Minority Representation in Proportional Representation Systems

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Abstract

The paper examines the effect of different forms of proportional representation (PR) on minority representation within Parliaments. We focus on the way candidates are ranked within party lists. Under closed list PR, rankings are decided by party leaders; under open list PR, the electorate determines the rankings by casting votes for individual candidates. The paper provides two main contributions to the literature: first, it goes beyond the standard distinction between open and closed list PR by considering variations in the number of candidates voters can select under open list. Second, it constitutes the first attempt to study endogenous policy positioning by individual candidates in PR systems. We consider a unidimensional and binary policy space and assume an asymmetric distribution of voters bliss points. Minority representation is defined as the number of elected candidates supporting the minority position. We show that minority representation is lowest under closed list PR and under open list PR when voters can select many candidates within a list. On the contrary, it is highest when voters can only approve a limited number of candidates. This suggests a form of non-monotonicity and discontinuity of minority representation as a function of voters' control on the selection of elected candidates.

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1 Introduction

Standard classifications of electoral systems in Political Economics and Comparative Politics are usually limited to the distinction between first-past-the-post (FPTP) and proportional representation (PR) rules. This distinction focuses on the way seats are attributed across parties, but completely ignores seat allocation *within* them. While this dimension can probably be neglected with little consequences for FPTP[1], it constitutes the source of large cross-country variation under PR.

In some countries (among many others, Argentina, Israel, Italy, Spain and South Africa) seats allocation within parties follows a predetermined ranking of candidates: if a party wins s seats, the first s candidates on the party list will be elected to the Parliament. Voters cannot modify the ranking and can only cast a vote for a party list as a whole. In some other countries (e.g. Belgium, The Netherlands and Sweden), voters can modify party rankings by expressing a *preference* for some candidates within the list. If some candidates obtain enough individual votes, they are elected independently of their position on the original ranking. Finally, there are countries (e.g. Finland and Norway) where rankings are uniquely determined by the electorate and the s seats won by a party are distributed to the s candidates obtaining the highest number of individual votes. The three specifications are known as *closed*, *flexible* (or *semi-open*) and *open list* PR systems, respectively.

Preferential votes in open and flexible list PR are equivalent to approval voting within the party list: voters decide to approve (and not rank) some of the politicians appearing on the ballot. The number of approval votes that can be expressed varies a lot form country to country. Just to mention some examples, this is equal to one in Sweden, Finland, Denmark and Brazil, to

[1] In FPTP systems, the presence of single-member-districts completely eliminates intra-party competition at the election stage. Such type of competition might however be enhanced by *primaries*. We will comment more about this in Section 6.

four in Greece and Czech Republic and to the total number of candidates on the list in Belgium.

The goal of this paper is to examine the effects of different forms of PR on minority representation within Parliaments. More precisely, we are interested in the consequences of allowing the electorate to vote for individual candidates within party lists. Our focus is both on the distinction between closed and open list PR and, within open list, on the number of candidates that can be approved by voters. Whether candidates are pre-ranked by party leaders (as in flexible list systems) or not is not particularly relevant to our purposes. Our results show that, in general, closed list PR is associated to lower minority representation within Parliaments than open list. However, minority representation under open list is decreasing in the number of candidates voters can "approve" in a party. Whenever this coincides with the total number of politicians on the list, open and closed list PR achieve the same level of minority representation.

Our basic setting is in line with a multi-candidate Downsian model with binary policy space and asymmetric distribution of voters' bliss points. During the electoral campaign preceding a Parliamentary election, office motivated candidates belonging to two different parties commit to support one of two possible policy positions if elected. One of the two positions is preferred by the majority of people, while less than half of the population is in favor of the other. Minority representation within a Parliament is defined as the number of elected candidates that promised to support the position of the minority.

Even if our focus is primarily on individual promises by candidates, we are not willing to disregard the possibility for parties to exert some control over their members. We introduce some mild form of control by assuming that candidates also care about the total number of seats won by their party. Furthermore, we assume that candidates within the same party are able to coordinate and consider equilibria that are robust to deviation by any coalition of candidates (of the same party).

Voters care about the final composition of the Parliament. They obtain some positive utility any time a candidate supporting their favorite platform is elected. Utility from the election of opposing candidates is normalized to zero. Voters' total utility is then the sum of the utilities received from each elected candidate. Such a modeling strategy is consistent with the idea that, even if decisions within the Parliament are taken by majority rule, electing candidates that support one particular policy position increases the chances for this policy to be adopted[2].

The electoral system is PR and parties are represented at the election by a list of candidates. Seats are distributed to parties in a way that is almost proportional to the number of votes they receive. Perfect proportionality is ruled out by the assumption of non-fractionally divisible seats. Once seats are assigned to parties, seat allocation within their lists follows different rules depending on whether the system is an open or closed list one, as explained above.

Our first result shows that the minority is never represented in Parliament under closed list PR. Under this system, candidates' objective almost coincides with maximizing the number of seats obtained by the party. This directly follows from the fact that politicians are ranked and that they care about the global performance of the party. To maximize the number of seats, candidates need to appeal to the majority of voters. Anytime one party contains a candidate supporting the minority position in its list, the best response for the candidates belonging to the other party is to compose a list that fully supports the majority. By doing so, they attract all the votes of this group.

In our second proposition, we show that minority representation under open list PR depends on the number of candidates voters can select within

[2]For example, when legislative bargaining is done on multiple topics, some representatives of the majority might accept to vote in favor of the minority issue in exchange for minority's support on another dimension.

party lists. Under this system, politicians' election only depends on the number of personal votes they receive. Most importantly, there always exists an implicit quota of personal votes that guarantees the assignment of a seat. Such quota is increasing in the number of candidates that can be approved by voters. When this is small, so that the quota is low enough, minority voters are numerous enough to guarantee the election of some candidates that represent them. In presence of a large majority however, increases in the number of candidates that can be approved by voters reduce minority representation in the Parliament.

It is interesting to notice the discontinuity and non-monotonicity of minority representation as a function of the number of candidates that can be approved by voters. When no preference for individual candidates within party lists can be expressed (i.e. under closed list PR), minority representation is very low. However, it jumps up to its maximum possible value as soon as the possibility of approving one candidate is introduced. As the number of approval votes increases, then, minority representation goes back to the closed list levels.

This paper clearly challenges the common belief that PR systems are associated to higher minority representation in Parliaments. Such belief is based on the idea of parties as unique and cohesive players of the electoral game. As our results show, however, the introduction of candidates as autonomous players significantly changes these conclusions. At the end of the paper, we adapt our model to the analysis of FPTP in presence or absence of primaries. We show that equilibria with positive minority representation can only exist when candidates are selected by party leaders (i.e., when primaries are not held). Most importantly, minority representation in these equilibria can be much higher than in any form of PR.

Our results allow to rank the different forms of PR according to the level of minority representation they guarantee. Normative evaluation and social welfare implications, however, strictly depend on the interpretation one gives to the minority. Our model is flexible enough to fit different applications, and two possible ones are proposed at the end of the paper. In the first, the minority-majority conflict is interpreted as a targeting vs public good provision problem (Myerson (1993b), Persson and Tabellini (1999), Lizzeri and Persico (2001), Milesi-Ferretti et al. (2002)). Public good provision is welfare maximizing and any increase in minority representation (i.e. targeting) only induces a suboptimal allocation of resources. Under this interpretation, closed list PR or open list PR with a high number of approval votes are the best systems, as they minimize minority representation within Parliaments.

