Turning the Page on Business Formats for Digital Platforms: Does the Agency Model Soften Competition?

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**Abstract:** The agency model is a business format where platform providers (like Apple and Google) delegate retail pricing to (upstream) content providers subject to a fixed revenue-sharing rule. In a non-cooperative setting with competition both upstream and downstream, we analyze whether the platform providers actually will undertake such delegation or rather set prices on their own. We show that even if industry-wide adoption of the agency model would lead to higher profits for all firms, there may exist equilibria without delegation. This is due to a prisoner’s dilemma situation between the platform providers. We demonstrate that this prisoner’s dilemma can be addressed through implementation of MFN clauses (used by Apple in the controversial e-book case) without the need for any collusion.
1 Introduction

Digital platform providers (such as Apple and Google) often adopt the agency model in their dealings with upstream content providers (such as e-book publishers and app developers). The agency model has two key ingredients. The first is that downstream platforms delegate retail pricing decisions to upstream content providers, and the second is that platform providers are compensated for their services via revenue-sharing. The revenue-sharing rule is typically fixed and common to all content providers.\(^1\) Thus, for example, the retail price of the popular game Angry Birds is controlled by its inventor, Rovio Entertainment, and e-book retail prices are determined by the publishers. Apple and Google keeps 30\% of the revenue created when a sale is made through its platform.\(^2\)

The agency model is controversial. One reason for this is that little is known about the competitive effects of allowing the content providers to set the retail prices. Although this feature of the agency model is similar to resale price maintenance (RPM), about which much has been written, the usual reasons for why RPM is adopted do not apply in these markets. Arguments that RPM can reduce free-riding on downstream services (Telser, 1960), stimulate inter-brand competition by providing quality certification (Marvel and McCafferty, 1984), and ensure that downstream firms have enough margins to maintain adequate supplies of inventory (Deneckere et al, 1996)) are not likely to be important issues in digital markets. The agency model is often used by dominant market players like Apple and Google, thus mitigating any role for quality certification. Moreover, there is no

\(^1\)The agency model is also used by other platforms like eBay and Amazon Marketplace (see e.g. Johnson, 2013b), and the practice is not new. It is also used in the market for mobile content messages, where mobile operators delegate control of the retail prices to the content providers and then split the revenues (Foros et al, 2009). See also the discussion of the European market for printed books below.

\(^2\)While Apple does not always delegate retail pricing to content providers (Steve Jobs dictated a retail price of 99-cent per song when Apple entered the market for music with iTunes Store; see Isaacson, 2011), the 70/30 revenue split is used regardless of type of content and the size of content providers. So News Corp (Murdoch) does not obtain a better deal than a small, insignificant e-book publisher or app developer (see Isaacson, 2011, and United States v. Apple Inc, 12 Civ. 2826 (DLC)). Other digital platform providers have used different revenue splits (e.g.; Google previously used a 80/20-split), but the majority of platforms seem to have evolved towards a 70/30 rule similar to Apple’s agency model. One exception is Microsoft (Windows Store). Initially Microsoft uses the 70/30 split, but if the app sells more than $ 25.000 the split will go to 80% for the content provider (MarketingWeek, 2011).
limit on the number of goods that can be sold on each platform, the platform providers do not incur any costs of holding inventories, and concerns that consumers might learn about a product’s features from one retailer and buy from another retailer are not applicable. Neither is double marginalization an issue when there is revenue sharing (and marginal costs are zero or close to zero).

One motivation for studying the agency model stems from the notoriety of the recent e-books investigations in the United States and Europe. Although Apple and Google both adopted the agency model without raising significant concerns from antitrust authorities when the first smartphones were introduced in 2008, this changed when Apple entered the e-books market in 2010. However, worries that RPM might increase book prices have been widespread long before the antitrust investigation towards Apple and major publishers in the e-book market. In several European markets for printed books, a business model with ingredients similar to the agency model (the combination of RPM and revenue-sharing) has been controversial for more than a century (Tosdal, 1915; see Canoy et al., 2006, for a more recent overview). As in the recent e-book case from the US, the conjecture has been

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3While Apple uses revenue-sharing also without RPM (Isaacson, 2011), the conventional business model for books in the US prior to Apple’s entry was the so-called wholesale model in the US. Then downstream firms determine retail prices while wholesale contracts consist of a unit wholesale price (see e.g. Buettner et al., 2013). When wholesale prices are above marginal costs, double marginalization may arise when retail prices are determined by downstream firms. Then, solution of double-marginalization problems may be an additional benefit of the agency model compared to the present model. However, it is questionable whether there exists a double marginalization problem under the wholesale model. In practice, the cover price may act as a maximum RPM (even though it is not formally binding).

4The judge in the e-books case ruled that Apple was guilty of conspiring with the publishers to fix e-books prices. The ruling is currently under appeal. A key issue is whether Amazon was pressured into using the agency model, or whether it would have adopted it anyway. See United States v. Apple, 12 Civ. 2826 (DLC). The publishers are HarperCollins, Hachette, Macmillan, Penguin and Simon & Schuster. See also discussion in Buettner et al. (2013) and Amici (2014).

5The case involved Apple and five of the major publishers in the US and in Europe. Department of Justice (2012) sued the six firms in April 2012. The publishers have reached settlements, while the in June 2013 the trial against Apple took place. In United States v. Apple Inc, 12 Civ. 2826 (DLC)) the judge found that Apple had engaged in a violation of the Sherman Act. In a parallel case, the European Commission found that Apple had violated the European competition law; see European Commission, Case COMP/AT.39847-E-Books, Commission Decision of 12/12/2012. Buettner et al. (2013) provide a survey of the case both in the US and within EU.

6Currently, RPM is used by large countries like France and Germany. Until 1995 it was also used in
that RPM raises prices towards consumers.

We consider the platform providers’ incentives to delegate control of the retail prices to the content providers. We do so in a non-cooperative setting. Thus, we intentionally rule out a priori any concern that delegating the price decisions to the upstream firms may be used by powerful retailers to facilitate a retail cartel (Yamey, 1954) or that it may facilitate upstream collusion when retail prices are transparent (Jullien and Rey, 2007), which are two leading explanations for why RPM may be anticompetitive. In the process, we focus on whether unilateral adoption of the agency model by a single downstream platform provider would be expected to lead to higher or lower retail prices all else being equal. We take as our but-for benchmark case revenue-sharing arrangements in which the platform providers themselves retain control of the retail prices (see footnote 2 and 3 above). Furthermore, we explicitly consider the role of MFN clauses (most-favored-nation clause”) as adopted by Apple in the market for e-books, which have recently come under scrutiny in the U.S. and Europe.

We set up a simple, single-period setting in which there are two (upstream) content providers and two (downstream) platform providers, and each platform provider sells the goods of both content providers. All retail prices are chosen simultaneously. We find that absent MFN clauses, delegating control of the retail prices to the upstream firms may lead to higher or lower retail prices. It depends on whether the platform providers offer the UK.

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8 Hence, we intentionally fix the number and the quality of the products offered in the market and abstract from asymmetric information and uncertainty (see further discussion in Conclusion).
9 The MFN requires that the publishers do not set higher retail prices at Apple than at other downstream firms, regardless of whether the latters’ prices are controlled by the publishers. The judge in the case against Apple (United States v. Apple Inc, 12 Civ. 2826 (DLC), page 47) wrote that "The MFN guaranteed that the e-books in Apple’s e-bookstore would be sold for the lowest retail price available in the marketplace." See also Department of Justice (2012), Johnson (2013b) and Buettner et al. (2013).
10 This is reflected in the recent papers by Gans (2013), Johnson (2013b), Atlee and Botteman (2013), Wu and Bigelow (2013), and Fletcher and Hviid (2014), and the joint public workshop on MFN clauses held by the Antitrust Division of the U.S. Department of Justice and the Federal Trade Commission on September 10, 2012.
11 This feature is critical for the kinds of markets we consider. Apple and Google, for instance, deal with the same set of content providers (e.g., Angry Birds is popular both on Android and Apple smartphones and tablets).
the same or different revenue-sharing splits and on the relative willingness of consumers to substitute between goods and between platforms.

We show that a platform provider that keeps a higher revenue share for itself will be saddled with higher retail prices, and retail prices will tend to be higher with than without the agency model if competitive pressures are relatively lower upstream than downstream. By giving control over the retail prices to the upstream firms, the downstream platforms can trade one type of pecuniary externality (arising from competition between platforms) for another (from competition between goods).

Recognizing that adopting the agency model is a choice, we also find that when the revenue splits are the same for both platforms, all firms (upstream as well as downstream) benefit from industry-wide adoption of the agency model if it increases prices. An interesting question therefore is whether one would expect the platform providers to voluntarily adopt the agency model in this case, or whether some kind of pressure might be necessary to avoid a prisoner’s dilemma (as was implicitly conjectured in the e-books case by the Department of Justice, 2012). To analyze this question, we extend the game to allow for an initial stage in which the platform providers simultaneously and independently choose whether to adopt the agency model. If industry-wide adoption of the agency model would lead to higher retail prices, multiple subgame-perfect equilibria may exist, and in equilibrium (depending on parameters), zero, one, or both firms adopt the agency model. In particular, we show that there may indeed be a prisoner’s dilemma in which all firms would be better off with industry-wide adoption of the agency model, but in which no downstream platform has an incentive to do so.

Lastly, we demonstrate that this prisoner’s dilemma can be addressed through the use of MFN clauses without the need for any collusion or “pressure”. We find that such clauses can nudge the industry toward agency adoption (by making it a weakly dominant strategy) and thereby lead to higher prices than would be the case if the agency model had not been adopted or had only been adopted by one firm. Moreover, this is so even if only one

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12 When Apple established its iTunes store for music, for example, it declined to delegate control of the retail prices (see footnote 2 above), opting instead to dictate that each song be offered for sale at a uniform price of 99-cents. Moreover, there is a clear indication that Apple viewed the agency model for apps and e-books as a second best solution: He [Jobs] had refused to offer the music companies the agency model and allow them to set their own prices. Why? Because he didn’t have to (from Steve Jobs’ biography, Isaacson, 2011, page xxx).
downstream platform has an MFN clause, and even when the MFN clause by assumption has no effect on the firms’ revenue-sharing splits.\footnote{This is consistent with the stylized facts in the e-books case, where, for instance, the revenue-sharing split that Apple imposed on the book publishers was the same as the revenue-sharing split that it used for music (where it did not delegate control of the retail prices and where it did make use of MFN clauses).}

In the addition to contributing to the literature on the agency model in digital markets, our analysis makes several contributions to the broader vertical-relations literature. First, most models of vertical-contracting assume either that the upstream or the downstream level is monopolized, or that there are pairs of upstream and downstream firms in which each upstream firm offers its good to only one downstream firm. In contrast, ours is one of only a handful of papers that allow for imperfect competition and strategic players at both levels, and in which each downstream firm can sell multiple upstream firms’ goods. The models in Dobson and Waterson (2007) and Johnson (2013a,b) are similar to ours in that regard, but they use linear wholesale prices as their but-for benchmark when analyzing the competitive effects of RPM. As a result, mitigation of double-marginalization is a key factor in their models.\footnote{Like us, Johnson’s analyses are motivated by the agency model, but in contrast to us, he finds that the upstream firms never benefit from the agency model (without MFN), and that delegating control of the retail prices to the upstream firms always reduces prices.}

Second, we show that RPM might reduce prices even if we disregard double marginalization. It is commonly argued that RPM might increase prices through facilitation of a cartel (which we discussed above), the softening of competition through a commitment to higher retail prices and/or retail markups (Shaffer, 1991; Rey and Verge, 2010; Foros et al, 2011), and the deterrence of an upstream potential entrant (Asker and Bar-Isaac, 2014). While this obviously is true, we argue that one should take into account the fact that the price effects of RPM depend fundamentally on whether the competitive pressures is weaker upstream than downstream.\footnote{Our analysis takes place in a non-cooperative, single-period setting, thus ruling out tacit or explicit collusion as an explanation, and there is no potential entrant at the upstream level, thus ruling out the concern in Asker and Bar-Isaac (2014). Moreover, there is no de facto first-mover advantage to exploit when some firms adopt RPM and others do not, as in Shaffer (1991) and Foros et al (2011), and no manipulation of the retail markups when both wholesale prices and fixed fees are feasible, as in Rey and Verge (2010) and Shaffer (2013).}

Third, we contribute to the literature on MFN clauses. It is well known that MFN clauses
clauses can be used as a commitment device to raise prices in inter-temporal settings (Cooper, 1986; Nielson and Winter, 1993; Schnitzer, 1994; Hviid and Shaffer, 2012). It is also well known that MFN clauses can lead to higher prices in bargaining settings in which contracts are negotiated sequentially (Cooper and Fries, 1991; and Neilson and Winter, 1994). Closer to us, Johnson (2013b) suggests that MFN clauses can remove the platforms’ incentives to provide higher revenue shares to content providers in order to induce a lower retail price (and a higher retail price for goods sold through the rival’s platform). Although this implication of MFN clauses also holds in our model, we find that MFN clauses may in addition adversely affect prices even if they have no effect on the firms’ revenue shares.

Fourth, we contribute to the large literature on strategic delegation that was inspired by the seminal works of McGuire and Staelin (1983), Moorthy (1988), Rey and Stiglitz (1988), and Bonanno and Vickers (1988), among others. Typically, in this literature, firms commit to taking an action which, if observable, dampens competition. The commitment device in our setting is the delegation of control of the retail prices to the upstream firms. Since this is a discrete choice, unlike the papers in the strategic-delegation literature, we find that the delegation we consider is not always profitable even if it would dampen competition.

