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Effects of Progressive Taxes under Decentralized Bargaining and
Heterogeneous Labor

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Abstract

We study effects of changes in income tax progressivity in an economy where workers' productivities differ and workers and firms bargain individually over wages. When employment is given, we show that a pure increase in tax progressivity reduces wages by reducing workers' relative bargaining power. When average taxes then also increase, after-tax wages are unambiguously reduced, while the effects on gross wages and firm profitability are ambiguous. We next consider an example where the income tax is linear and the productivity distribution uniform, and the linear tax is the government's only policy instrument. A first-best solution then cannot be implemented. A second-best solution can however be implemented using a whole family of tax functions, with different tax progressivity, and a more progressive tax implies a higher tax revenue to the government. The government may then achieve a higher tax revenue, and a more even distribution of after-tax income, without any additional disturbance to allocation.

1. Introduction

The taxation of income, and the degree of progressivity of the income tax system, have a number of consequences many of which have been analyzed extensively in the literature. In particular, much of the literature on optimum income taxation (see e.g. Mirrlees (1971, 1976), Atkinson and Stiglitz (1980), chapters 13-14), focuses on tradeoffs between equity and efficiency in increasing tax progressivity. The basic presumption is that a more progressive tax system, while possibly contributing to a more egalitarian distribution of after-tax income, distorts allocative decisions with respect to labor supply and demand, work effort and the accumulation of human capital, and may lead to other wasteful, fraudulent or rent-seeking activities (e.g. in the form of tax evasion and avoidance or reduced gains from labor specialization).²

²For a treatment of the implications of tax progressivity for tax evasion see Sandmo (1981), in the context of a “dual labor market” model see Sørensen (1998), and implications for allocation of labor in the household see Anderberg and Balestrino (1999) and Kelven et.al. (1999). Concerning the specialization of labor, it is also commonly recognized, in particular in the Scandinavian countries where overall marginal tax rates are among the highest, that an implication of the tax system is that many services, in particular those done within the household, are most profitably done by oneself instead of by paid outside labor, often at substantial loss of social efficiency.

In some of the recent literature it is however recognized that a more progressive tax system in many practical cases imply other beneficial effects, and thus overall does not necessarily reduce efficiency. At least three types of arguments are used to support such a claim.³ First, it is argued that labor supply and demand considerations make it efficient to raise the net labor income of low-wage workers, in particular women, and that a change in the tax system to a more progressive one usually will have just such an effect.⁴ A second argument is that when labor markets are not competitive, wages may be inefficiently high at the market equilibrium, and that progressive taxation may contribute to lower equilibrium wages.⁵ A third and related argument is that when wage setters have egalitarian objectives, progressive taxes may reduce the need for redistribution in pre-tax earnings, thus potentially increasing the demand for low-productivity workers.

In this paper we will study effects of progressive income taxes within a model focusing on the second of the three arguments just referred to above. We consider an economy where each worker's productivity is given, but productivities differ across individuals, and firms and workers bargain individually over wages. We assume "instantaneous matching" in the sense that hiring firms may immediately hire an unemployed worker, provided that they incur a positive hiring cost, and that firms cannot observe the productivity of individual workers prior to hiring them. There is then always some unemployment among all worker groups at equilibrium. In section 2 of the paper we study a "partial equilibrium" version of this model, where the rate of unemployment is taken as exogenously given. We here also assume that no workers are "unwanted", i.e., even workers at the lowest level of productivity are active, and employable, in the labor market. Given a general income tax

³These arguments are surveyed in Røed and Strøm (1999). See also Agell (1999), and references therein, who presents and surveys different but partly overlapping arguments in favor of wage compression, through the tax system or by other means.

⁴For labor supply see e.g. Hausman (1985), Juhn et.al. (1991) and Blundell (1997), and for labor demand OECD (1995).

⁵For analyses with union wage determination or union-firm bargaining, see Hersoug (1984) and Koskela and Vilmunen (1996), and with efficiency wage determination, Hoel (1990) and Andersen and Rasmussen (1999). See also Sørensen (1997), who concludes that imperfectly competitive models of wage setting typically predict a negative relationship between tax progressivity and equilibrium unemployment.

function, we study the effects of a more progressive tax system, on equilibrium gross and after-tax wages. We find that the after-tax wage is reduced when taxes become more progressive, except possibly for workers whose absolute tax rates are reduced substantially (most likely, those at the very low end of the wage scale). The effect on the pre-tax wage is more typically ambiguous, but is unambiguously negative for workers whose absolute tax rate is not increased when progression increases.

To better understand these effects, we can view a change in the tax system as split into a pure change in progressivity, and a pure change in average taxation. An increase in progressivity reduces both the net and the gross wage. Intuitively, such an increase leads to a reduction in the effective bargaining power of workers relative to that of the firm. A pure increase in the average tax rate reduces the net wage, but increases the gross wage. Intuitively, the “cake” to be split between the two parties, the worker and the firm, is then reduced, and this reduces both the firm’s and the worker’s net return. A reduction in the firm’s return implies in our model that the gross wage increases.

