Public Protests, Social Media and Policy Making

Abstract
We explore the limits of petitions and public protests as mechanisms to aggregate dispersed information. We show that if citizens’ signals are not sufficiently precise, information aggregation is impossible, even if the conflict with the policy-maker is small, no matter how large is the population of informed citizens. We characterize the conditions on conflict and the signal structure that guarantee information aggregation. When these conditions are satisfied, we show that full information aggregation is possible as population grows to infinity. When they are not satisfied, we show that information aggregation may still be possible if social media are available.

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1 Introduction

Petitions and public protests are a common feature of the U.S. political system. From the civil rights movement, to the war in Iraq, to the more recent debates on tax and health care reforms, petitioners and protesters have often attempted (and sometime succeeded) to influence policy making. Petitions and public protests play similarly important roles in other established democracies, in the private sector and even in non-democratic regimes. Recent examples illustrate the range and magnitude of these phenomena: in 2007, over 1.8 million UK citizens signed an online petition against “road pricing and car tracking,” prompting a reconsideration by the government; in 2012, United Airlines retracted its “PetSafe” policy restricting dogs from flying on the basis of their breed after over 46,000 customers protested using the online portal Change.org; in 2014 nearly 20,000 people signed an on-line petition on MoveOn.org against military intervention in Iraq.\(^1\) In other examples, fewer citizens are involved, but they represent an influential elite of the population: in 2003 over 400 economists, including 10 Nobel prize-winners, signed a petition published in the New York Times against the tax cuts of the Bush administration; in 2007, over 1500 active-duty military personnel and reservists signed an appeal for troop withdrawal from Iraq; in 2013, 30 prominent economists, including Nobel prize-winner Kenneth Arrow, signed a letter to congress to oppose health reform that did not include an individual mandate with subsidies.\(^2\)

In all these environments, it is rarely the case that active citizens and policy-makers share the same preferences; still, citizens believe that the “power of the numbers” allows them to overcome conflicts, change the policy-maker’s mind and affect public decisions.\(^3\)

Economic theory has attempted to explain public protests as signaling phenomena.\(^4\) The key assumption is that information, as first suggested by Condorcet \citeyear{condorcet1785}, is dispersed. In his original contribution, Condorcet conjectured that elections serve as a mechanism to aggregate dispersed private information and induce superior public policies. More recently, Lohmann \citeyear{lohmann1993,lohmann1994} observed that public protests may be another way for citizens to signal their private information.

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\(^1\) For the first example, see BBC News \citeyear{bbc2007}; for the second example, see Karp \citeyear{karp2012}; the petition on MoveOn.org is available on line at http://pac petitions.moveon.org/sign/no-new-us-war-in-iraq.

\(^2\) The petition of the first example was published in a paid page of the New York Times on February 11, 2003. For the second example, see Alvarez \citeyear{alvarez2007} in the New York Times and Shorn \citeyear{shorn2007} in 60 Minutes. For the third example see Pollack \citeyear{pollack2013}.

\(^3\) This belief appears to be shared by policy-makers too. Exploiting recent technological advances, a number of governments are attempting to “institutionalize” public protests by providing online tools to channel them. In 2011 the Obama administration created the web portal We the People, a platform that gives U.S. citizens a tool to propose and/or endorse petitions (see, https://petitions.whitehouse.gov/); similar portals have been opened by the UK government in 2010 (see, http://epetitions.direct.gov.uk/) and the German Bundestag in 2005 (see https://epetitionen.bundestag.de/). Private companies as Change.org, Avaaz.org and 38degrees.org.uk are also providing tools for on line campaigns and petitions.

\(^4\) From now on we use “public protest” as a generic term for a variety of forms of public dissent as petitions, street protests, etc.
to policy-makers: protesters “vote” in favor or against a policy with their “voice,” and the policy-makers react to the number of protesters as they would in an election. Yet existing theory has left a number of open questions. First, is it really true that the presence of a large number of informed citizens is sufficient for information transmission even if there is a conflict of interest between the policy-maker and the citizens? The Condorcet Jury Theorem (see Feddersen and Pesendorfer [1996, 1997], Myerson [1998b]) shows that under relatively weak assumptions the “power of the numbers” dominates in elections: even if a decision rule that is biased in favor to an alternative is used, elections allow aggregation of dispersed information and hence better policies are achieved with large electorates. It is, however, unclear from the existing literature the extent to which the same logic applies to protests in which the policy-maker cannot commit to a voting rule. Second, when protests allow information aggregation, how effective can they be, both in absolute and in relative terms, compared to other mechanisms such as elections? Under what conditions do they allow policy-makers to avoid mistakes by fully aggregating dispersed information? While protests are often visible and memorable events, they probably often do not have an impact on policy outcomes, and many policy decisions involve no protests at all. Understanding the conditions under which protests play an informative role and the limits of their role in improving public policies is important.

In this paper we propose a theory of public protests to address these questions. In our model, a policy-maker chooses between two polices, $A$ and $B$. The policy-maker and the citizens agree that policy $A$ is optimal in state $a$, and policy $B$ in the complementary state $b$. The policy-maker and the citizens have the same prior on these two states, but they disagree on the benefit of choosing policy $A$ in state $a$ and the cost of choosing policy $A$ in state $b$. A fraction of the citizens, moreover, receives an informative signal on the state of the world. With no protest, the policy-maker would chose $B$; citizens however can signal their dissent with public protests after observing their signal. The policy-maker chooses the policy after observing the citizens’ protests, but he/she is otherwise unconstrained by the protesters.

We first study the conditions under which public protest can serve as a mechanism to aggregate any information. Contrary to what happens in elections and what is suggested by the previous literature on public protests, we prove that information aggregation is not always possible: it depends on the signal structure and the conflict. We characterize a simple necessary and sufficient condition for information aggregation with protests and show that if citizens’ individual signals are not sufficiently precise, information aggregation is impossible, even if the conflict with the policy-maker is small, no matter how large is the population of informed citizens. In a sense, quality is more important than quantity, as far as the impact of citizens’ signals on public protests is concerned. While the literature on signalling and cheap talk has highlighted the importance
of conflict for communication, this literature has been limited to models with a small number of informed agents who directly observe the state of the world. To our knowledge this is the first paper that highlights the importance of the precision of individual signals for the possibility of communication in an environment with an arbitrarily large number of imperfectly informed agents.

We then study the limits of information aggregation when information aggregation is feasible. We show that a version of the Condorcet Jury Theorem, that we call the Weak Condorcet Jury Theorem, holds for public protests: if signals are sufficiently informative and/or the conflict is small enough such that information aggregation is possible, then the probability of a policy mistake converges to zero as the expected number of protesters grows to infinity. Indeed, we show that there are situations in which the probability of choosing the wrong policy is lower when the policy-maker does not pre-commit to a decision rule than when he/she commits (if the policy rule is not appropriately chosen). This implies that in some situations political protests can be more effective than voting. As said, the literature on the Condorcet Jury Theorem has proven full information aggregation results under a variety of conditions. This paper, however, is the first to show the possibility of full information aggregation when there is conflict and the policy-maker can not commit to a policy rule.

Finally, we study the role of social media, since it is often argued that social media is empowering masses that were previously ignored by policy-makers. To study the effect of social media, we enrich the model by assuming that each citizen is affiliated to a smaller social circle (a group of friends, a YouTube or Facebook page, a Twitter hashtag, a union or a political party). In the absence of social media, citizens act independently (or can coordinate only in very small social circles). Social media allows citizens to pool their information within their social circle and so be better informed; but, in the presence of conflict vis-a-vis the policy-maker, it potentially reduces their credibility since it allows social circles to coordinate the actions of its members and act as one. We show that improvements in social media expand the conditions under which protests can be effective.

The reason why signaling may be impossible in the face of large numbers of informed citizens and the reason why social media may improve communication can be explained intuitively. First, note that, as in voting models, a citizen’s decision to protest matters only when it is pivotal, that is when a marginal increase in the number of protesters induces a change in the policy-maker’s decision. Informative protests, therefore, are possible only if citizens are willing to act according to their signal, conditioning on being pivotal: in particular, it must be that at least the citizens who

5 See Casciani [2009], Kirkpatrick [2010] for examples of the effect of Facebook and other social media on public protests, and Section 5 for a more extensive list of references.
receive the signal that is most supportive of $B$ (and hence most in line with the default action of the policy-maker) are willing not to protest, otherwise all citizens choose to protest and no information is conveyed. This puts an upperbound on the citizens’ posterior on the state in which $A$ is optimal. Second, note that, in public protests, the number of protesters that makes a citizen pivotal (the pivotal event) is endogenous, since it is determined by the number of protesters at which the policy-maker is just indifferent between $A$ and $B$. To be indifferent, the policy-maker can not assign too low a probability that $A$ is optimal in states in which citizens are pivotal. This, in turn, puts a lowerbound on the citizens’ posterior on $a$. When the conflict of interest is small and/or citizens’ signals are sufficiently precise, these two constraints are compatible; but for sufficiently high conflict and/or insufficiently precise signals, they are not, even with an arbitrarily large number of citizens. In these cases no information can be conveyed by protesters. Social networks are useful because they relax the tension between the precision of individual signals and the conflict vis-a-vis the policy-maker.

Taken together, these results suggest that public protests can function as effective mechanisms to aggregate dispersed information. Large masses of poorly informed citizens, however, will not be able to aggregate information, even if they don’t have a significant conflict of interest with policymakers. Public protests are effective on issues where there is an elite of citizens who receives sufficiently precise signals, even if the fraction of population receiving these signals is small, as well as when social media allow the citizens to pool information and coordinate their actions.

The organization of the remainder of the paper is as follows. Section 2 outlines the model. Section 3 characterizes the necessary and sufficient condition under which public protests aggregate information. Section 4 studies information aggregation as the number of protesters becomes arbitrarily large, compares public protests with elections and provides a new interpretation of the Condorcet Jury Theorem in lights of this comparison. Section 5 studies the effects of social media on public protests. Section 6 presents a number of extensions of the basic model. A discussion of the related literature is presented in the remainder of this section.

**Related Literature** There is a vast literature empirically evaluating the effects of public protests on policy outcomes. The theoretical literature explaining why public protests affect policy is however more limited. The leading theory in economics and politics is the informative theory introduced by Lohmann [1993, 1994]. In her work, Lohmann proposes a model in which the preferences of the policy-maker coincide with the preferences of the median citizen. In this environment, conflict of interest with the policy-maker is not a problem when the population is

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large: although citizens are heterogeneous in preferences, the number of citizens with arbitrarily small conflict of interest with the policy-maker (or no conflict at all) becomes arbitrarily large as the size of population increases. These citizens have no incentive to misrepresent their signal if they choose to protest. The main focus of this model is to argue that when participation costs are sufficiently small, citizens with preferences close to the policy-maker choose to overcome their free rider problem.\footnote{The consequences of costs to participation to a generic collective decision-making process is also the focus of Osborne et al. [2000] and Osborne et al. [2010].} This paper, moreover, does not study how effective protests can be in aggregating information. In our work, we show that participation and information aggregation can fail even with no costs of participation; and we show that full information aggregation is feasible as population grows to infinity.

A more skeptical approach is taken by Banerjee and Somanathan [2001] who show that, when protesters and the policy-maker have different priors, information transmission can fail even (and indeed, in their model, especially) when the number of protesters is large. A key difference with respect to our work is that Banerjee and Somanathan assume that while the number of protesters can be large, only one protester receives valuable information. The focus of their work, therefore, is more on the ability of the informed citizen to communicate in spite of uninformed citizens than on aggregation of dispersed information; the focus of our work is, instead, on the possibility of information aggregation in the presence of conflict of interest.

Signaling models of information aggregation with conflict and dispersed information a’ la Condorcet are presented by Austen-Smith [1990, 1993] and Battaglini and Benabou [2003].\footnote{These works should be distinguished from the literature on cheap talk with multiple senders (see Gilligan and Krehbiel [1989], Krishna and Morgan [2001], Battaglini [2002] among others). This literature is also about communication with conflict, but it assumes that all senders observes the same realization of the signal, so it is not about information aggregation of dispersed signals. Another related but different literature is the literature on public polls, that studies information aggregation when the policy maker can choose an unbiased sample of citizens to poll (see McKelvey and Ordeshook [1985], Cukierman [1991], Morgan and Stocken [2008]). This literature shows that with polls full information aggregation is typically achieved.} Austen-Smith [1990] is perhaps the first paper to study information aggregation in a cheap talk model with multiple senders. He focuses on agenda determination and information aggregation in environments with few political actors. Battaglini and Benabou [2003] show that, in signaling models of political activism, typically two classes of equilibria coexist: equilibria with mass participation and poor information aggregation; and equilibria with more selective participation and superior information aggregation. Both papers, by focusing on environments with few players, do not explain the extent to which conflict can be mitigated by a large number of informed agents or the extent to which information can be aggregated as the number of protesters becomes large.

As highlighted above, the study of information aggregation with protests is closely connected to the study of information aggregation in elections. Information aggregation of dispersed in-
formation in elections was pioneered by Feddersen and Pesendorfer [1996, 1997], Austen-Smith and Banks [1996] and Myerson [1998b]. This literature shows that rational citizens typically have incentives to misrepresent their signal when casting a vote; and it studies conditions under which information aggregation is possible despite these incentives. In public protests, citizens face similar incentive problems. The key difference with elections is that the policy rule is not exogenous: the decision must be ex post optimal for the decision maker. To clearly explore the connection with our work, we model the signal structure in our work after the signal structure used in this literature. In particular we follow Myerson [1998b], who models the Condorcet Jury Theorem in the context of a Poisson game in which the number of citizens is uncertain.

To our knowledge, the only paper that has studied whether a voting rule is ex post optimal is Yildirim [2013]. Yildirim [2013] characterizes a condition for a voting rule to be ex post optimal when the policy-maker maximizes the citizens’ welfare; and shows that, even without conflict between the policy-maker and the citizens, not all voting rule are ex post optimal. In his environment, however, the ex-ante optimal voting rule is ex post optimal and so an informative equilibrium always exists. Because of this, the author does not study the conditions under which an informative equilibrium may fail to exist, and the limits of information aggregation as population grows to infinity, in the presence of conflict between the policy-maker and the citizens.

