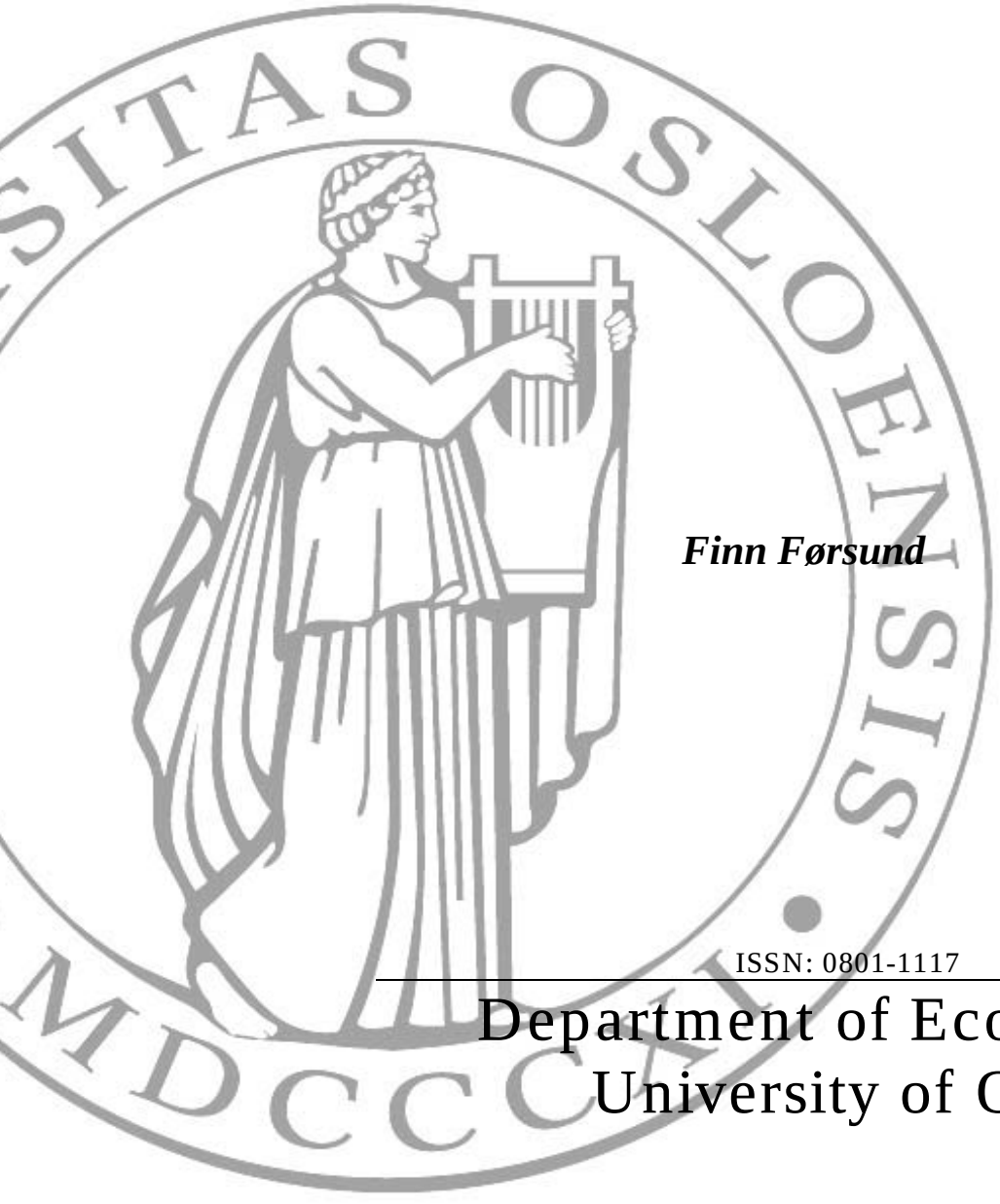
MEMORANDUM

No 37/99

Modelling Transboundary Air Pollution: The Rains Model Approach



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ISSN: 0801-1117

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This series is published by the
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MODELLING TRANSBOUNDARY AIR POLLUTION: THE RAINS MODEL APPROACH[1]*

by

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1. Background

The **R**egional **A**cidification **IN**formation and **S**imulation (RAINS) model is the best example so far of an environmental economics model actually being used in "the real world" to help countries design policies of reducing emissions of pollutants. RAINS was used when negotiating the second sulphur protocol, the Oslo Protocol, signed in Oslo in 1994, and is currently being used during negotiations to reduce emissions of sulphur, nitrogen, ammonia and volatile organic substances (VOC) for the target year of 2010. This "integrated assessment model" was developed at the **I**nternational **I**nstitute for **A**pplied **S**ystems **A**nalysis (IIASA), Laxenburg, Austria during an initial project period 1983 - 1988 (see Alcamo et al., 1990). The model basically integrates an atmospheric transportation model and data on purification costs on a country level. Work on improving and extending the model is still going on. The latest element to be included is the formation of ground-level ozone (see Amann et al., 1998).

The economic cost effectiveness principle behind the model scenarios used for negotiating the Oslo Protocol, together with the critical loads concept of natural science for environmental evaluation, have lead for the first time to a significant non-uniform distribution of reductions of a pollutant within an international agreement. This is in contrast to the 30 per cent uniform reduction agreement of the first sulphur protocol.

The empirical work on the European atmospheric transportation model started as an OECD project in 1972, the same year as the first UN conference on the human environment took

[1]I am indebted to Ove Wolfgang and Olav Bjerkholt for perceptive comments.

∗ This is an updated and extended version of March 2004 of Memorandum 37/99. I am indebted to Miss Qin Yan for excellent assistance in the updating.

place in Stockholm. This programme, the Co-operative Programme to Measure the Long-Range Transport of Air Pollution[2], was later taken over by the UN Economic Commission for Europe in 1979 under the Convention on Long-Range Transboundary Air Pollution (LRTAP) (signed by 35 countries, including Canada and the United States). An institutional structure was provided for the new programme, renamed the Co-operative Programme for the Monitoring and Evaluation of the Long-Range Transmission of Air Pollutants in Europe (EMEP). The Norwegian Meteorological Institute plays an important role here.

The purification cost functions have been established through the IIASA project with inputs from participating countries. It should be remembered that prior to the protocol negotiations the information available on costs and environmental effects at country levels was scarce. It is probably only due to the participation in convention related work over several years that the countries now themselves have a comprehensive picture of national cost functions, and that critical loads for ecosystems have been collected. Within a country the environmental authority is usually concerned with setting emission standards for firms on an individual basis without having a comprehensive national plan with respect to cost efficiency. Indeed, it is the participation in conventions that has brought national plans up as necessary policies.

The purpose of the paper is to relate the RAINS model to the standard micro model of environmental economics, and to study the structure of the model, and provide interpretations both of primal and dual solutions of the (stylised) model.

2. The micro model

The generic micro environmental economics model, with plants as units, has only two types of relations: the first type of relation connects economic activity (production and/or consumption) including its use of resources, and discharge of pollutants and the second type is an environmental service production function linking environmental services, M, and discharge of pollutants. One simple way of modelling the first type of relation (see Førsund (1998) for more general modelling) is to introduce a cost function in output, y, and emissions,

[2] 11 countries participated: Austria, Belgium, Denmark, Finland, France, the Federal Republic of Germany, the Netherlands, Norway, Sweden, Switzerland and the United Kingdom.

e, summarising all the information about transformation from inputs to outputs and purification options:

$$
\begin{aligned}
&c_i = c_i(y_i, e_i) \\
&M_{sj} = m_{sj}(e_1, ..., e_N), e_i = (e_{1i}, ..., e_{Ki}) \\
&\text{functional restrictions:} \\
&c_{iy}' > 0, c_{iyy}'' > 0, c_{ie_k}' < 0, c_{ie_k e_k}'' > 0, c_{iye_k} < 0 \qquad (1) \\
&m_{sjki}' \le 0, m_{sjki}' = 0 \text{ for } 0 \le e_{ki} \le e_{ki}^A, \\
&0 \le M_{sj} \le M_{sj}^{\max} \equiv m_{sj}(e_1, ..., e_N) \text{ for } e_{ki} \in \left[0, e_{ki}^A\right] \\
&j = 1, ..., R, s = 1, ..., S, k = 1, ..., K, i = 1, ..., N
\end{aligned}
$$

where we have specified $I = 1,..,N$ sources discharging $k = 1,..,K$ pollutants, e_{ik}, being transferred to $j = 1,..,R$ receptors in Nature, producing $s = 1,..,S$ services, M_{sj}.

The cost function is so general that we do not see explicitly purification options connected with inputs[3]. In order to make correct assumptions about partial derivatives and restrictions on the cost function (see Wolfgang, 1999), it is useful to make a distinction between abatement and purification:

Definition:
Abatement: reduction of pollutants discharged to the environment with endogenous output level.
Purification: reduction of pollutants discharged to the environment with output level fixed.

