What a Puzzle! Unravelling why UK Phillips Curves were Unstable

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Abstract

Between 1860 and 2021, UK Phillips curves linking wage inflation ($\Delta w$) and unemployment ($U_r$) exhibit every slope in sub-period regressions from strongly negative, slightly negative, flat, slightly positive and strongly positive. These sub-period outcomes are predicted by an econometric model of real-wage growth expressed in terms of $\Delta w$. Correcting $\Delta w$ for its regressors other than $U_r$, its sub-period regressions on $U_r$ all have the same negative slope. However, ‘shifts’ in the real-wage model’s variables do not explain the instabilities: surprisingly, the Phillips curves shift when some of the real-wage model’s sub-period regressors are insignificant.

\textbf{JEL classifications:} C2, C5, J3.

\textbf{Keywords:} Phillips Curves; Wages; Unemployment; Inflation; Structural Breaks.

1 Introduction

The instability over time in Phillips curves is well known and well documented by both academics and policy-makers; see Del Negro et al. (2020) and Haldane and Quah (1999) for the former and Powell (2019) and Cunliffe (2017) for the latter, both from a US and UK perspective. This instability is demonstrated for the UK in Figure 2 which records five subsample estimates of the Phillips curve using annual data from 1860–2021. Every slope in sub-period relationships between wage inflation ($\Delta w$, where lower case letters denote logs) and the unemployment rate ($U_r$) is observed, from strongly negative, slightly negative, flat, slightly positive and strongly positive. These outcomes are compared with those over the same sub-samples derived from the econometric model of real-wage growth in Castle and Hendry (2014) updated to 2021 in Castle et al. (2023), expressed in terms of $\Delta w$, revealing a close match in every sub-period. Thus, the constant real-wage growth model can account for the instabilities in the simple Phillips’ curve, a successful mis-specification encompassing result (see Hendry, 1995, Ch. 14).

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\footnote{Bill Phillips, whose amazing life is recounted by Alan Bollard (2016), is gratefully remembered by Hendry as his tutor at LSE in 1966–67 who successfully guided him through his initial struggles with econometrics.}
We first compare Phillips Curves in price and wage inflation over the original sample in Phillips (1958), then extend the estimates to 2021 to establish its well-known instability. Next, we derive the nominal-wage inflation-unemployment relation from the real-wage model in Castle et al. (2023). We use that transform to demonstrate that its fitted values $\Delta w_t$ closely replicate the shifts for every sub-period. Thus the additional regressors must explain the shifts. Confirmation that the underlying relationship between $\Delta w_t$ and $U_{r,t}$ is constant conditional on the other regressors is shown by the relation between $\Delta w_t$ and $U_{r,t}$ derived from the transformed real wage equation having essentially the same slope in every sub-period. However, it transpires that it is not ‘shifts’ in the real-wage model’s regressors that explain the instabilities, which is a puzzle.

The structure of this paper is as follows. Section 2 compares Phillips Curves in price and wage inflation and replicates the original Phillips (1958) non-linear relation of wage inflation to the unemployment rate. Section 3 derives the nominal-wage inflation-unemployment relation from the real-wage model in CHM and Section 4 analyzes its sub-period implications. Section 5 tests the validity of conditioning on $U_{r,t}$. Section 6 examines subsample equations to ascertain what caused the shifts. Section 7 concludes and the Appendix 8 provides definitions and sources of the data series used.

2 Comparing Phillips Curves in price and wage inflation

![Figure 1: Comparing Phillips Curve in $\Delta p$ with $\Delta w$.](image)

Phillips (1958) related changes in nominal wages to the unemployment rate as both are labour market variables, but many recent variants use price inflation (see Forder, 2014 and Hoover, 2015).

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2 Similar non-invariance of new-Keynesian Phillips curves (NKPCs) is found by Castle et al. (2014).
for historical perspectives). Consequently, Figure 1 compares nominal wage and price inflation using annual observations on his data period 1860–1913. Phillips defined wage inflation, $Dw$, as $Dw = 0.5(W_{t+1} - W_{t-1})/W_t$, whereas we use the standard definition of the change in the log (there is a difference in timing of the peaks and troughs, but the two series are highly correlated: see Hendry, 2001). Phillips was aware that discrete approximations created moving-average errors (see Phillips, 2000), but in 1958 these were nearly impossible to estimate. He also knew that the ‘loops’ around his long-run relation represented dynamic adjustments, so calculated his equation from subsets of unemployment levels within which the average over a business cycle should be close to zero (see Desai, 1975).