The signalling effects of voting behavior are also studied in the literature on “signal jamming” started by Piketty [2000] in which citizens, when casting a vote, care both about the outcome of the election and the inference that the winner and/or other citizens draw from the outcome of the election. In the environment of this literature, failures of information aggregation are generated by the fact that too many goals are assigned to a single instrument (the binary vote with or without abstention).

To focus on the signaling role of public protests, we ignore many important aspects that have been the object of other research. First, we assume that public protests affect the policy-maker only through the informative channel. There is a significant literature in economics and political science studying the strategic implications of the possibility of public protests, assuming that protests have an exogenous direct effect on policy-makers if participation is large enough.

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9 Li, Rosen and Suen [2001] present a more general model of information aggregation in which the decision rule can be a generic monotonic rule (and, therefore, not necessarily a voting rule). As in the voting literature, however, they assume that the decision rule is exogenous and given.

10 Coughlan [2000] and Austen-Smith and Feddersen [2006] present models in which there is a communication stage before an election. In these models, however, the electoral rule is given. The focus is on how different voting rule affect the incentives to reveal information in the pre-deliberation communication stage.

11 See also Castanheira [2003], Rabin [2003], Myatt [2012] and McMurray [2012], among others.

12 The literature on “Private Politics” (Baron [2003], Baron and Diermeier [2007]) studies how activists can change production practices of private companies threatening actions that directly impact a company’s profit like
Second, as in the literature on the Condorcet Jury Theorem, we adopt an individualistic approach, ignoring ethical concerns or other behavioral factors that may affect participation.\textsuperscript{13} Finally, we do not model how preferences determining political conflict are formed. A significant literature in political science is dedicated to the study of what kind of conflict is transformed in public protests; we assume preferences and conflict as exogenous variables. While the exclusive attention to the informative role of protests and an individualistic approach may limit the scope of our theory, it allows us to focus on and clarify an important channel. We leave to future research the task of integrating the informative theory developed here in a richer model of citizens’ behavior.

2 Model

Consider a model in which a policy-maker has to choose between two policies, \( A \) and \( B \). The policy-maker believes that policy \( A \) is optimal in state \( a \) and policy \( B \) in the complementary state \( b \). Formally, the policy-maker’s preference is \( \mu(p, \theta) \), where \( p = A, B \) is the policy and \( \theta = a, b \) is the state of the world. The prior probability that the state is \( \theta \) is \( \mu(\theta) \) with \( \mu(a) = \mu \). If we define \( V(\theta) = V(A, \theta) - V(B, \theta) \) to be the net expected benefit of \( A \) in state \( \theta \), then \( V(a) > 0 \) and \( V(b) < 0 \). The policy-maker is willing to choose \( A \) if

\[
\mu \geq -\frac{V(b)}{V(a) - V(b)} = \frac{1}{1 + V},
\]

where \( V = -V(a)/V(b) > 0 \). We define \( \mu^* = 1/(1 + V) \in (0, 1) \) and assume that \( \mu < \mu^* \). This implies that, with no additional information, the policy-maker chooses \( B \).

There is a population of informed citizens. The number of citizens is a Poisson random variable with mean \( n \).\textsuperscript{14} Citizens’ utilities are described by \( v(p, \theta) \), where \( p \) is the policy and \( \theta \) is the state of the world. Citizens agree that \( A \) is the best policy in state \( a \) and \( B \) is the best policy in state \( b \). If we define \( v(\theta) = v(A, \theta) - v(B, \theta) \), we have \( v(a) > 0 \) and \( v(b) < 0 \). A citizen is willing to choose \( A \) if

\[
\mu \geq -\frac{v(b)}{v(a) - v(b)} = \frac{1}{1 + v},
\]

where \( v = -v(a)/v(b) \). The policy-maker and the citizens have different willingness to choose \( A \). We assume \( v > V \), so \( \mu^{**} = 1/(1 + v) < \mu^* \). Citizens are therefore more partial to choosing \( A \) than a boycott. The literature on “Regime Change” assumes that a regime change occurs if a participation to a mass protest is higher than a given exogenous threshold. The focus is on the possibility of multiple equilibria due to coordination problems (Weingast [1997]); and on the political factors that can serve as coordination devices for political action: a violent revolutionary vanguard (Bueno de Mesquita [2010]), elections (Little et al. [2014]).

\textsuperscript{13} Formalizations of the effect of ethical concerns on participation to collective decision processes is presented by Coate and Conlin [2005] and Feddersen and Sandroni [2006].

\textsuperscript{14} The use of Poisson games to study large games with anonymous players has been pioneered by Myerson [2000] and [1998b] and it has become quite common since then. See Myerson [1998a] for a discussion of the advantages of this approach.
the policy-maker is. The difference between \( v \) and \( V \) (or equivalently \( \mu^* \) and \( \mu^{**} \)) provides a natural way to measure the conflict of interest between the policy-maker and the citizens.

Citizens observe a private informative signal \( t \) with distribution \( r(t; \theta) \), support \( T = \{1, \ldots, T\} \) with \( T \geq 2 \) and \( r(t; \theta) > 0 \) for any \( t, \theta \). For any \( t' \geq t \), we assume a standard monotone likelihood ratio property for \( r(t; \theta) \): for any \( t' > t \), \( r(t'; a)/r(t'; b) \geq r(t; a)/r(t; b) \) with strict inequality for some \( t' \) and \( t \). This implies that the posterior \( \mu(a; t) \) of a citizen with signal \( t \) is non decreasing in \( t \). After observing the private signal, each citizen chooses whether to protest against the policy-maker’s default policy \( B \) or to stay home. The policy-maker observes the number of protesters and chooses a policy that maximizes her utility.

This model is best suited to describe public protests in established democracies in which the overthrow of the political regime is out of question and the purpose of the protests is to convince the policy-maker to change policy, so purely informational. It fits the cases mentioned in the introduction of citizens in the US protesting against the war in Iraq on MoveOn.org in 2014; or the case of economists signing a petition against the tax cuts of the Bush administration in 2003; but it also fits well the case of customers protesting against a private company’s policies (as in the case of United and its “PetsSafe” policy in 2012). In some applications, it may be natural to assume that only a fraction of citizens receive informative signals, or that some citizens are better informed than others (as in the 2007 petition by 1500 active-duty military personnel and reservists against the war in Iraq, or the 2003 petitions by economists cited in the introduction). The framework described above allows for these possibilities. However, it should be noted that, in equilibrium, participation is endogenous so it is not necessarily only citizens who receive an informative signal who may choose to protest (as presumably happened in the larger 2014 petition on MoveOn.org also cited in the introduction). In other environments, it is likely that all citizens receive informative signals: as, for example, in the 2007 protest against “road pricing and car tracking” in the UK.

In Section 5 we discuss how we model social networks in this environment. Until then, we assume that citizens act independently. When social networks are not available, a strategy for the policy-maker is a function from the observed number of protesters to a probability of choosing \( A \), i.e., \( \rho : N \to [0, 1] \). A strategy for a citizen is a function from the signal to a probability of protesting, i.e., \( \sigma : T \to [0, 1] \). Given this, the probability that a citizen protests in state \( \theta \) when the strategy is \( \sigma \) is \( \phi(\theta; \sigma) = \sum_t r(t; \theta) \sigma(t) \). The posterior probability that the state is \( a \) if \( Q \)

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15 Citizens, however, are not necessarily assumed to prefer \( A \) to \( B \) at the ex-ante level, it may still be that \( \mu < \mu^{**} \).
16 Assume for example that \( T = 3 \) and \( r(2, a) = r(2, b) = p \), so signal 2 is “uninformative.” This model is equivalent to a model in which the citizens are uninformed with probability \( p \) and with probability \( 1 - p \) they receive one of two informative signals, 1 and 3.
The public protest game described above always has an equilibrium in which the policy-maker ignores the protesters and chooses $A$ with probability one: in such an equilibrium citizens use uninformative, state-uncontingent strategies. In the following we study the conditions under which the policy-maker’s decision is influenced by the “wisdom of the crowd,” i.e., the citizens’ actions. Naturally, citizens’ protests can affect the policy-maker’s action only if they are informative on the state of the world. We say that $\sigma, \rho$ is an informative equilibrium if citizens use informative strategies and so the probability of protesting is higher in state $a$, the state in which the policy-maker’s default policy is incorrect: $\phi (a; \sigma) > \phi (b; \sigma)$. In this case the probability of $a$ is increasing in $Q$ and there is a $Q^*$ such that the policy-maker is willing to choose $A$ if and only if $Q > Q^*$. We are interested in studying the existence and the properties of informative equilibria in environments with an arbitrarily large number of citizens. To formalize this point, we say that an informative equilibrium exists in a large society if there is a $n^*$ such that an informative equilibrium exists for all $n > n^*$.

Informativeness of an equilibrium is just a minimal requirement for public protests to be useful: even if public protests are informative, information transmission can be minimal and the policy-maker’s mistake can be significant; even when the population is arbitrarily large, informativeness may converge to zero as $n \to \infty$. The probability of a mistake in an informative equilibrium $\sigma, \rho$, is $E(\sigma, \rho) = (1 - \mu) \Pr (A, b; \sigma, \rho) + \mu \Pr (B, a; \sigma, \rho)$, where $\Pr (p, \theta; \sigma, \rho)$ is the probability that policy $p$ is chosen in state $\theta$. We say that full information aggregation is achievable if there is a sequence of informative equilibria $\sigma_n, \rho_n$ for environments with population $n$ such that $E(\sigma_n, \rho_n)$ converges to zero as $n \to \infty$.

To appreciate the peculiarities of the public protest game, it is useful to introduce a natural benchmark. If we assume that the policy-maker can commit to a response function, then the public protest game looks similar to a voting game. Citizens can “vote” against the policy-maker’s default policy $B$ by protesting, or “vote” for the policy-maker’s ex ante optimal policy by staying home. We can also have informative equilibria in which citizens “protest” to show support to the policy-maker and stay home to signal their disagreement: in this case $\phi (a; \sigma) < \phi (b; \sigma)$. For the purpose of this paper, there is no loss of generality to focus on the most natural case in which a protest is interpreted as a sign that citizens protests to induce a change in the policy maker’s action.

\[ \Gamma_n (a; Q, \sigma) = 1 + \frac{1}{\mu} \left( 1 - \frac{1}{\mu} e^{\frac{-n\phi(b; \sigma)}{\sigma}} (n\phi(b; \sigma))^Q \right). \]
silent. The similarities between the two games are not perfect (in an election voters either vote A, B or abstain, in the public protest game they can only protest or abstain); yet, the essence of the strategic interaction is similar. From the literature of the Condorcet Jury Theorem, it is well known that full information is achieved in large elections. A similar result is obtained in the model of public protests described above. We say that a policy rule is a cut-off rule if there is a threshold $\hat{Q}$ such that $A$ is chosen if and only if the fraction of protesters over the expected population $Q/n$ is larger than or equal to $\hat{Q}$. We have:

**Proposition 1.** Full information aggregation is achieved if the policy-maker can commit to a cut-off rule $\hat{Q} \in (0,1)$.

The key feature of the public protest game is that the policy-maker is unable to pre-commit to a policy rule. The inability of the policy-maker to commit imposes an additional equilibrium condition requiring that, given the citizens’ strategies, the policy-maker is willing to use the cut-off. In the following we study the implications of the inability to commit on the effectiveness of public protests in aggregating information in large societies.

### 3 Public protests and information aggregation

In this section we study the conditions under which an informative equilibrium exists in the game described in the previous section. To this end, we first characterize an equilibrium in terms of simple cut-off strategies and then we use the characterization to study when information aggregation is possible.

The policy-maker’s optimal choice naturally depends on his posterior belief $\Gamma_n(a;Q,\sigma)$ given the citizens’ strategy $\sigma$. If the citizens use informative strategies, $\Gamma_n(a;Q,\sigma)$ is increasing in $Q$ and the policy-maker always finds it optimal to follow a cut-off rule. Let $Q_n(\sigma,\rho)$ to be the minimal $Q$ such that:

$$\Gamma_n(a;Q,\sigma) \geq \mu^*.$$  \hspace{1cm} (2)

The policy-maker strictly prefers $B$ if $Q < Q_n(\sigma,\rho)$ and $A$ if $Q > Q_n(\sigma,\rho)$; if $Q = Q_n(\sigma,\rho)$ the policy-maker is indifferent if $Q_n(\sigma,\rho)$ satisfies (2) with equality and strictly prefers $A$ otherwise.

To account for the possibility of the policy-maker using mixed strategies, it is convenient to represent the planner’s strategy $\rho_n(Q)$ as a function of a threshold $q_n$ in the real line:

$$\rho_n(Q) = \begin{cases} 
0 & Q < \lfloor q_n \rfloor \\
\lfloor q_n \rfloor - q_n & Q = \lfloor q_n \rfloor \\
1 & Q > \lfloor q_n \rfloor 
\end{cases}.$$ \hspace{1cm} (3)
where \([x]\) and \([x]\) are, respectively, the largest integer lower or equal than \(x\) and the smallest integer larger than \(x\). When \(q_n\) is an integer, (3) describes a simple cut-off rule for action in pure strategies: type \(q_n\) is the smallest number of protestors that induces the policy-maker to choose \(A\) with probability one; so that \(B\) is chosen if and only if less than \(q_n\) citizens protest. When \(q_n\) is not an integer, then \([q_n]\) is the smallest number of protestors that induces the policy-maker to choose \(A\) with probability one. A policy-maker that observes \([q_n]\) chooses \(A\) with probability \([q_n] - q_n\); a policy-maker that observes less than \([q_n]\) chooses \(B\) with probability one. Following a strategy (3) is optimal for a policy-maker if and only if \(q_n \in [Q_n(\sigma, \rho), Q_n(\sigma, \rho) + 1]\), with \(q_n = Q_n(\sigma, \rho)\) if \(\Gamma_n(a; Q_n(\sigma, \rho), \sigma) > \mu^*\). In this case we say that \(q_n\) is optimal given \(\tau_n\).

