Equity weights in the allocation of health care:

The rank-dependent QALY model

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Abstract

This paper introduces the rank-dependent QALY model, a new method of aggregating individual QALY gains in economic evaluations of health care. The rank-dependent QALY model permits the formalization of important concepts of equity in the allocation of health care, such as the fair innings approach, and it includes as special cases many of the social welfare functions that have been proposed in the literature. We characterize the rank-dependent QALY model and argue that its central condition is normatively appealing. An important advantage of the rank-dependent QALY model is that it offers a straightforward procedure to estimate equity weights for QALYs.

Key Words: QALYs, equity, social welfare, rank-dependence.

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1 Introduction

The use of Quality Adjusted Life Years (QALYs)\(^1\) has become standard practice in the analysis of the cost-effectiveness of medical interventions. The use of the QALY measure is commonly associated with the assumption that health care resources should be allocated so as to achieve the maximal health gain as measured by additional QALYs. Many authors (Broome 1988, Dolan 1998, Culyer 1989, Harris 1988, Lockwood 1988, Wagstaff 1991) have raised concerns about the equity implications of this allocation rule.

The basic problem is presented by Williams (1997) in terms of the ‘fair innings’ argument. Is it equitable that an increase of, say, one QALY should be valued equally whether it accrues to someone who is already ‘rich’ in years or to someone whose life in the absence of treatment will be short and miserable? Williams suggests that an appropriate response may be to use ‘equity weights’, but expresses the concern (p. 28) that:

“there is a danger such weights become arbitrary and capricious and come to be used to fudge outcomes in ways that would not be acceptable if their basis were exposed. One safeguard against this is to have some underlying (or ‘over-arching’) general principle enunciated, which can be confronted with evidence so that its various implications can be explored in a quantitative way.”

Williams argues that the fair-innings principle may provide the basis for such an approach, but observes that much work needs to be done in developing the conceptual basis of a ‘fair innings’. In particular, it is important to determine the implications of

\(^1\) Although we speak of QALYs throughout the paper, our analysis also applies to other measures of health, such as healthy-year equivalents (HYEs).
any general principle, considered in isolation or in combination with other principles which may seem desirable. A set of principles may appear unexceptionable when considered separately, but may produce unpalatable implications when considered jointly, or may be mutually inconsistent. The best way to avoid such undesirable outcomes is through the derivation of equity weights from clearly stated conditions. The acceptability or otherwise of these conditions may then be assessed both in isolation and with respect to their joint implications.

The object of this paper is to present a general allocation rule which incorporates equity weights and to derive the conditions on which this rule depends. The equity weight assigned to an individual depends on his rank when individuals are ranked in terms of the number of QALYs they receive. That is, the equity weights depend on the relative positions of the individuals. We refer to this allocation rule as the rank-dependent QALY model. Models of rank-dependence are widely used in decision under uncertainty (Quiggin 1981, Yaari 1987, Schmeidler 1989, Tversky and Kahneman 1992) and the measurement of inequality (Weymark 1981, Ebert 1988, Yaari 1988). Rank-dependence was used to give a preference foundation for the QALY model in Bleichrodt and Quiggin (1997, 1999) and in Miyamoto (1999). Here we introduce the concept of rank-dependence for the social evaluation of QALY-profiles. An important special case of the rank-dependent QALY model is the ‘fair innings’ principle (Williams 1997).

We give a preference foundation for the rank-dependent QALY model and we will argue that the model has normative appeal. An important advantage of the rank-dependent QALY model is that it provides a simple way to estimate equity weights from individual choices. Williams and Cookson (2000) argue that the “great challenge [for health economists] is to bridge the gap between the economic requirement to estimate precisely targeted equity–efficiency trade-offs, and the psychological ca-
pabilities of respondents to think about equity and efficiency in such a tightly defined manner (p.1905).” The rank-dependent QALY model can bridge the gap to which Williams and Cookson refer. The model can incorporate both efficiency concerns and a wide array of equity arguments. Its empirical elicitation is straightforward and not cognitively demanding.