In the second application, the minority is instead defined as an ethnolinguistic group whose interest should be protected against the tyranny of the majority. Contrary to the previous application, high minority representation is now welfare maximizing. Clearly, the normative conclusions are completely reversed and open list PR with a limited amount of preference votes for individual candidates results to be the socially preferable system.

The remainder of the paper is organized as follows. Section 2 summarizes the literature that is relevant for our paper. The model is introduced in Section 3 and results are presented in the following one. A discussion of the possible applications is contained in Section 7. All proofs can be found in the Appendix.

2 Literature

As noticed in the introduction, the main distinction among electoral systems drawn by the literature is the one between FPTP and PR systems. The two electoral rules have been contrasted under a large variety of aspects, among which the probably most important ones are party creation (Duverger (1954), Taagepera and Shugart (1989), Palfrey (1989), Feddersen (1992), Lijphart (1994), Fey (1997), Morelli (2004), etc.), public good provision and transfers (Myerson (1993b), Persson and Tabellini (1999), Lizzeri and Persico (2001), Milesi-Ferretti et al. (2002)) and corruption (Myerson (1993a), Persson et al. (2003)).

Higher attention to the differences between open and closed list PR can be found in the political science literature concerned with the impact of electoral systems on the importance to cultivate candidates' personal reputation. These papers conclude that open list PR increases the value of personal reputation with respect to party reputation by enhancing intra-party competition and electoral uncertainty (Carey and Shugart (1995), Chang (2005)) or by inducing voters to focus more on candidates' characteristics and less on parties' positions (Shugart et al. (2005)). Ames' study of Brazilian electoral and political context (Ames (1995a) and Ames (1995b)) supports these conclusions and clearly highlights the very weak role played by national parties in the country. Cultivating personal reputation requires a consistent amount of resources and, if very valuable, might induce politicians to resort to illegal sources of financing. This explains why open list PR systems are found to be positively associated to corruption (Chang and Golden (2007)).

Another important variable in the analysis of PR systems is district magnitude (DM), that is the number of seats that are assigned in each district. Increases in DM are associated with higher importance of personal vote under open list PR and with lower under closed list (Shugart et al. (2005), Chang and Golden (2007)). Moderate values of DM seem to combine at best the need for stability of governments and representation of different preferences in the population (Carey and Hix (2011)). However, a study of the behavior of Swiss legislators by Portmann et al. (2012) suggests that they are more likely to support majority positions if elected in districts of low DM (but see also Carey and Hix (2013)).

Crutzen (2013) examines the effect of open and closed list PR on the effort exerted by politicians. He proves that the conclusion depends on candidate selection procedure used by parties. Candidate selection is competitive when it is based on the effort exerted by politicians, it is non competitive otherwise.

If candidate selection is competitive under PR, politicians compete both to be included in the list and to be placed in high positions. Under open list PR, the second source of competition is replaced by the need to attract a sufficiently high number of individual votes. Which of the two systems induces higher effort provision by politicians depends on voters' responsiveness to effort.

3 The Model

We consider the election of a Parliament composed of S seats, to be allocated in a unique national district. For simplicity, assume that S is an even number. There are $2S$ candidates, divided in two parties, A and B. Denote a candidate occupying position s on party A's and B's list by a_s and b_s, respectively. Then,

$$A = \{a_1, \dots, a_S\} \quad B = \{b_1, \dots, b_S\}$$

More generally, we let $\mathcal{C} = A \cup B$ be the set of all candidates and denote by $c \in \mathcal{C}$ a generic candidate within. Competition occurs over a binary policy issue $x \in \{x_1, x_2\}$. Before election, each candidate c chooses which position she will support, which we denote by $x_c \in \{x_1, x_2\}$. Candidates' primary goal is to win a seat for themselves. To introduce some form of party control, however, we assume that they (secondarily) care about the number of seats obtained by their party. In other words, whenever two strategies are indifferent in terms of individual outcome (i.e., they both guarantee election or none of them does), candidates will prefer the one that is best for their party.

The electorate is composed of N individuals. Each of them is characterized by a bliss point $x_i \in \{x_1, x_2\}$. More precisely, we distinguish between two groups of voters: a *majority* of them favors policy x_1, while a *minority* prefers x_2. Define

$$Maj = \{i \in N : x_i = x_1\}$$

and

$$Min = \{i \in N : x_i = x_2\}$$

where, with a slight abuse of notation, N denotes the the set of voters as well. We set $|Maj| = M > N/2$ and $|Min| = N - M < N/2$. For expositional convenience, let us relabel the two policy positions in a more intuitive way: we set $x_1 = x_{maj}$ and $x_2 = x_{min}$. Voters care about the final composition of the Parliament and want to elect as many candidates as possible that support their favorite position. Formally, letting W be the set of candidates obtaining a seat in the Parliament, voter i's total utility can be written as[3]

$$U_i = \sum_{c \in W} \beta_i \mathbb{1}_{[x_c = x_i]}$$

with

$$\beta_i = \begin{cases} \beta_{maj} & \text{if } i \in Maj \\ \beta_{min} & \text{if } i \in Min \end{cases}$$

Seats are allocated to parties according to a proportional representation rule. We assume a party obtains a seat every α votes received. For example, 2α votes guarantee two seats, 3α votes guarantee three seats and so on. More generally, denoting by V_P the amount of votes received by party $P \in \{A, B\}$, the number of seats P is entitled to receive can be computed as

$$S_P = \left\lfloor \frac{V_P}{\alpha} \right\rfloor$$

[3]This specific assumption on voters' utility is only made for expositional convenience. Our results would hold for any utility function of the form

$$U_i = \sum_{c \in W} u_i(x_c)$$

with $u_i(x_{maj}) > u_i(x_{min})$ for all $i \in Maj$ and $u_i(x_{maj}) < u_i(x_{min})$ for all $i \in Min$. Most importantly, $u_i(\cdot)$ and $u_j(\cdot)$ do not necessarily have to coincide for two individuals i and j belonging to the same social group.

Notice that this implies that, whenever a party obtains $s\alpha + \delta$ votes, with $\delta < \alpha$, the number of seats it receives is anyway equal to s. To keep the analysis simple and avoid dealing with remainders, we make the following assumption about the size of the two groups of voters

Assumption 1. *There exists $k_{maj} \in \mathbb{N}_{++}$ and $k_{min} \in \mathbb{N}_{++}$, with $k_{maj} + k_{min} = S$, such that $M = \alpha k_{maj}$ and $N - M = \alpha k_{min}$.*

In words, Assumption 1 states that the majority can be divided into (at most) k_{maj} groups of (exactly) α voters. Similarly, there are k_{min} groups of α voters within the minority[4]. For convenience, assume k_{maj} and k_{min} are both even. Our focus in this paper is on the way the S_P seats won by a party are distributed across candidates within its list. The two systems we consider are

Closed List PR. *Seats are assigned to the first S_P candidates appearing on the party list. Voters cannot modify this ranking and can only vote for a party.*

Open List PR. *Voters vote for a party and, within its list, they can approve up to $\pi \in \{1, \dots, S\}$ candidates. Seats are assigned to the S_P candidates that were approved by the highest number of voters.*

Thus, π denotes the number of individual candidates that can be selected by voters within a party list. For example, it is equal to one in Finland, Sweden or Brazil, to four in Greece and to S in Belgium. The timing of the game is the following:

($t = 0$ Party lists are created;)

$t = 1$ Candidates select their policy position $x_c \in \{x_{maj}, x_{min}\}$. Under closed list, they do so knowing their ranking on the list;

[4]Our results would not change if we assumed that $M = \alpha k_{maj} + \delta_{maj}$ and $N - M = \alpha k_{min} + \delta_{min}$, with $\delta_{maj} + \delta_{min} < \alpha$.

