The Friedman rule in an overlapping-generations model with nonlinear taxation and income misreporting

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Abstract

This paper models an overlapping-generations economy that includes money and is populated with individuals of different skills. They face a nonlinear income tax schedule and can engage in tax evasion. Money serves two purposes: the traditional one, modeled through a money-in-the-utility-function, and to facilitate tax evasion. It shows that income tax evasion leads to the violation of the Friedman rule that will otherwise hold.

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1 Introduction

Milton Friedman’s (1969) doctrine regarding the “optimum quantity of money” - according to which an optimal monetary policy would involve a steady contraction of the money supply at a rate sufficient to bring the nominal interest rate down to zero - is undoubtedly one of the most celebrated propositions in modern economic theory.\footnote{The classic reference for the Friedman rule is Friedman (1969). The earlier literature referred to it also as the Chicago rule; see Niehans (1978).}

This paper brings two strands of public finance literature to bear on the question of the Friedman rule (1969) for the optimal money supply. One is the optimal Mirrleesian taxation that started with Mirrlees (1971) and was popularized by Stiglitz (1982) in its simplified two-group version; the second is the tax evasion literature that followed the pioneering work of Allingham and Sandmo (1972). Our paper differs from the previous contributions on this topic with the “same” three ingredients in that it adopts a Mirrleesian rather than a Ramsey approach to optimal taxation.\footnote{Examples of the literature that examines the relevance of tax evasion for the Friedman rule from a Ramsey tax perspective include Nicolini (1998), Cavalcanti and Villamil (2003), Koreshkova (2006), and Arbex and Turdaliev (2011).}

It is now well-known that the Friedman rule is a first-best prescription and may or may not hold in second-best settings. This depends on the nature of the second-best (existence of distortionary taxes or intrinsic reasons for market failure), the set of tax instruments available to the government, and the structure of individuals’ preferences.\footnote{Non-optimality of Friedman rule in the presence of distortionary taxes was first discussed by Phelps (1973). A selective reference to other sources of distortion include: van der Ploeg and Alogoskoufis (1994) for an externality underlying endogenous growth; Ireland (1996) for monopolistic competition; Ereng \textit{et al.} (2000) and Khan \textit{et al.} (2003) for nominal wage and price settings; Schmitt-Grohe and Uribe (2004a,b) for imperfections in the goods market; and Shaw \textit{et al.} (2006) for imperfect competition as well as externality.}

Chari \textit{et al.} (1991, 1996), in the context of a model with identical and infinitely-lived individuals, related the optimality of Friedman rule in the presence of distortionary taxes to the uniform commodity tax result of Atkinson and Stiglitz (1972) and Sandmo (1974). This latter result states that if preferences are separable in labor supply and non-leisure goods, with the subutility for goods being homothetic, optimal commodity taxes are proportionately uniform. They showed that deviations from Friedman rule
violates this tax principle.\textsuperscript{4}

These studies, being carried out in an environment with identical individuals, are by construct silent on the validity of the Friedman rule when monetary policy has redistributive implications.\textsuperscript{5} A second related drawback of these studies is their reliance on the Ramsey tax framework, which assumes that all tax instruments, including the income tax, are set linearly.\textsuperscript{6}

In a recent contribution, da Costa and Werning (2008) break with this tradition and consider the optimality of the Friedman rule in a model with heterogeneous agents and allow the government to levy nonlinear income taxes. Interestingly, they show that the Friedman rule is optimal in their setting (for any social welfare function that redistributes from the rich to the poor). As with Chari \textit{et al.}'s (1991, 1996) earlier result, da Costa and Werning’s (2008) finding is also related to the uniform taxation result in public finance, albeit a different one. Whereas Chari \textit{et al.} (1991, 1996) draw on Sandmo’s tax uniformity (1974) result derived within a Ramsey setting, da Costa and Werning’s (2008) has its roots in Atkinson and Stiglitz (1976). This classic paper on the design of tax structures was particularly concerned with the usefulness of commodity taxes in the presence of a general income tax in economies with heterogeneous agents.\textsuperscript{7}

Atkinson and Stiglitz (1976) proved that with a general income tax, if preferences are weakly separable in labor supply and goods, then commodity taxes are not needed as instruments of optimal tax policy. With non-separability, one wants to tax the goods that are “substitutes” with labor supply and subsidize those that are “complements”

\textsuperscript{4}This uniformity result is derived within the context of the traditional one-consumer Ramsey problem. As such, the result embodies only efficiency considerations. Redistributive goals do not come into play.

\textsuperscript{5}With the exception of intergenerational redistributive issues that arise in overlapping generations models; see, e.g., Weiss (1980), Abel (1987), and Galvani (1988).


\textsuperscript{7}The ineffectiveness of commodity taxes and their proportionately uniform structure boil down to the same thing. In the absence of exogenous incomes, the government has an extra degree of freedom in setting its income and commodity tax instruments. This is because all demand and supply functions are homogeneous of degree zero in consumer prices. In consequence, the government can, without any loss of generality, set one of the commodity taxes at zero (i.e. set one of the commodity prices at one). Under this normalization, uniform rates imply absence of commodity taxes.
with labor supply. In da Costa and Werning (2008) the uniformity result, which implies a zero nominal interest rate, holds with preference separability. With non-separable preferences, da Costa and Werning assume that real cash balances and labor supply are complements so that cash balances should be subsidized. This implies that the optimal nominal interest rate is negative. But given the non-negativity of nominal interest rate, the zero interest rate emerges as the “optimal” policy.

da Costa and Werning’s (2008) results as well as the earlier Chari et al.’s (1991, 1996) results are all derived in settings that disregard tax evasion. Yet many empirical studies over the past few decades confirm that tax evasion is a widespread phenomenon all over the world; see Shaw et al. (2011) for a recent survey. However, introducing tax evasion into the optimal tax problem often invalidates policy lessons drawn ignoring this phenomenon. In the context of the uniform taxation results, for example, Cremer and Gahvari (1993) prove that the Ramsey results are no longer valid. Similarly, Boadway et al. (1994) show how the presence of tax evasion destroys the celebrated Atkinson and Stiglitz (1976) theorem on the redundancy of commodity taxes in the presence of Mirrleesian optimal income tax if preferences are weakly separable in labor supply and goods. One would then expect the same fate for the Friedman rule. There are indeed a number of papers that show this but they are all written in the context of Ramsey taxes.\(^8\) There are no such studies to date using the Mirrleesian tax framework.

We prove that the absence of tax evasion is crucial for da Costa and Werning’s (2008) results. First, when agents have access to a misreporting technology, which allows them to shelter part of their earned income from the tax authority, monetary policy becomes another useful instrument for redistribution. This is the case because income tax evasion invalidates the uniform commodity tax result of Atkinson and Stiglitz (1976) thus rendering the monetary growth rate a redistributive power that otherwise it does not possess. In particular, the presence of tax evasion invalidates da Costa and Werning’s (2008) result on the optimality of the Friedman rule as an interior solution if the conditions for Atkinson and Stiglitz (1976) theorem hold. Second, we show that

\(^8\)For a recent example, see Arbex and Turdaliev (2011) and references therein.
da Costa and Werning’s (2008) other result, on the optimality of the Friedman rule as a boundary solution if real cash balances and labor supply are complements, is no longer guaranteed either. This is because, in the presence of tax evasion, one does not know which type of agents supplies more labor (at the same level of reported income). Hence the complementarity assumption does not identify the type who demands more real cash balances.

Other interesting issues we discuss include the role of money in the economy. We allow for money to have two distinct usages. One is for the traditional (non-evading) reasons modeled by allowing real cash balances to enter the utility function; the other is to facilitate tax evasion. Another issue concerns the relevance of individual types who are the recipients of money injections. We show that, for a given monetary rate of growth, the fiscal authority can offset the redistributive effects of who gets the extra money by adjusting the individuals’ income tax payments. Put differently, in the presence of a general income tax, who receives the money injection is of no consequence. One other result is that even in the absence of tax evasion, complementarity of real cash balances and labor supply does not guarantee the optimality of the FR as a boundary solution. da Costa and Werning’s (2008) result to the contrary arises because there is no differential commodity taxes in their model. In a final section, we show that our results are robust with respect to the modeling of the number of agent types in the economy, the modeling of income tax evasion whether riskless by incurring a concealment cost or as a risky activity subject to audits, and the possibility of commodity tax evasion.

2 The model

Consider a two-period overlapping generations (hereafter OLG) model wherein individuals work in the first period and consume in both. There is no bequest motive. Prefer-
ences are represented by the strictly quasi-concave utility function \( U = u(c_t, d_{t+1}, x_t, L_t) \) where \( c \) denotes consumption when young, \( d \) consumption when old, \( x \) real money balances (held for non-evading activities)\(^{10}\), and \( L \) labor supply; subscript \( t \) denotes calendar time. While the utility function is assumed to be strictly increasing in \( c_t \) and \( d_{t+1} \), and strictly decreasing in \( L_t \), the possibility of satiation in real balances is not ruled out (i.e. \( \lim_{x \to x_{sat}} \frac{\partial u}{\partial x} = 0 \) at the “satiation level” \( x_{sat} \)). Each generation consists of two types of individuals who differ in skill levels (labor productivity). High-skilled workers are paid \( w^h_t \) and low-skilled workers \( w^l_t \); with \( w^h_t > w^l_t \). The proportion of agents of type \( j \), \( \pi^j_t \), \( j = h, l \), remains constant over time. Denote the number of young agents of type \( j \) born in period \( t \) by \( n^j_t \) and the total number of young agents by \( N_t \). We have \( n^j_t / N_t = \pi^j_t = \pi^j \). Population grows at a constant rate, \( g \).

Production takes place through a linear technology with different types of labor as inputs. Transfer of resources to the future occurs only through a storage technology with a fixed (real) rate of return, \( r \).\(^{11}\) We thus work with an OLG model à la Samuelson (1958) and assume away the issues related to capital accumulation.

### 2.1 Money and monetary policy

At the beginning of period \( t \), before consumption takes place, the young purchase all the existing stock of money, \( M_t \), from the old. Denote a young \( j \)-type agent’s purchases by \( m^j_t \). We have

\[
M_t = n^h_t m^h_t + n^l_t m^l_t.
\] (1)

The rate of return on money holdings (the nominal interest rate), \( i_{t+1} \), is related to the inflation rate, \( \varphi_{t+1} \), according to Fisher equation

\[
1 + i_{t+1} \equiv (1 + r)(1 + \varphi_{t+1}).
\] (2)

\(^ {10} \)Later in this section, we discuss the usage of money for evasion.

\(^ {11} \)An alternative assumption is that agents borrow and lend on international capital markets at an exogenously fixed interest rate.
Denote the price level at time $t$ by $p_t$; the inflation rate is defined as

$$1 + \varphi_{t+1} \equiv \frac{p_{t+1}}{p_t}. \quad (3)$$

The monetary authority injects money into (or retires money from) the economy at the constant rate of $\theta$. Money is given to (or taken from) the old—who hold all the stock of money—via lump-sum monetary transfers (or taxes). Thus a young $j$-type agent who purchases $m_j^t$ at the beginning of time $t$ receives $e_j^{t+1}$ at the beginning of period $t+1$. Clearly, $e_{t+1}^h$ and $e_{t+1}^\ell$ must satisfy the “money injection relationship”,

$$n_t^h e_{t+1}^h + n_t^\ell e_{t+1}^\ell = \theta M_t. \quad (4)$$

Beyond this, we do not specify how much of the extra money injection goes to which type. Indeed, an important message of our paper is to argue that this division is immaterial (as shown in subsection 4.1 below).

With money stock changing at the rate of $\theta$ in every period, $M_{t+1} = (1 + \theta) M_t$. Substitute for $M_t$ and $M_{t+1}$, from equation (1), into this relationship:

$$n_{t+1}^h m_{t+1}^h + n_{t+1}^\ell m_{t+1}^\ell = (1 + \theta) \left( n_t^h m_t^h + n_t^\ell m_t^\ell \right).$$

Given that the population of each type grows at the constant rate of $g$, one can rewrite this as$^{12}$

$$n_t^h \left( m_{t+1}^h - \frac{1 + \theta}{1 + g} m_t^h \right) + n_t^\ell \left( m_{t+1}^\ell - \frac{1 + \theta}{1 + g} m_t^\ell \right) = 0. \quad (5)$$

Assume that the money-holdings of each type changes in the same direction.$^{13}$ It follows from the above relationship that

$$m_{t+1}^j = \frac{1 + \theta}{1 + g} m_t^j. \quad \text{(5')}$$

$^{12}$Observe that $(1 + g) m_{t+1}^j$ is not necessarily equal to $m_t^j + e_{t+1}^j$. This will be the case only if $e_{t+1}^j = \theta m_t^j$. This assumption applies only to the sign and not the magnitude of such possible changes. Observe also that this is an assumption on the equilibrium money holdings as opposed to money purchases that may very well go in different directions depending on who gets the new money injections (or loses them). It is a natural assumption because there are no stochastic shocks in this model so that in going from one year to the next the opportunity sets and the prices faced by different agent types change in the same manner. Nor does the government follow a capricious redistributive policy changing the social welfare weights of different groups from one year to the next. If goods including real cash balances are normal, both types end up changing all their consumption levels in the same direction.
Finally, given the empirical observation that evasion is often associated with larger cash holdings, we posit that money has two usages. One is for the traditional (non-evading) reasons and modeled by allowing its “real” value, $x$, to enter the utility function; the other is used solely to facilitate tax evasion. This latter part is assumed proportional to the amount of income concealed from the tax authority. Let $a^j_t > 0$ denote income concealed by the $j$-type individual at time $t$. To make this possible, the evader must hold $\beta p_t a^j_t$ in cash over and above $p_t x^j_t$ that he holds for other reasons where $\beta$ is a positive constant less than one. Consequently, total “real money balances” in our model is equal to

$$x^j_t + \beta a^j_t = m^j_t / p_t. \quad (6)$$

### 2.2 Fiscal policy

The tax authority levies income and commodity taxes to maximize a social welfare function defined over the utility of all agents in the economy. The government knows the distribution of types in the population but it does not know the identity of the types. Consequently, type-specific lump-sum taxes are not implementable. Earned incomes are not publicly observable either. Income reported by agents for tax purposes may thus deviate from true earned income due to the possibility of income-misreporting. To model income-misreporting in the simplest possible way, we begin by following the riskless approach introduced by Usher (1986) and since then used in a number of subsequent contributions.\footnote{See, e.g., Mayshar (1991), Boadway et al. (1994), Kopczuk (2001), Slemrod (2001), and Chetty (2009).} Later on, in Section 7, we consider how our results may be affected if one were to model income misreporting as a risky activity, which can be discovered by the tax authority through costly audits and punished according to a penalty function. Under the riskless approach, instead, once agents have incurred some pecuniary cost that depends on the amount they misreport, they face no risk of detection. What the fiscal authority can rely on is thus taxing income reported by agents, which will be denoted by $I_t$, via a general nonlinear income tax $T(I_t)$.