The rest of the paper is organized as follows. In Section 2.1, we present the model and compare the retail prices that would arise when both firms adopt the agency model with the retail prices that would arise when neither firm adopts the agency model. In Section 2.2, we consider regimes in which only one firm adopts the agency model. In Section 3.3, we endogenize the choice of business formats. We then consider how MFN clauses might impact these choices and affect retail prices in Section 3.4. Section 4 concludes the paper.

2 The Model

We consider a market with two competing upstream firms, which are indexed by \( j = 1, 2 \) (superscripts on the variables), and two competing downstream firms, which are indexed by \( i = 1, 2 \) (subscripts on the variables). Each upstream firm \( j \) produces a single good, good \( j \), which it then distributes through the downstream firms for resale to the final consumers.

We assume that each upstream firm’s good is sold by both downstream firms. Thus, the demand for good \( j \) at downstream firm \( i \) will in general be a function of four prices:

\[
x_i^j = q_i^j(p_1^1, p_1^2, p_2^1, p_2^2).
\]
In the region of prices for which it is positive, we assume that $x^j_i$ is decreasing in $p^j_i$ (i.e., demand is downward sloping) and weakly increasing in each of the other prices (i.e., the products are gross substitutes). We also assume that the firms and the goods are symmetrically differentiated,\textsuperscript{16} and that the cost of making and selling each good is zero.

Writing $p = (p^1_1, p^2_1, p^2_2)$, downstream firm $i$’s profit given revenue share $s_i$ is thus

$$\Pi_{Di} = s_i \left( p^1_1 q^1_i(p) + p^2_2 q^2_i(p) \right).$$

(1)

In comparison, upstream firm $j$’s profit, given revenue shares $s_1$ and $s_2$, is given by

$$\Pi^{Uj} = (1 - s_1) p^j_1 q^j_i(p) + (1 - s_2) p^j_2 q^j_i(p).$$

(2)

In these profit expressions, downstream firm $D_i$ keeps $s_i \in [0, 1]$ share of the revenue it earns from selling goods 1 and 2, and upstream firm $U^j$ gets $1 - s_1$ share of the revenue $D_1$ earns from selling good $j$ and $1 - s_2$ share of the revenue $D_2$ earns from selling good $j$.

Our assumptions on the revenue-sharing splits make use of several stylized facts. First, for the markets we are interested in, the revenue-sharing splits do not appear to be negotiable.\textsuperscript{17} Thus, we assume they are taken as given by the upstream firms. Second, we assume the revenue-sharing splits are the same for both upstream firms at a given downstream location. This assumption reflects the fact that in practice digital platform providers use a “one size fits all” approach both within and across the various industries they participate in.\textsuperscript{18} It is beyond the scope of this paper to consider why the splits for music, for example, are the same as they are for games and e-books, or why the providers of games and other apps have the same terms as the providers of e-books. For now, we simply take this as given and assume the revenue splits are exogenous to the market at hand. Later, in the concluding section, we will speculate on the implications of our analysis for how the splits might optimally be set if it were feasible to vary them market by market.

\textsuperscript{16}Assuming that the downstream firms are symmetric implies that $q^j_i(a, b, c, d) = q^j_i(c, d, a, b)$. Assuming that the upstream firms’ goods are symmetric implies that $q^j_i(a, b, c, d) = q^j_i(b, a, d, c)$, for $i, j = 1, 2$.

\textsuperscript{17}The digital platform providers have significant bargaining power and can often dictate the terms at which they will do business. This is evident at numerous points in the judge’s decision in the e-books case (referenced in footnote 3). For example, after noting that HarperCollins, a large book publisher, suggested that Apple take a 20% commission (rather than a 30% commission), the judge wrote (p. 58) “Apple refused to budge. This was the same commission it charged in the App Store. It would give Apple only a single digit positive margin and, in Apple’s view, was necessary to generate the revenue Apple needed to build a great iBookstore. The 30% commission was ultimately adopted across all of Apple’s final Agreements.”

\textsuperscript{18}As we noted previously, Apple employs a 70/30 revenue-sharing split, which is the same for all providers regardless of their size, popularity, or type of content (apps, e-books, music, newspapers, magazines etc).
2.1 Comparing no RPM and RPM

We now compare the case in which the downstream firms determine the retail prices (no RPM) to the case in which the upstream firms do so (RPM). We will henceforth refer to the business format in which the upstream firms set the retail prices as the “agency model”.

In the no RPM case, downstream firm $i$’s optimization problem is given by

$$\max_{p_1^i, p_2^i} \Pi_{D_i}. \quad (3)$$

It follows from $D_i$’s problem that the system of first-order conditions, $i = 1, 2$, that characterizes the Bertrand equilibrium in this case can be written as

$$\frac{\partial \Pi_{D_i}}{\partial p_1^i} = s_i \left( p_1^i \frac{\partial q_1^i}{\partial p_1^i} + q_1^i + p_2^i \frac{\partial q_2^i}{\partial p_1^i} \right) = 0, \quad (4)$$

$$\frac{\partial \Pi_{D_i}}{\partial p_2^i} = s_i \left( p_1^i \frac{\partial q_1^i}{\partial p_2^i} + p_2^i \frac{\partial q_2^i}{\partial p_2^i} + q_2^i \right) = 0. \quad (5)$$

In contrast, in the RPM case, control over the retail prices is delegated to the upstream firms. Prices in this case will be chosen to maximize each upstream firm’s profit:

$$\max_{p_1^j, p_2^j} \Pi^{U_j}. \quad (6)$$

It follows from $U^j$’s problem that the system of first-order conditions, $j = 1, 2$, that characterizes the Bertrand equilibrium in the RPM case can be written as

$$\frac{\partial \Pi^{U_j}}{\partial p_1^j} = (1 - s_1) \left( p_1^j \frac{\partial q_1^j}{\partial p_1^j} + q_1^j \right) + (1 - s_2) \left( p_2^j \frac{\partial q_2^j}{\partial p_1^j} \right) = 0, \quad (7)$$

$$\frac{\partial \Pi^{U_j}}{\partial p_2^j} = (1 - s_1) \left( p_1^j \frac{\partial q_1^j}{\partial p_2^j} \right) + (1 - s_2) \left( p_2^j \frac{\partial q_2^j}{\partial p_2^j} + q_2^j \right) = 0. \quad (8)$$

In each case, we assume that a unique equilibrium exists. For existence, we assume that the demands $x_i^j$ are smooth whenever positive, that the Jacobian of the demand system is negative definite, and that each firm’s profit is quasi-concave in its choice variables. For uniqueness, we assume that own effects dominate the sum of the cross effects on profits.\(^\text{19}\)
Comparing the first-order conditions from (4) and (5) and the first-order conditions from (7) and (8), it can be seen that there are several significant differences. Note first that since $s_i$ is the same for both goods and enters the first-order conditions in (4) and (5) multiplicatively, the profit-maximizing prices set by each $D_i$ in the no RPM case will be independent of $s_i$. It follows that the Bertrand equilibrium prices in this case will also be independent of $s_i$, and thus that retail prices in the absence of RPM will not depend on whether the revenue-sharing splits that are offered by $D_1$ and $D_2$ are the same or different.

In contrast, when the upstream firms control the retail prices, prices would be expected to differ across firms if the downstream firms offer different revenue-sharing splits.\footnote{This follows immediately by noticing that simply dividing the left and the right-hand sides of (7) and (8) by $1 - s_i$ does not eliminate the left-hand sides’ dependence on $s_1$ and $s_2$ when $s_1$ differs from $s_2$.} In particular, we see from the second term in (7) that the marginal profitability of increasing $p_{i1}^j$ is decreasing in $s_2$ when consumers perceive the downstream firms as imperfect substitutes (because $\frac{\partial q_{i2}}{\partial p_{i1}^j} > 0$). This means that for a given $s_1$, the optimal $p_{i1}^j$ taking all other prices as given will be lower the higher is $s_2$ (analogously, we see from the first term in (8) that for a given $s_2$, the optimal $p_{i2}^j$ taking all other prices as given will be lower the higher is $s_1$). Conversely, the second term in (8) implies that the marginal profitability of increasing $p_{i2}^j$ is increasing in $s_2$,\footnote{Note that the second term in (8) must be negative because the first term, \( \frac{\partial q_{i1}}{\partial p_{i2}^j} \), is positive.} which means that for a given $s_1$, the optimal $p_{i2}^j$ taking all other prices as given will be higher the higher is $s_2$ (analogously, for a given $s_2$, the first term in (7) implies that the optimal $p_{i1}^j$ taking all other prices as given will be higher the higher is $s_1$).

These implications follow because if, for example, $s_i$ increases (i.e., $D_i$ opts to keep a larger share of the revenue for itself), then $U_i$’s incentive for a given $s_{-i}$ is to sell relatively more of its good through $D_i$’s rival and thus less through $D_i$. It can do this either by raising the price of good $j$ at $D_i$ by more than it raises the price of good $j$ at $D_i$’s rival, lowering the price of good $j$ at $D_i$’s rival by more than it lowers the price of good $j$ at $D_i$, or by both raising the price of good $j$ at $D_i$ and lowering the price of good $j$ at $D_i$’s rival.

The following proposition describes what can be said in general.

**Proposition 1** When the downstream firms control prices, equilibrium retail prices are independent of revenue shares. In contrast, when the upstream firms control prices, equilibrium retail prices are independent of revenue shares if and only if $s_i = s_{-i}$. For $s_i \neq s_{-i}$:

- $p_{i1}^j - p_{i-1}^j$ is increasing in $s_i$;
One might have thought that it would always be optimal for $U^j$ and $U^{-j}$ to adjust to an increase in $s_i$ by raising the prices of the goods that are sold through $D_i$ and lowering the prices of the goods that are sold through $D_{-i}$. However, Proposition 1 suggests that this need not be true in general unless the difference in the revenue shares of the downstream firms is sufficiently small. The reason is that when all prices adjust in equilibrium, an increase in the retail prices of the goods at $D_i$ (decrease in the retail prices of the goods at $D_{-i}$) may through a strategic-complements-like effect induce the upstream firms to set higher retail prices on the goods at $D_{-i}$ (lower retail prices on the goods at $D_i$) as well.

Nevertheless, as Proposition 1 shows, an increase in $D_i$’s revenue share $s_i$ will, for a given $s_{-i}$, cause the upstream firms to shift relative retail prices in favor of $D_{-i}$. It follows that if retail prices at $D_i$ were initially higher than at $D_{-i}$ before the increase in $s_i$, then the gap will widen, and if they were initially lower than at $D_{-i}$ before the increase in $s_i$, then the gap will narrow, and may even flip-flop, causing $D_i$’s prices to become higher.

We can extend our results by noting that when the upstream firms control the retail prices, each upstream firm will set the same retail price at $D_i$ regardless of the level of $s_i$ (because the goods are symmetrically differentiated), and moreover that the solution to (7), (8), and their analogs will yield identical prices across $D_i$ and $D_{-i}$ when $s_i = s_{-i}$ (because the downstream firms are symmetrically differentiated) but not when $s_i \neq s_{-i}$. In contrast, in the no RPM case, all retail prices will be the same in equilibrium, whether or not $s_i = s_{-i}$. Since $p_i^j - p_{-i}^j$ is increasing in $s_i$, we thus have the following implications:

**Proposition 2** When the downstream firms control prices, all prices will be the same. In contrast, when the upstream firms control prices, equilibrium retail prices are such that

\[ p_i^j = p_i^{-j} \text{ for all } s_i; \]
\[ p_i^j = p_{-i}^j \text{ if } s_i = s_{-i}; \]
\[ p_i^j > p_{-i}^j \text{ if and only if } s_i > s_{-i}. \]

In contrast to the case of no RPM, Proposition 2 implies that with RPM, the prices of all products will be the same in equilibrium if and only if revenue shares are the same. If
revenue shares are not the same, the firm whose revenue share is higher will have higher retail prices. How much higher depends on a variety of factors that affect the upstream firms’ decisions. Proposition 1 implies that the gap in prices will be increasing in the difference between \( s_i \) and \( s_{-i} \). It also implies that the upstream firms may respond to an increase in \( s_i \) by increasing the prices of the goods at \( D_i \) and decreasing the prices of the goods at \( D_{-i} \). This reflects the fact that an upstream firm which increases its retail price at one downstream firm and reduces it at the other need not lose much in sales — especially if the downstream firms are perceived to be close substitutes. An upstream firm which increases its price at one downstream firm may also not be disadvantaged much if the goods are not close substitutes. On the other hand, if the goods are close substitutes, then an upstream firm which unilaterally increases its price at one downstream firm may lose a significant share of its sales to its upstream rival. When this is so, competition may keep the retail prices of the firm with the higher revenue share from increasing as much.

Another significant difference between the no RPM and RPM cases (the first being the dependence or not of retail prices on revenue shares) is that the maximization problems in (3) and (6) can lead to different outcomes even if the revenue-sharing splits are the same. The reason is that the firms’ focus in the two cases are on different things. When the downstream firms choose the retail prices, each cares only about the number of sales it makes, not whether a particular sale at a given price comes from good 1 or good 2. In contrast, when the upstream firms choose the retail prices, assuming revenue shares are the same, each cares only about the number of sales of its own good, not where it is sold.