$t = 2$ Elections take place and seats are distributed first across and then within parties;

Time $t = 0$ is not part of our game and no strategic decision is made at this stage. It was introduced in the timing to make clear that policy choices by candidates occur in between list formation and elections.

We require the equilibria of the game to be subgame perfect and we additionally impose Strong Nash (Aumann (1959)) in the voting subgame. A profile of strategies is a Strong Nash equilibrium of a game if there exists no profitable deviation for any *coalition* of players. This equilibrium concept allows us to consistently reduce the number of equilibria of the game, making the analysis much more tractable[5].

Furthermore, we make the following two assumptions on voters' behavior under closed list PR.

Assumption 2. *Under closed list PR, if voters (or coalition of voters) are indifferent between the outcome of two voting strategies,*

i) They vote for the party with the highest number of preferred candidates

ii) If this is equal among the two parties, they vote for each of them with equal probability

The first part of Assumption 2 is in line with the idea of *partial honesty* proposed by Dutta and Sen (2012). With regard to the second part, we believe this is the most intuitive way of modeling voters' behavior in this situation. Our results partially depend on this assumption, in a way that will be discussed at the end of the paper.

[5]We are well aware of strength of the assumption we are making on voters' coordination ability. In our defense, we point out that the set of Strong Nash equilibria in our model coincides with the one obtained when using the less restrictive concept of Coalition-Proof Nash (Bernheim et al. (1987) and Bernheim and Whinston (1987)). It is anyway in our plans to consider a limited coordination power for the majority. This can be done by either assuming an upper limit for the size of the coalition or by introducing a cost of deviating that is increasing and convex in the number of deviating players.

Finally, to strengthen the idea that candidates belong to a party, we assume some form of coordination among them. In particular, we will select equilibria where no deviation by any coalition of candidates *within the same party* would be profitable.

4 Results

All the results we are going to present in this section are expressed in terms of *minority representation* (MR), which we define as the number of elected candidates supporting policy x_{min}, that is

$$MR = |\{c \in W : x_c = x_{min}\}|$$

Perfectly proportional representation of the minority is achieved when the share of members of Parliament representing the minority coincides with the size of the minority in the population, that is

$$\frac{MR}{S} = \frac{\text{Size of minority}}{\text{Total population}} = \frac{\alpha k_{min}}{\alpha(k_{maj} + k_{min})} = \frac{k_{min}}{S}$$

Thus, the minority is proportionally represented when $MR = k_{min}$. We begin the exposition of our results with closed list PR. Lemma 1 characterizes equilibrium voting behavior in subgames where one party fully supports the majority, while the other also contains minority candidates.

Lemma 1. *For all voting subgames where, for some $P \in \{A, B\}$, there exists $P_{min} \subseteq P$ such that $x_c = x_{min}$ for all $c \in P_{min}$ and $x_c = x_{maj}$ for all $c \in \mathcal{C} \setminus P_{min}$, equilibrium voting behavior is such that all voters in the minority vote for P and all voters in the majority vote for $P' \neq P$.*

Proof. Without loss of generality, let $P = A$. Voting for A is a weakly dominant strategy for the coalition of voters in the minority, as it is the only one that might allow to elect some candidates supporting x_{min}. By

Assumption 2, all minority voters will vote for A. Remember that $|Min| = \alpha k_{min}$ and a party receives a seat every α votes. This implies that party A will for sure obtain k_{min} seats and candidates $\{a_1, \ldots, a_{k_{min}}\}$ will be elected. Consider voters in the majority now. If αx of them vote for A, they will elect candidates $\{a_{k_{min}+1}, \ldots, a_{k_{min}+x}\}$. If $A_{min} \cap \{a_{k_{min}+1}, \ldots, a_{k_{min}+x}\} \neq \emptyset$, then deviating to B would be profitable, as no candidate in B supports x_{min}. If $A_{min} \cap \{a_{k_{min}+1}, \ldots, a_{k_{min}+x}\} = \emptyset$, then they are indifferent and, by Assumption 2, they should vote for B. This holds for all coalitions of αx voters in Maj, for all $x \in \{1, \ldots, k_{maj}\}$. □

Lemma 1 implies that in subgames where only party P contains candidates supporting the minority, $S_P = k_{min}$ and $S_{P'} = k_{maj}$. We will make large use of this result to prove our main result for closed list PR.

Proposition 1. *Under closed list PR, $MR = 0$ in equilibrium.*

To show that an equilibrium where $MR = 0$ exists, assume all candidates decide to support the majority, i.e. consider the voting subgame where $x_c = x_{maj}$ for all $c \in \mathcal{C}$. By Assumption 2, each party obtains $S_A = S_B = S/2$ seats in expectations. It is obvious that no coalition of voters can profitably deviate from this strategy. Proceeding backward, let us now consider possible deviations by coalitions of candidates within the same party. Without loss of generality, assume that a coalition $A_{min} \subseteq A$ of candidates in party A deviates to x_{min}. The new voting subgame is therefore one where

$$x'_c = \begin{cases} x_{maj} & \forall c \in B \cup \{A \setminus A_{min}\} \\ x_{min} & \forall c \in A_{min} \end{cases}$$

Lemma 1 immediately implies that

$$S'_A = k_{min} < S/2 \qquad S'_B = k_{maj} > S/2$$

Coalition A_{min}, therefore, cannot contain candidates occupying positions

$\{a_{k_{min}+1}, \ldots, a_{S/2}\}$. These are candidates that would lose the seat by deviating. For all other candidates, instead, the deviation does not affect whether they obtain or not the seat: candidates in $\{a_1, \ldots, a_{k_{min}}\}$ obtain it before and after the deviation, while those in $\{a_{S/2+1}, \ldots, a_S\}$ never obtain it anyway. Given that these candidates care about the total number of seats obtained by their party, they will never be part of A_{min}. Thus, it must be that $A_{min} = \emptyset$.

Let us now assume by contradiction that an equilibrium with $MR > 0$ exists. This happens in two possible situations: i) both parties' lists contain candidates supporting x_{min} or ii) only one of the two parties contains this type of candidates. Consider case i) first. In equilibrium, either $S_A = S_B = S/2$ or one of the two parties obtains more seats than the other. Assume without loss of generality that $S_B > S_A$. For both cases, assume that the coalition of candidates in A that was supporting the minority now deviates to x_{maj}. By Lemma 1, this deviation guarantees to the party $S'_A > S/2 \geq S_A$ seats. Thus, it is impossible that some candidates lose from the deviation. Consider the second case now and assume only party A has candidates supporting the minority. By Lemma 1 this party obtains $S_A = k_{min}$ seats. As before, a deviation to x_{maj} by all candidates supporting the minority, would allow the party to obtain $S/2 > k_{min}$ seats and would be profitable by all candidates within the party.

We now turn to open list PR. As already mentioned in the introduction, minority representation under this system depends on the number of candidates in a list that can be approved by voters, π. Proposition 2 provides results for the cases where π is either very large or very small. In the former, restrictions on the size of the majority are also introduced. The case of more general sizes of the majority requires some additional analysis and we suspect equilibria might fail to exist. We will further comment on this after we have introduced our results. To provide some intuition for Proposition 2, we here focus on the case of $\pi = 1$ and $\pi = S$. A formal proof of the Proposition can be found in the Appendix.

