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Abstract

Using a dynamic framework with strategic interactions we study the management of an exhaus-tible natural resource when property rights are generally weak. We define generally weak property rights as those under which both the stock of the resource and the revenues from exploiting it are imperfectly protected, due to trespassing and theft respectively. From the legitimate owner’s perspective, trespassing and theft have a fundamental effect on the inter-temporal trade-off governing the extraction decision: extracting the resource today protects it against trespassing but exposes it to theft. Our main results indicate that total depletion of the resource is always decreasing in the intensity of theft, and that when both the owner and the trespassers are affected by theft there is over (under) extraction in equilibrium if theft intensity is low (high).

Keywords: Depletion, exhaustible resources, trespassing, theft, weak property rights.

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1
1 Introduction

Property rights ought to be adequately defined and secured for economic interactions to lead to efficient outcomes. However, in the imperfect world we live in, ill-protected property rights are pervasive across economic activities. Such is the case of the management of exhaustible resources, where property rights have been for long central in the discussion of how to secure the optimal use of these resources. In this context, a deficient protection of property rights is often associated to a problem of common access to the stock of the resource. That is, it is generally assumed that when property rights are poorly protected, agents cannot be effectively excluded from accessing the pool of the resource and this ends up being over-exploited (a tragedy).

Besides the problem of common access to the pool of the resource, the weakness of the property rights system can also be present in other ways. Hotte, McFerrin, and Wills (2013) rightly point out that the failure to fully appropriate the benefits from own’s production, is another manifestation of property rights being insufficiently protected. Put differently, when property rights are weak the optimal management of a depletable resource may not only be threatened by insecure property rights over the stock of the resource, but also by inadequate property rights over the output generated from exploiting the resource.

Take for instance the anecdotes from Spindletop, the first major oil discovery in the U.S. (Yergin, 2008). January 10, 1901 marked the beginning of the Texas oil boom at this very hill in the south of the town of Beaumont. That day, the oil gusher caused by the first successful drilling in the area was so dramatically high that it did not take long for the news to spread across the country. In no time a mass flocked into Beaumont hoping to seize a share of its underground riches. As highlighted by Yergin (2008) “Within months, there were 214 wells jammed in on the hill, owned by at least a hundred different companies” (p. 70). This seemingly indiscriminate access to oil in the ground, would soon have its consequences, “by the middle of 1902... the underground pressure gave out at Spindletop because of overproduction, and specially because of all those derricks on postage-stamp sized plots, and production on the Big Hill plummeted”. At the same time as oil was being rapaciously extracted, the “fortunes” made at the top of the hill were scarcely protected in town; Beaumont was a far cry from a safe haven “... there were two or three murders a night... and there were endless frauds to make sure that money changed hands quickly” (p. 69).

Just as with the Texas oil boom, half a century before news of the gold discoveries in California took little time to spread across America. The California gold rush originally took place in a “stateless” environment. At the outset of the rush in 1848, California was yet to be admitted to the Union. The absence of a political construct defining the status of the geographical location, made it difficult to solve the coordination issue inherent to the public protection of property rights (Anderson & Libecap, 2014). In other words, the emergence of formal institutions in charge of the protection of property rights was hindered by the mere absence of a state, “not only were there no institutions to enforce the laws, there were no laws” (McDowell, 2002, p. 2). Although it has been argued that during the rush informal rules emerged to take the place of formal institutions regulating the access.

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1 A large fraction of first mineral discoveries in America occurred in a situation of “statelessness”. According to Couttenier, Grosjean, and Sangnier (2014) 35% of the counties where minerals were discovered between 1825 and WWII did not officially belong to a state or a colony at the time of the first discovery.
to private property, and that private efforts made up for the absence of publicly provided enforcem-
ment of property rights (e.g., Umbeck, 1977; McDowell, 2002), it is unclear whether these actually
served to restraint trespassing and other property and violent crimes. Clay and Wright (2005) con-
tend that this set of informal rules and enforcement bodies rather “... gave equal attention to the
rights of claim-jumpers as to claim holders, a balance that in practice generated chronic insecurity...”
(p. 155). Clay and Wright (2005) go further and propose that despite the existence of these informal
institutions and enforcement bodies, gold mining during the rush remained closer to an open access
regime. Besides claim-jumping (trespassing), “robberies and assaults also seemed to be on the rise”
(Rohrbough, 1997, p. 218), posing a direct threat on the miner’s output. Moreover, the absence of
a governing body coordinating law enforcement, implied that individual efforts had to be diverted
into the administration of protection and justice (Owens, 2002).

A present-day counterpart of the American gold rush of 1848 is the case of illegal/informal mining
in the developing world (Banchirigah, 2008; Hilson, 2002; Hilson & Potter, 2003). This activity, which
is by no means marginal, is a modern portrayal of generally weak property rights. By definition
the illegal aspect of the mining entails that the legitimate manager of the resource (the government)
cannot effectively exclude intruders from accessing the stock. On top of this all sorts of activities
related to the transformation of the mineral output into cash occur outside the boundaries of the
law. In practice, illegal miners are relatively unprotected against property and violent crime simply
because they cannot recur to the authorities for protection without threatening their own economic
activity.

These pieces of anecdotal evidence share as common theme the exploitation of an exhaustible
natural resource in an environment of weak property rights, where both the stock in the ground and
the output after extraction are at risk. With this as a background our paper analyzes the dynamic
management of an exhaustible resource when property rights are generally weak. Our paper is in
first instance related to a long tradition of contributions to the resource economics literature dealing
with resource management under insecure property rights. This literature has largely focused on the
“common access to the stock” side of the weak property rights story (e.g. Copeland & Taylor, 2009;
Hardin, 1968; Van Long, 2011; Ostrom, 2008).\(^2\) The usual finding is that individuals fail to internalize
the effect of their own use of the resource on the rest of the users, for which common access leads to
resource over-use. In this respect analyzing the effects of the interaction between two embodiments
of a weak property rights system in a resource management problem is interesting in itself. From
a general perspective, this type of analysis is an application of the second best theory (Lipsey &
Lancaster, 1956), in the sense that a new source of inefficiency (imperfectly protected output) is added
to an already imperfect world (in which the stock of the resource is imperfectly protected). The theory
indeed predicts that a world with more imperfections may be preferred from the social perspective.
A straightforward reason for this is that agents may react in opposite ways to different imperfections
and thus the aggregate effect is less intense than that of the separate imperfections.

After observing that the problem of weak property rights goes beyond the “common pool” issue,
Hotte et al. (2013) study a static production problem in which both the input used to produce (“the
common” in the resource literature) and the output are imperfectly protected. As an application of

\(^2\)Copeland & Taylor, 2009 present a model with a common renewable resource in which the property rights regime
emerges endogenously (individuals may decide to “cheat” and over-use the resource).
the second best theorem, their results indicate that in the presence of both sources of imperfection, production can be too high or too low from the social perspective.

Using the interaction of different market imperfections as a starting point, our paper studies the inter-play between two manifestations of a weak property rights system in a dynamic resource extraction model. Specifically, access to the resource stock is not fully secured and the benefits from extraction are imperfectly protected. To the best of our knowledge, this is the first contribution focusing on the interaction between these two types of imperfections in a dynamic setup, and so our main contribution is to explore the effects of an environment of generally weak property rights on the dynamic extraction path of an exhaustible resource. Following Hotte et al. (2013), we refer to the illegitimate extraction of the resource as trespassing and to the appropriation of someone else’s output as theft. At first glance, the dynamic nature of the resource management problem creates a clear distinction between these two. Trespassing affects the stock that is still in the ground, while theft reduces the value of the extracted flow. Therefore, from the legitimate owner’s perspective faster depletion would serve to protect the resource against trespassing, while increasing its exposure to theft. However intuitive, this line of argumentation misses out on some of the central, and fundamentally inter-temporal, trade-offs that lay behind the extraction decision.

The depletion of an exhaustible resource is in essence a consumption-saving problem, in which the benefits and costs from extracting today are weighted against the net benefits of leaving the resource in the ground for future use. Adopting a dynamic perspective generates new insights on the interaction between the two types of inefficiencies. For instance, theft not only reduces the value of what is being currently extracted, but it also reduces the value of what is still in the ground because it is eventually going to be extracted and will potentially be exposed to theft too. So, from the inter-temporal point of view the effect of theft on the extraction path actually depends on whether the intensity of theft changes over time. If theft is expected to remain constantly intense over time, the legitimate owner has no motive to distort her extraction path. However, if theft is expected to decrease in intensity, say because thieves are expected to be captured, the owner would adopt a more conservative position towards the extraction of the resource. Although completely absent in a static analysis, this sort of inter-temporal considerations remain central to any attempt to understand the dynamic channels affecting the management of an exhaustible resource.