The paper is organized as follows. Section 2 contains notation and model specification. We show that the rank-dependent QALY model can incorporate equity concerns into QALY-based decision making and that it has several important social welfare functions as special cases, including QALY-utilitarianism and Rawls’ maximin rule. An interesting feature of the rank-dependent QALY model is that it can be decomposed into an efficiency term and an equity term. This decomposition may be used to provide a welfare foundation for the measurement of equity in health and health care (van Doorslaer et al. 1993).

Our analysis relies to a large extent on the observation, based on the work of Harsanyi (1953, 1955), Atkinson (1970), and Rawls (1971), that there is a close connection between models of choice under uncertainty and models of social choice. Section 3 presents a translation of de Finetti’s (1931) famous book-making principle to the context of social choices over QALY-profiles. We use the book-making principle to derive conditions for the optimality of QALY-utilitarianism. Section 4 argues that the full form of the book-making principle is too restrictive in social choice. We introduce a weaker version, the comonotonic book-making principle. The comonotonic book-making principle is the central condition in our derivation of the rank-dependent QALY model. We argue that the comonotonic book-making principle is normatively appealing and thereby give a defense for the use of the rank-dependent QALY model in the economic evaluation of health care. Section 5 concludes. Proofs of the results presented throughout the paper are given in the appendix.
2 The Model

2.1 Background

Let \( n \) be the number of individuals in society. Let \( Q_i \) denote the number of QALYs received by individual \( i \). A QALY-profile \( (Q_1, \ldots, Q_n) \) specifies the number of QALYs received by each individual. We denote the set of all QALY-profiles by \( Q \). Occasionally we use the notation \( (p_1, Q_1; \ldots; p_m, Q_m) \). This notation means that proportion \( p_i \) of the population receives \( Q_i \) QALYs, \( i = 1 \ldots m \). A social preference relation \( \succ \) is defined over \( Q \), where \( \succ \) denotes “at least as preferred as.” The relation \( \succ \) reflects a policy maker’s preferences over QALY-profiles. More specific assumptions with respect to \( \succ \) will be made in Sections 3 and 4. A social welfare function \( W \) is a function from the set of QALY-profiles \( Q \) to the real numbers that represents \( \succ \). That is, if for all \( q_1, q_2 \in Q, W(q_1) \geq W(q_2) \) if and only if \( q_1 \succ q_2 \).

The best known social welfare function is the utilitarian form \( W(q) = \sum_{i=1}^{n} \frac{1}{n}Q_i \). This form is generally referred to as QALY-utilitarianism and is widely used in economic evaluations of health care. Harsanyi (1953) gave a defense of utilitarianism in terms of expected utility (EU). Harsanyi assumed that every person could occupy any position in society with equal (known) probability. He showed, under these assumptions, that people who are expected utility maximizers will choose to maximize the sum of utilities across society.\(^2\)

Critics of QALY-utilitarianism have frequently attacked the view that a given increment of QALYs should be valued equally, no matter to whom it accrues. These critics argue that we should rank individuals and apply different weights depending on the relative position of an individual. Rawls, for instance, proposes a principle of

\(^2\)To be precise, Harsanyi derived the function \( W(q) = \sum_{i=1}^{N} \frac{1}{N}U_i(Q_i) \).
justice focusing on the worst-off individual. Like Harsanyi, Rawls derived a social choice rule from preferences under uncertainty. However, Rawls attempted a more detailed consideration of how individual preferences under uncertainty might translate into social decisions. Rawls introduced the idea of a ‘veil of ignorance’, behind which individuals seek to agree on appropriate social rules without knowing what position they will hold in society or what particular preferences they will have. In particular, they do not know what their risk attitudes will be.

A key point of disagreement between Rawls and Harsanyi is that Rawls rejects EU as a basis for individual decision-making under uncertainty. Rawls’ criticisms of utilitarianism as a social choice criterion are closely related to his criticisms of EU as a theory of choice under uncertainty. In particular, Rawls’ (1971) claim that the aggregation process involved in utilitarianism ‘does not take individuals seriously’ follows directly from a rejection of the sure thing principle for choice under uncertainty. On the basis of his criticisms of EU, Rawls suggests the use of a maximin rule under uncertainty. That is, individuals should choose the action under which the harm suffered in the worst state of the world is as small as possible.