Let us start by assuming $\pi = 1$. In such a case, a candidate is sure to be elected whenever at least α voters "approve" her. To see why, assume that a candidate $c \in P$, $P \in \{A, B\}$, is approved by α voters. Then, since these voters must be voting for party P, the party receives at least one seat. Candidate c is not assigned this seat if and only if another candidate $c' \in P$ receives strictly more votes. Assume such candidate exists and, without loss of generality, assume she is approved by $\alpha+1$ voters. Then, the total number of voters voting for P is $2\alpha + 1$, with the consequence that the party must win two seats. If none of these seats is assigned to c, then there must be a third candidate $c'' \in P$ that obtains at least $\alpha + 1$ votes. But then, P must be assigned three seats. The same reasoning can be repeated till $S_P = S$, leading to a contradiction.

If giving α votes to a candidate is sufficient to elect her, it is immediate to see that, whenever there are $k_1 > k_{min}$ candidates with $x_c = x_{min}$, voters in the minority group will always be able to coordinate and assign α votes to exactly k_{min} of them. Similarly, whenever there are $k_2 > k_{maj}$ candidates supporting x_{maj}, the majority will be able to elect k_{maj} of them by equally distributing their approval votes among these candidates. In equilibrium, the total number of candidates (among the two parties) with $x_c = x_{min}$ cannot be lower than k_{min}. Assume it was, than any of the losing candidates could deviate and choose $x'_c = x_{min}$. By our discussion above, she would be elected for sure. The same reasoning holds for the number of candidates choosing x_{maj}. Thus, in equilibrium, exactly k_{min} candidates supporting the minority and k_{maj} candidates supporting the majority will be elected.

Now assume $\pi = S$. We here consider the case of a large majority, i.e. $k_{maj} > 2(k_{min} + 1)$. Assume that, in each party list, k_{maj} candidates choose x_{maj}, while the remaining ones choose x_{min}. Let voters in the majority equally split their votes between party A and B, so that each party receives at least $k_{maj}/2$ seats. Since $\pi = S$, majority voters are able to give

$$\alpha k_{maj}/2 > \alpha(k_{min} + 1)$$

votes to all the $k_{maj} < S$ candidates supporting x_{maj} in the two list. Now consider the voters belonging to the minority. The best strategy these voters can adopt is to concentrate their votes on a party and give to their favorite candidates within its list αk_{min} approval votes. Without loss of generality, assume they all vote for party A. Then, this party obtains $\alpha(k_{min} + k_{maj}/2)$ votes, so that $S_A = k_{min} + k_{maj}/2$. However, since there are $k_{maj} > S_A$ candidates supporting the majority and obtaining $\alpha k_{maj}/2 > \alpha k_{min}$ votes, no candidate supporting the minority will ever be elected. The same conclusion would be reached if minority voters deviated to party B.

A comparison between the case of $\pi = 1$ and the one where $\pi = S$, should give us the intuition behind Proposition 2.

Proposition 2. *Under open list PR,*

i. $MR = k_{min}$ *if* $\pi \leq k_{min}$, *for all values of* k_{maj}

ii. $MR = 0$ *if* $\pi \geq 2k_{min} + 1$ *and* $k_{maj} \geq 2(k_{min} + 1)$

One of the key points in our discussion of the case $\pi = S$ (and, more generally, in the proof we show in the Appendix) is that the majority should be sufficiently numerous to make it impossible for the minority to elect a candidate in *any* party. To better understand the importance of this assumption, let us consider a simple example. Assume there are $3\alpha + 1$ voters in the population[6], α of which belong to the minority and $2\alpha + 1$ to the majority. Thus, $k_{maj} = 2 < 2(k_{min} + 1) = 4$. Let $\pi = 2$ and assume that, within each party list, one candidate supports the minority and the other two support the majority. For simplicity, let us call these candidates a_{min}, a^1_{maj} and a^2_{maj} for party A and b_{min}, b^1_{maj} and b^2_{maj} for party B. If the α voters in the minority

[6]See footnote 3.

vote for a_{min}, then the best response for majority voters is to give $\alpha+1$ votes to a^1_{maj} and a^2_{maj} and let the other α individuals vote for, for example, b^1_{maj}. Such voting strategy guarantees the election of a^1_{maj}, a^2_{maj} and b^1_{maj}. Given such a behavior by the majority, the best reply for the minority is to vote for b_{min}. This would imply that $S_B = 2$, so that b_{min} and b^1_{maj} would win the seat. However, this is not an equilibrium either, since the best reply by the majority to the behavior of the minority is to give $\alpha + 1$ votes to b^1_{maj} and b^2_{maj} and let the other α voters vote for, for example, a^1_{maj}. Such a cycling behavior of voters' best responses prevents the existence of a pure strategy equilibrium.

5 Discussion

In this section, we discuss the effects of relaxing some of the assumptions we made in the benchmark model. In Section 5.1, we assume politicians are fully subject to party control and examine the case where party leaders can instruct their candidates on which policy to support. Section 5.2 removes the restriction on voters' behavior we imposed in Assumption 2. Finally, in Section 5.3, we consider the existence of more than two parties.

5.1 Party Leaders

Assume the only players of the game are the leaders of the two parties. Before election, they must instruct their candidates on which policy to support. That is, party P's leader chooses $x_c \in \{x_{maj}, x_{min}\}$ for all $c \in P$. Possibly, $x_c \neq x_{c'}$ for $c, c' \in P$.

Given our assumptions on candidates' utility in the benchmark model, results for closed list PR are not affected by this alternative setting. Indeed, party P's leader has a profitable deviation if an only if there exists a coalition of P's candidates that can profitably deviate in the benchmark model. To see why, consider a coalition $\tilde{P} \subseteq P$ of candidates in party P. Politicians belonging to this coalition can profitably deviate in two cases: first, if the deviation guarantees them a seat which they could not obtain before; second, if the deviation has no impact on whether they win a seat or not, but it helps the party to win a higher number of seats. Clearly, politicians' incentives in the second case are completely aligned to party leaders' ones. With regard to the first case, notice that a deviation allows a candidate to obtain a seat that she was not getting before if and only if it increases the number of seats won by the party. Thus, if a coalition $\tilde{P}$ is willing to deviate, party P's leader must have a profitable deviation as well, and vice versa.

Under open list PR with $\pi = 1$, this alternative setting increases the number and type of equilibria of the game. Providing a full characterization of the different classes of equilibria is beyond our goal here. We therefore limit ourselves to show an example of possible equilibrium that arises in this framework. In particular, we show that minority representation can be smaller than k_{min}. Key for the result is the partial misalignment of candidates' and party leaders' incentives arising under open list PR: while candidates primarily care about winning a seat, party leaders want to maximize the number of votes obtained. Given this misalignment, a deviation allowing a candidate to be elected while reducing the total number of seats won by the party was profitable in the benchmark model, but it is not any more in this new framework. The intuition of the example is then the following: starting from a subgame where less than k_{min} candidates supporting the minority are elected in the Parliament, it is always possible to find equilibria in the alternative (out-of-equilibrium) voting subgames such that any deviation by a party leader would reduce the number of seats obtained by the party.

