Price Regulations in a Multi-unit Uniform Price Auction*

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Abstract

Inspired by recent regulations in the New York ICAP market we examine the effect of different price regulations on a multi-unit uniform price auction. We investigate a bid cap and a bid floor. Given sufficiently high total capacities general bid caps always ensure that the market price does not exceed the price cap whereas a selective bid cap for only the larger firms, does not guarantee this outcome. A sufficiently high bid floor always destroys pure strategy equilibria with equilibrium prices above the marginal costs, no matter whether the floor applies to all or only to relatively small firms.

Keywords: Electricity, Capacity Markets, Price Regulation.
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1 Introduction

Multi-unit uniform price auctions are often used in electricity markets to determine market prices. In a typical day-ahead wholesale markets this

*Very preliminary version, please do not cite without the permission of the author.
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1The Electricity Pool in England and Wales before the reform in 2001, the Nord Pool in Scandinavia, the Spanish wholesale market, as well as the NYISO and the ERCOT
usually means that providers of electricity bid the price quantity combinations into the market that reflects their willingness to provide electricity to the market at a particular hour of the next day. These providers are mainly electricity generating firms but this could also be retailers who bought more electricity on future markets for that particular hour than their customers most likely demand or pure traders. The demand side of the market could either be represented by the system operator who confronts the aggregated supply schedules with a total demand forecast of that hour or it could be an aggregated demand schedule that the system operator constructs from price quantity bids representing the willingness to buy by, again electricity retail firms, large consumers of electricity, electricity generating firms which committed to higher production quantities in future markets for that particular hour than they most likely can or want to deliver, or pure traders.

Independent of the details of the particular market, the uniform price auction always means that the system operator determines the price where demand and supply is balanced. Finally all those providers who were willing to sell quantities below this balancing price will sell these quantities. Those providers who were willing to sell quantities exactly at the balancing price are rationed such that supply and demand equals. What makes the multi-unit auction a uniform price auction is that all traded quantities will be paid the balancing price, no matter whether the particular supplier demanded it or not. Those bidders who wanted to sell quantities at a higher price than the balancing price cannot sell anything.

For these auctions we want to derive in a very simple framework how different sorts of price regulations change the equilibrium price. The choice of the framework as well as the different types of price regulations are motivated by the introduction of different sorts of price regulations in the New York Installed Capacity Market (NYICAP) in 2008 which will be explained in more detail in the following section 2. The NYICAP is an instrument introduced into the New York electricity market in order to compensate power generating firms in the New York electricity market for a presumed lack of incentives to invest in power generating capacity.

The suboptimal level of investments in electricity generating capacity is usually motivated by the lack of informative scarcity signals implied in the

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See Bergman et al. (1999), Crampes and Fabra (2005), Newbery (2005), Hortaçsu and Puller (2008) and Zhang (2009).
unregulated electricity prices on wholesale markets (see e.g. Joskow (2008) and Cramton and Ockenfels (2012)) and might result in the *missing money problem*. On the one hand demand in wholesale electricity markets is very inelastic due to the fact that lots of consumers have contracts with their retailers where prices are fixed for longer periods.² On the other hand electricity cannot be stored which gives producers together with the inelasticity of demand an enormous amount of market power in periods with very high demand compared to the available capacity. In lots of electricity wholesale markets, especially in the US, system operators have implemented relatively low price caps which reduces the generators’ market power but also the potential rents from capacity investments.

Here we neither want to determine whether there is a missing money problem in the New York electricity market³ nor, given that the problem exists, whether the chosen instrument in New York is the most efficient one to solve it.⁴ Here our ambition is much more modest. We want to figure out in which way the outcome of a very simple multi-unit uniform price auction that resembles the NYICAP mechanism is influenced by price caps and price floors and how the effect is changed if the price cap or the price floor is only applied to a subset of firms active in the market instead of to the whole group of active firms. Motivated by the NYICAP example, we consider a price cap only for the firms with a relatively large generating capacity in the market and a price floor only for the firms with a relatively small capacity in the market.

We will consider a very simple model of a multi-unit uniform price auction without asymmetric information. It has first been introduced by von der Fehr and Harbord (1993) and forms also the basis of the mainly empirical contribution of Schwenen (2012) who shows that the NYICAP data from 2006-2008 are in line with such an auction model. The model used here is a linear version of the more general one presented in Moreno and Ubeda (2006) and Ubeda (2004). Its major assumptions are introduced in section 3 where we also identify some characteristics of the market outcome without

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²This will only change if more consumers get access to smart meters and are exposed to real-time prices.

³Léautier (2011) argues, for example, that the reduction in investments in capacity due to the missing money problem is small compared to the reduction in investments due to the lack of competition in the electricity market.

⁴See for a discussion of alternative instruments, e.g. Finon and Pignon (2008) and Cramton and Ockenfels (2012).
any price regulation. In section 4 we will investigate how the market outcome changes if the system operator either introduces general price caps or price caps only for firms with relatively large capacities, or if the system operator introduces general price floors or price floors only for firms with relatively small capacities.

Our main results are blub

Multi-unit uniform price auctions are also commonly used in treasury auctions (see e.g. Brenner et al. (2009)), emission permit auctions (see e.g. Betz et al. (2010)) and even in order to place IPOs on financial markets (see e.g. Degeorge et al. (2010)). Whether our results are relevant for these markets as well, will be discussed in the conclusions in section 5.

2 The New York Installed Capacity Market

The New York Installed Capacity Market (NYICAP) consists of three different regional markets for New York City, Long Island and the rest of the state of New York. For each of these markets the New York ISO conducts auctions for installed capacity for each month of the capability year (starting on May 1 and ending on April 30) at least 15 days before that year starts. The final auction for each month called the spot market takes place at least 2 days before the start of the month. Each load serving entity in the state of New York needs to buy capacity requirements that reflect the forecasted peak-load demand of its customers plus a reserve. The load serving entities can buy these requirements directly from electricity generators or via forward auctions. However, if they are not covered 30 days before the month starts the NYISO buys on behalf of them on the spot market.

The NYISO does so by constructing a demand curve for capacity that is derived from the minimum installed capacity requirement that should ideally be priced with a reference price. The latter should reflect the monthly going forward cost of a peaking facility adapted to whether the month is in the summer or the winter season. The minimum installed capacity requirement

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6The argument for using a downward sloping function instead of a simple step function that only relates to the reference price and the targeted capacity is that the capacity price is less volatile over time when the load growth is uncertain. See, e.g. the simulation model in Hobbs et al. (2007)
is derived from the forecasted peak-load in the particular region of the New York market in the considered capability year plus a reserve margin. The *going forward cost* is the annualized estimated cost of a peaking unit minus the estimated revenues that this peaking unit would generate on the New York electricity markets. The reference point defined by the reference price and the *minimum installed capacity requirement* together with 112% of the *minimum installed capacity requirement* and a price of zero define the linear demand for New York City plus a maximum price that works as a price cap. The linear demands for the other regions in the NYISO market are defined in a similar way but the slope is a bit flatter in absolute terms because the second point is 118% of the minimum installed capacity and a price of zero. In all regional NYICAP markets the linear demands are cut off at a maximum price where the inverse demand becomes horizontal. The latter can, of course, be interpreted as a general bidding cap applied to all suppliers of capacity.