$\Delta p_t$ is price inflation measured by the GDP deflator. As Figure 1 shows, a cubic spline fitted to $\Delta w$ matches Phillips’ non-linear form, whereas price inflation results in a nearly straight line. Thus, all our results relate to wage inflation. Note that since Phillips conducted his study, pre-World War I (WWI) data on unemployment have been substantially revised by Boyer and Hatton (2002), but our pre-WWI results are close to those Phillips reported.

However, the simple bivariate relation between $\Delta w$ and $U_r$ was not to last as shown in Figure 2. The five sub-periods plots of $\Delta w$ against $U_r$ are chosen as the original Phillips’ period 1860–1913 (before lags); WWI to the end of WWII; 1946–1980, namely post-war recovery till the end of the oil crisis; 1981–2011 which was the sample end in Castle and Hendry, 2014 and 2011–2021 which includes Brexit, the Covid-19 pandemic lockdowns and the UK government’s furlough scheme to prevent excessive unemployment.

![Figure 2: Shifts in wage inflation-unemployment relation.](image-url)
post-WWII periods, and flat since 2012 while \( U_r \) varied over 4%-8%, seem less excusable and confirms the unstable relation of \( \Delta w \) to \( U_r \). The ‘outliers’ from wars, oil crises, price controls, indexation and the ‘Great Depression’ are highlighted in ‘boxes’ as they derive from different extraneous causes at different times. The resulting sub-sample coefficient estimates \( \{ \hat{b} \} \) in the regression \( \Delta w_t = \hat{a} + \hat{b} U_{r,t} \), with their heteroscedasticity and autocorrelation consistent (HAC) standard errors (see Andrews, [1991]) are shown in Table 1.

<table>
<thead>
<tr>
<th>Period</th>
<th>( \hat{b} )</th>
<th>HAC standard errors</th>
</tr>
</thead>
<tbody>
<tr>
<td>1861–1913</td>
<td>–0.67</td>
<td>0.10</td>
</tr>
<tr>
<td>1914–1945</td>
<td>–1.30</td>
<td>0.41</td>
</tr>
<tr>
<td>1946–1980</td>
<td>2.14</td>
<td>0.36</td>
</tr>
<tr>
<td>1981–2011</td>
<td>0.53</td>
<td>0.13</td>
</tr>
<tr>
<td>2012–2021</td>
<td>–0.27</td>
<td>0.15</td>
</tr>
</tbody>
</table>

Table 1: Estimates and HAC standard errors of \( \hat{b} \) in the regression \( \Delta w_t = \hat{a} + \hat{b} U_{r,t} \)

While Figure 1 shows cubic splines graphically fitted to the data given the non-linear relation in Phillips (1958), these are close to the linear sub-sample estimated regressions in Figure 2 confirming coefficients of \( U_r \) change substantially over time.

3 Deriving a wage inflation-unemployment relation from a real-wage model

The natural explanation of such unstable estimates is that relevant variables not included in the simple model experience shifts. If all excluded variables were stationary and maintained a constant correlation with \( U_r \), its coefficient would be constant despite the omissions. Conversely, if the additional regressors included in the econometric model of real-wage growth in Castle et al. (2023) (reported for 1862–2021 in [1]) explained the shifts, its outcomes predicted for \( \Delta w_t \) in every sub-period should provide a close match. Figure 3 adds to Figure 2 the resulting sub-period plots of the full-sample \( \{ \Delta w_t \} \) against \( U_{r,t} \) and the close match confirms that the regressors in [1] model account for the instabilities in the simple Phillips’ curve. Since [1] is constant over the whole period \( T= 1862-2021 \) it successfully mis-specification encompasses the shifting Phillips curves.

