \\
& + (1 - q_c(n_c)) \cdot \Big\{ \mathbb{E}\left[\zeta_d | \xi_c < x(n_c)\right] + (n_d+1)\left[\lambda(n_d+1) - \tau(n_d+1)\right] \\
& - p_d(n_d) - t(n_d) + n_c \lambda(n_c) - d(n_d+1) + \tau(n_d+1)(n_d+1) \\
& + p_d(n_d) + t(n_d) + \delta_G \left[\frac{n_c}{N} g(n_c-1) + \frac{n_d+1}{N} g(n_c) \right] \Big\}
\end{aligned}
$$

The welfare measure is the expected value of two scenarios. First the case that the newborn consumer chooses the clean network. This happens with probability q_c. The value is then the expected idiosyncratic utility of the consumer, conditional on him choosing clean. Then we subtract the price he pays, we add the government tax revenue from the use of the dirty good and the consumers' network benefits, net of any flow tax paid. Further, we add the price revenue the clean network made from selling. Finally we add the expected continuation value, conditional on the clean network having been chosen today. Then the same exercise is repeated in the event the dirty network is chosen. The only difference is that we now also have to take into account the purchase tax $t(n_d)$ levied on the consumer.

3 Solving the model

The timing of the game in every period is as follows: First, the government sets taxes. Second, firms observe the current taxes, and then they compete in prices. Finally, the consumer makes his choice, knowing the prices and the current taxes. As we do not find it reasonable that the government can commit to future tax rates, we will only allow a stagewise leadership. Thus we are searching for a stochastic stagewise Stackelberg equilibrium.

What does this mean in practice? Our interpretation is that both the government and the firms announce a markovian rule that specifies the optimal response in every state. If both the industry and the current government believe the rules will be followed in all future periods, then it is optimal for the current government to follow it, too. This holds true in all periods, so the announced markovian strategies will indeed be followed.

To implement the equilibrium, we solve a set of dynamic programming problems by backwards induction. The consumer's choice problem is already solved by (4) and (5), which for given prices and taxes constitute a system of $2 * N$ equations with $2 * N$ unknowns e.g. $u_c(1)...u_c(N)$ and $u_d(1)...u_d(N)$. We also have everything we need to solve the firms' problems for given taxes. Lastly we solve the government's problem, taking into account the response functions of the firms and the consumers. But first, we derive the first best allocation.

3.1 First best

We start out by simplifying the expression for social welfare. First note that all taxes and prices paid are just transfers, and do not affect welfare under the additive welfare measure. This reduces eq. (7) to

$$\begin{aligned} g(n_c) = \Bigg\{ \mu + \sigma^2\phi(x(n_c)) + q_c(n_c) \cdot \Bigg[n_d\lambda(n_d) \\ + (n_c+1)\lambda(n_c+1) - d(n_d) + \delta_G \frac{n_c+1}{N} g(n_c) + \delta_G \frac{n_d}{N} g(n_c+1) \Bigg] \\ + (1-q_c(n_c)) \cdot \Bigg[(n_d+1)\lambda(n_d+1) + n_c\lambda(n_c) \\ - d(n_d+1) + \delta_G \frac{n_c}{N} g(n_c-1) + \delta_G \frac{n_d+1}{N} g(n_c) \Bigg] \Bigg\}, \end{aligned}$$

where the two first terms is the expected value of the idiosyncratic utility terms.[4] By gathering all the terms inside the square brackets, we compress the notation to

$$g(n_c) = \mu + \sigma^2\phi(x) + q_c(n_c)\Lambda^c(n_c) + (1-q_c(n_c))\Lambda^d(n_c), \tag{8}$$

where $\Lambda^c(n_c) = n_d\lambda(n_d) + (n_c+1)\lambda(n_c+1) - d(n_d) + \delta_G \frac{n_c+1}{N} g(n_c) + \delta_G \frac{n_d}{N} g(n_c+1)$ and $\Lambda^d(n_d) = (n_d+1)\lambda(n_d+1) + n_c\lambda(n_c) - d(n_d+1) + \delta_G \frac{n_c}{N} g(n_c-1) + \delta_G \frac{n_d+1}{N} g(n_c)$. Thus, the functions $\Lambda^k(n_k)$ are the social continuation values arising if the consumer chooses network k. They are made up of the current network benefits subtracted the environmental costs, and added the future values of $g(\cdot)$. These are independent of the current value of x.

What is the first-best allocation? The markovian allocation problem to solve, is which network the newborn consumer should join, given the currently observed market shares. Since consumers are born with stochastic taste parameters, this amounts to choosing the state-dependent cut-off value x for the taste parameter, which divides the pool of potential newborn consumers into clean adopters and dirty adopters. Some consumers might be born with strong preferences in favor of the clean technology, while others will favor the dirty technology. The first best allocation trades off these idiosyncratic preferences against the environmental damage and the network effects - current and future.

The first best is thus the policy rule $x^*(n_c)$ defined as

$$x^*(n_c) = \underset{x}{\operatorname{argmax}} \; g(n_c).$$

[4]The expression is derived in the Appendix. Note that the parameter μ is the expected value of the individual shocks ζ_k, σ^2 is the variance of the distribution of ξ_c.

