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Invariance Axioms and Functional Form Restrictions in Structural Models

by

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Abstract

The dominant practice in economics is to choose the mathematical specification of model relations on the basis of convenience, without much theoretical support. This paper discusses how quantitative model specifications can, in some cases, be given a more formal scientific underpinning in the sense of being based on a priori theory. I use an example from discrete choice theory to illustrate that it is sometimes possible to obtain a complete characterization of the choice model derived from a set of plausible axioms. Furthermore, I discuss how axioms can be tested non-parametrically, given that suitable Stated Preference data are available.

Keywords: Functional form, Theory of measurement, Invariance principles, Independence from Irrelevant Alternatives, Testing of inequality hypotheses

JEL classification: C40, C51, D12

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1. Introduction

A well-known problem in quantitative economic analysis is that economic theory provides limited guidance for the specification of functional forms of quantitative structural economic models. An unfortunate consequence is that it becomes difficult to discriminate between econometric model formulations based on the same theoretical framework which fit the data reasonably well but result in different counterfactual predictions. Given this state of affairs, the analyst is forced to choose between model specifications without adequate theoretical or empirical support.

Functional form assumptions are also crucial for obtaining identification in nonlinear microeconomic models and in macroeconomics. Examples are found in structural duration analysis (Flinn and Heckman, 1982, Heckman and Singer, 1982, 1984), such as the separation of heterogeneity from structural state dependence (Heckman, 1981, 1991), and in the analysis of social interaction (Manski, 1993, 2007). In macroeconomics the theoretical results obtained by Sonnenschein, Mantel and Debreu (see Kirman, 2010) show that there is no hope of obtaining a general result for stability nor uniqueness of equilibria in macro even if the individual agents satisfy standard axioms of rationality. Thus, without further assumptions about individual demand functions very little can be said about the corresponding aggregate demand function.

A major challenge within structural economic modeling is to find a way of establishing quantitative relations in a scientific sense that can be applied to allow precise counterfactual policy predictions. Here, it should be understood, the notion of "scientific" is used in a specific sense, similar to the one outlined by Frisch.[1] In today's practice it is quite striking how specification problems concerning quantitative behavioral relations are under-communicated and underestimated. Apart from properties such as monotonicity, concavity, separability, homogeneity and symmetry (which are without doubt very useful), there are often no further restrictions on functional form properties that follow from the theory.

A key feature of a rigorous scientific program is axiomatization. Axiomatization is a compact way of representing central properties of the theory (such as, for example, rational behavior). In an early pioneering approach, Frisch proposed elements of an axiomatic approach in order to characterize preference representations, and this set of axioms was later extended in a lecture series in Paris: see Bjerkholt (2012) and Bjerkholt and Dupont (2009). Frisch also proposed Stated Preference (SP) surveys to test his axioms.

[1] Frisch writes in the introduction of his Yale lectures in 1930: "If we take the words 'science' or 'scientific' in their old-fashioned restricted sense, we may perhaps say that astronomy is a field of study which is 'scientific' more than any of the other fields of study having as their object the exploration of the exterior world. The reason for this, it seems, is that in astronomy the fusion between theory and observation has been realized more perfectly than in the other fields of study. When astronomy is a science, it is not because it has an abstract theoretical structure, nor is it because it is built on minute prolonged observations, but it is because the astronomic observations are *filled into* the theoretical structure. It is this unification that raises astronomy to the dignity and significance of a true science. Also in economics we have had theoretical speculations, but most of the time it has not been that kind of theory which is built with the view to being verified by observations. Economic theory has not as yet received the stage where its fundamental notions are derived from the technique of observations." See Bjerkholt and Qin (2010).

In this paper I look at how statistical specifications (functional forms and distributions of unobservables) of empirical model relations can be justified on theoretical grounds. To this end I discuss an axiomatic approach in the spirit of Ragnar Frisch (see Bjerkholt, 2012), using an example from discrete choice theory that allows me to demonstrate how it is possible to use the axiomatic approach to obtain a complete characterization of the choice model. Although these axioms may have varying degrees of intuitive appeal, it is vitally important in this context to show that they can be tested non-parametrically and independently, using laboratory-type experiments or SP data.