These differences of perspective have implications for the comparison of retail prices. In the no RPM case, for example, conditions (4), (5), and their analogs imply that in equilibrium, the unique price \( p \) will satisfy

\[
p \frac{\partial q_i^j(p)}{\partial p_i^j} + q_i^j(p) + p \frac{\partial q_{-i}^j(p)}{\partial p_i^j} = 0,
\]

whereas when both firms adopt the agency model and revenue shares are the same, conditions (7), (8), and their analogs imply that in equilibrium, the unique price \( p \) will satisfy

\[
p \frac{\partial q_i^j(p)}{\partial p_i^j} + q_i^j(p) + p \frac{\partial q_{-i}^j(p)}{\partial p_i^j} = 0.
\]

Let \( p = p^* \) denote the solution to (9) and define \( p^* \equiv (p^*, p^*, p^*, p^*) \) to be the vector of equilibrium prices in the no RPM case. Then, because \( p = (p, p, p, p) \) in both cases, a
comparison of (9) and (10), using our assumptions on uniqueness, reveals the following:

**Proposition 3** When \( s_1 = s_2 \), equilibrium retail prices are

- the same in the no RPM and RPM cases if \( \frac{\partial q_{i,j}(p^*)}{\partial p^*_{i}} = \frac{\partial q_{i,j}(p^*)}{\partial p^*_{i}} \);
- lower in the RPM case than in the no RPM case if \( \frac{\partial q_{i,j}(p^*)}{\partial p^*_{i}} < \frac{\partial q_{i,j}(p^*)}{\partial p^*_{i}} \);
- higher in the RPM case than in the no RPM case if \( \frac{\partial q_{i,j}(p^*)}{\partial p^*_{i}} > \frac{\partial q_{i,j}(p^*)}{\partial p^*_{i}} \).

Proposition 3 implies that when revenue shares are the same, retail prices will be higher with RPM than without RPM if and only if there is more substitution at the downstream level (as measured by the price sensitivity of consumers between retailers) than at the upstream level (as measured by the price sensitivity of consumers between goods). Stated more formally, \( \frac{\partial q_{i,j}(p^*)}{\partial p^*_{i}} \) measures the sensitivity of demand for good \( j \) at \( D_i \) when \( D_i \)’s price on good \( j \) changes (i.e., substitution between retailers), and \( \frac{\partial q_{i,j}(p^*)}{\partial p^*_{i}} \) measures the sensitivity of demand for \( U^{-j} \)’s good at \( D_i \) when \( D_i \)’s price on good \( j \) changes (i.e., substitution between goods). Retail prices will be higher under RPM only if the former is greater.

Intuitively, when the downstream firms retain control of the retail prices, they do a good job of internalizing the substitution between goods at their respective stores, but succumb to head-to-head competition between themselves for the patronage of consumers. When store loyalty is relatively low, this can lead to fierce competition and result in low prices for consumers. In contrast, when the upstream firms have control of the retail prices, they do a good job of internalizing the substitution between retailers, but compete to get consumers to buy their good over their rival’s good. This too can lead to low prices for consumers, but only when the goods are relatively close substitutes. Thus, by giving control over the retail prices to the upstream firms, the downstream firms can effectively trade one type of externality (substitution between retailers) for another (substitution between goods). When the substitution between retailers is relatively high, they may jointly be better off transferring control to the upstream firms. Conversely, when the substitution between goods is relatively high, they may jointly be better off retaining control for themselves.

These implications follow because equilibrium retail prices under both formats (RPM and no RPM) will generally be below the level that maximizes industry profits. By choosing the format that induces the higher (symmetric) retail prices, therefore, the downstream
firms can move closer to the industry profit maximum. In the absence of mitigating factors (e.g., cost differences that might arise from implementing different formats), this implies:

**Corollary 1** When \( s_1 = s_2 \), the case of RPM would be expected to lead to higher industry profits relative to the case of no RPM when competitive pressures are lower upstream than downstream, and lower industry profits relative to case of no RPM when the opposite holds.

Transferring control over the retail prices to the level where competition is weakest would be expected to bring prices closer to the level that maximizes industry profit. Thus, all else being equal, one would expect the downstream firms to prefer the case of RPM to the case of no RPM when the competitive pressures are lower upstream than downstream.

However, there are at least two important caveats to this conclusion. One is that Proposition 3 and Corollary 1 presume that the downstream firms are offering the same revenue-sharing splits to the upstream firms. If this is not so, then prices in the RPM case will be affected. For example, if \( s_i > s_{-i} \), then relative to the case of \( s_1 = s_2 \), we know from Proposition 2 that in general both prices may increase, both may decrease, or \( p_i^j \) may increase and \( p_{-i}^j \) may decrease in the case of RPM. In contrast, retail prices do not depend on revenue shares in the case of no RPM. Thus, it follows that when \( s_1 \neq s_2 \), the comparison between the case of RPM and the case of no RPM may not be as clear-cut as in Proposition 3. The case of RPM may lead to higher industry profits relative to the case of no RPM even if competitive pressures are higher upstream (if both prices were to increase when \( s_i > s_{-i} \)) or lower industry profits relative to the case of no RPM even if competitive pressures are lower upstream (if both prices were to decrease when \( s_i > s_{-i} \)).

The second caveat is more subtle. In order to reach the business format that induces the higher industry profit (whether it be with RPM or not), both downstream firms must independently choose it. However, it is not obvious that they will do so (absent the ability to form a cartel), even though they are symmetric ex-ante. It may be that one of the firms would prefer to adopt the agency model (RPM) when the other retains control over its prices, or vice versa, or it may be that the firms will be stuck on the format that yields the lower industry profit because neither one wants to be the first to switch to a new format.
2.2 Mixed regimes

In a mixed regime, only one downstream firm adopts the agency model. There are two such regimes. Without loss of generality, let $D_i$ be the firm that has RPM. This means that $D_{-i}$ decides $p^1_{-i}$ and $p^2_{-i}$, while $U^1$ decides $p^1_i$ and $U^2$ decides $p^2_i$. We assume that all retail prices are chosen simultaneously.\footnote{Alternatively, we could assume that retail prices are chosen sequentially. However, this would add a strategic element — by giving the side moving first a first-mover advantage — that is not present in the other cases. It is also not clear why one side, or even which side, would have such a first-mover advantage.} The maximization problems are thus given by

\[
\max_{p^1_i} \Pi^U_1, \max_{p^2_i} \Pi^U_2, \tag{11}
\]

and

\[
\max_{p^1_{-i}, p^2_{-i}} \Pi_{D_{-i}}. \tag{12}
\]

From (11) and (12), we obtain the four first-order conditions that must be satisfied in equilibrium. Our assumptions imply that this equilibrium exists and is unique (and therefore that these conditions are sufficient). The two conditions that arise from $D_{-i}$’s problem are given by (4) and (5), with $i$ replaced by $-i$. The two conditions that arise from $U^1$ and $U^2$’s problem are given by (7) if $i = 1$ and (8) if $i = 2$, with $j = 1$ and $j = 2$.

It follows from the symmetry of the goods and firms that if $U^1$ and $U^2$’s prices were such that $p^1_i = p^2_i$, then $D_{-i}$ would set $p^1_{-i} = p^2_{-i}$, and vice versa. Thus, there exists a symmetric solution in which the prices of the goods within each firm are the same. And, since our assumptions imply that this solution must be unique, it follows that the four conditions that characterize the Bertrand equilibrium in the mixed regime can be reduced to the following two conditions, one that determines $p^j_{-i}$ and one that determines $p^j_i$:\footnote{Conditions (4) and (5), with subscript $i$ replaced by subscript $-i$, reduce to condition (13). Condition (7), with $j = 1$ and $j = 2$, reduces to condition (14), with subscripts $1, 2$ replaced by subscripts $i$ and $-i$.}

\[
p^j_{-i} \frac{\partial q^j_{-i}(p)}{\partial p^j_{-i}} + q^j_{-i}(p) + p^j_{-i} \frac{\partial q^j_{-i}(p)}{\partial p^j_{-i}} = 0, \tag{13}
\]

\[
(1 - s_i) \left( p^j_i \frac{\partial q^j_i(p)}{\partial p^j_i} + q^j_i(p) \right) + (1 - s_{-i}) \left( p^j_{-i} \frac{\partial q^j_{-i}(p)}{\partial p^j_{-i}} \right) = 0. \tag{14}
\]

These conditions not surprisingly have similarities with both the RPM and no RPM cases. It can be seen from (14) that, as in the RPM case, the optimal $p^j_i$ depends not only on $p^j_{-i}$ but also on $s_i$ and $s_{-i}$ when $s_i \neq s_{-i}$. However, unlike in the RPM case (but like in
the no RPM case), it can be seen from (13) that the optimal $p^j_{-i}$ depends only on $p^j_i$. This means that when $s_i \neq s_{-i}$, any impact of $s_i$ and $s_{-i}$ on $p^j_{-i}$ occurs only indirectly, through $p^j_i$. This fact allows us to obtain sharper implications than we obtained in Proposition 1:

**Proposition 4** In the mixed regime in which only $D_i$ adopts the agency model, equilibrium retail prices are independent of revenue shares if and only if $s_i = s_{-i}$. For $s_i \neq s_{-i}$:

- $p^j_i - p^j_{-i}$ is increasing in $s_i$;
- $p^j_i - p^j_{-i}$ is decreasing in $s_{-i}$;
- $p^j_i$ and $p^j_{-i}$ are increasing in $s_i$;
- $p^j_i$ and $p^j_{-i}$ are decreasing in $s_{-i}$.

The differences from Proposition 1 are as follows. Whereas in the RPM case, $p^j_i$ is increasing in $s_i$ in general only if the difference in the downstream firms’ revenue shares is sufficiently small, here $p^j_i$ is always increasing in $s_i$. And, whereas in the RPM case, $p^j_{-i}$ is decreasing in $s_i$ if the difference in the downstream firms’ revenue shares is sufficiently small, here $p^j_{-i}$ is always increasing in $s_i$. Intuitively, in the mixed regime in which only $D_i$ adopts the agency model, the upstream firms’ incentives are to adjust the prices of the products under their control so as to make the products which are sold at the downstream firm whose revenue share has increased relatively more expensive. Thus, it is optimal for $U_1$ and $U_2$ to adjust to an increase in $s_i$ by independently raising $p^1_i$ and $p^2_i$, respectively (in equilibrium, both prices will increase by the same amount), and conversely to adjust to an increase in $s_{-i}$ by independently lowering $p^1_i$ and $p^2_i$, respectively. In anticipation of this, $D_{-i}$ will also adjust its prices similarly – although not by as much (recall that $p^j_{-i}$ in this case only depends on $p^j_i$, and thus this follows because best replies are upward sloping). It follows therefore that $p^j_i$, $p^j_{-i}$, and $p^j_i - p^j_{-i}$ will be increasing in $s_i$ and decreasing in $s_{-i}$.

We can also see from (13) and (14) that the equilibrium $p^j_i$ will in general differ from the equilibrium $p^j_{-i}$, and that, after rearranging terms, the condition that determines which is larger depends on a comparison of the substitution between retailers and the substitution between goods – the same comparison that also drives the results in Proposition 3.

Our results for the equilibrium prices in the mixed regime can be summarized as follows:
Proposition 5  In the mixed regime in which only $D_i$ adopts the agency model, equilibrium retail prices are such that

- $p^j_i = p^{-j}_i$ for all $s_i$;
- $p^j_i = p^-_{-i}$ if $s_i = s_{-i}$ and $\frac{\partial q^j_i(p^*)}{\partial p^j_i} = \frac{\partial q^{-j}_i(p^*)}{\partial p^j_i}$;
- $p^j_i < p^-_{-i}$ if $s_i \leq s_{-i}$ and $\frac{\partial q^j_i(p^*)}{\partial p^j_i} < \frac{\partial q^{-j}_i(p^*)}{\partial p^j_i}$;
- $p^j_i > p^-_{-i}$ if $s_i \geq s_{-i}$ and $\frac{\partial q^j_i(p^*)}{\partial p^j_i} > \frac{\partial q^{-j}_i(p^*)}{\partial p^j_i}$.

Proposition 5 implies that the retail prices of the firm with RPM will be greater than the retail prices of the firm without RPM when the price sensitivity of consumers between downstream firms, as measured by $\frac{\partial q^j_i(p^*)}{\partial p^j_i}$, exceeds the price sensitivity of consumers between goods, as measured by $\frac{\partial q^{-j}_i(p^*)}{\partial p^j_i}$, and the revenue share of the firm with RPM is at least as large as the revenue share of the firm without RPM. The retail prices of the firm with RPM will be less than the retail prices of the firm without RPM when the opposite holds. Thus, for example, when the revenue shares of the two firms are the same, we would expect retail prices to be higher at the firm with RPM than at the firm without RPM if and only if there is more substitution at the downstream level than there is upstream.

The intuition in this case is similar to the intuition for the results in Proposition 3, with the difference being that there the comparison of retail prices was between both firms adopting the agency model and neither firm adopting the agency model, whereas here the comparison is between the firm that adopts the agency model and the firm that does not.

Notice that the ordering of the retail prices in Proposition 5 is not exhaustive. If $D_i$’s revenue share is lower than $D_{-i}$’s revenue share, but substitution is greater downstream than upstream, then we cannot say in general whether $p^j_i$ will be greater or less than $p^-_{-i}$.

