

Distortions and the Structure of the World Economy*

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Abstract

We model the world economy as one system of endogenous input-output relationships subject to distortions and study how the world's input-output structure and world's GDP change due to changes in distortions. We derive a sufficient statistic to identify distortions from the observed world input-output matrix, which we fully match for the year 2011. Our main empirical result is to determine how changes in internal distortions (affecting transactions across sectors within countries) impact the whole structure of the world's economy and show that they have a much larger effect on world's GDP than external distortions (affecting transactions across countries).

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1 Introduction

Wassily Leontief in his Nobel Lecture (Leontief 1974) argued: “The world economy, like the economy of a single country, can be visualized as a system of interdependent processes.” In this paper, we develop a model of the world economy as input-output relationships subject to distortions. We study the input-output relationships where the base unit is a country-sector pair. Rather than considering input-output relationships within countries and trade relationships across countries as is common in the literature, we model the world economy as one world input-output matrix. As a result we can study in a unified framework how the global production network is determined and affected by changes in distortions and TFPs across the world.

We start from the premise that the world input-output matrix is endogenous and that it responds to changes in prices, distortions, and TFPs. The endogenous production structure we model allows us to derive and compute the elasticity of the input-output structure and global GDP to the changes in distortions, and study their aggregate and distributional effects around the world. In addition, we show how our model fully matches the actual world input-output structure, which is crucial for quantitative analysis. Finally, by taking this global view of the world, we derive sufficient statistics to identify distortions and separate them from TFP. We show that with data on final and intermediate goods expenditures we can identify the evolution of distortions over time and, together with our framework, analyze how actual changes have shaped global production.

Our first analytical result is to derive the elasticity of the input-output expenditure shares to changes in distortions across countries and sectors. We show that this elasticity depends on the whole input-output structure. After that we match the world input-output table from the World Input-Output Database (WIOD, Timmer et al., 2015) and study how changes in internal distortions (affecting transactions across sectors within countries) influence the endogenous production structure of individual countries. As an illustrative example, for the United States’ input-output table we compute how its own input-output structure changes with respect to changes in its own internal distortions. We show that the quantitative effects follow the analytical results. We then compute the effects on the U.S. input-output structure from changes in internal distortions in China – the cross-elasticity of the U.S. economy structure to Chinese distortions.

We then take a global view and study the importance of the internal distortions for the world’s economy.¹ We do so by ranking the top 60 elasticities of the world’s GDP to

¹Our approach relates to Atkeson and Burstein (2017) who develop a sufficient statistic for the country aggregate effects of changes to innovation policy.

the distortions of a sector in a given country. The largest such internal elasticity - that of China's construction sector - has the same size as the elasticity of the world's GDP to *all* of the external distortions (affecting transactions across countries) in the United States. More generally, after computing the elasticity of world's GDP with respect to all external distortions we find that the elasticities of internal distortions are an order of magnitude larger than the elasticity of external distortions.²

Finally, we use our model together with the world input-output data to identify distortions. We propose a methodology to decompose distortions from sectoral TFPs. Specifically, we derive simple closed-form sufficient statistics for distortions as the functions of directly observable sectoral input expenditure shares and the final goods consumption shares. In doing so, we discuss the assumptions under which we can identify changes in distortions, and their levels.³

With our derived sufficient statistics and the input-output data from WIOD over the period 1995-2011 we compute more than half a million internal distortions over this period. Our main finding is that there is a significant heterogeneity in the growth rate of the distortions and their distribution across countries. Therefore, aggregate measures of relative distortions across countries can give only a partial view of the degree of distortions across countries, since the country level distortions mask the high heterogeneity at the more disaggregate level.⁴

We now briefly discuss the connection to the literature. Our model of the world input-output matrix is related to a recent work by Antras and de Gortari (2017), who develop a structural model that also matches the input-output structure of the world economy. While their model with multiple stages of production is aimed at studying the formation of global supply chains, our focus is on identifying internal distortions and studying their effects on the world's input-output structure. Our paper is also closely related to the macroeconomic literature that emphasizes domestic distortions (misallocation) across sectors and firms, (e.g. Restuccia and Rogerson 2008, 2013; Hsieh and Klenow 2009).⁵ This strand of the literature has encountered difficulties with regard to separating distortions from TFPs as argued in Jones (2011, 2013).⁶ We solve this identification problem by considering a model with the

²The closest in terms of this result is Tombe and Zhu (2015). They show that for China, the internal distortions play a much larger role for aggregate productivity and output than changes in trade costs.

³We also discuss that our methodology applies more broadly if one is interested in identifying the changes in the distortions and TFPs – the model then can have a number of additional dimensions of heterogeneity.

⁴Adamopoulos, Brandt, Leight, and Restuccia (2017) and Brandt, Kambourov, and Storesletten (2016) are the recent analyses of the evolution of distortions in China at the firm and sectoral level.

⁵Bartelme and Gorodnichenko (2015), Boehm (2015), Fadinger, Ghiglino, and Teteryatnikova (2015) develop various empirical proxies for distortions in an input-output setting.

⁶By using the United States as a benchmark undistorted country, Jones (2011, 2013) identify distortions as Hsieh and Klenow (2009). This, as he acknowledges, assumes that the U.S. same input-output structure

CES production for intermediate goods and the CES consumption structure rather than the Cobb-Douglas form considered in Jones (2011, 2013), Acemoglu, Carvalho, Ozdaglar, Tahbaz-Salehi (2012), and Caliendo and Parro (2015). This CES structure is precisely what defines an endogenous input-output structure of the economy that allows us to identify the distortions. Finally, our paper complements recent studies that show how local productivity shocks or distortions spread within a country (see e.g. Caliendo, Parro, Rossi-Hansberg, and Sarte 2017, Fajgelbaum, Morales, Serrato, and Zidar 2015) and to the literature on aggregate consequences of idiosyncratic shocks that emphasizes the departure from the Cobb-Douglas assumptions in the context of closed-economy models (Atalay 2017, Baqaee and Farhi 2017, and Carvalho, Nirei, Saito, and Tahbaz-Salehi 2017).⁷

The paper is organized as follows. In Section 2 we present our model of the input-output structure of the world economy. In Section 3 we study the changes in the input-output structure and world's GDP to changes in distortions. Section 4 derives the sufficient statistics to identify distortions and use data from an world input output matrix to measure them. Finally, Section 5 concludes. We relegate to an appendix all the mathematical derivations, and the exposition of additional results.

2 A Model of the World as Input-Output Relationships

The world economy consists of N countries (indexed by i, n). Each country has J sectors (indexed by j, k). Each country is endowed with one unit of equipped labor and the unit mass of agents.⁸ Goods from country i and sector j , Q_{ij} , are produced with a Cobb-Douglas production function:

$$Q_{ij} = A_{ij} L_{ij}^{\beta_{ij}} M_{ij}^{(1-\beta_{ij})},$$

where A_{ij} is the TFP, L_{ij} is amount of labor allocated to sector j , M_{ij} is the amount of materials used by the sector j , and $\beta_{ij} \in [0, 1]$ is the share of value added in gross output. Generically, goods Q_{ij} are used for final consumption as well as for the the production of materials, as we describe next.

Materials in country i and sector j , M_{ij} , are a CES aggregate of intermediate goods

applies to other countries. Tombe and Zhu (2015) identify and evaluate importance of both internal and external distortions in a model without the input-output structure. Bigio and La'O (2016) study labor and efficiency wedges in an input-output framework and exploit the constancy of the technological parameters over the business cycle to identify distortions.

⁷Our result also relates to Atkeson and Burstein (2010), who argue that only the microeconomic evidence on the firm responses are not sufficient to estimate the aggregate welfare effects of the changes in trade costs.

⁸The method we develop below to characterize distortions and TFPs does not depend on the size of population in each country. When computing the general equilibrium effects from changes in distortions, we account for the difference in population across countries.

from all sectors and locations, namely

$$M_{ij} = \left(\sum_{n,k} \iota_{nk} Q_{ij,nk}^{\frac{\theta}{1+\theta}} \right)^{\frac{1+\theta}{\theta}},$$

where $n = 1, \dots, N$; $k = 1, \dots, J$, and the sector i in the country j sources $Q_{ij,nk}$ of intermediate goods from the sector k in the country n . The input weight ι_{nk} captures the relative importance of different intermediate goods from sector k and country n in the production of materials in sector j and country i . There is perfect competition in all sectors.⁹

We assume free mobility of labor across the sectors within a country. The feasibility condition for labor is then given by:

$$\sum_{j=1, \dots, J} L_{ij} = 1, \quad \forall i \in \{1, \dots, N\}.$$

The unit price of good Q_{ij} is given by:

$$c_{ij} = \frac{1}{A_{ij}} w_i^{\beta_{ij}} P_{ij}^{(1-\beta_{ij})},$$

where w_i is the wage in country i , and P_{ij} denotes the price of materials from country i and sector j .

An agent in country i maximizes the CES utility:

$$U(C_i) = \left(\sum_{j=1, \dots, J} \chi_{ij} C_{ij}^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}},$$

where C_i is the composite consumption good, C_{ij} are the consumption goods from sector j in country i , and χ_{ij} are demand shifters applied to goods j in country i .

Sourcing a good from country n , sector k to country i , sector j entails a cost $\tau_{ij,nk} c_{nk}$. We call $\tau_{ij,nk}$ the distortion or the wedge. Note that we do not impose symmetry: $\tau_{ij,nk} \neq \tau_{nk,ij}$. We also assume $\tau_{ij,ij} = 1$, and therefore the distortions across sectors we identify later on are relative to the within-sector distortion.

We call the distortions that affect sectors within a country, $\tau_{ij,ik}$, *internal distortions*. Examples of such distortions are sector-specific taxes, regulations or policies that favor sourcing from one sector over another, or markups. We call the distortions that affect

⁹This assumption is also not essential as, for example, heterogenous markups represent themselves as the sector level distortions.

sectors across countries, $\tau_{ij,nk}$ ($i \neq n$), *external distortions*. Examples of such distortions are trade costs, implicit subsidies or tariffs for imports or exports, or differences in contract enforcement. We assume that such frictions do not depend on whether goods are used for final consumption or for production of intermediate goods. As a result, wedges in final goods are the same as in intermediate goods.

We further define two types of key statistics that we use throughout the paper. Denote the share of inputs from country n , sector k in total intermediate consumption in country i , sector j by $\gamma_{ij,nk}$. Formally,

$$\gamma_{ij,nk} \equiv \frac{X_{ij,nk}}{\sum_{m=1}^N \sum_{h=1}^J X_{ij,mh}},$$

where $X_{ij,nk}$ is expenditure in country i , sector j , on intermediate goods from country n , sector k . The intermediate expenditure shares $\gamma_{ij,nk}$ have a direct counterpart in the data as they are exactly the input-output shares that can be directly computed from any world input-output table, as we show later.

Denote the share of consumption of the consumption good from sector j in the aggregate consumption of country i by α_{ij} . Formally,

$$\alpha_{ij} \equiv \frac{P_{ij}C_{ij}}{\sum_{k=1}^J P_{ik}C_{ik}}.$$

Similarly to the intermediate expenditure shares $\gamma_{ij,nk}$, the final expenditure shares α_{ij} are directly observable in any world input-output matrix that contains data on final expenditure in a given country sourced from different sectors and countries.

In the next section, we use our model of the world input-output structure to illustrate how the structure of the world economy endogenously changes as a result of changes in distortions.