This paper is organized as follows. In the next section I discuss the extent to which neoclassical economic theory as applied to behavioral relations has empirical content. In section 3 I review central aspects of measurement theory. In section 4 I use an example from the field of discrete choice behavior to show how the full mathematical structure of the choice model (apart from a set of unknown parameters) follows from particular qualitative axioms. Section 5 discusses briefly if the Travel Demand Forecasting Project (TDFP) directed by McFadden prior to the construction of the Bay Area Rapid Transit system in the San Francisco Bay Area (McFadden, 2000, 2001) qualifies as a scientific program, as understood in this paper. Section 6 discusses how the type of axioms discussed in sections 4 and 5 can be tested by non-parametric statistical methods and section 7 offers some conclusions.

2. On the empirical content of neoclassical economics

A typical approach when analyzing economic behavior consists of postulating a utility-maximizing agent who faces an economic budget constraint, possibly under uncertainty, suitably adapted to the phenomenon being studied. To arrive at quantitative relations that can be applied in an empirical context, a reasonably flexible parametric functional form for the corresponding empirical relations is specified, including assumptions about the distribution of unobservables and the information that is taken to be available to the agent. Apart from qualitative properties such as monotonicity, homogeneity, concavity, separability and symmetry, the theory seldom implies further restrictions on functional form. The specification problem is of course not only restricted to the issue of functional form but also depends crucially on the theoretical framework that is adopted at the outset. One reason why structural economic analysis seems to have fallen into disrepute recently might be due to a combination of highly stylized theoretical representations of the phenomena under study combined with ad hoc econometric specification of the corresponding empirical model.[2] For further discussion on these aspects the reader is referred to Angrist (2001), Angrist and Krueger (1999), Angrist and Pischke (2010), Ginther (2010), Heckman (2010), Blundell (2010), Keane (2010a, b) and Rust (2010),

[2] One example that may serve as an illustration is the traditional textbook analysis of labor supply behavior. In this approach the theoretical starting point is a version of the theory of consumer behavior with two goods: namely, leisure and total consumption. Central aspects that matter to the workers, such as non-pecuniary job attributes, are typically neglected. Also neglected is the fact that hours of work are constrained, while the set of available jobs may be restricted as well.

and the references in their papers. Obviously, it is not a simple matter to assess whether or not a particular theoretical approach is too stylized to be acceptable, and it is beyond the scope of this paper to embark on a general discussion of assessments of various theoretical frameworks.

As indicated above, a researcher who wishes to establish a structural model in a scientific sense faces an extremely difficult challenge because only in a limited sense can conventional data on observed behavior be used to test the model. The problem of functional form is usually under-communicated within the economic research community. Perhaps one reason for this may be that some researchers seem to hold the view that it is pointless to try to achieve an a priori theoretical foundation for functional form specifications of quantitative empirical laws of the sort found in mechanics. Although advanced textbooks in economic theory and econometrics occasionally give the impression that it is possible to establish deep structural relations in a scientific sense, the problems inherent in this endeavor are seldom made explicit. Typically, the theory is discussed on a general, abstract level and, as already indicated, it is left to the econometrician to deal with the problem of establishing concrete empirical specifications. Econometrics is usually interpreted in a narrow sense – namely, as a collection of tools for carrying out statistical inference – with less focus on a priori theoretical concerns.