The citizens’ strategies depend on their posterior belief conditioning on being pivotal, i.e. affecting the policy-maker’s decision. To evaluate the citizens’ decision, define \(\varphi_n(\theta; \sigma, \rho)\) to be the pivot probability in state \(\theta\) given an expected size of population \(n\) and the strategies \(\sigma, \rho\). The pivot probability is the increase in the probability that \(A\) is chosen, as induced by a citizen’s decision to protest. The pivot probability in state \(\theta\) is:

\[
\varphi_n(\theta; \sigma, \rho) = \beta_n \cdot P(Q_n(\sigma, \rho) - 1, n\phi(\theta; \sigma)) + (1 - \beta_n) \cdot P(Q_n(\sigma, \rho), n\phi(\theta; \sigma))
\]

\[
= P(Q_n(\sigma, \rho), n\phi(\theta; \sigma)) \cdot \beta_n \cdot \frac{Q_n(\sigma, \rho)}{n\phi(\theta; \sigma)} + (1 - \beta_n),
\]

where \(P(\cdot, n\phi(\theta; \sigma))\) is a Poisson with mean \(n\phi(\theta; \sigma)\) and \(\beta_n\) is the probability that \(A\) is chosen if \(Q_n(\sigma, \rho)\) citizens are protesting. To interpret (4), note that a citizen is pivotal in only two events, when \(Q_n(\sigma, \rho) - 1\) or \(Q_n(\sigma, \rho)\) other citizens are protesting (corresponding, respectively, the first and second term in (4)). In the first event, a citizen’s protest increases the probability of \(A\) from zero to \(\beta_n\); in the second event, a citizen’s protest increases the probability of \(A\) from \(\beta_n\) to one.

A citizen chooses to protest if the expected benefit of the protest is non negative:

\[
\mu(a; t) v(a) \varphi_n(a; \sigma, \rho) + \mu(b; t) v(b) \varphi_n(b; \sigma, \rho) \geq 0.
\]

We can rewrite this condition as:

\[
\frac{\mu(a; t)}{\mu(b; t)} \geq -\frac{v(b) \varphi_n(b; \sigma, \rho)}{v(a) \varphi_n(a; \sigma, \rho)} = \frac{\varphi_n(b; \sigma, \rho)}{v \cdot \varphi_n(a; \sigma, \rho)}.
\]

The monotone likelihood assumption on citizens’ signals implies that there is a \(t_n(\sigma, \rho) \in [1, T]\) such that only citizens with \(t \geq t_n(\sigma, \rho)\) find it optimal to protest and citizens with \(t < t_n(\sigma, \rho)\) find it strictly optimal not to protest; if \(t_n(\sigma, \rho)\) satisfies (5) with equality then citizens with \(t = t_n(\sigma, \rho)\) are indifferent and are willing to randomize their action. As with the policy-maker, a citizen’s equilibrium strategy \(\sigma_n\) can be conveniently represented as a continuous function of a
threshold \( \tau_n \in [1, T + 1] \) as follows:

\[
\sigma_n(t) = \begin{cases} 
0 & t < \lceil \tau_n \rceil \\
\tau_n - \tau_n & t = \lfloor \tau_n \rfloor \\
1 & t > \lfloor \tau_n \rfloor
\end{cases}
\]  

(6)

Following a strategy (6) is optimal for a citizen if and only if \( \tau_n \in [\tau_n(\sigma, \rho), \tau_n(\sigma, \rho) + 1] \), with \( \tau_n = t_n(\sigma, \rho) \) if (5) is strict at \( \tau = t_n(\sigma, \rho) \). In this case, we say that \( \tau_n \) is optimal given \( q_n \).

The representation of the strategies in (3) and (6) allow to characterize an equilibrium in terms of two real numbers and simple cut-off strategies:

**Proposition 2.** An informative equilibrium is characterized by a pair of thresholds \( \tau_n^*, q_n^* \) such that \( q_n^* \) is optimal given \( \tau_n^* \), and \( \tau_n^* \) is optimal given \( q_n^* \).

As mentioned in Section 2, existence of an equilibrium is easily established. The real question is whether information transmission is possible in equilibrium. To appreciate the problems that may arise for the existence of an informative equilibrium, consider a simple example in which the citizens receive a binary signal \( T = \{1, 2\} \) with \( r(1, b) = r(2, a) = r > 1/2 \) and \( r(1, a) = r(2, b) = 1 - r \). If an informative equilibrium exists then there must be a threshold \( Q_n \) such that the policy-maker is willing to choose \( A \) if and only if the number of protesting citizens \( Q \) is at least \( Q_n \). At this threshold the policy-maker’s posterior probability must be sufficiently large: \( \Gamma_n(a; Q_n, \sigma) \geq \mu^* \). This inequality can be rewritten as:

\[
\frac{P(Q_n, n \phi(a; \sigma_n))}{P(Q_n, n \phi(b; \sigma_n))} \geq \frac{1}{1 - \mu^*} \left( \frac{1}{\mu} - 1 \right),
\]  

(7)

The equilibrium, however, is informative only if there is separation of the citizens’ types. This is possible only if, at the very minimum, the citizens with the lowest signal are willing to be inactive. By condition (5), we must have:

\[
\frac{\varphi_n(a; \sigma, \rho)}{\varphi_n(b; \sigma, \rho)} \leq \frac{1}{v} \left( \frac{1}{\mu(a; \sigma)} - 1 \right). 
\]  

(8)

The left hand sides of (7) and (8) are intimately connected. The left hand side of (7) is the ratio between the probabilities of having \( Q_n \) protesters in, respectively, state \( a \) and in state \( b \). The left hand side of (8) is the ratio of the pivot probabilities in, respectively, state \( a \) and \( b \). As it can be seen from (4), the pivot probability in state \( \theta \) is a convex combination of the probabilities that \( Q_n \) and \( Q_n - 1 \) citizens are active in state \( \theta \) (since a citizen is pivotal only in these two events). The weights in the convex combination depends on the policy-maker’s strategy (the probability of choosing \( A \) with \( Q_n \) protesters). There is therefore, a well defined relationship between the right
hand side of (7) and (8). As formally shown in the proof of Lemma 1, the relationship between them can be bounded as follows:

$$\frac{\varphi_n(a; \sigma, \rho)}{\varphi_n(b; \sigma, \rho)} \geq \frac{1 - r P(Q_n, n\phi(a; \sigma_n))}{r P(Q_n, n\phi(b; \sigma_n))}. \quad (9)$$

Using (9) we can now connect (7) and (8):

$$\frac{1}{v} \left( \frac{1}{\mu(a; 1)} - 1 \right) \geq \frac{\varphi_n(a; \sigma, \rho)}{\varphi_n(b; \sigma, \rho)} \geq \frac{1 - r P(Q_n, n\phi(a; \sigma_n))}{r P(Q_n, n\phi(b; \sigma_n))} \geq \frac{1 - r}{r V} \left( \frac{1}{\mu} - 1 \right)$$

The first and last inequality follow from (7)-(8), the second inequality follows from (9). We conclude that an informative equilibrium exists in our example only if:

$$V \geq \frac{(1 - r)(1/\mu - 1)}{r(1/\mu(a; 1) - 1)} \cdot v \quad (10)$$

Recall that the policy-maker is less inclined to choosing A than the citizens: so $V < v$ and the smaller is $V$, the larger is conflict. Condition (10) therefore, defines an upper bound on the conflict between the citizens’ and the policy-maker’s preferences: the right hand side is clearly smaller than $v$, so when $V$ is not sufficiently close to $v$, no informative equilibrium exists. If, for example, we assume $r = 2/3$ and a $v$ such that a citizen is indifferent when the posterior is $\mu^* = 1/(1 + v) = 0.5$, then no informative equilibrium exists if $V$ is such that the indifference threshold for the policy-maker is $\mu^* = 1/(1 + V) \geq 0.8$. Remarkably, when (10) is not satisfied, an informative equilibrium does not exist even if the number of informed citizens is arbitrarily large.

For the general case, we have:

**Lemma 1.** No informative equilibrium exists if $V < V_1(v)$ where:

$$V_1(v) = \frac{1/\mu(a; T) - 1}{1/\mu(b; 1) - 1} \cdot v. \quad (11)$$

Lemma 1 highlights a key difference between our public protests game and the voting games studied in the Condorcet Jury Theorem literature in which the policy-maker can commit to a response function: with commitment, as shown in Proposition 1, not only does an informative equilibrium exist, but full information aggregation is achieved as the size of the population increases. Conversely, when the policy-maker cannot commit to a response plan, the fact that citizens receive informative signals is not sufficient for information transmission. Indeed when the condition of Lemma 1 is satisfied, no information is transferred at all, no matter how large the number of informed citizens is.
Figure 1: The threshold $M(\mu^{**})$ in an example. The thresholds $M_1(\mu^{**}), M_2(\mu^{**})$ correspond to the upper- and lower-bounds on $M(\mu^{**})$ established in Lemmata 1 and 2.

Can, then, public protests be useful and allow the policy-maker to improve her choice when conflict is sufficiently small? The following result characterizes a simple sufficient condition for the existence of an informative equilibrium.

**Lemma 2.** An informative equilibrium exists if $V \geq V_2(v)$, where:

$$V_2(v) = \frac{1/\mu - 1}{1/\mu(a;1) - 1} v.$$  

It is easy to verify that $V_2(v)$ is positive and larger than $V_1(v)$. This implies that if conflict is sufficiently small, information transmission is possible for any size $n$ of the population.

The precise condition for the existence of an informative equilibrium naturally depends on the details of the environment, like the shape of the entire signal structure (as described by $r(t, \theta)$ for $t = 1, \ldots, T$), the number of agents, etc. The next result completes the analysis by showing that a simple threshold characterizes when information transmission is possible in a large society. Recall that we say a property holds in a large society if there is a $n^*$ such that it holds for all $n > n^*$.

**Proposition 3.** There is a threshold $V^*(v) \in [V_1(v), V_2(v)]$ such that an informative equilibrium exists in a large society if $V < V^*(v)$ and it does not exist if $V > V^*(v)$.

Two lessons on the effectiveness of public protests from Proposition 3 should be highlighted. First, the importance of the size of conflict between the citizens and the policy-maker. It is not surprising that the larger the conflict, the less effective public protests are; much more surprising is the fact that if the conflict is sufficiently large, then protests are completely ineffective even if
the number of informed citizens is arbitrarily large and each of them receives a strictly informative independent signal. The number of informed citizens is not a substitute for a small conflict of interest between the citizens and the policy-maker.

The second lesson has to do with the importance of the precision of the signals received by the citizens. As the precision of the citizens private information converges to zero, we have that:

\[ r(1;b)/r(1;a) \rightarrow r(T;b)/r(T;a) \]

As it is easy to see from (11), this implies that \( V_1(\nu) \) converges to \( v \): even if conflict is small, civic action may be ineffective if the citizens are not receiving sufficiently informative private signals.

The reason the existence of an informative equilibrium relies upon the precision of individual signals independently from the number of citizens can be intuitively explained. To have an informative equilibrium the policy-maker must be willing to choose \( A \) after observing \( Q_n \) protestors (i.e. the equilibrium threshold) and, at the same time, the citizens who receive the most “pro-B” signal (i.e. \( t = 0 \)) must be willing to refrain from protesting conditioning on the pivotal event in which \( Q_n \) or \( Q_n - 1 \) other citizens are protesting. How is this possible if citizens are more willing to choose \( A \) than the policy-maker? It is possible because the policy-maker conditions only on the number of protesters, while the citizen conditions on the number of other protesters and his own private signal. Information aggregation is possible if a low realization of the signal decreases the citizen’s expected benefit of \( A \) enough to compensate for the preference bias. When the signal received by each agent is not very informative, the difference in the posterior beliefs of the policy-maker and the citizens with the lowest realization of \( t \) is not sufficient to compensate for the conflict of interest: in this case information aggregation is impossible, even if the number of informed citizens is arbitrarily large.

To assess the level of conflict that is compatible with an informative role of public protests, it is useful to illustrate it in terms of beliefs (since beliefs are naturally normalized to be in \([0, 1]\)). Recall that \( \mu^* = 1/(1 + V) \) and \( \mu^{**} = 1/(1 + v) \) are the belief thresholds at which the policy-maker and the citizens are ex ante indifferent between the two policies. Proposition 3 can be restated as stating that there is a threshold \( M(\mu^{**}) \) such that an informative equilibrium exists if \( \|\mu^* - \mu^{**}\| < M(\mu^{**}) \) and does not exist if \( \|\mu^* - \mu^{**}\| > M(\mu^{**}) \). The left panel of Figure 1 illustrates these thresholds as functions of \( \mu^{**} \) in an example in which the signal distribution is \( r(t;\theta) = e^{-\alpha t}/\sum_{j=1}^{T} e^{-\alpha j} \) with \( \alpha_a = 1 \) and \( \alpha_b = 2.5 \). As it can be seen, information transmission can be feasible even in environments in which there is very significant conflict. The right panel of Figure 1 illustrates the thresholds as functions of \( \alpha \) in an example in which the signal distribution is \( r(t;\theta) = e^{-\alpha t}/\sum_{j=1}^{T} e^{-\alpha j} \) with \( \alpha_a = 1, \alpha_b = \alpha \) and \( \mu^{**} = 0.4 \): so that the
The values of the table represent the minimal and maximal equilibrium values of $Q/n$. The value n/a is reported when an informative equilibrium could not be found.

larger is $\alpha$, the more informative are the citizens’ signals.\(^{20}\)

The Table in Figure 2 illustrates how informative equilibria look like as we change $n$, $T$, and $\mu^*$.\(^{21}\) It is interesting to note that many equilibria typically exist; and, perhaps more importantly, they can be very different from each other. When $n = 1000$, $T = 3$, $\mu^{**} = 0.4$ and $\mu^* = 0.8$, for example, the policy-maker is “convinced” by protesters if a little more than 18% of the expected population chooses to protest in the smallest equilibrium, or a little more than 21% in the largest equilibrium. But when conflict is smaller and/or $T$ is larger, the range of equilibria is significantly larger. When $n = 1000$, $T = 3$ and $\mu^* = 0.65$ the policy-maker can be convinced if just 3.2% of the expected population protests in the smallest equilibrium, and 33% in the largest equilibrium.

