Working paper

DYNAMIC PROCUREMENT UNDER UNCERTAINTY: COMPETITION AND AUCTIONS[1]

February 28, 2017

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Abstract. We introduce risk aversion into the analysis of an optimal procurement contract for a basic service and an add-on whose costs are uncertain. We focus on the case of fixed-size projects that are allocated through competitive tenders. We show how the procurement agency can use the structure of profits to relax the cost of acquisition. We also highlight the ambiguous impact of risk aversion on bidding behavior and reserve prices.

Keywords. Auctions, procurement, dynamic mechanism design, asymmetric information, uncertainty, risk aversion.

1. INTRODUCTION

In the public sector in many countries, service provision is allocated through competitive bidding.[1] Yet, at the time of tender, bidders as well as the government agency organizing the auction may not be aware of specific needs and costs for future stages of the service. Actual bids may reflect how bidders perceive this future risk around the project.

Contract adjustments has traditionally been more common in private procurement as private firms are not bound by the same rules as the public sector in their procurement strategies and decisions. However, even in the public sector, there are many examples of contract adjustments. When the Norwegian Road Administration (Statens vegvesen) renovates and upgrades its infrastructure it sometimes requires additional, complementary, work from its contractors. When in 2015 road works prevented water provision to inhabitants living close to a construction site in northern Norway, a solution to this problem had to be included in the contract. In 2013 when the authorities realized that upgrading the ferry connections on one of the main roads on the west coast of Norway also required the provision of a temporary solution during the work period, this was seen as a "necessary consequence" of the initial contract and was added as a change order. Other examples includes unexpected additional work during renovation[2], extension of service

[1]We thank Antonio Miralles and Laurent Linnemer as well as seminar participants at Umeå University, X-CREST, University of Konstanz and HEC Montréal for valuable comments. All errors are ours.

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[1]For instance in the EU, open procedures, meaning first-price or first-score auctions which are open to any qualified bidder, constitute 73% of all tenders announced in the Official Journal (PWC (2011)).

[2]See KOFA case 2014/14 concerning renovation of a school in the municipality of Lenvik.

contracts to other public institutions[3] or the extension of a service provision to additional market segments[4]. Some of these contract adjustments are already within the scope of public procurement in Norway, and in the European Union more broadly. Other are not. For instance the extension of the service contract to a new public institution mentioned above, was considered an illegal direct procurement whereas the other examples were deemed to be legal extensions of an initial public contract. An additional example of such illegal extension of a public contract is the case of the port in Bodø in northern Norway where an initial contract for the installation and initial maintenance of floating docks was extended to also include protective breakwaters. However the new EU directives on public procurement[5] leave much more scope for contract adjustments in future public contracts. In particular, Art. 72 of the 2014 Directive on public procurement makes any contract adjustment below 10 % of the initial contract value for service and supply contracts and below 15 % of the initial contract value for works contract admissible as long as it does not change the overall nature of the contract.[6] This change in the policy on public procurement only adds to the multiple reasons why understanding the effects of such contract adjustments on contracting and firm behavior is of paramount importance.

In this paper we consider a competitive procurement environment for a basic service and a future, uncertain, and thus risky, add-on. We are interested in the effects of introducing risk aversion in this environment. We argue that even when contracts are allocated competitively to the most efficient provider using a first-price sealed-bid auction, the procurement agency can influence the cost of incentives via the payment structure. In fact, by shifting more of the expected payoff to the risky period, the procurement agency makes it less attractive for firms to exaggerate costs and the overall cost of procurement goes down. This profit shifting becomes more important the higher the cost realization of the basic service as higher costs influence the information rents of all lower cost realizations. Furthermore, we show how by committing to a prespecified payment structure within the contract, the procurement agency can indirectly control bidding strategies. We fully characterize the increasing, symmetric equilibrium bidding strategy in a first-price auction and show that risk aversion has two effects. The first one is in line with the literature on bidding behavior in auctions (see below) and states that risk-averse bidders bid more aggressively to reduce the risk of loosing. The second effect is due to the inherent risk within the contract. Accepting a risky contract requires a risk premium to be paid to a risk averse contractor. This pushes up the required payment and leads to less aggressive bidding.

[3]See KOFA case 2015/65 concerning the extension of a service provision contract for electronic documentation signed between the Norwegian Student Loan Administration and Maestro Soft AS to Innovasjon Norge.

[4]See KOFA case 2015/10 on sewage services in municipality of Oppdal.

[5]These include directives on public procurement, utilities procurement and concessions.

[6]Article 72 also opens the door for contract adjustments in other cases.

The empirical contracting literature has shown the importance of contract adjustments and change orders. Bajari et al (2014) estimate the adaptation costs in paving projects in California to be 8 to 14 percent of the winning bid. Jung et al (2016) show, using construction date from Vermont, that markups are higher in auctions with renegotiated tasks (and these tasks drive the higher markups). De Silva et al (2016) look at the effect on project modification on bidders' costs. In this paper we consider such contract adjustments in an optimal contracting model, and ask how contract adjustments in the form of an additional risky task affects bidding behavior and how the prinicpal should react to this in designing the optimal contract.

This paper is close to a small literature on contract design under adverse selection and risk aversion (Salanié (1990), Laffont and Rochet (1998) and Arve and Martimort (2016)). We analyze a similar environment to Arve and Martimort (2016). However, the focus in that paper is on the intensive margin and how risk aversion in a dynamic procurement environment with uncertainty affects output. In this paper we focus on the extensive margin. In fact, as opposed to Arve and Martimort (2016) we assume that both the basic service and the future add-on is of fixed size and show how the effects identified in that paper affect behavior in a competitive environment.

We also belong to a much larger literature on auctions with risk averse bidders. Holt (1980), Riley and Samuelson (1981), Maskin and Riley (1984) and Matthews (1984) compares standard auction formats under risk aversion and shows that the Revenue Equivalence Theorem (Myerson (1981) and Riley and Samuelson (1981)) fails. More recently the literature on risk aversion in auctions have analyzed the optimal reserve price (Hu et al, 2010; Hu, 2011) and asymmetries between bidders (asymmetric valuations in Menicucci (2003) and different risk attitudes in Maréchal and Morand (2011)). However, all of these papers focus on risk aversion in environments without an underlying risk such as the add-on in our model. By focusing on an environment with an uncertain component we are closer to McAfee and McMillan (1986) who look at the optimal contract in an environment where parts of the costs are unknown ex ante. However, they limit the analysis to linear contracts and, more importantly, they look at the trade-off between ex post screening and moral hazard in a static environment whereas we focus ad-ons and a dynamic contract rather than cost uncertainty. We also only consider adverse selection.

Esö and White (2004) were probably the first to focus on ex post risk in an auction environment. They show how bidders who exhibit decreasing absolute risk aversion (DARA) engage in precautionary bidding and, in a common value environment, reduce their bid by more than the corresponding risk premium. A similar bid reduction is present in our independent private value setting, but is not always dominating. Furthermore, we focus on the effects of the payment structure within the optimal contract, a component that is not present in the static environment in Esö and White (2004).

The rest of the paper is organized as follows. The model is presented in Section 2.

Contracting for the add-on and general incentive compatibility conditions are analyzed in Section 3. The optimal contract for the basic service is characterized in Section 4. Section 5 deals with equilibrium bidding strategies for the initial contract and the associated reserve price is derived and analyzed in Section 6. Proofs are relegated to an Appendix.

2. THE MODEL

We consider an environment with multiple firms competiting for the provision of fixed-size services in a procurement context. A public agency (henceforth *the principal*) organizes a tender to contract with one of $n+1$ firms for the provision of a service, which we refer to as the basic service. This basic service is a durable component which has to be provided over two periods. In the second-period, an add-on is also required. To simplify the modeling of the demand side[7], we assume that the size of these two components is fixed. In the examples mentioned in the Introduction, this means that the scope of the upgrade or renovation work is not up for discussion and the extension of the procurement contract to new segments or additional work is also not of variable size. Thus, the principal wants to procure only one unit of the basic service and one unit of the add-on. Uncertainty around the add-on puts the firm's returns at risk. We are interested in the impact of this risk on bidding behavior and on the intertemporal structure of these contracts.