Example 1 ($\pi = 1$*:* $MR < k_{min}$)**.** Assume the following values for the parameters of the model

$\lvert Maj \rvert$	$\lvert Min \rvert$	S	π
8α	4α	12	1

so that $k_{maj} = 8$ and $k_{min} = 4$. Furthermore, assume party leader A and B instruct exactly $k^A_{min} = k^B_{min} = 1$ candidates in their parties to support $x_c = x_{min}$. Consider a voting behavior such that $\alpha k^B_{min} = \alpha$ voters in the minority and $\alpha k_{maj}/2 = 4\alpha$ voters in the majority vote for B and the others vote for A (see Figure 1). This is an equilibrium as long as each subgroup of α voters assigns one approval vote to a different (preferred) candidate. Under this voting strategy, $S_B = k^B_{min} + k_{maj}/2 = 5$. Now assume party B's leader deviates and instructs exactly $k_{min} = 4$ candidates on the list to support the minority. For this voting subgame, consider the equilibrium voting behavior such that $\alpha k^A_{min} = \alpha$ voters in the minority and $\alpha k_{maj} = 8\alpha$ voters in the majority vote for A, while only $\alpha(k_{min} - k^A_{min}) = 3$ vote for B. Under this new equilibrium, $S'_B = k_{min} - k^A_{min} = 3 < k^B_{min} + k_{maj}/2 = 5$. Thus, this is not a profitable deviation for party B. The same reasoning applies to party A, proving that there exist an equilibrium where less than k_{min} candidates supporting the minority are elected under open list PR.

5.2 Voters' Behavior under Indifference

In this section, we examine the effects of relaxing the second part of Assumption 2. As this assumption was only used for the closed list case, we focus only on this system. The main consequence of relaxing this assumption is that multiple equilibria arise in the voting subgame where all candidates support position x_{maj}. Indeed, since voters are completely indifferent, any strategy profile is an equilibrium. Depending on the equilibrium we select in this subgame, we can therefore construct equilibria in the complete game where $k \in \{1, \ldots, k_{min}\}$ candidates supporting the minority are elected. As before, let us provide the intuition with an example (Figure 2 provides a graphical illustration).

Example 2 ($MR > 0$ *under closed list PR*)**.** Consider the subgame where the first $k \leq k_{min}$ candidates in A choose $x_c = x_{min}$, while all other candidates

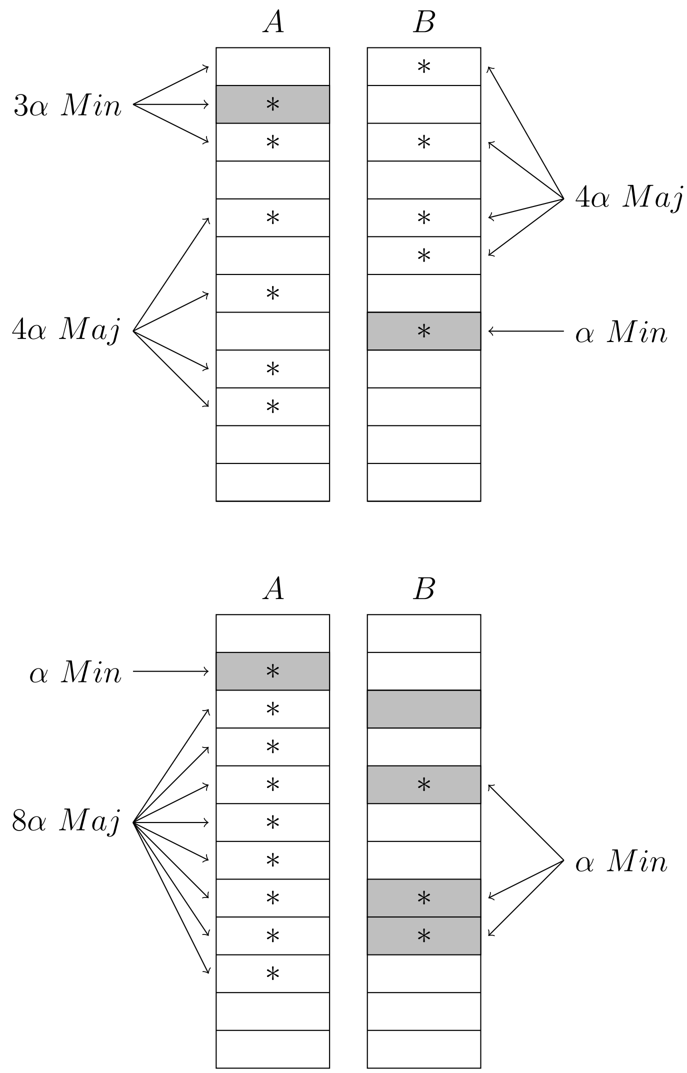

Figure 1: *(Example 1) The top figure represents the voting subgame on the equilibrium path: one candidate in each list was instructed by party leaders to choose* $x_c = x_{min}$ *(gray rectangles). Voters distribute their votes as indicated by the arrows: the majority equally splits between the two parties. The minority gives* 3α *votes to three candidates in* A *and* α *votes to one candidate in* B*. Each party wins* $S_A = 7$, $S_B = 5$ *seats (elected candidates are denoted by the symbol* $*$*). The bottom figure represents a possible voting equilibrium of the subgame where party leader* B *deviated and instructed 4 candidates to choose* $x_c = x_{min}$*. In this subgame, all majority voters vote for* A *and only* 3α *voters in the minority vote for* B*. Thus,* $S'_B = 3$.

(in A and B) decide to support the majority. Equilibrium voting behavior in this subgame is such that all majority voters vote for B, while (at least) αk voters in the minority vote for A. Consider a deviation by some candidates in A. As long as some candidates support the minority, all majority voters will vote for B in equilibrium. Thus, if a profitable deviation by a coalition exists, it must contain all k candidates supporting x_{min}. Assume they all deviate to x_{maj} and consider the new subgame where $x_c = x_{maj}$ for all $c \in \mathcal{C}$. A possible equilibrium of this subgame is the one where all voters vote for B. If candidates expect this to be the equilibrium in the voting subgame following their deviation, they will never deviate. This proves that an equilibrium with $k \leq k_{min}$ candidates supporting x_{min} always exists.

The number of elected candidates supporting the minority, however, cannot be greater than k_{min}.

Proposition 3. *If Assumption 2.ii) does not hold, $MR \leq k_{min}$ in all equilibria under closed list PR.*

Proof. Consider a subgame where only some (possibly all) candidates in party P support $x_c = x_{min}$, while all candidates in party P' choose x_{maj}. By Assumption 2.i), all majority voters will vote for P' so that the amount of seats won by P is $S_P \leq k_{min}$. Clearly, this implies that no more than k_{min} candidates supporting x_{min} can be elected. Now assume that candidates in both parties support x_{min}. Consider the party that wins $S_P \leq S/2$. If all candidates supporting the minority within this party deviate to x_{maj}, then $S'_P = k_{maj}$. By the same reasoning used before, this is a profitable deviation. But then, since $S_{P'} \leq k_{min}$, it must be that $MR \leq k_{min}$ as well. □

5.3 Higher number of Parties

Conclusions about minority representation in FPTP and PR systems rely on the idea that, for a given cost of creating a party, party formation is