To solve it, we differentiate wrt. x, and get

$$0 = \sigma^2 \phi'(x) - \phi(x)\Lambda^c + \phi(x)\Lambda^d$$
$$0 = -x\phi(x) - \phi(x)\Lambda^c + \phi(x)\Lambda^d$$
$$x^*(n_c) = \Lambda^d - \Lambda^c, \tag{9}$$

and the second-order condition is satisfied.

To gain more intuition for $x^*(n_c)$ we can rearrange (9). A consumer will only choose the dirty good if he has sufficiently strong preferences for it e.g. if $\xi_c = \zeta_c - \zeta_d < x(n_c)$. Now the tax restricts the value of $x(n_c)$ such that a consumer would only choose the dirty good if

$$\zeta_c - \zeta_d < x^*(n_c) = \overbrace{(n_c+1)\lambda(n_c+1) - n_c\lambda(n_c) + n_d\lambda(n_d) - (n_d+1)\lambda(n_d+1)}^{\text{Loss in consumer network benefits}} + \overbrace{d(n_d+1) - d(n_d)}^{\text{Increase in environmental damages}} + \overbrace{\delta_G\left(\frac{n_d}{N}g(n_c+1) + \frac{n_c-n_d}{N}g(n_c) - \frac{n_c}{N}g(n_c-1)\right)}^{\text{Change in continuation value}}.$$

That is, only if the personal gain to the consumer of choosing the dirty good over the clean good exceeds the costs to society of that same choice, should he be induced to make the choice. The costs to society include the shift in aggregate network benefits today, the increased emissions today and the change in the continuation value. Through the last term, the tax also takes into account that emissions will be higher in the future, not just because the consumer who chose dirty now will pollute in the future, but because he chose dirty today, he made the dirty good more attractive to future consumers too.

Note that this first-best allocation is independent of the underlying market structure. We now proceed to derive the MPE, starting out with the optimal pricing of firms.

3.2 Pricing monopolistic case

To derive the optimal price setting of the firms as a function of the given taxes when they act as monopolists, we maximize the firm value functions (6) with respect to $p_k(n_k)$. In solving this, the firms take the sequence of taxes, $\tau(n_d)$ and $t(n_d)$, as given, and the demand functions they face are the state- and price-dependent choice probabilities $q_k(n_k)$ of consumers. We can think of the firms deriving price rules for every state n_k. The first-order conditions with respect to price are:

$$q_k(n_k) + \frac{\partial q_k(n_k)}{\partial p_k(n_k)} p_k(n_k)$$
$$+ \frac{\partial q_k(n_k)}{\partial p_k(n_k)} \delta \left[\frac{n_{-k}}{N} v_k(n_k+1) + \frac{n_k - n_{-k}}{N} v_k(n_k) - \frac{n_k}{N} v_k(n_k - 1) \right] = 0,$$

which can be rewritten to

$$\frac{p_k(n_k;t,\tau) + w_k(n_k;t,\tau)}{p_k(n_k;t,\tau)} = \frac{q_k(n_k)}{-q'_k(n_k)p_k(n_k;t,\tau)} = \frac{1}{\epsilon_k}. \quad (10)$$

where $w_k(n_k;t,\tau) \equiv \delta \left[\frac{n_{-k}}{N} v_k(n_k+1) + \frac{n_k - n_{-k}}{N} v_k(n_k) - \frac{n_k}{N} v_k(n_k-1) \right]$. Equations (10) gives us $2*N$ equations which can be used to solve for the pricing rules of the two firms e.g. $p_c(0), p_c(1)...p_c(N-1)$ and $p_d(0), p_d(1)...p_d(N-1)$.

The function $w_k(n_k;t,\tau)$ has in the literature been dubbed the *discounted prize* of winning a sale, and it is the present value of the future excess revenue that stems from the current sale. By rewriting the expression, we see that it is positive if $v_k(n_k+1) > v_k(n_k) > v_k(n_k-1)$. Clearly, a technology sponsor may benefit from a larger network, since *ceteris paribus* the larger the network, the larger is the probability that the new consumer will choose the sponsor's network.

As Cabral pointed out, for a text book monopoly we would have $(P - MC)/P = 1/\epsilon$ where P is the price and MC is the marginal cost of the monopoly. Thus, in (10) the function $w_k(n_k;t,\tau)$ plays the role of a negative marginal cost in the firm's pricing problem. In other words, instead of experiencing a marginal cost associated with a sale, the firm experiences a more or less favorable distribution over the continuation values $v_k(\cdot)$. Hence, a technology sponsor could be willing to forgo profits today be setting a lower price, to the extent that $w_k(n_k;t,\tau) > 0$.