\[
\Delta (w - p)_t = 0.40 \Delta (y - l)_t + 0.13 \Delta (y - l)_{t-1} - 0.14 \Delta^2 p_t - 0.18 (U_{r,t} - 0.05) \\
+ 3.1 (U_{r,t} - 0.05)^2 - 0.22 \Delta U_{r,t} + 0.41 (\tilde{f}_t \times \Delta p_t) - 0.13 S_{1939} \\
+ 0.18 S_{1940} - 0.07 S_{1941} - 0.05 I_{1916} - 0.05 I_{1977} + 0.03 I_{WIII} \\
- 0.18 (w - p - y + l - \tilde{\mu})_{t-2} + 0.02 S_{2012} \\
\]

(1)

\( \hat{\sigma} = 1.1\% \) \( R^2 = 0.79 \) \( F_{ar}(2, 137) = 0.25 \) \( F_{arch}(1, 152) = 0.03 \)

\( \chi^2_{HED}(2) = 0.62 \) \( F_{Het}(19, 130) = 2.5\% \) \( F_{reset}(2, 137) = 2.34 \) \( F_{nl}(24, 121) = 1.09 \)

Coefficient standard errors are in parentheses (HAC in brackets), \( \hat{\sigma} \) is the residual standard deviation, \( F_{ar} \) tests residual autocorrelation (see Godfrey, [1978]), \( F_{arch} \) tests autoregressive conditional heteroscedasticity (see Engle, [1982]), \( F_{het} \) tests residual heteroskedasticity (see White,
1980, \chi^2_{nd}(2) tests non-Normality (see Doornik and Hansen, 2008), \textit{F}_{reset} tests non-linearity (see Ramsey, 1969), \textit{F}_{nl} also tests non-linearity (see Castle and Hendry, 2010), and \textit{F}_{chow} tests parameter constancy (see Chow, 1960). One star indicates test significance at 5%, two at 1%. In (1), \Delta (y - l), measures labour productivity and the labour share of income is given by (w - p - y + l)_{t}, where \hat{\mu} is its sample mean. S_{xxx} is a step indicator taking the value 1 till the date xxx and 0 after, and I_{xxx} is an indicator variable taking the value 1 for that observation only. Four selected consecutive impulse indicators during WWII are combined as their coefficients were equal and opposite signed \( (I_{WWII} = I_{1942} + I_{1943} - I_{1944} - I_{1945}) \) and:

\[ \tilde{\varphi}_t = \frac{1}{0.88} \left( \left[ 1 + \exp \left( -10 \left( 100 (\Delta p_t)^2 - 0.2 \right) \right) \right]^{-1} - 1 \right). \] (2)

is a logistic smooth transition function (see Luukkonen et al., 1988) where the scaling bounds the function between \([-1, 0]\) if \( \mu \) is its sample mean.

Most recently, (1) is constant over Brexit, the pandemic and the UK government’s furlough scheme during lockdowns, and also passes a test for super-exogeneity of its contemporaneous regressors (see Engle et al., 1983 and Engle and Hendry, 1993).

Figure 3: Comparing direct and derived Phillips Curves.

Expressing (1) in terms of \( \Delta w_t \) in relation to \( U_{r,t} \) plus other drivers shown as \([\cdot]\) yields:

\[
\Delta w_t = -0.71 U_{r,t} + \left[ 3.1 U^2_{r,t} + 0.22 U_{r,t-2} + 0.025 + 0.86 \Delta p_t + 0.14 \Delta p_{t-1} + 0.41 (\tilde{\varphi}_t \times \Delta p_t) + 0.41 \Delta (y - l)_{t-1} - 0.18 (w - p - y + l - \hat{\mu})_{t-2} - S_{1939} + 0.18 S_{1940} - 0.07 S_{1941} + 0.02 S_{2012} - 0.05 I_{1908} - 0.05 I_{1971} + 0.03 I_{WWII} \right] \] (3)

\^Castle and Hendry (2014) use a non-linear function of the form \( f_t = -1/(1 + 1000(\Delta p_t)^2) \) which yields similar results.
The coefficient of $\Delta p_t$ is carried over at unity from undoing $\Delta(w - p)_t$, so $0.86 = 1 - 0.14$ from (1), but is $0.96(0.08)$ if estimated unrestrictedly on the right-hand side of (1).