Technology, contracts and information. The basic service generates a gross surplus S_1 in each period. The winning firm provides this service at a constant cost θ_1. The gross surplus from providing the add-on in the second period is S_2 and the firm can provide this add-on at a constant cost θ_2.

The selection of the service provider is done using a first-price sealed-bid auction with a reserve price. The principal then offers a contract, consisting of payment specifications for the basic service over the two periods as a function of the winning firm's cost (announcement) as well as a menu of prices for the add-on. The fixed payments for the basic service are denoted by $b(\theta_1)$ and $b(\theta_1)+y(\theta_1)$ for periods 1 and 2 respectively. The premium $y(\theta_1)$ captures the possible non-stationarity of payments for this service. The second-period payment for the add-on is denoted by $p(\theta_1,\theta_2)$. The exact specifications required for the add-on are not completely known *ex ante* by the contracting parties and the principal therefore offers a menu of prices for this component, one for each state of the world.

At the time of tender each firm i has private information on its cost $\theta_{1,i}$ for providing the basic service.[8] These cost parameters are independently drawn from a common knowledge and atomless cumulative distribution $F(\cdot)$ with an everywhere positive density $f(\cdot)$ whose

[7]In this sense we are complementary to our previous paper in that this paper focuses on the extensive margin where the previous paper focused solely on the intensive margins.

[8]We will ignore the subscript i whenever possible.

support is $\Theta_1 = [\underline{\theta}_1, \overline{\theta}_1]$. Following a standard assumption in the screening literature,[9] the *monotone hazard rate property* holds:

Assumption 1 *Monotone hazard rate property:*

$$\frac{d}{d\theta_1}\left(\frac{F(\theta_1)}{f(\theta_1)}\right) \geq 0 \text{ for all } \theta_1 \in \Theta_1.$$

Firms are also symmetric in terms of the distribution of their second-period costs. To capture the idea that the add-on is not yet completely defined at the time of contracting, we assume that its cost is uncertain at this stage. *Ex ante*, there is symmetric but incomplete information on the cost parameter θ_2. However, before producing the add-on, the winning firm learns its own cost parameter θ_2. To maintain a tractable analysis, we consider the case where θ_2 is drawn from a common knowledge distribution on the discrete support $\Theta_2 = \{\underline{\theta}_2, \overline{\theta}_2\}$ (where $\Delta\theta_2 = \overline{\theta}_2 - \underline{\theta}_2 > 0$) with respective probabilities ν and $1-\nu$, where $\nu \in (0,1)$. We will assume that the value of the add-on, S_2, is large enough to ensure that this add-on is always valuable even under asymmetric information.[10]

First- and second-period cost parameters are independently drawn and, more generally, there is no technological linkage across periods. When deriving our results in this environment, any departure from the standard results well known in the case of risk neutrality comes from the fact that firms are risk averse in the second period.

Preferences. Denoting by $1-\beta$ and β the relative weights on the first and second period respectively, the principal's expected gains from dealing with a firm of type θ_1 which wins the tender can be written as:

$$S_1 - b(\theta_1) - \beta y(\theta_1) + \beta \mathbb{E}_{\theta_2}\left(S_2 - p(\theta_1, \theta_2)\right).$$

Denoting by $u_1(\theta_1) = b(\theta_1) - \theta_1$ the firm's first-period profit from the basic service and by $\mathcal{U}_2(\theta_1, \theta_2) = p(\theta_1, \theta_2) - \theta_2$ its second-period profit from the add-on, the principal's intertemporal payoff becomes:

$$S_1 - \theta_1 - u_1(\theta_1) - \beta y(\theta_1) + \beta \mathbb{E}_{\theta_2}\left(S_2 - \theta_2 - \mathcal{U}_2(\theta_1, \theta_2)\right). \tag{2.1}$$

This expression highlights the rent-efficiency trade-off that characterizes contracting under informational asymmetries. The principal cares about the social value of the project but would also like to minimize the share of that surplus that accrues to the firm subject to its participation to the tender mechanism.

We assume that firms are risk averse with respect to the second-period uncertainty.

[9] Bagnoli and Bergstrom (2005).

[10] Typically second-period surplus should cover second-period virtual costs (Myerson, 1981), i.e., $S_2 > \overline{\theta}_2 + \frac{\nu}{1-\nu}\Delta\theta_2$ gives a sufficient condition.

Accordingly, we express the winning firm's intertemporal payoff as:

$$(1-\beta)u_1(\theta_1) + \beta \mathbb{E}_{\theta_2}\left(v\left(u_1(\theta_1) + y(\theta_1) + \mathcal{U}_2(\theta_1,\theta_2)\right)\right),$$

where the firm's Bernoulli utility function $v(\cdot)$ is increasing and concave, ($v' > 0$, $v'' \leq 0$) with the normalizations $v(0) = 0$ and $v'(0) = 1$.

We are interested in the consequences of introducing uncertainty on the cost of the add-on. As can be seen from the winning firm's intertemporal payoff above, we assume that firms remain risk neutral w.r.t. first-period returns. However, studying how risk sharing impact on incentives requires to move away from more standard models of procurement and assume that firms are risk averse with respect to second-period returns. Our assumption that risk aversion changes over time is related to the idea that risk aversion should be viewed as a proxy for existing constraints that might limit the firm's access to the capital market when it raises outside funds to finance the necessary outlay investments associated with the services it is to provide. That the firm remains risk neutral with respect to the first-period returns thus captures the idea that returns on the basic service are well-known and stable enough to limit these costs of outside finance.

We also assume that $v(\cdot)$ satisfies standard properties in the risk literature:[11]

ASSUMPTION 2 *Decreasing (resp. constant) absolute risk aversion (DARA) (resp. CARA):*

$$\frac{d}{dz}\left(-\frac{v''(z)}{v'(z)}\right) < 0 \text{ (resp. } = 0) \quad \forall z.$$

The fact that the firm's preferences satisfy *DARA* can easily be motivated when risk aversion is viewed as a proxy for costly access to financial markets. Indeed, firms which already benefit from an activity (the basic service) that generates stable returns that can be used as pledgeable collateral also face less tight constraints and requirements on these markets.

Following the approach we developed in Arve and Martimort (2016), let $w(z,\varepsilon)$ be a utility function defined over wealth z and risk levels $\varepsilon \geq 0$ as:

$$w(z,\varepsilon) \equiv \nu v(z + (1-\nu)\varepsilon) + (1-\nu)v(z - \nu\varepsilon). \tag{2.2}$$

This function $w(\cdot)$ clearly inherits some important properties from the underlying utility function $v(\cdot)$ as it is also increasing and concave in z. It is also decreasing in ε which captures the fact that more background risk, represented by ε, reduces the firm's expected payoff. A last important property is that the cross derivative $w_{z\varepsilon}$ is non-negative as $v''' \geq 0$ and the firm exhibits *prudent* behavior when Assumption 2 holds. In other words, more

[11] Holt and Laury (2002).

background risk increases the firm's marginal value of income:[12]

$$w_{z\varepsilon}(z,\varepsilon) = \nu(1-\nu)(v''(z+(1-\nu)\varepsilon) - v''(z-\nu\varepsilon)) \geq 0.$$

We also let $\varphi(\zeta,\varepsilon)$ be the wealth level that guarantees ζ utils to the firm when the risk level is ε, i.e., $\zeta = w(\varphi(\zeta,\varepsilon),\varepsilon)$. The function $\varphi(\cdot)$ is increasing in ζ and ε.[13] Finally, we complete our setup by defining the function $H(\cdot)$ as:

$$H(z,\varepsilon) \equiv w_{z\varepsilon}(z,\varepsilon) - \frac{w_{zz}(z,\varepsilon)w_{\varepsilon}(z,\varepsilon)}{w_z(z,\varepsilon)}. \tag{2.3}$$

That $H(\cdot)$ is non-negative follows from Assumption 2.[14] Importantly, $\frac{dw_z}{d\varepsilon}(\varphi(\zeta,\varepsilon),\varepsilon) = H(\varphi(\zeta,\varepsilon),\varepsilon)$. Hence, the fact that $H(\cdot)$ remains non-negative means that the marginal utility of income increases with ε if the firm's utility is left unchanged by raising z. The sign of this total derivative plays an important role in understanding agency distortions because it shows how a change in second-period risk sharing impacts on the firm's intertemporal profits.