3 Endogenous World's Input-Output Structure

One important feature of our model is that the input-output matrix is endogenous, which results from our CES structure. In this section, we study how the world's input-output structure changes with changes in distortions. The counterpart of input-output shares $\gamma_{ij,nk}$ in our model is given by:

$$\gamma_{ij,nk} = \frac{(\tau_{ij,nk}c_{nk})^{-\theta} \iota_{nk}^{1+\theta}}{\sum_{m=1}^N \sum_{h=1}^J (\tau_{ij,mh}c_{mh})^{-\theta} \iota_{mh}^{1+\theta}}$$

which in turn can be written as:

$$\gamma_{ij,nk} = \frac{A_{nk}^\theta \tau_{ij,nk}^{-\theta} P_{nk}^{-\theta(1-\beta_{nk})} \iota_{nk}^{1+\theta}}{\left(P_{ij}/w_n^{\beta_{nk}}\right)^{-\theta}}, \quad (1)$$

where the sectoral price index is given by

$$P_{ij} = \left(\sum_{m=1}^N \sum_{h=1}^J A_{mh}^\theta \tau_{ij,mh}^{-\theta} w_m^{-\theta\beta_{mh}} P_{mh}^{-\theta(1-\beta_{mh})} \iota_{mh}^{1+\theta} \right)^{-1/\theta}.$$

It follows that the elasticity of input-output share $\gamma_{ij,nk}$ with respect to distortions $\tau_{ij,nk}$ for all i, j, n, k is given by

$$\frac{d \log \gamma_{ij,nk}}{d \log \tau_{ij,nk}} = -\theta + \theta \frac{d \log P_{ij}}{d \log \tau_{ij,nk}} - \theta(1 - \beta_{nk}) \frac{d \log P_{nk}}{d \log \tau_{ij,nk}} - \theta \beta_{nk} \frac{d \log w_n}{d \log \tau_{ij,nk}}.$$

Accordingly, the elasticity of input-output shares $\gamma_{ij,nk}$ with respect to changes in distortions depends on three forces. First, there is a direct effect on the sector-country that experiences the change in distortions, with an elasticity of $-\theta$. Second, it depends on the indirect price effects of changes in distortions anywhere. Third, it depends on the general equilibrium effects on local wages.

More generally, we can express the elasticity of the whole world's input-output structure with respect to changes in distortions and productivities in terms of the primitives of the model up to the change in the wages. Specifically, the change in the world's input-output structure to changes in distortions and productivities is given by the following equation (see Appendix A.1 for the derivation):

$$\Gamma = \mathcal{F}(\theta, \beta, \gamma)A + \mathcal{H}(\theta, \beta, \gamma)\tau + \mathcal{O}(\theta, \beta, \gamma)\omega, \quad (2)$$

where Γ , A , τ , and ω are vectors that contain the log changes in expenditure shares, productivities, distortions, and wages respectively. The elasticities $\mathcal{F}(\theta, \beta, \gamma)$, $\mathcal{H}(\theta, \beta, \gamma)$, and $\mathcal{O}(\theta, \beta, \gamma)$ are matrices that depend on the elasticity θ , the share of value added in gross output β across all countries and sectors, and all expenditure shares γ . These matrices provide the direct effects of distortions and productivities on the input-output shares as well as the indirect effects through changes in prices.

To reduce the notational burden and discuss our main insight from the input-output elasticity, we derive below this elasticity under some simplifying assumptions. Specifically, we assume one country i , and two sectors j, k . We normalize the wage in that country to

1, and study the elasticity of the input-output share $\gamma_{ij,ik}$ to a change in the distortions $\tau_{ij,ik}$ and $\tau_{ik,ij}$. Totally differentiating the expression for the prices and for the input-output shares, we have that the elasticities of the input-output shares with respect to distortions are given by (see Appendix A.1 for the derivation):

$$\frac{d\log\gamma_{ij,ik}}{d\log\tau_{ij,ik}} = -\theta \frac{1 - \tilde{\gamma}_{ij,ik}}{1 - \tilde{\gamma}_{ik,ij}(1-\beta_{ik})\tilde{\gamma}_{ij,ik}(1-\beta_{ij})}, \quad \frac{d\log\gamma_{ij,ik}}{d\log\tau_{ik,ij}} = (1-\beta_{ik})\tilde{\gamma}_{ik,ij} \frac{d\log\gamma_{ij,ik}}{d\log\tau_{ij,ik}}, \quad (3)$$

where $\tilde{\gamma}_{ij,ik} \equiv \frac{\gamma_{ij,ik}}{1-\gamma_{ij,ik}(1-\beta_{ij})}$.

Similarly, the elasticities of the input-output shares with respect to sectoral TFPs are given by:

$$\frac{d\log\gamma_{ij,ik}}{d\log A_{ij}} = \theta \frac{\tilde{\gamma}_{ij,ij}(\tilde{\gamma}_{ik,ij}(1-\beta_{ik})-1)}{1-\tilde{\gamma}_{ik,ij}(1-\beta_{ik})\tilde{\gamma}_{ij,ik}(1-\beta_{ij})}, \quad \frac{d\log\gamma_{ij,ik}}{d\log A_{ik}} = \theta\beta_{ij} \frac{\tilde{\gamma}_{ij,ij}+(1-\beta_{ik})\tilde{\gamma}_{ik,ik}\tilde{\gamma}_{ij,ij}}{1-\tilde{\gamma}_{ik,ij}(1-\beta_{ik})\tilde{\gamma}_{ij,ik}(1-\beta_{ij})}. \quad (4)$$

The main insight from these equations is that the input-output elasticities with respect to changes in distortions and productivities are not constant or mechanically driven by the elasticity θ , but they depend on the whole production network. In other words, computing the impact of a change in distortion anywhere in the world on a given input-output share requires keeping track of the whole world's input output matrix.

We now proceed to compute the world's input-output elasticities to changes in distortions. We first study how the input-output structure in a given country changes with a change in internal distortions. We also compute the effect of a change in internal distortions in a given country on the input-output structure of a different country. We then take a global view and study how changes in internal distortions in a given country and sector impact the world's GDP, which sectors and countries have a larger impact on the world's GDP, and how relevant internal versus external distortions are for the world's economy.

Before turning to the results, we close the model by imposing the market clearing condition given by:

$$w_i L_i = \sum_{k=1}^J \sum_{n=1}^N \frac{1-\beta_{nk}}{\beta_{nk}} \gamma_{nk,ij} X_{nk}. \quad (5)$$

The equilibrium of this economy is therefore defined by equations (5), (6), (8) (9), and .

3.1 Matching the World Input-Output Matrix

All the data used in the empirical analysis in this paper comes from the World Input-Output Database (WIOD, Timmer et al., 2015). The WIOD database traces the flow of goods

and services across 35 industries classified according to the NACE classification system, 40 countries, and a constructed rest of the world, over the period 1995-2011. This data is integrated into a world input-output table, which also provides gross output and value added for each sector and country. In Appendix A.2 we provide the exact list of the sectors and the countries.

The flow of goods and services includes domestic transactions of intermediate goods, that is, flows across industries in a given country, as well as cross-sector and cross-country flows. Using these flows, we are able to directly determine the bilateral expenditure shares $\gamma_{ij,nk}$ across all countries and sectors. The WIOD database also contains final expenditure across sectors and countries that we use to construct α_{ij} . In doing so, we leave out changes in inventories from the calculations. The shares of value added across sectors and countries, β_{ij} , are constructed using value added and gross output data for each sector and country. Finally, in our empirical exercises we use the standard values of the elasticities in the literature $\theta = 4$ and $\sigma = 4$.

Figure 1: Global expenditure shares across sectors and countries in 2011

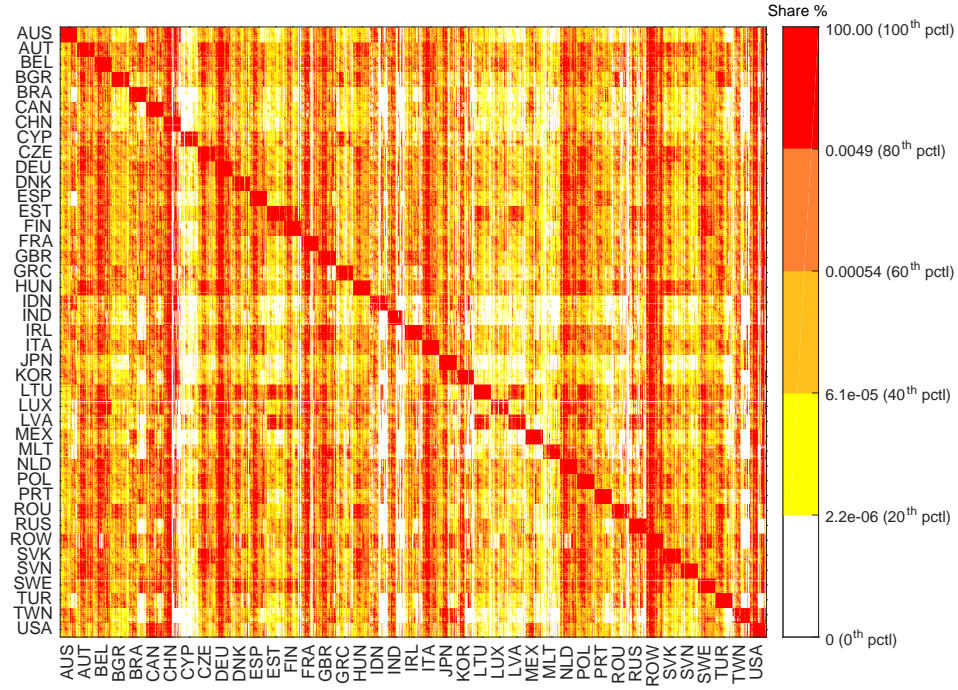


Figure 1 displays the heatmap of the world input-output table for the year 2011. Specifically, it shows the bilateral expenditure shares across all sectors and countries in the world,

where the y-axis (rows) presents the buyer unit, and the x-axis (columns) shows the seller unit. The colors in the figure represents different percentiles that are labeled on the right hand side of the figure, and the observations are ordered first by country and then by sector. For instance, the 80th percentile corresponds to an expenditure share of 0.0049, that is, the buyer unit is spending 0.0049 percent of its total expenditure on goods from that specific seller unit. Since observations are ordered first by country and then by sector, the first red square visualized in the figure displays the Australian domestic input-output coefficients, that is, expenditure share of each Australian sector on goods from every other industry in Australia.

We want to highlight two relevant features from the world input-output table. First, the diagonal is strong, that is, the red squares along the diagonal of the figures mean that the domestic input-output transactions tend to be stronger than the cross-country transactions. Second, the world is very interconnected, with countries such as China, the United States, and Germany playing a role of the important suppliers of inputs for all countries in the world. Later, we discuss further how these features shape the elasticities of GDP and expenditure shares with respect to changes in distortions.¹⁰

3.2 Input-Output and World's GDP Elasticities

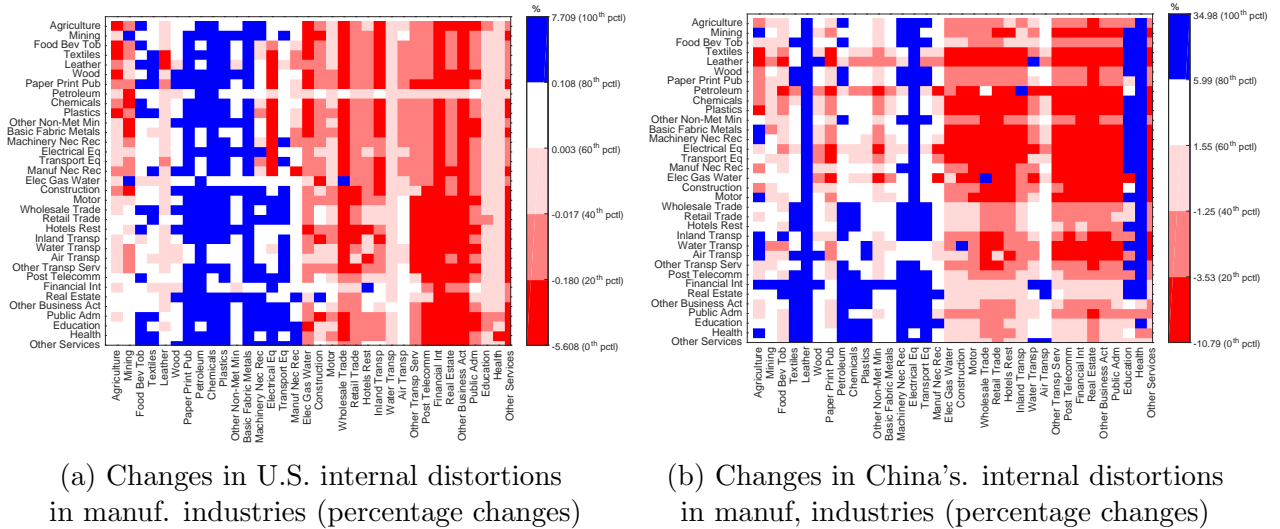
As discussed above, in our model changes in distortions impact wages and prices in the whole world economy, and the input-output shares across all sectors and all countries may change as a result. Given this, the world's input-output structure is endogenous to changes in distortions. Therefore, armed with our model and the data from the WIOD, we emphasize this point by computing the elasticity of the input-output shares to changes in internal distortions. We first focus on the U.S. and China input-output structure, and compute how the U.S. input-output structure changes with respect to changes in its own internal distortions. We also study the effects on the U.S. input-output structure from changes in internal distortions in China – the cross-elasticity of China's internal distortions on the United States. We then take a global view on the impact of internal distortions on the world's economy.