3.3 Pricing competitive case

When more firms are selling a durable good of the same type, we assume that these durables are perfect substitutes. Moreover, we assume that firms are symmetric with zero marginal cost. Hence price equal to zero in every period is clearly a symmetric equilibrium. With respect to sales in the current period, each firm has an incentive to undercut each other as long as prices are positive. Since this incentive is present in all future periods as well, no firm has an incentive to set a negative price, since it cannot reap the benefits of a larger network later on. This gives us the following pricing function for a competitive network:

$$p_k(n_k) = 0, \ \forall \ n_k. \quad (11)$$

As mentioned we study two competitive cases: Either $p_k(n_k) = 0$ for both clean and dirty, or only for the dirty network. Note that in the cases in which only the clean technology has a sponsor, the pricing rule for the clean technology (10) must still hold. The only difference is that we must insert $p_d = 0$ into the probability function $q_c(p_c)$.

Finally, when prices are zero in all periods, the continuation values $v_k(n_k)$ must be zero for all n_k.

3.4 Setting the optimal taxes

The problem now facing the government is a dynamic programming problem subject to two functional constraints on the prices. We write the dynamic programming problem of the goverment as follows:

$$g(n_c) = \max_{t(n_d),\tau(n_d+1)} \left\{ \mu + \sigma^2\phi(x) + q_c(n_c)\Lambda^c(n_c) + (1-q_c(n_c))\Lambda^d(n_c) \right\}, \tag{12}$$

$$\text{s.t. } p_k = f_k(n_k; t, \tau),\ \ k = c, d.$$

Through the constraint $p_k = f_k(n_k, t, \tau),\ k = c, d$, the government takes into account the optimal price responses of the firms (equation (10) or (11)). Note, however, that neither the prices nor the taxes enter the value function directly. They only enter indirectly through $x(n_c)$.

For a formal solution to this problem, see the appendix. Here we note that both the entry tax $t(n_d)$ and the use tax $\tau(n_d)$ affects $x(n_c)$. They do not work in the same manner, but they achieve the same goal, and implement $x^*(n_c)$ in the exact same way; by making one network or the other more or less attractive *ex ante*. The equivalence arises because there are no distortions from taxes: we force the market to be covered, and the use of the goods once they are acquired is perfectly inelastic. The result is therefore that it does not matter whether the government taxes entry to or participation in the dirty network. In the following we will focus on the entry tax, since in our model both externalities are directly linked to the choice of technology.[5]

Proposition 1 *The government can implement the first-best allocation.*

Since the prices and taxes only enter the welfare expression through the indifference parameter x, we find a first-order condition for the purchase tax, t, of the following form:

$$0 = \frac{dx}{dt} \cdot \phi(x) \cdot \left[\Lambda^d(n_c) - \Lambda^c(n_c) - x(n_c)\right]. \tag{13}$$

We can see that (9) is a solution to this. This means that even in this dynamic taxation game, when we have restricted the government to choose from only time consistent policies, the first best can be implemented. Since taxes are non-distortionary, the government can therefore credibly promise to set whatever future taxes are needed to implement the first best. If high taxes

[5]If the use intensity was not fixed, the use tax would introduce a distortion different from the entry tax. The government would then likely use both.

were very costly (for instance if consumers then would choose some outside good and disappear from the market in question), one could imagine that the government's threat to set high taxes in the future was not credible, and that it therefore might struggle to implement the first best today. This does not happen here.

For the same reason, it is easy to see that (9) will solve (13) for any configuration of sponsors. This leads to the following proposition:

Proposition 2 *The implemented real allocation between the clean and dirty network does not depend on the configuration of sponsors.*

The expression for the optimal tax rate $t(\cdot)$ can, unfortunately, not be derived in closed form. The tax must be set such that the post-tax price competition implements the above value of $x(\cdot)$, but that depends on solving for the fixed point in (9), on the form $x(n_c) = G(x(n_c))$. The function $G(\cdot)$ involves the cumulative distribution function of the taste parameters, and the implicitly defined value function. This fixed point has to be found numerically. We are however interested in to what extent the optimal tax departs from the Pigovian tax. In the Appendix we solve for the optimal entry tax without network effects and without market power, which we will denote t^{pig} and refer to as the Pigovian entry tax. The Pigovian entry tax is given by:

$$t^{pig} = \frac{d}{1-\delta\frac{N-1}{N}} \; \forall n_c, \tag{14}$$

where the right-hand side is the present value of the expected environmental damages of joining the dirty network. That is, each consumer pollutes to a marginal damage of d every period, and the discount rate is augmented to take into account that the consumer will die with probability $1/N$ each period. This expression is a constant, and hence the Pigovian entry tax is a constant. By expanding the expression for x from (1), we can write the optimal entry tax in the following manner:

$$t(n_d) = \Lambda^c(N-1-n_d) - \Lambda^d(n_d) + p_c(N-1-n_d) - p_d(n_d) - u_c(N-n_d) + u_d(n_d) \tag{15}$$

This can not be constant when there are network effects. Even in the competitive case in which p_c and p_d are equal to zero, the tax will have a different numerival value for every n_d. Thus, we have:

Proposition 3 *For all configurations of sponsors, the optimal entry tax departs from the Pigovian entry tax.*

And accordingly:

Corollary 4 *The optimal tax rule will depend on the configuration of sponsors.*

Since the prices p_c and p_d enter directly in (15), this must be the case. In other words, the government implements the same $x(n_c)$ for all configurations, but when setting the tax it takes into account the price responses by the firms marketing the two technologies.