With the true income being equal to $w_t L_t$, the amount of income concealed is equal
to $a_t = w_t L_t - I_t$. The cost of misreporting is expressed by means of the function $f(a_t)$. Assume that $f(\cdot)$ is non-negative, increasing in the absolute value of $a_t$ and strictly convex with $f(0) = f'(0) = 0$. Finally, assume that the information the tax authority has on transactions, including money holdings, is of anonymous nature; it does not know the identity of the purchasers. This assumption, which is made for realism, implies that goods can be taxed only linearly (possibly at different rates).

2.3 Constrained Pareto-efficient allocations

To characterize the (constrained) Pareto-efficient allocations, one has to account for the economy’s resource balance, the standard incentive compatibility constraints due to our informational structure, and the implementability constraints caused by linearity of commodity taxes—itself due to informational constraint, as well as the monetary expansion mechanism. To this end, we derive an optimal revelation mechanism. For our purpose, a mechanism consists of a set of type-specific before-tax reported labor incomes, $I^j_t$’s, “assigned” after-tax incomes, $z^j_t$’s, commodity tax rates on consumption when young and old, $\tau^c$ and $\tau^d$, a money supply growth rate, $\theta$, and a monetary distributive rule, $e^j_t$. This procedure determines $\tau^c, \tau^d, \theta$, and $e^j_{t+1}$ from the outset. A complete solution to the optimal tax problem per-se, i.e. determination of $I^j_t$ by the individuals via utility maximization, then requires only the design of a general income tax function $T(I_t)$ such that $z^j_t = I^j_t - T(I^j_t)$.

To proceed further, it is necessary to consider the optimization problem of an individual for a given mechanism $(\tau^c, \tau^d, \theta, e_{t+1}, z_t, I_t)$. This is necessitated by the fact that

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15While we often speak of under-reporting and tax evasion ($a_t > 0$), in principle, over-reporting ($a_t < 0$) is also possible. Over-reporting is an optimal strategy when an agent faces a negative marginal income tax rate. In turn, the possibility of a negative marginal income tax arises because of the existence of commodity taxes, and in our model also inflation, in the system (see, e.g., Edwards et al., 1994). None of our results depends on the sign of $a_t$.

Observe also that no extra cash holding is required with over-reporting. Consequently, if $a_t < 0$ then $\beta = 0$ so that $\beta a_t$ vanishes and equation (6) simplifies to $x^j_t = m^j_t / p_t$.

16This formulation assumes that consumption expenditures are not publicly observable at a personal level. Strictly speaking, this procedure does not characterize allocations as such; the optimization is over a mix of quantities and prices. However, given the commodity prices, utility maximizing households would choose the quantities themselves. We can thus think of the procedure as indirectly determining the final allocations.
the mechanism determines personal consumption levels only indirectly, namely through prices. The mechanism assigns the sextuple \((\tau^c, \tau^d, \theta, e_{t+1}, z_t^j, I_t^j)\) to a young individual who reports his type as \(j\). The individual will then allocate \(z_t^j\), and any other disposable income that he may have, between first- and second-period consumption, and real money balances.

Formally, given any vector \((\tau^c, \tau^d, \theta, e_{t+1}, z_t, I_t)\), an individual of type \(j\) chooses \(c_t, d_{t+1}, x_t\) and \(a_t\) to maximize

\[
u = u\left(c_t, d_{t+1}, x_t, \frac{I_t + a_t}{w_t^j}\right), \quad j = h, \ell,
\]

subject to the per-period budget constraints

\[
p_t \left(1 + \frac{\tau^c}{\tau^d} c_t + s_t\right) + m_t = p_t [z_t + a_t - f(a_t)], \tag{8}
\]

\[
p_{t+1} \left(1 + \tau^d\right) d_{t+1} = p_t s_t (1 + i_{t+1}) + m_t + e_{t+1}, \tag{9}
\]

where \(s_t\) is the level of real savings chosen by the agent. Observe that \(\theta\) does not explicitly appear in the problem above; it does so implicitly through its effect on \(i_{t+1}\). Equations (8)–(9) can be unified (see the Appendix) into the following intertemporal budget constraint for the young:

\[
(1 + \tau^c) c_t + \frac{(1 + \tau^d) d_{t+1}}{1 + r} + \frac{i_{t+1}}{1 + i_{t+1}} x_t = z_t + a_t - f(a_t) - \frac{i_{t+1}}{1 + i_{t+1}} \beta a_t + \frac{e_{t+1}}{p_{t+1} (1 + r)}. \tag{10}
\]

Observe that \(i_{t+1}/(1 + i_{t+1})\) is the opportunity cost of holding one dollar in cash so that \(f(a_t) + i_{t+1} \beta a_t / (1 + i_{t+1})\) is the total cost of concealing \(a_t\).

The problem of a young \(j\)-type, who is facing the sextuple \((\tau^c, \tau^d, \theta, e_{t+1}, z_t, I_t)\), is to choose \(c_t, d_{t+1}, x_t,\) and \(a_t\) in order to maximize (7) subject to (10). The first-order
conditions for this problem are
\[
\frac{\partial u \left( c_t, d_{t+1}, x_t, (I_t + a_t) / w_t \right)}{\partial d_{t+1}} = \frac{1 + \tau^d}{(1 + \tau^c)(1 + r)}
\]
(11)

\[
\frac{\partial u \left( c_t, d_{t+1}, x_t, (I_t + a_t) / w_t \right)}{\partial c_t}
\]
\[
= \frac{i_{t+1}}{(1 + \tau^c)(1 + i_{t+1})}
\]
(12)

\[
\frac{\partial u \left( c_t, d_{t+1}, x_t, (I_t + a_t) / w_t \right)}{\partial x_t} = - \frac{1 - f' (a_t) - \beta i_{t+1}/(1 + i_{t+1})}{(1 + \tau^c)} w_t^i
\]
(13)

Conditions (11)–(13), along with the individual’s intertemporal budget constraint (10),
yield the conditional demands for the \( j \)-type’s first- and second-period consumption,
real money balances, and the concealed labor income. For ease of notation, introduce

\[
q_t^c = 1 + \tau^c
\]
(14)

\[
q_{t+1}^d = \frac{1 + \tau^d}{1 + r}
\]
(15)

\[
q_t^x = \frac{i_{t+1}}{1 + i_{t+1}},
\]
(16)

\[
b_{t+1}^j = \frac{c_{t+1}}{p_{t+1}(1 + r)}
\]
(17)

One can then write the conditional demand functions, and the concealed labor income,
when facing \((\tau^c, \tau^d, \theta, c_{t+1}, z_t, I_t)\), as

\[
c_t^j = c \left( q_t^c, q_{t+1}^d, q_t^x, z_t + b_{t+1}, I_t, w_t^i \right),
\]
(18)

\[
d_{t+1}^j = d \left( q_t^c, q_{t+1}^d, q_t^x, z_t + b_{t+1}, I_t, w_t^i \right),
\]
(19)

\[
x_t^j = x \left( q_t^c, q_{t+1}^d, q_t^x, z_t + b_{t+1}, I_t, w_t^i \right),
\]
(20)

\[
a_t^j = a \left( q_t^c, q_{t+1}^d, q_t^x, z_t + b_{t+1}, I_t, w_t^i \right).
\]
(21)

The last equation also determines \( j \)-type’s labor supply, \((I_t + a_t^j) / w_t^i\). When incomes
are observable, there will not be such an equation so that \( a_t^j = 0 \). Assigning \( I_t \) to an
individual then determines his labor supply, \( I_t / w_t^i \).

As a final observation, it is crucially important to realize that, in this model, an
individual’s total expenditures on goods, his (actual) disposable income, is not just
$z_t + b_{t+1}$ as it would be the case in the absence of misreporting. Instead, it will be equal to

$$z_t + b_{t+1} + a_t - f(a_t) - \beta \frac{i_{t+1} a_t}{1 + i_{t+1}}, \quad (22)$$

which includes income evaded net of concealment costs (where concealment costs include the opportunity cost of holding money for concealment). It thus depends on whether or not a particular type evades and to what extent. We summarize our discussion thus far regarding the determination of the temporal equilibrium of this economy as,

**Proposition 1** Consider an overlapping-generations model à la Samuelson (1958) with money wherein money holdings are rationalized by a money-in-the-utility-function approach. There are two types of agents, skilled and unskilled workers, denoted by $h$ and $\ell$. Both types grow at a constant rate so that the proportion of each type in the total population remains constant over time. Let a young $j$-type individual face, at time $t$, the sextuple $(\tau^c, \tau^d, \theta, e^j_{t+1}, z^j_t, I^j_t)$, where $\tau^c$ is the tax rate on first-period consumption, $\tau^d$ is the tax rate on second-period consumption, $\theta$ is the money growth (or contraction) rate, $e^j_{t+1}$ is the $j$-type’s allotment of money injection (or money withdrawal) to be given in second period, $z^j_t$ is the $j$-type’s net-of-tax reported income, and $I^j_t$ is the $j$-type’s before-tax reported income; $j = h, \ell$. Reported income differs from actual earnings by the amount misreported, $a^j_t$. Under the perfect foresight assumption, the period by period equilibrium of this economy is characterized by equations (1)–(3), and (18)–(21), where the last four equations hold for both $j = h, \ell$.

**2.4 Mechanism designer**

It remains for us to specify how the mechanism designer chooses $(\tau^c, \tau^d, \theta, e^j_{t+1}, z^j_t, I^j_t)$. This will complete the characterization of the set of (constrained) Pareto-efficient allocations in every period under the perfect-foresight assumption.

Substituting the values of $e^j_t$, $d^j_{t+1}$, $x^j_t$ and $a^j_t$, from (18)–(21), in the young $j$-type’s utility function (7) facing $(\tau^c, \tau^d, \theta, e^j_{t+1}, z^j_t, I^j_t)$ yields his conditional indirect utility
function,
\[
v(q_t^c, q_{t+1}^d, q_t^r, z_t + b_{t+1}, I_t, w_t^l) \equiv \\
u \left( c\left(q_t^c, q_{t+1}^d, q_t^r, z_t + b_{t+1}, I_t, w_t^l\right), d\left(q_t^c, q_{t+1}^d, q_t^r, z_t + b_{t+1}, I_t, w_t^l\right), x\left(q_t^c, q_{t+1}^d, q_t^r, z_t + b_{t+1}, I_t, w_t^l\right), I_t + a(q_t^c, q_{t+1}^d, q_t^r, z_t + b_{t+1}, I_t, w_t^l) \right). \tag{23}
\]

Let \( \delta^j \)'s denote positive constants with the normalization \( \sum_{j = t, h} \delta^j = 1 \). The mechanism designer maximizes
\[
\sum_{j = t, h} \delta^j v(q_t^c, q_{t+1}^d, q_t^r, z_t^j + b_{t+1}^j, I_t^j, w_t^j),
\]
with respect to \( \tau^c, \tau^d, \theta, e_{t+1}^j, z_t^j \) and \( I_t^j \); subject to the government’s generational budget constraint,
\[
n_t^h \left( I_t^h - z_t^h \right) + n_t^l \left( I_t^l - z_t^l \right) + \tau^c \left( n_t^h e_t^h + n_t^l e_t^l \right) + \frac{\tau^d}{1+\tau} \left( n_t^h d_{t+1}^h + n_t^l d_{t+1}^l \right) \geq N_t \bar{R}, \tag{24}
\]
where \( \bar{R} \) is an exogenous per-young revenue requirement, the money injection relationship (4), and the self-selection constraints
\[
v(q_t^c, q_{t+1}^d, q_t^r, z_t^h + b_{t+1}^h, I_t^h, w_t^h) \geq v(q_t^c, q_{t+1}^d, q_t^r, z_t^l + b_{t+1}^l, I_t^l, w_t^l), \tag{25}
\]
\[
v(q_t^c, q_{t+1}^d, q_t^r, z_t^l + b_{t+1}^l, I_t^l, w_t^l) \geq v(q_t^c, q_{t+1}^d, q_t^r, z_t^h + b_{t+1}^h, I_t^h, w_t^h). \tag{26}
\]

The constraints (25)–(26) require that each type of agents must (weakly) prefer the bundle intended for them to that intended for the other type. An agent who misrepresent his true type by choosing the bundle intended for another type is called “mimicker”. In particular, in what follows, we shall refer to an agent of type \( j \) who mimics an agent of type \( k \) as a \( jk \)-agent or a \( jk \)-mimicker. Below, we will discuss the solution to the mechanism designer’s problem after it reaches its steady-state equilibrium (which we assume exists and is stable).\(^{17}\)

\(^{17}\)The questions of existence, uniqueness, and stability are endemic in OLG models. However, these are not the issues we are concerned with in this paper and thus ignore. For a discussion see, among others, Galvani (1988, pp 345–347) for models with money-in-the-utility-function and Galvani (2012, pp 795–797 and 813) for models with a cash-in-advance constraint.
3 Steady state

In the steady-state, individual holdings of real cash balances remains constant over time:

\[ x_{t+1}^j + \beta a_{t+1}^j = x_t^j + \beta a_t^j. \]

The constancy of \( x + \beta a \) implies,\(^{18}\)

\[ 1 + \phi = \frac{1 + \theta}{1 + g}. \]