In section 3 we study a model which is more general in some respects. Here the rate of employment is endogenized through a free-entry condition on jobs. We now also assume that the effective labor supply is endogenous: not all workers are “employable”, and that only those workers who earn a net wage in excess of the value of leisure, supply their labor to the market. We make the special assumption of a uniform distribution of worker productivities, and assume that the income tax is linear with constant marginal tax rate t_1 , and a positive standard deduction y , assuming no other taxes or subsidies. We first derive a (hypothetical) first-best solution for such an economy. Here all workers whose productivities are in excess of the value of leisure, plus the annualized value of the firm’s hiring cost, are employed. This solution cannot be implemented since the market solution requires there to be some unemployment at equilibrium, among all employable workers. We next derive the second-best allocation in this economy, given that the government only can use the tax scheme in question, and given that the government’s shadow value on tax receipts equals that on net

private-sector receipts. We then find that the second-best productivity cutoff level is lower than the first-best level. The second-best solution is implementable using a family of combinations of t_1 and y . A higher t_1 then implies a higher total tax revenue, and higher total taxes for all active workers. For all workers the after-tax wage is reduced when tax revenues increase, while firms' profits are unaffected.

A conclusion from our model example is that the government can make the tax system more progressive, and extract more total tax revenue, without any adverse consequences for the real allocation in the economy. Thus, in contrast to e.g. the standard optimal taxation literature, increasing government tax revenues through increased tax progressivity is a “free lunch”; it involves no allocational cost. One must however be careful in interpreting these results; in particular, they are based on the assumption that there are no disincentive effects on worker effort or human capital accumulation, of the types stressed in the traditional literature of e.g. Mirrlees and Atkinson-Stiglitz referred to above, from more progressive taxes. One can perhaps argue that some of our assumptions are “rigged” in favor of obtaining our main result, that there are no negative allocative effects of increased tax progressivity.

Most of the received literature dealing with progressive taxation departs from an assumption that all workers in question earn the same wage. Arguably, the issue of tax progressivity is more interesting when taxable incomes differ. We here provide a first consistent (albeit simple) such model, which may indicate some main effects and directions for future work. Discussions of potential weaknesses of the model, and future extensions of the current framework, are given in the final section 4.

2. Effects of tax progressivity in a partial equilibrium model with full employment

In this section we study consequences of changes in tax progressivity, in a partial equilibrium model, where the overall unemployment rate, and the overall number of firms in the economy, both are assumed exogenous,

and unaffected e.g. by changes in tax parameters. We also make the simplifying assumption that all worker types are “employable”, i.e., employment is mutually beneficial for a firm and a worker at the low end of the productivity scale. These simplifications make it possible to operate with a relatively general tax function. In section 3 below we will study a more general model, where instead the tax function is more specific.

Consider an economy where workers’ productivities differ and are fully general (i.e., a given worker has the same productivity in all firms).⁶ Following e.g. Pissarides (1990), each firm employs one worker, and all firms are identical. Denote the output produced by a worker by z . Across all workers in the economy, z has a continuous distribution function $F(z)$ with compact support $[z_0, z_m]$, where $z_0 \geq 0$. We assume that each firm knows the function F , that the productivity z_i of worker i cannot be identified at the time a worker is hired and hiring costs are sunk, but that z_i is discovered immediately after this time. Firms may establish freely, implying that the equilibrium value of establishing a firm is zero, but firms must incur a hiring cost H when taking on a worker. H is fully paid by the firm. Denote the gross wage of a worker by w . The net wage is then $w - t(w)$, where $t(w)$ is the income tax paid by the worker. The net surplus of a worker-firm match is then $z - t(w)$, and the current net firm return (after the hiring cost has been sunk) is $z - w$. Assume that, in each worker-firm match, the wage is determined by an asymmetric Nash bargain between the worker and the firm, with relative bargaining parameters β and $1-\beta$, respectively. In general, the outcome for the wage in an individual bargain will be a strictly increasing function of productivity z . This implies that we may write the resulting wage as a function of z , $w(z)$. Denote the discount rate (of both the firm and worker) by r , and assume that matches break up at an exogenous rate s . As a consequence turnover is assumed to be fully exogenous and not affected by any of the other variables in the model.

Denote the asset value for the firm, of having a worker with productivity z (after H is sunk), by $Q(z)$. We

⁶The model largely follows Strand (1999a). A difference from that paper, which will become apparent in section 3 below, relates to assumptions about the behavior of workers who are “unwanted” in the labor market. Since we in this section assume that no workers are unwanted, this difference is of no consequence here.

then have the Bellman equation $rQ(z) = z - w(z) - sQ(z)$, where it is recognized that the asset value of the firm

$$Q(z) = \frac{1}{r+s} [z - w(z)]. \quad (1)$$

is zero after a separation (by virtue of the free entry condition). This implies the following solution for Q:

Consider now the current value of employment for a worker with productivity z , $W(z)$. This is determined from a similar equation $rW(z) = w(z) - t(w(z)) + s[U(z) - W(z)]$, where $U(z)$ is the value of current unemployment

$$W(z) = \frac{1}{r+s} [w(z) - t(w(z)) + sU(z)]. \quad (2)$$

for a worker with productivity z . The solution for $W(z)$ in terms of $U(z)$ is

With identical firms, $U(z)$ is in turn given by $rU(z) = b + h[W(z) - U(z)]$, where b is a (constant and net) value of unemployment,⁷ h is the (constant) rate of hiring among unemployed workers. Since each individual worker's productivity cannot be identified by firms at the time of hiring, h must be the same for all workers and

$$U(z) = \frac{(r+s)b + h[w(z) - t(w(z))]}{r(r+s+h)} \quad (3)$$

thus independent of z . Using (2), $U(z)$ is then given by

Here U is a function of the market wage (which generally depends on z) and thus of z . In interpreting (3), U must be viewed as a function of the wage in other firms than the one in which the worker is currently employed.