Rawls’ approach seems to raise particular difficulties in the context of medical resource allocation. In this context, the worst off at any point in time are those at or near the point of death. The worst off in a lifetime sense are infants who die shortly after birth. There does not seem to be any limit to the resources that can be poured into the objective of achieving marginal extensions of life in circumstances of this kind. Hence, any allocation of medical resources to anyone other than the terminally ill would fail Rawls’ criterion of justice. As noted, Rawls would presumably not

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3In his informal discussion, Rawls suggests that the appropriate focus should be on the worst-off group in the community, representing, say, those in the bottom quintile of the income distribution, rather than on the worst-off person. This idea is not formalised, and seems easier to apply in relation to monetary income than in the case of health status.
advocate such an allocation in practice, but would seek to allocate resources to the worst-off group. But, it is difficult to see how to formalize this idea unless the maximin rule is abandoned.

2.2 The Rank-Dependent QALY Model

Rawls’ critique of utilitarianism has been subject to some reassessment in the light of the literature on generalized expected utility theory (Schmidt, 1998, Starmer, 2000). The general tendency of this literature is to endorse much of the critique of EU theory put forward both by Rawls and by earlier critics such as Allais (1953), but not to accept Rawls’ maximin alternative. Let us therefore formulate an alternative allocation rule that is sensitive to whom health benefits accrue but that is more flexible than Rawls’ maximin principle.

Suppose that the QALY-profile \( q \in Q \) is rank-ordered so that \( Q_1 \geq \ldots \geq Q_n \). Every QALY-profile can be written in this manner by reordering the individual QALY gains in decreasing order. The rank-dependent QALY model evaluates the rank-ordered QALY profile \( q \) as

\[
W(q) = \sum_{i=1}^{n} \pi_i Q_i
\]

where the \( \pi_i \) are equity weights that are defined as 
\[
\pi_1 = w(\frac{1}{n}), \quad \pi_j = w(\frac{j}{n}) - w(\sum_{i=1}^{j-1} \frac{i}{n}), \quad 1 < j \leq n.
\]

The equity weights indicate the attention given to individual \( i \) in the evaluation of QALY profile \( q \). The function \( w \) is defined from \([0,1]\) onto \([0,1]\) and satisfies \( w(0) = 0, w(1) = 1, \) and \( w(p) \geq w(q) \) if \( p > q \). It follows from the definition of \( w \) that the equity weights are nonnegative and sum to one.

The intuition behind the rank-dependent QALY model is that the policy maker’s concern for an individual (or group of individuals) depends on how badly-off the in-
individual is in comparison with the other individuals in society. To illustrate, consider a QALY profile \((1/3, 60; 1/3, 50; 1/3, 40)\) where one-third of the individuals receive 60 QALYs, one-third receive 50 QALYs and one-third receive 40 QALYs. Suppose that the policy maker is averse to inequalities in health. Then the policy maker shows more concern for the group getting 40 QALYs, than for either of the other two equally sized groups, because the group getting 40 QALYs is the worst-off. Say that the equity weight assigned to the group getting 40 QALYs, \(\pi_3\), is equal to 1/2. Then the policy maker will place the same value on an increment of one QALY accruing to the group getting 40 QALYs as on an increment of one QALY accruing to each of the other groups \((\pi_1 + \pi_2 = 1/2 \text{ if } \pi_3 = 1/2)\). Because the policy maker is inequality averse, he will pay most of his remaining attention to the group receiving 50 QALYs (say \(\pi_2 = 1/3\)) and the attention paid to the group with the largest QALY gain will be relatively small \((\pi_1 = 1/6)\). If the policy maker has another QALY to distribute then, being inequality averse, he will give it to the group getting 40 QALYs because they have the highest equity weight.

Suppose next that the group who first got 50 QALYs now only gets 30 QALYs, so that the QALY profile becomes \((1/3, 60; 1/3, 30; 1/3, 40)\). Under the rank-dependent QALY model, the weights accruing to the three groups depend on the ranking of the groups in terms of the number of QALYs they obtain. Since the ranking has changed, the equity weights also change, to \((1/6, 1/2, 1/3)\). Thus even though nothing has changed for the group receiving 40 QALYs (the amount of QALYs and the size of the group have not changed), their equity weight decreases because they are no longer the worst-off group.