This document is organized in four sections including this introduction. In section 2 the theoretical model is set up and solved. In section 3 the main results are analyzed and discussed. Finally, section 4 is devoted to the concluding remarks.

2 Model

2.1 Setup

The model presented here examines how the use of a non-renewable resource is affected by insecure property rights, where the imperfect protection is embodied by two types of distortions. First, the stock of the resource is imperfectly protected. That is, the rightful owner/user of the resource does not have exclusive access to the stock of the resource and other agents can trespass his property and exploit the remaining stock. Second, the proceeds from extraction are unprotected, and so other
agents can appropriate a fraction of the owner’s revenues from extraction.

To illustrate these two types of imperfections we build a continuous time infinite horizon model with three agents: owner \((i)\), trespasser \((j)\), and thief \((h)\). The owner is endowed with a stock \(S_0 > 0\) of an exhaustible resource; the trespasser (while active) also has access to this stock and can extract from it; and, the thief can put effort into appropriating a fraction of the owner’s revenues from extraction. In the following we describe the exact interactions entailed by each type of distortion.

**Trespassing** Initially both the owner and the trespasser have access to the stock of the resource, and they simultaneously decide how much of the resource to extract at each point in time. Instantaneous extraction is denoted by \(R_i\) and \(R_j\) respectively; by extracting \(R_n\) units the owner gets \(\frac{R_i}{1-\theta}\) while the trespasser gets \((1-\Omega) \frac{R_j}{1-\theta}\), with \(\theta \in (1, 2)\). \(\Omega \in \{\omega, 1\}\) reflects the level of institutional strength against trespassing. If \(\Omega = 1\), the trespasser has no incentives to deplete the resource and the resource is fully protected against trespassing. If \(\Omega = \omega < 1\), the trespasser actively participates in the depletion of the resource. Extraction depletes the resource over time: \(S(t) = -R_i(t) - R_j(t)\); and cumulative extraction is constrained by the remaining stock of the resource \(\int_0^\infty (R_i(v) + R_j(v)) \, dv \leq S(t)\).

The assumption here is that the owner and the trespasser individually face an extraction technology constraint. That is, the interaction between the owner and the trespasser is purely of inter-temporal nature (it goes through the depletion of the stock) but, trespassing does not pose an intra-temporal externality on the owner (i.e. trespassing does not drive down the owner’s marginal benefit from extraction). Instead of thinking of trespassing as a problem of a “common stock”, one could in principle approach it as problem of access to a “common market”. In that case, trespassing is equivalent to higher competition, which reduces the owner’s marginal return to extraction. Then, the externality imposed by trespassing is fundamentally intra-temporal. Given that our main interest is to focus on the inter-temporal tradeoffs, we abstract from the “common market” interpretation in order to preserve the transparency of the dynamic mechanisms.

**Theft** After extraction, the owner gets a gross revenue flow of \(\frac{R_i}{1-\theta}\). However, a fraction \(\tau\) of this flow can be appropriated by the thief. This fraction is endogenously determined by

\[
\tau(e_i, e_h) = \frac{(1-\Lambda) e_h}{\Lambda e_i + (1-\Lambda) e_h}
\]

where \(e_h\) is the effort that the thief puts into appropriation and \(e_i\) is the protecting effort by owner; \(e_h\) and \(e_i\) have the same exogenous unit cost of \(w\). The relative efficiency of the protective effort depends on the theft-specific dimension of institutional quality \(\Lambda \in \{\lambda, 1\}\). This is a measure of the de facto protection against theft, with \(\lambda = 1\) being perfect protection.

**Institutional quality** The institutional space in this economy is two-dimensional: \(\Omega\) determines how strong is the institutional environment against trespassing, while \(\Lambda\) determines the institutional

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\(^3\)The upper bound guarantees the existence of an equilibrium in linear strategies in the trespassing game.

\(^4\)The implicit assumption here is that extracted output cannot be securely stored.
strength against theft. Along these two dimensions we can define 4 different regimes of the general institutional quality: i. generally weak institutions, \( \Omega = \omega; \Lambda = \lambda \) (i.e. a regime with theft and trespassing); ii. weak protection of income, \( \Omega = 1; \Lambda = \lambda \) (i.e. a regime with only theft); iii. weak protection of wealth, \( \Omega = 1; \Lambda = \lambda \) (i.e. a regime with only trespassing); iv. strong institutions \( \Omega = 1; \Lambda = 1 \) (i.e. a regime without theft and trespassing).

We assume that initial state is one of generally weak property rights, and from there institutions improve at uncertain times. An institutional improvement in this context means \( \Omega \) or \( \Lambda \) becoming equal to one. Moreover \( \Omega = 1 \) and \( \Lambda = 1 \) are absorptive states, i.e. once institutions become strong in one dimension they remain strong. The speed and direction of the institutional improvement is determined by two types of parameters: i) \( \pi > 0 \) determines the overall speed of change that is, how likely are institutions to improve; ii) the probabilities \( p \in [0, 1] \) and \( q \in [0, 1] \), determine whether this improvement occurs along the trespassing dimension or the theft dimension respectively. More specifically, the hazard of \( \omega \) shifting from \( \omega \) to 1 is \( \pi p \), while the hazard of \( \lambda \) shifting from \( \lambda \) to 1 is \( \pi q \). \( \pi \) is an economy-wide measure of how fast institutions are likely to improve, while \( p \) and \( q \) are crime-specific and can be related to the specific development path of institutions. For instance, the legal system may evolve in such a way that it initially has a bias towards the protection of wealth (property), and eventually shifts its attention to the protection of income.

We assume that all the co-movement in the institutional improvement runs through \( \pi \) (i.e. \( p \) and \( q \) are not related to each other). This way of connecting the likelihood of a regime shift when multiple shifts are possible follows from Sakamoto (2014). Note that regime shifts in this setup are always beneficial for the legitimate owner, as a regime shift translates into the once and for all elimination of a type of crime.

**Objective and Equilibrium** The three agents seek to maximize the Net Present Value of revenues, using the exogenous rate \( r \) as a discount. We look at Markovian strategies, and rely on the Feedback Nash Equilibrium as equilibrium concept. Moreover, in the trespassing game we focus on linear strategies. That is, the extraction strategy of each agent is set to be a linear function of the remaining stock.

### 2.2 Solution

As mentioned above, there are four distinct regimes that can be analyzed depending on the strength of the each of the institutional dimensions. Initially institutions are generally weak and both types of criminals are active, and eventually institutions will become strong and both types of crime will vanish (provided that \( p \) and \( q \) are \( > 0 \)). We do not assume any specific sequence for the path of institutional improvement, meaning that institutions may first improve in any of the two dimensions. As a benchmark we first present the case with strong institutions, then we analyze the “weak protection of income” regime (i.e. when only the thief is active), then the “weak protection of wealth” regime (i.e. when only the trespasser is active), and finally the regime with generally weak institutions (i.e. when there is trespassing and theft).
2.2.1 Strong institutions — No trespassing and no theft)

The problem in this perfect protection regime is a standard one because once both types of crime have been eliminated, no further regime shifts can occur. Given the absence of property protection imperfections in this regime, extraction is optimal from the social perspective. The Hamilton-Jacobi-Bellman (HJB) equation of the owner’s problem is:

\[ rV_i(t) = \max_{R_i} \left\{ \frac{R_i(t)^{1-\frac{1}{\theta}}}{1-\frac{1}{\theta}} - V_i'(t)R_i(t) \right\} \]

Using standard techniques to solve, depletion (defined as \( \frac{R}{S} \)) is

\[ \frac{R_i(t)}{S(t)} = \theta r \]

and the value of the remaining stock is

\[ V_i(S(t)) = \frac{S(t)^{1-\frac{1}{\theta}}}{(1-\frac{1}{\theta})(\theta r)^{\frac{1}{\theta}}} \] (2.1)

2.2.2 Weak protection of income — Only theft [th]

Under this regime, the owner’s problem exhibits two main differences with respect to the perfect protection benchmark: i. the net flow of revenues needs to be adjusted by the total cost of theft (i.e. theft itself and protecting effort); ii. it needs to account for the possibility of a regime shift. As mentioned above, the thief faces the risk of her activity becoming unprofitable, therefore from the owner’s viewpoint, there is the “risk” that the regime will shift from a theft only environment to a fully protected environment.