### 4 Sub-period implications

To evaluate if correcting for the additional drivers produced stable subsample estimates, we calculated $\hat{x}_t$ as the sum of all the influences on $\Delta w_t$ in (1) other than $U_{r,t}$ in (3) to derive $(\Delta w_t | \hat{x}_t)$ as the residuals from the full-sample regression of $\Delta w_t$ on $\hat{x}_t$, shown in (4) with HAC standard errors.

\[
\begin{align*}
\Delta w_t & = -0.044 + 0.98 \hat{x}_t \\
\hat{\sigma} & = 2.6\% \quad R^2 = 0.80 \quad F_{arch}(2, 156) = 98^{**} \quad F_{het}(1, 158) = 85^{**} \\
\chi^2_{nd}(2) & = 3.3 \quad F_{het}(2, 157) = 2.13 \quad F_{reset}(2, 156) = 7.3^{**}
\end{align*}
\]

Thus, wage inflation is only corrected by a scalar which uses the same coefficients in all sub-periods, leading to the full-sample regression recorded in (5). The resulting sub-period regressions are shown in Figure 4 and are nearly identical across all sub-periods, with the full-sample regression in (5).

\[
(\Delta w_t | \hat{x}_t) = 0.039 - 0.73 U_{r,t}
\]

Table 2 reports the estimated coefficients of unemployment and their HAC standard errors, which stand in sharp contrast to the estimates in Table 1.

<table>
<thead>
<tr>
<th>Sub-period</th>
<th>Coefficients</th>
<th>HAC standard errors</th>
</tr>
</thead>
<tbody>
<tr>
<td>1860–1913</td>
<td>-0.76</td>
<td>0.07</td>
</tr>
<tr>
<td>1914–1945</td>
<td>-0.72</td>
<td>0.04</td>
</tr>
<tr>
<td>1946–1980</td>
<td>-0.85</td>
<td>0.15</td>
</tr>
<tr>
<td>1981–2011</td>
<td>-0.67</td>
<td>0.07</td>
</tr>
<tr>
<td>2012–2021</td>
<td>-0.84</td>
<td>0.11</td>
</tr>
</tbody>
</table>

Table 2: Coefficients of $U_{r,t}$ and HAC standard errors in the subsample $(\Delta w_t | \hat{x}_t)$ regressions

Although the Frisch and Waugh (1933) theorem suggests that $U_{r,t}$ should also be corrected for $\hat{x}_t$, it was essentially orthogonal to $\hat{x}_t$. This was a further surprise that the composite variable that explained most of the variance of real wages was not related to $U_{r,t}$. However, as shown in Table 3, while $U_{r,t}$ was uncorrelated with $\hat{x}_t$ on the full sample, $U_{r,t}$ was significantly correlated with $\hat{x}_t$ in those sub-samples where the Phillips curve shifted. The plot thickens...

In fact, the unconditional regression of $\Delta w_t$ on $U_{r,t}$ delivers a similar coefficient of $-0.72$ to (5):

\[
\begin{align*}
\Delta w_t & = 0.081 - 0.72 U_{r,t} \\
\hat{\sigma} & = 5.3\% \quad R^2 = 0.17 \quad F_{arch}(2, 156) = 81^{**} \quad F_{het}(1, 158) = 17.3^{**} \\
\chi^2_{nd}(2) & = 34.3^{**} \quad F_{het}(2, 157) = 0.28 \quad F_{reset}(2, 156) = 3.9^{*}
\end{align*}
\]
Corrected wage inflation

<table>
<thead>
<tr>
<th>Year</th>
<th>Coefficient</th>
<th>Standard Error</th>
<th>( \hat{\sigma} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1860–1913</td>
<td>0.002</td>
<td>0.05</td>
<td>0.033</td>
</tr>
<tr>
<td>1860–1913</td>
<td>0.11</td>
<td>0.18</td>
<td>0.019</td>
</tr>
<tr>
<td>1914–1945</td>
<td>-0.23</td>
<td>0.11</td>
<td>0.045</td>
</tr>
<tr>
<td>1946–1980</td>
<td>0.19</td>
<td>0.03</td>
<td>0.010</td>
</tr>
<tr>
<td>1981–2011</td>
<td>0.50</td>
<td>0.07</td>
<td>0.015</td>
</tr>
<tr>
<td>2012–2021</td>
<td>1.01</td>
<td>0.30</td>
<td>0.010</td>
</tr>
</tbody>
</table>

Table 3: Coefficients of \( \hat{x}_t \) and their standard errors with residual standard deviations \( \hat{\sigma} \) in the subsample \( U_{r,t} \) regressions.