We start by computing how the U.S. input-output structure would change with respect to a 10 percent decline in internal (within the United States) distortions in the manufacturing industries. The results are displayed in Figure 2 Panel (a). The figure shows the effects of the decline in the distortions of selling goods to manufacturing industries from any industry. We can see from the figure how the U.S. input-output table is impacted by changes in

¹⁰In Appendix A.5.1 we display the input-output structure of the United States and China to illustrate the heterogeneity of input-output linkages across different countries.

distortions in the manufacturing industries. In particular, we observe that manufacturing sectors buy more from other manufacturing sectors. Since it is cheap for manufacturing industries to buy intermediate inputs from everywhere, they buy proportionally more from the manufacturing industries because those industries experience a larger decline in the price index. Overall, this exercise makes it clear how changes in internal distortions in the manufacturing sector impact the whole U.S. production network. In Appendix A.5.2 Figure 10, we perform the same exercise but for a different economy, China, as a way of confirming whether the findings for the U.S. hold for another economy with a different input-output structure.

Figure 2: Change in U.S input-output structure



We now compute the cross-country effects of changes in internal distortions in the manufacturing sectors. Specifically, we compute the change in the U.S. input-output shares from a 10 percent reduction in the internal distortions in China. Figure 2 Panel (b) shows the results.¹¹ We find that the U.S. economy purchases proportionally more inputs from the manufacturing industries, and also manufacturing industries increase their importance as suppliers for the rest of the U.S. economy. This highlight the fact that the Chinese man-

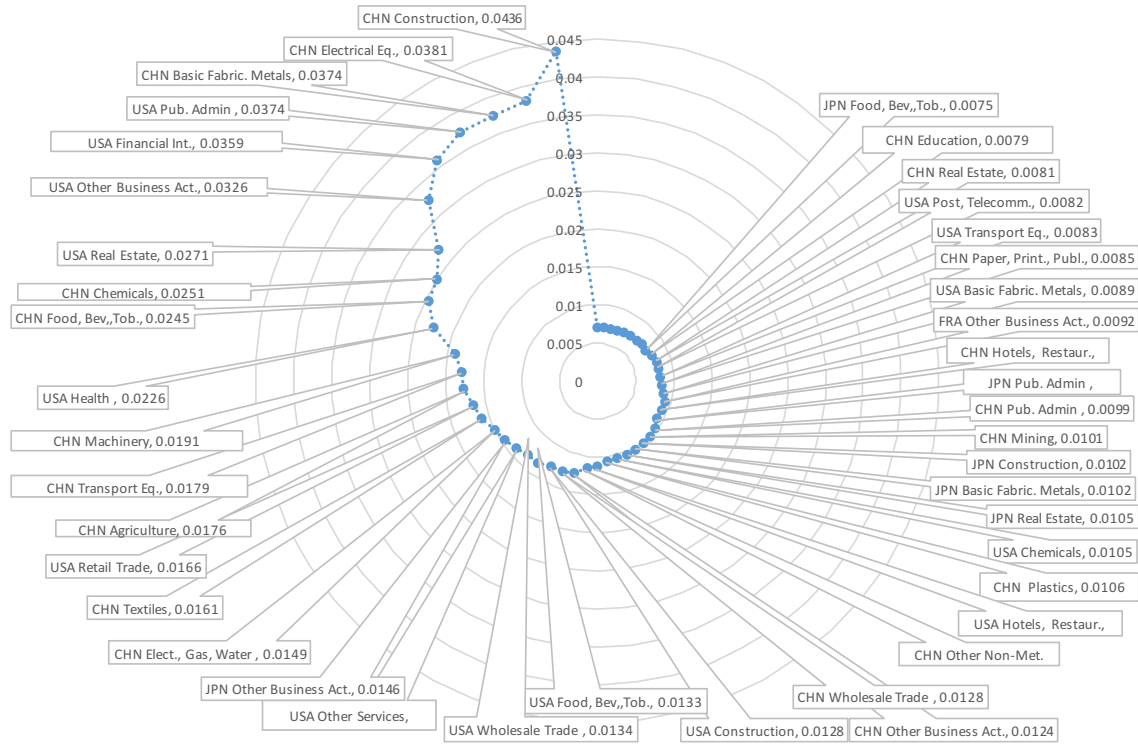
¹¹In Appendix A.5.2 we present the absolute change in the U.S. input-output structure to changes in internal distortions in the manufacturing industries in the United States and in China. We also present the effects on the U.S. input-output structure of the decline in distortions of buying goods from the manufacturing industries by any industry in the United States and in China.

ufacturing is more connected to the U.S. manufacturing sectors than to other industries.

These results highlight how complex the input-output relationships within countries and across countries are, and given this, how important it is to account for the endogenous changes in them as a results of changes in distortions anywhere in the world.

We now take a global perspective of the input-output shares and compute the elasticity of world's real GDP to changes in internal distortions. We next discuss the relative importance of the internal versus external distortions.

Figure 3: World's real GDP elasticity to changes in internal distortions (top 60 markets)



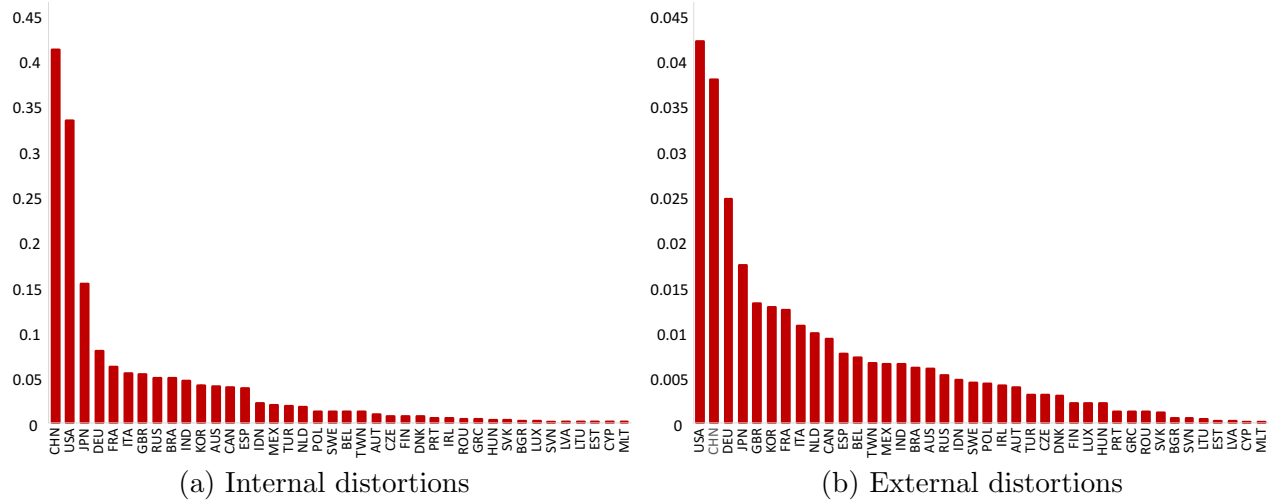
In Figure 3, we compute the world's real GDP elasticities with respect to changes in internal distortions in each country and study which sector/countries are more important for the world's economy. In particular, we rank top 60 elasticities of the world's GDP to a sector in a given country. The change in world's real GDP is computed by aggregating the changes in real wages across all countries using each country's GDP as weights.¹²

The importance of the internal distortions can be seen in Figure 3. The largest such

¹²In Appendix A.5.2 we present a heatmap that summarizes the world's GDP elasticity to changes in internal distortions in any country/sector in the world.

internal elasticity - that of China’s construction sector (0.0436) has the same size as the elasticity of the world’s GDP to all of the external distortions in the United States.

Figure 4: World’s real GDP elasticity to internal versus external distortions



In Figure 4, we compute the world’s real GDP elasticity to changes in internal distortions in a given country in Panel (a), and we then compare them with the world’s real GDP elasticity to changes in external distortions. Comparing the elasticities with respect to internal versus external distortions, our main result is that the elasticity with respect to the internal distortions is an order magnitude larger than that of external distortions.¹³ This finding can be connected to our findings in the previous section that domestic input-output relations tend to be more important than cross-country input-output relations. This different magnitude between internal versus external elasticities also sheds lights on the potential impact of changes in domestic versus external policy-related distortions.

4 Sufficient Statistics for Identification of Distortions

In this section we propose a method to separately decompose the TFPs and the distortions in the input-output economy. The key to the results is threefold. First, the CES structure of

¹³In Appendix A.3, we also present the normalized elasticity – the elasticity divided by the size of the country’s GDP. The ranking differs but the main insight of the much larger quantitative significance of the internal elasticities remains.

production of both the intermediate goods and consumption yields the expenditure shares γ and consumption shares α varying with TFP and distortions. Second, TFP and distortions affect the production and consumption shares differently, hence allowing to separately identify these two objects. Third, using the consumption shares allows us to substitute for prices with the consumption shares α .

We start with the production side of the economy. The share of the input of the sector k in the sector j is given by:

$$\gamma_{ij,ik} = \frac{A_{ik}^\theta \tau_{ij,ik}^{-\theta} P_{ik}^{-\theta(1-\beta_{ik})} \ell_{ik}^{1+\theta}}{\left(P_{ij}/w_i^{\beta_{ik}}\right)^{-\theta}}. \quad (6)$$

Dividing the input shares, $\gamma_{ik,ik}$ and $\gamma_{ij,ik}$, to cancel the sector k TFP, A_{ik} , and the wage, w_i , and using that $\tau_{ik,ik} = 1$, we get the expression for the distortion as a function of the sectoral prices:

$$\tau_{ij,ik} = \left(\frac{P_{ij}}{P_{ik}}\right) \left(\frac{\gamma_{ik,ik}}{\gamma_{ij,ik}}\right)^{\frac{1}{\theta}}. \quad (7)$$

Intuitively, the cross-sectoral variation in input-output shares contains information on the cross-sectoral variation in prices and on the distortions. Substituting for the definition of the sectoral price index, we obtain an expression for the composite of the distortion, the ratio of the sectoral TFPs, and the input weights:

$$\tilde{\tau}_{ij,ik} \equiv \tau_{ij,ik} \frac{\left(A_{ij} \ell_{ij}^{\frac{1+\theta}{\theta}}\right)^{1/\beta_{ij}}}{\left(A_{ik} \ell_{ik}^{\frac{1+\theta}{\theta}}\right)^{1/\beta_{ik}}} = \left(\frac{\gamma_{ij,ij}}{\gamma_{ik,ik}}\right)^{\frac{1}{\theta}} \left(\frac{\gamma_{ik,ik}}{\gamma_{ij,ik}}\right)^{\frac{1}{\theta}};$$

therefore, $\tilde{\tau}_{ij,ik}$ can be identified but not the TFPs and distortions separately. In other words, by only using the cross-sectoral variation in input-output production shares we can only identify a combination of distortions, TFPs, and input weights. In order to separate distortions from TFPs, we now turn to the consumption side of the economy.