The policy maker’s attitudes towards inequality in health outcomes can be characterized through the function \(w\). We say that a policy maker is inequality-averse if his preferences satisfy the Pigou-Dalton Principle of Transfers: a transfer of QALYs
from people who receive relatively many QALYs to those who receive relatively few QALYs is always desirable, as long as the transfer does not affect anybody’s position in the QALY ranking. The policy maker is *inequality seeking* if the above transfer is considered undesirable; he is *inequality neutral* if he is indifferent with respect to the above transfer.

Consider again the QALY profile \((1/3, 60; 1/3, 50; 1/3, 40)\) of the above example. We noted that if the policy maker is inequality-averse then \(\pi_1 = w(1/3) < 1/3\) and \(\pi_3 = 1 - w(2/3) > 1/3\), so that \(w(2/3) < 2/3\). Hence, to get the (inequality-averse) preferences of the above example we must have \(w(1/3) < 1/3\) and \(w(2/3) < 2/3\). This suggests as a general rule that to have inequality aversion we must have \(w(p) < p\) for all \(p \in (0, 1)\). It can be shown that this general rule indeed holds true (e.g. Quiggin 1993). Figure 1 displays a function \(w\) that has \(w(p) < p\) for all \(p \in (0, 1)\). Note that \(w\) is convex. Hence, we obtain as a general rule that inequality aversion corresponds to a convex function \(w\). It can be shown in a similar way that a concave \(w\) corresponds to inequality seeking. If \(w\) is linear, so that \(w(p) = p\) for all \(p \in (0, 1)\), then the policy maker is inequality neutral.

![Figure 1 here](image)

**2.3 Special Cases**

The rank-dependent QALY model is both consistent with QALY-utilitarianism (let \(w(j) = j\) for all \(j \in [0, 1]\)) and with Rawls’ maximin rule (let \(w(\sum_{i=1}^{n-1} \frac{1}{n}) = 0\)). It can also accommodate all social welfare functions that lie between QALY-utilitarianism and maximin, that is, social welfare functions that give different weight to individuals that are ranked differently but do not give all weight to the worst-off individual. For example, the rank-dependent QALY model is consistent with the social welfare
functions depicted in Figures 1, 3, 5, 8, 11 and 12 in Williams and Cookson (2000). The social welfare functions in Williams and Cookson (2000) that the rank-dependent QALY model cannot accommodate are those in which the equity weights are (partially) determined by other factors than the health of a person and those that have no maximand.

The rank-dependent QALY model is consistent with several of the proposals for weighting QALYs that have been put forward in the health economics literature. For example, if individuals are ranked according to lifetime QALYs, the rank-dependent QALY approach formalizes Williams' (1997) fair innings approach. The social welfare function proposed by Dolan (1998) is a special case of the rank-dependent QALY model if QALYs are not evaluated linearly but by the logarithmic function.

2.4 An Abbreviated Social Welfare Function

The rank-dependent QALY model permits to express the social welfare associated with a QALY profile as the product of the mean number of QALYs after implementation of the profile and a number that reflects the equality rating of the QALY profile. This decomposition makes explicit the point that there is an efficiency–equity trade-off in the rank-dependent QALY model.