This equation, along with the steady-state version of equation (2), establishes the relationship between \( r \) and the nominal interest rate, \( i \), which also remains constant over time. We have

\[ 1 + i = \frac{1 + r}{1 + g} (1 + \theta). \]  (27)

Observe also that \( q_t^j \) tends to, from (16),

\[ q^j \equiv \frac{i}{1 + i}. \]  (28)

In the steady state, the mechanism designer assigns

\[ I_{t+1}^j = I_t^j \equiv I^j, \quad z_{t+1}^j = z_t^j \equiv z^j, \]

and \( b_{t+1}^j = b_t^j \equiv b^j; \ j = h, \ell. \) The consumption levels and income misreports too will then remain constant over time:

\[ c_{t+1}^j = c_t^j \equiv c^j, \quad d_{t+1}^j = d_t^j \equiv d^j, \quad x_{t+1}^j = x_t^j \equiv x^j, \quad a_{t+1}^j = a_t^j \equiv a^j. \]

Introduce

\[ y^j \equiv z^j + b^j, \]  (29)

to denote the \( j \)-type’s aggregate “observable” disposable income. The steady-state versions of the equations for \( c_t^j, d_t^j, x_t^j \) and \( a_t^j \) are then given by,

\[ c^j \equiv c \left(q^j, q^d, q^r, y^j, I^j, w^j\right), \]  (30)
\[ d^j \equiv d \left(q^j, q^d, q^r, y^j, I^j, w^j\right), \]  (31)
\[ x^j \equiv x \left(q^j, q^d, q^r, y^j, I^j, w^j\right), \]  (32)
\[ a^j \equiv a \left(q^j, q^d, q^r, y^j, I^j, w^j\right). \]  (33)

\(^{18}\)To see this, substitute from equation (6) into equation (5) and divide the resulting equation by \( p_{t+1} \) to get

\[ (x_{t+1}^j + \beta a_{t+1}^j) = \frac{1 + \theta}{1 + g} \left(\frac{1 + \phi_{t+1}}{1 + \phi_{t+1}}\right). \]
Other equations of interest are the steady-state versions of the young \( j \)-type’s intertemporal budget constraint (10) and his conditional indirect utility function (23). These are given by

\[
q^c c^j + q^d d^j + q^x x^j = y^j + a^j - f(a^j) - q^x \beta a^j, \tag{34}
\]

\[
v^j = v\left(q^c, q^d, q^x, y^j, I^j, w^j\right), \tag{35}
\]

where \( y^j + a^j - f(a^j) - q^x \beta a^j \) is the \( j \)-type’s disposable income. To derive the steady-state version of the government’s budget constraint, divide equation (24) by \( N_t \) to write

\[
\sum_{j=\ell,h} \pi^j (I^j - z^j) + \tau^e \sum_{j=\ell,h} \pi^j c^j + \frac{\pi^d}{1 + \theta} \sum_{j=\ell,h} \pi^j d^j \geq \bar{R}. \tag{36}
\]

Additionally, there is a relationship between money disbursements in real terms, \( b^j \), and real cash balances, \( x^j + \beta a^j \). This is equal to (see the Appendix),

\[
\sum_{j=\ell,h} \pi^j b^j = \frac{1 + g}{1 + r} \frac{\theta}{\theta} \sum_{j=\ell,h} \pi^j (x^j + \beta a^j). \tag{37}
\]

Finally, one can write the “\( jk \)-mimicker’s” demand functions for \( c \) and \( d \), his concealed labor income, and his conditional indirect utility function as,

\[
c^{jk} = c\left(q^c, q^d, q^x, y^k, I^k, w^j\right), \tag{38}
\]

\[
d^{jk} = d\left(q^c, q^d, q^x, y^k, I^k, w^j\right), \tag{39}
\]

\[
x^{jk} = x\left(q^c, q^d, q^x, y^k, I^k, w^j\right), \tag{40}
\]

\[
a^{jk} = a\left(q^c, q^d, q^x, y^k, I^k, w^j\right), \tag{41}
\]

\[
v^{jk} = v\left(q^c, q^d, q^x, y^k, I^k, w^j\right). \tag{42}
\]

We have,

**Proposition 2** Consider the overlapping generations model of Proposition 1. Assuming that the model has a steady-state equilibrium, it is characterized by equations (27)–(33). Secondly, let \( v^j \) and \( v^{jk} \), defined by equations (35) and (42), denote the conditional indirect utility function of the young \( j \)-type and \( jk \)-type agents; \( j = h, \ell \) and \( k \neq j \).
Let $\delta^j$’s be positive constants with the normalization $\sum_{j=\ell,h} \delta^j = 1$. The constrained Pareto-efficient allocations are described by the maximization of $\sum_{j=\ell,h} \delta^j v^j$ with respect to $\tau^c, \tau^d, \theta, b^j, z^j$ and $I^j$; subject to the government’s budget constraint (36), the money injection constraint (37), and the self-selection constraints $v^h \geq v^{h\ell}$ and $v^f \geq v^{fh}$.

4 The overlap between fiscal and monetary instruments

We have seen that, in the steady state, the mechanism designer utilizes $(\tau^c, \tau^d, \theta, b^j, z^j, I^j)$ as his instruments but the welfare of the $j$-type is governed by $(q^c, q^d, q^x, y^j, I^j)$. This suggests that, in the presence of a general income tax schedule and commodity taxes, there is some overlap between fiscal and monetary instruments. This section addresses this question as it relates to the monetary distribution rule and monetary growth rate.

4.1 Monetary distribution rule

Consider, starting from any initial values for $b^h$ and $b^f$, a change in money disbursements to the $h$-type and the $\ell$-type equal to $db^h$ and $db^f$. Simultaneously, change $z^j$ according to $dz^j = -db^j$. Now, with $y^j = z^j + b^j$, $dy^j = 0$, and $(q^c, q^d, q^x, y^j, I^j)$ remains intact. Hence the utility of all agents in the economy including the mimickers, the $jk$-agents, remain the same. As a result, the incentive compatibility constraints continue to be satisfied.

Second, with $(q^c, q^d, q^x, y^j, I^j)$ remaining unchanged, the $j$-type’s demand for $x$ and choice of $a$ do not change either. Consequently, the changes in $b^j$ imply, from the money injection constraint (37), that

$$\sum_{j=\ell,h} \pi^j db^j = \frac{1 + g}{1 + r} \frac{\theta}{1 + \theta} \sum_{j=\ell,h} \pi^j (dx^j + \beta da^j) = 0. \quad (43)$$

Third, with $(q^c, q^d, q^x, y^j, I^j)$ remaining unchanged, the $j$-type’s demand for $c$ and $d$ does not change either. Hence, the only change in the government’s revenue requirement comes from the changes in $z^j$. From (36) and (43), we have

$$dR = - \left( \pi^h dz^h + \pi^f dz^f \right)$$
$$= \pi^h db^h + \pi^f db^f = 0.$$
We thus have shown that the considered changes satisfy all the constraints that the economy faces but leaves every agent as well off as he was before.

The import of all this is that the redistributive effects of increasing the monetary disbursements to one type of agents and reducing them to the other, such that the aggregate money injection to the economy remains the same, can always be offset by changes in the individuals’ income tax payments. The welfare of every agent remains unaffected. This holds true whether the initial equilibrium, corresponding to the initial values of \( b^h \) and \( b^l \), was optimal or not. This finding is summarized as

**Proposition 3** Consider the steady-state equilibrium of Proposition 2. For a given monetary rate of growth, the fiscal authority can offset the redistributive effects of who gets the extra money (or loses the money that is withdrawn from the economy), by adjusting the individuals’ income tax payments. All agents will continue to enjoy the same level of welfare.

We can now address the second dimension of monetary policy, i.e. the rate of growth of money.

### 4.2 Monetary growth rate

Consider now changing the monetary growth rate \( \theta \) by \( d\theta \) which also necessitates a change in \( b^j \) given by \( db^j \). The first-order effect of this change for a \( j \)-type individual is to change his effective price of cash holdings, \( q^x = i / (1 + i) \), and observable disposable income, \( y^j = z^j + b^j \). It is apparent that, whereas the fiscal authority can adjust \( z^j \) to keep \( y^j \) constant, it has no instrument at its disposal which enables it to prevent \( q^x \) from varying. Thus, this dimension of monetary policy has effects which cannot be neutralized by the fiscal authority.

That this aspect of monetary policy has a bite with no counterpart on fiscal side is due to the limitation of tax instruments. To understand this point, suppose one could expand the armory of tax instruments to include a tax rate \( \tau^x \) on all real cash balances
including cash held for concealment. With \( \tau^x \) as an additional instrument, the effective price of cash balances would become \( q^x = \frac{(i + \tau^x)}{(1 + i)} \). Now, concomitantly with the change in \( \theta \), assume the fiscal authority changes \( \tau^x \) and \( z^j \) to keep \( q^x \) and \( y^j \) constant. This would require a change in \( \tau^x \) according to

\[
d\tau^x = -\frac{1 - \tau^x}{1 + \theta} d\theta, \quad (44)
\]

and a change in \( z^j \) equal to \( dz^j = -db^j \). Given \( dq^j = dq^x = 0 \), and no change in \( I^j \), the instituted changes would leave the utility of the \( h \)-types and the \( \ell \)-types intact.

The instituted changes would not affect the utility of potential mimickers either so that the incentive compatibility constraints would remain satisfied as well. To ensure the feasibility of the prescribed reform, one would need only to check the economy’s resource constraint; or equivalently, the government’s budget constraint, which in the presence of \( \tau^x \) becomes:

\[
\sum_{j=\ell,h} \pi^j (I^j - z^j) + \tau^c \sum_{j=\ell,h} \pi^j c^j + \frac{\tau^d}{1+r} \sum_{j=\ell,h} \pi^j d^j + \frac{1 + g}{1 + r} \frac{\tau^x}{1 + \theta} \sum_{j=\ell,h} \pi^j (x^j + \beta a^j) \geq \bar{R}. \quad (45)
\]

---

19 This is of course an implausible assumption and why we have not included \( \tau^x \) in the model presented in Section 2. Nonetheless, assuming such a tax instrument is available is a useful pedagogical device for the purposes of this section.

20 Observe first that a change in \( \theta \) changes the nominal interest rate \( i \), from equation (27), by

\[
di = \frac{1 + r}{1 + g} d\theta.
\]

Now for \( q^x = \frac{(i + \tau^x)}{(1 + i)} \) to remain constant, one must have

\[
\frac{d \tau^x}{1 + i} = -d \frac{i}{1 + i} = d \frac{1}{1 + i}.
\]

This can be rewritten as

\[
\frac{(1 + i) d\tau^x - \tau^x di}{(1 + i)^2} = -di \frac{(1 + i)^2}{(1 + i)^2}.
\]

Simplifying and rearranging the terms results

\[
d\tau^x = \frac{1 - \tau^x}{1 + i} di = \frac{1 - \tau^x}{1 + i} \frac{1 + r}{1 + g} d\theta.
\]

Finally, substituting for \( 1 + i \) in this expression gives the stipulated value for \( d\tau^x \) in the text.

21 Observe that if \( \tau^x \) is levied on \( x \) only and not on cash balances kept for evasion, the effective price of the latter remains at \( i/(1 + i) \). Under this scenario, no change in \( \tau^x \) can keep utilities constant and tax policy cannot neutralize monetary policy.
With no change in consumption goods and holdings of real balances, it is easy to show that \( dR = 0 \).\(^{22}\) Consequently, one can always neutralize the effects of monetary policy through tax policy.

These findings are summarized as:

**Proposition 4** Consider the steady-state equilibrium of Proposition 2.

(i) The change in the rate of money growth has effects that cannot be offset by the tax authority.

(ii) The reason that tax policy cannot neutralize the effects of changes in the money growth rate is the limitation of tax instruments. If the tax authority could levy a direct tax \( \tau^x \) on all money holdings, including the part kept for evasion, the opportunity cost of holding cash balances, \( q^x \), would be determined jointly by the value of the monetary growth rate, \( \theta \), and the tax on cash holdings, \( \tau^x \), and given by \( q^x = (i + \tau^x) / (1 + i) \). Under this circumstance, the tax authority would have enough instruments to undo all the redistributive effects of monetary policy.

We are now ready to investigate whether, in our model with nonlinear income tax and income misreporting, the Friedman rule (hereafter FR) is part of an optimal policy or not. According to the FR, optimality requires a zero opportunity cost of holding real cash balances. This is often stated in terms of choosing a rate of growth for money supply

\[ dR = -\pi^h dz^h - \pi^i dz^i + \sum_{j=l,h} \pi^i \left(x^j + \beta a^i\right) \frac{1 + g}{1 + r} \frac{d\tau^x}{1 + \theta}. \]

\[ = \pi^h db^h + \pi^i db^i + \sum_{j=l,h} \pi^i \left(x^j + \beta a^i\right) \frac{1 + g (1 + \theta)}{1 + r} \frac{d\tau^x - \tau^x d\theta}{(1 + \theta)^2}. \]

\[ = \frac{1 + g}{1 + r} \left[ \sum_{j=l,h} \pi^i \left(x^j + \beta a^i\right) \right] d \frac{\theta}{1 + \theta} + \left[ \sum_{j=l,h} \pi^i \left(x^j + \beta a^i\right) \right] \frac{1 + g (1 + \theta)}{1 + r} \frac{d\tau^x - \tau^x d\theta}{(1 + \theta)^2}. \]

Substituting from (44) for \( d\tau^x \) in this relationship and simplifying yields \( dR = 0 \).
such that the nominal interest rate is equal to zero. By targeting on one instrument, the rate of growth of money supply, this presentation of the FR recognizes the absence of a tax on real cash balances. And with \( \tau^x = 0, \) \( q^x = i/(1 + i) \) so that \( i = 0 \) implies \( q^x = 0. \) Had it been possible to tax all real cash balances, the FR should have been stated as one of setting the money growth rate and the tax on real cash balances such that \( q^x = (i + \tau^x)/(1 + i) = 0; \) or \( i = -\tau^x. \) In what follows we recognize the infeasibility of setting a uniform tax on all cash balances and investigate the optimality of the FR in terms of \( i = 0. \) However, one can use our construct to investigate the FR in terms of \( q^x = 0. \)

5 Second-best characterization

In formulating the second-best optimization problem, we follow the common practice in the optimal income tax literature and ignore the “upward” incentive constraint, \( v^\ell \geq v^{th}; \) assuming that it is automatically satisfied. Thus, the only possible binding constraint will be that of the high-skilled agents mimicking low-skilled agents. Intuitively, this implies that we are concerned only with the realistic case of redistribution from the high-skilled to the low-skilled agents.