The consumer's problem yields the following consumption share in country i , sector j :

$$\alpha_{ij} = \frac{P_{ij} C_{ij}}{\sum_{k=1}^J P_{ik} C_{ik}} = \chi_{ij} \left(\frac{P_{ij}}{P_i}\right)^{1-\sigma}, \quad (8)$$

where

$$P_i = \left[\sum_{j=1, \dots, J} \chi_{ij} (P_{ij})^{1-\sigma} \right]^{1/(1-\sigma)} \quad (9)$$

is the ideal price index.

Dividing the shares of consumptions for sector j and k gives the ratio of sectoral prices:

$$\frac{P_{ij}}{P_{ik}} = \left(\frac{\alpha_{ij}/\chi_{ij}}{\alpha_{ik}/\chi_{ik}} \right)^{\frac{1}{1-\sigma}},$$

and after substitution the expression for the distortion we obtain (see Appendix A.4 for the derivation):

$$\tau_{ijk} = \left(\frac{\gamma_{ik,ik}}{\gamma_{ij,ik}} \right)^{\frac{1}{\theta}} \left(\frac{\alpha_{ij}/\chi_{ij}}{\alpha_{ik}/\chi_{ik}} \right)^{\frac{1}{1-\sigma}}.$$

Another way to gain intuition is to rewrite

$$\tau_{ij,ik} \left(\frac{\alpha_{ik}/\chi_{ik}}{\alpha_{ij}/\chi_{ij}} \right)^{\frac{1}{1-\sigma}} = \left(\frac{\gamma_{ik,ik}}{\gamma_{ij,ik}} \right)^{\frac{1}{\theta}}.$$

Hence, the undistorted economy equates the relative consumption shares to the input shares. The analysis of the external distortions is identical with the exception that the final expression would also include the ratio of ideal prices in countries i and n . Now, under the assumption that demand shifters are time invariant and orthogonal to distortions we can identify the changes in distortions over time using only intermediate and consumption shares. Namely, we can identify $\hat{\tau}$ where $\hat{\tau} \equiv \tau_{t+1}/\tau_t$, and more generally any variable with “hats” means changes over time.

We summarize the results with the following proposition:

Proposition 1. *In a world with N countries (indexed by i, n) and J sectors (indexed by j, k) the change in internal distortions is given by:*

$$\hat{\tau}_{ij,ik} = \frac{(\hat{\gamma}_{ik,ik}/\hat{\gamma}_{ij,ik})^{\frac{1}{\theta}}}{(\hat{\alpha}_{ik}/\hat{\alpha}_{ij})^{\frac{1}{1-\sigma}}}.$$

The change in external distortions is given by:

$$\hat{\tau}_{ij,nk} = \frac{(\hat{\gamma}_{nk,nk}/\hat{\gamma}_{ij,nk})^{\frac{1}{\theta}}}{(\hat{\alpha}_{nk}/\hat{\alpha}_{ij})^{\frac{1}{1-\sigma}}} \left(\frac{\hat{P}_i}{\hat{P}_n} \right).$$

The proposition above gives a very simple closed-form expression for the sufficient statistics formula to identify the change in external and internal distortions separately from the

TFPs. All of the formulas are expressed in terms of the small set of directly observable empirical counterparts from the world input-output matrix.

One generalization of Proposition 1 is for the case in which input weights are origin and destination specific, $\iota_{ij,nk}$. At the same time, assuming that the preference parameters χ_{ij} and production parameters $\iota_{ij,nk}$ are not changing over time, they would immediately drop out if we consider the change of the distortion across time. The same principle applies to other modifications of the model such as, for example, adding consumption distortions. If these additional elements do not change over time, one can still identify the changes in the distortions. However, if we impose further restrictions over the demand shifters we can also identify the level of distortions. This result is summarized in the following Corollary.

Corollary 1. *Under uniform demand shifters, in a world with N countries (indexed by i, n) and J sectors (indexed by j, k) the level of internal distortions are given by:*

$$\tau_{ij,ik} = \frac{(\gamma_{ik,ik}/\gamma_{ij,ik})^{\frac{1}{\theta}}}{(\alpha_{ik}/\alpha_{ij})^{\frac{1}{1-\sigma}}}.$$

The level of external distortions are given by:

$$\tau_{ij,nk} = \frac{(\gamma_{nk,nk}/\gamma_{ij,nk})^{\frac{1}{\theta}}}{(\alpha_{nk}/\alpha_{ij})^{\frac{1}{1-\sigma}}} \left(\frac{P_i}{P_n} \right).$$

Note that the result in Corollary 1 shows how to measure the level of distortions even under the presence of weights in the production function of intermediates. The result does rely on the assumption of uniform demand shifters. In what follows we compute growth rates in distortions; and therefore, these extensions to our model do not affect our quantitative results. Finally, we emphasize that in a nested CES framework with different elasticities of substitution across inputs, we can still identify distortions using production and consumption shares (of course, up to the relevant elasticity of substitution).

We comment on the two key difference with the results of the macroeconomic literature that focuses on misallocations and distortions – the use of the CES production function, and the use of the CES consumption to derive the ratio of the prices. In Hsieh and Klenow (2009), the production function is Cobb-Douglas. They identify the distortions by making reference to the undistorted country. This amounts in our context to assuming that either the net (of distortion) as well as gross (with distortion) prices are observable or that the country/sector pairs have the same Cobb-Douglas elasticities of inputs. Jones (2011, 2013), which are the closest to our study, also use the Cobb-Douglas production in the input-output

structure and hence cannot separate the TFPs from the distortions. Besides, Cobb-Douglas assumption implies that the expenditure and the consumption shares are exogenous and do not change with distortions and TFPs.¹⁴

Notice that to compute internal distortions, external distortions, we do not need to solve for the general equilibrium of the model, as they can be directly computed using data on intermediate and final expenditure shares, conditional on values for the elasticities σ and θ . As discussed above, we do not need to use price data or impose symmetry in distortions either, different from common approaches followed in the trade literature to compute external distortions.¹⁵ It is also immediately clear how to extend the analysis to the cases of the commonly used trade models. In particular, a wide class of trade models (e.g. Anderson and van Wincoop 2003, Eaton and Kortum 2002, Melitz 2003, among many others) deliver a gravity equation of type of equation (6), but for the case of cross-country flows in a given sector (that is, when $i \neq n$ and $k = j$). It immediately follows that our sufficient statistic can be directly mapped to gravity-trade models to infer trade costs.

One advantage of the sufficient statistic derived in this section is that it depends on input-output production shares, and on consumption shares, and both are directly observable on any world input output table. We use these sufficient statistics and the data from the WIOD to compute the evolution of internal distortions for the period of study. Figure 5 shows the distribution of the annual rate of growth of internal distortions for the world, and for selected countries. This graph shows several notable features. First, there is large heterogeneity in terms of the growth rate of distortions across sectors. For each country there are sectors in which the distortions grew and in which distortions decreased. Second, the heterogeneity differs across countries. The United States and Japan have relatively small dispersion compared to Europe, and all of these countries have much smaller dispersion compared to China.

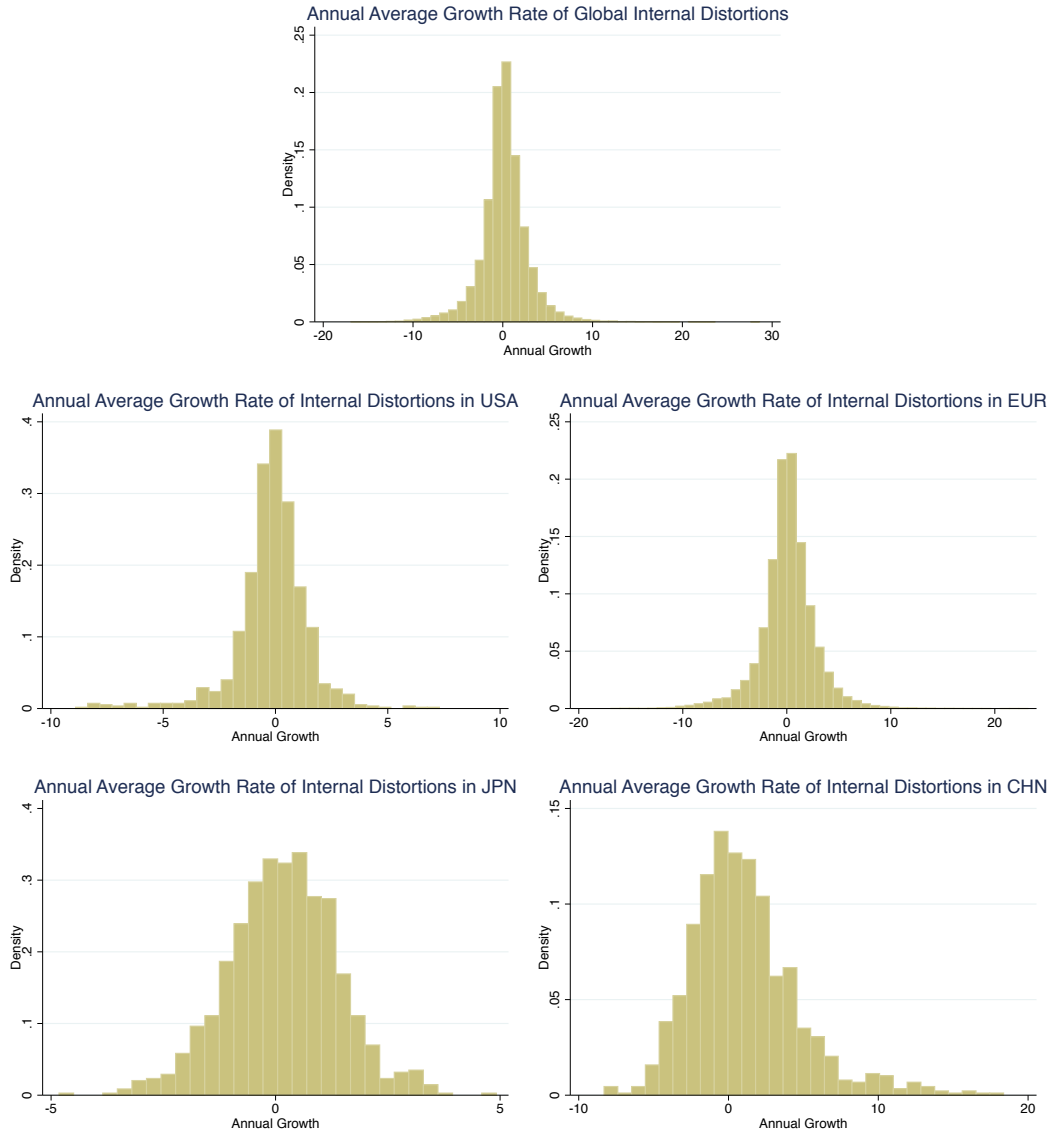
The main bottom line of these histograms is that, to a greater or lesser extent, there is heterogeneity in the changes in internal distortions in all countries, and some sectors

¹⁴Both CES and Cobb-Douglas production functions are subject to a similar limitation, that one cannot account for the formation of new input-output relations, that is, input-output shares that change from zero to positive. However, at the level of aggregation that we work in the empirical section, these cases are negligible.