Thus in the particular bargain relevant for setting w , $U(z)$ must be taken as exogenous. The Nash product to be maximized is given by $[W(z) - U(z)]^b Q(z)^{1-b}$. Maximizing with respect to w yields the first-order condition

⁷ b could here alternatively be interpreted as after-tax unemployment benefits or the value of leisure. In the welfare analysis in section 3 below the latter interpretation is the more natural and appropriate.

$$\beta(1-t')Q(z) = (1-\beta)[W(z) - U(z)], \quad (4)$$

where t' denotes dt/dw , i.e., the marginal tax rate on wage income. Manipulating the expressions for Q , W and U implies that we may derive the following expression for the before-tax wage, as a function of the average

$$w(z) = \frac{\mathbf{b}(r+s+h)(1-t')z + (1-\mathbf{b})(r+s)b}{\mathbf{b}(r+s+h)(1-t') + (1-\mathbf{b})(r+s)(1-t)}. \quad (5)$$

and marginal tax rates:

Whenever $t' \in [0, 1)$, w is here a strictly increasing function of z , and depends on the average and marginal tax rates, t and t' . Note that in general, both t and t' will vary with z . (5) generalizes an equivalent expression in Strand (1999a), to the case of income taxation (i.e., $t > 0$, $t' \neq 0$)⁸. To discuss the effects of changes in tax progressivity, define $(1-t')/(1-t) \equiv a$ as the degree of progressivity of the tax system.⁹ Following e.g. Jacobsson (1976) and Hersoug (1984), when $a = 1$, we then say that the tax system implies proportional taxes, while when $a < 1$, we say that the tax system is progressive, and more progressive the lower is a .

The net after-tax wage is given by $(1-t)w(z)$, and the current net compensation of the worker (over his alternative b) by

$$n(z) = w(z) - t(w(z)) - b = \frac{\mathbf{b}(r+s+h)a}{\mathbf{b}(r+s+h)a + (1-\mathbf{b})(r+s)} [(1-t)z - b]. \quad (6)$$

⁸More precisely, we generalize the version of Strand (1999a) where firing costs (F_0 , F_1 and F_2) are set equal to zero.

⁹This measure was first introduced by Musgrave and Thin (1948), and is what Atkinson and Stiglitz (1980, p. 37) refer to as the "residual income progression" parameter. Note that it is really a local, and not in general a global, measure of progressivity, as it cannot necessarily yield meaningful comparisons of progressivity at differing income levels, for a general tax function. For our purposes it is however useful, and will generally yield a consistent measure of progressivity for the particular class of linear tax functions considered in section 3 below.

Consider now effects of changes in the tax system on wages and net worker compensation, when the rate of rehiring h is kept constant. From (5) and (6), when h is constant, we may now derive the following

$$\frac{d w(z)}{d a} = \frac{(1-b)(r+s)}{(1-t)A^2} \left[\mathbf{b}(r+s+h)[(1-t)z-b] + \frac{A b d t}{(1-t)d a} \right] \quad (7)$$

expressions:

where $A = \beta(r+s+h)a + (1-\beta)(r+s)$. Consider here first the case where $dt/da = 0$ for a given worker, i.e., we

$$\frac{d n(z)}{d a} = \frac{\mathbf{b}(r+s+h)}{A^2} \left[(1-b)(r+s)[(1-t)z-b] - A \mathbf{a} z \frac{d t}{d a} \right], \quad (8)$$

consider a pure change in tax progressivity, without any change in the level of absolute taxes for that worker. Note that for all workers in question here, $(1-t)z - b \geq 0$: this condition must be fulfilled if all workers are to be gainfully employed (and, at equilibrium, prefer being employed) in the labor market, instead of enjoying leisure. A reduction in a here implies an increase in the progressivity of taxes. We then see that both w and n fall, i.e., a pure increase in progressivity reduces both the gross and the net wage of a worker experiencing such an increase. Moreover, when the tax rate is constant they must fall proportionately, w being reduced by more.