Focusing on the steady-state equilibrium, the mechanism designer’s problem can then be represented as:

\[
\max_{I^j, z^j, b^j, \tau^i, \tau^d, \theta} \sum_{j = \ell, h} \delta_j v \left( q^j, q^d, q^x, z^j + b^j, I^j, w^j \right),
\]

subject to the government’s budget constraint,

\[
\sum_{j = \ell, h} \pi^j \left( I^j - z^j + \tau^e c^j + \frac{\tau^d}{1 + r} d^j \right) \geq \tilde{R}, \quad (\mu)
\]

the money injection relationship (37),

\[
\sum_{j = \ell, h} \pi^j b^j = \frac{1 + g}{1 + r} \frac{\theta}{1 + \theta} \sum_{j = \ell, h} \pi^j \left( x^j + \beta a^j \right), \quad (\eta)
\]

19
the self-selection constraint
\[ v \left( q^c, q^d, q^x, z^h + b^h, I^h, w^h \right) \geq v \left( q^c, q^d, q^x, z^\ell + b^\ell, I^\ell, w^\ell \right), \]

and a final constraint for the non-negativity of the nominal interest rate \( i \),
\[ i \geq 0, \quad (\gamma), \]

where the Greek letters on the right-hand side of each constraint denotes its corresponding Lagrange multiplier.

Given the redundancy of one of the redistributive instruments \( b^h \) and \( b^\ell \), it is sufficient to carry out our optimization with respect to only \( b^h \) or \( b^\ell \). Without any loss of generality, we will choose \( b^h \). The mechanism designer then determines \( I^h, I^\ell, z^h, z^\ell, b^h, \tau^c, \tau^d \)
and \( \theta \). In turn, consumers determine their demands for consumption goods \( c, d \), real balances, \( x \), and the amount of income they conceal, \( a \) (thus determining their labor supply as well).

### 5.1 Tax characterization

With income misreporting one cannot rely on the standard argument in optimal tax models that justifies normalizing, without loss of generality, one of the commodity tax rates to zero.\footnote{The normalization argument is based on the observation that the demands for various goods are homogeneous of degree zero in consumer prices and disposable income. Thus, as long as relative prices of the various goods are kept fixed, any effect of a proportionately uniform increase or decrease in the vector of commodity tax rates can be offset via a proper adjustment in the income tax schedule. That with income misreporting this property no longer holds can be seen by inspecting the \( j \)-type’s optimization problem of subsection 2.3. The conditional demand functions (18)–(21), derived from maximization of (7) subject to (10) that yield first-order conditions (11)–(13), are not homogeneous of degree zero in prices \( q^t, q^{t+1}, q^t \), and income \( y_t = z_t + b_{t+1} \) (for a given \( I_t \) and \( w^t \)). For further discussion of this issue, see also footnote 28 below.} Consequently, the mechanism designer must optimize with respect to \( \tau^c, \tau^d \)
and \( \theta \). Denote compensated (Hicksian) variables by a “tilde”, so that, for instance, \( \tilde{x}^j \) denotes the \( j \)-type’s compensated demand for \( x \). The following Proposition, proved in the Appendix, characterizes the optimal policy with respect to the choice of \( \tau^c, \tau^d \)
and \( \theta \).
Proposition 5  Let $\alpha^{ht}$ denote the marginal utility of income for the $h\ell$-mimicker. In the steady-state equilibrium of Proposition 2, the optimal solution for $\tau^c$, $\tau^d$ and $\theta$ satisfy:

\begin{align*}
\sum_j \pi^j \left[ \tau^c \frac{\partial \tilde{\omega}_j}{\partial q^c} + \frac{\tau^d}{1 + r} \frac{\partial \tilde{\omega}_j}{\partial q^d} + \frac{1 + g}{1 + r} \frac{\theta}{1 + \theta} \left( \frac{\partial \tilde{\omega}_j}{\partial q^c} + \frac{\theta}{1 + \theta} \frac{\partial \tilde{\omega}_j}{\partial q^d} \right) \right] = \frac{\lambda \alpha^{ht}}{\mu} \left( c^\ell - c^{ht} \right), \quad (46) \\
\sum_j \pi^j \left[ \tau^c \frac{\partial \tilde{\omega}_j}{\partial q^c} + \frac{\tau^d}{1 + r} \frac{\partial \tilde{\omega}_j}{\partial q^d} + \frac{1 + g}{1 + r} \frac{\theta}{1 + \theta} \left( \frac{\partial \tilde{\omega}_j}{\partial q^c} + \frac{\theta}{1 + \theta} \frac{\partial \tilde{\omega}_j}{\partial q^d} \right) \right] = \frac{\lambda \alpha^{ht}}{\mu} \left( d^\ell - d^{ht} \right), \quad (47) \\
\sum_j \pi^j \left[ \tau^c \frac{\partial \tilde{\omega}_j}{\partial q^c} + \frac{\tau^d}{1 + r} \frac{\partial \tilde{\omega}_j}{\partial q^d} + \frac{1 + g}{1 + r} \frac{\theta}{1 + \theta} \left( \frac{\partial \tilde{\omega}_j}{\partial q^c} + \frac{\theta}{1 + \theta} \frac{\partial \tilde{\omega}_j}{\partial q^d} \right) \right] = \frac{\lambda \alpha^{ht}}{\mu} \left[ \left( x^\ell + \beta a^\ell \right) - \left( x^{ht} + \beta a^{ht} \right) \right] - \frac{\gamma}{\mu}, \quad (48)
\end{align*}

with either (i) $i \geq 0$ and $\gamma = 0$ if the constraint $i \geq 0$ is non-binding, or (ii) $i = 0$ and $\gamma > 0$ if the constraint $i \geq 0$ is binding.

Each of the equations (46)–(48) reflects an optimal trade-off that arises from a compensated marginal increase in a particular policy instrument: between public-budget effects (represented by the left-hand side), and mimicking-deterring effects (represented by the right-hand side\textsuperscript{24}). Note that, in reality, the last term within the square bracket on the left-hand sides of the three equations above captures the effect on the money-injection constraint. However, due to the fact that an optimizing planner always chooses the policy instruments in such a way as to achieve $\mu = -\eta$ (see the proof of Proposition 5 in the Appendix for details), one can re-interpret the effect on the money-injection constraint as a public-budget effect.

6 Is the Friedman rule optimal?

The literature on the golden rule has taught us that whenever the real interest rate $r$ differs from the population growth rate $g$, it is possible to exploit this difference to

\textsuperscript{24} Apart from $-\gamma/\mu$ that appears on the right-hand side of (48) whenever $\gamma \neq 0$ (so that $i = 0$ emerges as a boundary solution.)
increase the steady-state welfare through intergenerational wealth transfers.\textsuperscript{25} In the absence of generation-specific lump-sum taxes, one way to do this is by levying distortionary commodity taxes that entail intergenerational wealth transfers. An inflation tax, i.e. deviating from the FR as in Weiss (1980), is one such mechanism.\textsuperscript{26} Yet this reason for the suboptimality of the FR applies also in the absence of tax evasion (and even if the individuals within a generation are identical). Consequently, to isolate the implications of tax evasion, we proceed by assuming that $r = g$ so that the economy is operating at its golden rule level. We do this to abstract away from, and not be distracted by, the golden rule considerations.

Now setting $r = g$ implies, from (27), that $\theta = i$. Under this circumstance the optimality of the FR, $q^x = i/(1 + i) = 0$, is the same thing as the optimality of $\theta = 0$. It then follows from Proposition 5 that to have the FR satisfied, we must have

\[
\tau^c \sum_j \pi^j \frac{\partial \tilde{c}^j}{\partial q^c} + \tau^d \sum_j \pi^j \frac{\partial \tilde{d}^j}{\partial q^c} = \frac{\lambda \alpha h^f}{\mu} \left( c^f - c^{h^f} \right),
\]

(49)

\[
\tau^c \sum_j \pi^j \frac{\partial \tilde{c}^j}{\partial q^d} + \tau^d \sum_j \pi^j \frac{\partial \tilde{d}^j}{\partial q^d} = \frac{\lambda \alpha h^f}{\mu} \left( d^f - d^{h^f} \right),
\]

(50)

\[
\tau^c \sum_j \pi^j \frac{\partial \tilde{c}^j}{\partial q^x} + \tau^d \sum_j \pi^j \frac{\partial \tilde{d}^j}{\partial q^x} = \frac{\lambda \alpha h^f}{\mu} \left[ \left( x^f + \beta a^f \right) - \left( x^{h^f} + \beta a^{h^f} \right) \right] - \frac{\gamma}{\mu},
\]

(51)

with $\gamma = 0$ if the constraint $i \geq 0$ is non-binding and $\gamma > 0$ if the constraint is binding.

“Solve” equations (49)--(50) for $\tau^c$ and $\tau^d$, then substitute in (51), to get (see the Appendix):

\[
\gamma = \frac{\lambda \alpha h^f A}{\sum_j \pi^j \frac{\partial c^j}{\partial q} \sum_j \pi^j \frac{\partial d^j}{\partial q} - \sum_j \pi^j \frac{\partial \tilde{c}^j}{\partial q} \sum_j \pi^j \frac{\partial \tilde{d}^j}{\partial q}},
\]

(52)

\textsuperscript{25}The terminology and the original formulation of the golden rule, in the context of the neoclassical growth model, is due to Phelps (1961). For discussions in the context of overlapping-generations model, see, among others, Diamond (1965), Hamada (1972), and Pestieau (1974).

\textsuperscript{26}As shown by Galvani (1988), existence of generation-specific lump-sum taxes makes the use of such distortionary taxes unnecessary and restores the optimality of the FR.
where

$$A = \left[ \sum_j \pi^j \frac{\partial c^j}{\partial q^l} \sum_j \pi^j \frac{\partial d^j}{\partial q^x} - \sum_j \pi^j \frac{\partial d^j}{\partial q^d} \sum_j \pi^j \frac{\partial c^j}{\partial q^x} \right] \left( c^\ell - c^{h\ell} \right) +$$

$$\left[ \sum_j \pi^j \frac{\partial d^j}{\partial q^e} \sum_j \pi^j \frac{\partial c^j}{\partial q^e} - \sum_j \pi^j \frac{\partial c^j}{\partial q^e} \sum_j \pi^j \frac{\partial d^j}{\partial q^e} \right] \left( d^\ell - d^{h\ell} \right) +$$

$$\left[ \sum_j \pi^j \frac{\partial d^j}{\partial q^d} \sum_j \pi^j \frac{\partial c^j}{\partial q^d} - \sum_j \pi^j \frac{\partial c^j}{\partial q^d} \sum_j \pi^j \frac{\partial d^j}{\partial q^d} \right] \left[ \left( x^\ell + \beta d^\ell \right) - \left( x^{h\ell} + \beta a^{h\ell} \right) \right].$$

(53)

From the properties of the Slutsky matrix, the denominator in (52) is positive. Consequently, the FR holds as an interior solution if $A = 0$ and as a boundary solution if $A > 0$.

### 6.1 Absence of tax evasion

To set the stage for the discussion of the import of tax evasion, we first consider the applicability of the FR in our model in the absence of tax evasion. Under this circumstance $a^j = a^{jk} = 0$.

#### 6.1.1 Separable preferences

If preferences are separable in labor supply and goods, $U = U (u (c, d, x) - \varphi (L))$, Atkinson and Stiglitz (1976) theorem holds and $c^\ell = c^{h\ell}, d^\ell = d^{h\ell}, x^\ell = x^{h\ell}$. It then follows from (53) that $A = 0$. Consequently, as with da Costa and Werning’s (2008) result, the FR holds as an interior solution. Indeed, in this case, $\tau^c = \tau^d = \theta = 0$, coupled with $\gamma = 0$, constitutes a solution to (46)–(48) as required by the Atkinson and Stiglitz (1976) theorem.

#### 6.1.2 Non-separable preferences

In the absence of tax evasion tax normalization becomes possible. Setting $\tau^d = 0$, the tax optimization will be with respect to $\tau^c$ and $\theta$ only and we have, corresponding to
equations (49) and (51),
\[ \tau^c \sum_j \pi^j \frac{\partial \tilde{c}^j}{\partial q^c} = \frac{\lambda \alpha^{ht}}{\mu} \left( c^\ell - c^{ht} \right), \]  
(54)
\[ \tau^c \sum_j \pi^j \frac{\partial \tilde{c}^j}{\partial q^x} = \frac{\lambda \alpha^{ht}}{\mu} \left( x^\ell - x^{ht} \right) - \frac{\gamma}{\mu}, \]  
(55)
Eliminating \( \tau^c \) between these two equations yields,
\[ \gamma = \lambda \alpha^{ht} \left[ \left( x^\ell - x^{ht} \right) + \frac{\sum_j \pi^j \frac{\partial \tilde{c}^j}{\partial q^c}}{-\sum_j \pi^j \frac{\partial \tilde{c}^j}{\partial q^x}} \left( c^\ell - c^{ht} \right) \right]. \]  
(56)
Now recall that in da Costa and Werning (2008) the FR holds as a boundary solution if preferences are non-separable provided that real cash balances and labor supply are complements. This second result of da Costa and Werning (2008) does not hold in our setting. Their assumption of \( x^\ell > x^{ht} \) no longer guarantees that the right-hand side of (56) is positive so that \( \gamma > 0 \). An additional assumption such as \((c^\ell - c^{ht}) \sum_j \pi^j \left( \partial \tilde{c}^j / \partial q^c \right) \geq 0 \) is also required.