¹⁵Note that if sectoral prices were observable equation (7) could be used to identify the distortions. However, there are two major issues with this approach. First, using price data from existing databases is always going to constraint the computation of internal distortions due to the lack of consistent time series data or the lack of price data for non-tradable industries, the reduced number of countries in the database, the reduced number of sectors, or due to inconsistencies between the definition of prices in the price databases (for instance, deflators or PPI indexes), and their counterpart in the model or in the input-output data. Second, and perhaps more importantly, the measured prices do not take into account implicit price distortions such as differences in contract enforcement or implicit subsidies.

have become more distorted and other less distorted in some countries relative to others. Therefore, aggregate measures of relative distortions across countries can give only a partial view of the degree of distortions across countries, since they hide this high heterogeneity at the more disaggregate level.

Figure 5: Distribution of changes in internal distortions in selected countries



5 Conclusion

Our paper achieves several goals. First, we argue that it is fruitful to study the world economy as one interconnected input-output table with the country-sector pair as the base unit of analysis. Second, we show that the endogeneity of the input-output table due to the CES structure of production and consumption is important. This endogeneity allows us to resolve an important issue in the analysis of economies with distortions, namely, how to analytically and empirically separately identify distortions in the cases when the input-output relationships may differ across countries. Even more importantly, we show how to analytically compute the elasticities of distortions in a given country on the endogenous structure of the input-output relationships in that country, on other countries, and on the world's economy. We find that internal distortions play an important role in determining the structure of the economies of the individual countries and of the world economy. The elasticities of the internal distortions are an order of magnitude larger than those of the external distortions. Overall, with our paper we have highlighted the importance of modeling the world as a single endogenous production network. After all, the world economy, like the economy of a single country, can be thought of as a system of interdependent processes.

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Appendix

A.1 Derivation of the World's Input-Output Elasticities

In this Appendix, we derive the elasticity of the world's input-output structure with respect to changes in distortions and productivities. We start by deriving the input-output elasticities for the closed economy case with two sectors (equations (3) and (4) in the main text. The share of sector k in total intermediate consumption of sector j (where we have normalized wages to one) is given by

$$\gamma_{ij,ik} = \frac{A_{ik}^\theta \tau_{ij,ik}^{-\theta} P_{ik}^{-\theta(1-\beta_{ik})} \iota_{ik}^{1+\theta}}{(P_{ij})^{-\theta}}$$

where

$$P_{ij} = \left(A_{ij}^\theta P_{ij}^{-\theta(1-\beta_{ij})} \iota_{ij}^{1+\theta} + A_{ik}^\theta \tau_{ij,ik}^{-\theta} P_{ik}^{-\theta(1-\beta_{ik})} \iota_{ik}^{1+\theta} \right)^{-1/\theta}.$$

Totally differentiating the price equation and using the definition of the expenditure shares we get

$$-\theta d\log P_{ij} = \gamma_{ij,ij}(\theta d\log A_{ij} - \theta(1-\beta_{ij})d\log P_{ij}) + \gamma_{ij,k}(\theta d\log A_{ik} - \theta d\log \tau_{ij,ik} - \theta(1-\beta_{ik})d\log P_{ik}).$$

Let's define

$$\tilde{\gamma}_{ij,ij} = \frac{\gamma_{ij,ij}}{(1 - \gamma_{ij,ij}(1 - \beta_{ij}))}$$

$$\tilde{\gamma}_{ij,ik} = \frac{\gamma_{ij,ik}}{(1 - \gamma_{ij,ij}(1 - \beta_{ij}))},$$

then we have

$$d\log P_{ij} = -\tilde{\gamma}_{ij,ij}d\log A_{ij} - \tilde{\gamma}_{ij,ik}d\log A_{ik} + \tilde{\gamma}_{ij,ik}d\log \tau_{ij,ik} + \tilde{\gamma}_{ij,ik}(1 - \beta_{ik})d\log P_{ik}.$$

It is then trivial to show that

$$d\log P_{ik} = -\tilde{\gamma}_{ik,ik}d\log A_{ik} - \tilde{\gamma}_{ik,ij}d\log A_{ij} + \tilde{\gamma}_{ik,ij}d\log \tau_{ik,ij} + \tilde{\gamma}_{ik,ij}(1 - \beta_{ij})d\log P_{ij}.$$

Solving for prices we get

$$d\log P_{ij} = -\tilde{\gamma}_{ij,ij}d\log A_{ij} - \tilde{\gamma}_{ij,ik}d\log A_{ik} + \tilde{\gamma}_{ij,ik}d\log \tau_{ij,ik} - \tilde{\gamma}_{ik,ik}\tilde{\gamma}_{ij,ik}(1 - \beta_{ik})d\log A_{ik}$$

$$-\tilde{\gamma}_{ik,ij}\tilde{\gamma}_{ij,ik}(1-\beta_{ik})d\log A_{ij} + \tilde{\gamma}_{ik,ij}\tilde{\gamma}_{ij,ik}(1-\beta_{ik})d\log \tau_{ik,ij} + \tilde{\gamma}_{ik,ij}(1-\beta_{ij})\tilde{\gamma}_{ij,ik}(1-\beta_{ik})d\log P_{ij}.$$

$$d\log P_{ij} = -\frac{\tilde{\gamma}_{ij,ij} + \tilde{\gamma}_{ik,ij}\tilde{\gamma}_{ij,ik}(1-\beta_{ik})}{(1-\tilde{\gamma}_{ik,ij}(1-\beta_{ij})\tilde{\gamma}_{ij,ik}(1-\beta_{ik}))}d\log A_{ij} - \frac{\tilde{\gamma}_{ij,ik} + \tilde{\gamma}_{ik,ik}\tilde{\gamma}_{ij,ik}(1-\beta_{ik})}{(1-\tilde{\gamma}_{ik,ij}(1-\beta_{ij})\tilde{\gamma}_{ij,ik}(1-\beta_{ik}))}d\log A_{ik}$$

$$+ \frac{\tilde{\gamma}_{ij,ik}}{(1-\tilde{\gamma}_{ik,ij}(1-\beta_{ij})\tilde{\gamma}_{ij,ik}(1-\beta_{ik}))}d\log \tau_{ij,ik} + \frac{\tilde{\gamma}_{ik,ij}\tilde{\gamma}_{ij,ik}(1-\beta_{ik})}{(1-\tilde{\gamma}_{ik,ij}(1-\beta_{ij})\tilde{\gamma}_{ij,ik}(1-\beta_{ik}))}d\log \tau_{ik,ij}$$

Similarly

$$d\log P_{ik} = -\frac{\tilde{\gamma}_{ik,ik} + \tilde{\gamma}_{ij,ik}\tilde{\gamma}_{ik,ij}(1-\beta_{ij})}{(1-\tilde{\gamma}_{ik,ij}(1-\beta_{ij})\tilde{\gamma}_{ij,ik}(1-\beta_{ik}))}d\log A_{ik} - \frac{\tilde{\gamma}_{ik,ij} + \tilde{\gamma}_{ij,ij}\tilde{\gamma}_{ik,ij}(1-\beta_{ij})}{(1-\tilde{\gamma}_{ik,ij}(1-\beta_{ij})\tilde{\gamma}_{ij,ik}(1-\beta_{ik}))}d\log A_{ij}$$

$$+ \frac{\tilde{\gamma}_{ik,ij}}{(1-\tilde{\gamma}_{ik,ij}(1-\beta_{ij})\tilde{\gamma}_{ij,ik}(1-\beta_{ik}))}d\log \tau_{ik,ij} + \frac{\tilde{\gamma}_{ij,ik}\tilde{\gamma}_{ik,ij}(1-\beta_{ij})}{(1-\tilde{\gamma}_{ik,ij}(1-\beta_{ij})\tilde{\gamma}_{ij,ik}(1-\beta_{ik}))}d\log \tau_{ij,ik}$$

Totally differentiating the expenditure shares we have:

$$\gamma_{ij,ik} = \frac{A_{ik}^\theta \tau_{ij,ik}^{-\theta} P_{ik}^{-\theta(1-\beta_{ik})} \iota_{ik}^{1+\theta}}{(P_{ij})^{-\theta}}$$

$$d\log \gamma_{ij,ik} = \theta d\log A_{ik} - \theta d\log \tau_{ij,ik} - \theta(1-\beta_{ik})d\log P_{ik} + \theta d\log P_{ij}$$

Plugging the total differential for prices we have that

$$\frac{d\log \gamma_{ij,ik}}{d\log \tau_{ij,ik}} = -\theta - \theta(1-\beta_{ik}) \frac{\tilde{\gamma}_{ik,ij}(1-\beta_{ij})\tilde{\gamma}_{ij,ik}}{(1-\tilde{\gamma}_{ik,ij}(1-\beta_{ij})\tilde{\gamma}_{ij,ik}(1-\beta_{ik}))} + \theta \frac{\tilde{\gamma}_{ij,ik}}{(1-\tilde{\gamma}_{ik,ij}(1-\beta_{ij})\tilde{\gamma}_{ij,ik}(1-\beta_{ik}))}$$

$$\frac{d\log \gamma_{ij,ik}}{d\log \tau_{ij,ik}} = -\theta \left[\frac{1 - \tilde{\gamma}_{ij,ik}}{(1 - \tilde{\gamma}_{ik,ij}(1 - \beta_{ij})\tilde{\gamma}_{ij,ik}(1 - \beta_{ik}))} \right]$$

Likewise,

$$\frac{d\log \gamma_{ij,ik}}{d\log \tau_{ik,ij}} = (1 - \beta_{ik})\tilde{\gamma}_{ik,ij} \frac{d\log \gamma_{ij,ik}}{d\log \tau_{ij,ik}},$$

Similarly, we have

$$\frac{d\log \gamma_{ij,ik}}{d\log A_{ij}} = \theta(1-\beta_{ik}) \frac{\tilde{\gamma}_{ik,ij} + \tilde{\gamma}_{ij,ij}\tilde{\gamma}_{ik,ij}(1-\beta_{ij})}{(1-\tilde{\gamma}_{ik,ij}(1-\beta_{ij})\tilde{\gamma}_{ij,ik}(1-\beta_{ik}))} - \theta \frac{\tilde{\gamma}_{ij,ij} + \tilde{\gamma}_{ik,ij}\tilde{\gamma}_{ij,ik}(1-\beta_{ik})}{(1-\tilde{\gamma}_{ik,ij}(1-\beta_{ij})\tilde{\gamma}_{ij,ik}(1-\beta_{ik}))}$$

$$\frac{d\log\gamma_{ij,ik}}{d\log A_{ij}} = \theta \frac{(1 - \beta_{ik})\tilde{\gamma}_{ik,ij} + (1 - \beta_{ik})\tilde{\gamma}_{ij,ij}\tilde{\gamma}_{ik,ij}(1 - \beta_{ij}) - \tilde{\gamma}_{ij,ij} - \tilde{\gamma}_{ik,ij}\tilde{\gamma}_{ij,ik}(1 - \beta_{ik})}{(1 - \tilde{\gamma}_{ik,ij}(1 - \beta_{ij})\tilde{\gamma}_{ij,ik}(1 - \beta_{ik}))}$$

$$\frac{d\log\gamma_{ij,ik}}{d\log A_{ij}} = \theta \frac{(1 - \beta_{ik})\tilde{\gamma}_{ik,ij}\tilde{\gamma}_{ij,ij}\beta_{ij} + (1 - \beta_{ik})\tilde{\gamma}_{ij,ij}\tilde{\gamma}_{ik,ij}(1 - \beta_{ij}) - \tilde{\gamma}_{ij,ij}}{(1 - \tilde{\gamma}_{ik,ij}(1 - \beta_{ij})\tilde{\gamma}_{ij,ik}(1 - \beta_{ik}))}$$

$$\frac{d\log\gamma_{ij,ik}}{d\log A_{ij}} = \theta \frac{(1 - \beta_{ik})\tilde{\gamma}_{ik,ij}\tilde{\gamma}_{ij,ij} - \tilde{\gamma}_{ij,ij}}{(1 - \tilde{\gamma}_{ik,ij}(1 - \beta_{ij})\tilde{\gamma}_{ij,ik}(1 - \beta_{ik}))}$$

likewise

$$\frac{d\log\gamma_{ij,ik}}{d\log A_{ik}} = \theta\beta_{ij} \left(\frac{\tilde{\gamma}_{ij,ij} + (1 - \beta_{ik})\tilde{\gamma}_{ik,ik}\tilde{\gamma}_{ij,ij}}{(1 - \tilde{\gamma}_{ik,ij}(1 - \beta_{ij})\tilde{\gamma}_{ij,ik}(1 - \beta_{ik}))} \right).$$