The Lorenz curve for health is defined as $L\left(\frac{i}{n}\right) = \sum_{i=j}^{n} \frac{1}{n} \frac{Q_i}{\mu_q}$ where $\mu_q$ denotes the mean number of QALYs after the implementation of profile $q$. The Lorenz curve for health is comparable with the well-known Lorenz curve for income and indicates the shares of the total number of QALYs received by fractions $\frac{1}{n}, \frac{2}{n}, \ldots$ cumulated upwards from the individual (or group) who receives the lowest number of QALYs. Define $\gamma = \sum_{i=1}^{n} \frac{1}{n} \frac{Q_i}{\mu_q} \pi_i$. The term $\frac{1}{n} \frac{Q_i}{\mu_q}$ indicates the slope of the Lorenz curve for health at the point $\sum_{i=j}^{n} \frac{i}{n}$. The terms $\frac{1}{n} \frac{Q_i}{\mu_q} \pi_i$ indicate how much weight the policy maker gives to (changes in) QALY shares at different points of the Lorenz curve for health.
If the policy maker is inequality-averse then he will pay more attention to the QALY shares of those individuals who are worst-off. It follows that for two QALY profiles $q^1, q^2 \in Q$, if the policy maker is inequality-averse then $\gamma_{q^1} > \gamma_{q^2}$ indicates that $q^1$ is closer to complete equality than $q^2$. Hence, the term $\gamma_q$ can be interpreted as the equality rating of QALY profile $q$. Figure 2 gives a simple illustration that $\gamma$ can be interpreted as an equality rating. The figure shows the Lorenz curves for health corresponding to two QALY profiles $q^1$ and $q^2$. Profile $q^1$ is more equal than profile $q^2$ because its Lorenz curve for health $(L_{q^1})$ lies within the Lorenz curve for $q^2$ $(L_{q^2})$. The slope of $L_{q^1}$ exceeds that of $L_{q^2}$ for $p < 0.5$ and falls short of the slope of $L_{q^2}$ for $p > 0.5$. If the policy maker is inequality-averse, he gives more weight to the lower part of the Lorenz curve for health (the part for which $p < 0.5$) and therefore $\gamma_{q^1} > \gamma_{q^2}$. Hence, evaluated on the basis of $\gamma$, $q^1$ is more equal than $q^2$, and thus $\gamma$ can be interpreted as an equality rating.

[Figure 2 here]

The rank-dependent QALY model implies that the social welfare of QALY profile $q, W(q)$, is equal to $\mu_q \gamma_q$. That is, the social welfare of profile $q$ can be expressed as the product of an efficiency term (the mean number of QALYs after implementation of the profile) and an equity term (the equality rating of the profile). The most common instrument for assessing equality is the Gini coefficient. The Gini coefficient of equality is defined as twice the area under the Lorenz curve. It can be shown that if $\pi_i = \frac{2i-1}{n^2}$, the equality rating $\gamma$ is equal to the Gini coefficient of equality (Blackorby and Donaldson 1978). Hence, the rank-dependent QALY model contains the social welfare function underlying the Gini coefficient as a special case.\footnote{For this reason Weymark (1981) refers to the rank-dependent model as the generalized Gini model.}
Gini coefficient and related indices, such as the Kakwani index, are widely used in the measurement of equity in health and health care (van Doorslaer et al. 1993). This work has been criticized for having no foundation in welfare economics. The rank-dependent model can serve to provide such a welfare economic foundation. This topic, however, is beyond the scope of the present paper.

3 The Book-making Principle

Let us now turn to the characterization of the rank-dependent QALY model. Crucial in our characterization is a consistency condition, which is a reformulation of de Finetti’s book-making principle (de Finetti 1931) for social evaluations of QALY profiles. The book-making principle is based on the idea that a number of good decisions, when taken together, should still be good. In terms of the economic evaluation of health care, it means the following. Suppose that the policy maker has performed a given number, say \( m \), comparisons between QALY profiles. Abstracting from differences in costs, we may say that the policy maker has performed \( m \) different cost utility analyses. Suppose that in the first cost utility analysis profile \( q_1 \) is preferred to profile \( r_1 \) \( (q_1 \succ r_1) \); in the second cost utility analysis \( q_2 \) is preferred to \( r_2 \); and so on for all the \( m \) comparisons. Hence, for all \( m \) comparisons, implementing \( q_j \), \( j = 1, \ldots, m \), is a good thing. It then seems natural to expect that the joint implementation of \( q_1, \ldots, q_m \) will lead to a more favorable QALY profile than the joint implementation of \( r_1, \ldots, r_m \). If on the other hand the joint implementation of \( q_1, \ldots, q_m \) leads to a lower number of QALYs for each individual than the joint implementation of \( r_1, \ldots, r_m \), it appears that some inconsistency has arisen. The book-making principle excludes this apparent inconsistency. That is, the book-making principle excludes the possibility that when the recommendations of a number of cost utility analyses are carried out jointly, social welfare actually decreases. The book-making principle can
be considered as a consistency condition for cost-utility analysis. The book-making principle is implicitly assumed in the economic evaluation of health care. All we do here is to make this assumption explicit.