The main intuitive reason behind these effects is that a more progressive tax system makes it less advantageous for the firm and the worker together to set a high wage rate, since a greater share of the “cake” is then eaten up by taxes. This factor has the implication of reducing workers’ effective bargaining power, and increasing that of firms. One interpretation of the solution is obtained by noting that the “effective bargaining power” parameter of workers is $a\beta/[a\beta+(1-\beta)]$, and that of firms $(1-\beta)/[a\beta+(1-\beta)]$. Thus when $a = 1$ (i.e., under proportional taxation), we have the standard case where the respective bargaining powers are β and $1-\beta$. The other (hypothetical) extreme case of $a = 0$ (with a marginal tax rate equal to one). In this case

workers' bargaining power is reduced to nil. It is then futile for workers to try to obtain a wage exceeding b ; all the extra income is in any case eaten up by taxes.¹⁰

The case where a change in the tax system implies that only the marginal, and not the absolute, tax rate is changed, is very special. For most individuals a change in progressivity must be accompanied by a change in the rate of absolute taxation. In (7)-(8), this is represented by the last terms in the respective large square brackets. This term is positive in (7), and negative in (8). Assume first that an increase in progressivity goes together with an increase in the absolute tax level. Then $dt/da < 0$. In (7), the two main terms, representing respectively effects of a pure change in progression, and a change in the rate of absolute taxes, then go in opposite directions, while they go in the same direction in (8). Correspondingly, $dw(z)/da$ is ambiguous, while $dn(z)/da$ is unambiguously positive. If this is a "typical" case (which may be argued), the net wage will unambiguously fall when taxes become more progressive (a decreases), but we cannot in general say whether the gross wage will fall or increase. Intuitively, the increase in the absolute tax rate reduces the "cake" to be shared between the two parties. This "cake reduction" is split between the two parties, and implies that the firm's return is lower than under a constant t . This effect may be stronger than the positive effect on profits of the pure progressivity increase. If so, profits are affected negatively, and $w(z)$ positively. For $n(z)$, however, both the progressivity effect and the cake reduction effect are negative and thus go in the same direction.

In the opposite case, an increase in progressivity goes together with a reduction in the absolute level of taxation. Then a progressivity increase will reduce the gross wage unambiguously, while the effect on the after-tax wage is ambiguous.

In practice individuals with high incomes are likely to experience a higher absolute tax rate when the tax system is changed to a more progressive one, while individuals with low incomes could experience a lower tax

¹⁰This of course requires that for wages at b or lower, taxes are sufficiently low that $(1-t)z > b$ for all relevant z .

rate, at least when overall government revenues are not increased drastically as a result of the increase in progressivity. For high-income individuals we can then expect disposable after-tax wages to be reduced, while gross (before-tax) wages can change in either direction. For low-income individuals we can instead expect gross wages to fall, and net wages to go in either direction.

In most practical cases a will also differ at the outset between individuals, and be affected in different ways by changes in the tax system.¹¹ Thus the level and changes in tax progressivity cannot in general be represented by a single parameter.

To get a more concrete idea of how much tax progressivity may differ, consider the following special tax function, which will also be exploited in the following section 3:

$$T(w) = t_1 (w - y), \quad (9)$$

where T is total income tax paid. In (9) there is a constant marginal tax rate t_1 for income above a “standard deduction” level of y . We could here have $y > w$, implying a negative income tax. The progressivity parameter a can now be expressed as follows:

$$a = \frac{I}{I + \frac{t_1 y}{I - t_1 w}}. \quad (10)$$

This implies in particular that for $w = y$, $a = 1 - t_1$. In the cases of $t_1 = 0$, $y = 0$ or $w = \infty$, $a = 1$, i.e., the tax system is proportional.

Consider workers’ net take-home pay, $n(z)$ from (6), in two special cases, namely i) incomes very close

¹¹This will also be the case under the more specific example studied in section 3 below.

$$n(z) = \frac{\mathbf{b}(r+s+h)(1-t_1)}{\mathbf{b}(r+s+h)(1-t_1) + (1-\mathbf{b})(r+s)}(z-b). \quad (11)$$

to y , and ii) incomes approaching infinity. In case i) we find

$$n(z) = \frac{\mathbf{b}(r+s+h)}{\mathbf{b}(r+s+h) + (1-\mathbf{b})(r+s)}[(1-t_1)z - b]. \quad (12)$$

The equivalent expression in case ii) is

In (12), under case ii), b becomes inconsequential as z tends to infinity. This implies that an increase in t_1 (and thus increasing the progressivity of taxes at low income rates but not at high) reduces the share of take-home pay in total worker output by relatively more at high than at low income levels. This is the opposite of the effect of a pure progressivity change, discussed after equation (6) above. The reason is that the higher average tax rates at higher income levels reduce the “cake” to be shared between the worker and the firm by relatively more, and this effect tends to overwhelm the pure progressivity effect appearing via the term $(1-t_1)$ in (11).

3. Optimal tax schemes with endogenous employment and efficient self selection

3.1 Presentation of the model example

We will now consider a more elaborate model example. So far we have assumed that all worker types are being accepted by firms. This requires that $n(z) > 0$ for all relevant levels of z , which in turn requires $(1-t(z))z_0 > b$, where $t(z_0)$ now is the average tax level at the wage earned by a worker with productivity z_0 . We

now instead assume that $(1-t)z_0 < b$, which may occur when the average tax rate t is relatively high for workers at this wage level, or z_0 is low. We assume that only employable workers actually apply for jobs.¹² This implies that when a firm hires a worker, the condition $(1-t)z \geq b$ is fulfilled for that worker. Defining z_1 by $(1-t)z_1 = b$, we then have that all workers with z at or above z_1 apply for jobs, while workers with z below z_1 stay out of the active labor market.