We now derive the elasticity of the world's input-output structure with respect to changes in distortions and productivities (equation (2) in the main text). Analogous to the two-sector case, totally differentiating the price equations we get:

$$d\log P_{ij} = \sum_{m=1}^N \sum_{h=1}^J [-\tilde{\gamma}_{ij,mh}d\log A_{mh} + \tilde{\gamma}_{ij,mh}d\log \tau_{ij,mh} + (1 - \beta_{mh})\tilde{\gamma}_{ij,mh}d\log P_{mh} + \beta_m\tilde{\gamma}_{ij,mh}d\log w_m]$$

where

$$\tilde{\gamma}_{is,ip} \equiv \frac{\gamma_{is,ip}}{1 - \gamma_{is,is}(1 - \beta_{is})},$$

Let's define P be the vector that contains the log change in prices across all sectors and countries, that is

$$P_{(NJ \times 1)} = \begin{bmatrix} d\log P_{11} \\ \cdot \\ \cdot \\ d\log P_{1J} \\ \cdot \\ \cdot \\ d\log P_{NJ} \end{bmatrix}$$

Similarly, we define the vectors of log changes in TFPs and distortions as

$$\begin{aligned}
\underset{(NJ \times 1)}{A} &= \begin{bmatrix} d\log A_{11} \\ \cdot \\ \cdot \\ d\log A_{1J} \\ \cdot \\ \cdot \\ d\log A_{NJ} \end{bmatrix} & \underset{(NJNJ \times 1)}{\tau} &= \begin{bmatrix} d\log \tau_{11,11} \\ \cdot \\ \cdot \\ d\log \tau_{11,1j} \\ \cdot \\ \cdot \\ d\log \tau_{NJ,11} \\ \cdot \\ \cdot \\ d\log \tau_{NJ,NJ} \end{bmatrix}
\end{aligned}$$

Finally we define the vectors of log changes in input-output shares and wages as

$$\begin{aligned}
\underset{(NJNJ \times 1)}{\Gamma} &= \begin{bmatrix} d\log \gamma_{11,11} \\ \cdot \\ \cdot \\ d\log \gamma_{11,1J} \\ \cdot \\ d\log \gamma_{11,NJ} \\ d\log \gamma_{JN,11} \\ \cdot \\ \cdot \\ d\log \gamma_{JN,JN} \end{bmatrix} & \underset{(NJ \times 1)}{\omega} &= \begin{bmatrix} d\log w_1 \\ d\log w_1 \\ \cdot \\ \cdot \\ d\log w_n \\ d\log w_n \end{bmatrix} & \underset{(NJNJ \times 1)}{\tilde{\omega}} &= \begin{bmatrix} \omega \\ \omega \\ \cdot \\ \cdot \\ \omega \\ \omega \end{bmatrix}
\end{aligned}$$

Therefore, the log change in the expenditure share with respect to changes in distortions is given by

Similarly, let's define the matrices

$$\Gamma = \theta \tilde{A} - \theta \tau + \Sigma P + \theta \beta \tilde{\omega}$$

where we define Σ_{ij} to be a $J \times NJ$ matrix that is the sum of a $J \times NJ$ matrix that contains the element θ in the column $(i-1)J + j$ and zeros elsewhere, and another $J \times NJ$ matrix that contains the element $-\theta(1 - \beta_{mh})$ in the diagonal and zeros elsewhere.

Then we define,

$$\Sigma_{(NJ \times NJ)} = \begin{bmatrix} \Sigma_{11} \\ \cdot \\ \cdot \\ \Sigma_{1J} \\ \cdot \\ \cdot \\ \Sigma_{NJ} \end{bmatrix} \quad \text{and} \quad \tilde{A}_{(NJNJ \times 1)} = \begin{bmatrix} A \\ \cdot \\ \cdot \\ A \\ \cdot \\ \cdot \\ A \end{bmatrix}$$

To solve for the log change in prices, we first define the following matrices

$$Z_{NJ \times NJ} = \begin{bmatrix} 0 & \tilde{\gamma}_{11,12}(1 - \beta_{12}) & \cdot & \cdot & \tilde{\gamma}_{11,1J}(1 - \beta_{1J}) & \cdot & \cdot & \tilde{\gamma}_{11,NJ}(1 - \beta_{NJ}) \\ \tilde{\gamma}_{12,11}(1 - \beta_{11}) & 0 & \cdot & \cdot & \tilde{\gamma}_{12,1J}(1 - \beta_{1J}) & \cdot & \cdot & \tilde{\gamma}_{12,NJ}(1 - \beta_{NJ}) \\ \cdot & \cdot & 0 & \cdot & \cdot & \cdot & \cdot & \cdot \\ \tilde{\gamma}_{1J,11}(1 - \beta_{11}) & \tilde{\gamma}_{1J,12}(1 - \beta_{12}) & \cdot & \cdot & 0 & \cdot & \cdot & \tilde{\gamma}_{1J,NJ}(1 - \beta_{NJ}) \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \tilde{\gamma}_{NJ,11}(1 - \beta_{11}) & \tilde{\gamma}_{NJ,12}(1 - \beta_{12}) & \cdot & \cdot & \tilde{\gamma}_{NJ,1J}(1 - \beta_{1J}) & \cdot & \cdot & 0 \end{bmatrix}$$

$$\tilde{Z}_{NJ \times NJ} = \begin{bmatrix} \tilde{\gamma}_{11,11} & 0 & \cdot & 0 & \cdot & 0 \\ 0 & \tilde{\gamma}_{11,12} & \cdot & \cdot & 0 & \cdot & \cdot & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & \cdot & \cdot & \tilde{\gamma}_{NJ,11} & \cdot & 0 & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & \cdot & \cdot & 0 & \cdot & \cdot & \tilde{\gamma}_{NJ,NJ} \end{bmatrix}$$

and let $\Omega = I - Z$ and $\Upsilon = Z + \tilde{Z}$. Finally, we define the matrix

$$\Theta_{NJ \times NJNJ} = \begin{bmatrix} \tilde{\gamma}_{11,11} & \cdot & \tilde{\gamma}_{11,1J} & \cdot & \cdot & \tilde{\gamma}_{11,NJ} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \tilde{\gamma}_{12,11} & \cdot & \tilde{\gamma}_{12,NJ} & 0 & 0 & 0 & 0 & 0 & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \tilde{\gamma}_{NJ,11} & \cdot & \tilde{\gamma}_{NJ,NJ} \end{bmatrix}$$

$$\Omega P = -\Upsilon A + \Theta \tau + \beta \Upsilon \omega$$

and therefore

$$P = -\Omega^{-1}\Upsilon A + \Omega^{-1}\Theta \tau + \Omega^{-1}\beta \Upsilon \omega$$

Finally, we can obtain the change in input-output shares as

$$\Gamma = (\theta \tilde{A} - \Sigma \Omega^{-1} \Upsilon A) + (-\theta 1_{NJNJ \times 1} + \Sigma \Omega^{-1} \Theta) \tau + \Sigma \Omega^{-1} \beta \Upsilon \omega + \theta \beta \tilde{\omega}$$

where $1_{NJNJ \times 1}$ is a vector of ones of size $NJNJ \times 1$. This equation can be written more generally as

$$\Gamma = \mathcal{F}(\theta, \beta, \gamma) A + \mathcal{H}(\theta, \beta, \gamma) \tau + \mathcal{O}(\theta, \beta, \gamma) \omega$$

A.2 Data appendix

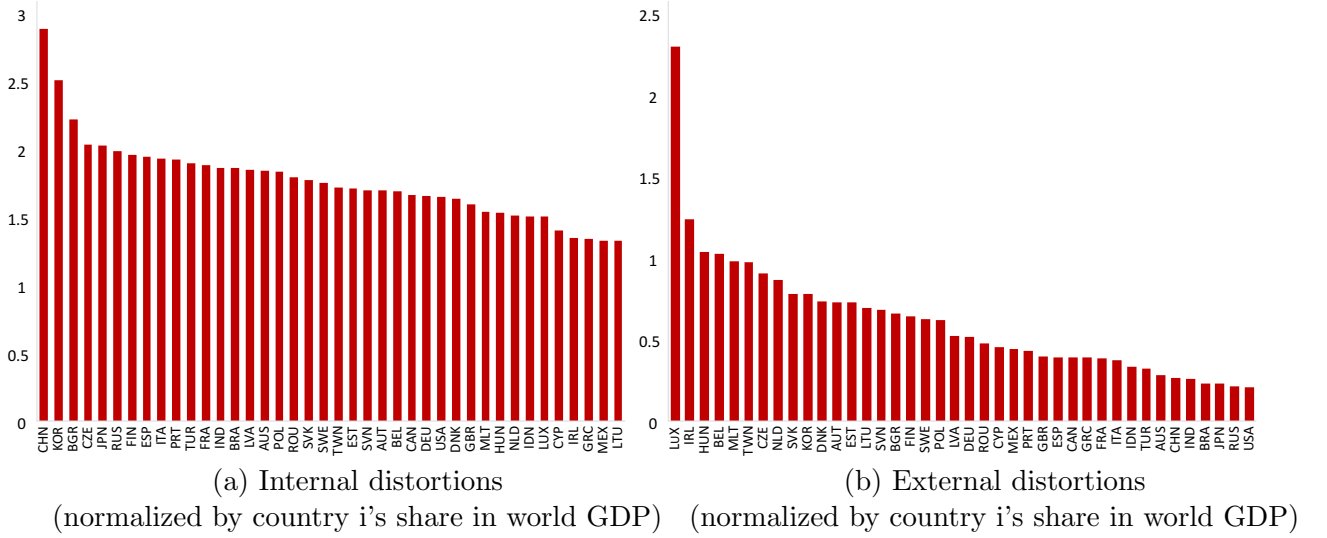
This Appendix describes the list of sectors and countries we included in the empirical analysis. The list of sectors is: Agriculture, Hunting, Forestry and Fishing (NACE AtB); Mining and Quarrying (NACE C); Food, Beverages and Tobacco (NACE 15t16); Textiles and Textile Products (NACE 17t18); Leather, Leather and Footwear (NACE 19); Wood and Products of Wood and Cork (NACE 20); Pulp, Paper, Paper, Printing and Publishing (NACE 21t22); Coke, Refined Petroleum and Nuclear Fuel (NACE 23); Chemicals and Chemical Products (NACE 24); Rubber and Plastics (NACE 25); Other Non-Metallic Mineral (NACE 26); Basic Metals and Fabricated Metal (NACE 27t28); Machinery, Nec (NACE 29); Electrical and Optical Equipment (NACE 30t33); Transport Equipment (NACE 34t35); Manufacturing, Nec; Recycling (NACE 36t37); Electricity, Gas and Water Supply (NACE E); Construction (NACE F); Sale, Maintenance and Repair of Motor Vehicles Retail Sale of Fuel (NACE 50); Wholesale Trade and Commission Trade, Except of Motor Vehicles (NACE 51); Retail Trade, Except of Motor Vehicles; Repair of Household Goods (NACE 52); Hotels and Restaurants (NACE H); Inland Transport (NACE 60); Water Transport (NACE 61); Air Transport (NACE 62); Other Supporting and Auxiliary Transport Activities; Activities of Travel Agencies (NACE 63); Post and Telecommunications (NACE 64); Financial Intermediation (NACE J); Real Estate Activities (NACE 70); Renting of M&Eq and Other Business Activities (NACE 71t74); Public Admin and Defense; Compulsory Social Security (NACE L); Education (NACE M); Health and Social Work (NACE N); Other Community, Social and Personal Services (NACE O); Private Households with Employed Persons (NACE P). We drop from the analysis the Private Households with Employed Persons as it presented generally incomplete data. The list of countries is: Australia,

Austria, Belgium, Brazil, Bulgaria, Canada, China, Cyprus, Czech Republic, Denmark, Estonia, Finland, France, Germany, Greece, Hungary, India, Indonesia, Ireland, Italy, Japan, Latvia, Lithuania, Luxembourg, Malta, Mexico, Netherlands, Poland, Portugal, Romania, Russia, Slovak Republic, Slovenia, South Korea, Spain, Sweden, Taiwan, Turkey, United Kingdom, United States, and the constructed Rest of the World.