### 4 Large public protests and information aggregation

Since Condorcet [1785] an important literature has been dedicated to the study of the conditions under which elections provide a mechanism for the aggregation of citizens’ private information. In its strongest form, the Condorcet Jury Theorem (henceforth CJT) claims that the outcome of

---

\(^{20}\) In both panels, we assume $n = 1000$, $T = 4$, and $\mu = 1/2$. The thresholds $M_1(\mu^{**})$, $M_2(\mu^{**})$ correspond to the upper-and lower-bounds on $M(\mu^{**})$ established in Lemmata 1 and 2: $M_1(\mu^{**}) = 1/(1 + V_1(1/\mu^{**} - 1)) - \mu^{**}$ and $M_2(\mu^{**}) = 1/(1 + V_2(1/\mu^{**} - 1)) - \mu^{**}$.

\(^{21}\) As in Figure 1, in Table 1 we assume $\mu = 1/2$ and $\mu^{**} = 0.4$ and signal distribution $r(t; \theta) = e^{-\alpha t}/\sum_{j=1}^{T} e^{-\alpha t}$ with $\alpha_a = 1$ and $\alpha_b = 2.5$. 

<table>
<thead>
<tr>
<th></th>
<th>$\mu^*=0.65$</th>
<th>$\mu^*=0.8$</th>
<th>$\mu^*=0.9$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>min</td>
<td>max</td>
<td>min</td>
</tr>
<tr>
<td>$T=2$</td>
<td>0.154</td>
<td>0.234</td>
<td>n/a</td>
</tr>
<tr>
<td>$T=3$</td>
<td>0.0321</td>
<td>0.338</td>
<td>0.186</td>
</tr>
<tr>
<td>$T=5$</td>
<td>0.0031</td>
<td>0.43</td>
<td>0.043</td>
</tr>
</tbody>
</table>

Figure 2: The values of the table represent the minimal and maximal equilibrium values of $Q/n$. The value n/a is reported when an informative equilibrium could not be found.
an election fully reflects all the information available to the citizens. An analogous question can be asked in the public protest game presented in the previous sections. Is there an equilibrium in which citizens’ actions are so informative that the policy-maker’s decision reflects all the information available to the citizens? As a consequence, is there a sequence of equilibria along which the probability of a policy mistake converges to zero as $n \to \infty$? This question pushes the line of inquiry of the Condorcet jury theorem one step further by asking not only whether it is optimal for the citizens to vote informatively, but also whether it is optimal for the policy-maker to commit to a rule consistent with the citizens’ actions. We refer to the conjecture that the policy-maker chooses the optimal action with probability one in a large society as the Weak Condorcet Jury Theorem (WCJT).

Proposition 3 immediately informs us that the answer to the question stated above can not be unequivocally positive: when $||v - V||$ is too large, no informative equilibrium exists. Proposition 3, however, does not address this question when an informative equilibrium exists: even if we have an informative equilibrium for every $n$, it is still possible that the level of informativeness converges to zero as population grows to infinity. The next result completes the analysis of Proposition 3 by proving the WCJT for our game of public protests. Recall from Section 2 that we say that full information aggregation is achievable if there is a sequence of informative equilibria along which the probability of mistake converges to zero.

**Proposition 4.** Full information aggregation is achievable if $V > V^*(v)$.

Figure 3 illustrates the equilibrium probability of committing an error as a function of $n$ in a specific example. The probability of error is computed as the probability of choosing $B$ in state $a$ plus the probability of choosing $A$ in state $b$: $E_n = \mu \Pr (B, a; \sigma_n, \rho_n) + (1 - \mu) \Pr (A, b; \sigma_n, \rho_n)$. The lower curve (in black) corresponds to the probability of committing an error in the best equilibrium, that is in the equilibrium with the lowest error. The intermediate curve (in red) corresponds to the probability of committing an error in the worst equilibrium. The higher curve (in blue) is a benchmark: it represents the expected error that we would have if the policy-maker could commit to choose $A$ if and only if more than 50% of the expected number of citizens protest. As the example shows the error converges to zero relatively quickly; still multiplicity of equilibria creates significant uncertainty when the number of citizens is not very large.

The intuition for Proposition 4 is as follows. As we know from Proposition 3, when $V > V^*(v)$ we have a sequence of informative equilibria $\tau_n, q_n$ as $n \to \infty$; let $\tau_\infty$ be the limit of $\tau_n$. If $\tau_\infty$ is interior, i.e. $\tau_\infty \in (1, T + 1)$, then the result is immediate. In this case, the monotone likelihood ratio assumption implies that $\phi(a; \tau_\infty) > \phi(b; \tau_\infty)$. This implies that the policy-maker is drawing

\[\text{As an additional reference, we should recall that the policy-maker with no information would commit a mistake with 50% probability when } \mu = 1/2, \text{ since with no information the policy-maker chooses } B \text{ with probability one.}\]
large samples from one of two distributions of independent random variables with different means: the result follows from a straightforward application of Chebyshev’s inequality. Problems arise when $\tau_\infty$ is not interior, but luckily this case can be ruled out.

Proposition 4 has significant implications on how to interpret the Condorcet Jury Theorem. A standard interpretation of the theorem is that it provides a positive theory of elections: elections are a good way to make public decisions because they have good informational properties (see for example Feddersen and Pesendorfer [1997]). An implied assumption in this interpretation is that other political processes, especially less democratic ones, would not have the same informative properties. Proposition 4 shows that this is not necessarily the case even in the least democratic of processes: a policy-maker with dictatorial powers and no commitment can achieve full information relying just on citizens voluntary signals as long as the ex-ante conflict is not too large.

---

23 As $n \to \infty$, the distribution of protesters looks approximately like a normal concentrated around $n\phi(a; \tau^*)$ in state $a$, and around $n\phi(b; \tau^*)$ in state $b$. Observing a fraction of protesters $Q/n$ lower or equal than $\phi(b; \tau^*)$ is arbitrarily more likely in state $b$ than in state $a$; and observing a fraction of protesters $Q/n$ larger or equal than $\phi(a; \tau^*)$ is arbitrarily more likely in state $a$. This implies that the equilibrium threshold $q^*$ that makes the policy-maker indifferent must be such that: $\phi(b; \tau^*) < q^*/n < \phi(a; \tau^*)$. Given this, the result follows from Chebyshev’s inequality: the probability that the fraction of protesters is larger than $q^*/n$ in state $b$ or lower than $q^*/n$ in state $a$ converges to zero as $n \to \infty$.

24 Indeed even a policy-maker with a significant conflict of interest with the citizens can do better than a voting
Proposition 4, however, can still provide a positive argument for the optimality of elections if we take a “behind the veil of ignorance” point of view. It is indeed natural to assume that citizens choose institutions at an ex-ante stage, before the issue to be decided upon is defined. At this stage it is plausible to assume that there is uncertainty regarding the conflict that may arise between the policy-maker and the citizens. Assume \( V \) is a random variable with density in \([0, v]\) and generic distribution \( F \) with full support. This represents an environment in which citizens and the policy-maker do not know what kind of issue they will have to face, so they are uncertain about the conflict of interest. Citizens have to choose between an election in which a non unanimous \( \hat{Q} \) rule is used (i.e., \( A \) wins if it receives a share \( \hat{Q}/n \) of votes lower than one) and a system in which the policy-maker has full authority but citizens can protest. In this case we have:

**Proposition 5.** If the most informative equilibrium is selected, there is a \( x^* \) such that for \( n > x^* \) both citizens and the policy-maker prefer to commit to elections rather than to leave the policy to the policy-maker’s discretion.

Proposition 5 puts very few constraints on the expected conflict between the policy-maker and the citizens. Still the result claims that the policy-maker would never find it optimal to retain discretion in policy making. The idea behind Proposition 5 is simple. When conflict is sufficiently small, public protests can perform as well or even better than elections. When the conflict is large public protests are ineffective but, by committing ex-ante to a voting rule, the policy-maker can achieve the first best in this case too. As \( n \) increases, any possible advantage of the policy-maker with discretional power fades away, since in both systems the probability of mistake converges to zero when \( V > V^*(v) \). In states in which \( V < V^*(v) \), on the contrary, the election does strictly better. The key observation following in Proposition 5 is that what makes elections valuable for information aggregation is not the specific voting rule, but it is the commitment implicit in the decision rule. We should expect citizens to commit to electoral systems when expected conflict is high and when population is high. When population is not particularly high and or expected conflict is not too high, a more informal system in which an individual is entrusted with decision power and the other citizens can public protest may be more efficient.

For example, Figure 3 suggests that the probability of a mistake in the worst equilibrium is significantly lower than the probability of mistake in the case in which the decision is taken by committing to a cut-off rule \( \hat{Q}/n = 0.5 \). Commitment to a cut off rule guarantees that the mistake converges to zero as population increases; if the cut-off rule is not appropriately chosen, however, it does not guarantee that the mistake converges faster than the equilibrium mistake in an equilibrium of the protest game.
5 Protests and social media

Many scholars have recently emphasized the importance of social media in fostering collective action. From a conceptual point of view, the effect of social media on the ability of citizens to signal their information is ambiguous. Social media allows groups to share information and coordinate the activity of their members, but it reduces the number of independent actors. By sharing information, protesters become better informed, but they can also coordinate on strategies that are more effective at influencing the policy-maker: this may reduce the policy-maker’s willingness to “trust them.”

To study the impact of social media on the ability of public protests to aggregate information, consider a slightly simplified version of the model presented above in which the individual signal \( t \) has binary support \( T = \{0, 1\} \), with \( r(1; a) = r = r(0; b) \) and \( r > 1/2 \). We model the effect of social media assuming that each citizen is affiliated to a group of size \( g \). The number of groups is a Poisson random variable with mean \( m \geq 2 \), so expected population is now \( n = m \cdot g \). The key assumption in this version is that groups’ members can communicate and share their information within their groups. Examples of relevant social groups are blogs, Facebook, YouTube, Twitter hashtags; or more old fashioned groups, like unions or parties. We say that citizens are in autarky when \( g = 1 \), as in the previous sections. We say that social groups are available if \( g \geq 2 \). In this model technological progress in social media corresponds to an increase in \( g \).

Consider the problem faced by the citizen in a social group. Each citizen in a group receives an informative signal corresponding to the number of citizens with a \( t = 1 \) (instead of \( t = 0 \)) realization. This aggregate signal \( \tilde{t} \) has support \( \tilde{T} = \{0, \ldots, g\} \) and distribution \( r_g(t; \theta) = B_g(t, \theta), \) where \( B_g(t, \theta) \) is a binomial with mean \( r_g \) when the state is \( a \), and \((1 - r)g\) when the state is \( b \). If we fix the strategies of the policy-maker, it is easy to see that citizens in a social circle find it optimal to coordinate their actions and act as a block. Similarly, if all groups act in a coordinated way, the policy-maker will find it optimal to treat each group as an individual agent. This implies that the extended game with \( m \) groups of size \( g \) can be treated as a game with \( m \) individual with signal \( r_g(t; \theta) \). The trade-off mentioned at the beginning of this section is now clear: on the one hand each group receives a more precise signal than any individual protesters; on the other

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25 Recent work include Wasserman et al. [2010] who discuss the importance of mobile phones and texting for collective action in Africa; Valenzuela et al. [2012] who document the correlation between activity on Facebook and protests in Chile; Bennett and Segerberg [2010] who document the role of social media by activists during the 2009 G20 London Summit; and Acemoglu et al. [2014] who document the use of Twitter during Egypt’s Arab spring.

26 This idea reflects the evidence on the working of activist groups. For example, Bennett and Segerberg [2010] report that during the 2009 G20 London Summit 160 distinct civil society groups were active; Valenzuela et al. [2012] report the case of a successful protest in which 118 independent Facebook pages were active. There is abundant evidence that social media allows these groups superior information sharing and coordination.

27 A formal proof of these claims is presented in the appendix in the proof of Proposition 6.
hand, however, we have only \( m \) independent groups, rather than \( n = mg \) independent individuals.

To see why groups may help protesters signal their information, note that the likelihood ratio of the signal received by a group is now \( r(t; a)/r(t; b) = (r/(1 - r))^{2t-g} \). As \( g \) increases, the posterior probability that the state is \( a \) after signals \( t = 0 \) converges to zero: these groups are going to be willing to abstain from protesting even when there is a very large conflict. The implications of this can be seen on \( V_2(v) \) that now becomes:

\[
V_2(v) = [(1 - r)/r]^g \cdot v
\]

As \( g \) increases, \( V_2(v) \) converges to zero for any \( v \) (and \( M_2(\mu^{**}) \) converges to one for any \( \mu^{**} \)), guaranteeing existence of an informative equilibrium no matter how large the conflict is. We have:

**Proposition 6.** For any \( r \) and \( m \), information aggregation is possible with social media if \( g > g^*(r) \), where \( g^*(r) = \log(V/v) / \log((1 - r)/r) \).

Proposition 6 clarifies the importance of the information sharing and coordination services provided by social media. Two familiar forces emerge from the threshold \( g^*(r) \). First the size of conflict. The ratio \( V/v \) is a measure of the size of the conflict: it is minimal when \( V = v \) (a case with no conflict) and it increases as the distance between \( v \) and \( V \) increases. For any \( r \), as the conflict converges to zero, \( g^*(r) \) converges to a value that is lower or equal than one: so for sufficiently small conflict, communication in groups is irrelevant. For a given level of conflict, however, the minimal group size compatible with information aggregation increases with \( r \): as signals become uninformative, the size of a required group grows to infinity. The size of social media required for information aggregation is determined by these two forces.

The analysis becomes interesting when we assume that no information can be aggregated in autarky. This always occurs when the individual signals are not too informative. From (11), we see that no information is aggregated if \( r < r^* \), where \( r^* = 1/\left[1 + \sqrt{V/v} \right] \). Define:

\[
r(g) = 1/\left[1 + (V/v)^{1/2} \right]
\]

Note that \( r(g) < r^* \) for any \( g \geq 2 \) and \( r(g) \to 1/2 \) and \( g \to \infty \). An immediate implication of Proposition 5 is the following result:

**Corollary 1.** For any \( g \), information aggregation is feasible with public protests if and only if social media is available when \( r \in [r(g), r^*] \).