Example: *CARA* preferences. Suppose that $v(\cdot)$ is *CARA*. Given the normalizations $v(0)=0$ and $v'(0)=1$, $v(z)=\frac{1}{\tau}(1-exp(-\tau z))$ and $w(z,\varepsilon)=\frac{1}{\tau}(1-exp(-\tau z)\eta(\tau,\varepsilon))$ where $\eta(\tau,\varepsilon)=\nu exp(-\tau(1-\nu)\varepsilon)+(1-\nu)exp(\tau\nu\varepsilon)$. Finally, $H(z,\varepsilon)=0$ for all (z,ε).

Auction and Contract Design. The principal runs a first-price sealed-bid auction and commits to a long-term contract that regulates the basic service and the add-on over both periods.[15] Because firms are all symmetric, we look for a symmetric equilibrium bidding strategy $b(\theta)$ that determines a per-period price for the basic service as a function of the firm's announcement of its costs.

Although the selection of the service providing firm is done using an auction in which the firms place bids on their required per-period payment for the contract, the principal can still choose the payment structure that is offered to the winning firm at the ex ante stage. Therefore, from the Revelation Principle (Myerson (1981), Baron and Besanko (1984), Myerson (1986)), there is no loss of generality in restricting the analysis to incentive-compatible direct revelation mechanisms. In this case, a mechanism, denoted by $\mathcal{C}$, stipulates payments for each period as a function of the firm's report of its cost type for the basic service (or, equivalently, from the firm's equilibrium bid for the basic service) as well as a menu of prices for the add-on. The latter can both be a function of the announced cost for the basic service and the second-period announcement of the cost for

[12]The concept of *prudence* goes back to Leland (1968) and Sandmo (1970).
Experimental evidence (Deck and Schlesinger (2014), Noussair et al (2014)) is in line with this assumption.

[13]We have $\varphi_\zeta(\zeta,\varepsilon) = \frac{1}{w_z(\varphi(\zeta,\varepsilon),\varepsilon)} > 0$, and $\varphi_\varepsilon(\zeta,\varepsilon) = -\frac{w_\varepsilon(\varphi(\zeta,\varepsilon),\varepsilon)}{w_z(\varphi(\zeta,\varepsilon),\varepsilon)} > 0$.

[14]See Arve and Martimort (2016)

[15]For a discussion of how our results extend to the case of incomplete contracts see Section 7.

the add-on. Thus, a mechanism can be defined as $\mathcal{C} = \left\{ b(\hat{\theta}_1), y(\hat{\theta}_1), \{p(\hat{\theta}_1, \hat{\theta}_2)\}_{\hat{\theta}_1 \in \Theta_2} \right\}_{\hat{\theta}_1 \in \Theta_1}$ where $\hat{\theta}_1$ is the firm's announcements of its cost parameter for the basic service and $\hat{\theta}_2$ the firm's announcements of its cost parameter for the add-on in the second period. These reports are of course truthful in equilibrium.

Timing. The contracting game unfolds as follows:

1. Firms privately learn their individual cost parameters $\theta_{1,i}$ for the basic service.
2. The principal announces the rules of the first-price auction. A reserve price as well as prices for the basic service and the add-on phase are stipulated in $\mathcal{C}$. We normalize, without loss of generality, reservation payoffs for all parties to zero.
3. Firms announce their cost $\hat{\theta}_{1i}$ and the lowest-cost firm, $\min_i \hat{\theta}_{1i}$, is awarded the contract. In case of a tie among several bidders (a zero-probability event), the winning firm is randomly selected with equal probabilities.
4. The winning firm learns the value of the cost of the add-on, θ_2. The winning firm then reports $\hat{\theta}_2$ and provides the add-on at the price stipulated in the contract.

Complete Information Benchmark. As a first pass, suppose that the first-period cost θ_{1i} and the cost of the add-on θ_2 are both common knowledge, but recall that at the time of contracting the cost θ_2 is not yet realized. The solution to the contracting problem is obvious. First, because costs for the basic service are known and firms are ex ante identical with respect to the cost of the add-on, the principal does not have to run an auction to select the most appropriate provider of the services. She will simply enter into a contract with the firm with the lowest cost of providing the basic service. Second, because transferring risk to a risk averse firm is costly, the principal should keep all risk associated with the add-on so as to perfectly ensure the firm against second-period cost uncertainty. Third, the firm must keep the same marginal utility of income in both periods so as to smooth the cost of subsidies over time. Given the normalization $v(0) = 0$ and $v'(0) = 1$, this means that, for all realizations of its costs parameters, the firm should make zero profit in each period. This normalization provides a convenient benchmark that allows us to conclude that any non-stationarity in payments and profits follows from asymmetric information.[16] Lastly, the principal only has to cover actual costs of providing the services. Payments are thus given by:[17]

$$b^{fb}(\theta_1) = \theta_1, \; y^{fb}(\theta_1) = 0 \text{ and } p^{fb}(\theta_2) = \theta_2.$$

[16] Had the firm also had the same concave utility function in the first period, the same result would hold. Profits would be zero in each period.

[17] The superscript fb stands for *first-best* and it indexes optimal variables in the complete information benchmark.

3. INCENTIVE COMPATIBILITY

Second-period incentive compatibility. For any first-period report $\hat{\theta}_1$ that a winning firm may have reported in the first period[18], the requirement of incentive compatibility implies that the second-period report, which is truthful from the Revelation Principle, should maximize the firm's continuation payoff $p(\hat{\theta}_1, \hat{\theta}_2) - \theta_2$. Because the second-period project is a fixed-size project and because it is always valuable, no quantity screening can be used to help rent extraction in the second period. The winning firm will be paid a fixed amount $p(\hat{\theta}_1)$ for the provision of the add-on.

The firm's second-period profit is thus:

$$\mathcal{U}_2(\theta_2) = p - \theta_2 \tag{3.1}$$

Furthermore, because any non-zero expected profit could, by a simple redefinition of payments, be incorporated into the second-period premium for the basic service, $y(\hat{\theta}_1)$, there is no loss of generality in assuming that the firm makes zero expected profit on the add-on. This means that the second-period price for the add-on covers the expected cost and is thus indepedent of the first-period announcement.

$$p(\hat{\theta}_1) = E[\theta_2], \ \forall \hat{\theta}_1 \in \Theta_1. \tag{3.2}$$

Second-period profits can thus be expressed as a random variable with zero mean:

$$\mathcal{U}_2(\underline{\theta}_2) = (1-\nu)\Delta\theta_2 \text{ and } \mathcal{U}_2(\overline{\theta}_2) = -\nu\Delta\theta_2. \tag{3.3}$$

In terms of our previous notation this implies

$$\varepsilon = \Delta\theta_2. \tag{3.4}$$

First-period incentive compatibility. Of course, the second-period risk impacts on first-period incentives. Assuming that an increasing, symmetric bidding strategy $b(\cdot)$, the probability that a firm who reports a first-period cost $\hat{\theta}_1$ wins the auction is $(1 - F(\hat{\theta}_1))^n$. This together with equation (3.4) and our previous definition of payoffs in terms of $w(\cdot)$ allow us to rewrite the requirement of incentive compatibility for a bidder with type θ_1 as:

$$\mathcal{U}(\theta_1) = \max_{\hat{\theta}_1 \in \Theta_1} (1 - F(\hat{\theta}_1))^n \left((1-\beta)(b(\hat{\theta}_1) - \theta_1) + \beta w(b(\hat{\theta}_1) - \theta_1 + y(\hat{\theta}_1), \Delta\theta_2) \right). \tag{3.5}$$

Using our previous definition of per-period fixed payoff from the basic service as $u_1(\theta_1) =$

[18] We omit the index i for simplicity.