It follows therefore that if the downstream firms’ revenue shares are unequal, the firm that adopts the agency model may in theory have higher retail prices than the firm that does not adopt the agency model even if competitive pressures are higher upstream, as long as its revenue share is sufficiently larger than its rival’s revenue share (or lower retail prices

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24The reason is that there would then be two forces pulling in opposite directions. The relatively greater substitution downstream, all else equal, would favor higher retail prices at $D_i$; however, $D_i$’s lower revenue share would, as shown in Proposition 4, favor lower retail prices. A similar ambiguity would arise if $D_i$’s revenue share were greater than $D_{-i}$’s revenue share, but substitution was relatively greater upstream.
than the firm that does not adopt the agency model even if competitive pressures are lower upstream as long as its revenue share is sufficiently smaller than its rival’s revenue share).

2.3 Choice of business format

We model the downstream firms’ choice of business format by introducing a stage in which the firms non-cooperatively decide whether to adopt the agency model. The possibilities are (i) both firms adopt the agency model (the RPM case), (ii) one firm adopts the agency model (mixed regime), or (iii) neither firm adopts the agency model (the no RPM case).

As a first step, we compare equilibrium prices across the different cases and regimes. Recall that we have defined $p = p^*$ to be the unique price that solves (9), and thus that the vector of equilibrium prices in the no RPM case is given by $p^* \equiv (p^*, p^*, p^*, p^*)$. Equilibrium prices in the other cases depend on $s_1$ and $s_2$. We let $p = p^a$ denote the unique price that solves (10), and note that the vector of equilibrium prices in the RPM case is thus given by $p^a = (p^a, p^a, p^a, p^a)$ when $s_1 = s_2$. Lastly, we let $p_1^{a*}, p_2^{a*}$ denote the unique pair of prices that solves (13) and (14) when $i = 1$, and we define $p^{a*} \equiv (p_1^{a*}, p_2^{a*}, p_2^{a*}, p_2^{a*})$ to be the vector of equilibrium prices in the mixed regime in which only $D_1$ has RPM (we also define prices $p_1^{a*}, p_2^{a*},$ and $p^{a*}$ analogously for the mixed regime in which only $D_2$ has RPM).

Using this notation, we can rank the equilibrium prices when $s_1 = s_2$ as follows:

**Proposition 6** When $s_1 = s_2$, equilibrium retail prices are such that

- $p^a = p_1^{a*} = p_2^{a*} = p_1^{a*} = p_2^{a*} = p^*$ if $\frac{\partial q_i^j(p^*)}{\partial p_i^j} = \frac{\partial q_i^j(p^*)}{\partial p_i^j}$;

- $p^a < p_1^{a*} < p_2^{a*} < p^*$ and $p^a < p_2^{a*} < p_1^{a*} < p^*$ if $\frac{\partial q_i^j(p^*)}{\partial p_i^j} < \frac{\partial q_i^j(p^*)}{\partial p_i^j}$;

- $p^a > p_1^{a*} > p_2^{a*} > p^*$ and $p^a > p_2^{a*} > p_1^{a*} > p^*$ if $\frac{\partial q_i^j(p^*)}{\partial p_i^j} > \frac{\partial q_i^j(p^*)}{\partial p_i^j}$.

Here we see that the ranking depends only on a comparison of the degree of substitution upstream (as measured by $\frac{\partial q_i^j(p^*)}{\partial p_i^j}$) and the degree of substitution downstream (as measured by $\frac{\partial q_i^j(p^*)}{\partial p_i^j}$). Proposition 6 implies that when $s_1 = s_2$, retail prices will be lower in the mixed regimes than in the symmetric RPM case, and higher in the mixed regimes than

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25 Note that $p_i^{a*}$ depends on $s_1$ and $s_2$. The arguments have been suppressed for ease of exposition.

26 If $s_1 \neq s_2$, then the gap between $s_1$ and $s_2$ as well as the sign of $s_1 - s_2$ would also need to be considered. See, for example, Propositions 1, 2 and 5, and the discussion of these propositions in the text.
in the no RPM case, if and only if the degree of substitution downstream is greater than
the degree of substitution upstream. It also implies that when \( s_1 = s_2 \), retail prices will
be higher in the mixed regimes than in the symmetric RPM case, and lower in the mixed
regimes than in the no RPM case, if and only if the opposite holds. It follows that the retail
prices that arise in equilibrium in the mixed regimes will be bounded by the retail prices
that arise in the symmetric RPM case (where both firms adopt the agency model) and the
retail prices that arise in the no RPM case (where neither adopts the agency model).\(^27\)

![Figure 1](image)

The intuition for the results when the degree of substitution downstream is greater than
the degree of substitution upstream can be seen from Figure 1. In this figure, \( BR^a_2(p^j_1) \)
represents the locus of prices \( p^j_2 \) that satisfy (13) when \( i = 1 \), and \( BR^a_1(p^j_2) \) is its mirror
image. The intersection of these best reply curves occurs at the price pair \((p^*, p^*)\), which
corresponds to the solution in the no RPM case in which all four products are sold at price

\(^27\)Within the mixed regimes, Proposition 6 implies that retail prices will be higher at the firm that has RPM if and only if the substitution is greater downstream (a result that is also implied in Proposition 5).
Similarly, $BR_1^a(p^2_j)$ represents the locus of prices $p^j_1$ that satisfy (14) when $i = 1$ and $s_1 = s_2$, and $BR_2^a(p^1_j)$ is its mirror image. The intersection of these best reply curves occurs at the price pair $(p^a, p^a)$, which corresponds to the solution in the symmetric RPM case. For the regime in which only $D_1$ has RPM and $s_1 = s_2$, the solution occurs at the price pair $(p^a_1, p^a_2)$, which is where the best reply curve $BR_2^a(p^1_j)$ intersects $BR_1^a(p^2_j)$. In this solution, $U^1$ and $U^2$ set price $p^*_1$ on $D_1$'s two products, and $D_2$ sets price $p^*_2$ on its two products. The results in the third bullet point of Proposition 6 then follow immediately.\(^{28}\)

One might think that the downstream firms would always opt for the business format that yields the highest equilibrium prices when given a choice (e.g., adopt the agency model in the setting depicted in Figure 1, where equilibrium prices are higher when one firm adopts the agency model, and highest when both firms do so). As we will now show, however, this intuition is only partially correct. When $s_1 = s_2$, it holds when the degree of substitution is relatively greater upstream, but not when the degree of substitution is relatively greater downstream. More generally, it holds when equilibrium prices would be the highest in the no RPM case but not when they would be the highest in the RPM case.

For our next result, we allow for any $s_1$ and $s_2$ and assume without loss of generality that if only one firm adopts the agency model, it will be $D_1$. We also define $p^a_1$ and $p^a_2$ to be the unique pair of prices that solve (7) and (8), respectively, given that $p^j_i = p^{-j}_i$, and note that the vector of equilibrium prices in the RPM case is thus given by $p^a = (p^a_1, p^a_1, p^a_2, p^a_2)$.

It follows that $p^a_1 = p^a_2 = p^a$ if $s_1 = s_2$ and, from Proposition 2, that $p^a_i > p^a_{-i}$ if $s_i > s_{-i}$.

The following proposition describes what can be said in general.

**Proposition 7** Suppose there is an initial stage of the game in which the downstream firms simultaneously and independently choose whether to adopt the agency model. Then,\(^{29}\)

- if the equilibrium retail prices in the subgames are such that $p^a_i < p^a_{-i} < p^a$, the unique equilibrium outcome is for neither firm to adopt the agency model;

- if the equilibrium retail prices in the subgames are such that $p^a_i > p^a_{-i} > p^a$, there are settings in which each outcome can arise in equilibrium. The equilibrium need not be unique, and there may be no equilibrium in which the agency model is adopted.

\(^{28}\)An analogous figure holds for the case in which the degree of substitution is greater upstream.

\(^{29}\)If $s_1 = s_2$, then, as implied in Proposition 6, a necessary and sufficient condition for the first bullet point to hold is that the degree of substitution must be greater upstream, and a necessary and sufficient condition for the second bullet point to hold is that the degree of substitution must be greater downstream.
To prove the first bullet point, note that if an equilibrium in which neither firm adopts the agency model is to exist, it must be that neither firm unilaterally wants to delegate control over its retail prices to the upstream firms. Or, in other words, it must be that $\Pi_{D_1}(p^*) \geq \Pi_{D_1}(p^{a*})$ and $\Pi_{D_2}(p^*) \geq \Pi_{D_2}(p^{a*})$. This is indeed the case, as we can see by forming the difference for $D_1$ (analogously for $D_2$) and making a simple substitution:

$$\Pi_{D_1}(p^*) - \Pi_{D_1}(p^{a*}) = 2s_1 \left(p^* q_1(p^*, p^*, p^*, p^*) - p_1^{a*} q_1^1(p_1^{a*}, p_1^{a*}, p_2^{a*}, p_2^{a*})\right)$$

$$\geq 2s_1 \left(p^* q_1(p^*, p^*, p^*, p^*) - p_1^{a*} q_1^1(p_1^{a*}, p_1^{a*}, p^*, p^*)\right)$$

$$> 0.$$  

The first inequality in (15) follows because $D_1$ and $D_2$ are substitutes and retail prices in this case are highest in the subgame where neither firm adopts the agency model. The last inequality follows because $p_1^1 = p_2^2 = p^*$ maximizes $D_1$’s profit when $D_2$ sets $p_2^1 = p_2^2 = p^*$.

No other outcome can arise in this case because to support a mixed regime in which only $D_1$ has RPM, it must be that $\Pi_{D_1}(p^{a*}) \geq \Pi_{D_1}(p^*)$, which we have just shown fails to hold (similarly when only $D_2$ has RPM). And, to support an outcome in which both firms adopt the agency model, it must be that $\Pi_{D_1}(p^a) \geq \Pi_{D_1}(p^{a*})$ and $\Pi_{D_2}(p^a) \geq \Pi_{D_2}(p^{a*})$, which we can show also fails to hold using reasoning similar to the reasoning from above.\(^{30}\)

Surprisingly, the analog is not true. The second bullet point implies that there may also be an equilibrium in which neither firm adopts the agency model even when the retail prices would be higher with RPM. The reason is that neither firm may want to be the only firm to switch. We can see from (15), for example, that if the setting were such that $p_i^{a*} > p^*$, the first inequality would be reversed, but the last inequality would still hold. This would imply that a firm’s gain from being the first to adopt the agency model would be weakly greater than some amount which is negative — which does not tell us much.

It turns out that when the equilibrium retail prices would be highest in the RPM case, there are settings in which all three outcomes — zero, one, or both firms adopting the agency model — can arise in equilibrium, and that for some of these settings, there may be no equilibrium in which the agency model is adopted. These results can be understood intuitively by noting that when retail prices would be highest with RPM, there are two opposing factors that a downstream firm, say $D_1$, must consider. On the one hand, adopt-

\(^{30}\)Note that $\Pi_{D_1}(p^*) - \Pi_{D_1}(p^{a*})$ is less than or equal to $\Pi_{D_1}(p_1^a, p_1^{a*}, p_2^{a}, p_2^{a*}) - \Pi_{D_1}(p_1^{a*}, p_1^{a*}, p_2^{a}, p_2^{a*})$, which is less than zero because $p_1^1 = p_2^2 = p_1^{a*}$ maximizes $D_1$’s profit when its rival sets $p_2^1 = p_2^2 = p_2^{a*}$.
ing the agency model (whether or not \( D_2 \) also adopts the agency model) will cause \( D_2 \)’s equilibrium prices to increase (which positively impacts \( D_1 \)’s profit because \( D_1 \) and \( D_2 \) are substitutes). This is depicted by the up arrow in Figure 1. On the other hand, delegating pricing control to the upstream firms means that \( D_1 \)’s prices will be chosen to maximize a different objective function than \( D_1 \) would otherwise have preferred. This is depicted by the right arrow in Figure 1 — which shows that the prices chosen for \( D_1 \) are higher than what \( D_1 \) would have chosen for the same \( p^*_2 \) if instead it had retained control. Whether this tradeoff is worth making to induce higher retail prices at \( D_2 \) depends. Using linear demands, it can be shown that for a given degree of substitution upstream, \( D_1 \) will find the sacrifice more likely to be profitable the greater is the degree of substitution downstream.

2.4 MFN’s as an ancillary restraint

We have shown that a prisoner’s dilemma may arise in which the agency model is not adopted by either firm even when industry-wide adoption would increase retail prices and industry profits. We now assess whether an MFN clause may help to avoid this situation.\(^{31}\)

We model an MFN clause, when imposed by \( D_i \), as requiring that \( U_1 \) ensure that \( p^1_i \) be such that \( p^1_i \leq p^1_{-i} \) and \( U^2 \) ensure that \( p^2_i \) be such that \( p^2_i \leq p^2_{-i} \). In the game thus modified, we continue to assume that all prices chosen by \( U^1 \), \( U^2 \), and \( D_{-i} \) (if applicable) are set simultaneously.\(^{32}\) To account for the possibility that \( D_{-i} \) might set lower retail prices than \( U^1 \) and \( U^2 \) anticipate, we further assume that \( p^1_i \) and \( p^2_i \) will adjust automatically to satisfy the MFN clause in the (out-of-equilibrium) event that the constraints fail to hold initially.