The functional forms of the environmental relations, $m_{sj}(.)$, include transportation, transfer or diffusion from source to receptor, including degradation and/or dilution on the way, and any synergy effect between substances. The general restriction on the partial derivatives is that they are non-positive, and zero for loads less than e_{ki}^A; the load limits for the *assimilative capacity*, or self-purification, of the environment. The service function is generally calibrated such that the environmental services attain their maximal values when pollutants are zero (or in general lower than the threshold values e_{ki}^A), as illustrated in Figure 1a. However, for some

[3] The natural assumption to make about the cost function that $c_e' < 0$ (see e.g. Kling and Rubin, 1997) is not so innocent. If we need the model for a comparison no regulation with regulation the assumption will create trouble defining a no control solution. Defining a maximal emission, or assuming the derivative to be zero for a finite value of e, or assuming that the sign of the derivative goes from negative to positive values can solve the problem.

environmental problems it may be relevant that the partial derivative of an emission of the service production function is positive. An example is formation of ground-level ozone. The substances nitrogen oxides and volatile organic compounds (VOC) together with sunshine produce ground-level ozone, but depending on the situation, decreased supply of e.g. nitrogen may increase the formation of ozone. To cover the ozone case we must also open up for the possibility that $m_{sjki}' > 0$, as shown in Figure 1b. The environmental service is decreasing in ozone concentration. Instead of the form in (1) of the environmental service function we have a two stage process with the substance giving environmental effects being produced by the emissions.

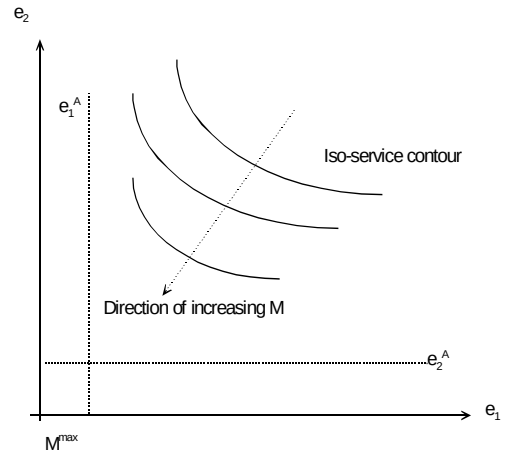

Figure1a. Iso -service contours of the environmental service function

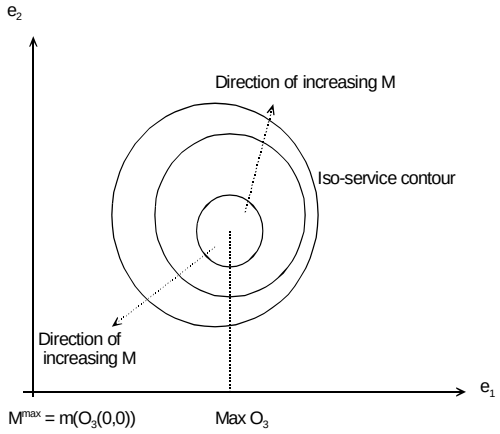

Figure 1b. Ozone hill

In order to write the environmental relation on a form that is more directly comparable to the RAINS model, we can lift the transportation in nature, degradation, dilution, etc. out of the functional form $m_{sj}(.)$, and take care of these natural processes by introducing explicitly transfer unit coefficients, a_{kij}:

$$m_{sj}(e_1,....,e_N) \equiv m_{sj}\left(\sum_{i=1}^{N} a_{1ij}e_{1i},....,\sum_{i=1}^{N} a_{kij}e_{ki},....,\sum_{i=1}^{N} a_{Kij}e_{Ki}\right),$$

$$\frac{\partial m_{sj}(e_1,....,e_N)}{\partial e_{ki}} = m_{sjk}'a_{kij}, m_{sjk}' \le 0, m_{sjk}' = 0 \text{ for } 0 \le e_{ki} \le e_{ki}^A \qquad (2)$$

$$j = 1,....,R, s = 1,....,S, k = 1,....,K, i = 1,....,N$$

where the transfer unit coefficient, a_{kij} , shows us the amount of a unit of pollutant k emitted by source i that reach receptor *j*.

3. The relations in the RAINS model

The environmental relation

Instead of naturally delimited areas of Nature as receptor, e.g. forest areas, lakes, air sheds, estuaries, etc. a grid is superimposed on the map of Europe. The sources, indexed $i = 1,..,N$ represent countries, and the receptors, indexed $j = 1,..,R$ are defines as a grid square of size 150x150 km (the type of pollutants index, k, is suppressed) illustrated in Figure 2. A country is not a point as indicated as source in Figure 2, but can be defined as a set of receptors. Receptors located in two or more countries are common to the countries in question. The country set of receptors is attributed the transfer coefficients a_{ij} as the source i. The RAINS model contains about 700 grid squares covering 38 sources, most of which are countries, but there are some sea regions (emissions from ships), and Russia is divided into several regions. It is assumed that the amount of a unit of emission from source i reaching receptor j , a_{ij}, is distributed uniformly over the square.

The unit transfer coefficients a_{ij} are calculated based on measurements at monitoring points and information on emission from actual stationary sources (e.g. fossil fuel electricity plants), and wind speed and precipitation. The coefficients are calculated as averages over a yearly cycle and over years, and assumed to be constants in the model calculations. The uncertainty due to meteorological variability is estimated to be in the range 9-15% (Alcamo et al., p. 127).

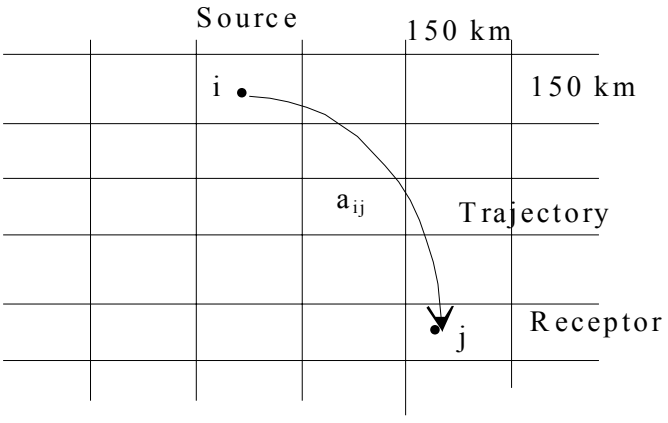

Figure 2. EMEP grids of Europe

The source - receptor (transport-, diffusion-, dispersion-) matrix covering N sources and R receptors is:

$$\begin{pmatrix} a_{11} & \dots & a_{1R} \\ .. & \dots & .. \\ a_{N1} & \dots & a_{NR} \end{pmatrix} \tag{3}$$

The distribution of emissions across a country is taken into consideration when calculating the transfer coefficients. When using the model the use of countries as a source implies that the relative distribution of emissions across a country is assumed constant.

For each EMEP grid square the following calculation of deposition, d_j, is done:

$$\sum_{i=1}^{N} a_{ij} e_i + b_j = d_j, j = 1, \dots, R \tag{4}$$

where b_j is the background deposition, i.e. depositions not accounted for by the N sources (e.g. emissions coming from the American continent, China, etc. and natural processes).

Comparing this relation with the environmental service production function in (1) we have that the connection between the physical registration of a deposition in the environment and the impact on environmental indicators (e.g. fish-and other biological populations) is missing. It is as if we measure the service M by $M_j = - d_j$.

The present version of the RAINS model is dealing with four substances; SO_2, NO_x, NH_3 and VOC. In addition the formation of ground-level ozone as a function of NO_x and VOC is included, but this is a non-linear process, and we will not include it in our discussion. The three environmental effects considered are acidity, eutrophication and ground-level ozone.

Acidity is created by depositions of SO_2, NO_x, NH_3 weighted together in a linear expression:

$$ss_j \left[\sum_{i=1}^{N} a_{sij} e_{si} + b_{sj} \right] + sn_j \left[\sum_{i=1}^{N} a_{nij} e_{ni} + b_{nj} + \sum_{i=1}^{N} a_{nhij} e_{nhi} + b_{nhj} \right] = d_j^A, j = 1, \dots, R \tag{5}$$

where the coefficients ss_j and sn_j are scaling sulphur and nitrogen into acidity units. Nitrogen depositions in the form of NO_x and NH_3 (sub index n and nh respectively) are directly added, and sulphur and nitrogen depositions are then weighted together with the scaling coefficients

ss_j and sn_j. The eutrophication load is expressed as a summation of the depositions of NO_x and NH_3. A superscript, *A*, for acidity is indicated on the deposition variable. In the sequel we will for ease of exposition work with the simple formulation (4).

Elaborations on the general cost function

In order to see the connection between the RAINS cost function and the general one, *c*(*y, e*) in (1), it may be useful first to make a specific additive decomposition of the latter:

$$c(y,e) = b(y) + c(g(y) - e, g(y)), c_{(g-e)}' , g' > 0, c_g' \text{ unrestricted}, e^{\min}(y) \le e \le g(y) \qquad (6)$$

The resource costs of providing *y* are covered by the term *b*(*y*). The purification options are limited to *end of pipe*. We have introduced a Frisch *product coupling* between primary emissions, *g(y)*, and output (see Førsund, 1998). Purification, *r*, in (6), defined as *r* = *g(y)* -*e*, means that e is reduced for a fixed *y* (and then primary emissions, *g(y)*, are fixed), abatement means that *e* may be reduced both through reduced *y* and through purification. Abatement is a broad economic concept, while purification is a technical, engineering concept. The level of primary emissions influence the shape of the purification cost function (see e.g. Førsund, 1992). We have introduced a lower limit on purification (i.e. complete purification is not possible), resulting in a positive lower limit on emissions. An illustration is provided in Figure 3.