$b(\theta_1)-\theta_1$ and $\mathcal{U}(\theta_1)=(1-F(\theta_1))^n\left((1-\beta)u_1(\theta_1)+\beta w(u_1(\theta_1)+y(\theta_1),\Delta\theta_2)\right)$, we can express the second-period profit, including the premium $y(\theta_1)$, in terms of other variables as:

$$u_1(\theta_1)+y(\theta_1)=\varphi\left(\frac{\frac{\mathcal{U}(\theta_1)}{(1-F(\theta_1))^n}-(1-\beta)u_1(\theta_1)}{\beta},\Delta\theta_2\right). \tag{3.6}$$

This condition tells us how the second-period profit $u_1(\theta_1)+y(\theta_1)$ on the basic service should be modified to keep the second-period utility $\frac{1}{\beta}(\mathcal{U}(\theta_1)-(1-\beta)u_1(\theta_1))$ constant for a given second-period risk $\Delta\theta_2$.

With this change of variables, any incentive-compatible allocation that can be achieved by a first-price auction cum an agreement on the provision of the add-on is equivalent to a pair $(\mathcal{U}(\theta_1),u_1(\theta_1))$ that stipulates an intertemporal rent and second-period fixed profit. Equipped with this dual specification of such incentive-compatible allocations, we can now present a lemma which provides necessary and sufficient conditions satisfied by any such allocation.

Lemma 1 Necessary condition. *Any incentive-compatible allocation* $(\mathcal{U}(\theta_1),u_1(\theta_1))$ *is such that* $\mathcal{U}(\theta_1)$ *is absolutely continuous in* θ_1 *(and thus almost everywhere differentiable) with at any point of differentiability:*

$$\dot{\mathcal{U}}(\theta_1)=-(1-F(\theta_1))^n\left(1-\beta+\beta w_z\left(\varphi\left(\frac{\frac{\mathcal{U}(\theta_1)}{(1-F(\theta_1))^n}-(1-\beta)u_1(\theta_1)}{\beta},\Delta\theta_2\right),\Delta\theta_2\right)\right). \tag{3.7}$$

Sufficient condition. *An allocation is incentive compatible if* $\mathcal{U}(\theta_1)$ *is absolutely continuous, satisfies (3.7) at any point of differentiability and is convex.*

To understand the envelope condition (3.7), it is useful to consider the benefits that a firm with first-period cost θ_1 gets when pretending to have a marginally higher cost $\theta_1+d\theta_1$. Doing so means that it can produce the requested basic service at a slightly lower cost and thus save an expected amount $(1-F(\theta_1+d\theta_1))^n d\theta_1\approx(1-F(\theta_1))^n d\theta_1$. This gain is evaluated at the margins $1-\beta$ for the first period and at β multiplied by the marginal utility of income in the second period. This marginal utility itself depends on the second-period profit for the basic service which roughly amounts to $u_1(\theta_1+d\theta_1)+y(\theta_1+d\theta_1)\approx u_1(\theta_1)+y_1(\theta_1)$. It also depends on how much risk is borne by the firm for the provision of the add-on. From second-period incentive compatibility, this amount of risk is fixed at $\varepsilon(\theta_1+d\theta_1)=\varepsilon(\theta_1)=\Delta\theta_2$. Putting these facts together, a firm with cost θ_1 is not tempted to mimic the behavior of a $\theta_1+d\theta_1$ type if it receives an extra rent $\mathcal{U}(\theta_1)-\mathcal{U}(\theta_1+d\theta_1)\approx -\dot{\mathcal{U}}(\theta_1)d\theta_1$ worth $(1-F(\theta_1))^n\left(1-\beta+\beta w_z(u_1(\theta_1)+y(\theta_1),\Delta\theta_2)\right)d\theta_1$. Simplifying yields (3.7).

The right-hand side of (3.7) shows the basic forces at play in the optimal contract. First, the familiar distortion in screening environments requires a firm to get an information rent to reveal this information. By exaggerating its first-period cost, the firm can get an extra benefit in case it is selected even if it is selected less often. The marginal benefit of exaggerating the costs in the first period is thus integrated by the probability that the firm still wins the tender.

However, as in Arve and Martimort (2016) there are two other, less familiar, effects. Indeed, among all intertemporal profiles of profits $(u_1(\theta_1), u_1(\theta_1) + y(\theta_1))$ that leaves the overall rent $\mathcal{U}(\theta_1)$ of a given type θ_1 unchanged, the principal benefits from shifting more of these profits towards the second period. Reducing the second-period marginal utility diminishes the slope $\dot{\mathcal{U}}(\theta_1)$ and the principal again saves on the rents and payments for all inframarginal types below θ_1.

Lastly, and still stemming from the concavity of the utility function, the risk $\varepsilon = \Delta\theta_2$ borne by the firm in the second period increases the cost of incentives by raising the required rent payment.

4. INTERTEMPORAL PRICING OF THE BASIC SERVICE

Due to the symmetry of bidders, everything happens as if the principal was actually dealing with a single firm (referred to as the *"winning firm"* from now on) but this firm would have a first-period cost drawn from the distribution of the minimum of $n+1$ independent variables. The corresponding distribution function is thus $G(\theta_1) = 1 - (1 - F(\theta_1))^{n+1}$ (with density $g(\theta_1) = (n+1)f(\theta_1)(1-F(\theta_1))^n$). This remark facilitates the derivation of the optimal contract under which the winning firm operates.

Furthermore, from the principal's viewpoint, the reserve price indirectly defines a cutoff for the winning bidder's cost for the basic service above which it is preferable not to engage in the long-term project. We denote by $\tilde{\theta}_1$ this cutoff.

Under asymmetric information the principal can a priori use two sets of instruments. First, she may play on the intertemporal profile of prices for each stage pf the project. Second, she can also use the reserve price. Although the principal cannot adjust quantities to screen types, she can still decide which firms can participate. On top, the principal can still shift profits to the second period to reduce the overall cost of information revelation. In fact, once a bidding strategy $b(\theta)$ is given, the rent profile $\mathcal{U}(\theta)$ is fully determined by the differential equation (3.7) for types $\theta \le \tilde{\theta}$ and the boundary condition that is implicitly defined by the reserve price:

$$\mathcal{U}(\tilde{\theta}) = 0. \tag{4.1}$$

PROPOSITION 1 *The second-period profit $u_1^{sb}(\theta_1) + y^{sb}(\theta_1)$ of the winning firm satisfies:*

$$(4.2)\quad w_z(u_1^{sb}(\theta_1)+y^{sb}(\theta_1),\Delta\theta_2)=1+\frac{F(\theta_1)}{f(\theta_1)}w_{zz}(u_1^{sb}(\theta_1)+y^{sb}(\theta_1),\Delta\theta_2)\le 1,\quad \forall\theta_1\le\tilde{\theta}_1^{sb}.$$

When Assumptions 2 holds, this profit is greater than when θ_2 is common knowledge.

As far as the first kind of distortion is concerned, Proposition 1 highlights how the basic service should be rewarded in the second period. In order to reduce the firm's marginal utility of income in the second period and make it less attractive to overstate costs to secure more profits for the basic service in the second period, the principal pays an extra premium for the basic service in this period. This *Income Effect* is stronger when there is also asymmetric information in the second period. Such asymmetry makes it more valuable to backload profits for precautionary purposes.