In order to say more about how the tax will depart from the Pigovian tax, we need to be more explicit about the network effects. Thus, in the next section we simulate a numerical version of the model.

4 Numerical simulations

We have chosen to concentrate on the competive case and the clean technology sponsor case. The results with two technology sponsors are very similar to the results with only a clean technology sponsor.

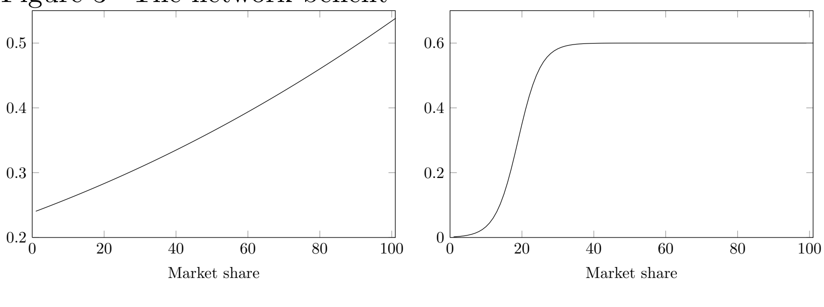
The model is not complicated to solve numerically. Given the solution to (13), we have a fixed point problem for all n_c, and we can simply iterate to fix the optimal $x(n_c)$. When the optimal $x(n_c)$ is determined, we have the implemented transition probabilities, and then the consumer values $u_k(n_k)$ follow. The prices are determined as the solution to the first-order conditions that implements the pre-determined $x(n_c)$, and then the firm values follow.

For the simulations, we represent the network effects $\lambda(n_k)$ by a logistic curve:

$$\lambda(n_k) = \frac{\psi}{1 + e^{\alpha - \beta \frac{n_k}{N}}} \tag{16}$$

where ψ, α and β are parameters. We study two parametrized examples of (16):

Figure 3 “The network benefit”



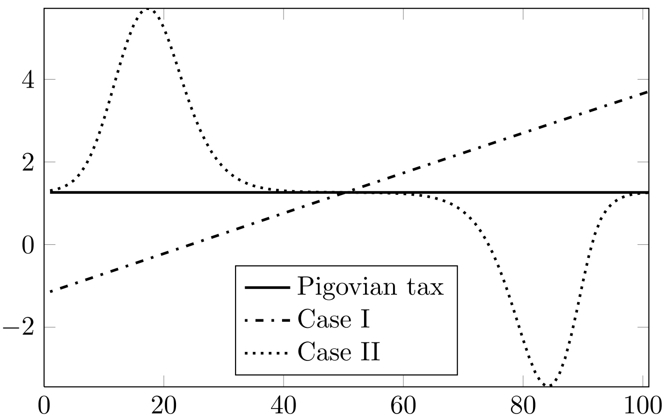
The market share of the network in question is shown on the x-axis, while the y-axis shows the value of the network benefit to each consumer. In the first case we have $\psi = 2$, $\alpha = 2$ and $\beta = 1$. This is almost linear, and intuitively we would expect full coordination to be efficient.

In the second case we have $\psi = 0.6$, $\alpha = 6$ and $\beta = 32$. The network benefit is then as important as in the first case, but it levels off at about $n_k \approx 30$. This is consistent with the explicit network model in Greaker and Heggedal (2010). With fixed costs for establishing the complimentary service, it seems reasonable that the technology must pass a threshold before this service takes off. Furthermore, like in the model of Greaker and Heggedal, there may be diminishing returns to a larger network after the service has taken off. In this example, we may keep both technologies in the market without incurring a loss of consumer utility. We will coin the situation in the left part of Figure 3 Case I, and the situation in the right part of Figure 3 Case II.

4.1 Optimal policy with no technology sponsors

We compare the optimal entry tax for the two cases with the Pigovian entry tax, i.e. $t^{pig} = \frac{d}{1-\delta\frac{N-1}{N}}$. We have used $\delta = 0.85$, and $d = 0.2$. Simulating the model we get the following optimal entry tax rules:

Figure 4 "Optimal entry tax rule with no sponsors"



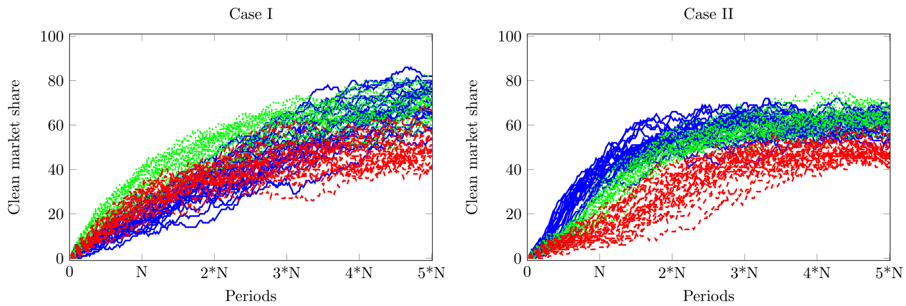
In Case I there is a big loss in network benefits to all existing dirty consumers if a newborn consumer chooses clean and the market is dominated by the dirty technology. This can be seen directly from the left part of Figure 3. For high market shares of the dirty good, this loss tends to outweight the potential reduced environmental damage from a new clean consumer instead of a new dirty consumer. The government wants the new born consumers to internalize this cost, and hence, sets a negative entry tax for the dirty good. However, when the market share of the clean technology is about 50%, there is no longer a loss in network benefits, and the government goes for the green network. Consequently, the government sets an entry tax in excess of the Pigovian rate.