To understand why social groups help public protests, it is useful to go back to why information aggregation fails in autarky. As we explained in Section 3, an informative equilibrium relied upon a citizen that receives a low signal choosing to stay home even if the policy-maker is indifferent.
and one additional protestor would be pivotal. However, when the signal received by each agent is not very informative (i.e. \( r < r^* \)), the difference in the posterior beliefs of the policy-maker and the citizens with the lowest realization of \( \tilde{t} \) is not sufficient to compensate for the conflict of interest: in this case information aggregation is impossible. When citizens share information in groups, however, they improve the precision of their aggregate signal. The worst possible signal now is not receiving \( t = 0 \) rather than \( t = 1 \), but receiving zero positive signals out of \( g \). If \( g \) is sufficiently large, precision in a group is sufficient to compensate for the conflict, and information aggregation becomes possible.

6 Discussion and Extensions

In the model analyzed in the previous sections we made many simplifying assumptions. In this section we relax these assumptions extending the basic framework in a number of directions. We show that the same framework can be used to study situations in which the conflict between the policy-maker and the citizens stems from different priors rather than different payoffs. We show that the model can be extended to allow for direct costs and/or benefits of participating in protests, for conflict of interests between citizens, and for public signals observed by all citizens and/or by the policy-maker. Finally, we use the model to explain and rationalize some recent empirical evidence on the effect of public protests.

Different priors. In the analysis presented above, citizens and the policy-maker assign the same prior probabilities on the states of the world. The origin of conflict between players lies on the fact that citizens and the policy-maker face different trade-offs between \( A \) and \( B \), as measured by \( V = -V(a)/V(b) \) and \( v = -v(a)/v(b) \). In their model of voice, Banerjee and Somanathan [2001] propose an alternative model in which citizens and the policy-maker assign the same payoffs to the effect of policies in the two states; the citizens and the policy-maker, however, assign different priors probabilities to the state of the world. To see the relationship of our model to Banerjee and Somanathan’s model, consider a modified version of the model presented in the previous sections in which \( V = v = \hat{V} \); the policy-maker assigns a prior probability \( \mu \in (0, 1) \) to state \( a \); citizens assign a probability \( \hat{\mu} \in (0, 1) \) with \( \hat{\mu} > \mu \). In this version, therefore, conflict arises because citizens assign a higher prior probability on \( a \) than the policy-maker.\(^{28}\)

To see the implications of this change, consider (5), the condition characterizing when it is

\(^{28}\) Of course we assume here that \( v = V \) just to make a cleaner comparison with the model of Section 2: we could assume both \( \hat{\mu} > \mu \) and \( v > V \) and have a model with different preferences and different prior probabilities.
optimal for a citizen to protest. With the new prior, it can be written as:

$$\frac{\hat{\mu}(a; t)}{\mu(b; t)} = \frac{\hat{\mu} \cdot r(t; a)}{(1 - \hat{\mu}) \cdot r(t; b)} \geq \frac{\varphi_n(b; \sigma, \rho)}{\varphi_n(a; \sigma, \rho)}$$

where $$\hat{\mu}(\theta; t)$$ is the posterior corresponding to the different prior $$\hat{\mu}$$. This condition can be easily rewritten as:

$$\frac{\mu(a; t)}{\mu(b; t)} \geq \frac{\varphi_n(b; \sigma, \rho)}{\varphi_n(a; \sigma, \rho)}$$  \hspace{1cm} (12)

where $$\mu(\theta; t)$$ is the posterior when the prior is $$\mu$$ and $$\hat{\mu} = \frac{\hat{\mu}(1-\mu)}{\mu(1-\hat{\mu})} \hat{V}$$. Condition (12) says that citizens with prior $$\hat{\mu}$$ and payoff parameter $$\hat{V}$$ are willing to protest if and only if citizens with prior $$\mu$$ and payoff parameter $$\hat{\mu} > \hat{V}$$ are willing to protest. This implies that the modified game in which citizens have different prior probabilities than the policy-maker is equivalent to the game studied above in which priors are the same but citizens have different payoffs than the policy-maker. The model presented in Section 2, therefore, can be reinterpreted as a model in which conflict originates from a disagreement in prior beliefs.

**Participation costs.** In all the examples presented in the introduction (the petition on Change.org by the customers of United, the petition of the 400 economists against president’s Bush tax cuts in 2003, for example), the costs of participating in the public protest are not particularly significant. There are however environments in which some citizens experience positive costs of participation, due to the effort of protesting; other citizens, on the contrary, experience a negative cost, if they enjoy voicing their concerns; others, finally, experience both costs and benefits, with a net that can be positive, negative or zero. To model all these possibilities, we follow the approach of Palfrey and Rosenthal [1985] who presented the first rigorous analysis of participation costs in elections. We assume each active citizen pays a token $$c$$ that has finite support with distribution $$G(c)$$. We assume that the minimal value in the support is $$c < 0$$, the maximal value is $$c > 0$$ (sufficiently large that citizens with this cost will never protest), and zero is in the support. We don’t impose other restrictions on the support or the relative probabilities assigned to the possible realizations.

With participation costs, the analysis is considerably more complicated since now the choice to protest depends both on the signal $$t$$ and on the cost $$c$$. In the previous analysis, citizens’ strategies could be described by a simple cut point $$\tau^*$$. Now citizens’ strategies can be described by a family of cut-points $$\tau^*(c)$$, one for each possible realization cost $$c$$. We generally have three classes of citizens: there is a threshold $$c^*$$ such that for $$c \geq c^*$$ citizens will chose to be inactive no matter what their signal is; there is a threshold $$c_\alpha$$, such that for $$c \leq c_\alpha$$ citizens will chose to protest no matter what their signal is; finally, for $$c \in (c_\alpha, c^*)$$, we have citizens who use informative
strategies. The results presented in the previous sections, however, can be extended to this more general version of the model. We have:

**Proposition 7.** There is a threshold \( V^G(v) \) such that an informative equilibrium exists in a large society if \( V > V^G(v) \) and it does not exist if \( V < V^G(v) \). When \( V > V^G(v) \) full information aggregation is achieved as \( n \to \infty \).

Although the cut point for the existence of an informative equilibrium may depend on the distribution of participation costs \( G \), what matters is that \( V^G(v) \in (V_1(v), V_2(v)) \), so the range of environments in which information aggregation is impossible because of the size of the conflict, and the range in which full information aggregation is feasible are non empty for any distribution of costs \( G \). What is remarkable is the fact that very little restrictions are required for the distribution of costs \( G \).

The intuition for this result is relatively straightforward. As in Palfrey and Rosenthal [1985], only the citizens with a sufficiently small cost/benefit of participation (i.e. a small \(|c|\)) will act according to their signal when population is large. This happens because the probability of being pivotal converges to zero as \( n \to \infty \), so the policy uncontingent payoff, \( c \), dominates the citizen’s decision. When \( V < V^G(v) \), however, not even the citizens with a very small \(|c|\) find it optimal to follow their signals: in this case, therefore, no citizen behaves informatively. When \( V > V^G(v) \) the logic is similar, with the only difference being that in this case the citizens with a very small cost/benefit of participation find it optimal to act according to their signal. Naturally the fact that full information aggregation is theoretically feasible in this case, doesn’t mean that the error will be zero or even small with finite population: the expected number of citizens who are willing to be informative may be drastically reduced, implying a reduction in the quality of information aggregation if \( n \) is finite.

**Conflict of interest between citizens.** Another assumption made in the previous sections is that all citizens have the same preferences as described by the single parameter \( v \). Naturally, in real life citizens have different preferences and potential conflicts among themselves. Extending the model to incorporate these conflicts is, at least conceptually, not difficult. Assume that there are \( K \) types of citizens, each characterized by a preference parameter \( v_i \) for \( i = 1, \ldots, K \); and that these types are independently distributed with probability \( (\xi_i)_{i=1}^K \). In this case, the number of citizens of type \( i \) is Poisson distributed with mean \( \xi_i n \). The analysis of this version of the model is similar to the analysis presented above: voters decide to protest based both on their signal and

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Note that in general \( c_* \) is strictly negative and \( c^* \) strictly positive since even if a protester likes (respectively, dislikes) the act of protesting, he/she may be reluctant to do it after a bad (good) signal.
their preference parameter; the equilibrium is described by a series of cut-points \((\tau^*_i)_{i=1}^K\) and a policy-maker’s decision rule \(q^\ast\).

A particularly convenient but still insightful version of the model with conflict is a model with partisans. Assume there are three types: type 1 with \(v_1 = v \in (0, \infty)\), who wants \(A\) in state \(a\) and \(B\) in state \(b\); type 2 with large \(v_2\), who finds \(A\) optimal in both states; and type 3 with \(v_3 = 0\), who finds \(B\) optimal in both states.\(^{30}\) Types 2 and 3 are “partisans” who have a dominant strategy and so their actions would never provide information to the policy-maker.\(^{31}\) Since type 3 agents would not find it optimal to protest, this model is equivalent to a model in which type 3 does not exist and \(n = (1 - \xi_3)n\). Since type 2 agents are always active, the policy-maker corrects his expectation for the fact that a share of agents \(\xi_3\) is always active. Assume that \(\tau^\ast, Q^\ast\) is the limit of an sequence of equilibria as \(n \to \infty\) with no partisans \((\xi_1 = 1)\). Then \(\tau^\ast, Q^{**}\) is a limit of equilibria of a model with partisans, where \(Q^{{**}} = Q^\ast\xi_1 + \xi_2\).

It is useful to consider this simple model because, although it leads to similar positive predictions, it may have different implications for welfare. Assume for example that \(\xi_3 = 0\) and \(\xi_2 > 0.5\). In this case, an election would lead to an uninformative outcome, since \(A\) would always win. The result is different in the public protest game. In this case, as \(n \to \infty\), public protests may lead the policy-maker to take the policy that maximizes his/her payoff in all states: i.e., we may have information aggregation.

**Public signals and other public events.** We have assumed that agents do not observe correlated events, but only conditionally independent signals. In real life, however, it is plausible to assume that agents can also receive a public signal, say \(S = \{\pi, \sigma\}\): \(S\) could be a set of public informative signals (for example, it could represent the speech of a foreign minister); or it could be a set of uninformative events observed by all agents (for example, weather conditions). Extending the analysis to these cases is technically straightforward: since the signal is public, the equilibrium in the subgame after the signal is observed can be characterized as in the analysis presented in the previous sections. An equilibrium can now be described by a pair of functions \(\tau^\ast(s), q^\ast(s)\) for \(s = \pi, \sigma\). The public signal has now two main effects on the equilibrium outcomes. First, even if the signal is completely uninformative, it enlarges the set of equilibrium payoffs since it allows the players to correlate their strategies. Second, if the signal is correlated with the state, it also affects the players’ priors.

**Theory and empirical evidence.** Madestam et al. [2013] use rainy days as an instrument

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\(^{30}\) Note that for \(v_2\) to be arbitrarily large we don’t need to have arbitrarily large payoffs. Since \(v_2 = -v(a)/v(b)\) it is sufficient to have \(v(b)\) close to zero.

\(^{31}\) The assumption of “partisan” citizens who prefer a policy no matter what the state is quite common in the literature on elections, see for example Feddersen and Pesendorfer [1996].
Figure 4: The effect of \( \gamma \) on the effectiveness of public protests.

to study the effect of tax day protests by tea party organizations on a variety of measures of policy outcomes. The idea is that bad weather during a public protest is negatively correlated to participation, but not directly correlated to policy variables. Rainy days are correlated with participation because during rainy days citizens are less likely to be able to participate in the protest. Madestam et al. [2013] indeed find that rain during public protests is negatively correlated with the with the outcomes desired by the tea party. The authors of this work, however, do not have a formal theory of why public protests should affect policy outcomes. Can the informative theory presented above explain this evidence? And what does the theory tell us about the identification strategy?

To model the effect of rain on the equilibrium, let us assume that the expected number of
citizens is \( n \) during sunny days and \( \gamma n \) during rainy days, with \( \gamma < 1 \). The key question is whether the theory predicts, first, that there is positive correlation between participation and rainy days and, second, between rainy days and the effectiveness of public protests.\(^{32}\) The top left panel of Figure 4 represents ex-ante expected participation as a function of \( \gamma \) for a computed example; the top right panel of Figure 4 represents the effectiveness of public protests as a function of different values of \( \gamma \) in the same example.\(^{33}\) Effectiveness is measured by the probability that the policy-maker changes his decision from \( B \) (i.e. the policy that would be chosen without protests) to \( A \). As it can be seen, the theory can easily explain both the link between “rainy days” and participation, and the indirect link between rainy days and effectiveness of public protests.

The theory however suggests that “rainy days” are not necessarily a perfectly valid instrument to test causality in the model. The problem is that rainy days may be directly correlated with the policy-maker’s reaction function, since the way the policy-maker interprets the citizens’ actions is endogenous. This can be seen from the bottom panels of Figure 4 where we represent \( Q_{\gamma} \), the number of active citizens required to convince the policy-maker to chose \( A \), as a function of \( \gamma \). The bottom left panel represents all equilibrium \( Q_{\gamma} \). Because of multiplicity, depending on the equilibrium selection, as \( \gamma \) decreases, the threshold chosen in equilibrium by the policy-maker may increase or decrease. In the bottom right panel we apply a plausible consistent selection of the equilibrium: we select \( \tilde{Q}_{\gamma} \) the equilibrium that maximizes the citizens’ utilities. Still, as it can be seen from the figure, the threshold changes with \( \gamma \). In general, the effect of \( \gamma \) on the effectiveness of the protests is a combination of both its equilibrium effect on the citizens’ strategies and on the effect of on the policy-maker’s strategies.