All generators who own generating capacity in the region can, after having the characteristics of their capacity registered with the NYISO, bid their capacity in the relevant regional NYICAP spot market as long as they have not sold it already on a forward capacity market. The NYISO demands the quantities according to the artificially constructed demand function minus the quantities that the load serving entities had already procured until the deadline via bilateral contracts or the forward market on behalf of the not yet covered load serving entities. The NYISO determines the price of the spot market via a uniform price auction. Those firms who sold capacities either via bilateral trade agreements with the load serving entities or in a future or the spot market for capacity are obliged to offer electricity equivalent to these capacities in the wholesale market for electricity. They can, however, always bid it at the highest possible price that is represented by the relevant price cap on the wholesale market.

The NYISO installed capacity market is not unique in the US. Similar set-ups also exist in other markets. Examples are the PJM and the New England capacity market. They do differ mainly by two characteristics. The auctions are held much longer in advance before the procured capacity

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7 These characteristics are for example typical maintenance times during a year where the facility does not work, etc., in order to determine for how much capacity a certain facility should count.

8 See, e.g. Joskow (2008) and Hobbs et al. (2007)
needs to be offered on the wholesale electricity market.\textsuperscript{9} In addition the demand function has more than one linear segment with different slopes and therefore more kinks.

Besides the general price cap inherent in the NYICAP demand function the NYISO has introduced selective price regulations in 2008 after observing that the spot market price for capacity coincided close to always with this price cap since 2006 (see also the empirical paper by Schwenen (2012) on 2006-2008 data). In order to mitigate market power suppliers who are considered to be \textit{pivotal} must now offer their capacity at a price not exceeding the reference price.\textsuperscript{10} A firm is \textit{pivotal} if its capacity is necessary for supplying the market with at least the reference quantity of capacity. In addition a price floor has been introduced for new capacity in the market in order to prevent inefficient entry. New capacity in the market cannot be supplied below 75\% of the net cost of new entry, meaning the cost of a peaking facility minus the revenues that can be realized with it in the new market.\textsuperscript{11}

In order to get a rough idea about how the market should respond to these regulations we will in the following consider a very simple model of a multi-unit uniform price auction with a certain linear demand as in the NYICAP market and given capacities of the participating firms. We first derive the potential equilibria for given capacities without any price caps. Then we show how these equilibria change if a general price cap is introduced which corresponds to the market situation in the NYICAP market before 2008. Then we analyze a price cap only for firms above a particular capacity and characterize again the resulting market equilibria. This pretty much resembles the selective price caps introduced into the NYICAP after 2008. Then we consider the introduction of a general price floor to all firms in the market. The latter cannot be observed in the NYICAP market. However then we compare it with a selective introduction only to a sub-group of firms that we assume to be characterized by small capacities. The latter is, of course, not exactly the same as being a new entrant in the market that would reflect the specific NYICAP regulation. However, it is just taken for granted that new entrants enter on a small scale.

\textsuperscript{9}This is supposed to provide more incentives to enter the market with new capacity.

\textsuperscript{10}Pivotal firms can apply for getting another firm specific bidding cap if they can prove to the ISO that their \textit{going forward costs} deviate substantially from the average in the market.

\textsuperscript{11}Again a firm that can prove that its entry cost substantially differs from this market wide determined index number can apply for an individually determined bidding floor.
3 The Model and the Equilibria without Price Regulation

Consider a market with a set of \( N = 1, 2, \ldots, n \) active firms with \( n > 2 \). Each firm \( i \in N \) owns a certain amount of capacity \( K_i \) that it potentially can supply on the market. For now it is assumed that supplying parts of the capacity or all of it on the market does not cause any costs.\(^{12}\) Assume that the firms are indexed such that \( K_i \geq K_{i+1} \). In addition define \( \bar{K} = \sum_{i=1}^{n} K_i \) as the total capacity available and \( \bar{K}_{-i} = \bar{K} - K_i \) as the total capacity available if firm \( i \) is not supplying its capacity \( K_i \). Market demand \( D(p) \) is assumed to be linear in the market price \( p \) with

\[
D(p) = \alpha - \beta p \text{ and } \alpha, \beta > 0. \tag{1}
\]

Each firm can submit a price bid \( b_i \geq 0 \) at or above which it is willing to supply its total capacity \( K_i \) to the market.\(^{13}\) The auctioneer sorts all bids according to the demanded price in an ascending order and forms an aggregate supply function. The equilibrium price is the price at which the supply function equals the ex ante publicly known demand. All firms that bid their capacity at a price below the equilibrium price sell their total capacity. Those, which bid above, sell nothing, whereas the marginal firm(s) that bids(bid) the equilibrium price might be rationed in order to balance supply and demand.\(^{14}\) The vector of all bids \( (b_1, \ldots, b_n) \) needs to be a Nash equilibrium in order to determine an equilibrium price in the auction.

Note that if we would allow our firms to split their total capacity into \( l \geq 1 \) discrete pieces for which they could demand different minimum prices to supply them to the market this would not change the potential equilibrium

\(^{12}\)In the NYICAP market the cost could be the obligation to supply the capacity on the electricity wholesale market at least at the maximum possible price. As long as the marginal cost of production of the considered capacity does not exceed the maximum possible price usually determined by the price cap on the wholesale market, withholding the capacity at the price cap cannot be more profitable than supplying the capacity at least at this high price. Only if the virtually calculated average running times used to calculate the maximum capacity of each provider on the capacity market is misspecified and does not allow the generating firm to fulfill the obligation, providing capacities would cause a cost because the firm would be punished by the NYISO.

\(^{13}\)The Model is the same as in Moreno and Ubeda (2006) and Ubeda (2004) only with a more specified linear demand and, for now, without any possibility to invest in capacity.

\(^{14}\)In the case of multiple marginal bidders we assume that they are rationed according to their relative share in the total capacity bid at the marginal price by the marginal bidders.
prices. Given that the demand is common knowledge and whatever the other firms bid, a single firm can never gain from withholding capacity and asking for a higher price if it is finally not one of the price setting firms. It can potentially only lose if it is substituted by another firm in the merit order and is bidding too high on part of its capacity in order to be necessary to satisfy demand. If the firm is a price setting marginal firm, then it does not matter whether it bids that marginal market clearing price on all its capacity and is rationed or on only part of its capacity and is not rationed. It is always less profitable to increase the price on part of the capacity such that another firm substitutes for that part of the capacity in the merit order than demanding just slightly less than that potentially substituting firm on one's total capacity.

When characterizing the potential market equilibria which depend on the capacities held by the firms it is worthwhile as in Moreno and Ubeda (2006) to split the firms according to their capacities in those who can potentially be price setting or marginal in an equilibrium and those who cannot. For the start define the two sets of firms

\[ Q = \{ j \in N | \bar{K}_{-j} < \alpha \} \text{ and } O = \{ j \in N | \bar{K}_{-j} \geq \alpha \}. \]  

(2)

**Proposition 1** The firms \( j \in O \) can only set the marginal price in a Nash equilibrium if \( Q = \emptyset \) and the equilibrium price is \( p = b_j = 0 \). If \( Q \neq \emptyset \) then the price in any pure strategy Nash equilibrium needs to be strictly larger than the marginal costs of zero.