Without accounting for price inflation, labour productivity, the wage share, non-linear wage-price spiral effects and exogenous shocks such as wars and oil crises, the Phillips curve is non-constant. These additional drivers obscure a constant nominal wage-unemployment rate relation over the last 160 years shown in Figure 5(b).

## 5 Testing the validity of conditioning on \( U_{r,t} \)

Valid conditioning in non-constant processes requires super exogeneity, see Engle et al. (1983) and Engle and Hendry (1993). Extending the automated test in Hendry and Santos (2010), we first apply impulse indicator saturation (IIS) and step indicator saturation (SIS) jointly at a significance level of 0.1% to the unconditional \( U_{r,t} \) regression on a constant, shown in (7), to select step shifts over 1862–2021 to match the estimation samples above which were shorter from lagged variables.
In both (7) and (8), the standard errors reported are HAC, but the conventional standard errors also confirm significance as do the $F_{\text{exclude}}$ tests of excluding the indicators. Importantly, none of the step indicators in (7) also enter (3).

The five selected indicators are highly significant when added to (4) as reported in (8), which is expected as they are a ‘big effects proxy’ for the missing role of $U_{r,t}$.

\[
\hat{U}_{r,t} = 0.069 - 0.048 S_{1920} + 0.081 S_{1938} - 0.033 S_{1974} - 0.058 S_{1980} + 0.039 S_{1984} \\
\hat{\sigma} = 0.02 \quad R^2 = 0.63 \quad F_{\text{ar}}(2, 152) = 143^{**} \quad F_{\text{arch}}(1, 158) = 91^{**} \quad \chi_{nd}^2(2) = 1.7 \quad F_{\text{reset}}(2, 151) = 0.00 \quad F_{\text{exclude}}(5, 154) = 51.2^{**}
\]

Figure 5: Full-sample Phillips Curves for (a) $\Delta w_t$ on $U_{r,t}$; (b) $\Delta w_t|x_t$ on $U_{r,t}$. 

\[
\hat{\Delta w}_t = - 0.05 + 0.99 x_t - 0.04 S_{1920} - 0.06 S_{1938} + 0.03 S_{1974} + 0.04 S_{1980} - 0.03 S_{1984} \\
\hat{\sigma} = 1.9% \quad R^2 = 0.90 \quad F_{\text{ar}}(2, 151) = 25.8^{**} \quad F_{\text{arch}}(1, 158) = 7.3^{*} \quad \chi_{nd}^2(2) = 0.13 \quad F_{\text{Het}}(7, 152) = 3.57^{**} \quad F_{\text{reset}}(2, 151) = 2.97 \quad F_{\text{exclude}}(5, 153) = 30.7^{**}
\]
However, all the step indicators become insignificant when added to (5) and no diagnostic tests reject. Thus the major shifts in \( U_{r,t} \) do not enter the regression of \( (\Delta w_t| \bar{x}_t) \) on \( U_{r,t} \), confirming it is super exogenous in that model.

\[
(\Delta w_t| \bar{x}_t) = 0.042 - 0.76 U_{r,t} - 0.001 S_{1920} + 0.004 S_{1938} + 0.002 S_{1974} \\
- 0.004 S_{1980} - 0.002 S_{1984}
\]

\( \bar{\sigma} = 1.1\% \) \( R^2 = 0.84 \) \( F_{ar}(2, 151) = 0.30 \) \( F_{arch}(1, 158) = 0.02 \) \( \chi^2_{ind}(2) = 1.3 \) \( F_{Het}(7, 152) = 1.74 \) \( F_{reset}(2, 151) = 0.09 \) \( F_{exclude}(5, 153) = 0.57 \)

These results are consistent with the evidence in Castle and Hendry (2014) that most UK unemployment has been involuntary. Nevertheless, (5) and (9) are projections from a multivariate relation determining real wages, where the nominal level is determined by the price equation as in Hendry (2001).