7 Conclusion

This paper has presented a theory of petitions and public protests to study when they may serve as mechanisms to aggregate information dispersed among citizens. We have shown that when citizens receive sufficiently precise signals and/or the conflict with the policy-maker is sufficiently small, public protests aggregate dispersed information and improve the policy-maker’s decisions. But when these conditions are not satisfied, no information aggregation is possible, even if the number of informed citizens is arbitrarily high. We have characterized the conditions for information aggregation and studied their properties as the number of citizens grows to infinity. For the

\(^{32}\) Note that none of these two “links” is obvious since it is possible, as we change the environment, that both the citizens’ participation cut point \( r_{\alpha}^* \) and the policy maker’s cut point \( q_{\alpha}^* \) change with it.

\(^{33}\) To compute Figure 4 we have assumed \( \mu = 1/2, n = 500 \) and signal distribution \( r(t; \theta) = e^{-\alpha_{\theta}t} / \sum_{j=1}^{T} e^{-A_{\theta}j} \) with \( \alpha_{\theta} = 1 \) and \( A_{\theta} = 1.5 \). Preferences are described by \( v \) and \( V \) such that \( \mu^{**} = 1/(1 + v) = 0.5 \) and \( \mu^* = 1/(1 + V) = 0.6 \).
case in which they are satisfied, we have shown that full information is possible. This means that as population grows to infinity, there is a sequence of equilibria in which the probability of a policy mistake converges to zero. For the case in which they are not satisfied, we have shown that information aggregation may still be possible if social media are available. Our theory, in particular, provides new insights on why social media may enhance the effectiveness of public activism.

There are many different directions in which the ideas presented here might usefully be developed. Perhaps the most important extension would be to fully develop a version of the model with heterogeneous preferences among citizens. As mentioned in Section 6, there are no conceptual issues in pursuing this extension, except for a more complicated model. This extension would allow the model to study how the informative role of public protests is affected by the distribution of preferences among citizens, and would make it easier to bring the theoretical prediction to the data. Numerical simulation based on the simple model with partisans presented in Section 6 suggest that heterogeneity in preferences reduces information aggregation. Addressing heterogeneity would also allow the model to study environments with social groups of different sizes. Another interesting extension would be to allow for the behavioral factors highlighted by Coate and Conlin [2005] and Feddersen and Sandroni [2006], including altruism and ethical motives. These factors may play an important role in determining political participation and may improve the effectiveness of public protests.
8 Appendix

The proofs of the results omitted in this section are in the on-line appendix. The on-line appendix is available for download at http://www.mbattaglini.com/public-protests.

8.1 Proposition 1

We prove here a result that is more general than the result stated in Proposition 1. We say that a policy rule is a weak cut-off rule if there is a threshold $Q$ such that $A$ is chosen if the fraction of protesters over the expected population $Q/n$ is larger than $Q$; $B$ is chosen if the fraction of protesters over the expected population $Q/n$ is smaller than $Q$; and $A$ is chosen with some probability $\mu \in [0, 1]$, if the fraction of protesters over the expected population $Q/n$ is equal to $Q$.

A cut-off rule is a special case of a weak cut-off rule in which $A$ is chosen with probability 1 if the fraction of protesters over the expected population $Q/n$ is greater than or equal to $Q$. We now proceed in two steps. We first prove the existence of a sequence of equilibria in correspondence of which citizens’ strategies are characterized by a cut point $\tau^*_n(Q) \in (\tau_b(Q), \tau_a(Q))$; then we prove that in correspondence to this sequence we have full information aggregation.

**Step 1.** Consider the pivot probability in state $\psi$, $\phi(\psi)$, as defined in (4); and the probability that a citizen is protesting in state $\psi$, $\phi(\psi)$, as defined in Section 2. By (3) and (6) we can represent them without loss of generality as continuous functions of the thresholds $\psi$ and $\rho$ as, respectively, $\psi(\psi, \rho)$ and $\phi(\psi, \rho)$. Fix $Q \in (0, 1)$ and define $\rho = Q \cdot n$ and $\tau_\theta(Q)$ as the solution:

$$\phi(\psi, \tau_\theta(Q)) = Q.$$

It is easy to verify that $\tau_\theta(Q)$ is uniquely defined and $\tau_\theta(Q) \in (0, 1)$; the monotone likelihood ratio property, moreover, implies that $\tau_a(Q) > \tau_b(Q)$. We start form the following useful lemmata.

**Lemma A1.** If the decision is taken by a weak cut-off rule $Q$, then there is an $n_1$ such that for $n > n_1$, $\log \left( \frac{\psi_\theta(k; \tau, \rho)}{\psi_\theta(a; \tau, \rho)} \right) / n$ is a strictly decreasing function of $\tau$ in $\tau \in [\tau_b(Q), \tau_a(Q)]$.

**Proof.** See on-line Appendix. ■

**Lemma A2.** There is an $n_2$ such that a $\tau_n(Q)$ satisfying $\log \left( \frac{\psi_\theta(k; \tau, \rho)}{\psi_\theta(a; \tau, \rho)} \right) / n = 0$ exists for any $n > n_2$. Moreover, $\tau(Q) = \lim_{n \to \infty} \tau_n(Q) \in (\tau_b(Q), \tau_a(Q))$ and

$$\log \left( \frac{\psi_\theta(k; \tau, \rho)}{\psi_\theta(a; \tau, \rho)} \right) / n > 0$$

(resp. $< 0$) if $\tau \in [\tau_b(Q), \tau_n(Q)]$ (resp. $\tau \in (\tau_n(Q), \tau_a(Q)]$).

**Proof.** See on-line Appendix. ■
Define \( r(t) = \frac{r(t; a)}{r(t; b)} \) for \( t = 1, \ldots, T \) and \( r(0) = 0, r(T + 1) = r(T) + 1 \). Define the following correspondence:

\[
\Xi(\tau) = \begin{cases} \\
\left[ r(\tau - 1), r(\tau) \right] & \text{if } \tau \text{ is an integer} \\
r(\lfloor \tau \rfloor) & \text{else} \end{cases}
\]  

(13)

We now show that, for any \( \mathcal{Q} \in (0, 1) \), there is a unique \( \tau_n^*(\mathcal{Q}) \in (\tau_b(\mathcal{Q}), \tau_a(\mathcal{Q})) \) such that:

\[
\frac{1}{v} \left( 1 - \mu \right) \varphi_n(b; \tau_n^*(\mathcal{Q}), q_n) \in \Xi(\tau_n^*(\mathcal{Q})),
\]

(14)

for sufficiently large \( n \). First, note that \( \Xi(\tau) \) is a convex, compact-valued, upper-hemi-continuous correspondence with \( \xi(\tau) = \min \{ x \text{ s.t. } x \in \Xi(\tau) \} > 0 \forall \tau \) and \( \bar{\xi}(\tau) = \max \{ x \text{ s.t. } x \in \Xi(\tau) \} \) bounded above \( \forall \tau \). Second, note that Lemma A2 implies that as \( n \to \infty \), \( \frac{\varphi_n(b; \tau, q_n)}{\varphi_n(a; \tau, q_n)} \to \infty \) for \( \tau \in [\tau_b(\mathcal{Q}), \tau_a(\mathcal{Q})) \) and \( \frac{\varphi_n(b; \tau, q_n)}{\varphi_n(a; \tau, q_n)} \to 0 \) for \( \tau \in (\tau_a(\mathcal{Q}), \tau_a(\mathcal{Q})) \). It follows that there is a \( n^* \) such that for \( n > n^* \),

\[
\frac{1}{v} \left( 1 - \mu \right) \varphi_n(b; \tau_b(\mathcal{Q}), q) > \bar{\xi}(\tau_b(\mathcal{Q})) \quad \text{and} \quad \frac{1}{v} \left( 1 - \mu \right) \varphi_n(b; \tau_a(\mathcal{Q}), q) < \xi(\tau_a(\mathcal{Q})),
\]

This implies that there is a \( \tau_n^*(\mathcal{Q}) \in (\tau_b(\mathcal{Q}), \tau_a(\mathcal{Q})) \) satisfying (14) for \( n > n^* \). Because \( \frac{\varphi_n(b; \tau, q_n)}{\varphi_n(a; \tau, q_n)} \) is strictly decreasing there is a unique point with this property.

We conclude this step by proving that there is a \( n^* \) such that for all \( n > n^* \), \( \tau_n^*(\mathcal{Q}) \) is an equilibrium of the protest game in which the policy maker commits to a response \( \rho_n = \mathcal{Q} \cdot n \).

Assume first that \( \tau_n^*(\mathcal{Q}) \) is an integer. If \( t < \tau_n^*(\mathcal{Q}) \), then we must have:

\[
\frac{r(t; a)}{r(t; b)} \leq \frac{r(\tau_n^*(\mathcal{Q}) - 1; a)}{r(\tau_n^*(\mathcal{Q}) - 1; b)} = \frac{1 - \mu}{v} \frac{\varphi_n(b; \tau_n^*(\mathcal{Q}), q_n)}{\varphi_n(a; \tau_n^*(\mathcal{Q}), q_n)} \leq \frac{1 - \mu}{v} \frac{\varphi_n(b; t, q_n)}{\varphi_n(a; t, q_n)}
\]

(15)

where the second follows from (14), and the third from Lemma A1. Condition (15) implies that

\[
\frac{\mu(a; t)}{\mu(b; t)} \leq \frac{\varphi_n(b; t, q_n)}{\varphi_n(a; t, q_n)}
\]

and so it is optimal to have \( \sigma(t) = 0 \). Similarly if \( t \geq \tau_n^*(\mathcal{Q}) \), then (14) implies it is optimal to have \( \sigma(t) = 1 \). Assume now that \( \tau_n^*(\mathcal{Q}) \) is not an integer, then

\[
\frac{r(\lfloor \tau_n^*(\mathcal{Q}) \rfloor; a)}{r(\lfloor \tau_n^*(\mathcal{Q}) \rfloor; b)} = \frac{1 - \mu}{v} \frac{\varphi_n(b; \tau_n^*(\mathcal{Q}), q_n)}{\varphi_n(a; \tau_n^*(\mathcal{Q}), q_n)}
\]

so type \( \lfloor \tau_n^*(\mathcal{Q}) \rfloor \) is indifferent. If follow that \( \sigma(t) = 0 \) for \( t < \lfloor \tau_n^*(\mathcal{Q}) \rfloor \), \( \sigma(t) = 1 \) for \( t > \tau_n^*(\mathcal{Q}) \) and \( \sigma(t) = (\lfloor \tau_n^*(\mathcal{Q}) \rfloor + 1 - \tau_n^*(\mathcal{Q})) \) for \( t = \lfloor \tau_n^*(\mathcal{Q}) \rfloor \) is an optimal reaction function. We conclude that the strategy described by (6) for \( \tau = \tau_n^*(\mathcal{Q}) \) is an equilibrium.

**Step 2.** We now prove that we have full information aggregation in correspondence to the sequence of equilibria \( \tau_n^*(\mathcal{Q}) \). Let \( \tau^*(\mathcal{Q}) = \lim_{n \to \infty} \tau_n^*(\mathcal{Q}) \) and \( \hat{\tau}(\mathcal{Q}) = \lim_{n \to \infty} \hat{\tau}_n(\mathcal{Q}) \). From the argument above we must have that \( \lim_{n \to \infty} \tau_n^*(\mathcal{Q}) = \hat{\tau}^*(\mathcal{Q}) \in (\tau_b(\mathcal{Q}), \tau_a(\mathcal{Q})) \). It follows that \( \tau^*(\mathcal{Q}) \) and that for \( \varepsilon > 0 \) sufficiently small we can find an \( n^* \) such that
\( \tau_n^*(Q) \in (\tau_0(Q) + \varepsilon, \tau_0(Q) - \varepsilon) \) for all \( n > n^* \). This implies that there is a \( \varepsilon > 0 \) such that \( Q \in (\phi(b; \tau_n^*(Q)) + \varepsilon, \phi(a; \tau_n^*(Q)) - \varepsilon) \) for \( n > n^* \). Let \( \eta_n = \min_{\theta = a, b} |Q - \phi(\theta; \tau_n^*(Q))| \) note that \( \eta = \lim_{n \to \infty} \eta_n > 0 \). The probability of a mistake on the sequence of equilibria can be bounded above as follows:

\[
\mathcal{M}(\tau_n^*(Q)) \leq \sum_{\theta} \Pr\left( \left| \frac{\bar{Q}_n}{n} - \phi(\theta; \tau_n^*(Q)) \right| > \eta_n \right) \leq \frac{\left( \sum_{\theta} \phi(\theta; \tau_n^*(Q))/\eta_n^2 \right)}{n} \tag{16}
\]

where \( \bar{Q}_n/n \) is the realized fraction of protesting citizens. The last inequality in (16) follows from the Chebyshev’s inequality recognizing that the fraction of protesting citizens in state \( \theta \) is a Poisson random variable with mean \( \phi(\theta; \tau_n^*(Q)) \) and standard deviation \( \sqrt{\phi(\theta; \tau_n^*(Q))/n} \). Condition (16) implies that \( \mathcal{M}(\tau_n^*(Q)) \to 0 \) as \( n \to \infty \). \( \blacksquare \)

### 8.2 Proof of Lemma 1

Assume by way of contradiction that an informative equilibrium exists and \( V < V_1(v) \). Define \( Q^* = \min_{Q \geq 0} \{ Q \text{ s.t. } \Gamma_n(a; Q, \sigma^*) \geq \mu^* \} \). In correspondence to an informative equilibrium, assuming its existence, it must be that \( Q^* \) is finite for any \( (finite) \) \( n \). By definition of \( \mu^* \), we must have:

\[
\Gamma_n(a; Q^*, \sigma^*)V(a) + (1 - \Gamma_n(a; Q^*, \sigma^*))V(b) \geq 0. \tag{17}
\]

By Bayes’ rule, we can rewrite (17) as:

\[
\frac{P(Q^*, n\phi(a; \sigma^*))V(a) + (1 - \mu)P(Q^*, n\phi(b; \sigma^*))V(b)}{P(Q^*, n\phi(a; \sigma^*))} \geq \frac{1}{V} \left( \frac{1}{\mu} - 1 \right),
\]

or:

\[
\frac{P(Q^*, n\phi(a; \sigma^*))V(a) + (1 - \mu)P(Q^*, n\phi(b; \sigma^*))V(b)}{P(Q^*, n\phi(b; \sigma^*))} \geq \frac{1}{V} \left( \frac{1}{\mu} - 1 \right), \tag{18}
\]