The reason for this discrepancy in results is that da Costa and Werning (2008) disregard differential commodity taxation and its impact on the consumption of real cash balances. Intuitively, the reason that \( i = 0 \) emerges as a boundary solution in their model is the desire to encourage the consumption of real balances by subsidizing it. Due to \( \tau^c \), however, the consumption of real cash balances might already be high enough as to making it raise further by pushing \( i \) down to zero undesirable. The additional \((c^\ell - c^{ht}) \sum_j \pi^j \left( \partial \tilde{c}^j / \partial q^x \right) \geq 0 \) constraint is meant to preclude this possibility. To see the argument, suppose the additional constraint is violated so that \((c^\ell - c^{ht}) \sum_j \pi^j \left( \partial \tilde{c}^j / \partial q^x \right) < 0 \). Given that from (54) \( \tau^c \) and \( c^\ell - c^{ht} \) are of opposite signs, this is equivalent to \( \tau^c \sum_j \pi^j \left( \partial \tilde{c}^j / \partial q^x \right) = \tau^c \sum_j \pi^j \left( \partial \tilde{c}^j / \partial q^c \right) > 0 \). In turn, this implies that \( \tau^c \) and \( \sum_j \pi^j \left( \partial \tilde{c}^j / \partial q^x \right) \) are of the same sign. Whether \( \tau^c \) is positive or negative, its presence implies a higher consumption level for real cash balances.

Finally, notice that with no differential commodity taxes in our model, \( \tau^c = \tau^d = 0 \) and \( \gamma \) reduces to
\[ \gamma = \lambda \alpha^{ht} \left( x^\ell - x^{ht} \right). \]
The $x^\ell > x^{h\ell}$ assumption is then sufficient for $\gamma > 0$.

6.2 Tax evasion

Lemma 1, proved in the Appendix, provides the key to understanding the import of tax evasion for the results concerning optimality of the FR (and redundancy of commodity taxation result in general).

Lemma 1 Faced with the mechanism $(\tau^c, \tau^d, b^h, z^h, z^\ell, I^h, I^\ell)$, the $h\ell$-mimicker conceals a larger amount of income ($a^{h\ell} > a^\ell$) and has a larger disposable income than the $\ell$-type.

Lemma 1 implies, among other things, that tax evasion leads to the breakdown of two common results in optimal Mirrleesian income tax models. One is the celebrated Atkinson and Stiglitz (1976) theorem. Suppose preferences are separable in labor supply and goods so that individuals’ marginal rates of substitution between goods are independent of their labor supplies. This implies that in the absence of tax evasion, the $h\ell$-mimicker and the $\ell$-type would have identical demand for goods (which makes differential commodity ineffectual for redistributive purposes). On the other hand, with tax evasion, the $h\ell$-mimicker and the $\ell$-type would have different levels of disposable income despite having the same $(z^\ell, I^\ell)$ bundle. As a result, they will have different demands for goods. This restores the usefulness of differential commodity taxes, and along with it, a role for deviating from the FR if preferences are separable in labor supply and goods.

The other is the result that the $h\ell$-mimicker has a lower labor supply than the true low-skilled agent. In the absence of tax evasion, this result follows because the mimicker, being more productive, works fewer hours to earn the same amount of income as the low-skilled. A result that allows one to compare the mimickers’ and the low-skilled agents’ demands for a particular good based on the complementarity/substitutability of that good with labor supply. That with income tax evasion $a^{h\ell} > a^\ell$ implies one can no longer conclude that the labor supply of a low-skilled agent, $L^\ell = (I^\ell + a^\ell) / w^\ell$,
is unambiguously larger than that of an \( h\ell \)-mimicker, \( L^{h\ell} = (I^\ell + a^{h\ell}) / u^h \).\(^{27}\) This ambiguity deprives the complementarity/substitutability assumption of its predictive power. We will see the relevance of this finding for the optimality of the FR in the discussion of the non-separable preferences.\(^{28}\)

### 6.2.1 Separable preferences

That Lemma 1 implies \( c^\ell \neq c^{h\ell}, d^\ell \neq d^{h\ell}, x^\ell \neq x^{h\ell} \) even if preferences are separable in labor supply and goods alerts us to the fact that in the presence of tax evasion expression \( A \) given by (53) can be equal to zero only by chance for some special type of preferences. That is, as a general rule, the FR does not hold as an interior solution. Indeed, one can easily prove that with separability if all pairs of goods are Hicksian substitutes, the FR can never hold as an interior solution. Interestingly, it cannot hold as a boundary solution either. To prove this, recall that from Lemma 1, \( a^{h\ell} > a^\ell \) so that an \( h\ell \)-mimicker has a larger disposable income than a true low-skilled agent:

\[
y^\ell + a^{h\ell} - f(a^{h\ell}) < y^\ell + a^\ell - f(a^\ell) - \beta_q^x a^\ell.
\]

Moreover, with separability, labor supply does not directly affect the way income is spent across goods. Thus, assuming

\(^{27}\)More precisely, the fact that a mimicker has a higher wage rate exerts both an income and a substitution effect on labor supply. The income effect is negative and tends to make the mimicker’s labor supply lower than that of a low-skilled. The substitution effect, on the other hand, may be either positive or negative (for a detailed analytical proof of this claim, see section 2.2 of Blomquist et al., 2011). If the substitution effect is positive and large enough, a mimicker’s labor supply will exceed the labor supply of a true low-skilled.

\(^{28}\)Lemma 1 also sheds light on the reason for our earlier observation regarding the relevance of tax normalization. With income misreporting, even a uniform commodity tax rate can have a bite and normalizing one of the commodity tax rates to zero is no longer a harmless assumption. To see this, note that according to this Lemma, income misreporting implies that a mimicker has a larger disposable income than a true low-skilled agent. Now start from an initial equilibrium where commodity taxes are not used and consider introducing a small uniform commodity tax at rate \( r \) on all goods, while at the same time raising the after-tax reported income \( z^j, j = \ell, h \), to leave the utility of the non-mimicking agents unchanged. [In our setting, this requires adjusting \( z^j \) by \( dz^j = [(c^j + d^j) / (1 + r)] \tau \).] With the \( h\ell \)-mimickers’ disposable income exceeding that of true low-skilled agents, and for simplicity assuming \( i = 0 \), the total expenditure of the mimicker on goods \( c \) and \( d \) exceeds that of a true low-skilled agent: \( c^{h\ell} + d^{h\ell} / (1 + r) > c^\ell + d^\ell / (1 + r) \). Consequently, the increase in \( z^j \) is not enough to fully compensate the mimicker for the introduction of the uniform commodity tax. As a result, this reform would make the \( h\ell \)-mimicker worse-off and slackens the previously binding self-selection constraint. This means that, in the presence of tax evasion, the absolute price levels of \( c \) and \( d \) matter and one cannot simply normalize one of the prices.
all goods are normal, \( c^{ht} > c^t \), \( d^{ht} > d^t \), and \( x^{ht} + \beta a^{ht} \geq x^t + \beta a^t \).\(^{29}\) Now, assuming all pairs of goods are Hicksian substitutes, the bracketed expressions to the left of \( (c^t - c^{ht}) \) and \( (d^t - d^{ht}) \) in (53) are positive. At the same time, from the properties of the Slutsky matrix, the bracketed expression appearing to the left of \([\left( x^t + \beta a^t \right) - \left( x^{ht} + \beta a^{ht} \right)]\) is also positive. Consequently, \( A < 0 \) and the FR holds neither as an interior solution nor as a boundary solution.

On the other hand, if at least one pair of goods are Hicksian complements, then the FR might hold as a boundary solution. For instance, if \( x \) is a Hicksian complement to both \( c \) and \( d \), while \( c \) and \( d \) are Hicksian substitutes, the first two terms in \( A \) are positive\(^{30}\) and the FR may hold as a boundary solution. To get an intuition for why this might happen, suppose one chooses \( \tau^c \) and \( \tau^d \) optimally conditional on \( i = \theta = 0 \). Both \( \tau^c \) and \( \tau^d \) would then be positive,\(^{31}\) thus indirectly discouraging the demand for real cash balances (since \( x \) is assumed to be a Hicksian complement to both \( c \) and \( d \)). If the induced downward distortion on the demand for cash balances proves to be suboptimally large, the social planner may want to counter it by pushing \( i \) all the way down to zero.

Another instance of income misreporting negating the optimality of the FR as an interior solution with separable preferences arises when the general income tax is the only instrument used (\( \tau^c = \tau^d = 0 \)). Under this circumstance, equations (46)–(47) in Proposition 5 no longer apply and the optimality condition for monetary growth rate is given by equation (48) only. Setting \( \gamma = 0 \) and \( \tau^c = \tau^d = 0 \) in equation (48) yields,

\[
\frac{\theta}{1 + \theta} \sum_j \tau_j^i \left( \frac{\partial \tilde{x}_j}{\partial q^t} + \beta \frac{\partial \tilde{x}_j}{\partial q^t} \right) = \frac{\lambda \alpha_{ht}}{\mu} \left[ \left( x^t + \beta a^t \right) - \left( x^{ht} + \beta a^{ht} \right) \right].
\]

The FR is thus violated in the presence of tax evasion because, as we have seen, the right-hand side of the above equation is negative.\(^{32}\) Indeed, given that \( \partial \tilde{x}_j / \partial q^t < 0 \)

\(^{29}\)The possibility for equality arises if agents are over-reporting so that \( \beta = 0 \) and if both \( x^t \) and \( x^{ht} \) happen to be at their satiation level.

\(^{30}\)Remember that, under separability and with income misreporting, \( c^t - c^{ht} < 0 \) and \( d^t - d^{ht} < 0 \).

\(^{31}\)This can be easily seen by setting \( \theta = 0 \) in (46)–(47) and remembering that \( c^t - c^{ht} < 0 \) and \( d^t - d^{ht} < 0 \).

\(^{32}\)Observe that in the absence of commodity taxes, low-ability agents (as well as \( h \ell \)-mimickers) face
and $\partial a^j / \partial q^r < 0$, the optimal solution for $\theta$ is positive. On the other hand, without tax evasion, the right-hand side collapses to zero so that $\theta = 0$ and the FR holds.

### 6.2.2 Non-separable preferences

With non-separable preferences, tax evasion implies that the FR may or may not hold as a boundary solution. Inspecting the various terms in $A$, as given by (53), reveals that $A$ may be positive or negative depending on the various substitutability/complementarity relationships between goods and between goods and labor supply. However, unlike the case without tax evasion, one cannot establish simple sufficient conditions for $A > 0$ (and the optimality of the FR as a boundary solution). The problem is that even full knowledge of the substitutability/complementarity relationship of labor supply to $c, d$, and $x$ leaves the signs of $(c^e - c^{ht}), (d^e - d^{ht})$, and $(x^e + \beta a^e) - (x^{ht} + \beta a^{ht})$ in $A$ indeterminate. In the presence of tax evasion, as we mentioned in our discussion of Lemma 1, either the low-skilled or the mimicker can have a larger labor supply.\(^\text{33}\)

The results of this section are summarized as:

**Proposition 6** Consider the steady-state equilibrium of Proposition 2 at its golden rule level:

(i) The Friedman rule does not generally hold as an interior solution.

(ii) The Friedman rule may or may not hold as a boundary solution.

(iii) A necessary condition for the FR to hold as a boundary solution under separability of preferences between labor supply and goods is that at least one pair of goods are Hicksian complements.

(iv) Whether or not complementarity between real cash balances and labor supply favors the optimality of the FR as a boundary solution depends on whether the labor supply of a low-skilled agent exceeds or falls short of the labor supply of an $ht$-mimicker.

\(^{33}\)The problem is simpler with separable preferences because we need not know which agents have a larger or smaller labor supply. It simply does not matter.
(v) Even in the absence of tax evasion, complementarity between labor supply and consumption of real cash balances does not guarantee the optimality of the Friedman rule as a boundary solution.

7 Robustness of the results

This Section briefly discusses how our results generalize to settings characterized by different modeling assumptions. In particular, we will consider the consequences of (i) increasing the number of agents’ types, (ii) modeling evasion as a risky activity, and (iii) allowing for the possibility of commodity tax evasion.

7.1 Increasing the number of agents’ types

Although our results are derived in the context of a two-group model, they generalize to models with more than two groups. The two-group specification is a simple yet useful device that allows economizing on notation while at the same time shedding light on the important mechanisms underlying the planner’s problem. Our main goal in this paper is to examine if the FR is optimal in a Mirrleesian tax problem with income misreporting. And what we have found is that the answer is in general in the negative with the underlying reason being that tax evasion allows mimickers to have a higher disposable income than the true low-skilled.\(^{34}\) Exploiting this property via deviating from the FR allows the mechanism designer to improve welfare by weakening an otherwise binding incentive compatibility constraint.

Specifically, with two types of agents, there is only one binding self-selection constraint. Under the so-called “normal” case where redistribution goes from the high- to the low-skilled, the binding self-selection constraint is the one requiring high-skilled agents not to have an incentive to mimic low-skilled agents by choosing the income point intended solely for the latter. The potential role for exploiting a deviation from the FR as a deterrent to mimicking comes, as we have seen, from the fact that with

\(^{34}\) Another reason, as we have seen earlier, is difference between \(r\) and \(g\). However, this has nothing to do with tax evasion and exists also in models without tax evasion.
income misreporting a high-skilled mimicker would have a larger disposable income than a low-skilled agent. In turn, this implies that the overall demand for cash balances by a mimicker is larger than that of a low-skilled agent.

In a more general model with \( n > 2 \) groups of agents, the same type of mechanism will be at work. It is true that one would have \( n - 1 > 1 \) binding self-selection constraints. However, these binding constraints would all be running downwards, linking pairs of adjacent types. And each of these constraints would share the properties of the single self-selection constraint characterizing our two-type model.\(^\text{35}\)

### 7.2 Evasion as a risky activity

Another assumption of our model has been that income misreporting is a costly but riskless activity. As an alternative, one may model income misreporting as a risky activity which can be detected through costly audits by the tax authority and punished along a penalty function. This is the approach taken, among others, by Cremer and Gahvari (1996) and Schroyen (1997) within the strand of the optimal general income tax literature.\(^\text{36}\) The question is if this latter modeling strategy may change the nature of our qualitative results regarding the optimal monetary policy. The answer to this question, we argue below, is in the negative.