A.3 Normalized Elasticities to Changes in Distortions

In this appendix, we present the normalized elasticities of world's real GDP with respect to changes in internal and external distortions, that is, the elasticities are divided by the size of the country's GDP.

Figure 6: World's real GDP elasticity to internal versus external distortions



A.4 Sufficient Statistics to Identify Internal Distortions

The share of the input of the sector k in the sector j is given by:

$$\gamma_{ij,ik} = \frac{A_{ik}^\theta \tau_{ij,ik}^{-\theta} P_{ik}^{-\theta(1-\beta_{ik})} \iota_{ik}^{1+\theta}}{\left(P_{ij}/w_i^{\beta_{ik}}\right)^{-\theta}}.$$

Then we have that

$$\frac{\gamma_{ij,ik}}{\gamma_{ik,ik}} = \tau_{ij,ik}^{-\theta} \left(\frac{P_{ij}}{P_{ik}}\right)^\theta$$

On the other hand, the consumer's problem yields the following consumption share in country i , sector j :

$$\alpha_{ij} = \chi_{ij} \left(\frac{P_{ij}}{P_i}\right)^{1-\sigma}$$

and therefore

$$\frac{\alpha_{ij}}{\alpha_{ik}} = \frac{\chi_{ij}}{\chi_{ik}} \left(\frac{P_{ij}}{P_{ik}} \right)^{1-\sigma}.$$

Therefore,

$$\frac{\gamma_{ij,ik}}{\gamma_{ik,ik}} = \tau_{ij,ik}^{-\theta} \left(\frac{\alpha_{ij} \chi_{ik}}{\alpha_{ik} \chi_{ij}} \right)^{\frac{\theta}{1-\sigma}},$$

and solving for the internal distortion we get

$$\tau_{ij,ik} = \left(\frac{\gamma_{ik,ik}}{\gamma_{ij,ik}} \right)^{1/\theta} \left(\frac{\alpha_{ij} \chi_{ik}}{\alpha_{ik} \chi_{ij}} \right)^{\frac{1}{1-\sigma}},$$

and the relative change in distortion is given by

$$\hat{\tau}_{ij,ik} = \left(\frac{\hat{\gamma}_{ik,ik}}{\hat{\gamma}_{ij,ik}} \right)^{1/\theta} \left(\frac{\hat{\alpha}_{ij}}{\hat{\alpha}_{ik}} \right)^{\frac{1}{1-\sigma}}.$$

A.5 Additional Results

A.5.1 Production network across countries

In Figure 7, we reproduce the input-output for the U.S. economy presented as a heatmap. The y-axis (rows) shows the buyer sectors from each seller sector in the x-axis (columns). In the figure, for a given seller industry, we aggregated U.S. purchases from all countries in the world in that industry. In other words, the bilateral expenditure shares displayed in the figure shows both domestic and international linkages. Similar to Figure 1, the colors represents different percentiles that are labeled on the right hand side of the figure. For instance, the 80th percentile in the figure corresponds to a U.S. sector that spends at least 3.3 percent of their total expenditure on goods from a given seller, including purchase from the United States, and from all other countries in the world.

We observe from the figure that the diagonal is strong, that is, a sector tends to buy more materials from itself rather than other sectors. Still, the interconnection across sectors is also relevant. In particular, some service sectors such as finance and business activities are an important source of intermediate inputs to other sectors, as well as some manufacturing sectors such as petroleum and paper. Figure 8 displays the input-output table for China. Similar to the United States, the diagonal is strong, but it is also evident from the figure how the input-output linkages across different sectors are different from those in the U.S. economy.

Figure 7: U.S. expenditure shares across sectors and countries in 2011

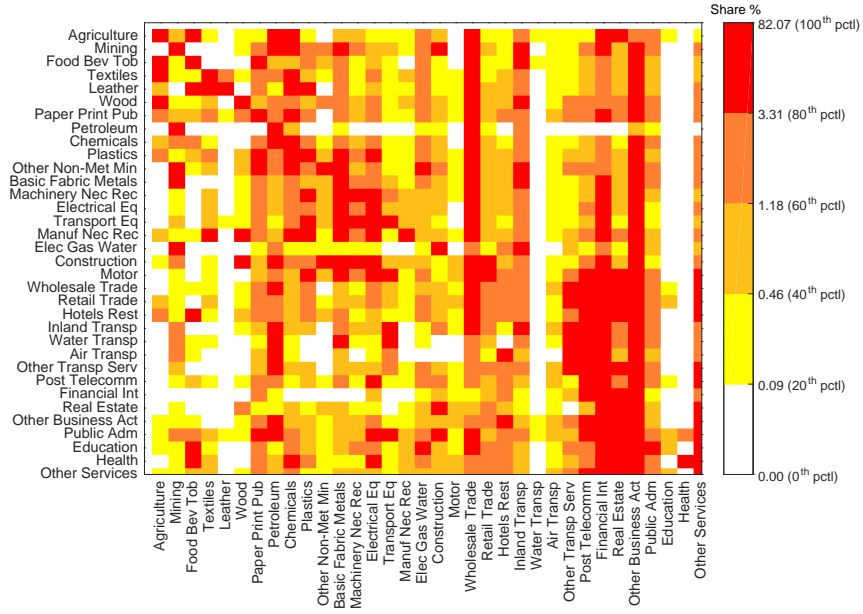
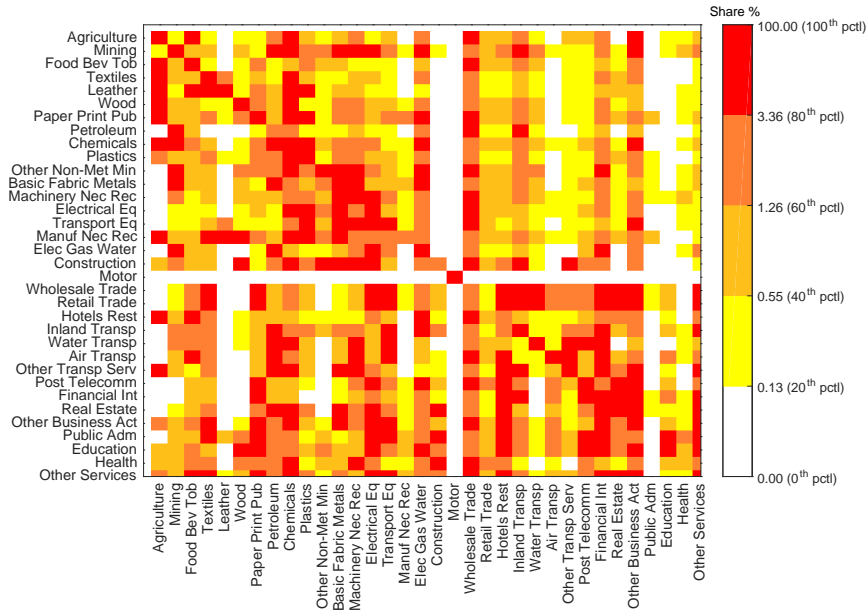


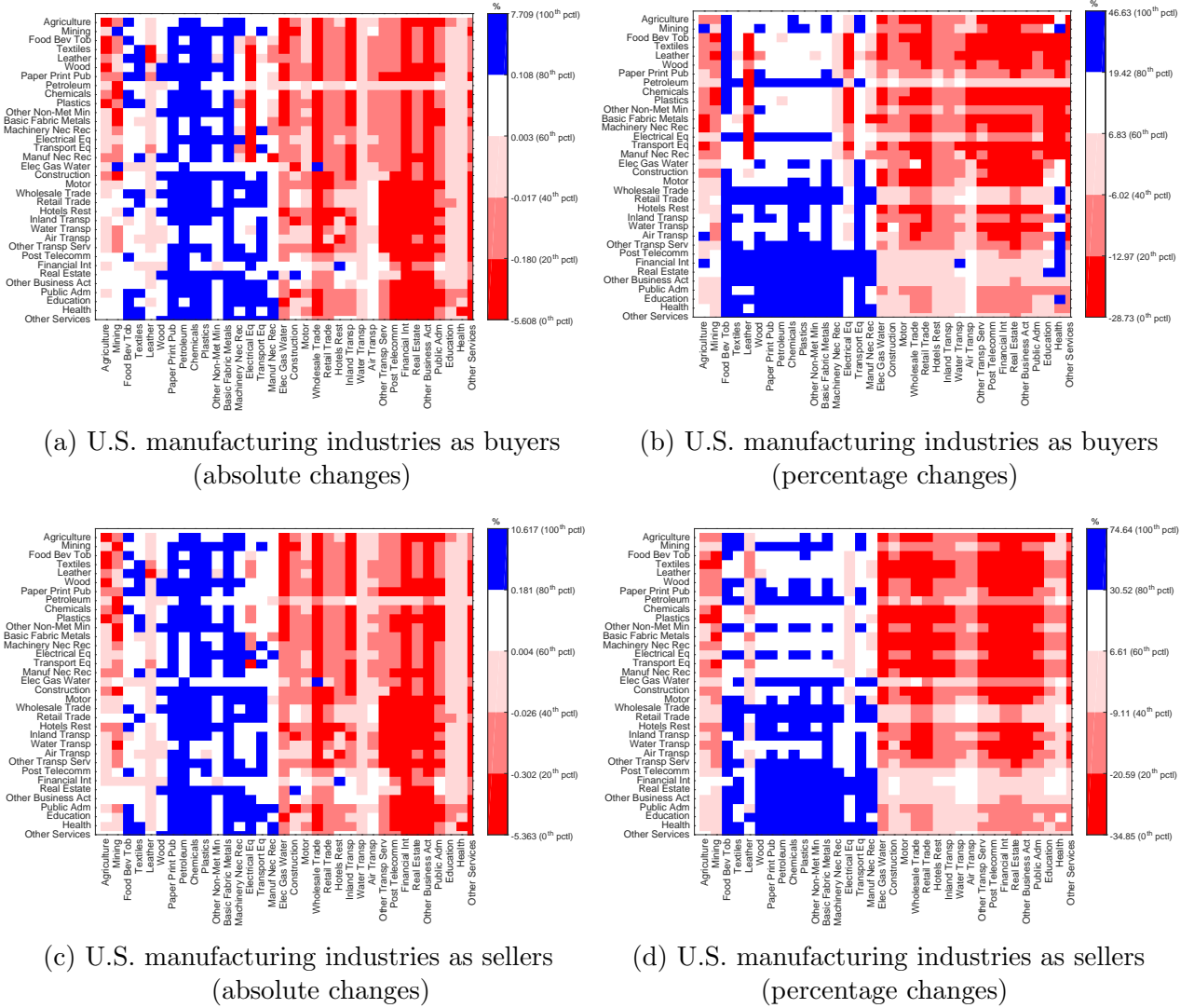
Figure 8: China's expenditure shares across sectors and countries in 2011



A.5.2 Endogenous input-output structure

In this section of the appendix, we present additional results on the effects of declines in internal distortions in the manufacturing industries in the United States and China. Figure 9 computes how the U.S. input-output structure would change with respect to a 10 percent decline in internal (within the United States) distortions in the manufacturing industries.