**Proof.** Suppose \( j \in O \) and \( b_j > 0 \) is the equilibrium price. This is only possible if the bids of all the other firms \( i \in N, i \neq j \), satisfy \( b_i \geq b_j \). However, then firm \( i \) has an incentive to undercut firm \( j \) slightly in a Bertrand fashion in order to increase its profit, no matter whether \( i \in O \) or \( i \in Q \) holds. If on the other hand \( j \in O \) and \( b_j = 0 \) then \( p = b_j = 0 \) cannot be an equilibrium price if there are still firms \( i \in Q \). These firms always have an incentive by unilaterally bidding \( \alpha > b_i > b_j = 0 \) to increase the price to \( b_i \) and, thus, increase their profit. This argument would not change if the low bidding firm with \( b_j = 0 \) were belongs to the set \( Q \) instead. ■

\(^{15}\text{See also Fabra et al. (2006) who prove this in their Lemma 2 for 2 competing firms and Schwenen (2012) who makes the intuitive argument presented here in a more formal way for } n > 2 \text{ firms.} \)
Not surprisingly firms whose capacity is so small that their capacity is not even needed to supply total demand at a price of zero \((j \in O)\) can never be price setters in equilibrium as long as other firms have capacities large enough that they are necessary to satisfy demand at a price of zero.

For the rest of the analysis we assume that the firms’ capacities are such that \(Q \neq \emptyset\) implying
\[
\bar{K} \geq K_j > \bar{K} - \alpha \quad \text{for some } j \in N.
\]
(3)

For the firms \(j \in Q\) we can define their residual demand
\[
D^r(p, \bar{K}_j) = \max\{\alpha - \bar{K} - \beta p, 0\}
\]
as the demand left to firm \(j\) if all other firms in the market offer their total capacities and if demand would be efficiently rationed. For firm \(j\)’s residual demand we can derive firm \(j\)’s monopoly price which is
\[
p_j = \arg\max\{pD^r(p, \bar{K}_j)\} = \frac{\alpha - \bar{K}_j}{2\beta}.
\]
(4)

The price \(p_j\) is only feasible as an equilibrium price if
\[
D^r(p_j, \bar{K}_j) < K_j \quad \text{or} \quad K_j > \alpha - \bar{K}.
\]
(5)

If for none of the firms \(j \in Q\) condition (5) holds, the only feasible equilibrium price is
\[
\bar{p} = \{p | D(p) = \bar{K}\} = \frac{\alpha - \bar{K}}{\beta}.
\]
(6)

We can prove the following proposition that characterizes the potential Nash equilibria in this multi-unit-uniform-price auction.

**Proposition 2** If \(Q \neq \emptyset\) then depending on the capacities of the firms \(j \in Q\) we can distinguish between two different cases.

(i) If \(K_j \leq \alpha - \bar{K}\) for all \(j \in Q\), there are infinitely many bidding equilibria in pure strategies where each firm \(j \in N\) bids \(b_j \leq \bar{p}\). The equilibrium price is \(p = \bar{p}\) regardless of the firms bids and all firms \(j\) in \(N\) sell their total capacity \(K_j\).
(ii) If \( K_j > \alpha - \bar{K} \) for some \( j \in Q \) there are potentially multiple equilibria in pure strategies where one of the firms \( j \in P \) with
\[
P = \{ j \in Q | K_j \geq \frac{(\alpha - \bar{K})^2 + K^2}{2K^1} \}
\]
bids \( b_j = p_j \) as defined in (4) and sells potentially only part of its capacity necessary to satisfy the residual demand \( D^r(p_j, \bar{K}_j) \). All other firms \( i \in N \setminus j \) bid \( b_i \leq b_j \) with
\[
b_j = \frac{(\alpha - \bar{K} + K_j)^2}{4\beta K_j} < p_j = b_j
\]
and sell their total capacity \( K_i \). The equilibrium price is identical with the bid \( b_j = p_j = p \) of the highest bidding firm \( j \).

**Proof.** See the argument in Appendix A. ■

Proposition 2 implies that if all firms \( j \in Q \) have a rather small capacity with \( K_j \leq \alpha - \bar{K} \), then the equilibrium price is always determined by the balance of total available capacity and the demand for capacity and is given by \( \bar{p} \) as defined in (6).\(^{16}\) If some firms \( j \in Q \) are larger with \( K_j > \alpha - \bar{K} \), the firm with the largest capacity, firm 1 would then necessarily belong to \( P \) as defined in (7), meaning it could potentially be a price setting firm with \( b_1 = p_1 = p \). However, whether firm 1 is the only possible price setting firms depends on the capacities of the other firms \( j \in Q \). Given \( K_1 > \alpha - \bar{K} \) and therefore also \( 1 \in P \), we can in principle distinguish two different situations characterized in figure 1 and 2.

Suppose \( \bar{K} < \alpha \) then all firms \( j \in N \) also belong to \( Q \) and \( O = \emptyset \). Note that if \( 0 < \bar{K} < \frac{\alpha}{2} \) we necessarily have \( \alpha - \bar{K} > \frac{\alpha}{2} > K_1 \) and neither firm 1 nor any other firm \( j \) with \( K_j < K_1 \) can be an element in \( P \), and be price setting in equilibrium. This situation is sketched on the left-hand side of figure 1. With \( \bar{K} < \alpha \) and \( K_1 \geq K_j \) with \( j \neq 1 \) only those combinations of \( K_1 \) and \( K_j \) can occur which lie to the east of the upward sloping and to the west of the downward sloping dashed 45\(^\circ\)-degree line. Thus, \( P = \emptyset \) and the equilibrium price is necessarily \( p = \bar{p} \). If \( \frac{\alpha}{2} < \bar{K} < \alpha \) then firm 1 can be a price setter in equilibrium, given \( K_1 > \alpha - \bar{K} \), and \( 1 \in P \). Whether other

\(^{16}\)In Figure 1 this is the case as soon as \( K_1 \leq \alpha - \bar{K} \) which necessarily implies \( K_j \leq K_1 \leq \alpha \) for all \( j \in Q \).
Figure 1: Characterization of Potentially Price Setting Firms with $\bar{K} < \alpha$

Figure 2: Characterization of Potentially Price Setting Firms with $\alpha \leq \bar{K}$
firms \( j \in Q \) could also be price setters depends on their capacity relative to the capacity of firm 1. If their smaller capacity is relative close to firm 1’s such that \( j \in P \), they can also be price setting firms and we could potentially also observe \( p = p_j \) as market equilibria. On the right-hand side in figure 1 all potentially price setting firms \( j \) with \( K_j \leq K_1 \) must have a capacity \( K_j \) above the bold curve, but since \( \bar{K} < \alpha \) also implies \( K_1 + K_j \leq \bar{K} < \alpha \) these firms capacities cannot be to the west of the dashed downward sloping 45°-line.

Now suppose \( \alpha \leq \bar{K} < 2\alpha \), then not all firms are necessarily an element of \( Q \). If \( Q = \emptyset \) this implies a relative even distribution of all capacities and proposition 1 applies with the equilibrium price being \( p = 0 \). However, if \( Q \neq \emptyset \), then firm 1 \( \in Q \) and firm 1 \( \in P \) is also necessarily satisfied which means, firm 1 can be a price setter and then the price would be \( p = p_1 \). In addition all those firms \( j \in Q \) with a capacity \( K_j \leq K_1 \) that exceeds the bold curved line on the left-hand side of figure 2 can also be price setters which would result in an equilibrium price of \( p = p_j \). Note that as soon as \( K_1 > \alpha \) holds, no other firm \( j \neq i \) can be an element of \( Q \) and therefore \( P \) because necessarily \( K_{-j} > \alpha \) and the equilibrium price can only be either \( p = p_1 \) if \( K_1 > \bar{K} - \alpha \), or \( p = 0 \) if \( K_1 \leq \bar{K} - \alpha \) and proposition 1 applies. These are the only two possible prices for \( \bar{K} > 2\alpha \), the case illustrated on the right-hand side of figure 2.