3.1 QALY-utilitarianism

We assume that the preference relation \( \succeq \) is a weak order; that is, it is complete (for all \( q, r \in Q \), either \( q \succeq r \) or \( r \succeq q \)) and transitive (for all \( q, r, s \in Q \), if \( q \succeq r \) and \( r \succeq s \), then \( q \succeq s \)). We further assume that there exists a constant equivalent for each QALY-profile. The constant equivalent of QALY profile \( q = (Q_1, ..., Q_n) \) is the number of QALYs \( c_1, ..., c_n \) which, if distributed equally, would give the same level of social welfare as \( q \). That is, \( (c_1, ..., c_n) \succeq (Q_1, ..., Q_n) \). The concept of a constant equivalent is commonly used in the welfare economics literature. For example, Kolm (1969) uses the notion of representative income which, if equally distributed among the individuals results in the same social welfare as the existing income distribution. Atkinson (1970) called this income level the equally distributed equivalent income.

Denote by \( \sum_{j=1}^{m} Q_i^j \) the number of QALYs that individual \( i \) obtains when QALY profiles \( q_1, ..., q_m \) are jointly implemented. Similarly, \( \sum_{j=1}^{m} R_i^j \) denotes the number of QALYs that individual \( i \) obtains when QALY profiles \( r_1, ..., r_m \) are jointly implemented.

**Definition 1** A book consists of the preferences \( q_i^j \succeq r_i^j \), \( j = 1, ..., m \), but \( \sum_{j=1}^{m} Q_i^j < \sum_{j=1}^{m} R_i^j \) for all individuals \( i \).

As we explained above, the aim of the book-making principle is to exclude the possibility of books. Hence, we say that the book-making principle holds if no book can be made.
Let $\rho$ be a permutation, that is, a function which specifies a reordering of the individuals in society. Let $q_\rho$ denote the QALY profile that is obtained after application of permutation $\rho$. For example, if $q = (2, 3)$ and $\rho(1) = 2$, $\rho(2) = 1$ then $q_\rho = (3, 2)$. The preference relation satisfies anonymity if for all permutations $\rho$ and for all QALY profiles $q \in Q$, $q \prec q_\rho$. In words, anonymity says that permuting the QALY gains among the individuals does not affect social preference. It implies that social preference does not depend on the identity of a QALY recipient. Anonymity is widely used in social choice theory and is generally considered appealing (Sen 1970).

We can now state:

**Theorem 2** The following two statements are equivalent.

1. QALY utilitarianism holds.

2. The preference relation $\succ$ is a weak order that satisfies anonymity, for each QALY profile there exists a constant equivalent, and the book-making principle holds.

\[ \square \]

A proof of Theorem 2 is given in the appendix.

## 4 The Comonotonic Book-making Principle

Theorem 2 is a powerful result, perhaps surprisingly so. The assumptions of weak order, anonymity, and the existence of a constant equivalent are widely accepted, while the book-making argument seems a necessary consistency requirement for economic evaluations of health care. However, Theorem 2 shows that these four, seemingly innocuous, conditions imply that the social welfare function must take the
QALY-utilitarian form. Hence, Theorem 1 provides a rationale for the use of QALY-utilitarianism in the economic evaluation of health care. To justify the use of equity weighting QALYs, at least one of the four conditions must be modified. We will consider modifying the book-making principle, because the other three conditions are reasonable. We will argue that the book-making principle stated above, although plausible at first sight, is too strong because it does not pay attention to the relative position of individuals and therefore ignores equity considerations. Instead we will propose a weaker version of the book-making principle, which takes account of the ranking of individuals in terms of QALYs and thereby incorporates equity concerns.

Let us first give an example, which illustrates why we believe that the (unrestricted) book-making principle is too strong.