Whether \( D_i \)’s MFN clause would have any effect on \( U^1 \) and \( U^2 \)’s choices in this modified game will depend, of course, on whether it would be binding in equilibrium. We found in Proposition 5, for example, that even without an MFN clause, \( D_i \)’s retail prices would be lower than \( D_{-i} \)’s retail prices in the mixed regime if \( s_i \leq s_{-i} \) and the degree of substitution is greater upstream than downstream. However, in other settings, we found that it would be binding (e.g., if \( s_i \geq s_{-i} \) and the degree of substitution is instead greater downstream).

To see what prices can be supported in equilibrium when \( D_i \)’s MFN clause is binding,

\(^{31}\)As quoted in DOJ (2012; p. xxx), “The MFN here required each publisher to guarantee that it would lower the retail price of each e-book in Apple’s iBookstore to match the lowest price offered by any other retailer, even if the Publisher Defendant did not control that other retailer’s ultimate consumer price.”

\(^{32}\)Alternatively, we could just as well assume that \( D_{-i} \) (assuming it has control) sets its prices before \( U_1 \) and \( U_2 \) simultaneously set their respective prices \( p^1_i \) and \( p^2_i \). This would not affect our qualitative results.
consider first the case of a mixed regime in which \( D_{-i} \) retains control over its pricing. Note first that because of \( D_i \)'s MFN clause, \( D_{-i} \) cannot undercut the retail prices set by \( U^1 \) and \( U^2 \) through \( D_i \). Any attempt to do so would only force the upstream firms to follow suit with their own price cuts. Such a strategy would therefore be profitable for \( D_{-i} \) only for retail prices that were above \( p' \), the industry profit-maximizing price. Note next that for any price \( p_{-i}^j = \hat{p} \) set by \( D_{-i} \) such that (14) would be negative when evaluated at \( p_{i}^j = \hat{p} \) (i.e., when the best reply of \( U^j \) is to set \( p_{i}^j < \hat{p} \) when all other prices are equal to \( \hat{p} \)), the upstream firms would both want to undercut \( D_i \)'s price and would be free to do so. Note finally that for any price \( p_{-i}^j = \hat{p} \) set by \( D_{-i} \) such that (14) would be positive when evaluated at \( p_{i}^j = \hat{p} \) (i.e., when the best reply of \( U^j \) is to set \( p_{i}^j > \hat{p} \) when all other prices are equal to \( \hat{p} \)), the upstream firms would want to charge higher prices but would be forced by \( D_i \)'s MFN clause to match \( D_{-i} \)'s lower price. It follows that if \( p^m \) denotes the retail price that solves (14) when all prices are the same (note that \( p^m \) depends on \( s_1 \) and \( s_2 \); for example, when \( s_i = s_{-i} \), \( p^m = p^a \), and when \( s_i > s_{-i} \), \( p^m > p^a \) and \( p^m > p_{i}^a > p_{-i}^a \)), then the minimum of \( p' \) and \( p^m \) is the highest retail price that can be supported in equilibrium.

Other prices, however, can also be supported. For example, equal prices in the neighborhood below \( p^m \) can also be supported when \( p^m \leq p' \) because at these prices, the upstream firms would ideally like to set higher retail prices but are constrained by \( D_i \)'s MFN clause, and \( D_{-i} \) will not find it profitable to deviate because although it would ideally like to set lower retail prices, this would only cause the upstream firms to reduce their prices as well.

The following proposition describes what can be said in general.

**Proposition 8** In the mixed regime in which only \( D_i \) has adopted the agency model,

- \( D_i \)'s MFN clause with \( U^1 \) and \( U^2 \) will have no effect on equilibrium retail prices if \( s_i \leq s_{-i} \) and the degree of substitution is greater upstream than downstream;

- \( D_i \)'s MFN clause with \( U^1 \) and \( U^2 \) will be binding if \( s_i \geq s_{-i} \) and the degree of substitution is greater downstream than upstream. The vector of retail prices \( \mathbf{p} = (p, p, p, p) \) can be supported in equilibrium if and only if \( p \in [p^*, \min\{p^m, p'\}] \).

There are multiple equilibria in this regime because when \( D_i \)'s MFN clause binds, a kink point is created in the best replies of the upstream firms and \( D_{-i} \). The upstream firms cannot set higher prices than \( D_{-i} \) sets, and \( D_{-i} \) knows this. The clause thus works
to mitigate $D_{-i}$'s incentive to set low prices. At the same time, however, if $D_{-i}$ anticipates that the upstream firms will set relatively low prices, then it may be optimal for $D_{-i}$ also to set relatively low prices, and the relatively low prices may then become self-supporting.

Figure 2 illustrates the equilibrium outcomes for the mixed regime in which only $D_{i}$ adopts the agency model, $s_1 = s_2$, and the degree of substitution is greater downstream. At the lower bound of what can be supported when $D_{i}$ has an MFN clause, retail prices are at $p^*$, the same as if $D_{i}$ had not adopted the agency model. In all other outcomes, however, retail prices are strictly higher. The same is true when $s_i > s_{-i}$ and the degree of substitution is greater downstream. It follows, therefore, that if the game is further modified to allow for an initial stage in which the downstream firms simultaneously and independently choose whether to adopt the agency model and impose an MFN clause, and if the degree of substitution is relatively greater downstream, then adopting the agency model and imposing an MFN clause is a best response for $D_{i}$ (it is a strict best response for all but the least advantageous outcome) when $D_{-i}$ does not adopt the agency model.

This leads to our first main result of this subsection. Using Pareto optimality to select
among equilibria in the subgames with MFN clauses, there is no equilibrium in the initial
stage of the game in which the agency model is adopted by either firm when conditions are
such that industry-wide adoption would increase retail prices and industry profits. This
follows because \( p = \min\{p^m, p^I\} \) in the Pareto-optimal equilibrium of the continuation
game in which only \( D_i \) adopts the agency model and \( D_i \) has an MFN clause. \( D_i \)'s profit
in this case would therefore either be \( \Pi_{D_i}(p^m, p^m, p^m, p^m) \) or \( \Pi_{D_i}(p^I, p^I, p^I, p^I) \), which exceeds
what \( D_i \) could earn if instead it did not adopt the agency model, \( \Pi_{D_i}(p^*, p^*, p^*, p^*) \). It
follows that MFN clauses can indeed help to avoid a prisoner's dilemma in such settings.

We can further extend our results by noting that the multiplicity of equilibria that
plagues the mixed regimes with MFN clauses disappears when both downstream firms
adopt the agency model and at least one firm has an MFN clause. The reason is that the
upstream firms then control the prices of their goods at both locations and thus can satisfy
their MFN clause(s) without having to anticipate the prices of an independent retailer. In
these settings, if only \( D_i \) has an MFN clause, then \( U^j \) chooses \( p^i \) and \( p^i_{-j} \) to maximize its
profit subject to \( p^i < p^i_{-j} \). And if both \( D_i \) and \( D_{-i} \) have MFN clauses, then \( U^j \) chooses \( p^i \)
and \( p^i_{-j} \) to maximize its profit subject to \( p^i = p^i_{-j} \). Our assumptions ensure that there will
be a unique equilibrium outcome whether or not the constraints turn out to be binding.

We have already considered the case in which the constraints are not binding. In the
case in which the constraints imposed by the MFN clause(s) are binding, the first-order
conditions, \( j = 1, 2 \), that characterize the Bertrand equilibrium when both downstream
firms adopt the agency model and at least one firm has an MFN clause can be written as

\[
\frac{\partial \Pi^{U^j}_{p^i}}{\partial p^i_j} \Big|_{p^i_j=p^a} = (1-s_i) \left( p^i_j \left( \frac{\partial q^i_j}{\partial p^i_j} + \frac{\partial q^i_{-j}}{\partial p^i_{-j}} \right) + q^i_j \right) + (1-s_{-i}) \left( p^i_{-j} \left( \frac{\partial q^i_{-j}}{\partial p^i_j} + \frac{\partial q^i_{-j}}{\partial p^i_{-j}} \right) + q^i_{-j} \right) = 0. \tag{16}
\]

Solving these conditions to obtain the equilibrium \( p^i_j \) and \( p^i_{-j} \) yields a surprising impli-
cation. If \( U^{1-j} \) is setting \( p^i_{-j} = p^{1-j} = p^a \), then the best \( U^j \) can do is also to set \( p^i_j = p^i_{-j} = p^a \)
(recall that \( p^a \) is the price that solves (10)), and vice versa. This suggests that not only
is the multiplicity of equilibria in the mixed regimes gone, but the inability of \( U^1 \) and
\( U^2 \) to discriminate between retailers implies that when the MFN clause(s) are binding (as
they will be if both \( D_i \) and \( D_{-i} \) have MFN clauses, or if only \( D_i \) has an MFN clause and
\( s_i \geq s_{-i} \)), the upstream firms lose their ability to disadvantage any downstream firm that
has a relatively high revenue share. Thus, unlike in the case of RPM without MFN clauses, with binding MFN clauses, the unique equilibrium outcome when both firms adopt the agency model calls for all prices to be set equal to $p^a$, whether or not $s_i$ is equal to $s_{-i}$.$^{33}$

This leads to our second main result of this subsection. In any equilibrium in the initial stage in which at least one firm adopts the agency model and has a binding MFN clause, retail prices on all products will be at least as high as $p^a$. This result does not depend on whether Pareto optimality is used to select among equilibria in the subgames with MFN clauses, and it holds because if $D_i$ adopts the agency model and imposes an MFN clause, then $D_{-i}$ can ensure that the unique equilibrium price vector in the continuation game will be $p = (p^a, p^a, p^a, p^a)$ by also adopting the agency model and having an MFN clause.

It remains to consider whether and under what conditions equilibria exist in which at least one firm adopts the agency model and has a binding MFN clause. The following proposition summarizes our results thus far and describes what can be said in general.

**Proposition 9** Suppose there is an initial stage of the game in which the downstream firms simultaneously and independently choose whether to adopt the agency model and have an MFN clause. Then, if the degree of substitution is greater downstream than upstream,

- there exists an equilibrium in which at least one firm adopts the agency model and has an MFN clause. In every such equilibrium, retail prices are the same and equal to $p^a$ when $s_i = s_{-i}$, and the same and greater than or equal to $p^a$ when $s_i > s_{-i}$;

- there exists an equilibrium in which $D_i$ adopts the agency model and has an MFN clause, and retail prices are the same and strictly exceed $\max\{p_i^a, p^a\}$ when $s_i > s_{-i}$;

- there does not exist an equilibrium in which neither firm adopts the agency model if Pareto optimality is used to select among equilibria in the mixed-regime subgames.

It follows from Proposition 9 that MFN clauses can have significant effects when they are binding (e.g., they can help to avoid a prisoner’s dilemma).$^{34}$ If, for example, the

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$^{33}$We note that if the disparity in revenue shares becomes too large, it may be in the interests of the upstream firms not to sell to both downstream firms, a consideration that we have abstracted from here.

$^{34}$It should be noted, however, that in some well defined settings, MFN clauses need not be binding. If the degree of substitution is relatively greater upstream and $s_i = s_{-i}$, for example, then even without an MFN clause, $D_i$’s retail prices would be lower than $D_{-i}$’s retail prices when only $D_i$ adopts the agency model. In this case, MFN clauses would affect neither equilibrium prices, nor the choice of business formats.
degree of substitution is relatively greater downstream and Pareto optimality is used to select among equilibria in the mixed regimes in which only one firm adopts the agency model and has an MFN clause, then there is no equilibrium in which neither firm adopts the agency model. Not adopting the agency model when the rival firm does not adopt the agency model ensures that equilibrium prices in the continuation game will be \( p^* \), whereas by adopting the agency model and imposing an MFN clause, a deviating firm can ensure that equilibrium prices (and profits) in the continuation game will be strictly higher. An MFN clause can thus ensure that at least one firm will find it profitable to engage in RPM.

We can also see from Proposition 9 that MFN clauses can affect equilibrium prices. This occurs in some well-defined settings. When \( s_i = s_{-i} \) and the degree of substitution is relatively greater downstream, it occurs when the equilibrium in the initial stage of the game in the absence of MFN clauses is for either zero or one firms to adopt the agency model. In the first instance, retail prices in the continuation game are given by \( \mathbf{p} = (p^*, p^*, p^*, p^*) \), and in the second instance, they are given by \( \mathbf{p} = (p_1^a, p_1^a, p_2^a, p_2^a) \) if only \( D_1 \) adopts the agency model (and analogously if only \( D_2 \) adopts the agency model). In both instances, the retail prices are strictly less than \( p^a \) (see Proposition 6). It follows from Proposition 9, therefore, that MFN clauses would lead to strictly higher retail prices in these settings.

MFN clauses can also lead to higher retail prices when \( s_i > s_{-i} \) and the degree of substitution is relatively greater downstream. In this setting, we know from Proposition 9 that an equilibrium in which \( D_i \) adopts the agency model and has an MFN clause exists, and that retail prices on all products in this equilibrium are the same and strictly greater than \( \max\{p_i^a, p^a\} \). For example, using Pareto optimality to select from among equilibria in the mixed regimes, it is straightforward to show that such an equilibrium exists and that the equilibrium retail prices in this case are equal to \( \min\{p^m, p^f\} > \max\{p_i^a, p^a\} \). The conclusion that MFN clauses can lead to higher retail prices in this case then follows on recognizing that \( \max\{p_i^a, p^a\} \) is the maximum price that can arise in equilibrium otherwise.