For any informative equilibrium, moreover, we need that type \( t = 1 \) is willing to stay inactive, otherwise all types would be active and no information would be revealed by the citizens’ actions. This requires:

\[
\frac{1}{v} \left( \frac{1}{\mu(a; 1)} - 1 \right) \geq \frac{\varphi_n(a; \sigma^*, \rho^*)}{\varphi_n(b; \sigma^*, \rho^*)}, \tag{19}
\]

for any \( n \). Observe that we can write:

\[
\frac{\varphi_n(a; \sigma^*, \rho^*)}{\varphi_n(b; \sigma^*, \rho^*)} = \frac{\rho(Q^*) \cdot P(Q^* - 1, n\phi(a; \sigma^*)) + (1 - \rho(Q^*)) \cdot P(Q^*, n\phi(a; \sigma^*))}{\rho(Q^*) \cdot P(Q^* - 1, n\phi(b; \sigma^*)) + (1 - \rho(Q^*)) \cdot P(Q^*, n\phi(b; \sigma^*))} = \frac{e^{-n\phi(a; \sigma^*)} (n\phi(a; \sigma^*))^{Q^*}/(Q^!)!}{e^{-n\phi(b; \sigma^*)} (n\phi(b; \sigma^*))^{Q^*}/(Q^!)!} \cdot \frac{(1 - \rho(Q^*)) + \rho(Q^*) \cdot \frac{Q^*}{n\phi(a; \sigma^*)}}{(1 - \rho(Q^*)) + \rho(Q^*) \cdot \frac{Q^*}{n\phi(b; \sigma^*)}}
\]

The last inequality follows from the fact that \( \frac{Q^*}{n\phi(a; \sigma^*)} \) is non increasing in \( \rho(Q^*) \). The following lemma is useful to complete the argument:
Lemma A3. For any pair of strategies $\sigma, \rho$, we have: \( \frac{\phi(b; \sigma, \rho)}{\phi(a; \sigma, \rho)} \geq \frac{r(T; b)}{r(T; a)} \).

Proof. See on-line Appendix. 

From Lemma A3 and (20) we have:

\[
\frac{\varphi_n(a; \sigma^*, \rho^*)}{\varphi_n(b; \sigma^*, \rho^*)} = \frac{e^{-n\phi(a; \sigma^*)}(n\phi(a; \sigma^*))^{Q^*}}{e^{-n\phi(b; \sigma^*)}(n\phi(b; \sigma^*))^{Q^*}} \frac{r(T; b)}{r(T; a)}
\]

\[
= \frac{e^{-n\phi(a; \sigma^*)}(n\phi(a; \sigma^*))^{Q^*}}{e^{-n\phi(b; \sigma^*)}(n\phi(b; \sigma^*))^{Q^*}} \frac{\mu(b; T)\mu}{\mu(a; T)(1-\mu)} = \left(1 - \frac{1}{\mu(a; T)} - 1 \right) \frac{P(Q^*, n\phi(a; \sigma^*))}{P(Q^*, n\phi(b; \sigma^*))}
\]

We conclude from (22) that: \( \frac{P(Q^*, n\phi(a; \sigma^*))}{P(Q^*, n\phi(b; \sigma^*))} \leq \frac{\varphi_n(a; \sigma^*, \rho^*)}{\varphi_n(b; \sigma^*, \rho^*)} \frac{1}{\mu(a; T) - 1} \). Combining this inequality with (18) and (19), we have:

\[
\frac{1}{v} \left( \frac{1}{\mu(b; T) - 1} \right) \geq \frac{\varphi_n(a; \sigma^*, \rho^*)}{\varphi_n(b; \sigma^*, \rho^*)} \geq \frac{1}{\mu(a; T) - 1} \frac{P(Q^*, n\phi(a; \sigma^*))}{P(Q^*, n\phi(b; \sigma^*))} \geq \frac{1}{\mu(a; T) - 1} \frac{1}{V} \left( \frac{1}{\mu - 1} \right) = \frac{1}{V} \left( \frac{1}{\mu(a; T) - 1} \right).
\]

This implies that $V \geq V_1(v)$, a contradiction. 

8.3 Lemma 2

We proceed in two steps. In Step 1 we consider a modified game in which we force the lowest type to be inactive and the highest type to be active. We prove that an informative equilibrium exists in this modified game. In Step 2 we prove that if $V \geq V_2(v)$, then any equilibrium of the modified game is also an equilibrium of the original game. Recall that strategies $\sigma, \rho$ can be represented by two thresholds $\tau, q$ with $\tau \in [1, T+1]$ and $q \in [0, \infty)$. In the rest of this section, we will represent the policy-maker’s posterior $\Gamma_n(\theta; Q, \sigma)$ and the pivot probabilities $\varphi_n(\theta; \sigma, \rho)$ as, respectively, $\Gamma_n(\theta; Q, \tau)$ and $\varphi_n(\theta; \tau, q)$.

Step 1. Restrict the strategy space imposing $\tau \in [2, T]$. Let $Q(\tau) = \max_Q \{ Q \text{ s.t. } \Gamma_n(a; Q, \tau) \leq \mu^* \}$ and $\hat{Q} = \max_{\tau \in [2, T]} Q(\tau)$. It is easy to see $\hat{Q} < \infty$. Restrict the set of strategies for the policy-maker to $q \in [0, \hat{Q} + 2]$. We now have a modified game in which $\tau \in [1, T]$ and $q \in [0, \hat{Q} + 2]$.

Given a strategy $\tau, q$, define $t(\tau, q)$ as follows: $t(\tau, q) = 1$ if $\frac{\mu(a; t)}{\mu(b; t)} > \frac{\varphi_n(b; \tau, q)}{\varphi_n(a; \tau, q)} \text{ for all } t \geq 2$ and $t(\tau, q) = \max \left\{ t \in \{2, \ldots, T\} \text{ s.t. } \frac{\mu(a; t)}{\mu(b; t)} \leq \frac{\varphi_n(b; \tau, q)}{\varphi_n(a; \tau, q)} \right\}$ otherwise. Using the notation introduced in Section 2, a citizen’s strategy described by $\tau$ is optimal, given the other players’ strategies $\tau, q$, if and only if $\tau \in R_1(\tau, q)$ where $R_1(\tau, q)$ is
Let the function given

We now prove that the equilibrium of the restricted game \((\tau^*, q^*)\) is optimal for the planner given \((\tau, q)\) if and only if

Similarly, define \(Q(\tau)\) as \(Q(\tau) = -1\) if \(\Gamma_n(a; Q, \tau) > \mu^*\) for all \(Q \in \{0, ..., \hat{Q} + 2\}\) and

otherwise. The policy-maker’s strategy described by \(q\) is optimal, given the other players’ strategies \(\tau, q\), if and only if \(q \in R_2(\tau)\) where \(R_2(\tau)\) is defined as:

Let \(X = [2, T] \times [0, \hat{Q} + 2]\). Define \(R : X \rightarrow X\) as \(R = R_1(\tau, q) \times R_2(\tau)\). We have:

\[ R \text{ has a closed graph.} \]

\[ \text{Proof. See on-line Appendix.} \]

It is easy to verify that, in addition to being closed valued, \(R\) is non empty and convex valued. The Kakutani fixed point theorem implies that there is a fixed point \((\tau^*, q^*)\) in \(R(\tau^*, q^*)\): this fixed point is an equilibrium of the modified game.

**Step 2.** We now prove that the equilibrium of the restricted game \((\tau^*, q^*)\) is an equilibrium of the full game if \(V \geq V_2(v)\). By definition of \(\hat{Q}\), \(q^* < \hat{Q} + 1\). Given this, the strategy described by \(q^*\) is optimal for the planner given \((\tau^*, q^*)\). To show that the strategy described by \(\tau^*\) is optimal, we proceed in three steps. Assume first that \(\tau^* \in (1, T)\). In this case, by construction types \(t < t(\tau^*, q^*)\) and type \(t = t(\tau^*, q^*)\) if \(\mu(a; t(\tau^*, q^*)) < 1\) find it optimal to abstain; type \(t = t(\tau^*, q^*)\) if \(\mu(a; t(\tau^*, q^*)) \geq 1\) is indifferent; and types \(t > t(\tau^*, q^*)\) find it optimal to be active: this is exactly the action prescribed by \(\tau^*\). It follows that \(\tau^*\) is an optimal reaction function given \((\tau^*, q^*)\). We conclude that \((\tau^*, q^*)\) is a Nash equilibrium of the full game.

Assume now that \(\tau^* = 2\), to prove that \((\tau^*, q^*)\) is an equilibrium we only need to prove that the lowest type finds it optimal to abstain, i.e. \(\sigma(1) = 0\) is optimal. For type 1 voters it is optimal to stay inactive for type 1 if:

\[ \frac{1}{v} \left( \frac{1}{\mu(a; 1)} - 1 \right) \leq \frac{\varphi_n(a; \tau^*, q^*)}{\varphi_n(b; \tau^*, q^*)} \]

To verify this inequality, note that:

\[ \frac{\varphi_n(a; \tau^*, q^*)}{\varphi_n(b; \tau^*, q^*)} = \frac{\rho(Q(\tau^*)) \Pr(Q(\tau^*) - 1; a) + (1 - \rho(Q(\tau^*))) \Pr(Q(\tau^*); a)}{\rho(Q(\tau^*)) \Pr(Q(\tau^*) - 1; b) + (1 - \rho(Q(\tau^*))) \Pr(Q(\tau^*); b)} \]
\[
\frac{P(Q(\tau^*), n\phi(a; \tau^*))}{P(Q(\tau^*), n\phi(b; \tau^*))} \leq \frac{1}{V} \left( \frac{1}{\mu(a; 1)} - 1 \right) \geq \frac{1}{V} \left( \frac{1}{\mu} - 1 \right) \geq \frac{\varphi_n(a; \tau^*, q^*)}{\varphi_n(b; \tau^*, q^*)}
\]

The last inequality in (24) follows from the fact that, by construction of the strategies and the monotone likelihood ratio property assumption, \( \phi(a; \tau^*) \geq \phi(b; \tau^*) \). By definition of \( Q(\tau^*) \), we must have:

\[
\frac{P(Q(\tau^*), n\phi(a; \tau^*))}{P(Q(\tau^*), n\phi(b; \tau^*))} \leq \frac{1}{V} \left( \frac{1}{\mu} - 1 \right)
\]

Conditions (24) and (25), together with \( V \geq V_2(v) \), then imply:

\[
\frac{1}{v} \left( \frac{1}{\mu(a; 1)} - 1 \right) \geq \frac{1}{V} \left( \frac{1}{\mu} - 1 \right) \geq \frac{\varphi_n(a; \tau^*, q^*)}{\varphi_n(b; \tau^*, q^*)}
\]

and so (23) is satisfied.

Assume now that \( \tau^* = T \), to prove that \( (\tau^*, q^*) \) is an equilibrium we only need to prove that the highest type finds it optimal to be active, i.e. \( \sigma(T) = 1 \) is optimal. Define \( \tilde{Q}(\tau^*) = \min \{ Q \in \{0, \ldots, \tilde{Q} + 2\} \text{ a.t. } \Gamma_n(a; Q, \tau^*) \geq \mu^* \} \). Naturally \( \tilde{Q}(\tau^*) \leq \tilde{Q} + 2 \) and, since \( \mu < \mu^* \), \( \tilde{Q}(\tau^*) > 0 \). Consider the problem faced by a voter of type \( T \). It is optimal to stay active for type \( T \) if:

\[
\frac{1}{v} \left( \frac{1}{\mu(a; T)} - 1 \right) \leq \frac{\varphi_n(a; \tau^*, q^*)}{\varphi_n(b; \tau^*, q^*)}
\]

(26)

Using the same steps as in (22) we can show that:

\[
\frac{\varphi_n(a; \tau^*, q^*)}{\varphi_n(b; \tau^*, q^*)} \geq \left[ \left( \frac{1}{\mu(a; T)} - 1 \right) / \left( \frac{1}{\mu} - 1 \right) \right] \cdot \frac{P(\tilde{Q}(\tau^*), n\phi(a; \tau^*))}{P(\tilde{Q}(\tau^*), n\phi(b; \tau^*))}
\]

(27)

We conclude that:

\[
\frac{1}{v} \leq \frac{1}{V} \leq \frac{P(\tilde{Q}(\tau^*), n\phi(a; \tau^*))}{P(\tilde{Q}(\tau^*), n\phi(b; \tau^*))} \leq \frac{1}{\mu(a; T)} \leq \frac{\varphi_n(a; \tau^*, q^*)}{\varphi_n(b; \tau^*, q^*)} \cdot \left( \frac{1}{\mu(a; T)} - 1 \right)
\]

This implies (26). We conclude that the equilibrium of the modified game is an equilibrium of the original game.

### 8.4 Proof of Proposition 3

We proceed in two steps. In Step 1 we prove that if information aggregation is possible in a large society when the policy-maker’s preference parameter is \( V \), then information aggregation is possible in a large society if the preference parameter is \( V' \geq V \) as well. In Step 2 we prove that Step 1 plus Lemmata 1 and 2 implies the result.
Step 1. Assume that information aggregation is possible in a large society when the policy-maker’s preference parameter is $V$. Then there is a $n_1$ such that for any $n > n_1$ there is an informative equilibrium $\tau_n^*, q_n^*$ with $\phi(a; \tau_n^*) > \phi(b; \tau_n^*)$, where $\phi(\theta; \tau_n^*)$ is the expected probability that a random citizen chooses to protest given strategy $\tau_n^*$ in state $\theta$. Let $Q^* = \lim_{n \to \infty} q_n^*/n$.