So far we have assumed that h is kept constant. In reality h is endogenous and determined by supply and demand conditions in the labor market as a whole, and thus e.g. also by the progressivity of the tax system. In order to endogenize h , we must make a few additional assumptions. First, we will make the simplifying assumption that a new job which is posted by a firm is always filled immediately (perfect matching of workers would imply $h = \infty$ and thus $w = z$ from (5), which is impossible since no firms then would enter, given that entering firms need to put up a hiring cost H). The current expected profit to the firm, associated with having

$$Ep = \int_{z_1}^{z_m} [z - w(z)] dF(z). \quad (13)$$

a job filled after posting it, is given by

The free entry condition can now be expressed as follows:

¹²Here the model differs from Strand (1999a). There I assumed that also workers whose productivities are below this level, are active in the labor market by applying for jobs. Such behavior “contaminates” the labor force when (as here) firms cannot observe individual workers’ productivities prior to hiring, since hiring cost need to be incurred also for workers that firms do not wish to keep. Nonemployable workers were there assumed to be economically active e.g. since active labor market participation was a prerequisite for collecting unemployment benefits. When disregarding the concern for unemployment benefits, or when the payment of such benefits is not contingent on proven active labor force participation, there is no logical reason why workers whose productivities are below z_1 should apply for jobs.

$$E_p = (r+s)H, \quad (14)$$

where the latter term expresses the annualized cost (as distributed over the worker's expected attachment time to the firm) of the recruiting cost H . We thereby obtain an additional condition which, together with the set of conditions (5), endogenizes $w(z)$ and h . Finally we will assume that the overall number of workers is constant.

In the following analysis in this section we will concentrate on a particular example with a uniform productivity distribution function, on $[0, 1]$, i.e., $z_0 = 0$, and $z_m = 1$, and a linear tax function $T = t_1(w-y)$, exposed in section 2 above. This implies that all active workers are subject to the marginal tax rate t_1 . As already discussed above, $a < 1$ implies a progressive tax system. Note then again that the larger the last term in the denominator of (10) is, the more progressive is the tax system by our definition. Thus a higher marginal tax rate t_1 , and a higher ratio y/w , imply more progressive taxes. Correspondingly, for higher w taxes become less progressive. There is then less progression for high-wage (and thus high-productivity) workers than for low-wage workers.

As a consequence of our assumptions about the distribution of productivities across workers, when considering the entire stock of workers the expected productivity is given by $Ez = 1/2$. In general only workers with productivities above some minimum level $z_1 > 0$ will however be employed. The average productivity of workers who are actually employed is then given by $E_c z = (1+z_1)/2$.

Under the same bargaining model over wages as that used in section 2 above, the equilibrium wage function can now be expressed as follows:

$$w(z) = \frac{\mathbf{b}(1-t_1)(r+s+h)z - (1-\mathbf{b})(r+s)t_1 y + (1-\mathbf{b})(r+s)\mathbf{b}}{(1-t_1)(r+s+\mathbf{b}h)}. \quad (15)$$

The cutoff level z_1 above which workers will be active is now given by $w(z_1) - T(w(z_1)) = (1-t_1)w(z_1) + t_1 y = \mathbf{b}$, which implies that $z_1 = w(z_1)$. z_1 can then be written as

$$z_1 = \frac{b - t_1 y}{1 - t_1}. \quad (16).$$

A necessary condition is here that z_1 always be between zero and one. This requires that $t_1 < (1-b)/(1-y)$, which is assumed to be fulfilled in the following.¹³

Firms hire workers randomly from the pool of the unemployed, on the productivity range $[z_1, 1]$. As noted the distribution of the unemployed on this range has the same shape as the underlying distribution for the entire population of workers, and the firms' sampling distribution will thus also be uniform. Using (15), (16) and the condition $E_c z = (1+z_1)/2$, we may then express the expected current profit of a firm from a hired worker, E_p ,

in the following two alternative ways:

E_p here decreases in h , for given other parameters. In fact, taking the tax parameters y and t_1 as given (implying that z_1 is endogenously determined through (16)), h is the only endogenous variable in (17), and it is determined by the free-entry condition (14). A requirement for the existence of a solution is then that $E_p > (r+s)H$ for $h = 0$. This condition puts an upper bound on H which is here assumed to hold. For given h , and assuming $b > y$, E_p is seen to depend on other key parameters in the following way:

- E_p falls when either β or b is raised. Intuitively, increases in both these parameters raise the bargaining power of workers relative to that of the firm, thereby increasing wage.

¹³The condition imposed on t_1 is here not particularly strong. With natural assumptions about b , say, $b = 1/4$ (implying that the output of the most productive worker is four times the common value of unemployment), the requirement is that $t_1 < 0.75$, regardless of y . In practical cases t_1 will perhaps be in the range 0.2-0.5. The consequences for z_1 can also readily be calculated. Note here that z_1 will generally always be greater than b when $t_1 > 0$ and $b > y$. E.g., consider the case where $t_1 = 0.5$, $b = 0.25$ and $y = 0.20$. This implies $z_1 = 0.30$.