A partial change in output now generates two effects: a change in primary pollutants generation and a shift in the purification cost curve. The change in purification costs is the

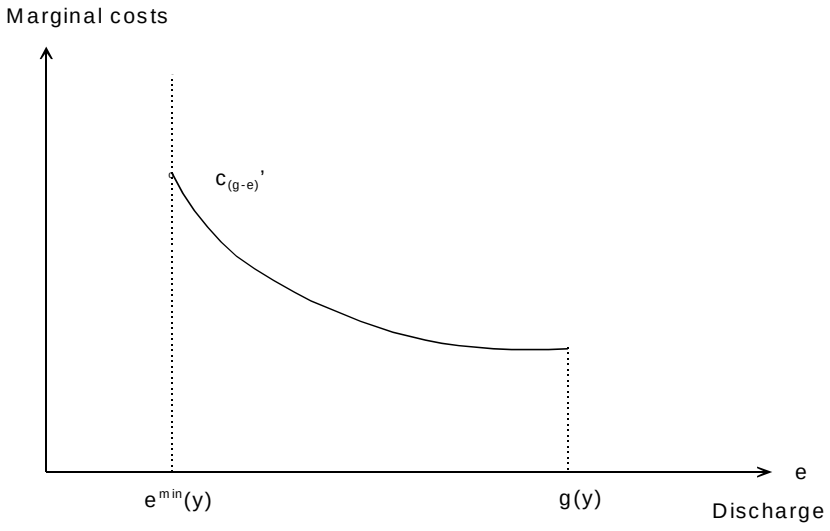

Figure 3. Marginal cost curve

opportunity cost of reducing output instead of increasing purification:

$$\frac{\partial c(g(y)-e,g(y))}{\partial y} = c'_{(g-e)}g' + c'_g g' = (c_{(g-e)}' + c_g')g' \tag{7}$$

The first term shows the marginal purification cost effect of changing output, while the second term shows the shift effect. The sign of the latter may be difficult to determine in general. A positive sign means that marginal costs increase when increasing the reference emission, e_i^o, a sort of diseconomies of scale. But there may be economies of scale effects in purification implying a negative sign. The question is how the structural change behind change in output is composed. There may be different relative and absolute changes in activities with different purification options. The sign of the shift term may depend on the specific location on the purification cost curve. A shift of the purification cost curve in Figure 3 when output is reduced is illustrated in Figure 4. The starting- and end points shifts in general to the left, while the particular shift illustrated the marginal cost curve shifts in such a way that initial purification is more expensive, but the most expensive options get a downward shift.

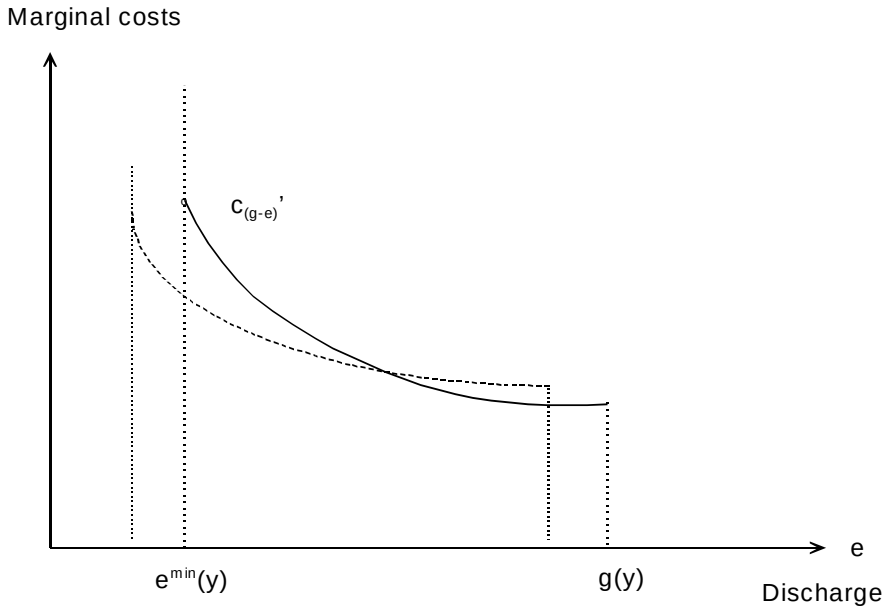

Figure 4. Shift due to a reduction in output

The RAINS purification cost curve

The RAINS purification cost curve has a country as the unit, and is built up from sectoral information. It is basically the same functional form as in (6) for the micro (plant) level at a higher level of aggregation:

$$c(e^0 - e, e^0), c_{(e^0-e)}' \equiv c' > 0, c_{e^0}' \text{ unrestricted}, 0 < e^{\min}(e^0) \le e \le e^0 \quad (8)$$

where e^o = reference value of emissions. The reference value is a function of projected energy use, F^o, and level of outputs, y^o, involving process emissions caused by non-energy inputs:

$$e^0 = f(F^0, y^0), f_F', f_y' > 0 \quad (9)$$

The function $f(.)$ represents the emission factors, and encompasses both the state of technology and current legislation as to emission standards. The cost function (8) is calibrated on the latest available data reflecting BAT (Best Available Technology, see Førsund, 1992), and then updated to a future point in time by replacing plants, like fossil fuel electricity plants, with BAT capacity according to information about age and physical life times. Capacity is also expanded with BAT plants if necessary, i.e. if we have growth and capacity becomes exhausted.

The RAINS cost function is a piecewise linear representation of (8) and is based on a set of purification possibilities. For each country, indexed *i*, the purification possibilities consist of a certain number, L_i, of measures. Each element represents a measure for reducing the emissions. It is assumed that each measure has a constant average cost per unit purified equal to the marginal cost, calculated as the total costs of applying the measure divided by the total reduction. Examples of measures in the RAINS model to remove sulphur are low sulphur coal and coke, low sulphur heavy oil, low sulphur gas oil, limestone injection / fluidised bed combustion, flue gas desulphurisation (FGD) and advanced flue gas desulphurisation.

An illustration is provided in Figure 5a, where three purification measures are shown. By definition the emissions are related to purification:

$$e_{il} = r_{il}^{\max} - r_{il}, e_{il}^{\min} = e_{il}^{\max} - r_{il}^{\max} > 0, l = 1, ..., L_i, i = 1, ..., N \quad (10)$$

The emission, $e_{il}^{\max}$, corresponds to the reference emissions attributed the sources in question for measure *l*. Corresponding to maximal purification, r_{il}^{max}, applying the measure *l* we have minimal emissions, e_{il}^{min}. Sorting the purification options in a cost merit order and substituting emissions produce a piecewise linear cost function in emissions analogous to the continuous

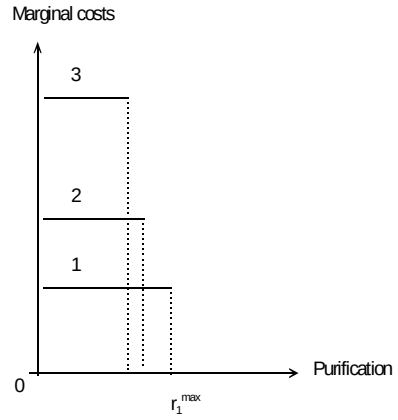

Figure 5a. Purification measures

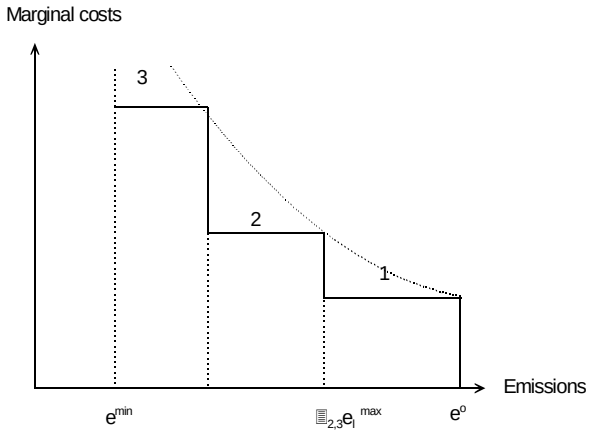

Figure 5b. Merit order. Piecewise linear marginal cost curve

function (8), as shown in Figure 5b for the same three options. The dotted line along the end points may indicate the connection between these two cost curves[4].