Thus, it becomes obvious that the potential market prices and highest potential bids in the market depend, on the one hand, on the level of the total capacity but, on the other hand, also on how unequally distributed the capacities of the different generators are. The upper and lower limit of the market price are characterized in the following corollary where we mainly apply proposition 1 and the definition of \( P \) in equation (7) of proposition 2 on the market price defined in equation (4).

**Corollary 1**  
Observed market prices depend on the total capacity and on the inequality of the capacities in the following way.

(i) For \( 0 \leq \bar{K} < \frac{\alpha}{2} \) the market price observed should satisfy

\[
p = \frac{\alpha - \bar{K}}{\beta} = \bar{p}.
\]

(ii) For \( \frac{\alpha}{2} \leq \bar{K} < \alpha \) the market prices observed should satisfy

\[
p = \frac{\alpha - \bar{K}}{\beta} \text{ for } K_1 < \alpha - \bar{K} \text{ and }
\]
\[ p \in \left[ \frac{(\alpha - \bar{K} + K_1)^2}{4\beta K_1}, \frac{\alpha - \bar{K} + K_1}{2\beta} \right] \text{ otherwise.} \]

(iii) For \( \alpha \leq \bar{K} < 2\alpha \) the market prices observed should satisfy

\[ p = 0 \text{ for } K_1 < \bar{K} - \alpha, \]

\[ p \in \left[ \frac{(\alpha - \bar{K} + K_1)^2}{4\beta K_1}, \frac{\alpha - \bar{K} + K_1}{2\beta} \right] \text{ for } \bar{K} - \alpha \leq K_1 < \alpha \text{ and} \]

\[ p = \frac{\alpha - \bar{K} + K_1}{2\beta} \text{ otherwise.} \]

(iv) For \( 2\alpha \leq \bar{K} \) the market prices observed should satisfy

\[ p = 0 \text{ for } K_1 < \bar{K} - \alpha \text{ and} \]

\[ p = \frac{\alpha - \bar{K} + K_1}{2\beta} \text{ otherwise.} \]

By applying proposition 1 as well as the definition of \( P \) given in equation (7) to the upper limit of the non-marginal bids, given in (8) of proposition 2, we can also characterize the non-marginal bids.

**Corollary 2** The non-marginal bids in equilibrium need to satisfy the following conditions.

(i) For \( 0 \leq \bar{K} < \frac{\alpha}{2} \) all bids can be non-marginal and need to satisfy

\[ b_j \leq \frac{\alpha - \bar{K}}{\beta} \text{ for all } j \in N. \]

(ii) For \( \frac{\alpha}{2} \leq \bar{K} < \alpha \) again all bids need to satisfy

\[ b_j \leq \frac{\alpha - \bar{K}}{\beta} \text{ for } K_1 < \alpha - \bar{K}. \]

The non-marginal bids need to satisfy

\[ b_i \leq b \text{ with } b \in \left[ \frac{(\alpha - \bar{K} + K_1)^4}{8\beta K_1[(\alpha - \bar{K})^2 + K_1^2]}, \frac{(\alpha - \bar{K} + K_1)^2}{4\beta K_1} \right] \text{ otherwise.} \]
(iii) For $\alpha \leq \bar{K} < 2\alpha$ all bids are marginal and must satisfy

$$b_j = 0 \text{ for all } j \in N \text{ if } K_1 < \bar{K} - \alpha.$$ 

The non-marginal bids need to satisfy

$$b_i \leq b_j \text{ with } b_j \in \left[ \frac{(\alpha - \bar{K} + K_1)^4}{8\beta K_1((\alpha - K)^2 + K_1^2)^2}, \frac{(\alpha - \bar{K} + K_1)^2}{4\beta K_1} \right] \text{ if } \bar{K} - \alpha \leq K_1 < \alpha$$

and

$$b_i \leq \frac{(\alpha - \bar{K} + K_1)^2}{4\beta K_1} = b_1 \text{ otherwise.}$$

(iv) For $2\alpha \leq \bar{K}$ again all bids are marginal and must satisfy

$$b_j = 0 \text{ for } j \in N \text{ and } K_1 < \bar{K} - \alpha.$$ 

For $K_1 \geq \bar{K} - \alpha$ the non-marginal bids need to satisfy

$$b_i \leq \frac{(\alpha - \bar{K} + K_1)^2}{4\beta K_1}.$$ 

4 The Effect of Price Regulations on Market Outcomes

We are now in the position to analyze the effects of price regulations on market outcomes. We first focus on maximum prices in the form of bid caps and later on price floors or minimum bids. In both cases we start with either general bid caps or general bid floors and later consider the effect of selective price regulations, meaning bid caps only for firms with large capacities and bid floors only for firms with low capacities. The NYICAP market by which this analysis is inspired has general and selectively more stringent bid caps, but only selective and no general bid floors.\footnote{See section 2 for more details on this.}
4.1 The Effect of Maximum Prices

Obviously, a general bid cap can only have an effect if it forces at least some firms to change their bidding behaviour. From proposition 1 we know that if \( Q = \emptyset \) then all firms \( j \in O = N \) bid \( b_j = 0 \) and the equilibrium price is \( p = 0 \). This bidding behaviour would not be changed by any bid cap that forces all firms \( j \in N \) to bid \( b_j \leq \hat{b} \) with \( \hat{b} \geq 0 \). This obviously changes if \( Q \neq \emptyset \) and proposition 2 would apply without a bid cap.

If \( K_j \leq \alpha - \bar{K} \) for all firms \( j \in Q \) then the bid cap might change the bidding behaviour if \( \hat{b} < \bar{p} \) where the latter is defined in equation (6). It would however not change the market price because total supply and total demand cannot be balanced at any price \( p = \hat{b} < \bar{p} \). The auctioneer needs to elevate the market price from the potentially highest bid \( \hat{b} \) to \( p = \bar{p} \) to balance total supply and demand. The same is true if some firms \( j \in Q \) have a capacity with \( K_j > \alpha - \bar{K} \) but the bid cap is still set such that \( \hat{b} < \bar{p} \) then all firms bid at or below the threshold \( \hat{b} \), but the final market price will always be \( p = \bar{p} \).

If \( K_j > \alpha - \bar{K} \) for some \( j \in Q \) then a bid cap with \( \hat{b} > \bar{p} \) does not only have potential to change the bidding behaviour but also to change the market outcome. For having an effect on the bidding behaviour and this way also on the market price it is necessary that the bid cap is either below the bid of the marginal bidder or changes the set of potentially marginal bidders \( P \) as given in (7) in proposition 2. We can prove the following propositions.

**Proposition 3** If \( Q = \emptyset \) and all firms \( j \in N \) also satisfy \( j \in O \), then a bid cap \( \hat{b} \geq 0 \) which only allows the firms to bid a price \( b_j \leq \hat{b} \) for their total capacity, does not change the firms’ bidding behaviour. All firms \( j \in N \) still bid \( b_j = 0 \) and thus the market price is still \( p = b_j = 0 \). If \( Q \neq \emptyset \) and \( \hat{b} \leq \bar{p} \) the bid cap might reduce the bid of some bidders \( j \in Q \), and it will always result in a market price of \( p = \bar{p} \) as defined in (6).