- E_p increases when y is raised. An increase in y implies a lower average tax for a given marginal tax, which implies that the total “cake” to be divided between the worker and firm is increased, part of which is captured by the firm.

$$\frac{dE_p}{d t_1} = -k \left(\frac{1}{1-t_1} \right)^2 (b-y) < 0, \quad (19)$$

- A partial increase in t_1 reduces E_p for given h . This is seen from differentiating (14), as follows:

where k is a positive constant (which however depends on the endogenous variable h). An increase in the marginal tax rate here also implies that average taxes are raised, parts of which are generally paid for by the firm through lower profits. Note however that a) an increase in t_1 implies that the relative bargaining power of workers is reduced (a increases), thus reducing the negative effect on profits; and b) that an increase in t_1 implies that the minimum required worker productivity level, z_1 , increases. The latter is seen from differentiating (16), as follows:

$$\frac{d z_1}{d t_1} = \left(\frac{1}{1-t_1} \right)^2 (b-y) > 0. \quad (20)$$

Consequently, a partial increase in t_1 implies that the average quality of employed workers increases. We still find that within our specific model the overall effect of increased t_1 on E_p is negative. This is however not a general result, but depends on the parametric assumptions utilized in this section.

Notice that via (18), expected profits can (for given h) be written as a function of the minimum hiring standard z_1 , while the tax parameters y and t_1 do not directly enter this expression. The same must hold for the expression for the expected wage across all active workers, which is correspondingly:

$$E w = \frac{1}{2} \frac{\mathbf{b}(r+s+h)(1+z_1) + 2(1-\mathbf{b})(r+s)z_1}{r+s+\mathbf{b}h}. \quad (21)$$

Thus the distribution of total value added between profits and gross wages is not affected by the tax parameters

whenever the minimum worker quality standard is kept constant. The latter condition requires the particular relationship between the tax parameters derived in (16). This conclusion is important for the discussion below, on optimal government tax policy.

3.2 First-best welfare analysis

We will now study welfare properties of the solution derived in section 3.1 above, and how welfare depends on the parameters of the tax function, t_1 and y . We initially consider a first-best case where the government is in full control of production. When the total number of workers in the economy is normalized to one, and b interpreted as the social value of workers' leisure, a utilitarian government will then at an initial

$$R(fb) = \frac{h}{s+h} \int_{z=z_b}^{z_m} \left(\frac{z-b}{r+s} \right) dF(z). \quad (22)$$

point of time seek to establish a number of jobs so as to maximize the following expression:

The government is here viewed as directly maximizing first-best welfare, $R(fb)$, with respect to h and z_b , where the latter is interpreted as the minimum quality standard on workers corresponding to the first-best solution. $h/(s+h)$ expresses the fraction of the stock of workers, in excess of the minimum quality standard, that are to be employed at such a solution. The solution to this maximization problem is simple: set $h = \infty$ (corresponding to full employment among wanted workers), and z_b to make the bracket in (22) equal to zero, i.e.,

$$z_b = b + (r+s)H. \quad (23)$$

Optimally, thus, the minimum worker quality standard should compensate both for workers' leisure value, b ,

and for firms' total current costs associated with hiring a worker, $(r+s)H$.¹⁴

3.3 Second-best analysis with no concern for government tax revenue

We will now study government policies in the market solution, where the government no longer is in direct control of production, but must rely on its existing market instruments to affect it. Our objective is here to seek an optimal combination of t_l and y , which maximizes welfare given that employment is determined in the market, by the model exposed in subsection 3.1 above. We will in this subsection assume that the government has no concern for tax revenue per se. Moreover, taxes create no distortions apart from those directly described by the our model. We may here e.g. view the government as taxing labor income, and disbursing the tax revenues in the form of a uniform lump-sum subsidy to all individuals.¹⁵ The government has no instruments apart from these two tax parameters (e.g., profits cannot be taxed, and tax revenues cannot be used to subsidize firms or job creation). The welfare function of a utilitarian government in this case, $R(m)$, can

be written on the form

$$R(m) = \frac{h}{s+h} \frac{1}{r+s} (1 - z_c) \left[\frac{1}{2} (1 + z_c) - b - (r+s)H \right], \quad (24)$$

where z_c now denotes the constrained optimal level of minimum productivity in this case. Note that when h is held fixed, maximizing $R(m)$ with respect to z_c yields $z_c = z_b$, i.e., the government would choose a first-best

¹⁴ rH can here be considered the current interest costs, and sH the current cost of servicing the principal, provided that the firm borrows to finance H , and such that these costs are incurred only as long as a given worker is actually employed.

¹⁵This of course raises the issue of why the government at all collects taxes in this economy. We will not worry about this issue here, but will come back to it in subsection 3.4 below.

productivity cutoff level. The government's problem is however complicated by the fact that h is endogenous and affected by z_c . The market solution must then obey (14) and (17)-(18), with z_1 replaced by z_c , implying

$$(r + s + \mathbf{b}h)H = \frac{1 - \mathbf{b}}{2}(1 - z_c). \quad (25)$$

The constrained optimal solutions for h and z_c can now be derived maximizing (24) under the constraint (25).