Observe also that (4.2) is independent of the number of competing firms. Competition plays no role in determining second-period profits which are identical to those achieved when there is a single provider. The amount of profits that is backloaded to the second-period is independent of how competitive the environment is. The intuition for this result is similar to that of the result in Myerson (1981) and Riley and Samuelson (1981) who show that the reserve price in an optimal auction is independent of the number of bidders. In our case, the independence is because the payment structure is only relevant for this one winning bidder. Of course, bids depend on the magnitude of competition as can be seen from the equilibrium bidding function in (5.1).

To further illustrate this effect and stress the role of second-period uncertainty, we use (4.2) and the fact that $w_{z\varepsilon}\ge 0$, to get the following string of inequalities:

$$1\ge w_z(u_1^{sb}(\theta_1)+y^{sb}(\theta_1),\varepsilon^{sb}(\theta_1))\ge w_z(u_1^{sb}(\theta_1)+y^{sb}(\theta_1),0)=v'(u_1^{sb}(\theta_1)+y^{sb}(\theta_1)).$$

This in turn implies that the second-period profit from the basic service $u_1^{sb}(\theta_1)+y^{sb}(\theta_1)$ is always non-negative. In fact, the following corollary establishes that $u_1^{sb}(\theta_1)+y^{sb}(\theta_1)$ is in fact increasing.

COROLLARY 1 *The second-period payoff $u_1^{sb}(\theta)+y^{sb}(\theta)$ is increasing in θ_1:*

$$(4.3)\quad \dot{u}_1^{sb}(\theta_1)+\dot{y}^{sb}(\theta_1)\ge 0.$$

More profit is backloaded for higher types so that the overall cost of incentives goes down.

Consider now a type θ_1 slightly lower than the cut-off type $\tilde{\theta}_1$ that is indifferent between participating or not. Because the rent profile is decreasing, rent minimization calls for leaving this worst type just indifferent between participating or not, i.e., $\mathcal{U}(\tilde{\theta}_1)=0$. Putting together this condition with the fact that second-period profits for the basic service are always positive gives

$$u_1^{sb}(\theta_1) \leq 0 \leq u_1^{sb}(\theta_1) + y^{sb}(\theta_1)$$

for such a type. To deter the most efficient firms from mimicking those with large first-period costs, the optimal contract stipulates a first-period loss if large costs are reported and this loss is only recouped later on.

5. FIRST-PERIOD BIDDING STRATEGIES

We look for a symmetric equilibrium bidding strategy $b(\theta_1)$ that determines a fixed per-period price for the basic service as a function of the firm's announcement of its costs. Of course, this strategy takes into account the second-period premium specified in the contract as well as the reserve price set by the principal.

The optimal bidding strategy $b_0(\cdot)$ of a risk-neutral firm in a first-price auction with reserve price $\tilde{\theta}_1$ is well known to be:

$$b_0(\theta_1) = \theta_1 + \frac{1}{(1 - F(\theta_1))^n} \int_{\theta_1}^{\tilde{\theta}_1} (1 - F(s))^n ds.$$

This strategy is an important benchmark to evaluate the firms' bidding strategy $b(\theta_1)$ in our dynamic context. As a preliminary remark, notice that any payment profile $(u_1(\theta_1), y(\theta_1))$ that is chosen by the principal indirectly controls the firms' bidding strategy $b(\theta_1)$. Using (3.6) and (3.7), the equilibrium bidding strategy can easily be obtained in terms of the second-period profit $u_1(\theta_1) + y(\theta_1)$ as:

$$\begin{aligned} b^*(\theta_1) =& b_0(\theta_1) + \\ & \frac{\beta}{1-\beta} \left(\int_{\theta_1}^{\tilde{\theta}_1} \frac{(1 - F(s))^n}{(1 - F(\theta_1))^n} w_z \left(u_1(s) + y(s), \Delta\theta_2\right) ds - w(u_1(\theta_1) + y(\theta_1), \Delta\theta_2) \right). \end{aligned} \tag{5.1}$$

The equilibrium bidding strategy in (5.1) shows that risk aversion in the second period has two effects. First, it reduces the bids as can be seen in the second part of the second line of (5.1). This is in line with the literature where Holt (1980) (see also Krishna (2002)) shows that in a standard first-price auction, risk-averse bidders bid more aggressively than their risk-neutral counterpart. This is because, for a risk-averse bidder compared to a risk-neutral bidder, the risk of losing the auction from a small increase in the bid has a larger effect on expected utility than the loss of profits from a slightly lower bid. A risk-averse bidder would thus be willing to lower his bid more than the risk-neutral bidder to reduce the risk of loosing the auction.

However, our analysis also unveils an effect that goes in the opposite direction. In fact the first part of the second line of (5.1) suggests that the equilibrium bid for a risk-averse bidder is higher than the risk-neutral equivalent. In fact on top of the risk related to winning or loosing the auction that is present in Holt (1980), our environment also contains an inherent risk related to the add-on. To accept this risk, the winning firm will require a risk premium and this naturally pushes up firms' bids.

Consider a bidder with costs close to the cut-off $\tilde{\theta}$. For such a bidder, the first effect dominates and the equilibrium bid under risk aversion is, as in the literature, lower than its bid under risk neutrality. However, the next example shows that this need not be the case for low enough costs.

Figure 5 illustrates the firms' equilibrium biddings strategies in the CARA case. Notice that for costs close enough to the reserve price (0.94 in this example), the bid of a risk-averse bidder is actually below costs (illustrated by the dotted line). This does of course not mean that the bidder will make a loss if he wins. However, the chosen payment structure in the optimal contract will be such that the premium $y(\theta_1)$ is sufficiently large so that even if $u_1(\theta_1) = b^*(\theta_1) - \theta_1$ is negative, overall expect utility is positive. (The figure takes into account this optimal payment structure).

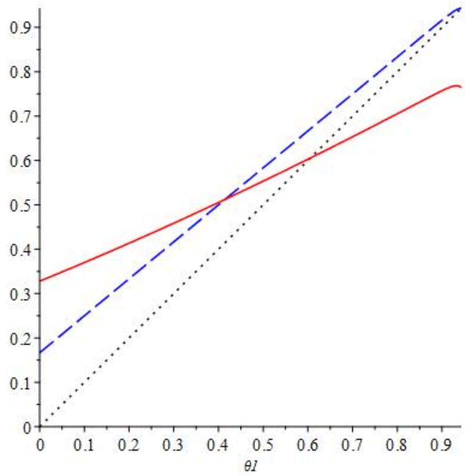

FIGURE 1.— Equilibrium bidding strategy for risk-averse bidders (red, solid) and risk-neutral bidders (blue, dashed)

6. OPTIMAL RESERVE PRICE

Let us now turn to the optimal reserve price or, more precisely, its consequences on participation. For the sake of the comparison, it is useful to recall the value of the cutoff $\tilde{\theta}_1^{rn}$ that would be achieved had the firm been risk neutral. When fixing this reserve price, the principal trades off the overall value of the project (including the expected benefits from the add-on) and its cost, taking into account information rents left to the winning firm. It is routine to verify that the cutoff $\tilde{\theta}_1^{rn}$ solves:

$$S_1 + \beta S_2 = \tilde{\theta}_1^{rn} + \frac{F(\tilde{\theta}_1^{rn})}{f(\tilde{\theta}_1^{rn})} + \beta E[\theta_2]. \tag{6.1}$$

This condition simply means that, for the cutoff $\tilde{\theta}_1^{rn}$, the overall value of the projects (the left-hand side of (6.1)) is equal to its *virtual costs* (the right-hand side). To ensure an interior solution $\tilde{\theta}_1^{rn}$, we will from now on assume:

Assumption 3

$$\underline{\theta}_1 < S_1 + \beta(S_2 - E[\theta_2]) < \overline{\theta}_1 + \frac{1}{f(\overline{\theta}_1)}.$$

We also define $\tilde{\theta}_1^i$ as the cutoff when θ_2 is common knowledge and ε can be set to zero. In that case the take-it-or-leave it offer for the add-on, simply offers a price equal to the cost realization θ_2. We now turn to the characterization of the optimal cutoff as well as its comparison to $\tilde{\theta}^{rn}$ and $\tilde{\theta}^i$.