3 Conclusion

This paper analyzes the competitive effects of the agency model in a structure with both upstream and downstream competition, and we fix both the number and quality of products offered in the marketplace and abstract from issues of asymmetric information and
uncertainty. Furthermore, we treat the revenue shares as exogenously given. In so doing, we keep the paper’s focus solely on the pricing decisions of the firms, and how these decisions would impact adoption of the agency model all else being equal, recognizing that the analysis can only provide a partial depiction of the complex forces that govern the relationships in these markets. For example, content providers may be better informed than platform providers about the market potential for their goods. In such cases, there might be efficiency gains from letting content providers determine retail prices (Foros, Kind and Hagen, 2009). Hagiu and Wright (2013) analyze the efficient choice of business format when firms engage in non-contractible marketing activities (they abstract from pricing and do not focus on the agency model). Gans (2012) considers the hold-up problem that may arise if consumers must undertake specific investments in order to have platform access prior to the upstream firms’ choosing prices. In the market for e-books there may be a negative externality from e-books to printed books (Abhishek et al., 2012). Letting publishers decide on retail prices allow them to internalize this externality. However, from digital platform providers’ perspective, publishers may then have incentives to set retail prices too high. In fact, Apple included price caps (maximum RPM) in the proposed agency model towards publishers. These price caps were nailed to the recommended prices for printed books, and the motivation was to prevent publishers to set e-book prices prohibitively high. Apple and publishers finally agreed to set the price caps 30-50% above Amazon’s retail price ($9.99).\footnote{See Buettner et al. (2013) for more details.}

Other externalities may work against transferring control of the retail pricing to the upstream content providers. Content in the form of music, apps and e-books, for example, are complementary products to tablets and smartphones. Such complementarity would all else being equal work in favor of the retail prices being determined by the platform providers (Gaudin and White, 2013).

Finally, throughout we have treated the revenue shares as exogenously given. The reason is that most platforms offer a “one size fits all” revenue split. Apple behaves towards the content providers as though the 70/30 split is carved in stone (see Isaacson, 2011). If platforms endogenously set revenue splits, for each market, as take-it-or-leave-it contracts prior to the upstream firms’ determination of retail prices in our set-up, one might at the outset expect that the upstream firms will be left with a zero revenue-share (abstracting
from fixed costs). However, this is presumably not correct. This follows from the fact that the greater is the revenue share a downstream firm requires, the higher retail prices will the upstream firms tend to charge through that platform. Thereby the rival downstream firm will get a competitive advantage. This problem could partly be circumvented by imposing an MFN clause, because that will weaken the incentive to compete in revenue shares (see also Johnson 2013 a,b). However, if one downstream firm sets a much higher revenue share than its rival, then upstream firms may optimally foreclose it. Thus, we should always expect upstream revenue shares to be positive, also with MFN.
Appendix

Proof of Proposition 1: We have already shown in the text that equilibrium retail prices are independent of revenue shares when the downstream firms control prices. We have also shown that when the upstream firms control prices, equilibrium retail prices are independent of revenue shares if and only if $s_1 = s_2$ (see footnote 20). This leaves only the three bullet points, regarding the effects of different revenue-sharing splits, to be proven.

Since the goods are symmetrically differentiated, each upstream firm will set the same retail price for $D_i$ in equilibrium. This means that $p_i^1 = p_i^{-1}$ and $p_i^2 = p_i^{-2}$. Therefore, let $p_1$ denote the common price at retailer 1 and $p_2$ denote the common price at retailer 2. The four first-order conditions that characterize the Bertrand equilibrium in the agency model (i.e., (7) and (8), $j = 1, 2$) can therefore be reduced to the following two conditions:

\begin{align}
(1 - s_1) \left(p_1 \frac{\partial q_1^1}{\partial p_1^1} + q_1^1\right) + (1 - s_2) \left(p_2 \frac{\partial q_2^1}{\partial p_1^1}\right) &= 0, \\
(1 - s_1) \left(p_1 \frac{\partial q_1^2}{\partial p_2^2}\right) + (1 - s_2) \left(p_2 \frac{\partial q_2^2}{\partial p_1^2} + q_2^1\right) &= 0.
\end{align}

We can define best-reply curves as follows. Let $p_1 = BR_1^a(p_2, s_1, s_2)$ be the solution to (A.1) and $p_2 = BR_2^a(p_1, s_1, s_2)$ be the solution to (A.2). Then, it must be that in equilibrium, retail prices satisfy $p_1 = BR_1^a(BR_2^a(p_1, s_1, s_2), s_1, s_2)$ and $p_2 = BR_2^a(BR_1^a(p_2, s_1, s_2), s_1, s_2)$.

Consider first the equilibrium $p_1$. Taking the derivative with respect to $s_2$ yields

\[\frac{\partial p_1}{\partial s_2} = \frac{\partial BR_1^a}{\partial p_2} \left(\frac{\partial BR_2^a}{\partial p_1} \frac{\partial p_1}{\partial s_2} + \frac{\partial BR_2^a}{\partial s_2}\right) + \frac{\partial BR_1^a}{\partial s_2}.\]

Rearranging this yields

\[\frac{\partial p_1}{\partial s_2} = \frac{\frac{\partial BR_1^a}{\partial p_2} \frac{\partial BR_2^a}{\partial p_1} \frac{\partial p_1}{\partial s_2} + \frac{\partial BR_1^a}{\partial s_2}}{(1 - \frac{\partial BR_1^a}{\partial p_2} \frac{\partial BR_2^a}{\partial p_1})}.\]

Our assumptions on uniqueness imply that $\frac{\partial BR_1^a}{\partial p_2}$ and $\frac{\partial BR_2^a}{\partial p_1}$ are less than one in absolute value. It follows that the sign of $\frac{\partial p_1}{\partial s_2}$ will be the same as the sign of the numerator in (A.4):

\[\frac{\partial BR_1^a}{\partial p_2} \frac{\partial BR_2^a}{\partial p_1} \frac{\partial p_1}{\partial s_2} + \frac{\partial BR_1^a}{\partial s_2}.\]

In general, the sign of (A.5) is ambiguous. The term $\frac{\partial BR_1^a}{\partial p_2} \frac{\partial BR_2^a}{\partial s_2}$ is positive (we assume best-response curves are upward sloping, and $\frac{\partial BR_2^a}{\partial s_2} > 0$), but the term $\frac{\partial BR_1^a}{\partial s_2}$ is negative. Nevertheless, we can show that in the limit, as $s_1$ goes to $s_2$, the sign is strictly negative.
To see this, note that
\[
\frac{\partial BR_1^a}{\partial s_2} = \frac{p_2 \frac{\partial q_1^2}{\partial p_2}}{\Delta p_1} \quad \text{and} \quad \frac{\partial BR_2^a}{\partial s_2} = \left( \frac{p_2 \frac{\partial q_1^2}{\partial p_2} + q_1^2}{\Delta p_2} \right),
\] (A.6)
where \(\Delta p_1\) is the partial derivative of the left-hand side of (A.1) with respect to \(p_1\) and \(\Delta p_2\) is the partial derivative of the left-hand side of (A.2) with respect to \(p_2\). Our assumptions on the existence and uniqueness of equilibrium imply that \(\Delta p_1 < 0\) and \(\Delta p_2 < 0\).

Substituting from (A.6) into (A.5) yields
\[
\frac{\partial BR_1^a}{\partial p_2} \left( p_2 \frac{\partial q_1^2}{\partial p_2} + q_1^2 \right) - \frac{\partial BR_1^a}{\partial s_2} \left( 1 - s_2 \right) \left( p_1 \frac{\partial q_1^1}{\partial p_1} \right) + \frac{p_2 \frac{\partial q_1^2}{\partial p_2}}{\Delta p_1},
\] (A.7)
which, using (A.2), can be rewritten as
\[
- \frac{\partial BR_1^a}{\partial p_2} \left( 1 - s_1 \right) \left( p_1 \frac{\partial q_1^1}{\partial p_1} \right) + \frac{p_2 \frac{\partial q_1^2}{\partial p_2}}{\Delta p_1} \Rightarrow 0.
\] (A.8)
For \(s_1\) sufficiently close to \(s_2\), the sign of (A.8) is negative because in the limit
\[
- \left( 1 - s_1 \right) \left( p_1 \frac{\partial q_1^1}{\partial p_1} \right) \Rightarrow 0.
\] (A.9)
and \(\left| \frac{\partial BR_1^a}{\partial p_2} \right| < 1\). Thus, we have established the third bullet point in Proposition 1.

Now consider the equilibrium \(p_2\). Taking the derivative with respect to \(s_2\) yields
\[
\frac{\partial p_2}{\partial s_2} = \frac{\partial BR_2^a}{\partial p_1} \left( \frac{\partial BR_1^a \partial p_2}{\partial p_2} \frac{\partial BR_1^a}{\partial s_2} + \frac{\partial BR_1^a}{\partial s_2} \right) + \frac{\partial BR_2^a}{\partial s_2}.
\] (A.10)
Rearranging this yields
\[
\frac{\partial p_2}{\partial s_2} = \frac{\partial BR_2^a}{\partial p_1} \frac{\partial BR_1^a}{\partial s_2} + \frac{\partial BR_2^a}{\partial s_2} \left( 1 - \frac{\partial BR_2^a}{\partial p_1} \frac{\partial BR_1^a}{\partial s_2} \right). \quad (A.11)
\]
Our assumptions on uniqueness imply that \(\frac{\partial BR_1^a}{\partial p_2}\) and \(\frac{\partial BR_1^a}{\partial p_1}\) are less than one in absolute value. It follows that the sign of \(\frac{\partial p_2}{\partial s_2}\) will be the same as the sign of the numerator in A.11:
\[
\frac{\partial BR_2^a}{\partial p_1} \frac{\partial BR_1^a}{\partial s_2} + \frac{\partial BR_2^a}{\partial s_2}. \quad (A.12)
\]
In general, the sign of (A.12) is ambiguous. The term \(\frac{\partial BR_2^a}{\partial p_1} \frac{\partial BR_1^a}{\partial s_2}\) is negative (we assume best-response curves are upward sloping, and \(\frac{\partial BR_2^a}{\partial s_2} < 0\)), but the term \(\frac{\partial BR_2^a}{\partial s_2}\) is positive.
Nevertheless, using the same reasoning as was used in (A.6) through (A.9) above, it is straightforward to show that for $s_1$ sufficiently close to $s_2$, the sign of (A.12) is positive.

It remains to establish that the difference $p_i^j - p_{-i}^j$ is increasing in $s_i$. To do this, we construct $p_2 - p_1$ and look at the sign of $\frac{\partial p_2}{\partial s_2} - \frac{\partial p_1}{\partial s_2}$. We want to show that it is increasing.

Using (A.11) and (A.4), we note that the sign of $\frac{\partial p_2}{\partial s_2} - \frac{\partial p_1}{\partial s_2}$ is the same as the sign of

$$\frac{\partial BR_2^a}{\partial p_1} \frac{\partial BR_2^a}{\partial s_2} + \frac{\partial BR_1^a}{\partial p_2} \frac{\partial BR_2^a}{\partial s_2} - \left( \frac{\partial BR_1^a}{\partial p_2} \frac{\partial BR_2^a}{\partial s_2} + \frac{\partial BR_1^a}{\partial s_2} \right),$$

which can be rewritten as

$$\left(1 - \frac{\partial BR_1^a}{\partial p_2}\right) \frac{\partial BR_2^a}{\partial s_2} - \left(1 - \frac{\partial BR_1^a}{\partial p_1}\right) \frac{\partial BR_1^a}{\partial s_2}.$$

(A.13)

(A.14)

Using the fact that $|\frac{\partial BR_1^a}{\partial p_2}| < 1, |\frac{\partial BR_2^a}{\partial p_1}| < 1, \frac{\partial BR_2^a}{\partial s_2} > 0$, and $\frac{\partial BR_1^a}{\partial s_2} < 0$, it follows that the sign of (A.14) is indeed positive. This establishes the first bullet point in Proposition 1. Q.E.D.

**Proof of Proposition 2:** The claim that all prices will be the same when the downstream firms control prices (no RPM case) follows from the fact that both the firms and the goods are symmetrically differentiated, and noting that the solution to (4), (5), and their analogs does not depend on revenue shares (from Proposition 1). The first bullet point follows from the fact that since the goods are symmetrically differentiated, each upstream firm will set the same retail price for $D_i$ in equilibrium. The second bullet point follows from the fact that since the downstream firms are symmetrically differentiated, each upstream firm will set $p_i^j = p_{-i}^j$ when $s_i = s_{-i}$. The third bullet point follows from the first two bullet points and the fact that $p_i^j - p_{-i}^j$ is increasing in $s_i$ (from Proposition 1). Q.E.D.

**Proof of Proposition 3:** We have shown in the text that in the no RPM case, conditions (4), (5), and their analogs imply that in equilibrium, the unique price $p$ will satisfy

$$p \frac{\partial q_i^j(p)}{\partial p_i^j} + q_i^j(p) + p \frac{\partial q_{-i}^j(p)}{\partial p_i^j} = 0,$$

(A.15)

whereas when both firms adopt the agency model and revenue shares are the same, conditions (7), (8), and their analogs imply that in equilibrium, the unique price $p$ will satisfy

$$p \frac{\partial q_i^j(p)}{\partial p_i^j} + q_i^j(p) + p \frac{\partial q_{-i}^j(p)}{\partial p_i^j} = 0.$$
In each case, the price vector $\mathbf{p}$ is evaluated at the same four prices: $\mathbf{p} = (p, p, p, p)$.