6 Subsample equations: what caused the shifts?

Figure 6 plots the time series of \( \Delta w_t \) and \( U_{r,t} \) with the sub-periods shown, where the bars mark the two world wars. Their patterns within each sub-period are very different, so it is unsurprising that the original Phillips curve would not be constant across the five subsamples, as shown in Figure 2.

Table 4 records the regression coefficient values with \( |t| \geq 2 \) from fitting the general model to subsamples to examine what changes were due to which variables \( (e_{t-2} = (w - p - y - l - \bar{\mu})_{t-2}) \).

The first two sub-periods are similar to the full sample, but the next two differ in many respects (the final sample is too short to be reliable). In particular, the impacts of \( (U_{r,t} - 0.05) = \bar{U}_{r,t} \) and \( (\bar{U}_{r,t})^2 \) are then insignificant, as are the non-linear inflation reactions. These absent impacts on wage inflation explain the upward slopes.

<table>
<thead>
<tr>
<th>Variable</th>
<th>( \Delta (y - l)_t )</th>
<th>( \Delta (y - l)_{t-1} )</th>
<th>( \Delta^2 p_t )</th>
<th>( U_{r,t} )</th>
<th>( (\bar{U}_{r,t})^2 )</th>
<th>( \Delta^2 U_{r,t} )</th>
<th>( f_t \times \Delta p_t )</th>
<th>( e_{t-2} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1860–2021</td>
<td>0.41</td>
<td>0.13</td>
<td>-0.14</td>
<td>-0.18</td>
<td>3.1</td>
<td>-0.22</td>
<td>0.41</td>
<td>-0.18</td>
</tr>
<tr>
<td>1860–1913</td>
<td>0.18</td>
<td>0.17</td>
<td>-0.30</td>
<td>-0.20</td>
<td>3.3</td>
<td>-0.21</td>
<td>0.44</td>
<td>-0.10</td>
</tr>
<tr>
<td>1914–1945</td>
<td>0.34</td>
<td>0</td>
<td>-0.12</td>
<td>-0.22</td>
<td>3.4</td>
<td>-0.38</td>
<td>0.65</td>
<td>0</td>
</tr>
<tr>
<td>1946–1980</td>
<td>0.63</td>
<td>0.45</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>-0.47</td>
<td>0</td>
</tr>
<tr>
<td>1981–2011</td>
<td>0.49</td>
<td>0</td>
<td>-0.50</td>
<td>0</td>
<td>0</td>
<td>-0.47</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2012–2021</td>
<td>0.23</td>
<td>0.42</td>
<td>-1.1</td>
<td>0.16</td>
<td>32</td>
<td>0</td>
<td>0.47</td>
<td>-0.61</td>
</tr>
</tbody>
</table>

Table 4: Coefficients with \( |t| \geq 2 \) in the subsample regressions for the general model of \( \Delta (w - p)_t \).

Imposing the whole sample coefficient estimates for the zero values over 1946–1980 in Table 4 yields an equation standard error of \( \bar{\sigma} = 1.17\% \) as against the unrestricted fit of \( \bar{\sigma} = 1.11\% \). Similarly, for 1981–2011, \( \bar{\sigma} = 0.91\% \) versus \( \bar{\sigma} = 0.98\% \) (the lower constrained value is an artefact of not counting restricted coefficients in the degrees of freedom). In neither case were any mis-specification tests significant for the constrained models, consistent with the overall constancy of the general model despite subsample estimate variations. Thus, the slope changes in the original Phillips curve are due to the lack of variability of the real-wage model regressors in the sub-samples: its stability needed hidden co-breaking (Hendry and Massmann, 2007) between the variables in the real-wage model, and the absence of some impacts stopped that occurring.
Figure 6: Time series of $\Delta w_t$ and $U_{r,t}$ with the two World Wars shaded and the other subsamples shown by vertical lines.

7 Conclusion

The UK Phillips’ curve relating changes in the log of nominal wages to unemployment is unstable. Sub-period relationships between wage inflation ($\Delta w$) and unemployment ($U_r$) can be strongly negative, slightly negative, flat, slightly positive and strongly positive in a time series from 1860 to 2021. Such behavior prompted five puzzles:

Puzzle 1: what caused the Phillips curve slopes to change so much?