Proposition 2 *The optimal cut-off $\tilde{\theta}_1^{sb}$ is given by the following equation*

$$\begin{aligned}(6.2)\qquad S + \beta(S_2 - E[\theta_2]) &= \tilde{\theta}_1^{sb} + \frac{F(\tilde{\theta}_1^{sb})}{f(\tilde{\theta}_1^{sb})} \\ &+ \beta\left(u^{sb}(\tilde{\theta}_1^{sb}) + y^{sb}(\tilde{\theta}_1^{sb}) - w\left(u^{sb}(\tilde{\theta}_1^{sb}) + y^{sb}(\tilde{\theta}_1^{sb}), \Delta\theta_2\right)\right) \\ &+ \beta\left(\frac{F(\tilde{\theta}_1^{sb})}{f(\tilde{\theta}_1^{sb})}(w_z(u^{sb}(\tilde{\theta}_1^{sb}) + y^{sb}(\tilde{\theta}_1^{sb}), \Delta\theta_2) - 1)\right).\end{aligned}$$

The optimal reserve price in the first-price auction is thus $b^*(\tilde{\theta}_1^{sb})$.

The two distortions away from the risk-neutral bidding strategy identified in the previous section remain present here. From Proposition 1, the last part of equation (6.2) is negative. This is because the optimal payment structure shifts parts of the payoff to the second period and reduces the overall cost of incentives. Thus the optimal cutoff is reduced.

However, since the firm is risk averse, a payment in the second-period is evaluated at a lower utility than its monetary equivalent. This means that, for the principal, providing a certain utility level in the second period becomes more expensive when the firm is risk averse. This pushes up the principal's costs and increases the cut-off.

Finally, we can compare the thresholds obtained under different assumptions.

Proposition 3 *Assuming that θ_2 is common knowledge, the Income Effect increases participation:*

$$(6.3)\qquad \tilde{\theta}_1^i \geq \tilde{\theta}_1^{rn}.$$

When θ_2 is private information and Assumptions 2 holds, the Risk Effect decreases par-

ticipation:

$$\tilde{\theta}_1^{sb} \leq \tilde{\theta}_1^{i}. \tag{6.4}$$

The *Income Effect* makes it less attractive to exaggerate first-period costs since payments are now backloaded. The principal can thus raise the optimal reserve price beyond its value had firms been risk neutral and thereby foster more participation. However, the impact of second-period uncertainty on that reserve price goes in the other direction. First, the *Risk Effect* requires an extra risk premium to be paid to ensure firms' participation. This calls for a lower reserve price and reduces participation. Second, risk on the add-on increases the marginal utility of income (since $w_{z\varepsilon} \geq 0$) and makes first-period incentive compatibility more costly. This also pushes towards a lower reserve price and further reduces participation.

EXAMPLE ($CARA$ PREFERENCES - CONTINUED). This example allows us to quantify the relative impact of both effects and show that whether more risk on the add-on hardens or exacerbates participation is ambiguous. First, observe that (4.2) now gives us the following closed-form expression of second-period profits:

$$u_1^{sb}(\theta_1) + y^{sb}(\theta_1) = \frac{1}{r} ln\left(\eta(r, \Delta\theta_2)\right) + \frac{1}{r} ln\left(1 + r\frac{F(\theta_1)}{f(\theta_1)}\right), \quad \forall \theta_1 \in \Theta_1.$$

Inserting this expression into (3.7) and taking into account (4.1), we obtain:

$$\mathcal{U}_1^{sb}(\theta_1) = \int_{\theta_1}^{\tilde{\theta}_1^{sb}} (1 - F(s))^n \left(1 - \beta + \frac{\beta}{1 + r\frac{F(s)}{f(s)}}\right) ds, \quad \forall \theta_1 \in \Theta_1,$$

where $\tilde{\theta}_1^{sb}$ solves:

$$S_1 + \beta(S_2 - E[\theta_2])) = \tilde{\theta}_1^{sb} + \frac{F(\tilde{\theta}_1^{sb})}{f(\tilde{\theta}_1^{sb})} + \beta\left(\frac{1}{r} ln\left(\eta(r, \Delta\theta_2)\right) + \frac{1}{r} ln\left(1 + r\frac{F(\tilde{\theta}_1^{sb})}{f(\tilde{\theta}_1^{sb})}\right) - \frac{F(\tilde{\theta}_1^{sb})}{f(\tilde{\theta}_1^{sb})}\right).$$

When $\Delta\theta_2$ is sufficiently small, the risk-premium $\frac{1}{r} ln\left(\eta(r, \Delta\theta_2)\right)$ that is required to induce the firm's participation is also small and the bracket on the right-hand side remains negative. The *Income Effect* drives the direction of the distortion and second-period risk increases participation. ■

7. EXTENSION: INCOMPLETE CONTRACTS

This section analyzes the possible costs that parties incur when they are not able to perfectly commit ex ante to a complete contract with a single firm in charge of providing both the basic service and the add-on. Such scenarios are meant to capture the highly incomplete contracting environments that may surround long-term contracts, a concern that has repeatedly been brought forward by practitioners in the PPPs sector.[19] The examples related to the Norwegian Road Adminsitration and their contracts are also examples where the add-on is not clearly included in the initial contract.

In practice, parties might face unforeseen contingencies that could not be anticipated and written into the initial contract, especially if this contract covers the provision of a basic service over many years. To model such settings, we now suppose that ex ante parties can only agree on a highly incomplete long-term contract which does not even specify payments and output requirements for the add-on. Of course and in accordance with the incomplete contracting literature,[20] the mere opportunity of such additional projects can be anticipated. In the remainder of this section we argue that contracting on the add-on on the spot entails entails the same results as our complete contracting framework.

In fact, even if parties can only contract on the add-on at the interim stage, the same allocation as in the optimal long-term contract $\mathcal{C}^{sb}$ can still be implemented. To see how, consider a long-term agreement $\left\{b(\hat{\theta}_1), y(\hat{\theta}_1) - \nu\Delta\theta_2\right\}_{\hat{\theta}_1\in\Theta_1}$ that regulates the basic service over the whole relationship and, as such, does not specify any risk premium nor any add-on specification. At the beginning of the second period, parties agree on a spot contract to regulate this add-on. This spot contract specifies a price $p(\hat{\theta}_1, \hat{\theta}_2)$ for the add-on as a function of announced costs. This spot contract does not modify the firm's risk attitude and, even though it is anticipated by parties, it has no impact on first-period incentives. Compounding the impact of this spot contract with the initial contract for the basic service replicates the optimal long-term contract.

PROPOSITION 4 *There is no loss of generality in contracting for the add-on only in the second period.*

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APPENDIX A: PROOFS

Proof of Lemma 1: Necessity. From Theorem 2 and Corollary 1 in Milgrom and Segal (2002), it immediately follows that $\mathcal{U}(\theta_1)$ is absolutely continuous and thus almost everywhere differentiable with (3.7) holding at any point of differentiability.