**Proof.** See the arguments above. ■

**Proposition 4** If \( Q \neq \emptyset \), if some firms \( j \in Q \) have a capacity \( K_j > \alpha - \bar{K} \) and if the bid cap satisfies \( \hat{b} > \bar{p} \) then the bid cap has only an impact on the bidding behaviour of the firms if it constrains the potential bids of some firms \( j \in Q \) meaning \( p_j > \hat{b} \Leftrightarrow K_j > 2\beta \hat{b} - (\alpha - \bar{K}) \).

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(i) If $K_j > 2\hat{\beta}b - (\alpha - \bar{K})$ holds for all firms $j \in Q$ then there is a set of Nash equilibria where any of the firms $j \in Q$ with $K_j > 2\hat{\beta}b - (\alpha - \bar{K})$ bids $b_j = \hat{b}$ and all other firms $i \in N \setminus j$ set $b_i \leq \hat{b}_j < b$ with

$$
\hat{b}_j = \frac{\hat{b}(\alpha + \hat{K} - \bar{K} - \beta\hat{b})}{K_j} < b,
$$

and $\hat{b}_j$ being defined in (8). The market price will be $p = \hat{b}$.

(ii) If only for some firms $j \in Q$ the condition $K_j > 2\hat{\beta}b - (\alpha - \bar{K})$ holds, the equilibria described in (i) still exist, but in addition other Nash equilibria can exist. In these additional equilibria one of the other firms $j \in Q$ with $K_j \leq 2\hat{\beta}b - (\alpha - \bar{K})$ bids its monopoly price on the residual demand $b_j = p_j < \hat{b}$ and all the other firms $i \in N \setminus j$ bid $b_i \leq b_j$. They exist if $j \in \hat{P}$ with

$$
\hat{P} = \left\{ j \in Q | K_j \geq \frac{2\hat{\beta}b(\alpha - \beta\hat{b} + K_1 - \bar{K}) - K_1(\alpha - \bar{K})}{K_1} \right\}.
$$

(iii) If for all firms $j \in Q$ the condition $K_j \leq 2\hat{\beta}b - (\alpha - \bar{K})$ holds, then the bid cap has no effect on the firms bidding behaviour in equilibrium and proposition 2 still applies.

Proof. See the the proof in Appendix B.

Note that the threshold that the capacity of a firm $j$ with $K_j \leq 2\hat{\beta}b - (\alpha - \bar{K})$ needs to exceed in order to be potentially price setting in case (ii) ($j \in \hat{P}$) is, despite the same equilibrium price of $p = p_j$, lower than the threshold that defined whether firm $j$ could be a price setter without the bid cap ($j \in P$ defined in proposition 2). The reason for that is that other firms $i \in Q$ with $K_i > K_j$ find it less attractive to overbid firm $j$ if they are restrained to the bid cap and can no longer bid the monopoly price on their residual demand because $p_i > \hat{b}$. Thus, firms $j \in Q$ with smaller capacities as before can be price setters in an equilibrium.

Now let us assume that $K_1 > 2\hat{\beta}b - (\alpha - \bar{K})$ and $p_1 > \hat{b} > \bar{p}$ implying that firm 1 with the largest capacity is potentially constrained by the bid cap, but also that $K_1 > \alpha - \bar{K}$. Note that this excludes the case that $0 < \bar{K} < \frac{\alpha}{2}$ or case (i) as well as part of case (ii) from corollary 1. The market prices that we might observe in this case are characterized in corollary 3.
Corollary 3 If $Q \neq \emptyset$ and $K_1 \leq 2\hat{b} - (\alpha - \bar{K})$ then a bid cap $\hat{b} > \bar{p}$ should not change the observed prices in the market as characterized in corollary 1. If, however, $Q \neq \emptyset$ and $K_1 > 2\hat{b} - (\alpha - \bar{K})$ then a bid cap $\hat{b} > \bar{p}$ changes the observed market prices in the following way.

(i) For $\frac{\alpha}{2} \leq \bar{K} < \alpha$ the market prices observed should satisfy

$$p \in \left[\hat{b} - \beta \hat{b} + K_1 - \bar{K}, \hat{b}\right].$$

(ii) For $\alpha \leq \bar{K} < 2\alpha$ the market prices observed should satisfy

$$p = 0 \text{ for } K_1 < \bar{K} - \alpha,$$

$$p \in \left[\hat{b} - \beta \hat{b} + K_1 - \bar{K}, \hat{b}\right] \text{ for } \bar{K} - \alpha \leq K_1 < \alpha \text{ and }$$

$$p = \hat{b} \text{ otherwise.}$$

(iii) For $2\alpha \leq \bar{K}$ the market prices observed should satisfy

$$p = 0 \text{ for } K_1 < \bar{K} - \alpha \text{ and }$$

$$p = \hat{b} \text{ otherwise.}$$

Since the threshold for non-marginal bids $\hat{b}_j$, given in (9), is monotonously increasing in the capacity $K_j$ of the price setting firm $j$ ($\partial \hat{b}_j / \partial K_j > 0$) we can again also characterize the non-marginal bids.

Corollary 4 Given $Q \neq \emptyset$ and $K_1 > 2\hat{b} - (\alpha - \bar{K})$ then a bid cap $\hat{b} > \bar{p}$ results in the following observed non-marginal bids.

(i) For $\frac{\alpha}{2} \leq \bar{K} < \alpha$ the non-marginal bids need to satisfy

$$b_i \leq \hat{b}_j \text{ with } \hat{b}_j \in \left[\hat{b}(\alpha - \beta \hat{b} + K_1 - \bar{K}) - K_1(\alpha - \bar{K}), K_1 \hat{b} + K_1 - K - \beta \hat{b}\right].$$
(ii) For $\alpha \leq \bar{K} < 2\alpha$ the bids must satisfy

$$b_j = 0 \text{ for all } j \in N \text{ if } K_1 < \bar{K} - \alpha.$$ 

The non-marginal bids need to satisfy

$$b_i \leq \hat{b}_j \text{ with } \hat{b}_j \in \left[ \frac{\beta^2 \hat{b} [2(\alpha - \bar{K} - \beta \hat{b}) + K_1]}{2 \beta \hat{b}(\alpha - \beta \hat{b} + K_1 - \bar{K}) - K_1(\alpha - \bar{K})}, \frac{\hat{b}(\alpha + K_1 - \bar{K} - \beta \hat{b})}{K_1} \right]$$

if $\bar{K} - \alpha \leq K_1 < \alpha$ and

$$b_i \leq \frac{\hat{b}(\alpha + K_1 - \bar{K} - \beta \hat{b})}{K_1} = \hat{b}_1 \text{ otherwise.}$$

(iii) For $2\alpha \leq \bar{K}$ the bids must satisfy

$$b_j = 0 \text{ for all } j \in N \text{ if } K_1 < \bar{K} - \alpha.$$ 

The non-marginal bids need to satisfy

$$b_i \leq \frac{\hat{b}(\alpha + K_1 - \bar{K} - \beta \hat{b})}{K_1} = \hat{b}_1 \text{ otherwise.}$$

Now what happens if the bid cap is only selectively applied to firms with $K_j > \hat{K} > 0$ instead of to all firms participating in the auction.\(^{18}\) Note that, if the bid cap $\hat{b}$ applies only to firms with $K_j > \hat{K}$ with $\hat{K} < \bar{K} - \alpha$, this would not change our results because this would mean it would still apply to all potentially price setting firms $j \in Q$. Thus, all our results summarized in proposition 3 and 4 as well as in corollary 3 and 4 would still hold.