To address i. remember that the contest over the flow of revenues is characterized by \( \tau (e_i, e_h) = \frac{(1-\Lambda)e_h}{\Lambda e_i + (1-\Lambda)e_h} \). This is, at every point in time the owner and the thief, after observing the flow of revenues that the former gets from extraction, engage in a contest over these revenues. Specifically, the owner keeps a fraction \( 1 - \tau \) of the flow of revenues, while \( \tau \) goes to the thief. These fractions are endogenously determined by the contesting efforts \( (e_i, e_h) \) which are chosen simultaneously, after observing the flow of revenues. As for ii., one can introduce the effect of a regime shift taking into account that the effective hazard of a shift is constant and equal to \( \pi q \), and that the continuation value for the owner is the stock’s value under perfect protection and for the thief is 0. The HJB equations for the owner and the thief respectively are:

\[ (r + \pi q) V_{ih}^h(t) = \max_{\{R_i, e_i\}} \left\{ (1 - \tau (e_i(t), e_h(t))) \frac{R_i(t)^{1-\frac{1}{\theta}}}{1-\frac{1}{\theta}} - w e_i(t) - V_{ih}^h(t)R_i(t) + \pi q V_i(t) \right\} \]

\[ (r + \pi q) V_{ih}^i(t) = \max_{\{e_h\}} \left\{ \tau (e_i(t), e_h(t)) \frac{R_i(t)^{1-\frac{1}{\theta}}}{1-\frac{1}{\theta}} - w e_h(t) - V_{ih}^h(t)R_i(t) \right\} \]
Note that the superindex “th” in the value function stands for theft only, and the absence of it indicates perfect protection. The FOCs with respect to the contesting efforts ($e_i$ and $e_h$) reveal that this is in essence a static problem (with dynamic consequences). If $\Lambda = 1$, there is no contest and the owner retains all the revenues from extraction. If $\Lambda = \lambda$, the optimal appropriation and protection efforts are determined by the following FOCs:

\[ [i] : \frac{(1 - \lambda) \lambda e_h (t)}{(\lambda e_i (t) + (1 - \lambda) e_h (t))^2} \frac{R_i (t)^{1 - \frac{1}{\theta}}}{1 - \frac{1}{\theta}} - w = 0 \]

\[ [h] : \frac{(1 - \lambda) \lambda e_i (t)}{(\lambda e_i (t) + (1 - \lambda) e_h (t))^2} \frac{R_i (t)^{1 - \frac{1}{\theta}}}{1 - \frac{1}{\theta}} - w = 0 \]

In equilibrium

\[ e_i^{th} (t) = e_h^{th} (t) = (1 - \lambda) \lambda \frac{R_i (t)^{1 - \frac{1}{\theta}}}{(1 - \frac{1}{\theta}) w} \]

Therefore $1 - \tau^{th} = \lambda$. With $\Lambda = \lambda$ the owner and the thief effectively engage in a contest over the revenues, and in equilibrium the owner retains a fraction $\lambda$ of the revenues. The owner’s revenues net of theft and the cost of protection are given by

\[ (1 - \tau^{th}) \frac{R_i (t)^{1 - \frac{1}{\theta}}}{1 - \frac{1}{\theta}} - w e_i^{th} (t) = \lambda \frac{R_i (t)^{1 - \frac{1}{\theta}}}{1 - \frac{1}{\theta}} - w e_h^{th} (t) = \lambda^2 \frac{R_i (t)^{1 - \frac{1}{\theta}}}{1 - \frac{1}{\theta}} \]

Using this, the owner’s extraction problem is then characterized by

\[ (r + \pi q) V_i^{th} (t) = \max_{R_i} \left\{ \lambda^2 \frac{R_i (t)^{1 - \frac{1}{\theta}}}{1 - \frac{1}{\theta}} - V_i^{th'} (R_i (t)) + \pi q \left( V_i (t) \right) \right\} \]

The FOC of this problem is

\[ \lambda^2 R_i^{1 - \frac{1}{\theta}} = V_i^{th'} \]

which back into the HJB equation leads to

\[ (r + \pi q) V_i^{th} = \lambda^2 \frac{V_i^{th'1 - \theta}}{1 - \frac{1}{\theta}} + \pi q \left( V_i \right) \]

Using

\[ V_i^{th} = k_i^{th} \frac{\lambda^2 S (t)^{1 - \frac{1}{\theta}}}{1 - \frac{1}{\theta}} \]  \hspace{1cm} (2.2) \]

as a guess for the value function $V_i^{th} (t)$; $V_i (t)$ comes from (2.1). The solution for the $k_i^{th}$ constant, which is also the depletion rate in equilibrium $R / S$ (see FOC), is implicitly given by

\[ z_i^{th} (k_i^{th}) = k_i^{th} + \frac{\theta \pi q}{\lambda^2 (\theta r)^{\frac{1}{\theta}}} k_i^{th} = \theta (r + \pi q) \]  \hspace{1cm} (2.3) \]

From this expression it becomes evident that (as expected) depletion is less rapacious in a more theft-
prone environment: i.e. \( k_i^{th} \) is increasing in \( \lambda \). This follows directly from \( z_i^{th} \) being strictly increasing in \( k_i^{th} \) and decreasing in \( \lambda \). Intuitively, the lower \( \lambda \) the more harmful theft is, and thus the more is there to win from preserving the resource until after theft is eliminated (\( \lambda = 1 \)). Note that with \( \lambda = 1 \), the depletion rate corresponds to the social optimum level \( \theta r \).

**Proposition 1.** Depletion in the theft only regime (\( \Lambda = \lambda < 1 \)) is below the social optimum: \( k_i^{th} \leq \theta r \).

*Proof: See Appendix A.1.*

**Proposition 2.** The more likely is protection against theft to improve the higher the owner’s incentives to preserve the resource: \( k_i^{th} \) is decreasing in \( \pi q \).

*Proof: See Appendix A.1.*

The interpretation of these two propositions is straightforward and stems from: i. a regime shift is favorable from the owner’s viewpoint (i.e. shifting to a world without theft is good news for the owner); and, ii. the problem of the owner is a typical consumption-savings trade-off. Saving the resource (not extracting today) comes at the cost of not consuming today but, has the potential advantage of leaving the resource to be extracted in a safer environment (theft will no longer be a treat at some point in the future). The intuition behind proposition 1 is that the owner slows down extraction while theft is a treat because it reduces marginal benefit of extraction but, this reduction is expected to have a finite end date (the thief is expected to be captured in finite time), thus the owner preserves the resource today with the objective of extracting a larger fraction of it in a potentially safer environment. As to proposition 2, the higher \( \pi q \) the less the owner expects to wait for the protection against theft to improve, and therefore the more willing the owner is to preserve the resource.

### 2.2.3 Weak protection of wealth — Only trespassing (TR)

In the presence of the trespasser two elements need to be accounted for: i. total extraction depends on how much both the owner and the trespasser extract; and ii. as with the thief, the trespasser faces the risk her activity becoming unprofitable (i.e. \( \Omega \) becoming 1).

The HJB equation of the owner’s problem is:

\[
[i]: (r + \pi p) V^{TR}_i (t) = \max_{R_i} \left\{ \frac{R_i (t)^{1-\beta}}{1 - \frac{1}{\beta}} - V^{TR'}_i \left( R_i (t) + R_j (t) \right) + \pi p V_i (t) \right\}
\]

where \( \pi p \) is the effective hazard that \( \Omega \) becomes 1, and \( V_i (t) \) is the continuation value for the owner in case this occurs (see 2.1). The superscript “TR” stands for TRespassing only.

The trespasser’s HJB equation, while \( \Omega = \omega \) is:

\[
[j]: (r + \pi p) V^{TR}_j (t) = \max_{R_j} \left\{ (1 - \omega) \frac{R_j (t)^{1-\beta}}{1 - \frac{1}{\beta}} - V^{TR'}_j \left( R_i (t) + R_j (t) \right) \right\}
\]

Note that the continuation value for the trespasser is 0 (i.e. the NPV of a shift to \( \Omega = 1 \) is 0).

Both agents choose their extraction simultaneously in a non-cooperative way, and they base their extraction decisions on how much of the stock is still in the ground. The FOC with respect to extraction is
\( R_i^{-\frac{1}{\theta}} = V_i' \); \((1 - \omega) R_j^{-\frac{1}{\theta}} = V_j' \)

Plugging this back in the owner’s value function,

\[(r + \pi p) V_i^{TR} = \frac{V_i^{TR1-\theta}}{\theta - 1} - (1 - \omega) \theta V_i^{TR'} V_j^{TR' - \theta} + \pi p V_i(t)\]

and similarly for the trespasser one gets

\[(r + \pi p) V_j^{TR} = (1 - \omega) \theta V_i^{TR'} V_j^{TR' - \theta}\]

Now, as in the previous cases, one can guess value functions for both agents of the form

\[V_i^{TR} = k_i^{TR - \frac{1}{\theta}} \frac{S(t)^{1-\frac{1}{\theta}}}{1 - \frac{1}{\theta}} \]

(2.4)

\[V_j^{TR} = k_j^{TR - \frac{1}{\theta}} (1 - \omega) \theta \frac{S(t)^{1-\frac{1}{\theta}}}{1 - \frac{1}{\theta}} \]