Sufficiency. $\forall(\theta_1,\hat\theta_1)$, we rewrite (3.5) as:

$$\begin{aligned}(A.1)\qquad \mathcal{U}(\theta_1) \geq & \mathcal{U}(\hat\theta_1) + (1-F(\hat\theta_1))^n \Big[(1-\beta)(\hat\theta_1-\theta_1) \\ & +\beta\left(w(u(\hat\theta_1)+y(\hat\theta_1)+(\hat\theta_1-\theta_1),\Delta\theta_2)-w(u(\hat\theta_1)+y(\hat\theta_1),\Delta\theta_2)\right)\Big].\end{aligned}$$

Using (3.7) and absolute continuity, the rent profile $\mathcal{U}(\theta_1)$ satisfies:

$$\mathcal{U}(\theta_1)-\mathcal{U}(\hat\theta_1) = \int_{\theta_1}^{\hat\theta_1}(1-F(s))^n\,(1-\beta+\beta w_z(u(s)+y(s),\Delta\theta_2))\,ds, \quad \forall(\theta_1,\hat\theta_1)\in\Theta^2.$$

Condition (A.1) thus holds when:

$$\int_{\theta_1}^{\hat\theta_1}(1-F(s))^n\,(1-\beta+\beta w_z(u(s)+y(s),\Delta\theta_2))\,ds \geq$$

$$(1-F(\hat\theta_1))^n\left[(1-\beta)(\hat\theta_1-\theta_1)+\beta\left(w(u(\hat\theta_1)+y(\hat\theta_1)+(\hat\theta_1-\theta_1),\Delta\theta_2)-w(u(\hat\theta_1)+y(\hat\theta_1),\Delta\theta_2)\right)\right].$$

Because $w(\cdot)$ is concave in its first argument, we have:

$$w(u(\hat\theta_1)+y(\hat\theta_1)+(\hat\theta_1-\theta_1),\Delta\theta_2)-w(u(\hat\theta_1)+y(\hat\theta_1),\Delta\theta_2)\leq(\hat\theta_1-\theta_1)w_z(u(\hat\theta_1)+y(\hat\theta_1),\Delta\theta_2).$$

A sufficient condition for (A.1) to hold is thus:

$$\begin{aligned}(A.2)\qquad & \int_{\theta_1}^{\hat\theta_1}(1-F(s))^n\,(1-\beta+\beta w_z(u(s)+y(s),\Delta\theta_2))\,ds \geq \\ & \qquad (\hat\theta_1-\theta_1)(1-F(\hat\theta_1))^n\left(1-\beta+\beta w_z(u(\hat\theta_1)+y(\hat\theta_1),\Delta\theta_2\right).\end{aligned}$$

Observe now that $(1-F(\theta_1))^n(1-\beta+\beta w_z(u(\theta_1)+y(\theta_1),\Delta\theta_2))$ weakly decreasing implies

(A.2). Hence, a sufficient condition to get (A.2) and thus (A.1) is given by:

$$(1-F(\theta_1))^n(1-\beta+\beta w_z(u(\theta_1)+y(\theta_1),\Delta\theta_2)) \text{ weakly decreasing.}$$

Inserting into (3.7), this condition amounts to having $\mathcal{U}(\cdot)$ convex. *Q.E.D.*

Proof of Propositions 1, 2 and 3 : The principal's intertemporal payoff when dealing with this firm can be written as:

$$\mathcal{W}(u_1(\theta_1),\mathcal{U}(\theta_1)) = S_1-\theta_1+\beta(S_2-E[\theta_2])-(1-\beta)u_1(\theta_1)-\beta\varphi\left(\frac{\frac{\mathcal{U}(\theta_1)}{(1-F(\theta_1))^n}-(1-\beta)u_1(\theta_1)}{\beta},\Delta\theta_2\right).$$

Given the type distribution $G(\cdot)$ of the winning firm's bid, the problem with this representative firm can now be written as follows:

$$(\mathcal{P}^{as}): \quad \max_{(u(\theta),\mathcal{U}(\theta),\tilde{\theta})} \int_{\underline{\theta}_1}^{\tilde{\theta}_1} \mathcal{W}(u_1(\theta_1),\mathcal{U}(\theta_1))g(\theta_1)d\theta_1 \text{ subject to (3.7)-(4.1).}$$

This is a relaxed optimization problem since incentive compatibility has been reduced to its necessary condition (3.7). Equipped with this expression, and denoting by λ the costate variable for (3.7) we can now write the Hamiltonian for problem $(\mathcal{P}^{as})$ as:

$$\begin{aligned}\mathcal{H}(u_1,\mathcal{U},\lambda,\theta_1) &= (n+1)f(\theta_1)(1-F(\theta_1))^n\mathcal{W}(u_1(\theta_1),\mathcal{U}(\theta_1))\\ &\quad -\lambda(1-F(\theta_1))^n\left(1-\beta+\beta w_z\left(\varphi\left(\frac{\frac{\mathcal{U}(\theta_1)}{(1-F(\theta_1))^n}-(1-\beta)u_1(\theta_1)}{\beta},\Delta\theta_2\right),\Delta\theta_2\right)\right).\end{aligned}$$

Since $\mathcal{H}(u_1,\mathcal{U},\theta_1)$ is concave in $(u_1,\mathcal{U})$[21], we can use the Pontryagyn Principle to get necessary and sufficient conditions for the optimum. These necessary and sufficient conditions are listed below.

• *Costate variable.* There exists λ, continuous and differentiable, such that:

$$\text{(A.3)} \quad \dot{\lambda}(\theta_1) = \left((n+1)f(\theta_1)+\lambda(\theta_1)w_{zz}\left(\varphi\left(\frac{\frac{\mathcal{U}(\theta_1)}{(1-F(\theta_1))^n}-(1-\beta)u_1(\theta_1)}{\beta},\Delta\theta_2\right),\Delta\theta_2\right)\right)\times \varphi_\zeta\left(\frac{\frac{\mathcal{U}(\theta_1)}{(1-F(\theta_1))^n}-(1-\beta)u_1(\theta_1)}{\beta},\Delta\theta_2\right).$$

• *Transversality condition.* Because there is no boundary condition on $\mathcal{U}$ at $\underline{\theta}$, the transver-

[21]It is straightforward to show that $\mathcal{H}(u_1,\mathcal{U},\theta_1)$ is concave in $(u_1,\mathcal{U})$ if and only if it is concave in $\mathcal{U}$. The condition for concavity in $\mathcal{U}$ simplifies to $1 \geq \frac{\lambda(\theta_1)}{(n+1)f(\theta_1)}w_{zz}\left(\varphi\left(\frac{\frac{\mathcal{U}(\theta_1)}{(1-F(\theta_1))^n}-(1-\beta)u_1(\theta_1)}{\beta},\Delta\theta_2\right),\Delta\theta_2\right)$, which always holds given the value of $\lambda(\theta_1)$ found below and given that $w_{zz}\leq 0$.

sality condition is given by:

(A.4) $\lambda(\underline{\theta}) = 0.$

• *Optimality condition with respect to* u_1. Using the first-order condition, we find:

$$(A.5)\quad 1 = \varphi_\zeta\left(\frac{\frac{\mathcal{U}(\theta_1)}{(1-F(\theta_1))^n} - (1-\beta)u_1(\theta_1)}{\beta}, \Delta\theta_2\right)$$
$$\left(1 + \frac{\lambda(\theta_1)}{(n+1)f(\theta_1)} w_{zz}\left(\varphi\left(\frac{\frac{\mathcal{U}(\theta_1)}{(1-F(\theta_1))^n} - (1-\beta)u(\theta)}{\delta}, \Delta\theta_2\right), \Delta\theta_2\right)\right).$$

We now use these optimality conditions to derive more specific results.

• *Proposition 1.* Inserting (A.5) into (A.3) yields $\dot{\lambda}(\theta_1) = (n+1)f(\theta_1)$. Taking into account (A.4) yields $\lambda(\theta_1) = (n+1)F(\theta_1)$. Inserting this expression into (A.5), and simplifying yields (4.2).