To this end, we adapt Cremer and Gahvari’s (1995) approach to our setting. Let \( p^j \) denote the probability that an agent reporting income \( I^j \) is audited and \( P^j \) denote the additional payment he has to make upon being audited. If \( P^j > 0 \) this is the unpaid tax plus the penalty; if \( P^j < 0 \) this is a reward for being subjected to an audit while

\(^{35}\)There is one caveat. Whereas in a two-type model a pooling equilibrium—with both types of agents choosing the same income point—is necessarily suboptimal, in a model with more than two types the optimum may deviate from a fully separating equilibrium and can feature partial pooling. If this is the case, an additional advantage of deviating from the FR is that it may help separating the agents’ types who are pooled at the pure income tax optimum.

\(^{36}\)Both papers ignore commodity taxes and focus on the design of optimal nonlinear income tax and audit policy in a two-group model of risk-averse agents who, as in our paper, differ only in terms of market ability (skill). The main difference between the two contributions concerns the modeling of the penalty function. In Cremer and Galvari (1995), the penalty policy is not restricted and is designed as part of the optimal tax/audit/fine policy. This allows them to directly control the underreporting behavior of the agents which implies that in equilibrium no agent has an incentive to evade. In Schroyen (1997), on the other hand, the penalty system is predetermined with fines being proportional to the evaded income. Hence evasion may occur at the solution to the planner’s optimal tax problem.
innocent (i.e. earning one’s reported income). \(37\) Moreover, as in Cremer and Gahvari (1995), assume that, if found guilty, an agent will have all his income confiscated by the tax authority. Further, for simplicity, ignore commodity taxes.

Assume preferences are separable in labor supply and goods, and define:

\[
EU^j = (1 - p^j) v(q^x, z^j + b^j) + p^j v(q^x, z^j + b^j - P^j) + \phi \left( \frac{I^j}{w^j} \right),
\]

\[
EU^{jk} = (1 - p^k) v(q^x, z^k + b^k) + p^k v(q^x, z^k + b^k - P^k) + \phi \left( \frac{I^k}{w^j} \right),
\]

\[
V^j = (1 - p^j) v(q^x, z^j + b^j + a^j - q^x \beta a^j) + \phi \left( \frac{I^j + a^j}{w^j} \right),
\]

\[
V^{jk} = (1 - p^k) v(q^x, z^k + b^k + a^{jk} - q^x \beta a^{jk}) + \phi \left( \frac{I^j + a^{jk}}{w^k} \right).
\]

Expressions (57)–(58) represent the expected utilities of agents who earn the income level they report (i.e. do not engage in tax evasion); but may or may not report their type truthfully. Specifically, equation (57) represents the expected utility of an agent of type \(j\) who chooses to report income \(I^j\) and does not engage in tax evasion. Expression (58) represents the expected utility of a \(j\)-type agent who masquerades as a \(k\)-type by reporting and earning income \(I^k\). Observe that these individuals receive a reward if their income is audited so that \(-P^j\) and \(-P^k\) that appear in these equations are positive.

Expressions (59)–(60) represent the expected utilities of agents who earn a different level of income from what they report (i.e. engage in tax evasion). Thus equation (59) represents the expected utility of a \(j\)-type agent who reports his income to be \(I^j\) but earns \(I^j + a^j\) (concealing \(a^j\) units of income from the tax authority). Similarly, expression (60) represents the expected utility of a \(j\)-type agent who masquerades as a \(k\)-type by reporting income \(I^k\) but earns \(I^k + a^{jk}\) (concealing \(a^{jk}\) units of income).

Observe that in writing these expressions, we have normalized \(v(q^x, 0) = 0\).

Let \(\Theta \left( \sum_{j=\ell,h} \pi^j p^j \right)\) denote audit costs where \(\Theta(\cdot)\) is increasing and convex. Using _______________________________

\(37\) As shown in Cremer and Gahvari (1995), it is optimal to reward honesty so that \(P^j < 0\) is a possibility in this model. This is a standard result in the auditing literature; see, e.g., Border and Sobel (1987) and Mookherjee and Png (1989) in other contexts.
expressions (57)–(60), one can write the planner’s problem as:

$$\max_{I^j, z^j, b^j, p^j, P^j} \sum_{j=\ell, h} \delta^j EU^j,$$

subject to the government’s budget constraint,

$$\sum_{j=\ell, h} \pi^j (1 - p^j) (I^j - z^j) + p^j \pi^j (I^j - z^j + P^j) \geq \tilde{R} + \Theta \left( \sum_{j=\ell, h} \pi^j p^j \right), \quad (\mu)$$

the money injection relationship (37),

$$\sum_{j=\ell, h} \pi^j b^j = \frac{1 + g}{1 + \theta} \frac{\theta}{1 + \theta} \sum_{j=\ell, h} \pi^j (x^j + \beta a^j), \quad (\eta)$$

the self-selection constraint

$$EU^h \geq EU^{h\ell}, \quad (\lambda_1)$$

and the following set of “moral hazard” constraints:

$$EU^h \geq V^h, \quad (\lambda_2)$$

$$EU^h \geq V^{h\ell}, \quad (\lambda_3)$$

$$EU^\ell \geq V^\ell. \quad (\lambda_4)$$

Notice in particular the appearance in the above problem of three new constraints, the moral hazard constraints, requiring that in equilibrium no agent has an incentive to conceal income from the tax authority. As shown by Cremer and Gahvari (1995), in their model without money, in all the solutions to the planner’s optimal tax problem either $EU^h \geq V^{h\ell}$ binds, $EU^\ell \geq V^\ell$ binds, or both. Which implies, as we argue below, that in our model with money the FR is never optimal.

To understand the point, start from an initial equilibrium when the above problem is solved subject to the additional constraint that $i = 0$. Then consider a marginal increase

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38 Specifying all, there are four possible regimes: (i) $\lambda_1 > 0, \lambda_2 = 0, \lambda_3 > 0, \lambda_4 = 0$; (ii) $\lambda_1 > 0, \lambda_2 = 0, \lambda_3 = 0, \lambda_4 > 0$; (iii) $\lambda_1 > 0, \lambda_2 = 0, \lambda_3 > 0, \lambda_4 > 0$; (iv) $\lambda_1 = 0, \lambda_2 = 0, \lambda_3 > 0, \lambda_4 = 0$. 

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in $i$ accompanied with the necessary adjustments in $b^j$ and $z^j$, $j = h, \ell$, that ensures the money injection relationship and the government’s budget constraint continue to be satisfied while the expected utilities of non-mimicking agents, $EU^h$ and $EU^\ell$, remain unchanged.\(^{39}\) While this reform has, by construct, no first-order welfare effect on the $h$- and the $\ell$-types, it makes the tax evaders worse off. This is because a person who reports $I^j$, $j = h, \ell$, but earns a different level of income has a larger disposable income, and thus a larger consumption of cash balances, than the corresponding non-evading agent. He thus needs more compensation, to remain at his previous utility level, than that required by the corresponding non-evading agent. Consequently, $V^h$, $V^\ell$, and $V^{ht}$ all decline. Given that at least one of the constraints $EU^h \geq V^{ht}$ and $EU^\ell \geq V^\ell$ is always binding, that binding constraint slackens. Increasing the growth rate of money supply will then lead to a higher level of welfare.

Thus, the main difference between modeling income misreporting as a riskless or a risky activity is that in the latter case, allowing for a nonlinear penalty policy, no agent is misreporting income at the optimum. Nonetheless, the results obtained for the riskless case, on which our paper has focused, generalize to the case where evasion is a risky activity. In the riskless case, deviating from the FR helps weaken the binding self-selection constraint; in the risky case, deviating from the FR helps weaken the binding moral hazard constraints.

\subsection{Commodity tax evasion}

In our analysis we have disregarded the possibility of commodity tax evasion. But suppose that instead, in addition to income misreporting, consumers can evade paying sales taxes by concealing their purchases from the tax authority. Assume also that, as in Cremer and Gahvari (1993), they are able to do so if they incur a concealment cost which is commodity-specific, increasing and convex in the amount concealed. For any given tax rate on a commodity, consumers will conceal their purchases up to a point where the private marginal resource cost of concealing coincides with the commodity

\(^{39}\)There are four instruments to ensure the four constraints remain satisfied.
tax rate. Beyond this point, any additional purchases of the good will not be concealed and will be subject to its commodity tax rate.

For the points our paper is making, allowing for commodity taxation makes no difference except that the distortionary effect of taxes is perceived differently by the social planner. A marginal compensated increase in the tax rate on a given good will have two effects on the tax revenue generated from that good. A first standard effect comes from the reduction of the demand for that good (depending on the own-price substitution effect). The second, non standard, effect comes from a further reduction in the units of the good that are taxed (due to the fact that concealment becomes more beneficial to consumers). To sum, allowing for the possibility of commodity tax evasion affects the magnitude of the derivatives of the compensated demands appearing in the formulas presented in Section 6 but it does not ultimately alter the qualitative results concerning the desirability to deviate from the FR.

8 Summary and conclusion

This paper has developed a version of Samuelson’s (1958) overlapping-generations model that (i) includes money, (ii) is populated with agents who are heterogeneous in terms of earning ability, and (iii) allows for tax evasion. Money is used for two reasons. One is the traditional (non-evading) usage captured by money-in-the-utility-function; the other is to facilitate tax evasion. Money supply increases, or contracts, at a fixed rate per year through lump-sum money transfers to individuals. The policy-maker has information on the distribution of abilities in the population, on individuals’ preferences, and on the technology used by agents to shelter income from the tax authority. Reported incomes are subject to a nonlinear tax schedule.

The paper has studied the nature of the economy’s perfect-foresight temporal equilibrium as well as its steady state. It has characterized the informationally constrained Pareto-efficient allocations of this economy and the properties of the optimal commodity taxes that implement them. It has shown that, for a given monetary rate of growth, the fiscal authority can offset the redistributive effects of who gets the extra money (or
loses the money that is withdrawn from the economy), by adjusting the individuals’ income tax payments. All agents will continue to enjoy the same level of welfare. This result holds with or without tax evasion. On the other hand, tax evasion, despite the existence of a general income tax, has drastic implications for the redistributive power of the monetary growth rate.

In the absence of tax evasion, as long as preferences are separable in labor supply and goods, Atkinson and Stiglitz (1976) theorem applies and the Friedman rule becomes optimal for precisely the same reason that commodity taxes become redundant in such a setting. With non-separable preferences, if consumption of real cash balances is positively related to labor supply, one would want to encourage the demand for cash balances. Absent (differentiated) commodity taxation, this implies that the optimal nominal interest rate is negative, and therefore, given the non-negativity of nominal interest rate, the Friedman re-emerges as a boundary solution. With optimally differentiated commodity taxes, however, complementarity between real cash balances and labor supply is no longer a sufficient condition for the optimality of the Friedman rule. If goods that are Hicksian substitutes for real cash balances are taxed at relatively high rates, deviating from the Friedman rule may be part of an optimal policy even with positively correlated real cash balances and labor supply. The same is true if goods that are Hicksian complements to real cash balances are taxed at relatively low rates.

The presence of tax evasion has important consequences for the desirability of the Friedman rule. The reason is that although the agents who are high-skilled but pretend to be low-skilled and true low-skilled agents have the same before-tax reported labor income, they will nevertheless have different disposable incomes. A situation that cannot happen without tax evasion. Specifically, the “mimickers” conceal a larger amount of income which results in their having a larger disposable income than the true low-skilled agents. This difference breathes a new life into the redistributive power of monetary growth rate and the Friedman rule does not generally hold as an interior solution if preferences are separable in labor supply and goods. Moreover, because of tax evasion, the labor supply of a high-skilled mimicker can exceed that of a low-skilled agent. This
implies that complementarity of real cash balances and labor supply may even weaken the case for the optimality of the Friedman rule as a boundary solution.

The paper concludes by showing that its results are robust on many fronts: the number of types in the economy, modeling income tax evasion as a risky activity subject to audits, and the possibility of commodity tax evasion.
Appendix

Derivation of (10): Substitute \( z_t + a_t - f(a_t) - (1 + \tau^c) c_t - m_t/p_t \) for \( s_t \) from (8) into (9) to get:

\[
p_{t+1} \left(1 + \tau^d\right) d_{t+1} = p_t \left[z_t + a_t - f(a_t) - (1 + \tau^c) c_t - m_t/p_t\right] (1 + i_{t+1}) + m_t + e_{t+1}
\]

\[
= p_{t+1} \left[z_t + a_t - f(a_t) - (1 + \tau^c) c_t - m_t/p_t\right] (1 + r) + m_t + e_{t+1},
\]

where in deriving the last equality we have substituted \( p_t (x_t + \beta a_t) \) for \( m_t \) from equation (6). Divide the above expression by \( p_{t+1} (1 + r) \) and substitute \( 1/(1 + i) \) for \( p_t/p_{t+1} (1 + r) \) to arrive at

\[
\frac{(1 + \tau^d) d_{t+1}}{1 + r} = [z_t + a_t - f(a_t) - (1 + \tau^c) c_t - x_t - \beta a_t] + \frac{x_t + \beta a_t}{1 + i_t} + \frac{e_{t+1}}{p_{t+1} (1 + r)}
\]

Rearranging the terms and simplifying leads to (10).