This implies the following additional condition:

$$(1 - \mathbf{b})s(1 - z_c) \left[\frac{1 + z_c}{2} - z_b \right] = 2\mathbf{b}h(s + h)H(z_b - z_c). \quad (26)$$

Here z_b is given by (23). (25)-(26) now together solve for the endogenous variables h and z_c . Although we do not have closed-form solutions for these variables, we may note a number of interesting properties of the constrained optimal solution. Among these are the following:

1. The tax parameters t_1 and y do not directly enter into the set of conditions (25)-(26); these parameters are “suppressed” in the sense of only entering the expression for z_c (or rather z_1 , in (16)). The implication of this is that any combination of t_1 and y is compatible with a constrained optimal solution, only provided that z_c is constrained optimal.
2. The solution for h must be finite.¹⁶ This implies that there must always be some unemployment among “wanted” workers at equilibrium, while first-best efficiency requires that there be no such unemployment. Intuitively, if there were no unemployment among the wanted worker group, workers would have all bargaining power at equilibrium (since they would always immediately find a new job after being fired), and equilibrium firm profits would be zero. The latter would imply no firm establishment.

¹⁶This is seen directly from (14) and the equilibrium condition $E_p = (r+s)H$. Fulfillment of this condition requires a

finite h.

3. From (26) we see that $z_b \in (z_c, (1+z_c)/2)$ (since both sides of (26) must be positive). Thus in particular, $z_b > z_c$, stating that the constrained optimal cutoff productivity level is lower than the unconstrained optimal level. Tax parameters must consequently be set “low” in order to make z_c is less than z_b . The intuitive reason for this is that low taxes are necessary in order to spur firm entry, and thus employment, since there is underemployment (relative to the first best) at equilibrium, as a consequence of h being finite as noted under point 2 above. There is thus an equilibrium tradeoff, between some inefficient unemployment among those groups of workers who are employed, and some inefficient employment of workers with low productivities (in the range between z_c and z_b).

3.4 The case with government concern for tax revenues

While we so far have not required that the government collect tax revenues, in practical cases the government will be concerned with the amount of taxes collected. It is then of interest to study how total tax revenues vary when the tax parameters t_1 and y change in such a way that z_c is kept constant (and equal to z_1 as given by (16)). Using (16) to solve for y in terms of z_c , we may express collected taxes per worker as a function of t_1 and z_c . For a worker with productivity z we then find the following expression:

$$T(z) = \frac{b(r+s+h)}{r+s+bh} t_1 (z - z_c) + z_c - b. \quad (27)$$

We consequently find that an increase in tax progressivity (whereby t_1 increases) in this case implies that all workers pay higher taxes, and that the increase in taxes is greater for higher-productivity workers.¹⁷ Total tax

¹⁷From (27) we see that a sufficient condition for $T(z) > 0$ for all z is that $z_c > b$ (and in the special case of $z_c = b$ we find, from (16), that $y = b$). Now the system (25)-(26) does not in general guarantee that $z_c > b$. In such cases (27) will imply a negative income tax for individuals at the “low end” of the productivity distribution.

revenues must then increase in t_1 , for given h and z_c . The gross wage, $w(z)$, and the net take-home pay (net of leisure), $n(z)$, can now be expressed as follows:

$$w(z) = \frac{\mathbf{b}(r + s + h)z + (1 - \mathbf{b})(r + s)z_c}{r + s + \mathbf{b}h}. \quad (28)$$

$$n(z) = \frac{\mathbf{b}(r + s + h)}{r + s + \mathbf{b}h}(1 - t_1)(z - z_c). \quad (29)$$

(28) may here be interpreted in terms of the formula derived in (7), where we recall that the two terms in (7) express the effect of a pure increase in progressivity, and the effect of pure increase in the tax burden. The effect of the first factor was as noted always negative, and that of the second positive when an increase in progressivity also implies an increase in the absolute tax rate. As seen from (27), taxes must increase for all individuals when progressivity increases, which here implies that the average tax rate increases for all. In fact, in our particular case these two effects turn out to exactly cancel for all individuals, so as to leave the gross wage constant for all; this is also seen directly from (28), since t_1 does not enter into the expression for $w(z)$. From (8), net pay is always reduced when tax progressivity and the average tax rate both increase. This is confirmed by (29), and the absolute reduction in net pay is greater for workers with higher productivities (it is a linear function of $z - z_c$).

Since workers' gross wages are independent of the tax function for given h and z_1 , firms' total expected profits per worker must also be constant, something that is also seen directly from (18). Thus firm entry is not affected, and h is unaffected by the tax parameters t_1 and y as long as z_1 is given. This has an important and interesting implication, namely that an increase in tax progressivity, whereby t_1 and y are simultaneously increased so as to keep z_1 constant in (16), has no effect on the real allocation in the economy. In particular, employment is unaffected by such a change. But from (27), total tax revenues are thereby increased. In other

words, the government can raise its revenues costlessly, by changing the tax system to a more progressive one, as long as the tax system obeys (16) with z_1 constant ($= z_0$). Revenues are thereby shifted from wages to taxes, and workers' after-tax earnings are reduced in proportion to their productivities.

4. Conclusions and final comments

We have studied an economy with instantaneous worker-job matching where heterogeneous workers bargain individually with their employers over wages, and have introduced a progressive income tax scheme into this economy. The following main results stand out.