As in the text, let $p = p^*$ denote the solution to (A.15) and define $\mathbf{p}^* \equiv (p^*, p^*, p^*, p^*)$ to be the vector of equilibrium prices. Then, by the definition of $p^*$, it follows that

$$p^* \frac{\partial q_j^i(\mathbf{p}^*)}{\partial p_i^j} + q_j^i(\mathbf{p}^*) + p^* \frac{\partial q_{-i}^{-j}(\mathbf{p}^*)}{\partial p_i^j} = 0.$$  \hspace{1cm} (A.17)

Evaluating the left-hand side of (A.16) at $\mathbf{p} = \mathbf{p}^*$ for all $i, j$, and using (A.17) yields

$$p^* \left( \frac{\partial q_{-i}^{-j}(\mathbf{p}^*)}{\partial p_i^j} - \frac{\partial q_{-i}^{-j}(\mathbf{p}^*)}{\partial p_i^j} \right),$$ \hspace{1cm} (A.18)

which can be either positive, negative, or zero, depending on the sign of $\frac{\partial q_{-i}^{-j}(\mathbf{p}^*)}{\partial p_i^j} - \frac{\partial q_{-i}^{-j}(\mathbf{p}^*)}{\partial p_i^j}$.

The three bullet points in Proposition 2 follow immediately on noting that our assumptions imply that the left-hand side of (A.16) is decreasing in $\mathbf{p}$ when $\mathbf{p} = (p, p, p, p)$. Thus, for example, if the sign of (A.18) is positive, then the left-hand side of (A.16) is positive at $\mathbf{p} = \mathbf{p}^*$, implying that $p = p^*$ is less than the (symmetric) equilibrium RPM price, and if the sign of (A.18) is negative, then the left-hand side of (A.16) is negative at $\mathbf{p} = \mathbf{p}^*$, implying that $p = p^*$ is greater than the (symmetric) equilibrium RPM price. \hspace{1cm} Q.E.D.

**Proof of Proposition 4:** We have shown in the text that the four first-order conditions that characterize the Bertrand equilibrium in the mixed regime when $D_i$ has RPM can be reduced to the following two conditions, one to determine $p_{-i}^j$ and one to determine $p_i^j$:

$$p_{-i}^j \frac{\partial q_{-i}^{-j}(\mathbf{p})}{\partial p_{-i}^j} + q_{-i}^{-j}(\mathbf{p}) + p_{-i}^j \frac{\partial q_{-i}^{-j}(\mathbf{p})}{\partial p_{-i}^j} = 0,$$ \hspace{1cm} (A.19)

$$(1 - s_i) \left( p_i^j \frac{\partial q_i^j(\mathbf{p})}{\partial p_i^j} + q_i^j(\mathbf{p}) \right) + (1 - s_{-i}) \left( p_{-i}^j \frac{\partial q_{-i}^j(\mathbf{p})}{\partial p_i^j} \right) = 0.$$ \hspace{1cm} (A.20)

Let $p_{-i}^j = BR_{-i}^*(p_{-i}^j)$ be the solution to (A.19) and let $p_i^j = BR_i^*(p_i^j, s_i, s_{-i})$ be the solution to (A.20). Then, it must be that in the equilibrium of the mixed regime in which $D_i$ has RPM, retail prices satisfy $p_{-i}^j = BR_{-i}^*(BR_i^*(p_{-i}^j, s_i, s_{-i}))$ and $p_i^j = BR_i^*(BR_{-i}^*(p_i^j), s_i, s_{-i})$.

Consider first the equilibrium $p_i^j$. Taking the derivative with respect to $s_i$ yields

$$\frac{\partial p_i^j}{\partial s_i} = \frac{\partial BR_i^*}{\partial p_{-i}^j} \frac{\partial BR_{-i}^*}{\partial p_i^j} \frac{\partial p_{-i}^j}{\partial s_i} + \frac{\partial BR_i^*}{\partial s_i}.$$  \hspace{1cm} (A.21)
Rearranging this yields
\[
\frac{\partial p^j_i}{\partial s_i} = \frac{\partial BR^e_i}{\partial s_i} \left( 1 - \frac{\partial BR^e_i \partial BR^{*,-}_i}{\partial p^j_{i-1} \partial p^j_i} \right).
\] (A.22)

Our assumptions on uniqueness imply that \(\frac{\partial p^j_i}{\partial s_i}\) and \(\frac{\partial BR^e_i}{\partial p^j_i}\) are less than one in absolute value. It follows that the sign of \(\frac{\partial p^j_i}{\partial s_i}\) will be the same as the sign of the numerator in (A.22), which is positive because the direct effect of an increase in \(s_i\) is to increase \(p^j_i\).

Next, consider the equilibrium \(p^j_{-i}\). Taking the derivative with respect to \(s_i\) yields
\[
\frac{\partial p^j_{-i}}{\partial s_i} = \frac{\partial BR^{*,-}_i}{\partial p^j_i} \left( \frac{\partial BR^e_i \partial p^j_{-i}}{\partial p^j_{-i} \partial s_i} + \frac{\partial BR^e_i}{\partial s_i} \right).
\] (A.23)

Rearranging this yields
\[
\frac{\partial p^j_{-i}}{\partial s_i} = \frac{\frac{\partial BR^{*,-}_i \partial BR^e_i}{\partial p^j_i \partial s_i}}{\left( 1 - \frac{\partial BR^{*,-}_i \partial BR^e_i}{\partial p^j_i \partial p^j_{-i}} \right)}.
\] (A.24)

Our assumptions on uniqueness imply that \(\frac{\partial BR^{*,-}_i}{\partial p^j_i}\) and \(\frac{\partial BR^e_i}{\partial p^j_i}\) are less than one in absolute value. It follows that the sign of \(\frac{\partial p^j_{-i}}{\partial s_i}\) will be the same as the sign of the numerator in (A.24), which is positive because best-response curves are upward sloping and \(\frac{\partial BR^e_i}{\partial s_i} > 0\).

It remains to establish that the difference \(p^j_i - p^j_{-i}\) is increasing in \(s_i\). Using (A.22) and (A.24), we note that the sign of \(\frac{\partial p^j_i}{\partial s_i} - \frac{\partial p^j_{-i}}{\partial s_i}\) will be the same as the sign of \( \left( 1 - \frac{\partial BR^{*,-}_i}{\partial p^j_i} \right) \frac{\partial BR^e_i}{\partial s_i} \). Using the fact that \(\frac{\partial BR^{*,-}_i}{\partial p^j_i} < 1\) and \(\frac{\partial BR^e_i}{\partial s_i} > 0\), it follows that this sign is indeed positive.

This establishes the first and third bullet points in Proposition 4. The proofs of the remaining bullet points, regarding the effects of an increase in \(s_{-i}\), are analogous. Q.E.D.

**Proof of Proposition 5:** We have already shown in the text that \(p^j_i = p^j_{-i}\) for all \(s_i\). To establish the second through fourth bullet points, consider first the case in which \(s_i = s_{-i}\). In this case, the two conditions that characterize the Bertrand equilibrium in the mixed regime when only \(D_i\) has RPM, (13) and (14), simplify to the following two conditions:

\[
p^j_{-i} \frac{\partial q^j_{-i}(p)}{\partial p^j_{-i}} + q^j_{-i}(p) + p^j_{-i} \frac{\partial q^j_{-i}(p)}{\partial p^j_{-i}} = 0,
\] (A.25)

\[
p^j_i \frac{\partial q^j_i(p)}{\partial p^j_i} + q^j_i(p) + p^j_i \frac{\partial q^j_i(p)}{\partial p^j_i} = 0.
\] (A.26)
Recall that we have defined \( p_{i}^{\hat{j}} = BR_{i}^{*}(p_{i}^{\hat{j}}) \) to be the solution to (A.25) (see the proof of Proposition 4). Then, using the definition of \( p^{\ast} \) (see the proof of Proposition 3), the fact that \( BR_{i}^{*}(p_{i}^{\hat{j}}) \) is increasing in \( p_{i}^{\hat{j}} \), and \( |\frac{\partial BR_{i}}{\partial p_{i}^{\hat{j}}}| < 1 \), we have (i) for \( p_{i}^{\hat{j}} = p^{\ast} \), \( p_{i}^{\hat{j}} = BR_{i}^{*}(p_{i}^{\hat{j}}) \), (ii) for all \( p_{i}^{\hat{j}} < p^{\ast} \), \( p_{i}^{\hat{j}} < BR_{i}^{*}(p_{i}^{\hat{j}}) < p^{\ast} \), and (iii) for all \( p_{i}^{\hat{j}} > p^{\ast} \), \( p_{i}^{\hat{j}} > BR_{i}^{*}(p_{i}^{\hat{j}}) > p^{\ast} \).

Thus, to establish the second bullet point when \( s_{i} = s_{-i} \), we need only to establish that \( \frac{\partial q_{i}^{\ast}(p^{\ast})}{\partial p_{i}^{\hat{j}}} = \frac{\partial q_{i}^{-}(p^{\ast})}{\partial p_{i}^{\hat{j}}} \) implies that in equilibrium \( p_{i}^{\hat{j}} \) is equal to \( p^{\ast} \). To establish the third bullet point when \( s_{i} = s_{-i} \), we need only to establish that \( \frac{\partial q_{i}^{\ast}(p^{\ast})}{\partial p_{i}^{\hat{j}}} < \frac{\partial q_{i}^{-}(p^{\ast})}{\partial p_{i}^{\hat{j}}} \) implies that in equilibrium \( p_{i}^{\hat{j}} \) is less than \( p^{\ast} \). And, to establish the fourth bullet point when \( s_{i} = s_{-i} \), we need only to establish that \( \frac{\partial q_{i}^{\ast}(p^{\ast})}{\partial p_{i}^{\hat{j}}} > \frac{\partial q_{i}^{-}(p^{\ast})}{\partial p_{i}^{\hat{j}}} \) implies that in equilibrium \( p_{i}^{\hat{j}} \) is greater than \( p^{\ast} \). But this is precisely what we found in the proof of Proposition 3 when we determined that the left-hand side of (A.26) (in Proposition 3, it was (A.16)), when evaluated at \( p^{\ast} \) for all prices was either zero, negative, or positive, depending on the sign of \( \frac{\partial q_{i}^{\ast}(p^{\ast})}{\partial p_{i}^{\hat{j}}} - \frac{\partial q_{i}^{-}(p^{\ast})}{\partial p_{i}^{\hat{j}}} \).

In particular, we found that when \( \frac{\partial q_{i}^{\ast}(p^{\ast})}{\partial p_{i}^{\hat{j}}} = \frac{\partial q_{i}^{-}(p^{\ast})}{\partial p_{i}^{\hat{j}}} \), the left-hand side of (A.26) is zero when evaluated at \( p^{\ast} = (p^{\ast}, p^{\ast}, p^{\ast}, p^{\ast}) \). This means that the \( p_{i}^{\hat{j}} \) that solves (A.26) given \( p_{i}^{\hat{j}} = p^{\ast} \) is equal to \( p^{\ast} \), and thus in equilibrium we know that \( p_{i}^{\hat{j}} = p^{\ast} \). When \( \frac{\partial q_{i}^{\ast}(p^{\ast})}{\partial p_{i}^{\hat{j}}} < \frac{\partial q_{i}^{-}(p^{\ast})}{\partial p_{i}^{\hat{j}}} \), the left-hand side of (A.26) is negative at \( p^{\ast} = (p^{\ast}, p^{\ast}, p^{\ast}, p^{\ast}) \). This means that the \( p_{i}^{\hat{j}} \) that solves (A.26) given \( p_{i}^{\hat{j}} = p^{\ast} \) is less than \( p^{\ast} \), and thus in equilibrium we know that \( p_{i}^{\hat{j}} < p^{\ast} \) (since \( p_{i}^{\hat{j}} \) will only further decrease when \( p_{i}^{\hat{j}} \) decreases). Lastly, when \( \frac{\partial q_{i}^{\ast}(p^{\ast})}{\partial p_{i}^{\hat{j}}} > \frac{\partial q_{i}^{-}(p^{\ast})}{\partial p_{i}^{\hat{j}}} \), the left-hand side of (A.26) is positive at \( p^{\ast} = (p^{\ast}, p^{\ast}, p^{\ast}, p^{\ast}) \). This means that the \( p_{i}^{\hat{j}} \) that solves (A.26) given \( p_{i}^{\hat{j}} = p^{\ast} \) is greater than \( p^{\ast} \), and thus in equilibrium we know that \( p_{i}^{\hat{j}} > p^{\ast} \) (since \( p_{i}^{\hat{j}} \) will only further increase when \( p_{i}^{\hat{j}} \) increases).

In the case of \( s_{i} < s_{-i} \), we know from Proposition 4 that the equilibrium \( p_{i}^{\hat{j}} \) will be even further below \( p_{i}^{\hat{j}} \) relative to the case of \( s_{i} = s_{-i} \) when \( \frac{\partial q_{i}^{\ast}(p^{\ast})}{\partial p_{i}^{\hat{j}}} < \frac{\partial q_{i}^{-}(p^{\ast})}{\partial p_{i}^{\hat{j}}} \), thus establishing the third bullet point for all \( s_{i} \leq s_{-i} \), and in the case of \( s_{i} > s_{-i} \), we know from Proposition 4 that the equilibrium \( p_{i}^{\hat{j}} \) will be even further above \( p_{i}^{\hat{j}} \) relative to the case of \( s_{i} = s_{-i} \) when \( \frac{\partial q_{i}^{\ast}(p^{\ast})}{\partial p_{i}^{\hat{j}}} > \frac{\partial q_{i}^{-}(p^{\ast})}{\partial p_{i}^{\hat{j}}} \), thus establishing the fourth bullet point for all \( s_{i} \geq s_{-i} \). Q.E.D.