(2.5)

where \(k_n^{TR}\) is an agent-specific constant. This specification of the value function and the FOCs imply that individual depletion \((R_n/S)\) is given by \(k_n^{TR}\). Using the guessed value functions and (2.1):

\[\theta \frac{(r + \pi p) k_i^{TR - \frac{1}{\theta}}}{} = k_i^{TR1-\frac{1}{\theta}} - (\theta - 1) k_i^{TR - \frac{1}{\theta}} k_j^{TR} + \frac{\theta \pi p}{(\theta r)^{\frac{1}{\theta}}} \]

and

\[\theta \frac{(r + \pi p) k_j^{TR - \frac{1}{\theta}}}{} = k_j^{TR1-\frac{1}{\theta}} - (\theta - 1) k_j^{TR - \frac{1}{\theta}} k_i^{TR} \]

Rearranging, the equilibrium values of \(k_i^{TR}\) and \(k_j^{TR}\) are implicitly given by

\[z_i^{TR} \left( k_i^{TR} \right) = (2 - \theta) k_i^{TR} + \frac{\pi p}{(\theta r)^{\frac{1}{\theta}}} k_i^{TR} = \theta \frac{(r + \pi p)}{} \]

(2.6)

\[k_j^{TR} \left( k_i^{TR} \right) = (\theta - 1) k_i^{TR} + \theta \frac{(r + \pi p)}{} \]

(2.7)

Note that because the continuation value for the trespasser is 0, \(\omega\) does not play a role in determining the speed of extraction by any of the two agents, it only affects the trespasser’s valuation of the resource. Moreover, \(\theta > 1\) implies strategic complementarity in the extraction game. This follows from the fact that \(\theta\) is a measure of the curvature of the revenue function. The higher \(\theta\) the less concave the function, and thus the higher the substitution between extraction today and in the future (i.e. the lower the need to smooth individual extraction out). The presence of another agent with access to the stock of the resource lowers the “return” to preserve it, because part of what is left in the ground is going to be extracted by the other agent; in that sense extracting today protects the resource against future trespassing. When \(\theta\) is relatively high the lower need for smooth extraction implies that the “return” motive dominates, therefore more rapacious depletion from one agent results also in more
rapacious depletion by the other.

**Lemma 1.** Given $\theta < 2$, there exists a unique pair of positive constants $k_{i}^{TR}$, $k_{j}^{TR}$ that fulfills the FOCs of the TR problem.

*Proof: See Appendix A.2.*

**Proposition 3.** i) The owner depletes the resource above the socially optimal depletion rate $\theta$; ii) before trespassing becomes unprofitable ($\Omega = 1$) the trespasser depletes the resource faster than the owner.

*Proof: See Appendix A.1.*

$k_{j}^{TR} > k_{i}^{TR}$ is associated to the fact that, as opposed to the owner, the trespasser faces the risk of losing access to the resource. This on the one hand makes the trespasser effectively more impatient than the owner (i.e. the regime shift is costly for the trespasser). On the other hand, it means that the owner attaches a positive probability to the emergence of a regime free of trespassing in finite time; the scrap value of the resource once the trespasser is captured is an increasing function of the remaining stock, which creates incentives for the owner to preserve the resource.

**Corollary 1.** The competition for the depletable stock exacerbates the over-extraction problem pushing the trespasser to deplete the resource even faster than the rate suggested by the “inflated” effective discount (i.e. $\frac{R_{i}}{\tau} > \theta (r + \pi p)$).

**Proposition 4.** The more likely it is that trespassing becomes unprofitable the slower the owner extracts: i.e. higher $\pi p$ implies lower $k_{i}^{TR}$

*Proof: See Appendix A.1.*

At first glance higher $\pi p$ is good news for the owner because of the better prospect of a future free of trespassing. This implies that the owner has stronger incentives to preserve the resource. However, a higher $\pi p$ makes the trespasser effectively more impatient, because of the higher risk of loosing access to the resource. As a result the trespasser becomes more rapacious, which reduces the return to savings for the owner (while active the trespasser extracts a larger fraction of what is left in the ground). Thus, there are two opposing forces determining what the owner should do. On the one hand, the owner wants to preserve the resource for the “trespassing-free” future; on the other hand, the owner does not want to leave the resource exposed to more rapacious trespassing. Which one of the two dominates depends on how concave the revenue function is, and thus on how feasible is the substitution between present and future extraction. With a moderately concave revenue function (i.e. $\theta > 1$), future extraction is a good substitute for extracting today, and delaying extraction is a good strategy: more patience triumphs over lower returns (i.e. the owner prefers to wait until after the trespasser is no longer around).

**2.2.4 Generally weak institutions — Trespassing and Theft (TRth)**

In the initial regime both trespassing and theft are active treats. This regime has three essential characteristics: i. the trespasser extracts from the owner’s stock, so total extraction is the sum of the owner’s and the trespasser’s extraction ($R_{i} + R_{j}$); ii. the thief appropriates a fraction $\tau$ of the owner’s...
revenues, where revenues net of the total cost of theft are $\lambda^2 \frac{R_i^{1-\frac{1}{\beta}}}{1-\frac{1}{\beta}}$; iii. the regime can shift in any of three directions namely, weak protection of revenues with hazard $\pi p (1 - q)$, weak protection of wealth with hazard $\pi q (1 - p)$, and strong institutions with hazard $\pi pq$. Taking these three features into account the HJB equation of the owner’s problem is (suppressing the time dependency to save notation):

$$[i] : (r + \pi) V_i^{TRth} =$$

$$\max_{R_i} \left\{ \lambda^2 \frac{R_i^{1-\frac{1}{\beta}}}{1-\frac{1}{\beta}} - V_i^{TRth'} (R_i + R_j) + \pi \left( p (1 - q) V_i^{th} + q (1 - p) V_i^{TR} + pq V_i + (1 - p) (1 - q) V_i^{TRth} \right) \right\}$$

Using equations (2.1)-(2.6) the owner’s HJB equation reduces to

$$(r + \pi^{TRth}) V_i^{TRth} = \max_{R_i} \left\{ \lambda^2 \frac{R_i^{1-\frac{1}{\beta}}}{1-\frac{1}{\beta}} - V_i^{TRth'} (R_i + R_j) + \pi \kappa_i \frac{S_i^{1-\frac{1}{\beta}}}{1-\frac{1}{\beta}} \right\}$$

(2.8)

where $\pi^{TRth} \equiv \pi (p + q - pq)$ is the effective hazard of “a” regime shift. Moreover, $\kappa_i \frac{S_i^{1-\frac{1}{\beta}}}{1-\frac{1}{\beta}}$ is the owner’s expected continuation value of “a” regime shift (given the three potential directions in which a shift can occur) with $\kappa_i \equiv p (1 - q) \lambda^2 k_i^{th-\frac{1}{\beta}} + q (1 - p) k_i^{TR-\frac{1}{\beta}} + pq (\theta r)^{1-\frac{1}{\beta}}$; $\kappa'_i > 0$. The sign of the derivative with respect to $\lambda$ follows from the equilibrium condition for $k_i^{th}$ (2.3).

In a similarly fashion the trespasser’s HJB equation is

$$[j] : (r + \pi) V_j^{TRth} =$$

$$\max_{R_j} \left\{ (1 - \omega) \frac{R_j^{1-\frac{1}{\beta}}}{1-\frac{1}{\beta}} - V_j^{TRth'} (R_i + R_j) + \pi \left( p (1 - q) V_j^{th} + q (1 - p) V_j^{TR} + pq V_j + (1 - p) (1 - q) V_j^{TRth} \right) \right\}$$

which from equations (2.4) and (2.7) and noting that the continuation value of trespassing becoming unprofitable (i.e. shifting to a regime only with theft or with perfect protection) is 0, can be rewritten as

$$(r + \pi^{TRth}) V_j^{TRth} = \max_{R_j} \left\{ (1 - \omega) \frac{R_j^{1-\frac{1}{\beta}}}{1-\frac{1}{\beta}} - V_j^{TRth'} (R_i + R_j) + \pi \kappa_j \frac{S_j^{1-\frac{1}{\beta}}}{1-\frac{1}{\beta}} \right\}$$