Puzzle 2: why is the Phillips curve over the very turbulent period 1914–1945 similar to the original over the relatively stable 1860–1913?

Puzzle 3: can a constant-parameter real-wage model successfully encompass a shifting nominal wage equation?

Puzzle 4: does the lack of correlation of $U_r$ with the full-sample estimated coefficient linear combination of the variables $\tilde{x}_t$ that explains changes in nominal wages, hold in subsamples?

Puzzle 5: why does correcting $\Delta w_t$ by $\tilde{x}_t$ produce a near constant subsample set of equations for $\Delta w_t|\tilde{x}_t$ on $U_{r,t}$?

These puzzles concerning aspects of what caused the shifts in subsample Phillips curves can all be resolved as follows.

In Section 3, mis-specification encompassing (see Hendry and Nielsen, 2007, Ch. 13) revealed that the shifts in the subsample Phillips curves could be accounted for by a constant parameter real-wage equation.

In Section 4, partialling out from nominal wages the full-sample estimated coefficient linear combination of the regressors $\tilde{x}_t$, other than unemployment, showed that the resulting subsample equa-
otions had essentially the same downward slopes of between $-0.67$ to $-0.85$; In Section 5, the validity of conditioning $(\Delta w_t | \tilde{x}_t)$ on $U_{r,t}$ was confirmed; In Section 6, the insignificance of estimated coefficients in subsample real-wage models in Table 4, matched when the Phillips curve shifted, as did the significance of the correlation of $U_{r,t}$ with $\tilde{x}_t$ in subsamples as in Table 3. Imposing the full-sample estimated values on insignificant subsample coefficients produced constant equations with no deterioration in fit, identifying the culprits behind the instability by their absence. Quite a surprise that the constancy of the nominal wage change-unemployment rate relationship depended on co-breaking of all the variables that it omitted from the constant real-wage model, then failed only when those lacked significance.

Although the whole sample regression of $\Delta w_t$ on $U_{r,t}$ delivers the same coefficient as in a much more general constant parameter equation, it is not a useful way to model the inflation-unemployment relation important to economic policy. Instead, useful policy implications require taking account of the constant parameter, multivariate, non-linear, dynamic relationship for real wages that encompasses the original Phillips curve, interacted with a price inflation model to determine the overall level and persistence of inflation, as in Castle et al. (2023).

References


8 Appendix: data definitions and sources

\[ Y_t = \text{real GDP, £million, 1985 prices} \quad [6], \text{p.836, [5]a (1993), ONS code: YBHH at 2005 prices, [9].} \]
\[ P_t = \text{implicit deflator of GDP, (1860=1)} \quad [3], \text{p.836, [5]a (1993), ONS code: ABML, [9].} \]
\[ U_t = \text{unemployment} \quad [4], [5]c (1993), \text{ONS code: MGSC.} \]
\[ W_{\text{pop}}t = \text{working population} \quad [4], [5]c (1993), \text{ONS code: MGSC.} \]
\[ U_{r,t} = \frac{U_t}{W_{\text{pop}}t} \text{ (unemployment rate, fraction)} \]
\[ L_t = \text{employment (=} W_{\text{pop}}t - U_t \) \quad [1], [5] \]
\[ W_t = \text{average weekly wage earnings} \quad [7], [8], \text{ONS code: LNMM} \]
\[ W_{r,t} = \text{nominal wage rates} \quad [2], [6], [8] \]
\[ \Delta z_t = (z_t - z_{t-1}) \text{ for any variable } z_t \]
\[ \Delta^2 z_t = \Delta z_t - \Delta z_{t-1} \]

Sources:
[2] Phillips (1958);
[4] Feinstein (1972) and Boyer and Hatton (2002);
[5] Bean ((a) Economic Trends Annual Supplements, (b) Annual Abstract of Statistics, (c) Department of Employment Gazette and (d) National Income and Expenditure, as well as other sources cited here);
[7] Crafts and Mills (1994);
[8] Feinstein (1990);
[9] ONS

Hendry and Ericsson (1991) and Hendry (2001) provide detailed information about many of these series.