In addition only those firms with $K_j > 2\beta \hat{b} - (\alpha - \bar{K})$ are potentially constrained in their price setting behaviour by a bid cap $\hat{b} > \frac{\alpha - \bar{K}}{\beta} = \bar{p}$ because the constraint would not influence the behaviour of the firms $j$ with $\hat{K} \leq K_j \leq 2\beta \hat{b} - (\alpha - \bar{K})$. This implies that if $\hat{K} \leq 2\beta \hat{b} - (\alpha - \bar{K})$ holds, then again nothing would change compared to the situation characterized in proposition 3 and 4 as well as in corollary 3 and 4.

\(^{18}\)Note that in the NYICAP market $\hat{K}$ is determined such that $\hat{K} = \bar{K} - \bar{K} < \alpha - \bar{K}$ with $\bar{K} \in (0, \alpha)$ being the reference quantity. The firms with $K_j > \bar{K} = \bar{K} - \bar{K}$ cannot bid their capacity at a higher price than the reference price that satisfies $\bar{K} = \alpha - \beta \hat{b} \Leftrightarrow \hat{b} = \frac{\alpha - \bar{K}}{\beta}$.\(^{18}\)
Changes can only occur if $\hat{K} > 2\beta\hat{b} + \bar{K} - \alpha$ holds. This would imply that not all firms with $K_j > 2\beta\hat{b} - (\alpha - \bar{K})$ will be constrained in their price setting behaviour. Firms with $\hat{K} > K_j > 2\beta\hat{b} - (\alpha - \bar{K})$ can, on the one hand, still bid their monopoly price on the residual demand, although $p_j > \hat{b}$ holds, and, on the other hand, their overbidding option, in case a firm with a smaller capacity places the highest bid, is still to set $b_j = p_j > \hat{b}$. Taking this into account we can derive the following proposition.

**Proposition 5** Suppose $Q \neq \emptyset$ and that a bid cap $\hat{b} > \bar{p}$, defined in (6), is imposed only on firms $j \in N$ with a capacity $K_j > \hat{K}$.

(i) This would not change the market equilibrium compared to proposition 4 if either $\hat{K} < 2\beta\hat{b} + \bar{K} - \alpha$ or if all the firms $j \in Q$ have a capacity $K_j > \hat{K}$.

(ii) If there are, however, firms $j \in Q$ with $\hat{K} > K_j > 2\beta\hat{b} - (\alpha - \bar{K})$ and at least firm 1 is constrained with $K_1 > \hat{K} > 2\beta\hat{b} - (\alpha - \bar{K})$, then the equilibria with firm $j \in Q$ with $K_j > \hat{K}$ bidding $b$ and all the other firms $i \in N \setminus j$ bidding $b_i \leq \hat{b}_j$ defined in (9) can still exist if there does not exist any other firm $i$ with $K_i \in (2\beta\hat{b} + \bar{K} - \alpha + 2\sqrt{\beta b(\beta b + \bar{K} - \alpha), \hat{K}})$. In addition equilibria with firm $j \in Q$ with $2\beta\hat{b} + \bar{K} - \alpha < K_j < \hat{K}$ bidding its monopoly price on its residual demand $p_j > \hat{b}$ and all other firms $i \in N \setminus j$ bidding $b_i \leq b_j$ defined in (8) exist if

$$K_j \geq \frac{K_i^2 + (\alpha - \bar{K})^2}{2K_i} \quad \text{for all } K_i \in (K_j, \hat{K}). \quad (11)$$

Equilibria with firm $j \in Q$ with $K_j \in [\max\{\bar{K} - a, 0\}, 2\beta\hat{b} + \bar{K} - \alpha]$ bidding its monopoly price on its residual demand $p_j \leq b$ and all other firms $i \in N \setminus j$ bidding $b_i \leq b_j$ can also exist if firm $j$’s capacity satisfies not only condition (11) but also $K_j \in \hat{P}$ as defined in (10) of proposition 4.

(iii) If $K_1 < 2\beta\hat{b} - (\alpha - \bar{K})$ then the selective bid cap does not have any influence on the equilibrium behaviour of any of the firms and proposition 2 still applies.

**Proof.** See Appendix C. ■

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Note that proposition 5 differs only in case (ii) from proposition 4. This has three major implications for the potential equilibrium. First, equilibria where the firms with the largest capacities bid the bid cap do not always exist even if $K_j > \hat{K}$ for some $K_j \in Q$. If there are firms $i \in Q$ with $K_i \in (2\beta \hat{b} + \tilde{K} - \alpha + 2\sqrt{\beta \hat{b}(\beta \hat{b} + \tilde{K} - \alpha)}, \hat{K})$ these firms would have an incentive to overbid any firm $j \in Q$ with $K_j > \hat{K}$ that would bid $b_j = \hat{b}$.

Second, prices above the bid cap can occur in equilibrium if there are firms $j \in Q$ with $\hat{K} > K_j > 2\beta \hat{b} - (\alpha - \hat{K})$. They can be price setters in equilibrium with $p = p_j > \hat{b}$ given that condition (11) holds for all larger firms $i$ which are not covered by the selective bid cap. Note that condition (11) is not a pure subset of $P$ defined in (7) of proposition 2 because it includes also firms $j \notin P$ with smaller capacities since $K_i < \hat{K} < K_1$ holds. Third, prices below the bid cap might occur in equilibrium if for some firms $j \in Q$ with $K_j \leq 2\beta \hat{b} + \tilde{K} - \alpha$ both conditions (11) as well as $K_j \in \hat{P}$ are satisfied. The latter ensure that neither the larger constrained firms $i \in Q$ with $K_i > \hat{K}$ nor the larger non constrained firms with $K_i \leq \hat{K}$ want to overbid firm $j$.

Which of the two constraints is binding depends on the level of the bid cap $\hat{b}$ and the critical capacity level $\hat{K}$ above which firms are constrained.

If we still want to give boundaries for the equilibrium prices resembling those given in corollary 1 and 3 we arrive at the following corollary.

**Corollary 5** Given $Q \neq \emptyset$ and some firms $j \in Q$ have a capacity $K_j > 2\beta \hat{b} - (\alpha - \hat{K})$ then a bid cap $\hat{b} > \bar{b}$ only applied to firms $j \in Q$ with $K_j > \hat{K} > 2\beta \hat{b} - (\alpha - \hat{K})$ results in the same observed market prices as in corollary 3 if there are no firms $j$ with $\hat{K} > K_j > 2\beta \hat{b} - (\alpha - \hat{K})$. If such firms exist and if at least $K_1 > \hat{K}$ holds, the boundaries for the market price change in the following way.