(2.9)

with $\kappa_j \equiv q (1 - p) k_j^{TR-\frac{1}{\beta}}$ and $\pi^{TRth} \equiv \pi (p + q - pq)$ denoting the effective hazard of a regime shift (i.e. the hazard adjusted by the probability that at institutions improve in at least one of the two dimensions).
Using the system of HJB equations (2.8) and (2.9), it is obtained that the FOCs with respect to extraction are
\[ \lambda^2 R_i^{-1 \frac{1}{\theta}} = V_i^{TRh'}, \quad (1 - \omega) R_j^{-1 \frac{1}{\theta}} = V_j^{TRh'} \]
plugging this back into the value functions (2.8) and (2.9)
\[
\left( r + \pi^{TRh} \right) V_i^{TRh} = \lambda^2 \theta \frac{V_i^{TRh'1 - \theta}}{\theta - 1} - (1 - \omega)^{\theta} V_i^{TRh'} V_j^{TRh'} + \pi \kappa_i \frac{S_{1 - \frac{1}{\theta}}}{1 - \frac{1}{\theta}}
\]
\[
\left( r + \pi^{TRh} \right) V_j^{TRh} = (1 - \omega)^{\theta} \frac{V_j^{TRh'1 - \theta}}{\theta - 1} - \lambda^2 \theta V_i^{TRh'} V_j^{TRh'} + \pi \kappa_j \frac{S_{1 - \frac{1}{\theta}}}{1 - \frac{1}{\theta}}
\]
Now, by using \( V_i^{TRh} = k_i^{TRh} s_i \frac{S_{1 - \frac{1}{\theta}}}{1 - \frac{1}{\theta}} = 1 \) and \( V_j^{TRh} = k_j^{TRh} (1 - \omega) s_j \frac{S_{1 - \frac{1}{\theta}}}{1 - \frac{1}{\theta}} = 1 \) as guesses for each of the two value functions to solve this system of Ordinary Differential Equations (ODEs), the following equilibrium system is obtained:

\[
z_i^{TRh} \left( k_i^{TRh}, k_j^{TRh} \right) = k_i^{TRh} + \frac{\theta \pi \kappa_i k_i^{TRh1 - \frac{1}{\theta}}}{\lambda^2} - (\theta - 1) k_j^{TRh} = \theta \left( r + \pi^{TRh} \right) \tag{2.10}
\]
and

\[
z_j^{TRh} \left( k_i^{TRh}, k_j^{TRh} \right) = k_j^{TRh} + \frac{\theta \pi \kappa_j k_j^{TRh1 - \frac{1}{\theta}}}{1 - \omega} - (\theta - 1) k_i^{TRh} = \theta \left( r + \pi^{TRh} \right) \tag{2.11}
\]

**Lemma 2.** If \( \theta < 2 \), there is a unique pair \( \left( k_i^{TRh}, k_j^{TRh} \right) \in \mathcal{R}_+^2 \) that solves the \( z_i^{TRh} = z_j^{TRh} = \theta \left( r + \pi^{TRh} \right) \) system: the equilibrium extraction strategies exist and are unique.

**Proof:** See Appendix A.2.

**Lemma 3.** When only the owner is affected by theft i) \( k_i^{TRh} \) and \( k_j^{TRh} \) are increasing in \( \lambda \).

**Proof:** See Appendix A.2.

## 3 Analysis and discussion

### 3.1 Analysis

The depletion rates under different regimes \( k_i^{th}, k_i^{TR}, k_j^{TR}, k_i^{TRh}, k_j^{TRh} \) and \( k_j^{TRh} \) can be obtained by solving the non-linear system (2.3), (2.6), (2.7), (2.10), and (2.11). Figure 3.1, depicts a numerical example of the depletion rate under all the possible regimes for both the owner and the trespasser, as the theft intensity \( \lambda \) goes from 0 to 1 (i.e. as the distortion imposed by the imperfect protection of revenue flows decreases). The rest of the parameters are set to: \( \theta = 1.5 \) (i.e. this is an illustration of \( \theta > 1 \)), \( p = q = 0.5 \), and \( r = \pi = 0.1 \).
As expected $\lambda$ has no effect on the “TR” regime depletion rates (it does not enter the problem), the “TRth” depletion rates ($k_{TRth}^i$ and $k_{TRth}^j$) are increasing in $\lambda$ and converge to their “TR” counterparts ($k_{TR}^i$ and $k_{TR}^j$) because theft becomes less distortive as $\lambda \to 1$. With respect to the social optimum level of depletion: $k_{TR}^i$, $k_{TR}^j$, and $k_{TRth}^j$ are always above the social optimum $\theta r$; $k_{th}^j$ is always below; and $k_{TRth}^i$ is below $\theta r$ for low values of $\lambda$ (i.e. when theft is very distortive) and it is above $\theta r$ when $\lambda$ is high (i.e. when theft is less distortive).

![Figure 3.1: Depletion for different $\lambda$s](image)

### 3.1.1 Owner and trespasser subject to theft

If both the owner and the trespasser face the threat of theft, and assuming that trespasser and thief engage in exactly the same type of contest over revenues as the owner and the thief, the ODE for the trespasser in the TRth is simply going to be symmetric to that of the owner, where the one for the owner is still given by (2.10) and the trespasser’s becomes:

$$z_{j}^{TRth} \left( k_{TRth}^i, k_{TRth}^j \right) = k_{TRth}^j + \frac{\theta \pi \kappa_j}{\lambda^2 (1 - \omega)} k_{TRth}^j + (\theta - 1) k_{TRth}^j = \theta \left( r + \pi^{TRth} \right)$$

The fundamental difference between this case, and the one in which only the owner is affected by theft is that the de facto protection against theft ($\lambda$) has a direct effect on the trespassers depletion rate (instead of running solely through the effect on the owner’s depletion).
Figure 3.2: Depletion for different $\lambda$s. Both owner and trespasser face theft.

Figure 3.2 depicts $k^T_{TRthi}$, $k^T_{TRthj}$ and the sum of the two, when theft affects both the owner and the trespasser. Evidently, when the distortion induced by theft is large (i.e. $\lambda$ is low), the depletion by both the owner and the trespasser is low, in fact total depletion of the resource is below the social optimum ($\theta r$). On the contrary, if the theft distortion is mild, then the total depletion is too high from the social perspective ($> \theta r$). This means that if theft affects both the owner and the trespasser, in the presence of theft and trespassing the socially optimum level of depletion is attainable if theft occurs in the right measure (i.e. the effects of the two distortions exactly cancel out).

3.1.2 The illegal mining model: only the trespasser is subject to theft

The case of illegal mining seems to be better portrayed by a case in which only the trespasser is directly affected by theft. In this case $\lambda$ affects only the net revenues from extraction for the trespasser. Again we end up with a system of equations from which one can solve the equilibrium levels of depletion. From the owner’s HJB one gets

$$z^{TRth}_i \left(k^T_{TRthi}, k^T_{TRthj}\right) = k^T_{TRthi} + \theta \pi_k k^T_{TRthi} - (\theta - 1) k^T_{TRthj} = \theta \left(r + \pi^{TRth}\right)$$

where $\kappa_i = q \left(1 - p \right) k^{TR}_i \frac{1}{\lambda} + p \left(\theta r\right) \frac{1}{\theta}$. From this expression it is clear that once the trespasser is captured (which conditional on a regime shift occurs with probability $p$) the fate of the thief is irrelevant for the owner’s problem. While from the trespasser’s HJB it is obtained that

$$z^{TRth}_j \left(k^T_{TRthi}, k^T_{TRthj}\right) = k^T_{TRthj} + \frac{\theta \pi_k k^T_j}{\lambda^2 \left(1 - \omega\right)} k^{TRthj} \frac{1}{\theta} - (\theta - 1) k^T_{TRthi} = \theta \left(r + \pi^{TRth}\right)$$

where $\kappa_j = q \left(1 - p \right) k^{TR}_j \frac{1}{\lambda} \frac{1}{\theta} \frac{1}{\theta}$ remains the unchanged. Once the thief is captured, the trespasser has the same valuation for the future.

**Lemma 4.** When only the trespasser is affected by theft, $k^T_{TRthi}$ and $k^T_{TRthj}$ are increasing in $\lambda$ irrespective of $\theta$.

*Proof: See Appendix A.2.*
Interestingly, even if theft is strong enough to completely discourage trespassing (i.e. $\lambda = 0$) the owner’s extraction path in the TRth regime would still be distorted. More precisely, if $\lambda = 0$ and thus $k_{TRth}^i = 0$, $k_{TRth}^j$ is above $\theta_r$ because $k_{TR}^i$ is above $\theta$. From the owner’s perspective, the scenario with $\lambda = 0$ can be described solely in terms of trespassing: initially during the TRth regime there is no trespassing; then if there is no theft trespassing becomes again an active threat; finally the economy would move back to a regime with inactive trespassing. Even if in the TRth regime theft is such that trespassing plays no immediate role on the owner’s problem, the potential for a future regime in which trespassing is active affects current extraction. The mechanism for the distortion in the TRth regime is therefore purely dynamic, and it is actually driven by the same forces that delineate the extraction path in the TR regime. That is, as the owner responds to trespassing by engaging in over-extraction, the owner also over-extracts if there is no current trespassing but there is a potential shift towards a regime with active trespassing.