• *Corollary 1.* Straightforward differentiation of (4.2) yields:

$$(A.6)\quad (\dot{u}_1(\theta_1) + \dot{y}(\theta_1)) = \frac{\frac{d}{d\theta_1}\left(\frac{F(\theta_1)}{f(\theta_1)}\right) w_{zz}(u_1^{sb}(\theta_1) + y^{sb}(\theta_1), \Delta\theta_2)}{w_{zz}(u_1^{sb}(\theta_1) + y^{sb}(\theta_1), \Delta\theta_2) - \frac{F}{f}(\theta_1) w_{zzz}(u_1^{sb}(\theta_1) + y^{sb}(\theta_1), \Delta\theta_2)}.$$

Risk aversion implies that $w_{zz}(\cdot)$ is negative, while Assumption 1 ensures that the derivative of the hazard rate is non-negative. Finally, Assumption 2 implies $w_{zzz} \geq 0$ and yields the result in Corollary 1.

• *Proposition 2.* From Seierstad and Sydsaeter (1987) the optimality condition with respect to $\tilde{\theta}_1$ writes as:

(A.7) $\mathcal{H}(u_1(\tilde{\theta}_1), \mathcal{U}(\tilde{\theta}_1), \lambda(\tilde{\theta}_1), \tilde{\theta}_1) = 0.$

Taking into account (4.1), this optimality condition can be expressed as:

$$S_1 + \beta(S_2 - E[\theta_2]) = \tilde{\theta}_1$$
$$+(1-\beta)u_1(\tilde{\theta}_1) + \beta\varphi\left(\frac{-(1-\beta)}{\beta}u_1(\tilde{\theta}_1), \Delta\theta_2\right) + \frac{F(\tilde{\theta}_1)}{f(\tilde{\theta}_1)}(1-\beta+\beta w_z(u_1(\tilde{\theta}_1)+y(\tilde{\theta}_1), \Delta\theta_2)).$$

Using the definition of $\varphi(\cdot)$, this condition can be simplified to (6.2)

• *Proposition 3. Mutatis mutandis*, we can also use the same conditions as above when θ_2 is common knowledge. In this case, the firm is paid $p(\hat{\theta}_1, \theta_2) = \theta_2$. It is thus enough to replace $\Delta\theta_2$ by 0 in (6.2) to get the following expression for $\tilde{\theta}^i$ that is chosen when θ_2 is common knowledge:

$$\text{(A.8)} \quad S_1 + \beta(S_2 - E[\theta_2]) = \tilde{\theta}^i + \frac{F(\tilde{\theta}^i)}{f(\tilde{\theta}^i)}$$
$$+\beta\left(u_1(\tilde{\theta}^i) + y(\tilde{\theta}^i) - v(u_1(\tilde{\theta}^i) + y(\tilde{\theta}^i)) + \frac{F(\tilde{\theta}^i)}{f(\tilde{\theta}^i)}(v'(u_1(\tilde{\theta}^i) + y(\tilde{\theta}^i)) - 1)\right).$$

Define $\mu(\theta_1) \equiv u_1^i(\theta) + y^i(\theta)$. From (4.2), we know that $\mu(\theta_1)$ solves:

$$\text{(A.9)} \quad v'(\mu(\theta_1)) = 1 + \frac{F(\theta_1)}{f(\theta_1)} v''(\mu(\theta_1)).$$

We will use the function $\mu(\theta_1)$ to define $J(\theta_1)$ as:

$$\text{(A.10)} \quad J(\theta_1) = \mu(\theta_1) - v(\mu(\theta_1)) + \frac{F(\theta_1)}{f(\theta_1)}(v'(\mu(\theta_1)) - 1).$$

First observe that from (4.2) and the normalizations made on $v(\cdot)$, $J(\underline{\theta}_1) = 0$. Second, differentiating and taking into account (A.9) yields:

$$\text{(A.11)} \quad \dot{J}(\theta_1) = \frac{d}{d\theta}\left(\frac{F(\theta_1)}{f(\theta_1)}\right)\frac{F(\theta_1)}{f(\theta_1)} v''(\mu(\theta_1)) \leq 0,$$

where the last inequality follows from Assumption 1. From this, it follows that $J(\tilde{\theta}^i) < 0$ when $\tilde{\theta}^i > \underline{\theta}_1$. Inserting into (6.2), we deduce that:

$$S_1 + \beta(S_2 - E[\theta_2]) < \tilde{\theta}^i + \frac{F(\tilde{\theta}^i)}{f(\tilde{\theta}^i)}.$$

This gives us (6.3).

From Assumption 2, we have:

$$-w\left(u_1(\theta_1) + y(\theta_1), \Delta\theta_2\right) + \frac{F(\theta_1)}{f(\theta_1)} w_z(u_1(\theta_1) + y(\theta_1), \Delta\theta_2) \geq -v(u_1(\theta_1) + y(\theta_1)) + \frac{F(\theta_1)}{f(\theta_1)} v'(u_1(\theta_1) + y(\theta_1)).$$

Using this for $\theta_1 = \tilde{\theta}_1^{sb}$ and inserting it into (6.2) yields:

$$\text{(A.12)} \quad S_1 + \beta(S_2 - E[\theta_2]) \geq$$
$$\tilde{\theta}_1^{sb} + \frac{F(\tilde{\theta}_1^{sb})}{f(\tilde{\theta}_1^{sb})} + \beta\left(u_1(\tilde{\theta}_1^{sb}) + y(\tilde{\theta}_1^{sb}) - v(u_1(\tilde{\theta}_1^{sb}) + y(\tilde{\theta}_1^{sb})) + \frac{F(\tilde{\theta}_1^{sb})}{f(\tilde{\theta}_1^{sb})}(v'(u_1(\tilde{\theta}_1^{sb}) + y(\tilde{\theta}_1^{sb})) - 1)\right).$$

This implies (6.4). *Q.E.D.*

PROOF OF PROPOSITION 4: Suppose that the contract $(b^{sb}(\hat{\theta}_1), y^{sb}(\hat{\theta}_1) - \nu\Delta\theta_2)$ regulates the basic service over the two periods of the relationship. Following the same arguments as for second-period incentive compatibility in Section 3, it is straightforward to conclude that, ex post, for the firm to accept the spot contract for all values of θ_2, we must have $p = \bar{\theta}_2$, $\forall(\theta_1, \theta_2)$.

This implies that the second-period profit from the add-on is:

$$\mathcal{U}_2(\underline{\theta}_2) = \Delta\theta_2,$$
$$\mathcal{U}_2(\bar{\theta}_2) = 0.$$

Using these values of $\mathcal{U}_2$, we can write the firm's second period expected utility as:

$$\begin{aligned} &E\left[\, v(b^{sb}(\hat{\theta}_1) - \theta_1 + y^{sb}(\hat{\theta}_1) - \nu\Delta\theta_2) + \mathcal{U}_2(\theta_2)\right] \\ &\quad = \nu v(b^{sb}(\hat{\theta}_1) - \theta_1 + y^{sb}(\hat{\theta}_1) + (1-\nu)\Delta\theta_2) + (1-\nu)v(b^{sb}(\hat{\theta}_1) - \theta_1 + y^{sb}(\hat{\theta}_1) - \nu\Delta\theta_2)) \\ &\quad = w(b^{sb}(\hat{\theta}_1) - \theta_1 + y^{sb}(\hat{\theta}_1), \Delta\theta_2). \end{aligned}$$

This condition means that the firm, anticipating acceptance of the spot contract $p = \bar{\theta}_2$, also truthfully reveals its type to get the same payoff as in the second-best contract $\mathcal{C}^{sb}$, namely:

$$\mathcal{U}^{sb}(\theta_1) = \max_{\hat{\theta}_1 \in \Theta_1} (1-\beta)(b^{sb}(\hat{\theta}_1) - \theta_1) + \beta w(b^{sb}(\hat{\theta}_1) - \theta_1 + y^{sb}(\hat{\theta}_1), \Delta\theta_2).$$

This shows that the same outcome as with $\mathcal{C}^{sb}$ can be obtained even if the add-on cannot be contracted within the intial ex ante contract. *Q.E.D.*