Derivation of (37): Substitute for \( M_t \) from equation (1) into (4), divide by \( N_t p_t \), and substitute for \( m_j^t \) from (6) into the resulting equation to get:

\[
\pi^h \epsilon^h_{t+1} \frac{p_t}{p_{t+1}} + \pi^\ell \epsilon^\ell_{t+1} \frac{p_t}{p_{t+1}} = \theta \left( \pi^h \frac{m^h}{p_t} + \pi^\ell \frac{m^\ell}{p_t} \right)
\]

\[
= \theta \left[ \pi^h \left( x^h_t + \beta a^h_t \right) + \pi^\ell \left( x^\ell_t + \beta a^\ell_t \right) \right].
\]

Then, using the relationship between \( \epsilon^j_{t+1} \) and \( b^j_{t+1} \) given by (17), rewrite the above equation in terms of \( b^j_{t+1} \), and substitute \( (1 + i_{t+1}) \) for \( (1 + r) p_{t+1}/p_t \):

\[
(1 + i_{t+1}) \left( \pi^h b^h_{t+1} + \pi^\ell b^\ell_{t+1} \right) = \theta \left[ \pi^h \left( x^h_t + \beta a^h_t \right) + \pi^\ell \left( x^\ell_t + \beta a^\ell_t \right) \right].
\]

The steady version of this equation is,

\[
(1 + i) \sum_{j=\ell,h} \pi^j \beta^j = \theta \sum_{j=\ell,h} \pi^j \left( x^j + \beta a^j \right).
\]

Dividing this equation by \( (1 + i) \) and substituting for \( (1 + i) \) from (27) in it yields (37).
Proof of Proposition 5: The mechanism designer’s problem can be described by means of the Lagrangian:

\[
L = \sum_{j=l,h} \delta^j v^j + \lambda \left( v^h - v^l \right) + \eta \left[ \sum_{j=l,h} \pi^j v^j - \frac{1 + g}{1 + r} \frac{\theta}{1 + \theta} \sum_{j=l,h} \pi^j \left( x^j + \beta a^j \right) \right] \\
+ \mu \left[ \sum_{j=l,h} \pi^j \left( I^j - z^j + \tau^c c^j + \frac{\tau^d d^j}{1 + r} d^j \right) - \bar{R} \right] + \gamma i.
\]

Given the redundancy of one of the redistributive instruments \( b^h \) and \( b^l \), it is sufficient to carry out our optimization with respect to only \( b^h \) or \( b^l \). Without any loss of generality, we will choose \( b^h \). Then, the first order conditions of this problem are:

\[
\frac{\partial L}{\partial I^h} = \left( \delta^h + \lambda \right) \frac{\partial v^h}{\partial I^h} - \eta \left( 1 + g \right) \frac{\theta}{1 + r} \frac{1 + \theta}{1 + \theta} \pi^h \left( \frac{\partial x^h}{\partial I^h} + \beta \frac{\partial a^h}{\partial I^h} \right) + \mu \pi^h \left( 1 + \tau^c \frac{\partial c^h}{\partial I^h} + \frac{\tau^d}{1 + r} \frac{\partial d^h}{\partial I^h} \right) = 0,
\]

\[
(A1) \quad \frac{\partial L}{\partial I^l} = \delta^l \frac{\partial v^l}{\partial I^l} - \lambda \frac{\partial v^l}{\partial I^l} - \eta \left( 1 + g \right) \frac{\theta}{1 + r} \frac{1 + \theta}{1 + \theta} \pi^l \left( \frac{\partial x^l}{\partial I^l} + \beta \frac{\partial a^l}{\partial I^l} \right) + \mu \pi^l \left( 1 + \tau^c \frac{\partial c^l}{\partial I^l} + \frac{\tau^d}{1 + r} \frac{\partial d^l}{\partial I^l} \right) = 0,
\]

\[
(A2) \quad \frac{\partial L}{\partial z^h} = \left( \delta^h + \lambda \right) \frac{\partial v^h}{\partial y^h} - \eta \left( 1 + g \right) \frac{\theta}{1 + r} \frac{1 + \theta}{1 + \theta} \pi^h \left( \frac{\partial x^h}{\partial y^h} + \beta \frac{\partial a^h}{\partial y^h} \right) - \mu \pi^h \left( 1 - \tau^c \frac{\partial c^h}{\partial y^h} - \frac{\tau^d}{1 + r} \frac{\partial d^h}{\partial y^h} \right) = 0,
\]

\[
(A3) \quad \frac{\partial L}{\partial z^l} = \delta^l \frac{\partial v^l}{\partial y^l} - \lambda \frac{\partial v^l}{\partial y^l} - \eta \left( 1 + g \right) \frac{\theta}{1 + r} \frac{1 + \theta}{1 + \theta} \pi^l \left( \frac{\partial x^l}{\partial y^l} + \beta \frac{\partial a^l}{\partial y^l} \right) - \mu \pi^l \left( 1 - \tau^c \frac{\partial c^l}{\partial y^l} - \frac{\tau^d}{1 + r} \frac{\partial d^l}{\partial y^l} \right) = 0,
\]

\[
(A4) \quad \frac{\partial L}{\partial b^h} = \left( \delta^h + \lambda \right) \frac{\partial v^h}{\partial b^h} + \eta \left( 1 - \frac{1 + g}{1 + r} \frac{\theta}{1 + \theta} \left( \frac{\partial x^h}{\partial b^h} + \beta \frac{\partial a^h}{\partial b^h} \right) \right) + \mu \pi^h \left( \tau^c \frac{\partial c^h}{\partial b^h} + \frac{\tau^d}{1 + r} \frac{\partial d^h}{\partial b^h} \right) = 0,
\]

\[
(A5) \quad \frac{\partial L}{\partial c^c} = \sum_j \delta^j \frac{\partial v^j}{\partial c^c} + \lambda \left( \frac{\partial v^h}{\partial c^c} - \frac{\partial v^l}{\partial c^c} \right) - \eta \left( 1 + g \right) \frac{\theta}{1 + r} \frac{1 + \theta}{1 + \theta} \sum_j \pi^j \left( \frac{\partial x^j}{\partial c^c} + \beta \frac{\partial a^j}{\partial c^c} \right) + \mu \pi^j \left( \tau^c \frac{\partial c^j}{\partial c^c} + \frac{\tau^d}{1 + r} \frac{\partial d^j}{\partial c^c} \right) = 0,
\]

\[
(A6) \quad \frac{\partial L}{\partial d^d} = \sum_j \delta^j \frac{\partial v^j}{\partial d^d} + \lambda \left( \frac{\partial v^h}{\partial d^d} - \frac{\partial v^l}{\partial d^d} \right) - \eta \left( 1 + g \right) \frac{\theta}{1 + r} \frac{1 + \theta}{1 + \theta} \sum_j \pi^j \left( \frac{\partial x^j}{\partial d^d} + \beta \frac{\partial a^j}{\partial d^d} \right) + \mu \pi^j \left( \tau^c \frac{\partial c^j}{\partial d^d} + \frac{\tau^d}{1 + r} \frac{\partial d^j}{\partial d^d} \right) = 0,
\]

\[
(A7) \quad \frac{\partial L}{\partial \tau^c} = \sum_j \delta^j \frac{\partial v^j}{\partial \tau^c} + \lambda \left( \frac{\partial v^h}{\partial \tau^c} - \frac{\partial v^l}{\partial \tau^c} \right) - \eta \left( 1 + g \right) \frac{\theta}{1 + r} \frac{1 + \theta}{1 + \theta} \sum_j \pi^j \left( \frac{\partial x^j}{\partial \tau^c} + \beta \frac{\partial a^j}{\partial \tau^c} \right) + \mu \pi^j \left( \tau^c \frac{\partial c^j}{\partial \tau^c} + \frac{\tau^d}{1 + r} \frac{\partial d^j}{\partial \tau^c} \right) = 0,
\]

38
\[
\frac{\partial \mathcal{L}}{\partial \theta} = \sum_j \delta^j \frac{\partial v^j}{\partial \theta} + \lambda \left( \frac{\partial v^h}{\partial \theta} - \frac{\partial v^{h\ell}}{\partial \theta} \right) + \mu \sum_j \pi^j \left( \tau^x \frac{\partial \phi^j}{\partial \theta} + \frac{\tau^d}{1 + r} \frac{\partial \psi}{\partial \theta} \right) - \eta \frac{1 + g}{1 + r} \left[ \frac{1}{(1 + \theta)^2} \sum_j \pi^j \left( x^j + \beta \alpha^j \right) + \frac{\theta}{1 + \theta} \sum_j \pi^j \left( \frac{\partial x^j}{\partial \theta} + \beta \frac{\partial \alpha^j}{\partial \theta} \right) \right] + \gamma \frac{1 + r}{1 + g} = 0,
\]

(A8)

where comparing (A3) with (A5) reveals that \( \mu = -\eta \).

By way of substituting for \( i \) from (27) in (16),

\[
q^x = \frac{(1 + r) (1 + \theta) - (1 + g)}{(1 + r) (1 + \theta)} = 1 - \frac{1 + g}{(1 + r) (1 + \theta)}.
\]

Differentiating with respect to \( \theta \) yields

\[
\frac{\partial q^x}{\partial \theta} = \frac{1 + g}{1 + r} \frac{1}{(1 + \theta)^2}.
\]

(A9)

Using \( \partial x^j / \partial \theta = (\partial x^j / \partial q^x) (\partial q^x / \partial \theta) \) and \( \partial \alpha^j / \partial \theta = (\partial \alpha^j / \partial q^x) (\partial q^x / \partial \theta) \), one can then derive the following expressions,

\[
\frac{\partial x^j}{\partial \theta} = \frac{1 + g}{1 + r} \frac{1}{(1 + \theta)^2} \frac{\partial x^j}{\partial q^x},
\]

\[
\frac{\partial \alpha^j}{\partial \theta} = \frac{1 + g}{1 + r} \frac{1}{(1 + \theta)^2} \frac{\partial \alpha^j}{\partial q^x}.
\]

Similarly,

\[
\frac{\partial \phi^j}{\partial \theta} = \frac{1 + g}{1 + r} \frac{1}{(1 + \theta)^2} \frac{\partial \phi^j}{\partial q^x},
\]

\[
\frac{\partial \psi}{\partial \theta} = \frac{1 + g}{1 + r} \frac{1}{(1 + \theta)^2} \frac{\partial \psi}{\partial q^x}.
\]

Next, differentiate \( v^j \) and \( v^{jk} \), as specified by equations (35) and (42), with respect to \( z^j, b^j, z^k, b^k \). We have,

\[
\frac{\partial v^j}{\partial z^j} \bigg|_{q^r, q^s, z^j, I^j} = \frac{\partial v^j}{\partial b^j} \bigg|_{q^r, q^s, z^j, I^j} = \frac{\partial v^j}{\partial y^j} \bigg|_{q^r, q^s, q^t, I^j} \equiv \alpha^j, \quad (A10)
\]

\[
\frac{\partial v^{jk}}{\partial z^j} \bigg|_{q^r, q^s, z^k, I^j} = \frac{\partial v^{jk}}{\partial b^k} \bigg|_{q^r, q^s, z^k, I^k} = \frac{\partial v^{jk}}{\partial y^k} \bigg|_{q^r, q^s, q^t, I^k} \equiv \alpha^{jk}. \quad (A11)
\]
Similarly, differentiate $v^j$ and $v^{jk}$ with respect to $\theta$, using (A9) and Roy’s identity, to get
\begin{align}
\frac{\partial v^j}{\partial \theta} \big|_{q^j, q^{j^2}, q^{j^3}, l^j} &= \frac{\partial v^j}{\partial q^j} \big|_{q^j, q^{j^2}, q^{j^3}, l^j} \frac{\partial q^j}{\partial \theta} = -\alpha^j (x^j + \beta a^j) \frac{1 + g}{1 + r (1 + \theta)^2}, \\
\frac{\partial v^{jk}}{\partial \theta} \big|_{q^j, q^{j^2}, q^{j^3}, l^k} &= \frac{\partial v^{jk}}{\partial q^k} \big|_{q^k, q^{k^2}, q^{k^3}, l^k} \frac{\partial q^k}{\partial \theta} = -\alpha^{jk} (x^{jk} + \beta a^{jk}) \frac{1 + g}{1 + r (1 + \theta)^2}.
\end{align}

Moreover, with $\partial q^j / \partial r^j = 1$ and $\partial q^d / \partial r^d = (1 + r)^{-1}$, we also have,
\begin{align}
\frac{\partial v^j}{\partial r^c} \big|_{q^j, q^{j^2}, q^{j^3}, l^j} &= \frac{\partial v^j}{\partial q^j} \big|_{q^j, q^{j^2}, q^{j^3}, l^j} \frac{\partial q^j}{\partial r^c} = -\alpha^j c^j, \\
\frac{\partial v^{jk}}{\partial r^c} \big|_{q^j, q^{j^2}, q^{j^3}, l^k} &= \frac{\partial v^{jk}}{\partial q^k} \big|_{q^k, q^{k^2}, q^{k^3}, l^k} \frac{\partial q^k}{\partial r^c} = -\alpha^{jk} c^{jk}, \\
\frac{\partial v^j}{\partial r^d} \big|_{q^j, q^{j^2}, q^{j^3}, l^j} &= \frac{\partial v^j}{\partial q^j} \big|_{q^j, q^{j^2}, q^{j^3}, l^j} \frac{\partial q^j}{\partial r^d} = -\alpha^j d^j \frac{1}{1 + r}, \\
\frac{\partial v^{jk}}{\partial r^d} \big|_{q^j, q^{j^2}, q^{j^3}, l^k} &= \frac{\partial v^{jk}}{\partial q^k} \big|_{q^k, q^{k^2}, q^{k^3}, l^k} \frac{\partial q^k}{\partial r^d} = -\alpha^{jk} d^{jk} \frac{1}{1 + r}.
\end{align}