3.1.3 Total extraction and institutional quality

Irrespective of whether the owner, the trespasser, or both are affected by theft, total extraction is increasing in the de facto protection against theft $\lambda$. The agent(s) directly affected by theft has incentives to deplete the resource faster if theft is weaker; that is the case because the net marginal return to extraction is higher the weaker theft is; this tilts the inter-temporal trade-off towards current depletion. Due to the strategic interaction between the owner and the trespasser, the agent that is not directly affected by theft also reacts to changes in $\lambda$. Specifically, if $\lambda$ increases the agent that is not directly affected by theft also accelerates depletion, therefore total depletion increases with $\lambda$.

**Proposition 5.** Whenever the trespasser and the thief are active: i) total depletion is increasing in $\lambda$ irrespective of which agent (the owner, the trespasser, or both) is subject to theft; ii) if only the owner is directly affected by theft total depletion is always above the social optimum level $\theta_r$; iii) if both the owner and the trespasser are subject to theft there exists a $\lambda^* \in (0, 1)$ such that if $\lambda \geq \lambda^*$ total depletion is $\geq \theta_r$; iv) if only the trespasser is subject to theft total depletion is $> \theta_r$ for any $\lambda > 0$.

*Proof: See Appendix A.1.*

Thus, an improvement in terms of how prone to theft the environment is always leads to more rapacious depletion. This means that in case that only the owner is affected by theft, an improvement in the institutional environment (in terms of a higher $\lambda$) exacerbates the over-extraction problem. Actually, a necessary condition for there to be an imperfect level of institutional quality $\lambda$ such that depletion is optimal from the social perspective is that the trespassers is directly affected by theft. However, this does not mean that the outcome is a first best because resources are inefficiently diverted into protection and theft.

3.2 Discussion

As mentioned above the existence of $\lambda^*$ does not imply that first best is achievable, only that the total depletion would be at is first best level if $\lambda = \lambda^*$. The reason for the the second best nature of the problem under the TRth regime with $\lambda = \lambda^*$ is that even if total extraction is optimal, costly effort is
diverted into the protection-theft contest. Taking into account the total net gain from the protection-theft contest (i.e. sum of revenues net of total effort), the contest is the least efficient when $\lambda = .5$ and the most efficient when $\lambda$ is either 0 or 1.\(^5\) This is the case because the more symmetric the contest the more effort each agent puts into it (i.e. the higher the stakes), but when the de facto protection is clearly favorable to one agent, both agents face little incentives to engage in the contest.

The total distortion imposed by trespassing and theft comes both in the form of a potentially distorted depletion path and of diverted effort into the protection-theft contest. The former being of inter-temporal nature and the latter of intra-temporal one. We know that the closer we are to $\lambda^*$ the less strong the inter-temporal distortion becomes (i.e. the closer is total depletion to the optimum level $\theta r$), and the further away we are from $\lambda = \frac{1}{2}$ the less intense the intra-temporal distortion is. This means that if $\lambda^* < \frac{1}{2}$ (>$\frac{1}{2}$), and $\lambda \in (\lambda^*, .5)$ a reduction (an increase) in $\lambda$ is necessarily efficiency improving.

Coming back to the origin of the trespassing problem, in our model we approach it as a common stock issue. However, one could also study it as a “common market” (e.g. Boyce & Vojtassak, 2008; Datta & Mirman, 1999; Salo & Tahvonen, 2001; Sandal & Steinshamn, 2004) problem issue. However, as opposed to the inter-temporal nature of the “common stock” externality, the “common market” one is essentially intra-temporal. Arguably, the “common market” externality mechanically implies that the owner slows down extraction because of the lower marginal return. In such case the resource “over-use” arises as consequence of a larger number of suppliers; that is, there is no individual “over-use” but, there is “over-use” in the aggregate because of the coordination failure.

In contrast, in the “common stock” case the inter-temporal externality creates two opposing forces delineating the owner’s behavior. On the one hand, it reduces the incentives to preserve the resource because the stock left in the ground would be shared with the trespasser in the future. On the other hand, for extraction-smoothing purposes the owner may act more conservatively to counter the excessively high depletion induced by trespassing; in terms of the consumption-saving tradeoff, the former means that the return to savings is lower reducing the incentives to save, while the latter implies that the owner cannot fully appropriate her own “savings” in the future, hence there is an increased need for “savings” to finance future “consumption”.

4 Concluding remarks

Weak property rights in the management of an exhaustible resource go beyond the “common access” (or “trespassing”) problem typically explored in the resource economics literature. The history of resource rushes (e.g. oil and gold) provides prominent examples of cases in which the legitimate owners (users) of a resource not only had to deal with trespassing but also with theft. The interaction of these two types of distortions, one affecting the stock in the ground and the other affecting the stream of revenues from extracting it, has a significant effect on the inter-temporal trade-off governing the choice of an extraction path.

\[^5\]If one does not take the revenues of the thief into account the contest is the less distortive when $\lambda = 1$ and the most distortive when $\lambda = 0$.\]
resource protects it from trespassing, but exposes it to theft. The dynamic model that we develop in this paper highlights that the dynamic implications, of an environment with generally weak property rights, are rich and transcend this intuitive trade-off. These implications are rooted in the dynamic strategic interactions between agents, as well as in the possibility of shifts towards regimes with better property rights (i.e. regimes with no theft or no trespassing). Among the results we find that an improvement in the institutional quality in terms of a higher probability of catching the trespasser, exacerbates the over-extraction imposed by trespassing itself, if the owner finds it hard to substitute revenues across time. Moreover, an improvement in the institutional quality in terms of a reduction of the theft intensity always leads to more rapacious depletion of the resource. Finally in terms of efficiency, if the trespasser is affected by theft the optimal level of extraction may be achieved. However, the waste inherent to the protection-theft contest implies that we would be in a second best. Efficiency unambiguously improves as the parameter determining the theft intensity in equilibrium moves away from the level that maximizes wasteful effort and towards the level that allows for optimal depletion. Thus in some cases, a more theft-prone environment may be more desirable in terms of efficiency.

As a potential avenue for future research using this type of dynamic framework, with a broader source of imperfect property rights and multiple regime shifts, one could think of how the interaction between governments and private extraction companies is affected by the different alternatives that the former has to get a share of the latter’s revenues. In such a framework instead of trespassing, expropriation would be the source of insecure stocks; and instead of theft, revenues would be subject to taxation. An interesting feature of a setup along these lines is the dual role of the government as both “trespasser” and “thief”. Such a model may shed light on the type of tools that a government should use when trying to maximize the net present revenues that it gets from the riches in the ground once the strategic response of the firm is taken into account: when should the government pursue more or less aggressive taxation? when (if ever) is expropriation a better choice?
References


Appendix

A Proofs

A.1 Propositions

Proposition 1

Proof. Evaluating \( z_i^{th} (.) \) at \( \theta r \)

\[
z_i^{th} (\theta r) = \theta \left( r + \frac{\pi q}{\lambda^2} \right) \geq \theta (r + \pi q) = z_i^{th} (k_i^{th})
\]

With \( z_i^{th} \) being strictly increasing, it follows that \( k_i^{th} \leq \theta r \), with strict inequality whenever theft is a relevant (\( \lambda < 1 \)) regime with finite expected end date (\( q > 0 \)).

Proposition 2

Proof. In equilibrium

\[
\begin{align*}
\frac{\partial k_i^{th}}{\partial \pi q} + \frac{\theta k_i^{th} \frac{1}{\lambda^2 (\theta r)^{\frac{3}{2}}} + \pi q k_i^{th} \frac{1}{\lambda^2 (\theta r)^{\frac{3}{2}}} \frac{\partial k_i^{th}}{\partial \pi q}}{\lambda^2 (\theta r)^{\frac{3}{2}}} &= \theta \\
\left( 1 + \frac{\pi q k_i^{th} \frac{1}{\lambda^2 (\theta r)^{\frac{3}{2}}} \frac{\partial k_i^{th}}{\partial \pi q}}{\lambda^2 (\theta r)^{\frac{3}{2}}} \right) \frac{\partial k_i^{th}}{\partial \pi q} &= \theta \left( 1 - \frac{k_i^{th} \frac{1}{\lambda^2 (\theta r)^{\frac{3}{2}}}}{\lambda^2 (\theta r)^{\frac{3}{2}}} \right)
\end{align*}
\]

Thus, \( \text{sign} \left( \frac{\partial k_i^{th}}{\partial \pi q} \right) = \text{sign} \left( \lambda^{2\theta} \theta r - k \right) \). Evaluating \( z_i^{th} (.) \) at \( \lambda^{2\theta} \theta r \)

\[
z_i^{th} \left( \lambda^{2\theta} \theta r \right) = \lambda^{2\theta} \theta r + \theta \pi q = \theta \left( \lambda^{2\theta} r + \pi q \right) < \theta (r + \pi q) = z_i^{th} \left( k_i^{th} \right)
\]

Given that \( z_i^{th} \) is strictly increasing, it follows that \( k_i^{th} > \lambda^{2\theta} \theta r \longrightarrow \frac{\partial k_i^{th}}{\partial \pi q} < 0 \)