In Case II there is no loss in network benefits to the existing dirty consumers if a newborn consumer chooses clean and the market is dominated by the dirty technology. Instead if there already are a few existing clean consumers, the gain in network benefits to them is high. Consequently, the government sets an entry tax in excess of the Pigovian rate already from the introduction of the clean technology. On the other hand, when the clean technology has reached a high market share, additional clean consumers start to impose losses on the existing dirty consumers. This is reflected in the negative tax rates at the right end of Figure 4.

Looking at the industry dynamics also gives some insight into the optimal tax rules. In the competitive situation, without any intervention by the government, the market will for both Case 1 and 2 over time move towards a $50-50$ split independent of the intial situation. The reason is that from time to time consumers with a high preference for one of the technologies is born, and will choose that technology independent of the network sizes.

We start in a situation in which the market is dominated by the dirty technology. The red lines are for the situation without any tax. The green lines are the situation with the Pigovian entry tax, and finally, the blue lines are the development with the optimal entry tax:

Figure 5 “Industry dynamics competitive situation”



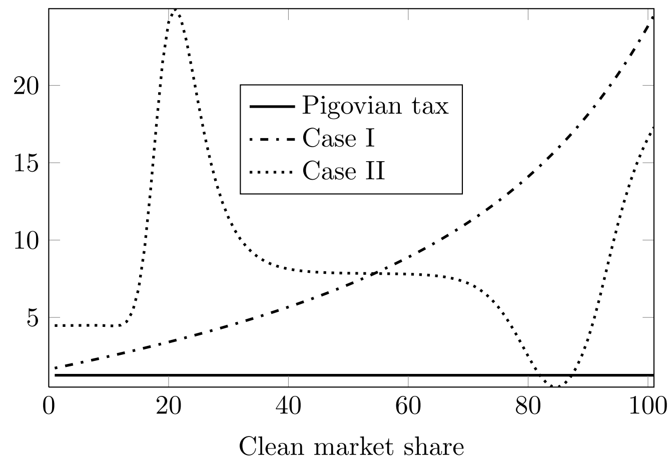
For Case I we note that the effect of the optimal policy is two-fold: Intially it slows the transition to the clean technology, but in the end it ensures a higher market share for the clean technology than without the optimal policy. Also, for the time frame we are looking at (five complete product cycles), the Pigovian tax is insufficient to ensure a market dominance for the clean technology.

In Case II the policy has the oppsite effect: It speeds up the market penetration of the clean technology. On the other hand, the optimal policy also tries to ensure that the dirty technology is left with a higher mass of consumers than in Case I.

4.2 Optimal policy with a clean technology sponsor

Figure 6 plots the different taxes for the clean sponsor simulations. Recall that the government implements the same probabillity of choosing the clean good for all configurations of sponsors. Hence, any difference in the optimal entry tax rates between Figure 4 and 6 is due to the pricing of the clean technology supplier. The price of the clean technology supplier must satisfy equation (10). Since we start off with only the dirty technology in place, the clean sponsor could use its price to increase its network. From the optimal entry tax rule, we see that this to some extent happens for low market shares:

Figure 6 “Optimal entry tax rule with a clean technology sponsors”



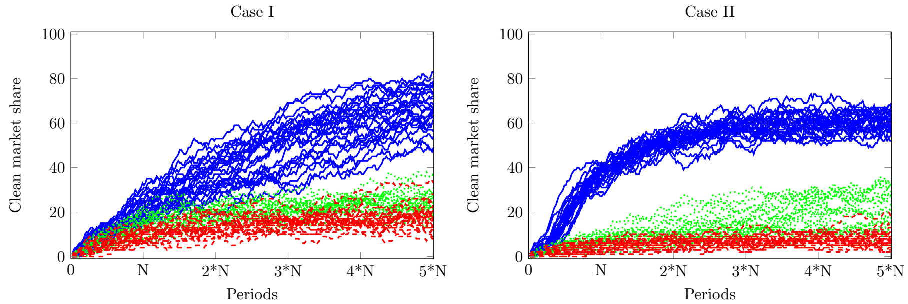
First, note that the variation in the tax is much higher compared to the competitive situation. This is a result of the price responses by the clean technology sponsor; as the government increases its tax, the clean technology sponsor responds by increasing her price, then the tax has to increase again and so on.