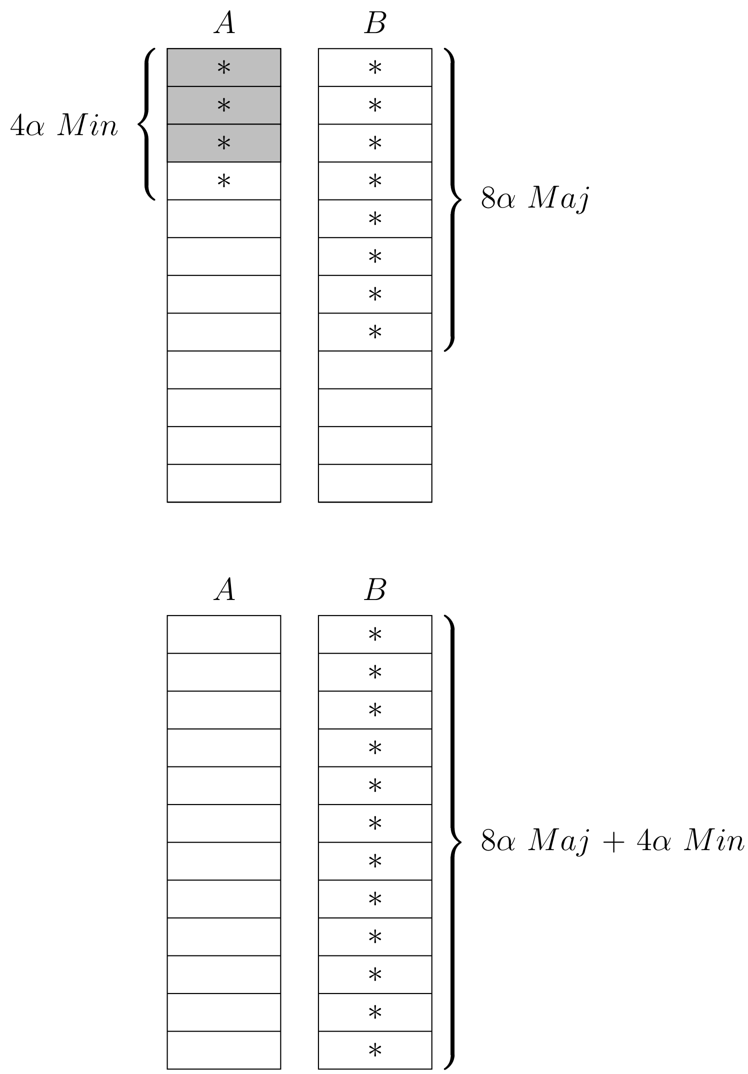

Figure 2: *(Example 2) As for Figure 1, we set* $k_{maj} = 8$ *and* $k_{min} = 4$*. The top figure represents the voting subgame on the equilibrium path: the first* $k = 3 < k_{min}$ *candidates in party* A *support* x_{min}*, all other candidates support the majority. Minority voters all vote for party* A*, while majority voters all vote for* B*. All the symbols and colors should be interpreted as in Figure 1. The bottom figure represents an alternative subgame where all candidates supporting* x_{min} *in* A *deviated to* x_{maj}*. All voters voting for party* B *is an equilibrium of this subgame.*

more rewarding under PR than FPTP. Higher minority representation in the former is therefore associated with the possibility of creating *minority parties* that support the interests of this group.

Given that the number of parties is fixed in our model, our results might not seem directly comparable with those ones. This section shows that, for sufficiently small sizes of the minority, our conclusions do not change when we allow for the creation of one (or more) additional parties. Our focus will be on the systems where $MR = 0$ in equilibrium (i.e. closed list and open list with large π), since these are those which mostly challenge standard conclusions. Let us begin with an example for closed list PR and consider the possibility that a third party is created. Our question is, can there be an equilibrium where this party supports the minority? Remark 1 shows that, if the minority is sufficiently small, the answer is always no.

Remark 1 (Closed list PR: creation of a third party)**.** Let $|Min| < N/3$ and assume a third party C is created. Furthermore, assume it contains candidates supporting the minority. Then, three possible situations can occur:

i) only C contains candidates supporting the minority.

ii) C and another party, say A, contains candidates supporting the minority, B fully supports the majority;

iii) all parties A, B and C contain candidates supporting the minority;

Consider case i) first. By Assumption 2, majority voters will randomize between A and B, so that $S_C \leq k_{min}$. If all the candidates supporting the minority deviated to x_{maj}, then each party would get $S/3 > k_{min}$ seats. Thus, i) cannot happen in equilibrium. In case ii), party B is the only party that obtains the votes of the majority. The other two parties compete for the αk_{min} votes of the minority. As before, then, $S_C \leq k_{min}$. Deviating to x_{maj} is again a profitable deviation for minority candidates in C (or A), as it would imply $S'_C = S'_B = S/2$ (as in case i)). Finally, let us consider case iii). Notice that in this case there must exist a party that obtains $S_P \leq S/3$. If the minority candidates in this party deviated to x_{maj}, the party would increase the number of seats to $S'_P = k_{maj}$. Thus, no equilibrium can exist where C (or, more generally, one of the three parties) contains candidates supporting the minority.

To conclude, assume all candidates support the majority. Then, $S_P = S/3$ for all P. Any deviation by a coalition of politician in party P would lead to the seat outcome equivalent to the one described in case i). That is, $S'_P \leq k_{min} < S/3$. This proves that $x_c = x_{maj}$ for all $c \in A \cup B \cup C$ is part of an equilibrium strategy profile.

A generalization of Remark 1 proves the following proposition.

Proposition 4. *If $|Min| < N/\bar{Q}$, for some $\bar{Q} \in \mathbb{N}_{++}$, $\bar{Q} < N$, and the number of parties is $Q \leq \bar{Q}$, then $MR = 0$ in equilibrium under closed list PR.*

The analysis of open list PR with large π is more complicated than the one for closed list PR and we here only provide a numerical example. The general intuition is that if some candidates within a party promise to support the minority, it is profitable for other politicians within the same party to appeal to the majority. Indeed, some majority voters will vote for them in order to prevent the election of the minority candidates. Given the large π and the small size of the minority, majority voters can always outnumber minority ones.

Example 3 (Open list PR with $\pi = S$ and three parties)**.** As before, assume a third party C exists and consider the following numerical values for the parameters of our model

$\|Maj\|$	$\|Min\|$	π
12α	2α	S

Thus, $k_{maj} = 12$, $k_{min} = 2$ and $S = 14$. Now assume that, within each party list, at least 6 candidates choose x_{maj} and consider the behavior of the majority. In particular, assume they equally divide their votes across all three parties and, within the party, they approve all the candidates that chose x_{maj}. Then, in each party, at least 6 candidates will receive 4α individual votes. Now consider the behavior of the minority. The reasoning is identical to the main section of the model: if all minority voters vote for one party, P, they can give at most 2α individual votes to all the candidates supporting x_{min}. However, since $S_P = 6$ and there are at least 6 candidates supporting the majority with more than 2α individual votes, the minority will never be able to elect any of the candidates.

Now consider the incentives for politicians to choose x_{maj}. Assume less than 6 candidates in C decide to support the majority. Then, the equilibrium voting behavior will be such that at least one of them is elected. Indeed, the majority would not have enough candidates to vote for to subtract the seat from minority candidates. However, any coalition of losing candidates could deviate to x_{maj} in order to reach at least 5 candidates supporting that platform. The reasoning above proves that this is a profitable deviation.

Even if we still cannot prove a complete proposition, we strongly suspect that $k_{maj} > \bar{Q}(k_{min} + 1)$ and $\pi > 2(k_{min} + 1)$ are sufficient conditions for having $MR = 0$ in all equilibria of open list PR with $Q < \bar{Q}$ parties. Indeed, these are the parameters of the model that allow majority voters to distribute enough votes across all the Q parties to prevent the election of minority candidates. Formally, if the majority voters equally divide their votes across all parties, they are able to $\alpha(k_{min} + 1)$ approval votes to

$$q = \frac{\pi \alpha k_{maj}/Q}{\alpha(k_{min} + 1)}$$

candidates[7]. If $q > S_P$, for each party P, then no minority candidate will

[7] If the majority equally divides its votes across the Q parties, the total number of

ever be elected. Given the behavior of the majority, S_P is highest when all minority voters vote for P, so that $S_P = k_{min} + k_{maj}/Q$. Substituting for q and S_P, we find that $q > S_P$ if

$$\pi > \left(\frac{Qk_{min}}{k_{maj}} + 1\right)(k_{min} + 1) = \tilde{\pi}$$

If $k_{maj} > \bar{Q}(k_{min} + 1) > Q(k_{min} + 1)$, then

$$\tilde{\pi} < 2k_{min} + 1$$

so that imposing $\pi > 2k_{min} + 1$ is a sufficient condition for the result.