**Proof of Proposition 6:** We have already established the relationship between \( p^{\alpha} \) and \( p^{\ast} \) (see Proposition 3) and of that between \( p_{1}^{\ast} \) and \( p_{2}^{\ast} \) (see Proposition 5). It thus remains only to establish the relationship that holds between \( p^{\alpha} \) and \( p_{1}^{\ast} \) and between \( p_{2}^{\ast} \) and \( p^{\ast} \).

Consider first the relationship between \( p^{\alpha} \) and \( p_{1}^{\ast} \). When \( s_{1} = s_{2} \), the conditions that characterize the Bertrand equilibrium when only \( D_{i} \) has RPM are given by (A.25) and
(A.26). Evaluating (A.25) and (A.26) at \( \mathbf{p} = (p^a, p^\alpha, p^\beta, p^\gamma) \), we see that the left-hand side of (A.26) is zero (recall that \( p = p^a \) is the unique price that solves (10)), but the left-hand side of (A.25) may be zero, negative, or positive depending on the sign of \( \frac{\partial q_i^a (p^\alpha)}{\partial p_i^\alpha} - \frac{\partial q_i^a (p^\beta)}{\partial p_i^\beta} \).

If \( \frac{\partial q_i^a (p^a)}{\partial p_i^a} = \frac{\partial q_i^a (p^\alpha)}{\partial p_i^\alpha} \), then the left-hand side of (A.25) is also zero when evaluated at \( \mathbf{p} = (p^a, p^\alpha, p^\beta, p^\gamma) \), which implies that \( p^a = p_1^a = p_2^a \) is the unique solution to (A.25) and (A.26). If \( \frac{\partial q_i^a (p^a)}{\partial p_i^a} < \frac{\partial q_i^a (p^\alpha)}{\partial p_i^\alpha} \), then the left-hand side of (A.25) is positive at \( \mathbf{p} = (p^a, p^\alpha, p^\beta, p^\gamma) \). This means that the \( p_{\alpha i}^a \) that solves (A.25) given \( p_i^a = p^a \) is greater than \( p^a \), and thus in equilibrium we know that \( p_{\alpha i}^a > p^a \) (since \( p_i^a \) will increase when \( p_{\alpha i}^a \) increases). And, finally, if \( \frac{\partial q_i^a (p^a)}{\partial p_i^a} > \frac{\partial q_i^a (p^\alpha)}{\partial p_i^\alpha} \), then the left-hand side of (A.25) is negative at \( \mathbf{p} = (p^a, p^\alpha, p^\beta, p^\gamma) \). This means that the \( p_{\alpha i}^a \) that solves (A.25) given \( p_i^a = p^a \) is less than \( p^a \), and thus in equilibrium we know that \( p_{\alpha i}^a < p^a \) (since \( p_i^a \) will decrease when \( p_{\alpha i}^a \) decreases).

Now consider the relationship between \( p_{\alpha i}^a \) and \( p^a \). Evaluating (A.25) and (A.26) at \( \mathbf{p} = (p^*, p^*, p^*, p^*) \), we see that the left-hand side of (A.25) is zero, but the left-hand side of (A.26) may be zero, negative, or positive depending on the sign of \( \frac{\partial q_i^a (p^*)}{\partial p_i^a} - \frac{\partial q_i^a (p^*)}{\partial p_i^*} \).

If \( \frac{\partial q_i^a (p^*)}{\partial p_i^a} = \frac{\partial q_i^a (p^*)}{\partial p_i^*} \), then the left-hand side of (A.26) is also zero when evaluated at \( \mathbf{p} = (p^*, p^*, p^*, p^*) \), which implies that \( p^* = p_1^a = p_2^a \) is the unique solution to (A.25) and (A.26). If \( \frac{\partial q_i^a (p^*)}{\partial p_i^a} > \frac{\partial q_i^a (p^*)}{\partial p_i^*} \), then the left-hand side of (A.26) is positive at \( \mathbf{p} = (p^*, p^*, p^*, p^*) \). This means that the \( p_i^a \) that solves (A.26) given \( p_{\alpha i}^a = p^a \) is greater than \( p^a \), and thus in equilibrium we know that \( p_{\alpha i}^a > p^a \) (since \( p_i^a \) will increase when \( p_{\alpha i}^a \) increases). And, finally, if \( \frac{\partial q_i^a (p^*)}{\partial p_i^a} < \frac{\partial q_i^a (p^*)}{\partial p_i^*} \), then the left-hand side of (A.26) is negative at \( \mathbf{p} = (p^*, p^*, p^*, p^*) \). This means that the \( p_i^a \) that solves (A.26) given \( p_{\alpha i}^a = p^a \) is less than \( p^a \), and thus in equilibrium we know that \( p_{\alpha i}^a < p^a \) (since \( p_i^a \) will decrease when \( p_{\alpha i}^a \) decreases).

Noting that the sign of \( \frac{\partial q_i^a (p^*)}{\partial p_i^a} - \frac{\partial q_i^a (p^*)}{\partial p_i^*} \) is the same as the sign of \( \frac{\partial q_i^a (p^*)}{\partial p_i^a} - \frac{\partial q_i^a (p^*)}{\partial p_i^*} \) (this is a consequence of the definition of \( p^* \) as the price \( p \) that solves (9) and of \( p^a \) as the price \( p \) that solves (10), and our assumption that the left-hand sides of (9) and (10) are strictly decreasing in \( p \)), and using the results from Propositions 3 and 5, we have thus shown that (i) if \( \frac{\partial q_i^a (p^*)}{\partial p_i^a} = \frac{\partial q_i^a (p^*)}{\partial p_i^*} \), then \( p^* = p_1^a = p_2^a = p^a \), (ii) if \( \frac{\partial q_i^a (p^*)}{\partial p_i^a} < \frac{\partial q_i^a (p^*)}{\partial p_i^*} \), then \( p^* < p_1^a < p_2^a < p^a \), and (iii) if \( \frac{\partial q_i^a (p^*)}{\partial p_i^a} > \frac{\partial q_i^a (p^*)}{\partial p_i^*} \), then \( p^a > p_1^a > p_2^a > p^* \). Q.E.D.

**Proof of Proposition 7:** The proof of the first bullet point is in the text. To prove the second bullet point, we construct a linear-demand example to show that (i) there are
settings in which each outcome can arise in equilibrium, (ii) the equilibrium need not be unique, and (iii) for some of these settings, there is no equilibrium outcome in which the agency model is adopted. The proof of the second bullet point is available on request.

Q.E.D.

**Proof of Proposition 8:** We have shown (see Proposition 5) that even in the absence of $D_i$’s MFN clause, $U^1$ and $U^2$ would set $p^1_i < p^2_i$ in equilibrium when the conditions in the first bullet point in Proposition 8 are satisfied. Hence, $D_i$’s MFN clause would have no effect in this case. However, we also showed in Proposition 5 that when the conditions in the second bullet point in Proposition 8 are satisfied (i.e., when $s_i \geq s_{-i}$ and the degree of substitution downstream is greater than the degree of substitution upstream), $D_i$’s MFN clause would be binding (because otherwise $U^1$ and $U^2$ would set $p^1_i > p^2_i$ in equilibrium).

Thus, in this latter case, it must be that all four prices are the same in any equilibrium.

We now show that no vector of prices $\tilde{p} \equiv (\tilde{p}, \tilde{p}, \tilde{p}, \tilde{p})$ can be supported in equilibrium if $\tilde{p}$ is greater than $\min\{p^m, p^f\}$ or less than $p^*$. Consider first the case in which $\tilde{p} > \min\{p^m, p^f\}$. In this case, either $D_{-i}$ would want to deviate by undercutting $\tilde{p}$, knowing that this would force $U^1$ and $U^2$ to match its price cuts, or $U_j$ would want to deviate by undercutting $\tilde{p}$. In the first instance, this follows because if $\tilde{p} > \min\{p^m, p^f\} = p^f$, then by deviating to $p^1_i = p^2_i = p^f$ and having $U^1$ and $U^2$ match its prices, $D_{-i}$ would earn $\Pi_{D_{-i}}(p^f) > \Pi_{D_{-i}}(\tilde{p})$. In the second instance, this follows because if $\tilde{p} > \min\{p^m, p^f\} = p^m$, then the left-hand side of (14) is negative when all prices are equal to $\tilde{p}$ (recall that (14) is satisfied with equality when all prices are equal to $p^m$). Now consider the case in which $\tilde{p} < p^*$. In this case, $D_{-i}$ would want to deviate by charging a higher price on its goods. This follows because the left-hand side of (13) is positive when all prices are equal and less than $p^*$ (recall that (13) is satisfied with equality when all prices are equal to $p^*$).

Lastly, we show that the vector of prices $\tilde{p} \equiv (\tilde{p}, \tilde{p}, \tilde{p}, \tilde{p})$ can be supported in equilibrium if $\tilde{p} \in [p^*, \min\{p^m, p^f\}]$. To see this, note that for all $\tilde{p}$ in this set, $U_j$ would ideally want to increase its price above $\tilde{p}$ (this follows because for all $\tilde{p} \leq p^m$, the left-hand side of (14) is weakly positive when all prices are equal to $\tilde{p}$) but cannot do so because of the MFN clause, and $D_{-i}$ would ideally want to charge a lower price on its goods (this follows because for all $\tilde{p} \geq p^*$ the left-hand side of (13) is weakly negative when all prices are equal to $\tilde{p}$) but cannot do so without causing $U^1$ and $U^2$ to match its price cuts. Q.E.D.
Proof of Proposition 9: We have already seen that if the degree of substitution is relatively greater downstream, then retail prices will be the same on all products in any equilibrium with MFN clauses. We now establish the rest of the first two bullet points.

To prove that there exists an equilibrium in which at least one firm adopts the agency model and has an MFN clause, suppose without loss of generality that $s_i \geq s_{-i}$, and consider the candidate equilibrium in which $D_i$ has an MFN clause and only $D_i$ adopts the agency model. Suppose that in the equilibrium of the continuation game, $p = \min\{p^m, p^f\}$. Then, it follows that if $s_i = s_{-i}$, $p = p^a$, and if $s_i > s_{-i}$, $p > \max\{p_i^a, p^a\}$. It also follows that $D_i$’s profit is equal to $\Pi_{D_i}(p^m, p^m, p^m, p^m)$ if $p^m \leq p^f$ and $\Pi_{D_i}(p^f, p^f, p^f, p^f)$ if $p^m > p^f$.

To check for profitable deviations, note that if $D_i$ does not adopt the agency model in this case, it will earn $\Pi_{D_i}(p^*, p^*, p^*, p^*)$, which is less than both $\Pi_{D_i}(p^m, p^m, p^m, p^m)$ and $\Pi_{D_i}(p^f, p^f, p^f, p^f)$. And, if it does adopt the agency model but does not have an MFN clause, it will earn $\Pi_{D_i}(p_i^a, p_i^a, p_i^a, p_i^a)$, which is also less than both $\Pi_{D_i}(p^m, p^m, p^m, p^m)$ and $\Pi_{D_i}(p^f, p^f, p^f, p^f)$. It follows that $D_i$ does not have a profitable deviation. Now consider whether $D_{-i}$ has a profitable deviation. If it adopts the agency model it will earn $\Pi_{D_i}(p^a, p^a, p^a, p^a)$ whether or not it also has an MFN clause. But this too is less than (or equal to) its profit in the candidate equilibrium. It follows that $D_{-i}$, like $D_i$, does not have a profitable deviation, and thus the candidate equilibrium is indeed an equilibrium.

By construction, this establishes the second bullet point. To prove the rest of the first bullet point, we still need to show that in any equilibrium in which at least one firm adopts the agency model and has an MFN clause, retail prices are the same and equal to $p^a$ when $s_i = s_{-i}$, and the same and greater than or equal to $p^a$ when $s_i > s_{-i}$. But this is so because we have already shown that retail prices cannot be the same and less than $p^a$ in any equilibrium involving MFN clauses, nor can they be the same and greater than $p^a$ in any equilibrium involving MFN clauses when $s_i = s_{-i}$ (because $p^m = p^a$ when $s_i = s_{-i}$).

To establish the third bullet point, we suppose without loss of generality that $s_i \geq s_{-i}$ and note that Pareto optimality implies that in any equilibrium in which $D_i$ has an MFN clause and only $D_i$ adopts the agency model, retail prices will be the same on all products and equal to $p = \min\{p^m, p^f\}$ in the continuation game. As in the discussion above, this leads to a profit for $D_i$ of $\Pi_{D_i}(p^m, p^m, p^m, p^m)$ if $p^m \leq p^f$ and $\Pi_{D_i}(p^f, p^f, p^f, p^f)$ if $p^m > p^f$. In either case, $D_i$’s profits are strictly greater than they would be if neither firm adopted the agency model, which implies that $D_i$ would thus have a profitable deviation. Q.E.D.
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