1. Given that overall employment is constant (assumed in section 2), we show that a pure increase in tax progression implies lower wages, since such an increase reduces workers' effective bargaining power relative to that of firms. Higher average taxes also reduce (after-tax) wages. A shift to a more progressive tax scheme then unambiguously reduces net after-tax wages at high wage levels, since these two effects then go in the same direction. For workers at low wage levels a more progressive tax scheme may imply lower average taxes, in which case the overall effect on after-tax wages are ambiguous.

2. With a linear tax scheme, uniform productivity distribution, and endogenous employment under free entry of identical jobs (studied in section 3), we show that the first-best solution implies that all workers above a certain minimum productivity standard are employed. This solution is however not implementable in the market, since there must be some unemployment among "employable" worker groups at equilibrium. When the government is constrained to using a linear tax scheme, and employment market determined, the second-best minimum productivity standard of workers is lower than the first-best level. This tax scheme trades off a "too low" employment among the employables, against a "too great" productivity range for workers to be hired.

3. The second-best solution just described above can be implemented using a whole family of linear tax schemes, with different degrees of progression. A more progressive scheme within this family will increase the government's tax revenue, and reduce each worker's net after-tax wage accordingly. This implies that there are no allocative effects of more progressive taxes, only distributional effects due to income redistribution from workers to the government. With a positive government value on net tax receipts an increase in tax progressivity should then be desirable for the government. The attractiveness of such a change is enhanced further if the government prefers a more even income distribution, since the dispersion of after-tax incomes is then also reduced.¹⁸

The bottom line of our analysis boils down to two main conclusions. First, when there is wage bargaining between individual workers and firms, a more progressive system of income taxation reduces the equilibrium wage by effectively reducing the relative bargaining power of workers. Secondly, under such circumstances a more progressive tax system does not necessarily lead to increased allocative disturbances. This has been shown in the context of a very simple model example, where among other things moral hazard considerations are unimportant. In our model, an increase in progression in fact leaves allocation intact and just raises government revenues, which may be viewed as desirable.

To my knowledge this is the first model in the literature where consequences of progressive income taxation are studied in labor market equilibrium with heterogeneous workers. In our model of section 3, labor demand and supply are both endogenous. The supply side is modelled by assuming that only workers whose wage

¹⁸Note that the government could in principle extract an arbitrarily high share of wage income in excess of b , by setting t_1 sufficiently close to one (and y sufficiently close to b , by (16)). In practice factors not dealt with directly in this paper would likely prevent such an extreme solution; in particular, the moral hazard effects stressed in the traditional optimal tax literature would then come to play with full force. This however does not override our main conclusion, that an increase in progressivity starting from a "low" progressivity level should be viewed favorably by the government.

exceeds their opportunity cost actually supply their labor to the market. Due to the many simplifications the model must still be viewed as an initial simple attempt to deal with such issues. First, we have assumed that there are no “unemployable” workers in the pool of unemployed seeking work at any given time. Having “unemployables” in this pool complicates the situation for firms since they then must immediately dismiss some of the workers hired. It then matters what mechanisms firms have to screen out unwanted workers before engaging them. Such a mechanism, based on information about workers' past labor market history, was proposed by Strand (1987). Strand (1999a) studies a model related to the current one where there is no screening whatsoever, and demonstrates that this leads to considerable complications due e.g. to externalities in the hiring process, and to distorting effects of dismissal costs (which are not considered here). Moreover, while we here assume that all productivity differences are purely individual-specific, in practice there will always be both a firm-specific and an individual-specific component, the implications of which should be explored in future work.

Another important issue is the mechanism for wage determination. Here we have assumed individual bargaining, but obvious alternatives exist such as centralized union-firm bargaining and firm wage determination (wage posting). A model where both centralized and individual wage bargaining are considered is Strand (1999b); a result here is that a more even productivity distribution across workers makes it more likely that a sector with centralized bargaining can coexist with a sector with individual bargaining. This tendency could be stronger under progressive taxation, but this remains to be studied explicitly. In related work Ellingsen and Rosén (1997) demonstrate that also wage posting is a more stable institution versus individual bargaining when the productivity distribution is less spread out. Inderst (1999) studies wage posting under heterogeneous labor, and shows that this is generally not renegotiation proof when the alternative is individual wage bargaining, but may be so when workers are organized in a union. It will be of interest to study consequences of progressive taxation in such frameworks. A prior hypothesis could then be that individual bargaining is more likely to

survive, the less progressive taxes are.

Finally, and perhaps most importantly, we have avoided all problems related to moral hazard. This clearly biases our analysis in favor of a too positive view of progressive taxation. It is however also clear that combining the current adverse selection approach with moral hazard will imply additional analytical complications which are difficult to fully foresee. One likely implication is that efficiency wage considerations will come into play and thus that wage posting may play a larger role, as in the traditional moral hazard based efficiency wage literature (Shapiro and Stiglitz (1984), Strand (1987), Albrecht and Vroman (1998, 1999)). The implications for the equilibrium wage distribution and for the effects of progressive income taxation are more difficult to predict, but are interesting topics for future research.

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