Finally, use the result $\mu = -\eta$ and equations (A10)–(A17) to simplify and reduce the first-order conditions (A1)–(A8) into the following seven equations:
\begin{align}
\left(\delta^h + \lambda\right) \frac{\partial v^h}{\partial l^h} + \mu \pi^h \left[ \frac{1}{1 + r} \left( 1 + g \right) \theta \left( \frac{\partial x^h}{\partial l^h} + \beta \frac{\partial a^h}{\partial l^h} \right) + 1 + r \frac{\partial c^h}{\partial l^h} + \frac{\tau^d}{1 + r} \frac{\partial d^h}{\partial l^h} \right] &= 0, \\
\delta^\ell \frac{\partial v^\ell}{\partial l^\ell} - \lambda \frac{\partial v^{h\ell}}{\partial l^\ell} + \mu \pi^\ell \left[ \frac{1}{1 + r} \left( 1 + g \right) \theta \left( \frac{\partial x^\ell}{\partial l^\ell} + \beta \frac{\partial a^\ell}{\partial l^\ell} \right) + 1 + r \frac{\partial c^\ell}{\partial l^\ell} + \frac{\tau^d}{1 + r} \frac{\partial d^\ell}{\partial l^\ell} \right] &= 0, \\
\left(\delta^h + \lambda\right) \alpha^h + \mu \pi^h \left[ \frac{1}{1 + r} \left( 1 + g \right) \theta \left( \frac{\partial x^h}{\partial y^h} + \beta \frac{\partial a^h}{\partial y^h} \right) - 1 + r \frac{\partial c^h}{\partial y^h} + \frac{\tau^d}{1 + r} \frac{\partial d^h}{\partial y^h} \right] &= 0, \\
\delta^\ell \alpha^\ell - \lambda \alpha^{h\ell} + \mu \pi^\ell \left[ \frac{1}{1 + r} \left( 1 + g \right) \theta \left( \frac{\partial x^\ell}{\partial y^\ell} + \beta \frac{\partial a^\ell}{\partial y^\ell} \right) - 1 + r \frac{\partial c^\ell}{\partial y^\ell} + \frac{\tau^d}{1 + r} \frac{\partial d^\ell}{\partial y^\ell} \right] &= 0, \\
- \sum_j \delta^j \alpha^j c^j + \lambda \left( \alpha^{h\ell} c^{h\ell} - \alpha^{h^2} c^{h^2} \right) + \\
\mu \sum_j \pi^j \left\{ c^j + r^c \frac{\partial c^j}{\partial q^j} + \frac{1}{1 + r} \left[ \tau^d \frac{\partial d^j}{\partial q^j} + \left( 1 + g \right) \theta \left( \frac{\partial x^j}{\partial q^j} + \beta \frac{\partial a^j}{\partial q^j} \right) \right] \right\} &= 0.
\end{align}
\[ -\sum_j \delta^j \alpha^j d^j + \lambda \left( \alpha^{h\ell} d^{h\ell} - \alpha^h d^h \right) \]
\[ + \mu \sum_j \pi^j \left\{ d^j \left[ \tau^c \frac{\partial \zeta^j}{\partial q^c} + \frac{\tau^d}{1 + r} \frac{\partial \tilde{d}^j}{\partial q^d} + \frac{(1 + g) \theta}{(1 + \theta)(1 + r)} \left( \frac{\partial x^j}{\partial q^c} + \beta \frac{\partial a^j}{\partial q^d} \right) \right] \right\} = 0, \]  
(A23)

\[ -\sum_j \delta^j \alpha^j (x^j + \beta a^j) + \lambda \left[ \alpha^{h\ell} \left( x^{h\ell} + \beta a^{h\ell} \right) - \alpha^h \left( x^h + \beta a^h \right) \right] + \gamma \frac{(1 + r)^2 (1 + \theta)^2}{(1 + g)^2} \]
\[ + \mu \sum_j \pi^j \left\{ (x^j + \beta a^j) \left[ \tau^c \frac{\partial \zeta^j}{\partial q^c} + \frac{\tau^d}{1 + r} \frac{\partial \tilde{d}^j}{\partial q^d} + \frac{1}{1 + r} \frac{(1 + g) \theta}{1 + \theta} \left( \frac{\partial x^j}{\partial q^c} + \beta \frac{\partial a^j}{\partial q^d} \right) \right] \right\} = 0. \]  
(A24)

Let \( \bar{\zeta}, \bar{d}, \bar{x}^j \) and \( \bar{a}^j \) denote the compensated versions of \( \zeta^j, d^j, x^j \) and \( a^j \). Use the Slutsky equation to rewrite equations (A22)–(A24). Rearranging the terms and using (27),

\[ -\sum_j \delta^j \alpha^j c^j + \lambda \left[ \alpha^{h\ell} c^{h\ell} - \alpha^h c^h \right] + \mu \sum_j \pi^j \left\{ \tau^c \frac{\partial \bar{\zeta}^j}{\partial q^c} + \frac{\tau^d}{1 + r} \frac{\partial \bar{d}^j}{\partial q^d} + \frac{(1 + g) \theta}{(1 + \theta)(1 + r)} \left( \frac{\partial x^j}{\partial q^c} + \beta \frac{\partial a^j}{\partial q^d} \right) \right\} = 0, \]  
(A25)

\[ -\sum_j \delta^j \alpha^j d^j + \lambda \left[ \alpha^{h\ell} d^{h\ell} - \alpha^h d^h \right] + \mu \sum_j \pi^j \left\{ \tau^c \frac{\partial \bar{\zeta}^j}{\partial q^c} + \frac{\tau^d}{1 + r} \frac{\partial \bar{d}^j}{\partial q^d} + \frac{(1 + g) \theta}{(1 + \theta)(1 + r)} \left( \frac{\partial x^j}{\partial q^c} + \beta \frac{\partial a^j}{\partial q^d} \right) \right\} = 0, \]  
(A26)

\[ -\sum_j \delta^j \alpha^j (x^j + \beta a^j) + \lambda \left[ \alpha^{h\ell} \left( x^{h\ell} + \beta a^{h\ell} \right) - \alpha^h \left( x^h + \beta a^h \right) \right] \]
\[ + \mu \sum_j \pi^j \left\{ \tau^c \frac{\partial \bar{\zeta}^j}{\partial q^c} + \frac{\tau^d}{1 + r} \frac{\partial \bar{d}^j}{\partial q^d} + \frac{(1 + g) \theta}{(1 + \theta)(1 + r)} \left( \frac{\partial x^j}{\partial q^c} + \beta \frac{\partial a^j}{\partial q^d} \right) \right\} + \gamma (1 + r)^2 \]
\[ + \mu \sum_j \pi^j \left\{ (x^j + \beta a^j) \left[ \tau^c \frac{\partial x^j}{\partial q^c} + \frac{\tau^d}{1 + r} \frac{\partial \bar{d}^j}{\partial q^d} + \frac{1}{1 + r} \frac{(1 + g) \theta}{1 + \theta} \left( \frac{\partial x^j}{\partial q^c} + \beta \frac{\partial a^j}{\partial q^d} \right) \right] \right\} = 0. \]  
(A27)
Next multiply equation (A20) by \( c^h \) and equation (A21) by \( c^f \) and add them together; similarly, multiply equation (A20) by \( d^h \) and equation (A21) by \( d^f \) and add them together, and multiply equation (A20) by \( x^h + \beta a^h \) and equation (A21) by \( x^f + \beta a^f \) and add them together. We get

\[
\mu \sum_j \pi^j c^j \left\{ 1 - \left[ \tau^c \frac{\partial c^j}{\partial y^j} + \frac{\tau^d}{1 + r} \frac{\partial d^j}{\partial y^j} + \frac{(1 + g) \theta}{(1 + \theta)(1 + r)} \left( \frac{\partial x^j}{\partial y^j} + \beta \frac{\partial a^j}{\partial y^j} \right) \right] \right\} = 
\left( \delta^h + \lambda \right) \alpha^h c^h + \delta^f \alpha^f c^f - \lambda \alpha^{ht} c^f,
\]

(A28)

\[
\mu \sum_j \pi^j d^j \left\{ 1 - \left[ \tau^c \frac{\partial c^j}{\partial y^j} + \frac{\tau^d}{1 + r} \frac{\partial d^j}{\partial y^j} + \frac{(1 + g) \theta}{(1 + \theta)(1 + r)} \left( \frac{\partial x^j}{\partial y^j} + \beta \frac{\partial a^j}{\partial y^j} \right) \right] \right\} = 
\left( \delta^h + \lambda \right) \alpha^h d^h + \delta^f \alpha^f d^f - \lambda \alpha^{ht} d^f,
\]

(A29)

\[
\mu \sum_j \pi^j (x^j + \beta a^j) \left\{ 1 - \left[ \tau^c \frac{\partial c^j}{\partial y^j} + \frac{\tau^d}{1 + r} \frac{\partial d^j}{\partial y^j} + \frac{(1 + g) \theta}{(1 + \theta)(1 + r)} \left( \frac{\partial x^j}{\partial y^j} + \beta \frac{\partial a^j}{\partial y^j} \right) \right] \right\} = 
\left( \delta^h + \lambda \right) \alpha^h (x^h + \beta a^h) + \delta^f \alpha^f (x^f + \beta a^f) - \lambda \alpha^{ht} (x^f + \beta a^f).
\]

(A30)

Equations (46)–(48) are obtained substituting from (A28)–(A30) into (A25)–(A27), simplifying terms and taking into account that \( \gamma (1 + i)^2 = \gamma \) (since \( \gamma = 0 \) for \( i > 0 \)).

**Proof of (53):** “Solving” (49)–(50) for \( \tau^c \) and \( \tau^d \), one gets

\[
\tau^c = \frac{\lambda \alpha^{ht} (c^f - c^h) \sum_j \pi^j \frac{\partial d^j}{\partial q} - (d^f - d^h) \sum_j \pi^j \frac{\partial c^j}{\partial q}}{\mu \sum_j \pi^j \frac{\partial c^j}{\partial q} \sum_j \pi^j \frac{\partial d^j}{\partial q} - \sum_j \pi^j \frac{\partial c^j}{\partial q} \sum_j \pi^j \frac{\partial d^j}{\partial q}},
\]

(A31)

\[
\frac{\tau^d}{1 + r} = \frac{\lambda \alpha^{ht} (d^f - d^h) \sum_j \pi^j \frac{\partial c^j}{\partial q} - (c^f - c^h) \sum_j \pi^j \frac{\partial d^j}{\partial q}}{\mu \sum_j \pi^j \frac{\partial c^j}{\partial q} \sum_j \pi^j \frac{\partial d^j}{\partial q} - \sum_j \pi^j \frac{\partial c^j}{\partial q} \sum_j \pi^j \frac{\partial d^j}{\partial q}}.
\]

(A32)

Substituting in (51) and simplifying leads to (53).

**Proof of Lemma 1:** Consider the problem of a \( j \)-type individual choosing \( c, d, \) and \( x \) to maximize \( u(c, d, x, (I + a) / w^j) \) subject to the budget constraint

\[
q^c c + q^d d + q^f x = y + a - f(a) - \beta q^f a,
\]

conditional on a given value for \( a \). This optimization problem yields

\[
c = c(y, I, w^j; a); \quad d = d(y, I, w^j; a); \quad x = x(y, I, w^j; a),
\]

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where for ease in notation we have suppressed $q^c, q^d, q^x$ from the list of arguments. This allows one, through the composite commodity theorem, to define a new utility function in the $(y, I)$ space:

$$U(y, I, w^j; a) = u(c(y, I, w^j; a), d(y, I, w^j; a), x(y, I, w^j; a), (I + a)/w^j).$$

Observe that normality of $c, d,$ and $x$ in $u(\cdot)$ ensures normality of $y$ in $U(\cdot)$. In turn, this ensures that the “single-crossing” property is satisfied for $U(\cdot)$.

Next define the marginal rate of substitution between observable disposable income, $y$, and before-tax reported labor income, $I$, for an agent of type $j$ as,

$$MRS_{yI}(y, I, w^j; a) = \frac{\partial U(y, I, w^j; a)/\partial I}{\partial U(y, I, w^j; a)/\partial y} = \frac{1 + \tau^c u_L(c(y, I, w^j; a), d(y, I, w^j; a), x(y, I, w^j; a), (I + a)/w^j)}{w^j a_c(c(y, I, w^j; a), d(y, I, w^j; a), x(y, I, w^j; a), (I + a)/w^j)}.$$ 

(A33)

Observe that the normality of $y$ also implies that $MRS_{yI}(y, I, w^j; a)$ is increasing in $a$. This happens both because an increase in $a$, for a given $I$ and $w^j$, implies a higher labor supply, and because it implies a larger disposable income, $y + a - f(a) - \beta q^x a$.

Finally, from equation (13),

$$MRS_{yI}(y, I, w^j; a) = 1 - f'(a) - \beta q^x.$$ 

Hence a low-skilled agent when faced with the quadruple $(q^c, q^d, q^x, y^\ell, I^\ell)$, implied by the mechanism $(\tau^c, \tau^d, \theta^h, b^h, z^h, z^\ell, I^h, I^\ell)$, chooses $a$ to satisfy,

$$MRS_{yI}(y^\ell, I^\ell, w^h; a) = 1 - f'(a) - \beta q^x.$$ 

(A34)

On the other hand, the $h\ell$-mimicker chooses $a$ such that

$$MRS_{yI}(y^\ell, I^\ell, w^h; a) = 1 - f'(a) - \beta q^x.$$ 

(A35)

\footnote{The single-crossing or “agent monotonicity” condition requires that the marginal rate of substitution between consumption and income, $y$ and $I$ in this case, to be decreasing in wage so that at any $(y, I)$ bundle, the high-ability agent will have a flatter indifference curve than the low-ability agent. In this way, they can cross only once. See, e.g., Salanié (2011).}
Denote the solution to (A34) by \(a^\ell\) and the solution to (A35) by \(a^{h\ell}\). It follows from (A34)–(A35) that
\[
MRS_{yI}(y^\ell, I^\ell, w^\ell; a^\ell) + f'(a^\ell) = MRS_{yI}(y^\ell, I^\ell, w^h; a^{h\ell}) + f'(a^{h\ell}). \tag{A36}
\]
At the same time, the single-crossing property implies that for the same value of \(a\),
\[
MRS_{yI}(y^\ell, I^\ell, w^\ell; a) > MRS_{yI}(y^\ell, I^\ell, w^h; a); \text{ or}
\]
\[
MRS_{yI}(y^\ell, I^\ell, w^\ell; a^\ell) + f'(a^\ell) > MRS_{yI}(y^\ell, I^\ell, w^h; a^\ell) + f'(a^\ell). \tag{A37}
\]
Substituting from (A36) for the left-hand side of (A37),
\[
MRS_{yI}(y^\ell, I^\ell, w^h; a^{h\ell}) + f'(a^{h\ell}) > MRS_{yI}(y^\ell, I^\ell, w^h; a^\ell) + f'(a^{h\ell}).
\]
Now with \(MRS_{yI}(y, I, w^j; a)\) increasing in \(a\) as shown earlier and \(f'(a)\) increasing in \(a\), due to convexity of \(f(a)\), it follows from the above inequality that \(a^{h\ell} > a^\ell\).
References