**Example 3** Consider two individuals who are identical in all respects. Let \( q_1 = (20, 0) \). That is, QALY-profile \( q_1 \) gives 20 QALYs to the first individual and nothing to the second. Let \( q_2 = (0, 20) \) be the “mirror-image” of \( q_1 \), that is, the QALY-profile that gives 20 QALYs to individual 2 and nothing to individual 1. If the policy maker is inequality-averse then the following preferences are natural:

\[
(9, 9) \succ (20, 0)
\]

and

\[
(9, 9) \succ (0, 20)
\]

But \( (9, 9) + (9, 9) = (18, 18) \) \( \prec (20, 20) = (20, 0) + (0, 20) \) in violation of the book-making principle.

The book-making principle is violated in Example 3 because the inequality in \( q_1 \) is neutralized by the inequality in \( q_2 \) and vice versa. In other words, the two profiles are complementary. To exclude such violations of the book-making principle, we must...
exclude the possibility of complementarity between the profiles. Complementarity cannot arise if the rank positions of individuals (when they are ranked in terms of their QALYs) do not vary across profiles. QALY-profiles in which each individual occupies the same rank position are comonotonic. Formally, two QALY profiles \( q^1 = (Q^1_1, ..., Q^1_n) \) and \( q^2 = (Q^2_1, ..., Q^2_n) \) are comonotonic if there are no \( i, j \in \{1, ..., n\} \) such that \( Q^1_i > Q^1_j \) and \( Q^2_i < Q^2_j \). It is easily verified that the profiles \( q^1 \) and \( q^2 \) in Example 3 are not comonotonic. Comonotonicity was introduced by Schmeidler (1989) for decision under uncertainty. Ben-Porath and Gilboa (1994) used comonotonicity to characterize preferences over income distributions.

Example 3 suggests a generalization of the book-making principle. A comonotonic book is a book as in Definition 1 with the extra restriction that the QALY profiles \( \{q^1, ..., q^m, r^1, ..., r^m\} \) are comonotonic. The comonotonic book-making principle then says that no comonotonic books are possible. Because the profiles \( q^1 \) and \( q^2 \) in Example 3 are not comonotonic, Example 3 does not violate the comonotonic book-making principle. The comonotonic book-making principle only excludes the existence of books for profiles that are comonotonic, that is, for profiles for which it is not possible to neutralize the inequality in one profile by the inequality in the other.

We are now in a position to characterize the rank-dependent QALY model.

**Theorem 4** The following two statements are equivalent.

1. The rank-dependent QALY model holds.

2. The preference relation \( \succcurlyeq \) is a weak order, for each QALY profile there exists a constant equivalent, and the comonotonic book-making principle holds.
A proof of Theorem 4 is given in the appendix. Note that we no longer impose anonymity. Anonymity is implied by the other three conditions. In the rank-dependent QALY model the weight an individual receives in social welfare evaluations depends on his rank. Individuals that occupy the same rank will get the same weight regardless of who they are. Hence, the identity of individuals plays no role in social welfare judgments, which is what anonymity asserts.

Theorem 4 offers a convincing case for the use of the rank-dependent QALY model in economic evaluations of health care. As noted before, weak order and the existence of a constant equivalent are weak and generally accepted conditions. The crucial condition in the representation is therefore the comonotonic book-making principle. The comonotonic book-making principle implies that if we follow the recommendations of $m$ cost utility analyses that do not affect the relative positions of individuals in society, society cannot be made worse off (in the sense that every individual is worse off). That is, profiles are compared only if they yield the same ranking of individuals. We may say, somewhat loosely, that the comonotonic book-making principle is equivalent to the book-making principle restricted to cases when no equity concerns apply. We believe that this is a normatively appealing principle.

5 Conclusion

This paper has introduced a new model to evaluate QALY-profiles, referred to as the rank-dependent QALY model. The rank-dependent QALY model provides a way to incorporate equity principles into cost-utility analysis. For example, it allows the specification of decision rules that treat people differently depending on their number of lifetime QALYs and can therefore formalize the ideas of Williams (1997). As such, the rank-dependent QALY model may serve as a framework within which a range of ideas about the equity weighting of QALYs can be evaluated and compared. The
The rank-dependent QALY model is consistent with many types of social welfare function that have been proposed in the literature, including QALY-utilitarianism, Rawls’ maximin, and the Gini social welfare function.