In Case I the government no longer subsidizes the dirty technology. This is not necessary since the clean technology sponsor sets a positive price and the price on the dirty technology is zero. Moreover, as the clean market share increases, the tax on the dirty has to increase. The reason is that the clean sponsor starts harvesting the benefits of his installed base. Since he has a large market share, his network is attractive, and he increases the price beyond what the government finds optimal. As a result, the government has to increase its tax manyfold compared to the competitive situation.

In Case II the optimal entry tax rule also has a similar shape as in the competitive situation. Note however, the low entry tax when the market share is about 15. The clean technology sponsor keeps the price low since the market for the clean technology is about to take off. The government can then keep the low entry tax for the dirty good somewhat longer. On the other hand, once the clean technology has taken off, the clean technoly sponsor starts to harvest the market. Consequently the entry tax on the dirty good has to be very high when the market share of the clean good is between 20 and 70.

In neither of the two cases the clean technology sponsor and the government agree with respect to the optimal diffusion of the clean technology as can be seen from the next figure.

Figure 7 “Industry dynamics, clean technology sponsor only”



As we can see from the green and red lines above, the clean technology sponsor tends to settle with a far too low market share in both Case I and II, and only under the optimal entry tax rule does the clean technology properly penetrate the market.[6] Case II is the most striking case. Without the optimal entry tax, the market for the clean technology may not develop. We can see from the figure that the market share stays low for up to $5 * N$ periods even if the government has introduced a Pigovian tax. Note that in our model, the same result could be obtained with a high entry subsidy for the clean network.

5 Conclusion

As far as we know this is the first paper that treats environmental policy when there are network externalities in a dynamic model with optimizing firms and consumers. We have found that governments should intervene with a tax that no longer equals the social cost of emissions, independent of whether the green technology has a sponsor or not. The optimal tax takes into account the network effect, the mark-up pricing of the potential technology sponsors and the environmental externality. Still, the optimal tax depends on the nature of the network effects and the amount of damages from emissions, so we cannot advise governments to always support the clean network from the start. This generalizes, and moderates, the results of Sartzetakis and Tsigaris (2005).

In our opinion the most striking finding is that non-intervention may lead to carbon lock-in as hypothesized by a number of earlier papers mentioned above. In our simulations, a Pigovian tax might not be enough to escape from the dirty technology. In the clean technology sponsor case this result is most pronounced: Instead of pricing low in order to obtain a high market share, the clean technology sponsor earns higher profit on the customers willing to pay a high price for it’s product. Since the social value of clean good adoption exceeds the private value to the clean sponsor, he settles for a suboptimally low market share, from a social point of view. However, it may also happen to some extent with competive suppliers.

Is it likely that the government sets taxes and the firms set prices each time a consumer arrives? No, but the interpretation is that both the government

[6]This is also the case with two sponsors, but not so pronounced. Figures can be obtained from the authors upon request.

and the firms set a rule. Such rule setting is for instance the case in Norway with respect to electric cars. The government has put in place various kinds of subsidies, and have stated that these subsidies will gradually disappear as the market share of electric cars pick up.

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6 The conditional expected private utility

We want an expression for $\mathbb{E}(X|X-Z>a)$, where $X, Z : i.i.d. \sim N(\mu_X, \sigma_X)$. The distribution of $(X|X-Z>a)$ is called *skew normal* The expectation is derived in Birnbaum (1950). Relabel $Y \equiv X - Z$, and we have

$$\mathbb{E}(X|Y>a) = \mu_X + \rho_{X,Y}\sigma_X \frac{f_Y(\frac{a-\mu_Y}{\sigma_Y})}{1 - F_Y(\frac{a-\mu_Y}{\sigma_Y})}$$

where $f_Y(\cdot)$ is standard normal. Then we replace the standard normal with the $\phi(\cdot)$-distribution $Y \sim N(0, \sigma_Y)$, which gives us

$$\mathbb{E}(X|Y>a) = \mu_X + \rho_{X,Y}\sigma_X\sigma_Y \frac{\phi(a)}{1-\Phi(a)}$$

Now

$$\rho_{X,Y} = \frac{cov(X,Y)}{\sigma_X\sigma_Y}$$

which means that $\rho_{X,Y} \cdot \sigma_X \cdot \sigma_Y = cov(X,Y)$, and we have that $cov(X,Y) = cov(X, X-Z) = var(X) - cov(X,Z) = var(X) = \sigma_X^2$. Thus we get

$$\mathbb{E}(X|Y>a) = \mu_X + \sigma_X^2 \frac{\phi(a)}{1-\Phi(a)}$$

Our in our notation:

$$\mathbb{E}(\zeta_c|\xi_c > x(n_c)) = \mu_\zeta + \sigma_\zeta^2 \frac{\phi(x(n_c))}{1-\Phi(x(n_c))}$$

similarily

$$\mathbb{E}(\zeta_d|\xi_c < x(n_c)) = \mu_\zeta + \sigma_\zeta^2 \frac{\phi(x(n_c))}{\Phi(x(n_c))}$$

In our government value function, we want so sum

$$\begin{aligned} q_c(n_c)\mathbb{E}(\zeta_c|\xi_c > x(n_c)) &+ [1-q_c(n_c)]\,\mathbb{E}(\zeta_d|\xi_c < x(n_c)) \\ &= [1-\Phi(x(n_c))]\left[\mu_\zeta + \sigma_\zeta^2 \frac{\phi(x(n_c))}{1-\Phi(x(n_c))}\right] + \Phi(x(n_c))\left[\mu_\zeta + \sigma_\zeta^2 \frac{\phi(x(n_c))}{\Phi(x(n_c))}\right] \\ &= \mu_\zeta + 2\sigma_\zeta^2\phi(x(n_c)) \\ &= \mu_\zeta + \sigma_{\xi_c}^2\phi(x(n_c)) \end{aligned}$$

which is what we use in ().