Proposition 3

Proof. i) Evaluate \( z_i^{TR} \) at \( \theta r \): \( z_i^{TR} \left( k_i^{TR} \right) \equiv \theta (r + \pi p) \geq \theta (2 - \theta \theta r + \pi p) = z_i^{TR} \left( \theta r \right) \leftrightarrow \theta \geq \frac{1}{2} \). Given that \( z' > 0 \) then \( k_i^{TR} \geq \frac{\theta (r + \pi p)}{2 - \theta} \).

ii) \( k_j^{TR} \left( k_i^{TR} \right) > k_i^{TR} \) requires \( k_i^{TR} < \frac{\theta (r + \pi p)}{2 - \theta} \). Evaluating \( z_i^{TR} \) at \( \frac{\theta (r + \pi p)}{2 - \theta} \), it is obtained that \( z_i^{TR} \left( \frac{\theta (r + \pi p)}{2 - \theta} \right) = \theta (r + \pi p) + \left( \frac{r + \pi p}{2 - \theta} \right) ^{\frac{1}{2}} > \theta (r + \pi p) \equiv z_i^{TR} \left( k_i^{TR} \right) \). Following the same argument as above \( k_i^{TR} \geq \frac{\theta (r + \pi p)}{2 - \theta} \) and so \( k_j^{TR} > k_i^{TR} \).
Proposition 4

Proof. From \( z_i^{TR} (k_i^{TR}) = \theta (r + \pi p) \) one can obtain:

\[
\frac{\partial k_i^{TR}}{\partial \pi p} = \frac{\theta - \left( k_i^{TR} \right)^{\frac{1}{\beta}}}{(2 - \theta) + \frac{\pi p}{k_i^{TR}} \left( k_i^{TR} \right)^{\frac{1}{\beta}}}
\]

Therefore, the sign of \( \frac{\partial k_i^{TR}}{\partial \pi p} \) is equal to the sign of \( \theta - \left( k_i^{TR} \right)^{\frac{1}{\beta}} \). Using \( z_i^{TR} (k_i^{TR}) \):

\[
\theta \overset{\text{–}}{\sim} \left( k_i^{TR} \right)^{\frac{1}{\beta}} \iff \theta \pi p \overset{\text{–}}{\sim} \theta (r + \pi p) - (2 - \theta) k_i^{TR} \iff k_i^{TR} \overset{\text{–}}{\sim} \frac{\theta r}{2 - \theta}
\]

From \( z_i^{TR} (.) \) and \( z' > 0 \)

\[
k_i^{TR} \overset{\text{–}}{\sim} \frac{\theta r}{2 - \theta} \iff \theta (r + \pi) \overset{\text{–}}{\sim} z_i^{TR} \left( \frac{\theta r}{2 - \theta} \right) \iff \theta \overset{\text{–}}{\sim} \frac{1}{(2 - \theta)^\beta}
\]

\( \theta < \frac{1}{(2 - \theta)^\beta} \) for any \( \theta \in (1, 2) \); therefore, \( k_i^{TR} < \frac{\theta r}{2 - \theta} \) and \( \frac{\partial k_i^{TR}}{\partial \pi p} < 0. \)

Proposition 5

Proof. i) Expressing the equilibrium conditions \( z_i^{TRh} \) and \( z_j^{TRh} \) more generally as [and assuming \( \omega = 0 \) to save notation]:

\[
z_i^{TRh} \left( k_i^{TRh}, k_j^{TRh} \right) = k_i^{TRh} + \theta \pi \mu_i k_i^{TRh} - (\theta - 1) k_i^{TRh} = \theta \left( r + \pi^{TRh} \right)
\]

\[
z_j^{TRh} \left( k_i^{TRh}, k_j^{TRh} \right) = k_j^{TRh} + \theta \pi \mu_j k_j^{TRh} - (\theta - 1) k_j^{TRh} = \theta \left( r + \pi^{TRh} \right)
\]

Where \( \mu_s = \lambda \frac{1}{\beta} \) if agent \( s \) is subject to theft, and \( \mu_s = 1 \) otherwise. Moreover, \( k_i \equiv p (1 - q) \lambda^{2} k_i^{TRh} \cdot \theta + q \left(1 - p\right) k_i^{TRh} \cdot \theta \) if the owner is subject to theft and \( k_i \equiv q \left(1 - p\right) k_i^{TRh} \cdot \theta \) otherwise; \( k_j \equiv q \left(1 - p\right) k_j^{TRh} \cdot \theta \) remains the unchanged across scenarios.

Taking the derivatives of \( z_i^{TRh} \) and \( z_j^{TRh} \) with respect to \( \lambda \) and rearranging terms:

\[
[z_i] : a_1 \frac{\partial k_i^{TRh}}{\partial \lambda} - (\theta - 1) \frac{\partial k_i^{TRh}}{\partial \lambda} = -\theta \pi k_i^{TRh} \frac{\partial (\mu_i k_i)}{\partial \lambda}
\]

\[
[z_j] : a_1 \frac{\partial k_j^{TRh}}{\partial \lambda} - (\theta - 1) \frac{\partial k_j^{TRh}}{\partial \lambda} = -\theta \pi k_j^{TRh} \frac{\partial (\mu_j k_j)}{\partial \lambda}
\]

Where \( a_s \equiv 1 + \pi \mu_s \kappa_s k_s^{TRh} \cdot \theta \) > 1. Using the former in the latter and rearranging terms:
\[
\frac{\partial k_{TRh}}{\partial \lambda} = -\theta \pi \left( k_i^{TRh} \frac{\partial (\mu_i)}{\partial \lambda} + \frac{\theta - 1}{a_i} k_j^{TRh} \frac{\partial (\mu_j)}{\partial \lambda} \right)
\]

by symmetry

\[
\frac{\partial k_j^{TRh}}{\partial \lambda} = -\theta \pi \left( k_j^{TRh} \frac{\partial (\mu_j)}{\partial \lambda} + \frac{\theta - 1}{a_i} k_i^{TRh} \frac{\partial (\mu_i)}{\partial \lambda} \right)
\]

Adding the last two up

\[
\frac{\partial (k_i^{TRh} + k_j^{TRh})}{\partial \lambda} = -\theta \pi \left( (a_j + \theta - 1) k_i^{TRh} \frac{\partial (\mu_i)}{\partial \lambda} + (a_i + \theta - 1) k_j^{TRh} \frac{\partial (\mu_j)}{\partial \lambda} \right)
\]

\[
\frac{a_i a_j - (\theta - 1)^2}{\partial \lambda} > 0
\]

The sign follows from \(\theta < 2\), and \(a_s > 1\) and \(\frac{\partial (\mu_s)}{\partial \lambda} < 0\) for \(s \in \{i, j\}\).

ii) When only the owner is subject to theft:

If \(\lambda = 0\) and therefore \(k_i^{TRh} = 0, k_j^{TRh}\) in equilibrium is given by

\[
z_j^{TRh} (0, k_j^{TRh}) = k_j^{TRh} + \theta \pi k_i^{TRh} = \theta (r + \pi^{TRh})
\]

Evaluating \(z_j^{TRh}\) in \((0, \theta r)\) and comparing with \(z_j^{TRh} (0, k_j^{TRh})\):

\[
z_j^{TRh} (0, \theta r) = \theta r + \theta \pi k_j (\theta r)^\frac{1}{\theta} \geq \theta (r + \pi^{TRh}) = z_j^{TRh} (0, k_j^{TRh})
\]

using the definitions of \(\kappa_j\) and \(\pi^{TRh}\)

\[
\leftarrow q (1 - p) \left( \frac{\theta r}{k_j^{TRh}} \right) \geq q (1 - p) + p
\]

From proposition 3 \(k_j^{TR} > \theta r\), hence \(q (1 - p) \left( \frac{\theta r}{k_j^{TR}} \right) < q (1 - p) + p\). Given that \(z_j\) is increasing in \(k_j\), it is immediate that \(k_j^{TRh} \bigg|_{\lambda=0} > \theta r\). Given that \(k_j^{TRh} \bigg|_{\lambda=1} = k_j^{TR}\), it also follows from3 that \(k_j^{TRh} \bigg|_{\lambda=1} > \theta r\). As \(k_j^{TRh}\) is continuous and monotonic in \(\lambda\) (increasing if \(\theta > 1\) and decreasing if \(\theta < 1\), see lemma 3) and \(k_j^{TRh} \bigg|_{\lambda=0} < k_j^{TRh} \bigg|_{\lambda=1} > \theta r\), the intermediate value theorem implies that \(k_j^{TRh} > \theta r\) for any \(\lambda \in [0, 1]\). Hence total depletion \((k_i^{TRh} + k_j^{TRh})\) is always above the social optimum level.