A piecewise linear cost function for a country can be formulated as follows:

$$\sum_{l=1}^{L_i} c_{il}(e_{il}^{\max} - e_{il}), c_{il} > 0, e_{il}^{\min} \le e_{il} \le e_{il}^{\max}, l = 1,...., L_i, i = 1,...., N \qquad (11)$$

The total maximal emissions from each source, e_i^o, are given exogenously. This quantity and the total minimum emissions feasible, corresponding to $e^{min}(e^o)$ in (8), are found by summation:

$$e_i^0 = \sum_{l=1}^{L_i} e_{il}^{\max}(e_i^0), e_i^{\min} = \sum_{l=1}^{L_i} e_{il}^{\min}(e_i^0), i = 1,...., N \qquad (12)$$

Corresponding to the role of primary emissions, $g(y)$, in the general model (6), the minimal emission and maximal measure emissions are in general functions of the reference emission values, as indicated in (12).

In Figure 5b the measures are ranked in merit order according to marginal costs (= average costs). But a further refinement makes such rankings conditional. In the actual RAINS model we have the situation that some measures exclude each other, i.e. flue gas desulphurisation and advanced flue gas desulphurisation. The problem with a merit order ranking is illustrated in Figure 6. We have four measures, but measures 2 and 3 are alternatives: for total emissions in between using measures 1 and 2 to full extent measure 3 is not used. Measures 2 and 3

[4] The original piecewise linear cost function in RAINS is actually approximated with such a smooth curve in the ozone version, see Amann et al. (1998).

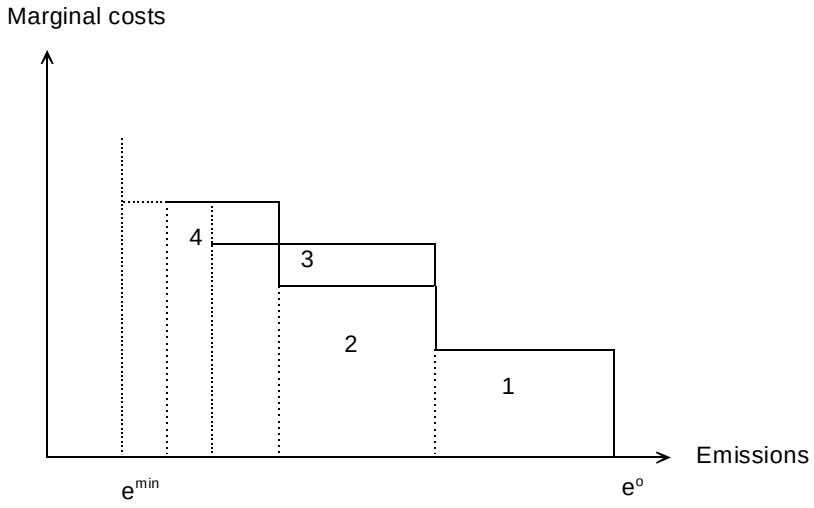

Figure 6. Competing measures

apply to the same plants. But if emissions are to be reduced beyond the level given by measures 1, 2 and 4, measure 3 has to be used. Measure 2 is then dropped completely. The remaining part of the cost curve is shifted to the left. So for maximal feasible reduction, measures 1, 3 and 4 are used.

4. Cost-effective solutions

The environmental objectives are taken care of by specifying constraints on the maximal depositions in each grid square. A cost effective cooperative solution is then obtained by finding a pattern of emissions that minimise total purification costs, measured in a common currency, Euro, subject to environmental constraints. Using first the smooth version (8) of the cost functions we have:

$$
\begin{aligned}
&Min_{\,e_i} \sum_{i=1}^{N} c_i(e_i^0 - e_i, e_i^0) \\
&\text{subject to} \\
&e_i^{\min}(e_i^0) \le e_i \le e_i^0, i = 1,....,N \\
&\sum_{i=1}^{N} a_{ij} e_i + b_j \le d_j^*, j = 1,....,R
\end{aligned}
\qquad (13)
$$

The variables d_j* reflect the environmental objectives and are termed *target loads*. Crucial values of the target loads are the values when there will be no (known) significant

environmental effects in the long run, i.e. the ecosystems should function normally as to reproduction and biomass stability. These values are called *critical loads* (*CL*). They correspond to a level of environmental services in (1) where either M_j remains at its maximal value, i.e. $m_j' = 0$, or the reduction in M_j is so small as to be called "not significant". Obviously we have target loads greater or equal to critical loads: $d_j^* \geq CL_j$

The long-term goal of the countries cooperating under the LRTAP convention is to obtain critical loads (or better) in all receptors. However, in the short or medium term we have two problems; feasibility using the critical loads as constraints, and the level of total costs (we have in reality only the last one, since output is locked in the RAINS formulation).

Gap closure

When target values greater than critical loads are adapted the question is how a fair environmental policy should be formulated. No participating country has any problem with critical loads as a fair principle for formulating environmental policy, but it is not so easy to see how countries can agree on principles for a policy of some environmental degradation. The acceptable solution is the so called gap *closure principle*[5]. Different principles have been used to calculate the targets (see Posch et al. (1999) and (2001) for definitions and a discussion of the principles). For the Oslo Protocol, the target loads are set as the same percentage for all grid squares of the difference between observed (or predicted) deposition in a benchmark year (t^o) and the critical loads:

The gap: $d_j(t^0) - CL_j \geq 0$

Gap closure with x percentage: (14)

$$\frac{d_j(t^0) - d_j^*}{d_j(t^0) - CL_j} = x \Rightarrow d_j^* = xCL_j + (1-x)d_j(t^0), j = 1,...,R$$

where x is the gap closure percentage, e.g. 0.6 (60%) as in the calculations for the Oslo Protocol (with 1980 as t^o).

It may be argued that depositions following from the predicted emissions with "business as usual" for the future first year of the protocol would be best as benchmark values. In general gap closure means that all grids experience the same percentage improvement as to loads, and this would apply more logically to the start year. However, using an existing year it is not

[5]The Norwegian meteorologist Anton Eliassen introduced the idea.

possible to influence the data in view of the gap closure rule. If structural adjustments and measures to reduce emissions in the years in between the benchmark year and starting year have not been too different between countries relatively the target loads should ideally keep the same relative proportions.

Other principles introduced later have been *ecosystem area* gap closure and *average accumulated exceedance* (AAE) gap closure. The latter principle is in use now and is also illustrated in Figure 7. One grid cell is considered and it is assumed to have eight eco-systems, and they are ordered according to increasing value of CL. Ecosystems 1 to 6 are unprotected at the benchmark deposition level $d(t^o)$, and eco-systems 7, 8 are protected. The horizontal bars for each system from the vertical axis to the CL-values are proportional to the eco-system areas. The CL cumulative distribution function is represented by the bold step-curve in Figure 7. Grid -cells may actually contain from a few to several thousands of eco-systems. (The most problematic Norwegian grid has 112 eco-systems.) The main types are forests of different tree species, lakes, grassland, bogs, moors, and tundra[6].

To apply a deposition gap closure principle the CL for the grid-cell has to be defined. It has been usual to define the CL for a grid-cell by having five percent of eco-system area unprotected. Let us assume that eco-system No.1 has an area share of five percent. The CL for

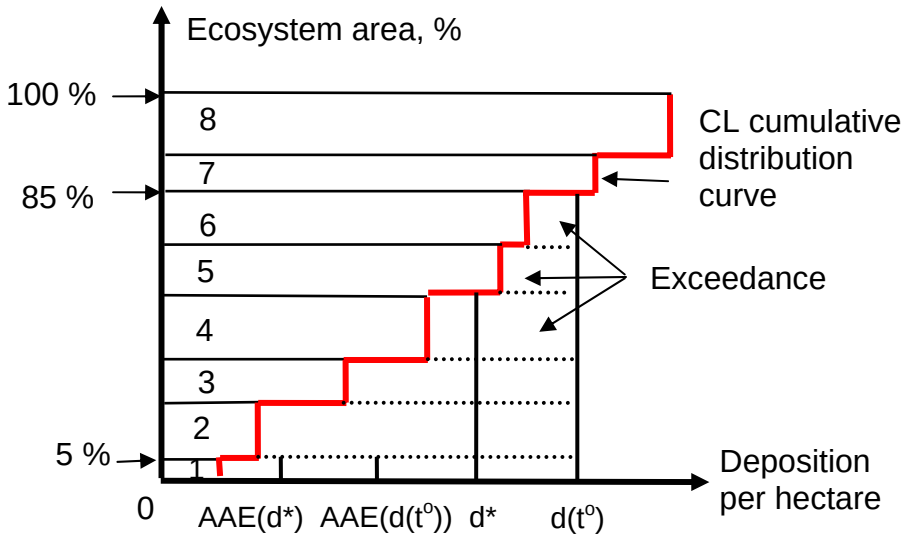

Figure 7. Critical load (CL), Gap closure and Average Accumulated Exceedance

6 Details of distribution of types of eco-systems on countries and area covered are found in Amann et al. (1998b) and Hettelingh et al. (2001).

eco-system No. 2 in the figure then determines the CL for the whole grid-cell. The gap to be closed with a given fraction (or percentage), x, is ($d(t^0)– CL_2$). The target deposition will then be $d^* = xCL_2 + (1-x)d(t^0)$. In figure 7 a gap closure of about x = 1/3 has been used, resulting in deposition d^*, protecting systems 5 and 6 in addition to 7 and 8 already protected at deposition level $d(t^0)$.