7 Deriving the first-order conditions

The government's dynamic programming problem gives the following first-order conditions:

$$\text{FOC } t(n_d)\text{: } 0 = \sigma^2_{\xi_c}\phi'(x(n_c))\frac{dx(n_c)}{dt(n_d)} + \phi(x(n_c))\frac{dx(n_c)}{dt(n_d)}\left[\Lambda^d(n_d) - \Lambda^c(n_c)\right]$$

$$\text{FOC } \tau(n_d+1)\text{: } 0 = \sigma^2_{\xi_c}\phi'(x(n_c))\frac{dx(n_c)}{d\tau(n_d+1)} + \phi(x(n_c))\frac{dx(n_c)}{d\tau(n_d+1)}\left[\Lambda^d(n_d) - \Lambda^c(n_c)\right]$$

If we restrict our attention to the entry tax $t(n_d)$, we get

$$\left[\sigma^2_{\xi_c}\phi'(x(n_c)) + \phi(x(n_c))(\Lambda^d(n_d) - \Lambda^c(n_c))\right] \cdot \frac{dx(n_c)}{dt(n_d)} = 0 \tag{17}$$

and when $\phi(\cdot)$ is the normal density, we have that

$$\phi'(x(n_c)) = -\frac{x(n_c)}{\sigma^2}\phi(x(n_c)) \tag{18}$$

Rewriting (17) we obtain

$$\left[(\Lambda^d(n_d) - \Lambda^c(n_c)) - x(n_c)\right] \cdot \frac{dx(n_c)}{dt(n_d)} \cdot \phi(x(n_c)) = 0$$

So clearly, the first best ($x(n_c) = \Lambda^d(n_d) - \Lambda^c(n_c)$) is also an optimal policy for the government.

To demonstrate that it is the only solution to the problem, we must show that $\frac{dx(n_c)}{dt(n_d)} \neq 0$. We have three cases to check: both networks competitive, both sponsored, and only one network sponsored. In general, we have that

$$\frac{dx(n_c)}{dt(n_d)} = \frac{\partial p_c(n_c)}{\partial t(n_d)} - \frac{\partial p_d(n_d)}{\partial t(n_d)} - 1. \tag{19}$$

$$\tag{20}$$

We first look at the case with two sponsors. Inserting for the optimal price (response function) of the firms (10), we have that

$$\begin{aligned}\frac{dx(n_c)}{dt(n_d)} &= 1 - \frac{q_c(n_c)}{-q_c'(n_c)}\frac{x(n_c)}{\sigma^2} - \left(-1 - \frac{q_d(n_d)}{-q_d'(n_d)}\frac{x(n_c)}{\sigma^2}\right) - 1 \\ &= 1 - \frac{x(n_c)}{\sigma^2}\left(\frac{q_c(n_c)}{-q_c'(n_c)} - \frac{q_d(n_d)}{-q_d'(n_d)}\right) = 1 - \frac{x(n_c)}{\sigma^2}\frac{1 - 2\Phi(x(n_c))}{\phi(x(n_c))}.\end{aligned}$$

Assume $\frac{dx(n_c)}{dt(n_d)} = 0$. This implies:

$$x(n_c) = \sigma^2\frac{\phi(x(n_c))}{1 - 2\Phi(x(n_c))} \tag{21}$$

If we take the derivative of the right hand side (RHS), we get

$$\frac{\partial \text{RHS}}{\partial x} = \frac{\phi'(x)\left[1-2\Phi(x)\right]+2\phi(x)^2}{\left[1-2\Phi(x)\right]^2}\begin{cases} >0 & \text{if } x<0 \\ \text{not defined} & \text{if } x=0 \\ >0 & \text{if } x>0 \end{cases}$$

and as we have that RHS$(-\infty)=0$, RHS$(x\nearrow 0)=+\infty$, while RHS$(x\swarrow 0)=-\infty$ and RHS$(+\infty)=0$, we can see that (21) can never be satisfied.

In the competitive case we have $\frac{\partial p_c(n_c)}{\partial t(n_d)}=\frac{\partial p_d(n_d)}{\partial t(n_d)}=0$. Hence, $\frac{dx(n_c)}{dt(n_d)}=-1$.