The central preference condition in the characterization of the rank-dependent QALY model is the comonotonic book-making principle. We believe that the comonotonic book-making principle is normatively appealing. The fact that the rank-dependent QALY model depends on normatively appealing conditions lends further support to its use in economic evaluations of health care.

Let us next turn to the empirical implications of the rank-dependent QALY model. An important advantage of the rank-dependent QALY model is that it offers a straightforward procedure to estimate equity weights from preferences over comonotonic QALY profiles. Several papers have argued that equity weights should be applied to QALYs, but relatively little empirical work has been done (Williams and Cookson, 2000). The work that has been done is somewhat ad hoc in nature. Typically, a specific social welfare function is selected a priori and the equity weights are estimated under the assumption that this function represents social preferences. Scarce attention is paid to the equity implications of the choice of the social welfare function. We have argued that the rank-dependent QALY model has normative appeal and is consistent with important concepts of equity in health. We therefore believe that future empirical studies should base their measurement of equity weights on the rank-dependent QALY model.

A way to extend the rank-dependent QALY model is to make the equity weights depend not only on individuals’ ranks but also on the number of QALYs they receive. This allows, for example, to have equity weights that vary with the severity of illness. A disadvantage of doing so is that the equity weights come to depend more on the outcomes, i.e. QALYs. This dependence makes the estimation of the equity weights
more complicated. In the rank-dependent QALY model the equity weights have to be elicited once and can then be applied to all social evaluations. This is no longer possible if the equity weights depend also on the number of QALYs. Hence, there is a trade-off between greater generality and practical applicability.

Williams and Cookson (2000, p.1907) conclude that “it will be a major step forward to find a way of translating people’s considered preferences about equity in health into measurements that can give clear and concrete guidance to social decision-makers in this field.” The rank-dependent QALY model provides such a translation.

6 Appendix

6.1 Proof of Theorem 2

That (1) implies (2) is straightforward. Hence, we assume (2) and derive (1). By the proof of Theorem 5 in Diecidue and Wakker (2001), weak order, the existence of a constant equivalent, and the book-making principle imply that \(W(q) = \sum_{i=1}^{n} p_i Q_i\) for nonnegative and uniquely determined \(p_i\) that sum to one. Let \(i, j \in \{1, ..., n\}\) and let \(q\) be a QALY profile that has \(Q_i \neq Q_j\). Consider the profile \(q^1\) which is obtained from \(q\) by giving \(Q_i\) to individual \(j\) and \(Q_j\) to individual \(i\) and leaving the QALY gain of all other individuals unchanged. By anonymity, \(q \sim q^1\), and, because \(Q_i \neq Q_j\), we must have \(p_i = p_j\). Because \(i\) and \(j\) were chosen arbitrarily, \(p_i = p_j\) for all \(i, j \in \{1, ..., n\}\). This establishes QALY-utilitarianism.

\(\square\)

6.2 Proof of Theorem 4

That (1) implies (2) is straightforward. Hence, we assume (2) and derive (1). Let \(\rho\) be a permutation of \(\{1, ..., n\}\). Define \(Q^\rho = \{q \in Q : Q_{\rho(1)} \succeq ... \succeq Q_{\rho(n)}\}\). Because all
\( q \in Q^\rho \) are comonotonic, the book-making principle holds on \( Q^\rho \), and, by Theorem 5 in Diecidue and Wakker (2001), there exists an additive representation \( W^\rho(q) = \sum_{i=1}^{n} p_i^\rho Q_i \) for \( \succ \) on \( Q^\rho \) with the \( p_i^\rho \) uniquely determined, nonnegative, and summing to one. Define \( w^\rho(\frac{j}{n}) = \sum_{i=1}^{j} p_i^\rho \). This gives the rank-dependent QALY model on \( Q^\rho \).

For any two permutations \( \rho \) and \( \rho' \), the common domain \( Q^\rho \cap Q^{\rho'} \) is nonempty. (It contains for example the constant QALY profiles.) The additive representations \( W^\rho(q) \) and \( W^{\rho'}(q) \) coincide on this common domain. This implies that \( w^\rho = w^{\rho'} = w \) for all permutations \( \rho, \rho' \). Hence, the rank-dependent QALY model holds on \( Q \).

\[ \square \]

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**References**


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