A weakness with this principle is that the CL value representing the whole grid depends on only one observation on the CL distribution curve, and the resulting target load d^*, is also one point on the curve. For different shapes of the CL distribution curve, but going through both the same CL grid value and the benchmark deposition value, $d(t^0)$, we may then have quite different eco-system area protection.

The area gap closure principle focuses on reducing the unprotected area with a certain percentage. The percentage unprotected area in a grid is the share of the ecosystem area where the critical loads are less than the deposition. Let A_{ij} be the area of eco-system i in grid-cell j, and let S_j be the set of eco-systems in grid-cell j. We will partition this set into the unprotected eco-systems, $S_j^-(d) = \{i : CL_{ij} < d\}$, and the protected systems, $S_j^+(d) = \{i : CL_{ij} \geq d\}$ and obviously we have $S_j^- \cup S_j^+ = S_j$. In Figure 7 the ecosystems 1 to 6 belong to the unprotected set at the benchmark deposition level $d(t^o)$, and eco-systems 7, 8 belong to the protected set. Eco-system area gap closure is usually interpreted as finding the deposition, d^*, that corresponds to a given percentage, x, reduction of unprotected area at a benchmark deposition, $d(t^o)$. Using our notation it means finding the maximal level of deposition d^* satisfying

$$\sum_{i \in S_j^-(d_j^*)} A_{ij} \leq (1-x) \sum_{i \in S_j^-(d_j(t^o))} A_{ij} \qquad (14a)$$

In Figure 7 eco-systems 5 and 6 have about 45% of the total area of systems 1-6. Applying a gap closure fraction of e.g. 0.4 results in protecting systems 5 and 6, i.e. more than 40%, and corresponds to a deposition exactly equal to the CL of system 5. It is regarded as a weakness of the principle (see Posch et al., 2001) that since the CL distribution curve is a step curve, equality will in general not hold in the equation above when calculating target depositions, d^*. This means that target depositions may vary over grid-cells also located in different countries,

which may create problems of fairness. Also left out is the distribution of the *degree* of excess of depositions over eco- systems, it is just a question of protected or unprotected eco- systems.

The *AAE principle* focuses on the exceedance in each eco-system of a grid-cell. The exceedances are accumulated (added) over the eco-systems in a cell and then averaged with eco-system area as weights. For a formal definition of the AAE for a grid-cell, consider ecosystem i in grid-cell j. Let A_{ij} be the area of this eco-system, and let S_j be the set of eco-systems in grid-cell j. The *excess* for an eco-system, i, is the difference between a benchmark deposition, $d_j(t^o)$, and the critical load, CL_{ij} when the difference is positive, and the exceedance is set to zero when the difference is negative. The average accumulated exceedance, AAE_j, is calculated by weighing each eco-system excess with eco-system area share:

$$AAE_j = \sum_{i \in S_j} Max\left\{ \frac{A_{ij}}{\sum_{i \in S_j} A_{ij}} (d_j(t^o) - CL_{ij}), 0) \right\} = AAE_j(d_j(t^o)),$$

$$AAE_j(d_j(t^o)) > 0, AAE_j' > 0 \ \textit{for}\ d_j(t^o) > Min\ CL_{ij}, \qquad (14b)$$

$$AAE_j(d_j(t^o)) = 0, AAE_j' = 0 \ \textit{for}\ 0 \le d_j(t^o) \le Min\ CL_{ij},\ i \in S_j,\ j = 1,..,R$$

The minimum critical load is the CL for the first eco-system, i.e. No. 1 in Figure 7, since CL is the cumulative distribution function. In Figure 7 systems 1 to 6 have exceedances illustrated by the continuation of the area bars with broken lines from the CL- values up to the d^o level, while systems 7 and 8 obtain the value of zero according to the definition in (14b). There is a one to one correspondence between average accumulated exceedances and deposition d^o through the *AAE*-function for deposition values above the minimum CL of a grid-cell's eco-systems[7]. This value is indicated on the horizontal axis in the figure as $AAE(d^o)$. It is measured in the same units as depositions, and obviously we must have $AAE(d(t^o)) < d(t^o)$ (as long as $d(t^o) > CL_1$, see (14b)). The target for accumulated excess for a grid-cell, j, with x as the gap closure fraction (or expressed as per cent), may be calculated as:

$$AAE_j^* = (1-x)AAE_j(d_j(t^o)),\ j = 1,..,R, \qquad (14c)$$

which implicitly gives a target also for depositions. Assuming d^* in Figure 7 is such a calculated target, the target for average accumulated exceedance, $AAE(d^*)$, is indicated in the figure. We must have $AAE(d^*) < AAE(d(t^o))$ for $d^* < d(t^o)$.

7 It should be mentioned that when addressing acidification in RAINS due to both sulphur and nitrogen unique CL *values* do not longer exist, but the problem is solved by measuring excess as the shortest distance from the deposition point for the substances to the now relevant concept of a critical load *function*, see Posch et al. (1999) and (2001).

A measure based on average accumulated exceedances of depositions is more robust as to the location of the *CL*-function than previous gap closure principles and also takes into consideration the whole distribution of excess. Depositions have to be reduced to a level less or equal to the CL of the most sensitive eco-system for no environmental pollution to occur. But notice that it might not be unproblematic to sum together exceedances in different ecosystems. It implies that damages from exceedances are assumed to be directly comparable, as would only be the case if damages were the same linear function of exceedance for all eco-systems in a grid-cell.

The computations of gap closure and target load are somewhat more complicated when we are dealing with two or more substances as for acidification. The critical load function for acidity is represented by the solid curve (two segments here, in general we may have several segments) in Figure 8. The synergy effects are seen by the CL for acidity being limited by

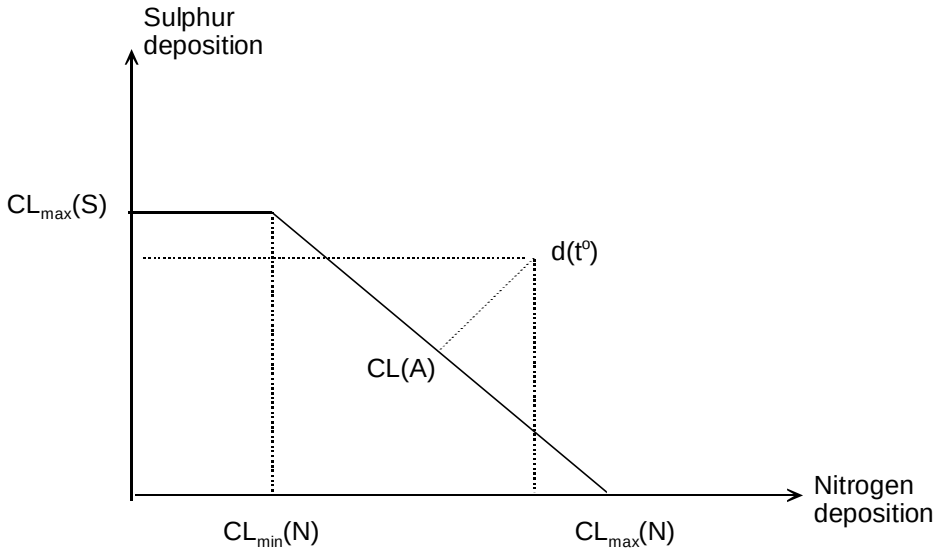

Figure 8. Critical load for aciditiy

either sulphur or nitrogen, $CL_{max}(S)$ and $CL_{max}(N)$. For levels of nitrogen below $CL_{min}(N)$ acidity is determined by the sulphur deposition. The exceedance represented by a benchmark observation $d(t^o)$ is measured as the shortest distance to the critical load function (marked *CL(A)*). Gap closure is then based on closing the gap of accumulated exceedances (summed over ecosystems) with *x* per cent.

Optimality conditions for the smooth purification cost function

The Lagrangian for the cost minimisation problem (13) is:

$$
\begin{aligned}
L = & \\
& -\sum_{i=1}^{N} c_i(e_i^0 - e_i, e_i^0) \\
& -\sum_{j=1}^{R} \lambda_j (\sum_{i=1}^{N} a_{ij} e_i + b_j - d_j^*) \\
& -\sum_{i=1}^{N} \mu_i (e_i - e_i^0) \\
& -\sum_{i=1}^{N} \gamma_i (-e_i + e^{\min}(e_i^0))
\end{aligned}
\tag{15}
$$

The necessary first order conditions are:

$$
c_i' - \sum_{j=1}^{R} \lambda_j a_{ij} - \mu_i + \gamma_i \leq 0, i = 1, ..., N \tag{16}
$$

The total marginal evaluation of depositions from emissions from a source *i* is taken into account. The shadow prices on the environmental standards are only positive if the corresponding constraint is binding. If we have typical upstream-downstream configurations it is to be expected that many standards will be over fulfilled (not binding). The shadow prices on the upper and lower constraints on emissions from a source cannot both be positive at the same time. If we are at the upper boundary μ_i will be positive and γ_i zero, and vice versa at the lower boundary. For an interior solution both are zero. We have the standard condition: It is necessary for an optimal emission level that marginal purification cost equals the total marginal shadow value of unit depositions.