6 Ideology and FPTP

Our model could be easily extended to a more general one, where candidates are not *ex ante* identical. For example, they might belong to leftist or rightist parties and therefore be associated by voters to a specific ideological position. A straightforward way to extend our results to this case is to assume the existence of two parties for each ideology and apply our model to the interaction between them.

Formally, let A_1, A_2, B_1, B_2 denote the four parties. As before, let each of them be composed of S candidates. There are two policy dimensions, $x \in \{x_{min}, x_{maj}\}$ and $y \in \{y_A, y_B\}$. Candidates are *ex ante* differentiated over y. More precisely, denoting by y_c candidate c's position on y, we assume

$$y_c = \begin{cases} y_A & \text{if } c \in A_1 \cup A_2 \\ y_B & \text{if } c \in B_1 \cup B_2 \end{cases}$$

approval votes that can be expressed by this group for candidates within one party is $\pi\alpha k_{maj}/Q$. q is the maximum number of candidates within one party that can receive $\alpha(k_{min} + 1)$ votes form the majority.

Candidates compete as before by selecting only their position on x. We now distinguish four groups of voters in the population, which we denote by Min_A, Min_B, Maj_A and Maj_B, with $|Min_A| < |Maj_A|$, $|Min_B| < |Maj_B|$ and $|Min_A| + |Maj_A| = |Min_B| + |Maj_B|$. As before, they care about the composition of the Parliament and we assume their utility function now depends also on candidates' position over y.

$$U_i = \sum_{c \in W} u_i(x_c, y_c)$$

Our benchmark model can be straightforwardly extended to this case if we assume that $u_i(x_c, y_c)$ represents the following preferences over candidate c type, (x_c, y_c)

Min_A	Min_B	Maj_A	Maj_B
(x_{min}, y_A)	(x_{min}, y_B)	(x_{maj}, y_A)	(x_{maj}, y_B)
(x_{maj}, y_A)	(x_{maj}, y_B)	(x_{min}, y_A)	(x_{min}, y_B)
(x_{min}, y_B)	(x_{min}, y_A)	(x_{maj}, y_B)	(x_{maj}, y_A)
(x_{maj}, y_B)	(x_{maj}, y_A)	(x_{min}, y_B)	(x_{min}, y_A)

In words, voters value dimension y more than dimension x: a candidate proposing their favorite position on y will always be preferred to one proposing the alternative position on that dimension, independently of her choice on x. Clearly, this implies that voters preferring y_A will never vote for candidates in $B_1 \cup B_2$ and voters supporting y_B will never vote for candidates in $A_1 \cup A_2$. The model therefore reduces to two separate and identical submodels where two parties (A_1, A_2 and B_1, B_2) compete on x. This is precisely the framework of our benchmark case and the results provided before can be replicated here.

Under this new framework, it is interesting to compare minority representation under PR and under FPTP systems. We here consider FPTP with and without primaries[8]. Assume the S seats are allocated by majority rule in S single member districts. There are two national parties A and B, represented by a candidate a_s and b_s in each district s. As before, candidates in A support party position y_A on dimension y and they can freely choose where to stand on x. Given that the election of one candidate is completely independent of the behavior of other candidates of the same party, we can simply assume that politicians care about winning the seat in their own district (i.e., their utility does not depend on the total number of seats obtained by their parties). The population of voters is identical to the one we just described for PR, but is replicated S times across the different districts.

We here consider two different ways of selecting the candidates that will represent party P at the election. Let (a_s^1, a_s^2) and (b_s^1, b_s^2) be two couples of politicians competing to become the candidate representing party A and B, respectively, for district s. Then, in presence of *primaries*

FPTP with primaries. *Before elections, voters within each district select the candidate that will represent the party. We assume that only voters in* $Min_P \cup Maj_P$ *can vote at the primaries of party* P.

If no primaries are held, instead,

FPTP without primaries. *Party leaders select the candidates in order to maximize the probability of winning the seat.*

The timing of the events is such that politicians decide their position on x before candidate selection takes place. Our first result for FPTP shows that, if no primary is held, there always exist equilibria with $MR > 0$.

Proposition 5. *If candidate selection is done by parties,* $MR \in \{0, 1, \ldots, S\}$ *in equilibrium.*

[8]The conclusions we find here hold under more general assumptions about groups' sizes and preferences. The same results were obtained using a probabilistic voting model. Furthermore, these are in line with Hirano et al. (2013)'s analysis of public good provision and targeting in presence of primaries. The modeling strategy we present here was chosen to be consistent with the rest of the paper.

Proof. Consider a generic district s. At the voting stage, only two candidates a_s and b_s compete for the seat. By our assumptions on voters' preferences, Maj_A and Min_A voters will vote for a_s, while Maj_B and Min_B voters will vote for b_s. Thus, each candidate wins the seat with probability one-half, independently of their position on x. Now consider candidate selection stage. Given that the seat is won with probability one-half independently of candidate's type, party leaders are indifferent about which politician they should select. A strategy that selects a politician supporting x_{min} (whenever such a politician exists) is therefore a legitimate equilibrium strategy. At the the first stage, if politicians expect that party leaders will select a candidate supporting x_{min}, they will both choose that position. This reasoning shows that a candidate supporting the minority could always be elected in equilibrium. Extending this result to all S districts, it is easy to see that MR can take any value in between 0 and S. □

If primaries are held, instead, there can be no equilibrium where $MR > 0$.

Proposition 6. *If candidate selection is done through primaries,* $MR = 0$ *in equilibrium.*

Proof. Consider a generic district s. As before, independently of their position on x, candidates a_s and b_s win with probability one-half. In this specific district, voters' expected utility is therefore

$$EU_i(x_{a_s}, x_{b_s}) = \frac{1}{2}u_i(x_{a_s}, y_A) + \frac{1}{2}u_i(x_{b_s}, y_B)$$

At the primaries stage, all voters in the majority prefer candidates supporting x_{maj}. More precisely,

$$EU_i(x_{maj}, x_{b_s}) > EU_i(x_{min}, x_{b_s}), \text{ for all } x_{b_s}$$

for all $i \in Maj_A$, and

$$EU_i(x_{a_s}, x_{maj}) > EU_i(x_{a_s}, x_{min}), \text{ for all } x_{a_s}$$

for all $i \in Maj_B$. Thus, in equilibrium, Maj_A and Maj_B voters will always vote for a candidate supporting x_{maj} (if there exists one). Since they are the majority in the population, they will always be able to elect their favorite candidate. At the first stage now, candidates know that they have some positive probability of winning only if they support x_{maj}. Therefore, no candidate will ever choose x_{min} in equilibrium. Since the same reasoning applies to all districts, $MR = 0$. □

It is interesting to notice how increasing voters' control on candidate selection can lead to opposite results under FPTP and PR. In the former, minority representation decreases when primaries are introduced. Under PR, instead, as long as the number of approval votes that can be expressed by voters is not too large, minority representation increases. More generally, our analysis clearly shows that the simple distinction between FPTP and PR is not enough to capture the patterns of minority representation in Parliaments. Minorities are always (weakly) more represented in PR systems than in FPTP with primaries; however, FPTP without primaries could guarantee a potentially much higher minority representation than any form of PR considered here[9].