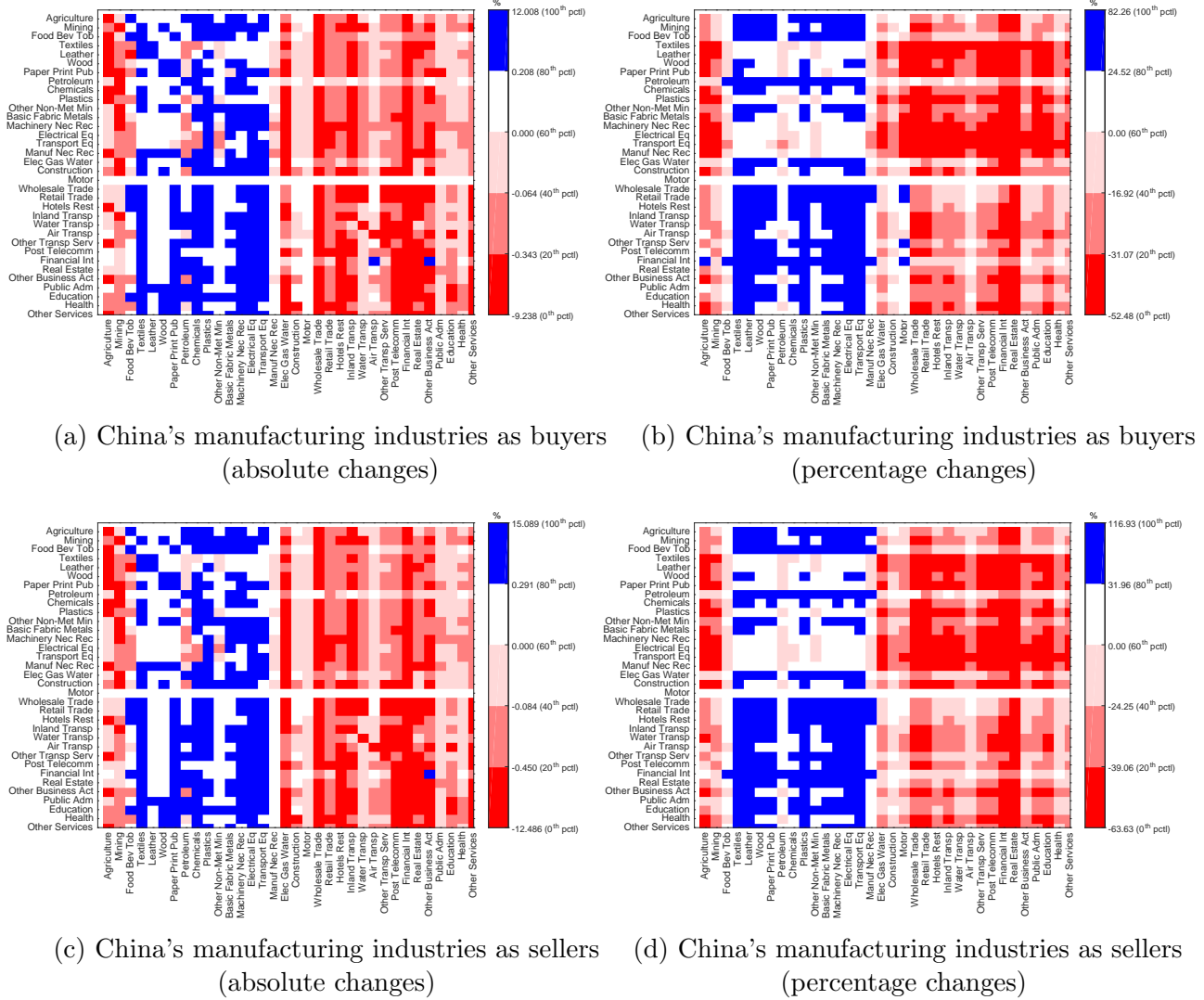
Figure 9: Change in U.S. expenditure shares to U.S. internal distortions in manufacturing



The upper two panels show the effects of the decline in the distortions of selling goods to manufacturing industries from any industry. The lower two panels show the effects

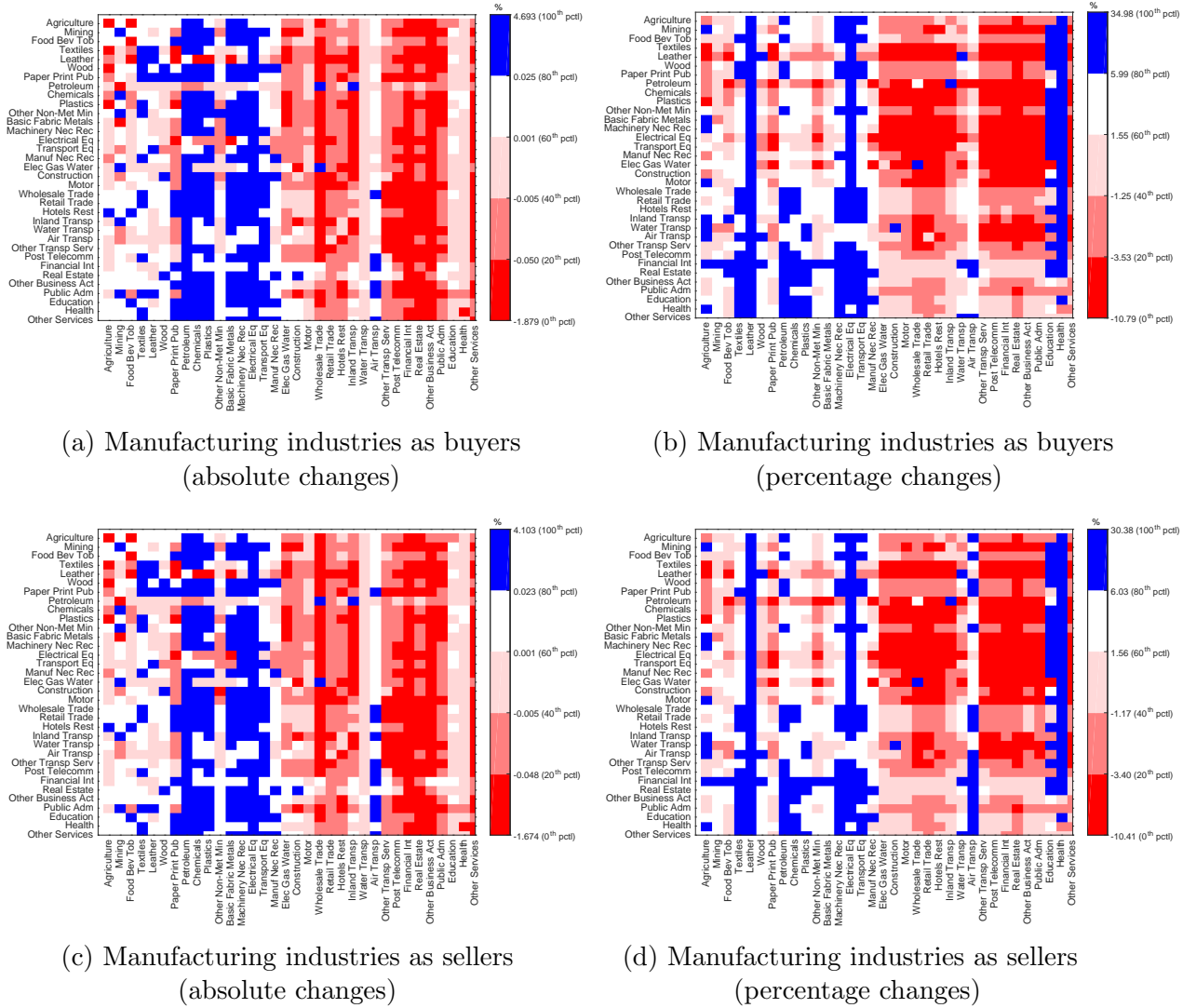
of the decline in distortions of buying goods from the manufacturing industries by any industry. We show the change in expenditure shares to changes in internal distortions in absolute terms on the left panels, and in percent changes on the right panels. Similar to the discussion in the main text, in the upper panels we see how the U.S. input-output table is impacted by changes in distortions in the manufacturing industries. In particular, we observe that manufacturing sectors buy more from other manufacturing sectors. In the lower panels, we observe that all sectors, and especially non-manufacturing industries, substitute inputs from non-manufacturing industries for manufacturing inputs, as the latter are now a cheaper source of inputs after the decline in distortions.

Figure 10: Change in China's expenditure shares to China's internal distortions in manuf.



In Figure 10, we perform the same exercise but for a different economy, China, as a way of confirming whether the findings for the U.S. hold for another economy with a different input-output structure. The four panels in Figure 10 follow the same structure as in Figure 9. In a nutshell, we can see qualitatively similar effects of the changes in internal distortions in China on the Chinese input-output structure. The upper panels show that manufacturing industries tend to buy more from manufacturing sectors, and the lower panels show that manufacturing industries increase their importance as suppliers for the rest of the economy.

Figure 11: Change in U.S. expenditure shares to China's internal distortions in manufact.



These findings confirm a common pattern across countries, in line with the predictions of

our closed-form elasticities. However, we emphasize the fact that the input-output structure of the U.S. economy is different from the Chinese one, and therefore the effects of the changes in distortions at the more disaggregate level are different in these two economies. For instance, the share of manufacturing purchases in total expenditure increases by 9.3 percent in absolute terms in the United States, and 12.3 percent in China. The sectors that increase more the purchases of manufacturing in the United States are Leather and Textiles, while in China the sectors that experience the largest increase in manufacturing purchases are transportation equipment and metals. Overall, these exercises show how our model and the derived elasticities emphasize these heterogenous input-output effects from changes in distortions across countries.

We next present additional measures on the cross-country input-output elasticities. In particular, Figure 11, upper left panel, presents the absolute change in the U.S. input-output structure to changes in internal distortions in the manufacturing industries. The lower panels present the effects of the decline in distortions of buying goods from the manufacturing industries by any industry. These additional measures the cross-country input-output elasticities show similar patterns to the ones discussed in the main text.

A.5.3 World’s GDP elasticity to changes in internal distortions

In Figure 12, we compute the world’s real GDP elasticities with respect to changes in internal distortions in each country. Specifically, each unit in the figure shows the percent change in world’s real GDP from a one percent reduction in distortions in a given industry and country of buying goods from all industries in that country. The change in world’s real GDP is computed by aggregating the changes in real wages across all countries using each country’s GDP as weights. As in the previous figures, each color represents a given percentile. For instance, the blue cells correspond to the country-sectors whose changes in internal distortions would have a larger impact in the aggregate world’s economy. This is the case of most of the sectors in United States and China, as well as some important sectors such as real estate, finance, business activities, and public administration in some developed countries such as Germany, France, Italy, and Japan. Overall, the elasticity of world’s real GDP is affected by both the size of the industry and how interconnected is a given industry in a given country with the rest of the economy, and with the rest of the world, and as a result, the bottom line of this exercise is to shed light on how quantitatively significant the impact on the world’s economy as a whole would be to changes in internal distortions in specific sectors and countries.

Figure 12: World's real GDP elasticity to changes in internal distortions