The environmental standard shadow prices have the interpretation as the change in the objective function of a marginal change in the constraint (evaluated at the optimal solution):

$$
\frac{\partial \left[-\sum_{i=1}^{N} c_i(e_i^0 - e_i^*, e_i^0) \right]}{\partial d_j^*} = \lambda_j \Rightarrow \frac{\partial \left[\sum_{i=1}^{N} c_i(e_i^0 - e_i^*, e_i^0) \right]}{\partial d_j^*} = -\lambda_j, j = 1, ..., R \tag{17}
$$

Relaxing a binding constraint will in general improve the optimal value of the objective function; in our case decrease the total purification costs. Tightening the environmental

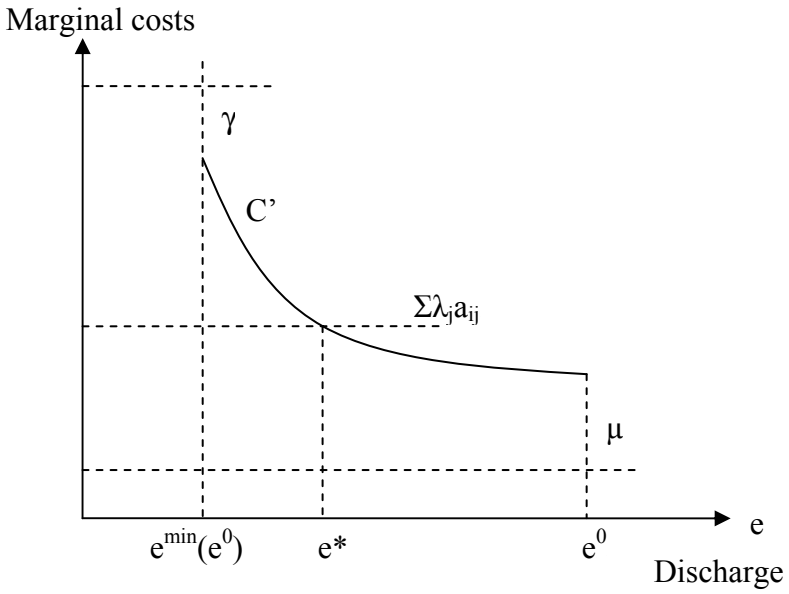

Figure 9. Optimal solutions

standard, i.e. lowering the target loads d_j* imposes a positive cost on the participating countries.

As pointed out above decreasing e^o is an alternative to purification. The envelope theorem yields:

$$\frac{\partial\left[-\sum_{i=1}^{N} c_i(e_i^0 - e_i^*, e_i^0)\right]}{\partial e_i^0} = \frac{\partial\left[L(e_i^0, e_i^*, \lambda_j, \mu_i, \gamma_i)\right]}{\partial e_i^0} = \tag{18}$$

$$-(c_i' + c_{ie^0}') + \mu_i - \gamma_i \frac{\partial e^{\min}(e^0)}{\partial e^0}, i = 1, ..., N$$

In the case of an interior solution both μ_i and γ_i are zero, and if the upper constraint on emissions is binding μ_i is positive and γ_i zero, and if the lower constraint is binding μ_i is zero and γ_i positive. For the first two cases we have:

$$\frac{\partial\left[\sum_{i=1}^{N} c_i(e_i^0 - e_i^*, e_i^0)\right]}{\partial e_i^0} = c_i' + c_{ie^0}' - \mu_i = \sum_{j=1}^{R} \lambda_j a_{ij} + c_{ie^0}', i = 1, ..., N \tag{19}$$

where we have substituted from the first order condition (16) in the last expression. The first two terms in the first expression show the cost change of purification due to the changes of the cost function itself. The last term shows the partial reduction in purification costs due to

increased emissions: the maximal emission constraint is relaxed and there is less purification at the margin. We are evaluating the change at the right-hand end point of the cost curve corresponding to the end point in Figure 9. The last expression shows that increasing the reference emission e_i^o imposes environmental costs, because emissions are increased at the margin. The shift effect remains, and may possibly imply a negative total effect on purification costs. If the upper constraint is not binding, μ_i is zero and there is no offsetting effect from relaxing this constraint. The environmental shadow cost is equal to marginal purification costs, and the shift effect remains the same.

In the case of the lower restriction being binding we have:

$$\frac{\partial\left[\sum_{i=1}^{N} c_i(e_i^0 - e_i^*, e_i^0)\right]}{\partial e_i^0} = c_i' + c_{ie^0}' - \gamma_i \frac{\partial e^{\min}(e^0)}{\partial e^0} = \sum_{j=1}^{R} \lambda_j a_{ij} + c_{ie^0}' - \gamma_i \left(\frac{\partial e^{\min}(e^0)}{\partial e^0} - 1\right), i = 1, \ldots, N \quad (20)$$

The case of $\partial e_i^{min} / \partial e_i^o = 1$ may be termed pure parallel shift, and we then have the same interpretation of the change in costs as above. In the general case the sign of $\partial e_i^{min} / \partial e_i^o$ may be difficult to determine, but a positive sign may seem to be the normal case, since it seems reasonable that the minimum emission should increase with the total reference level. In the opposite case it would be an environmental advantage to start with a high reference case as to the possibility to reduce emissions maximally. This does not seem to reflect engineering reality. Whether $\partial e_i^{min} / \partial e_i^o$ is greater or less than one will depend on the structure of reference emission increase. A proportional increase from various sources supports the marginal impact to be one, while increases in emissions for sources with higher (lower) minimum emissions tend to make the derivative greater (smaller) than one.

If we are talking of reducing the reference emissions as alternative to purification, all signs are reversed in the discussions above.

Optimality condition for a piecewise linear purification cost function

Corresponding to the continuous cost function case with the maximal emission influencing the shape and location of the marginal cost curve, we have in the piecewise linear case that the length of each step of the marginal cost function may be a function of this maximal quantity, i.e. we may have $e_{il}^{max}(e_i^o)$. In fact, the list itself of purification options may be

changed when changing e^o. But these features and the case of alternative measures are disregarded in the exercise above to keep the exposition reasonably simple. It is straightforward to work out the generalisations. Using the piecewise linear purification cost functions in (11) the linear programming formulation is:

$$Min\left[\sum_{i=1}^{N}\sum_{l=}^{L_i} c_{il}(e_{il}^{\max} - e_{il})\right],$$

subject to

$$\sum_{i=1}^{N} a_{ij}\sum_{l=1}^{L_i} e_{il} + b_j \le d_j^*, j = 1,....,R \qquad (21)$$

$$e_{il}^{\min} \le e_{il} \le e_{il}^{\max}, l = 1,....,L_i, i = 1,....,N$$

The Lagrangian for the problem is:

$$L =$$

$$-\sum_{i=1}^{N}\sum_{l=1}^{L_i} c_{il}(e_{il}^{\max} - e_{il})$$

$$-\sum_{j=1}^{R}\lambda_j(\sum_{i=1}^{N} a_{ij}\sum_{l=1}^{L_i} e_{il} + b_j - d_j^*) \qquad (22)$$

$$-\sum_{i=1}^{N}\sum_{l=1}^{L_i}\mu_{il}(e_{il} - e_{il}^{\max})$$

$$-\sum_{i=1}^{N}\sum_{l=1}^{L_i}\gamma_{il}(-e_{il} + e_{il}^{\min})$$

The necessary first order conditions are:

$$c_{il} - \sum_{j=1}^{R}\lambda_j a_{ij} - \mu_{il} + \gamma_{il} \le 0, i = 1,...,N, l = 1,...,L_i \qquad (23)$$

The shadow price on the maximal emission associated with each purification option is zero if emission is less than the maximal, and positive (strictly speaking non-negative) if the emission is equal to the maximal possible. The shadow price on the minimal emission associated with each purification option is zero if emission is greater than the minimal, and positive (strictly speaking non-negative) if the emission is equal to the minimal possible. The shadow prices cannot be positive at the same time, and will both be zero for an interior solution (if such a solution will exist). The condition is similar to the one for the smooth continuous case (16) with purification cost measure numbers added. One way of interpreting the condition is to compare the cost coefficients with the number for the "shadow evaluation" of the depositions from source (country) i of the optimal solution. If the cost coefficient is

greater than this number, then the measure is not in use, and the emission is maximal, i.e. $e_{il} = e_{il}^{max}$, and the shadow price μ_{il} is positive (the shadow price γ_{il} is then zero). The exact value is determined in (23) such that it holds with equality. If the cost coefficient is less than the shadow value of depositions, then the measure should be adopted to full extent, i.e. $e_{il} = e_{il}^{min}$. The shadow price γ_{il} is then positive (and μ_{il} zero), and determined such that (23) holds with equality.