(i) For $\frac{\alpha}{2} \leq \hat{K} < 2\alpha$ the market prices observed should satisfy

$$p \in \left[ \max \left\{ b \frac{\alpha - \beta \hat{b} + K_1 - \tilde{K}}{K_1}, \frac{(\alpha - \hat{K} + \hat{K})^2}{4\beta \hat{K}} \right\}, \frac{\alpha - \hat{K} + \hat{K}}{2\beta} \right].$$

(ii) For $2\alpha \leq \hat{K}$ the market prices observed should satisfy

$$p \in \left[ \max \left\{ b \frac{\alpha - \beta \hat{b} + K_1 - \tilde{K}}{K_1}, \frac{(\alpha - \hat{K} + \hat{K})^2}{4\beta \hat{K}} \right\}, \frac{\alpha - \hat{K} + \hat{K}}{2\beta} \right].$$
for $\bar{K} - \alpha \leq \hat{K} < \alpha$ and $p = \frac{\alpha - \bar{K} + \hat{K}}{2\beta}$ if $\hat{K} > \alpha$.

If $K_1 \leq \hat{K}$ hold then corollary 1 still applies.

Since $\hat{b} < \frac{\alpha - \bar{K} + \hat{K}}{2\beta} < \frac{\alpha - K + K_1}{2\beta}$ holds, it is possible to see a market price above the bid cap in the market, but it is strictly below the potential upper level in the case without any bid cap described in corollary 1. Given the restrictions $K_1 > a$, $\hat{K} > 2\beta \hat{b} - (\alpha - \hat{K})$ and $\hat{b} > \hat{p}$, $p = 0$ cannot be an equilibrium any more. In addition $p = \frac{\alpha - \bar{K} + \hat{K}}{2\beta}$ can only be a unique equilibrium outcome if $\hat{K} \geq 2a$ because otherwise $K_1$ and $\hat{K}$ cannot both exceed $\alpha$.

4.2 The Effect of Price Floors

For bid floors set above $\hat{b}$ for some price setting firms $j$ destroys the equilibria where these firms are price setting independent of whether this bid floor is generally or only selectively applied.

Needs to be completed.

5 Conclusions

Summary and relation to NYICAP as well as to other multi-unit price auctions.

Needs to be completed.

Appendix

A Proof of Proposition 2

A.1 Characterization of the Nash Equilibria for Less Capacity Constrained Firms

Assume $Q \neq \emptyset$ and consider first the case with $K_j > \alpha - \bar{K}$ for some $j \in Q$. This implies that these firms can potentially bid their monopoly price on the residual demand $p_j$ and be price setting with $b_j = p_j = \hat{p}$ because it is feasible. Note that $p_j > p_i$ if $K_j > K_i$. Firm $j$ bidding $b_j = p_j = \hat{p}$ and all
other firms bidding lower can only be a Nash equilibrium if no low bidding firm wants to overbid and firm \( j \) does not want to underbid the second highest bid. Let us first consider overbidding by other firms. If \( K_i < K_j \) then optimally overbidding \( b_j = p_j \) implies \( b_i = p_j + \varepsilon \) with \( \varepsilon \to 0 \) and a profit of \( p_j D^r(p_j, K_{-i}) \) because \( p_i < p_j \). One can show that for all \( i > j \) the profit from overbidding is smaller than the profit from either underbidding or matching \( b_j = p_j \) if \( K_j > \alpha - \bar{K} \). Now suppose \( K_i > K_j > \alpha - \bar{K} \) then the optimal overbidding strategy for firm \( i \) would be to bid \( b_i = p_i \). However, firm \( i \) only wants to overbid firm \( j \) with \( b_j = p_j \) if the profit from doing so is higher than matching or underbidding firm \( j \). Note that matching is always weakly dominated by undercutting. So firm \( i \) only has an incentive to overbid if

\[
p_i D^r(p_i, K_{-i}) > p_j K_i \iff K_j < \frac{(\alpha - \bar{K})^2 + K_j^2}{2K_i}.
\]

This implies that \( b_j = p_j = p \) and all other firms bidding lower can only be an equilibrium strategy if the condition above holds for none of the firms with \( K_i > K_j \). Since the right hand side of this inequality increases in \( K_i \) for \( K_i > K_j > \alpha - \bar{K} \), this is ensured if it holds for the firm with the highest capacity \( K_1 \), meaning (7) holds. Now let us check under which condition underbidding the second lowest bid by firm \( j \) is more profitable for firm \( j \). Suppose \( b_i < b_j = p_j \) is the second highest bid. Firm \( j \) is only tempted to undercut if

\[
b_i K_j > p_j D^r(p_j, K_{-j}) \iff b_i > \frac{(\alpha - \bar{K} + K_j)^2}{4\beta K_j}.
\]

Thus, if (8) holds firm \( j \) does not have an incentive to undercut and the equilibria characterised in proposition 2 do exist for \( K_j > \alpha - \bar{K} \).

### A.2 Characterization of the Nash Equilibria for More Capacity Constrained Firms

Assume again that \( Q \neq \emptyset \), but consider now the case with \( K_j \leq \alpha - \bar{K} \) for all \( j \in Q \). Then for all the potentially price setting firms \( j \in Q \) the monopoly price on their specific residual demand is smaller than the price that balances total capacity with total demand, meaning \( p_j \leq \bar{p} \). Suppose now that firm \( i \in N \) bids the highest price \( b_i \geq b_j \) for all \( j \in N \) with \( j \neq i \) for its capacity
As long as $b_i \leq \bar{p}$ the equilibrium price would be $\bar{p}$ because the auctioneer needs to increase the price from $b_i$ to $\bar{p}$ in order to balance supply and demand and firm $i$ would sell all its capacity $K_i$ at the price $p = \bar{p}$. None of the other firms $j \neq i$ would like to overbid firm $i$. For all $\bar{p} \geq b_j > b_i$ the auction price would not change and any firm $j$ would still sell its total capacity at $p = \bar{p}$, whereas firm $j$ would lose profits as soon as it sets $b_j > \bar{p} \geq b_i$ because then firm $j$ would generate its monopoly profit on its residual demand which is due to $\bar{p} > p_j$ monotonously decreasing for all $p_j = \bar{p} > p_j$. In addition firm $i$ with $b_i \leq \bar{p}$ has neither an incentive to underbid any of its competitors nor to increase its bid $b_i > \bar{p}$. In the first case nothing would change for firm $i$. It would still sell its total capacity at a price of $p = \bar{p}$. In the second case firm $i$ would also decrease its profits because of the same argument made before for an overbidding firm $j$. Thus, there are infinitely many equilibria in pure strategies where all the firms $j \in N$ bid $b_j \leq \bar{P}$.

**B  Proof of Proposition 4**

Assume again that $Q \neq \emptyset$ holds and that $K_j < \alpha - \bar{K}$ holds for some $j \in Q$. Assume in addition that the bid cap can potentially have an influence on the market price because $\hat{b} > \frac{\alpha - \bar{K}}{\beta} = \bar{p}$. Those firms $j$ for which the monopoly price on their residual demand exceeds the bid cap $p_j > \hat{b} \iff K_j > 2\beta \hat{b} - (\alpha - \bar{K})$ can at most bid $\hat{b}$ and they are willing to do so as long as

$$\hat{b}D^e(\hat{b}, K_{-j}) \geq b_iK_j \quad \text{for all } i \in N \setminus j.$$ 

This implies that all the other firms $i \in N \setminus j$ bid such that $b_i \leq \hat{b}$ where $\hat{b}$ is defined in equation (9) in proposition 4 and $\hat{b}$ in equation (8) in proposition 2.