iii) When the owner and the trespasser are subject to theft:

If \(\lambda = 0\) \(k_i^{TRh} \bigg|_{\lambda=0} = k_j^{TRh} \bigg|_{\lambda=0} = 0\); if \(\lambda = 1\) \(k_i^{TRh} \bigg|_{\lambda=1} + k_j^{TRh} \bigg|_{\lambda=1} > \theta r\), which follows from proposition 3. This means that total depletion in the TRth regime is below \(\theta r\) when \(\lambda \to 0\) (it goes to 0) and it is above \(\theta r\) when \(\lambda \to 1\). As total depletion is continuous and monotonically increasing in \(\lambda\) (see i), from the intermediate value theorem there is a unique \(\lambda^* \in (0, 1)\) such that total depletion is
\[ z_i^0 \leq \theta r \text{ if } \lambda \geq \lambda^*. \]

When only the trespasser is subject to theft:

\[ iv) \]

Assuming \( \lambda = 0 \) (and thus \( k_j^{TRth} = 0 \)), and evaluating \( z_i \) in \( \theta r \) when only the trespasser is affected by theft:

\[ z_i^{TRth}(\theta r, 0) = k_i^{TRth} + \theta \pi k_i^{TRth} \frac{1}{\theta} \geq \theta \left( r + \pi^{TRth} \right) \equiv z_i^{TRth}(k_i^{TRth}, 0) \iff k_i^{TR} \geq \theta r \]

From proposition 3 we know that \( k_i^{TR} > \theta r \). Using the fact that \( z_i \) is increasing in \( k_i^{TRth}, k_i^{TRth} > \theta r \), and thus \( k_i^{TRth} + k_j^{TRth} > \theta r \), for \( \lambda = 0 \). Moreover, as \( k_i^{TRth} \) is increasing in \( \lambda \) (see lemma 4) both \( k_i^{TRth} \) and \( k_i^{TRth} + k_j^{TRth} \) are above \( \theta r \) for any value of \( \lambda > 0 \)

\[ \square \]

A.2 Lemmas

Lemma 1

**Proof.** Note that the \( z_i^{TR}(k_i) \) is strictly increasing in \( k_i \); furthermore, \( z_i^{TR}(0) = 0 \). Thus, there is a unique value \( k_i^{TR} > 0 \) such that \( z_i^{TR}(k_i^{TR}) = \theta (r + \pi p) \). Moreover, for each \( k_i^{TR} \) there is a unique \( k_j^{TR} \); \( k_j^{TR} = (\theta - 1) k_i^{TR} + \theta (r + \pi p) \). If \( \theta > 1 \), \( k_i^{TR} > 0 \) implies \( k_j^{TR} > 0 \). If \( \theta < 1 \), it is required that \( k_i^{TR} = \frac{\theta(r + \pi p)}{1 - \theta} \). This is the case because

\[ z_i^{TR}\left(\frac{\theta (r + \pi p)}{1 - \theta}\right) = (2 - \theta) \frac{\theta (r + \pi p)}{1 - \theta} + \pi \left(\frac{\theta (r + \pi p)}{1 - \theta}\right)^{\frac{1}{\theta}} > \theta (r + \pi p) = z_i^{TR}(k_i^{TR}) \]

which from \( z_i' > 0 \) implies \( k_i^{TR} < \frac{\theta(r + \pi p)}{1 - \theta} \)

\[ \square \]

Lemma 2

**Proof.** If \( \theta > 1 \):

First note that \( \frac{\partial k_j}{\partial k_i} \bigg|_{z_i} > 0 \) and \( \frac{\partial k_i}{\partial k_i} \bigg|_{z_j} > 0 \). Moreover, \( z_i^{TRth}(0, k_j) \rightarrow k_j < 0 \) and \( z_i^{TRth}(0, k_i) \rightarrow k_j > 0 \). Then, it suffices to show that \( \frac{\partial k_j}{\partial k_i} \bigg|_{z_i} > \frac{\partial k_i}{\partial k_i} \bigg|_{z_j} \) always holds.

Differentiating \( z_i^{TRth} \) with respect to \( k_i \):

\[ 1 + \frac{\pi K_i i^{\frac{1}{2} - 1}}{\lambda} - (\theta - 1) \frac{\partial k_j}{\partial k_i} = 0 \]

\[ \frac{\partial k_j}{\partial k_i} \bigg|_{z_i} = \frac{1 + \frac{\pi K_i i^{\frac{1}{2} - 1}}{\theta - 1}}{\theta - 1} > \frac{1}{\theta - 1}; \; \forall k_i > 0 \]

Differentiating \( z_i^{TRth} \) with respect to \( k_j \):
\[
\left. \frac{\partial k_i}{\partial k_i} \right|_{z_j} = \frac{\theta - 1}{\pi \kappa_i k_i^{\text{TR}h_{-1}^{1/2}} - 1 - \omega} > 0 \quad \forall k_i > 0
\]

Then, if \( \theta < 2 \) then \( \left. \frac{\partial k_i}{\partial \kappa_i} \right|_{z_j} > \frac{1}{\theta - 1} > \left. \frac{\partial k_i}{\partial \kappa_i} \right|_{z_j}\) which implies that \( z_i \) and \( z_j \) have a single crossing in \( \mathcal{R}_+^2 \).

\[\]

**Lemma 3**

**Proof.** Taking the derivatives of \( z_i^{\text{TR}h} \) and \( z_j^{\text{TR}h} \) with respect to \( \lambda \) and rearranging terms:

\[
[z_i]: \left. \frac{\partial k_i^{\text{TR}h}}{\partial \lambda} \right|_{\lambda} = \frac{\theta - 1}{1 + \pi \kappa_i k_i^{\text{TR}h_{-1}^{1/2}}} < \theta - 1 \quad \forall k_i > 0 \quad \theta (r + \pi^{\text{TR}h})
\]

\[\]

\[
[z_i]: \left. \frac{\partial k_i^{\text{TR}h}}{\partial \lambda} \right|_{\lambda} = \theta - 1 \quad \frac{\partial k_i^{\text{TR}h}}{\partial \lambda} = -\theta \pi \kappa_i k_i^{\text{TR}h_{-1}^{1/2}} \frac{\partial (k_i / \lambda^2)}{\partial \lambda}
\]

\[\]

\[
\left. \frac{\partial k_i^{\text{TR}h}}{\partial \lambda} \right|_{\lambda} = \frac{-\theta \pi \kappa_i k_i^{\text{TR}h_{-1}^{1/2}} \frac{\partial (k_i / \lambda^2)}{\partial \lambda}}{1 + \pi \kappa_i k_i^{\text{TR}h_{-1}^{1/2}}} > 0
\]

\[\]

This expression is positive because both the numerator and the denominator are positive. From the definition of \( \kappa_i \), \( \frac{\partial k_i}{\partial \kappa_i} \) is decreasing in \( \lambda \), while \( 1 > \frac{(\theta - 1)^2}{\pi \kappa_i k_i^{\text{TR}h_{-1}^{1/2}}} \) for any \( \theta \in (1, 2) \). Using this and \( \left. \frac{\partial k_i^{\text{TR}h}}{\partial \lambda} \right|_{\lambda} \), \( \theta > 1 \) implies \( \left. \frac{\partial k_i^{\text{TR}h}}{\partial \lambda} \right|_{\lambda} > 0 \). That is, as \( \lambda \) increases \( k_i^{\text{TR}h} \) and \( k_j^{\text{TR}h} \) increase.

\[\]

**Lemma 4**

**Proof.** Taking the derivatives of \( z_i^{\text{TR}h} \) and \( z_j^{\text{TR}h} \) with respect to \( \lambda \) and rearranging terms:

\[
[z_j]: \left. \frac{\partial k_j^{\text{TR}h}}{\partial \lambda} \right|_{\lambda} = \frac{\theta - 1}{1 + \pi \kappa_j k_j^{\text{TR}h_{-1}^{1/2}}} \frac{\partial k_j^{\text{TR}h}}{\partial \lambda}
\]

\[\]

\[
[z_j]: \left. \frac{\partial k_j^{\text{TR}h}}{\partial \lambda} \right|_{\lambda} = \frac{\theta - 1}{1 + \pi \kappa_j k_j^{\text{TR}h_{-1}^{1/2}}} \frac{\partial k_j^{\text{TR}h}}{\partial \lambda} = 2 \frac{\theta \pi \kappa_j k_j^{\text{TR}h_{-1}^{1/2}}}{\lambda^3 (1 - \omega)} > 0
\]

\[\]

This together with \( \theta > 1 \) implies \( \left. \frac{\partial k_j^{\text{TR}h}}{\partial \lambda} \right|_{\lambda} > 0 \). That is, as \( \lambda \) increases both \( k_i^{\text{TR}h} \) and \( k_j^{\text{TR}h} \) increase.