Inspecting all the purification measures we get delimitation as to the status of measures:
Activated measures:

$$c_{il} < \sum_{j=1}^{R} \lambda_j a_{ij}, i = 1,...,N, l = 1,...,L_i \tag{24}$$

Measures not in use:

$$c_{il} > \sum_{j=1}^{R} \lambda_j a_{ij}, i = 1,...,N, l = 1,...,L_i \tag{25}$$

Measures partially in use, or not in use, or fully used:

$$c_{il} = \sum_{j=1}^{R} \lambda_j a_{ij}, i = 1,...,N, l = 1,...,L_i \tag{26}$$

A solution is illustrated in Figure 10. We see that the first two measures (looking to the left from e^o) should be used, but not the third one. If the shadow values of depositions should

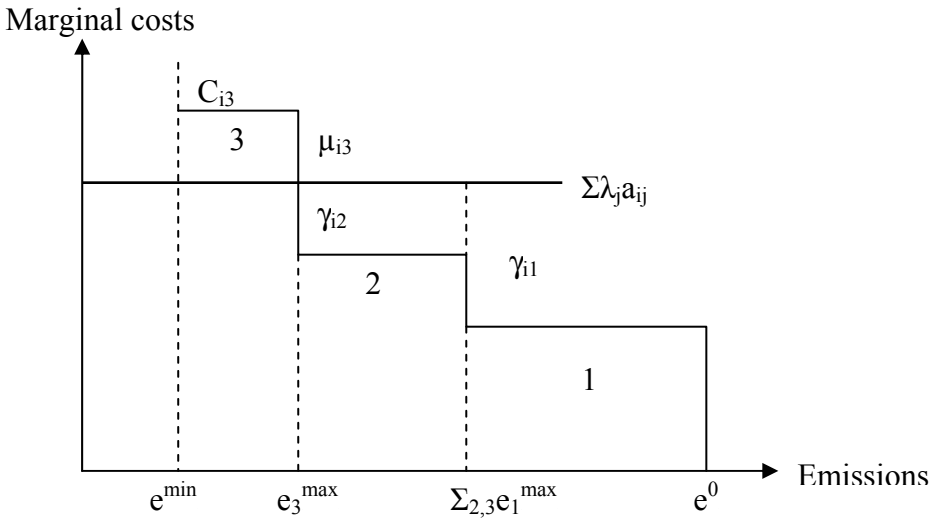

Figure 10. The optimal selection of measures

happen to coincide exactly with a cost coefficient value, the solution for emissions cannot be seen from the figure. If the cost coefficients for the same measure are different for all countries, then such equality can only happen for one country. Since wages are country specific, measures involving labour costs will be different in principle (although the differences may be very small). The amount emitted in the one country with equality will then be determined by the deposition constraints that are binding by a residual calculation.

The effect of changing the maximal emissions (generated by a change in the reference emissions or technical change) is similar to the continuous case taking the simplification mentioned above into consideration. The envelope theorem yields:

$$\frac{\partial\left[-\sum_{i=1}^{N}\sum_{l=1}^{L_i} c_{il}(e_{il}^{\max}-e_{il}^{*})\right]}{\partial e_{il}^{\max}} = \frac{\partial\left[L(e_{il}^{\max}, e_{il}^{\min}, e_{il}^{*}, \lambda_j, \mu_{il}, \gamma_{il})\right]}{\partial e_{il}^{\max}} = -c_{il}' + \mu_{il} \qquad (27)$$

$$l = 1,...,L_i, i = 1,...,N$$

Utilising the first order condition (23) we have:

$$\frac{\partial\left[-\sum_{i=1}^{N}\sum_{l=1}^{L_i} c_{il}(e_{il}^{\max}-e_{il}^{*})\right]}{\partial e_{il}^{\max}} = c_{il}^{*} - \mu_{il} = \sum_{j=1}^{R}\lambda_j a_{ij}, l = 1,...,L_i, i = 1,...,N \qquad (28)$$

Increasing the maximal emission for measures both in use and not in use implies increased emissions (purification is constant) and imposes the same positive shadow-valued cost.

Changing the minimal emissions for a purification measure has the straightforward shadow value interpretation (using (22)):

$$\frac{\partial\left[-\sum_{i=1}^{N}\sum_{l=1}^{L_i} c_{il}(e_{il}^{\max}-e_{il}^{*})\right]}{\partial e_{il}^{\min}} = \gamma_{il}, l = 1,...,L_i, i = 1,...,N \qquad (29)$$

Increasing the minimal emissions from measures in maximal use increases costs because the constraint is tightened; alternative purification is more costly. Reducing the minimal emissions decrease cost as more can be purified with a cheaper measure.

The dual

Since the problem (21) is a linear programming problem inspecting the dual may increase our understanding of the nature of the solution. Since the terms e_{il}^{max} are constants the objective function of the primal problem (21) can be simplified to:

$$Min_{e_{il}}\left[\sum_{i=1}^{N}\sum_{l=1}^{L_i} c_{il}e_{il}^{\max} - \sum_{i=1}^{N}\sum_{l=1}^{L_i} c_{il}e_{il}\right] = Max_{e_{il}}\left[\sum_{i=1}^{N}\sum_{l=1}^{L_i} c_{il}e_{il}\right] - \sum_{i=1}^{N}\sum_{l=1}^{L_i} c_{il}e_{il}^{\max} \tag{30}$$

The linear programming tableau for the coefficients of the endogenous variables, e_{il}, is:

$$\begin{array}{lllll}
\ldots & +c_{il}e_{il} & +\ldots & +c_{iL_i}e_{iL_i} & +\ldots \\
\ldots & +a_{il}e_{il} & +\ldots & +a_{iL_i} & +\ldots \quad \le d_1^* - b_1 \\
\ldots & +\ldots & +\ldots & +\ldots & +\ldots \quad \le \ldots \\
\ldots & +a_{iR}e_{il} & +\ldots & +a_{iR}e_{iL_i} & +\ldots \quad \le d_R^* - b_R \\
 & e_{il} & & & \le e_{il}^{\max} \\
 & & \ldots & & \le \ldots \\
 & & & e_{iL_i} & \le e_{iL_i}^{\max} \\
 & -e_{il} & & & \le -e_{il}^{\min} \\
 & & \ldots & & \le \ldots \\
 & & & -e_{iL_i} & \le -e_{iL_i}^{\min}
\end{array} \tag{31}$$

We have shown only the structure for one source, i, i=1,..,N for the L_i purification measures. Extending (31) to a full tableau is straightforward.

The dual problem corresponding to the primal with (30) as objective function and (31) as the coefficient tableau (see e.g. Berck and Sydsæter, 1991) is:

$$Min_{\lambda_j,\mu_{il},\gamma_{il}}\left[\sum_{j=1}^{R}(d_j^* - b_j)\lambda_j + \sum_{i=1}^{N}\sum_{l=1}^{L_i}\mu_{il}e_{il}^{\max} - \sum_{i=1}^{N}\sum_{l=1}^{L_i}\gamma_{il}e_{il}^{\min}\right]$$

subject to (32)

$$\sum_{j=1}^{R} a_{ij}\lambda_j + \mu_{il} - \gamma_{il} \ge c_{il}, l = 1,...,L_i, i = 1,....,N$$

The exogenous constraint variables in the primal problem become the coefficients of the dual objective function, and the shadow prices on the constraints in the primal problem become the endogenous variables in the dual objective function. The exogenous coefficients in the primal objective function become the constraint variables in the dual, and the coefficients for the

endogenous variables in the dual are the transposed coefficients of the constraint coefficient matrix of the primal (see (31)). Relaxations of constraints in the primal contribute to higher value of the objective function of the dual, i.e. a detrimental impact on the objective function. This is the case for target loads for depositions and maximal emissions according to the reference path, while increase in minimum emissions decrease the objective function of the dual because *decrease* of minimal emissions implies a relaxation of constraints in the primal.

The LP tableau for the $R+2\sum_i L_i$ endogenous variables, λ_j and μ_{il}, γ_{il} is:

$$\begin{array}{lllll} (d_1^*-b_1)\lambda_1 & +\ldots & +\ldots & +(d_R^*-b_R)\lambda_R & +\sum_{i=1}^{N}\sum_{l=1}^{L_i}\mu_{il}e_{il}^{\max} \quad -\sum_{i=1}^{N}\sum_{l=1}^{L_i}\gamma_{il}e_{il}^{\min} \\ a_{il}\lambda_1 & +\ldots & +\ldots & +a_{iR}\lambda_R & +\mu_{il} \quad -\gamma_{il} \quad \geq c_{il} \\ \ldots & +\ldots & +\ldots & +\ldots & +\ldots \quad -\ldots \quad \geq \ldots \\ a_{il}\lambda_1 & +\ldots & +\ldots & +a_{iR}\lambda_R & +\mu_{iL_i} \quad -\gamma_{iL_i} \quad \geq c_{iL_i} \end{array} \qquad (33)$$

In the tableau only the coefficient structure of the cost function for source *i* is shown. The generalisation is straightforward.

The primal problem of finding a cost effective solution given environmental constraints has turned into the dual one of minimising costs. The cost items are the maximal depositions (target loads subtracted background depositions) in the receptors, evaluated at prices λ_j, plus costs of emitting maximal quantities at each purification option, evaluated at prices μ_{il}, subtracted costs of emitting minimal quantities at each purification option, evaluated at prices γ_{il}. The constraints are that for each cost segment interval the value of depositions, evaluated at prices λ_j, plus the cost, μ_{il}, of emitting the maximal amount at the cost segment, minus the cost, γ_{il}, of emitting the minimal amount at the cost segment, should be greater or equal to the marginal purification cost at the segment. The objective function is now expressing the environmental evaluation through the shadow-price evaluation. We note that target load depositions and emissions are evaluated *separately*, and that maximal emissions increase costs while minimal emissions reduce the cost. Cost efficiency is taken care of by the constraints.