We now show that there is a $n_2 \geq n_1$ such that there is an informative equilibrium $\tau_n^{**}, q_n^{**}$ with $\phi(a; \tau_n^{**}) > \phi(b; \tau_n^{**})$ for $n > n_2$ when the policy-maker’s preference parameter is $V' \geq V$. To this end, consider a modified game in which $q_n$ is forced to be in $[0, q_n^*]$. Following the proof of Lemma 2 it is easy to show that this modified game has an equilibrium $\tilde{\tau}_n, \tilde{q}_n$ for any $n$. Moreover it can be easily verified that in no equilibrium of the modified game we can have $\tilde{q}_n = 0$. We now prove that any equilibrium of the modified game is also an informative equilibrium of the original game when the policy-maker’s preference parameter is $V' \geq V$.

Assume first that there is a $n_2$ such that for any $n > n_2$, $\tilde{q}_n < q_n^*$. This implies that $\tilde{\tau}_n, \tilde{q}_n$ is an equilibrium of the original game since $\tilde{q}_n < q_n^*$ implies that it is optimal for the planner to choose $A$ with probability ($[q_n^*] - q_n^*$) if $Q = [q_n^*]$ and with probability one if $Q > [q_n^*]$. So the restriction on the modified game is irrelevant and an equilibrium of the modified game is an equilibrium of the original game as well.

Assume now that for any $n_2$ there is a $n > n_2$ such that $\tilde{\tau}_n, q_n^*$ is an equilibrium of the modified game. We can therefore find a sequence of equilibria $\tilde{\tau}_n, q_n^*$ for $n \to \infty$. We have:

**Lemma A5.** If $\tilde{\tau}_n, q_n^*$ is a sequence of equilibria of the modified game, then $\lim_{n \to \infty} \tilde{\tau}_n = \lim_{n \to \infty} \tau_n^*$.

**Proof.** See on-line Appendix. ■

Since $\tau_n^*, q_n^*$ is an equilibrium of the original game, we must have:

$$
\frac{P(q_n^*, n\phi(a; \sigma_n^*))}{P(q_n^*, n\phi(b; \sigma_n^*))} \geq \frac{1}{V} \left(1 - \frac{1}{\mu}\right) \geq \frac{P([q_n^*] - 1, n\phi(a; \sigma_n^*))}{P([q_n^*] - 1, n\phi(b; \sigma_n^*))}
$$

(28)

This implies that there must be a $n'$ such that for $n > n'$:

$$
\frac{P(q_n^*, n\phi(a; \sigma_n^*))}{P(q_n^*, n\phi(b; \sigma_n^*))} \geq \frac{1}{V} \left(1 - \frac{1}{\mu}\right) \geq \frac{P([q_n^*] - 1, n\phi(a; \sigma_n^*))}{P([q_n^*] - 1, n\phi(b; \sigma_n^*))}
$$

(29)

The first inequality follows from the fact that $\frac{P([q_n^*] - 1, n\phi(a; \sigma_n^*))}{P([q_n^*] - 1, n\phi(b; \sigma_n^*))} \to 0$, (7) and $V' > V$.

The second from the fact that if $\frac{1}{V} \left(1 - \frac{1}{\mu}\right) < \frac{P([q_n^*] - 1, n\phi(a; \sigma_n^*))}{P([q_n^*] - 1, n\phi(b; \sigma_n^*))}$, then $q_n^*$ would not be optimal for the policy-maker on the sequence of modified equilibria: so $\tilde{\tau}_n, q_n^*$ would not be an equilibrium of the modified game, a contradiction. The inequalities in (29) imply that $q_n^*$ is an optimal reaction function for the policy-maker even in the original game for any $n$ such that $\tilde{\tau}_n, q_n^*$ is an equilibrium of the modified game.
We conclude that for any sequence of equilibria \( \tilde{\tau}_n, \tilde{q}_n \) of the modified game, it is a sequence of equilibria of the original game for \( n \) sufficiently large. Since \( \tilde{q}_n \in (0, q_n^*] \) we must have \( \tilde{\tau}_n \in (1, T + 1) \) for any \( n \), implying that \( \phi(a; \tilde{\tau}_n) > \phi(b; \tilde{\tau}_n) \) for any \( n \) sufficiently large.

**Step 2.** Define \( V^* \) as the infimum of the set of \( V \)s such that an informative equilibrium exists in a large society. By Lemma A1 and A2 we must have that \( V^* \in \{V_1(v), V_2(v)\} \). By definition of \( V^* \) if \( V < V^* \), then public protests cannot be informative in a large society. Assume \( V > V^* \). Then by the definition of \( V^* \) there is a \( V' \in (V^*, V) \) such that public protests are informative in a large society when the policy-maker’s preference parameter is \( V' \). Since \( V > V' \), Step 1 implies that we have an informative equilibrium in a large election when the policy-maker’s preference parameter is \( V' \). \( \blacksquare \)

### 8.5 Proposition 4

We consider two cases.

**Case 1.** Assume first that \( V \geq V_2(v) \). We first prove that we must have a sequence of equilibria \( \tilde{\tau}_n, \tilde{q}_n \) such that \( \tilde{Q} \in (\phi(b; \tilde{\tau}), \phi(a; \tilde{\tau})) \), where \( \tilde{\tau} = \lim_{n \to \infty} \tilde{\tau}_n \) and \( \tilde{Q} = \lim_{n \to \infty} \tilde{q}_n/n \). In Lemma 2 we have constructed a sequence of equilibria \( \tilde{\tau}_n, \tilde{q}_n \) such that \( \tilde{\tau}_n \in [2, T] \). It follows that \( \tilde{\tau} = \lim_{n \to \infty} \tilde{\tau}_n \in [2, T] \). This fact and the monotone likelihood ratio imply that \( \phi(a; \tilde{\tau}) > \phi(b; \tilde{\tau}) > 0 \).

Note that we can write:

\[
\varphi_n(\theta; \tau_n, q_n) = \frac{e^{-n\phi(\theta; \tau_n)}(n\phi(\theta; \tau_n))^\tilde{Q}_n}{\tilde{Q}_n!} \cdot \beta_n \frac{\tilde{Q}_n}{n\phi(\theta; \tau_n)} + (1 - \beta_n) \\
= \frac{e^{-n\phi(\theta; \tau_n)}(n\phi(\theta; \tau_n))^\tilde{Q}_n}{\tilde{Q}_n(n/e)^\tilde{Q}_n} \cdot \frac{\beta_n \tilde{Q}_n}{n\phi(\theta; \tau_n)} + (1 - \beta_n) \\
= \frac{e^{n\phi(\theta; \tau_n)\Psi(\tilde{Q}_n/n)}}{\tilde{Q}_n} \cdot \frac{\beta_n \tilde{Q}_n/n}{\phi(\theta; \tau_n)} + (1 - \beta_n)
\]

where \( \tilde{Q}_n = [\tilde{q}_n], \Psi(x) = x(1 - \log x) - 1 \) and \( \iota(Q) = Q!/(Q/e)^Q \sqrt{2\pi Q + \pi/3} \). Taking the log of the pivot probabilities in the two states and the limit as \( n \to \infty \), we have:

\[
\lim_{n \to \infty} \log \frac{\varphi_n(a; \tau, n \cdot Q)}{\varphi_n(b; \tau, n \cdot Q)} / n = \Omega(Q)
\]

(30)

where \( \Omega(Q) = \phi(a; \tau) \Psi(Q/\phi(a; \tau)) - \phi(b; \tau) \Psi(Q/\phi(b; \tau)) \).\(^{34}\) Note that \( \Omega(Q) \) is increasing in \( Q \) and:

\[
\Omega(\phi(a; \tau)) = -\phi(b; \tau) \Psi(\phi(a; \tau)/\phi(b; \tau)) > 0 \\
\Omega(\phi(b; \tau)) = \phi(a; \tau) \Psi(\phi(b; \tau)/\phi(a; \tau)) < 0.
\]

\(^{34}\) Details of this derivation are presented in Lemma A1 in the on-line Appendix.
This implies that if \( \bar{Q} = \lim_{n \to \infty} \bar{q}_n/n \leq \phi(b; \hat{\tau}) \), then \( \varphi_n(a; \tau, \bar{Q}_n)/\varphi_n(b; \tau, \bar{Q}_n) \) converges to zero, and so \( \Gamma_n(a; [\bar{q}_n], \sigma^*) < \mu^*, \) a contradiction: we conclude that we must have \( \bar{Q} > \phi(b; \hat{\tau}) \). Similarly, we can prove that we must have \( \bar{Q} < \phi(a; \hat{\tau}) \). We conclude that \( \bar{Q} \in (\phi(b; \hat{\tau}), \phi(a; \hat{\tau})) \).

Given this, the proof that the probability of a mistake converges to zero on the sequence of equilibria \( \hat{\tau}_n, \bar{q}_n \) as \( n \to \infty \) follows the same argument as in Step 2 of Proposition 1.

**Case 2.** Assume now that \( V \in (V^*(v), V_2(v)) \). If \( \hat{\tau}_n \to \hat{\tau}_\infty \in (1, T+1) \), then again we have \( \bar{Q} \in (\phi(b; \hat{\tau}), \phi(a; \hat{\tau})) \); and the result is proven as in Step 2 of Proposition 1. To complete the proof, we therefore only need to prove that we can neither have \( \hat{\tau}_n \to 1 \) nor \( \hat{\tau}_n \to T+1 \).

**Step 2.1.** Assume first that \( \hat{\tau}_n \to T+1 \), then it must be that \( \hat{\tau}_n > T \) for \( n \) sufficiently large, implying that citizens of type \( T \) must be indifferent between \( B \) and \( A \). First note that \( V > V^*(v) \) implies \( V > V_1(v) \), so:

\[
\frac{1}{v} \left( \frac{1}{\mu(a; 1)} - 1 \right) > \frac{1}{V} \left( \frac{1}{\mu(a; T)} - 1 \right) \geq \frac{1}{V} \left( \frac{1}{\mu} - 1 \right) \tag{31}
\]

where the first inequality follows from \( V > V_1(v) \) and the second from \( \mu \geq \mu(a; T) \). It follows that, for sufficiently large \( n \), we must have:

\[
\frac{1}{V} \left( \frac{1}{\mu} - 1 \right) < \frac{1}{v} \left( \frac{1}{\mu(a; 1)} - 1 \right) = \frac{\varphi_n(a; \hat{\tau}_n, \bar{q}_n)}{\varphi_n(b; \hat{\tau}_n, \bar{q}_n)} \cdot \frac{1/\mu(a; 1) - 1}{1/\mu(a; T) - 1} < \frac{P([\bar{q}_n] - 1, n\phi(a; \tilde{\tau}_n))}{P([\bar{q}_n] - 1, n\phi(b; \tilde{\tau}_n))}
\]

where the first inequality follows from (31); the equality follows from the fact that type \( T \) is indifferent so

\[
\frac{1}{v} = \frac{\varphi_n(a; \hat{\tau}_n, \bar{q}_n)}{\varphi_n(b; \hat{\tau}_n, \bar{q}_n)} \left( \frac{1}{\mu(a; T) - 1} \right);
\]

the last inequality follows from the fact that \( \mu(a; 1) > \mu(a; T) \) and the fact that, since \( \hat{\tau}_n \to T+1 \), then \( \phi(b; \hat{\tau}_n) \to 1, \phi(a; \hat{\tau}_n) \to 1 \), implying \( \frac{\varphi_n(a; \tilde{\tau}_n, \bar{q}_n)}{\varphi_n(b; \tilde{\tau}_n, \bar{q}_n)} \to \frac{P([\hat{q}_n] - 1, n\phi(a; \hat{\tau}_n))}{P([\hat{q}_n] - 1, n\phi(b; \hat{\tau}_n))} \to 0 \) as \( n \to \infty \). We conclude that:

\[
\frac{1}{V} \left( \frac{1}{\mu} - 1 \right) < \frac{P([\bar{q}_n] - 1, n\phi(a; \hat{\tau}_n))}{P([\bar{q}_n] - 1, n\phi(b; \hat{\tau}_n))}
\]

This implies that \( \Gamma_n(a; [\bar{q}_n] - 1, \hat{\tau}_n) > \mu^* \), a contradiction.

**Step 2.2.** Assume now that \( \hat{\tau}_n \to 1 \). It must be that \( \hat{\tau}_n < 2 \) for \( n \) sufficiently large, implying that citizens of type 1 must be indifferent between \( B \) and \( A \). This implies that there is a \( \varepsilon > 0 \) such that:

\[
\frac{1}{V} \left( \frac{1}{\mu} - 1 \right) - \varepsilon > \frac{1}{v} \left( \frac{1}{\mu(a; 1)} - 1 \right) = \frac{\varphi_n(a; \tilde{\tau}_n, \bar{q}_n)}{\varphi_n(b; \tilde{\tau}_n, \bar{q}_n)} \geq \frac{P([\bar{q}_n] - 1, n\phi(a; \hat{\tau}_n))}{P([\bar{q}_n] - 1, n\phi(b; \hat{\tau}_n))} - \varepsilon
\]

for \( n \) sufficiently large. The first inequality follows from the fact that \( V < V_2(v) \); the equality follows from the fact that type 1 is indifferent; the last inequality follows from the fact that \( \hat{\tau}_n \to 1 \),
implying that \( \phi(b; \hat{\tau}_n) / \phi(a; \hat{\tau}_n) \rightarrow 1 \). This, in turn, implies that
\[
\frac{\varphi_n(a; \hat{\tau}_n, \hat{\varphi}_n) - P(\hat{q}_n, n \phi(a; \hat{\tau}_n))}{P(\hat{q}_n, n \phi(b; \hat{\tau}_n))} \rightarrow 0
\]
as \( n \rightarrow \infty \). We conclude that:
\[
\frac{1}{\mu^*} \left( \frac{1}{\mu} - 1 \right) > \frac{P(\hat{q}_n, n \phi(a; \hat{\tau}_n))}{P(\hat{q}_n, n \phi(b; \hat{\tau}_n))},
\]
implying that \( \Gamma_n(a; [\hat{q}_n], \sigma) < \mu^* \), a contradiction. ■

8.6 Proposition 5

See on-line Appendix. ■

8.7 Proof of Proposition 6

See on-line Appendix. ■

8.8 Proof of Proposition 7

See on-line Appendix. ■
References