7 Applications

We here provide two possible applications of our model. The key difference between the two is the interpretation given to the minority. For the first application, assume that each candidate is endowed with some resources

[9]Our comparison between FPTP and PR system is related to Huber (2012) analysis of ethnic voting in the two systems. Contrary to common beliefs, the author finds that ethnic voting is higher in FPTP than in PR.

that she can use to contribute to the provision of a public good or to target a specific segment of the population. In this case, x_{maj} corresponds to public good provision, while x_{min} should be interpreted as targeting. Whenever

$$\beta_{maj}M > \beta_{min}(N - M)$$

public good provision by all candidates is the social welfare maximizing electoral outcome. Under this interpretation, one would conclude that the electoral systems that maximize social welfare are either closed list PR or open list PR with a large amount of approval votes to be expressed by voters (this is however only true if the majority is sufficiently numerous). Open list PR with a limited number of approval votes would guarantee the election of candidates suboptimally targeting resources to small groups in the population.

The second application we can consider is instead the one where an ethnolinguistic minority should be protected against the tyranny of the majority (x_{min} and x_{maj} should be interpreted accordingly). Reversing the assumptions on the parameters and setting

$$\beta_{maj}M < \beta_{min}(N - M)$$

we can transform minority protection as the welfare maximizing outcome. If this is the problem we face, then our model would predict that the best performing systems is open list PR with a limited number of approval votes to be expressed by the population.

An additional application (that would however require some small changes in the model and we therefore only suggest here) can be the problem of women representation in Parliaments. Even though women constitute the majority in the society, the number of people in favor of more women representation in the Parliaments might still be small enough to be considered as a minority (with regard to this, see Frechette et al. (2008)). Our model suggests that, under these circumstances, the best system to increase women representation (clearly, without resorting to gender quotas) is open list PR with small values of π.

8 Conclusion

The paper provides a comparison of different specifications of proportional representation systems in terms of the degree of minority representation they can achieve. Minority representation is here defined as the number of elected politicians that support a policy position favoring the minority. We have proven that minority representation is highest under open list PR when voters can approve only a limited number of candidates. This is because, under this system, the minority is able to assign enough individual votes to specific candidates to let them be elected *independently* of other voters' behavior. When voters are allowed to approve a higher number of candidates, the minority is unable to compete against the majority in the number of individual votes that can be assigned and, therefore, is never represented in the Parliament.

Closed list PR achieves the same low level of minority representation as open list with a large number of approval votes. The intuition behind this result however relies on parties' incentives to attract different segments of the population. Since party leaders want to maximize the number of seats obtained, appealing to the majority of voters is the best strategy in their electoral campaign.

The paper suggests that minority representation is a non-monotonous and discontinuous function of the number of candidates that can be selected by voters within party lists: the impossibility to vote for any candidate (i.e. closed list PR) and the possibility to approve a consistent number of them lead to the same level of minority representation, while moving from zero to one approval vote determines a sudden increase in its value.

The comparison we make with FPTP systems in presence or absence of party primaries shows that, contrary to common beliefs, minorities might be better represented under FPTP (with no primaries) than PR systems.

Before concluding, a final remark should be made about endogenous party formation. Even though this issue was partially addressed in the discussion of the model, we did not provide a full characterization of the equilibrium number of parties in the different systems. Most importantly, our results suggest that, under closed list PR, endogenous party creation might lead to the formation of a party which only supports the interest of the minority. Two different comments are important about this point. First, looking at our model from a different perspective, one might consider it as an explanation for the creation of minority parties under closed list systems: the creation of such parties is the only possible way for the minority to be represented in the Parliament[10]. Secondly, even though we only consider a unidimensional policy space, it is clear that electoral competition is a multidimensional phenomenon. If more than one policy dimension is present, (successful) creation of a minority party is not necessarily guaranteed. For example, such party would probably fail to attract the votes of the minority if voters within it are strongly divided on a second (and more important) dimension.

A Appendix

Proof of Proposition 2

Proof. Let us begin by proving the first point. Assume $\pi \in \{1, \dots, k_{min}\}$ and consider a subgame where at least k_{min} candidates within a party chose to support x_{min}. Without loss of generality, let them be candidates in A and

[10]With regard to this point, it might be interesting to see whether open list PR with high number of approval votes and closed list PR are associated to higher number of parties.

denote the set composed of such candidates by

$$A_{min} = \{c \in A : x_c = x_{min}\} \subseteq A$$

Clearly, if $A_{min} = A$, the result follows trivially. So $A_{min} \subset A$. Let voter within the minority adopt a voting strategy that assigns π individual votes to k_{min} candidates in A_{min} and consider the behavior of voters within the majority. If these voters want to prevent the election of at least one of the candidates supported by the minority, they must give at least $\pi\alpha + 1$ votes to a number of candidates greater then the number of extra seats the party A would win with their votes. Formally, let at least $\alpha q + \delta$ voters in the majority vote for party A, with $\delta < \alpha$, so that the total amount of seats won by the party is $S_A = k_{min} + q$. Furthermore, let k be the number of candidates in $A \setminus A_{min}$ obtaining more than $\pi\alpha + 1$ votes. Then only the remaining $S_A - k$ seats will be assigned to candidates supporting the minority. To prevent the election of $e > 1$ candidates, it must be that $k > q + e$. Each of the k candidates has to obtain at least $\pi\alpha + 1$ votes. Since each of the $\alpha q + \delta$ voters in the majority can vote for π candidates, the total amount of approval votes expressed by these voters is $\pi(\alpha q + \delta)$. Then q must be such that

$$\pi(\alpha q + \delta) = (\pi\alpha + 1)k \tag{1}$$

and since $k > q + e$, (1) becomes

$$\pi(\alpha q + \delta) = (\pi\alpha + 1)k > (\pi\alpha + 1)(q + e)$$

which is never satisfied since $\delta < \alpha$ and $e > 1$. Thus, if $\pi \in \{1, \ldots, k_{min}\}$, $MR = k_{min}$.

Let us now turn to second part of the proposition and assume $k_{maj} > 2(k_{min} + 1)$. Consider a subgame where, in each party, $k_{min} + k_{maj}/2$ candidates support the position of the majority and $k_{maj}/2$ the one of the minority. As before, the best strategy for minority voters is to concentrate their votes on a unique party and assign αk_{min} votes to all the candidates supporting x_{min}. Assume they all vote for party A. Now consider voters in the majority and assume they equally split their votes among party A and B, so that A wins $S_A = k_{min} + k_{maj}/2$ seats. The majority has $\pi\alpha k_{maj}/2$ approval votes to distribute among candidates of each in each party. This implies that they are able to give $\alpha(k_{min}+1)$ votes to

$$q = \frac{\pi\alpha k_{maj}/2}{\alpha(k_{min}+1)}$$

candidates. If $q > S_A$, then no minority candidate will ever be elected. Substituting for q and S_A, we find that this happens if

$$\pi > \left(\frac{2k_{min}}{k_{maj}} + 1\right)(k_{min}+1) = \tilde{\pi}$$

Now notice that, by our assumptions on the size of the majority,

$$\tilde{\pi} < 2k_{min} + 1$$

Thus, whenever $k_{maj} > 2(k_{min}+1)$ and $\pi > 2k_{min}+1$, no candidate supporting the minority can be elected.

Proceeding backward to the analysis of parties and candidate's behavior, it is easy to check that having less than $k_{min} + k_{maj}/2$ candidates supporting the majority is never optimal for party leaders. □

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