In the case with only a clean sponsor, we have

$$\frac{dx(n_c)}{dt(n_d)}=\frac{\partial p_c(n_c)}{\partial t(n_d)}-1=\frac{1-\Phi\left[x(n_c)\right]}{-\phi(n_c)}\frac{x(n_c)}{\sigma^2}$$

which is zero only if $x(n_c)=0$. This implies that the arriving consumer will choose either of the networks with equal probability for any network size and for any entry price the clean producer might set. This solution cannot be an optimum given that the dirty network pollutes. Hence, we conclude that the first-order condition requires:

$$x(n_c)=\Lambda^d(n_d)-\Lambda^c(n_c).$$

We can now turn to the second-order condition. We differentiate (17) to get

$$\begin{aligned}
&\sigma^2\phi''(x)\left(\frac{dx}{dt}\right)^2+\sigma^2\phi'(x)\frac{d^2x}{dt^2}\\
&+\left(\Lambda^d(n_d)-\Lambda^c(n_c)\right)\cdot\left[\phi'(x)\left(\frac{dx}{dt}\right)^2+\phi(x)\frac{d^2x}{dt^2}\right]\\
&=\sigma^2\left(\frac{dx}{dt}\right)^2\left(\frac{-\phi(x)}{\sigma^2}-\frac{x}{\sigma^2}\phi'(x)\right)-x\phi(x)\frac{d^2x}{dt^2}\\
&+\left(\Lambda^d(n_d)-\Lambda^c(n_c)\right)\cdot\left[-\frac{x}{\sigma^2}\phi(x)\left(\frac{dx}{dt}\right)^2+\phi(x)\frac{d^2x}{dt^2}\right]\\
&=\left(\frac{dx}{dt}\right)\left(\frac{-x}{\sigma^2}\right)\overbrace{\left[\sigma^2\phi'(x)\frac{dx}{dt}+\left(\Lambda^d(n_d)-\Lambda^c(n_c)\right)\phi(x)\frac{dx}{dt}\right]}^{=0}\\
&-\phi(x)\left(\frac{dx}{dt}\right)^2+\phi(x)\frac{d^2x}{dt^2}\left[\underbrace{\Lambda^d(n_d)-\Lambda^c(n_c)-x}\right]_{=0}\\
&=-\phi(x)\left(\frac{dx}{dt}\right)^2<0
\end{aligned}$$

We have a globally defined function, everywhere differentiable in x with only one stationary point, and this point is a local max. Hence it is also a global max.

Note, however, that this does not allow us to claim a unique equilibrium.

8 The Pigovian entry tax

In the numerical simulations, we use the Pigovian entry tax rate as a benchmark. This rate can be found by looking at the outcome of the model when all network effects are absent. The choice probabilities of the consumers will then only depend on the current prices and taxes, and not on the future expected sizes of the networks. The firm continuation values will therefore also be independent of the network sizes e.g. firms cannot increase the probability of a future sale by increasing their current network.

Assume there exists an equilibrium in constant prices and taxes. Then, the firms' value functions will be constant across states, and equal to $v_k(n_k) = \frac{1}{1-\delta} q_k p_k$. The optimal prices are then given by:

$$p_c(t,\tau) = \frac{1-\Phi(x_c)}{\phi(x_c)},\ p_d(t,\tau) = \frac{\Phi(x_c)}{\phi(x_c)} \tag{22}$$

where

$$x_c = p_c - p_d - t - \tau(1-\delta\frac{N-1}{N})^{-1} \tag{23}$$

We note from (22) and (23) that if the two taxes are kept constant, prices must also be kept constant partly confirming that we are on the right track. When everything is constant and the environmental damage is linear, we can 'guess and verify' a linear government value function:

$$g(n_c) = \frac{1}{1-\delta_G}\left[\sigma^2\phi(x) - N\cdot d + q_c(x)\frac{d}{1-\delta_G\frac{N-1}{N}}\right] + \frac{d}{1-\delta_G\frac{N-1}{N}} n_c.$$

Solving for the optimal $x(n_c)$, we find that the government implements the following:

$$\bar{x}(n_c) = \frac{-d}{1-\delta_G\frac{N-1}{N}},\ \forall n_c$$

where the right-hand side is the present value of expected environmental damages of joining the dirty network. Each consumer pollutes to a marginal damage of d every period, and the discount rate is augmented to take into account that the consumer will die with probability $1/N$ each period. This expression is a constant, confirming that we have an equilibrium of the model.

From (23) we note that the two tax instruments are "perfect substitutes". Hence, the government needs only one of the taxes. To find the entry tax, we use that firms set prices according to (22), and we can calculate the entry tax to be:

$$\bar{t} = -\bar{x} + p_c - p_d = \frac{d}{1-\delta_G\frac{N-1}{N}} + \frac{1-2\Phi(\bar{x})}{\phi(\bar{x})}.$$

Since the firms have market power, their prices differ from the marginal costs, and so the tax differs from the environmental damage by the constant term $\frac{1-2\Phi(\bar{x})}{\phi(\bar{x})}$. If prices were set to marginal costs, then this discrepancy would disappear, and the tax would equal the damages.

In the simulations we use $\bar{t} = \frac{d}{1-\delta_G \frac{N-1}{N}}$ for all cases. Hence, our Pigovian tax rate does not adjust for market power.