Note that overbidding is not an option for all firms $i \in N \setminus j$ and matching the bid $b_j = \hat{b}$ would clearly reduce their profits. Thus, for all firms $j \in Q$ with $K_j > 2\beta \hat{b} - (\alpha - \bar{K})$ bidding $\hat{b}$ and all other firms $i \in N \setminus j$ bidding $b_i \leq \hat{b}$ are Nash equilibria. Now assume that the firm $j \in Q$ with $K_j \leq 2\beta \hat{b} - (\alpha - \bar{K})$ sets its monopoly price $p_j$ on the residual demand. As long as there are no other firms $i \in N \setminus j$ for which the bid cap could potentially bind, meaning $K_i \leq 2\beta \hat{b} - (\alpha - \bar{K})$ for all $i \in N \setminus j$ the bid cap has no impact and proposition 2 still applies. However if there are other firms $i$ with $K_i > 2\beta \hat{b} - (\alpha - \bar{K}) \geq K_j$ then firm $j$ bidding $p_j$ and all other firms $i \in N \setminus j$ bidding $b_i < p_j$ can only be
part of a Nash equilibrium if none of the firms with $K_i > 2\hat{\beta}\hat{b} - (\alpha - \bar{K}) \geq K_j$ has an incentive to overbid with $b_i = \hat{b}$ meaning

$$\hat{b}D^r(\hat{b}, K_{-i}) \leq p_jK_i \Leftrightarrow K_j \geq \frac{2\hat{\beta}\hat{b}(\alpha - \beta\hat{b} + K_i - \bar{K}) - K_i(\alpha - \bar{K})}{K_i}.$$  

When taking into account that the condition above needs to hold for all firms with $K_i > 2\hat{\beta}\hat{b} - (\alpha - \bar{K}) \geq K_j$ and that the right-hand side of the condition increases in $K_i$ it becomes obvious that it must be satisfied for the largest potentially restrained bidder $K_i = K_1 > 2\hat{\beta}\hat{b} - (\alpha - \bar{K}) \geq K_j$ for the Nash equilibrium with $b_j = p_j$ and $b_i \leq \hat{b}$ to exist for $K_i > 2\hat{\beta}\hat{b} - (\alpha - \bar{K}) \geq K_j$ which implies (10), given in proposition 4.

\section*{C Proof of proposition 5}

If $\hat{K} < 2\hat{\beta}\hat{b} + \bar{K} - \alpha$ then the bid constraint $b_j < \hat{b}$ does not constrain a firm with capacity $K_i = \hat{K}$ because its monopoly price on the residual demand $p_i$ does not exceed $\hat{b}$ but potentially some of the firms $j \in Q$ with $K_j > \hat{K}$. On the other hand if all the firms $j \in Q$ have a capacity $K_j > \hat{K}$ all of them are potentially constrained. In both cases the selective bid cap has the same effect as if it would not have been tied to the condition that $K_j \geq \hat{K}$ and proposition 4 would still apply.

If there are, however, firms $j \in Q$ with $\hat{K} > K_j > 2\hat{\beta}\hat{b} - (\alpha - \bar{K})$ and at least $K_1 > \hat{K}$ then these firms $j$ are not constrained by the bid cap, although they would be if $\hat{b}$ would not have been applied only selectively. These firms can still bid $b_j = p_j$ whereas at least firm 1 is constrained and can only bid $b_1 \leq \hat{b} < p_1$.

Of course all the firms $j \in Q$ with $K_j > \hat{K}$ can still be price setting in equilibrium if

$$\hat{b}D^r(\hat{b}, \bar{K}_{-j}) \geq b_iK_j \text{ for all } i \in N \setminus j \Leftrightarrow b_i \leq \hat{b}_j$$

with $\hat{b}_j$ defined in (9) and none of the firms $i \in Q$ with $K_i \in (2\hat{\beta}\hat{b} + \bar{K} - \alpha, \hat{K})$
wants to overbid firm $j$ by bidding $b_i = p_i > \hat{b}$. The latter holds true if
\[ \hat{b}K_i \geq p_iD^r(p_i, K_{-i}) \iff K_i \leq 2\hat{b}\beta + \hat{K} - \alpha + 2\sqrt{\beta\hat{b}(\beta\hat{b} + \hat{K} - \alpha)} \] (12)
for all $i$ with $K_i \in (2\hat{b}\hat{b} + \hat{K} - \alpha, \hat{K})$.
Thus if (12) holds, the equilibria where any of the firms $j$ with $K_j > \hat{K}$ is bidding $\hat{b}$ and all the other firms $i \in N \setminus j$ bid $b_i \leq \hat{b}$ still exists. Note that with $\hat{K} \leq 2\hat{b}\beta + \hat{K} - \alpha + 2\sqrt{\beta\hat{b}(\beta\hat{b} + \hat{K} - \alpha)}$ condition (12) is necessarily satisfied.

In addition the unconstrained firms $j$ with $K_j \in (2\beta\hat{b} - (\alpha - \hat{K}), \hat{K})$ could bid $b_j = p_j > \hat{b}$ and be price setting. This could only be an equilibrium if firm $j$ could not generate more profits by undercutting another firm $i$’s bid $b_i$, meaning all other firms need to bid $b_i \leq \hat{b}_j$ which is defined in (8). At the same point in time no other firm $i$ with $K_i \in (K_j, \hat{K})$, should be tempted to overbid firm $j$ with $b_i = p_i$. Thus, $b_j = p_j$ for firm $j \in Q$ with $\hat{K} > K_j > 2\hat{b}\beta - (\alpha - \hat{K})$ and $b_i \leq \hat{b}_j$ for all $i \in N \setminus j$ can only be an equilibrium if
\[ p_jK_i \geq p_iD^r(p_i, K_{-i}) \iff K_j \geq \frac{K_i^2 + (\alpha - \hat{K})^2}{2K_i} \] for all $K_i \in (K_j, \hat{K})$. (13)
Note that the latter inequality is increasing on the right-hand side in $K_i$ and is necessarily satisfied if $\frac{K_i^2 + (\alpha - \hat{K})^2}{2K_i}$.

Also firms $j \in Q$ with $\hat{K} > 2\beta\hat{b} - (\alpha - \hat{K}) \geq K_j$ can potentially be price setting and bid $b_j = p_j \leq \hat{b}$ in equilibrium. In this case the other firms $i \in N \setminus j$ need to bid again $b_i \leq \hat{b}_j < p_j$. However, now all firms $i \in Q$ with $K_i \geq K_i > K_j$ could potentially overbid firm $j$ with either $b_i = \hat{b}$ if $K_i \in [\hat{K}, K_1]$ or with $b_i = p_i$ if $K_i \in (K_j, \hat{K})$. In an equilibrium they should not have an incentive to do so. The latter implies not only that (13) needs to hold but also
\[ p_jK_i \geq \hat{b}D^r(\hat{b}, K_{-i}) \iff K_j \geq \frac{2\hat{b}(\alpha - \hat{b} + K_i - \hat{K}) - K_i(\alpha - \hat{K})}{K_i} \] for all $K_i \in [\hat{K}, K_1]$.

\textsuperscript{19} Note that the firms $i \in Q$ with $K_i \notin (2\beta\beta + \hat{K} - \alpha, \hat{K})$ do not want to overbid $b_j = \hat{b}$ because $p_i < \hat{b}$.

\textsuperscript{20} Note that the firms $i$ with $K_i \in [\hat{K}, K_1]$ cannot overbid firm $j$’s bid $b_j = p_j$ and, as before no firm with $K_i < K_j$ will ever be tempted to overbid.
This restriction is again increasing in $K_i$ and is therefore equivalent with
$K_j \in \hat{P}$ as defined in (10).

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