From duality theory we know that the optimal value of the dual objective function must be equal to the optimal value of the primal objective function (provided unique optimal values

exist). The evaluation of depositions and emissions is equal to the total purification cost. Therefore, the evaluation expressed in the dual objective function has nothing to do with proper evaluation of the environment in the receptors.

Let us solve the dual problem explicitly. From the duality theory we know that the shadow prices on the constraints in (32) are the emissions, e_{il}. The Lagrangian for the dual problem (32) is (the problem is formulated as a maximisation problem, following Berck and Sydsæter (1991), and constraints changed accordingly):

$$\begin{aligned} L = & \left[-\sum_{j=1}^{R}(d_j^* - b_j)\lambda_j - \sum_{i=1}^{N}\sum_{l=1}^{L_i}\mu_{il}e_{il}^{\max} + \sum_{i=1}^{N}\sum_{l=1}^{L_i}\gamma_{il}e_{il}^{\min} \right] \\ & -\sum_{i=1}^{N}\sum_{l=1}^{L_i}e_{il}(-\sum_{j=1}^{R}a_{ij}\lambda_j - \mu_{il} + \gamma_{il} + c_{il}) \end{aligned} \tag{34}$$

The first order conditions for the endogenous variables, the shadow prices λ_j , μ_{il} and γ_{il} are:

$$\begin{aligned} & -(d_j^* - b_j) + \sum_{i=1}^{N}\sum_{l=1}^{L_i}e_{il}a_{ij} \le 0, j = 1,...,R \\ & -e_{il}^{\max} + e_{il} \le 0 \\ & e_{il}^{\min} - e_{il} \le 0, i = 1,...,N, l = 1,...,L_i \end{aligned} \tag{35}$$

We know that the "shadow price" e_{il} must be non-negative, and that the product of this price and the corresponding constraint in (32) must be zero (complementary slackness):

$$e_{il}(-\sum_{j=1}^{R}a_{ij}\lambda_j - \mu_{il} + \gamma_{il} + c_{il}) = 0, i = 1,....,N \tag{36}$$

We have assumed that $e_{il}{}^{min} > 0$, so we know that the second expression in (36) equals zero. There are three possible values for e_{il}; the two extreme values and an interior value. Let us assume that the minimum value is obtained, $e_{il} = e_{il}{}^{min}$. The second necessary condition in (35) of the dual problem then tells us that this relation must be fulfilled with inequality. This implies that the variable μ_{il} must attain its lowest value, i.e. the corner solution of zero. The third necessary condition tells us that this relation must be fulfilled with equality, i.e. $\gamma_{il} > 0$ (or = 0 as the limiting case). We then have that:

$$-\sum_{j=1}^{R}a_{ij}\lambda_j + c_{il} + \gamma_{il} = 0 \Rightarrow \sum_{j=1}^{R}a_{ij}\lambda_j \ge c_{il}, i = 1,...,N, l = 1,...,L_i \tag{37}$$

All the fully utilised measures satisfy (37). Equality is the arbitrary limiting case.

In the case of an interior value for e_{il} we have that both μ_{il} and γ_{il} are zero. We then get the last expression in (37) satisfied with equality.

Let us now consider the case of $e_{il} = e_{il}^{max}$. We must then have $\gamma_{il} = 0$ and $\mu_{il} > 0$ (or = 0 as the limiting case), yielding:

$$(-\sum_{j=1}^{R} a_{ij}\lambda_j - \mu_{il} + \gamma_{il}) = 0 \Rightarrow \sum_{j=1}^{R} a_{ij}\lambda_j \le c_{il}, i = 1,...,N, l = 1,...,L_i \quad (38)$$

For measures not in use (38) will be valid.

We have the problem that equality in (37) and (38) may mean both that the measure is in maximal use, partly in use or not in use. Consider the situation for a measure should it be partly in use. We then have that both μ_{il} and γ_{il} are zero, and this is exactly the same first order condition as in the primal problem when the cost coefficient is equal to the shadow evaluation of binding constraints.

Concerning the determination of the shadow price λ_j we have that if the optimal value is zero, then the first condition in (35) is fulfilled with inequality (or equality as the limiting case); depositions from current emissions are less than the target ($d_j^* - b_j$). In the case of a positive interior solution we have that depositions from current emissions are equal to the target.

In the dual objective function it is only the binding constraints that have positive shadow prices and count when calculating the value of the objective function.

Exceedance minimisation

As an alternative to cost minimisation with environmental constraints, scenario calculations with minimisation of exceedences; i.e. actual emissions minus critical loads, subject to the same environmental constraints and a total cost constraint has been performed by research groups from Imperial College London and Stockholm Environmental Institute (SEI) (see Gough et al., 1994). The main idea has been to control explicitly for total purification costs, and get the "best possible environment" per Euro committed. Although this formulation has environmental quantities in the objective function and costs as constraint, it is important to realise that it is not the dual to the RAINS model, as exposed above. A typical model

formulation with no weights on location of exceedences, reverting to the smooth representation as in (8), is:

$$Min_{e_i} \sum_{j=1}^{R} (d_j - CL_j)$$

subject to

$$e_i^{\min}(e_i^0) \le e_i \le e_i^0, i = 1,...., N \tag{39}$$

$$\sum_{i=1}^{N} a_{ij} e_i + b_j \equiv d_j \le d_j^*, j = 1,..., R$$

$$\sum_{i=1}^{N} c_i (e_i^0 - e_i, e_i^0) \le C^*$$

The Lagrangian is:

$$\begin{aligned} L = & \\ & -\sum_{j=1}^{R} (\sum_{i=1}^{N} a_{ij} e_i + b_j - CL_j) \\ & -\sum_{j=1}^{R} \lambda_j (\sum_{i=1}^{N} a_{ij} e_i + b_j - d_j^*) \\ & -\sum_{i=1}^{N} \mu_i (e_i - e_i^0) \\ & -\sum_{i=1}^{N} \gamma_i (-e_i + e^{\min}(e_i^0)) \\ & -\beta (\sum_{i=1}^{N} c_i (e_i^0 - e_i, e_i^0) - C^*) \end{aligned} \tag{40}$$

where we have used the same symbols for the shadow prices as in (15). The necessary first order conditions are:

$$\begin{aligned} & -\sum_{j=1}^{R} a_{ij} - \sum_{j=1}^{R} \lambda_j a_{ij} - \mu_i + \gamma_i + \beta c_i' \le 0 \Rightarrow \\ & \beta c_i' - \sum_{j=1}^{R} (1 + \lambda_j) a_{ij} - \mu_i + \gamma_i \le 0, i = 1,..., N \end{aligned} \tag{41}$$

Comparing (16) and (41) we have that we now have a shadow price in front of the marginal cost, and that the shadow prices on the depositions are different. Although the gap closure fairness principle applies setting the target loads, focussing on the total sum of exceedences in the objective function distorts the regional application of the principle. In the case of an interior solution for emissions we have that the shadow price is now $(1+\lambda_j)/\beta$ instead of λ_j. The set of shadow prices will in general differ. The prices in the two situations can only be the same if they all have the same value independent of location *j*. The shadow prices will

tend to be relatively more different the smaller the solution for λ_j in the present model (39). This means that if we use the optimised cost figures from solving the RAINS model as a constraint above, and keep the same target loads, the geographical distribution will differ from the RAINS model solution.

5. Concluding remarks

Taking a stylised view, the armchair approach of economists is to introduce just two functions when addressing pollution problems: one function showing the benefits of pollution and another function showing environmental damages. But even this extremely simple approach is difficult to apply to real situations because of the problems of quantifying the relations.

In the RAINS model the benefit function of pollution is quantified as purification cost curves. Instead of environmental damage functions environmental standards are introduced linked to the deposition of pollutants. There is an impressive research effort behind the establishment of data for *critical loads* for current pollution loads in the environment. It is a significant achievement of environmental economics modelling and cooperation across research fields that a large-scale empirical model is operational and actually used at international negotiations.

Recently the popularity in academia of applying game theory to international environmental agreements has increased rapidly (see the seminal paper by Mäler (1989) and an overview in e.g. Hanley et al., 1997). However, in contrast to results stressing free rider behaviour and side payments to prevent blocking of agreements, etc., what we observe is that side payments have not been used, and that countries obviously pay more than following a short-term non-cooperative behaviour.

As to observed cooperation on transboundary pollution problems one should note that the UN conference in 1972 adopted a Principle 21 to the effect that states have

> ... the responsibility to ensure that activities within their jurisdiction or control do not cause damage to the environment of other states or of areas beyond the limits of national jurisdiction

(Quoted from Alcamo et al. , p. 46).

Another international agreement with significance for international cooperation is the OECD rule of the Polluters Pay Principle (PPP), see OECD (1975). This principle excludes the use of side payments. Incorporating these rules of behaviour and then explaining international agreements is a challenge to game theory.

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