To illustrate this point further, I will consider the theory of consumer behavior as it is presented in, for example, Varian (1992). Here the rational consumer is supposed to have smooth preferences over a set of consumption bundles, from which the properties of the demand functions, $x_j = x_j(p, y),$ follow, where $x_j(p, y)$ is the demanded quantity of good *j*, given a vector of prices and income (*p*, *y*), *j* = 1, 2,…, *m*. The theory of consumer behavior implies that the so-called substitution matrix *A* with elements $A_{jk} = \partial x_j / \partial p_k + x_k \partial x_j / \partial y$ is symmetric and negative semi-definite. That is, for any vector $z = (z_1, z_2, ..., z_m)',$ the theory implies that $z' Az \leq 0.$ This restriction is certainly not very strong and unfortunately it is the only observational restriction on preferences that follows from standard economic theory: that is, the theory has no additional implication for the functional form of the demand functions. To test whether or not these restrictions hold, one cannot use data from a cross-section, because the demand functions may differ across individuals or households. Without further assumptions, even panel data cannot be applied to identify and test restrictions on *A*. But suppose the researcher were to be in the lucky position of having sufficient data to allow testing of the non-negative restrictions above: for example, by using data from SP surveys to achieve repeated observations of choices of the same consumer facing different prices and incomes. Unfortunately, even such data are not sufficient for establishing the "correct" functional forms and properties of the distributions of unobservables. The reason is that the theory is not constructive in the sense of providing sufficient guidance on the precise family of relevant functional forms and distributions of unobservables. See also Chiappori (1990) and Hey (2005) for similar discussions.

Another fundamental problem relates to the analysis of intertemporal choice behavior. Here a typical assumption is that the lifetime utility function is a discounted sum of period-specific utilities. Unless utility has a money-metric representation it seems at first glance ad hoc to assert that utilities can be added in the same way as money. However, additive separability has an axiomatic foundation (see Blackorby, Primont and Russel, 1978) but it is routinely made for convenience and seldom tested independently of other assumptions of the model.

A major problem is that in practice it is only possible to obtain data on agents' behavior from a limited set of combinations of prices, incomes, population and product characteristics. Because data are limited, this results in delicate identification and specification problems. Suppose, for the sake of argument, that one actually has access to data for outcomes of every relevant counterfactual policy experiment. In principle one could think of conducting a large number of natural experiments intended to cover all possible counterfactual policy settings of interest. This would allow the researcher to construct a very large collection of tables that could be used to predict the effect of a chosen policy reform. No model would be needed as long as there was perfect consistency between the conditions of the reform and the conditions underlying the corresponding outcomes reported in the tables. But even in this ideal setting, which would never occur in reality, the situation would be unsatisfactory, because the collection of such tables would not contribute to a scientific explanation and understanding of the causal mechanisms at work.[3] One is therefore forced to rely on theory, as represented by derived quantitative structural relations, beyond what can be validated empirically from the available data: see Hausman (1992, pp. 166–169). Unfortunately, for models to be operational empirically, additional auxiliary assumptions are needed which are not derived from first principles. Hence, when ad hoc auxiliary assumptions are present, counterfactual predictions will remain unconvincing and often controversial. In a controversial paper, Friedman (1953) claimed that the realism of the assumptions the model is based on is not important. What matters is that the model is able to predict well. But as discussed above, this point of view is not tenable because it is often the case that the model is intended for predicting effects from counterfactual reforms in cases where only fragmentary data- or no data at all- exist. Friedman's essay may have had a negative influence on the economic profession by providing support for developing sophisticated structural models that rest on extremely stylized and unrealistic assumptions. For example, the so-called micro foundation of modern macroeconomic models is based on the postulate that aggregate behavior can be represented by the behavior of a representative agent although it is known that a representative agent does not exists unless extremely restrictive conditions on the functional forms involved are fulfilled (Lewbel, 1989, Kirman, 1992).

In other words, in order to establish rigorous structural relations the researcher often faces a very challenging identification problem that cannot be "solved" without making theoretical progress (Blundell, 2010). A few authors have considered the link between neoclassical economics and physics – in particular, mechanics. For example, Mirowski (1984, 1989) has discussed how economists have

[3] In the history of science, Kepler's discovery of his famous three laws is a striking example of how theoretical principles serve to explain almost perfectly astronomical observations – Tycho Brahe's in this case.

borrowed concepts and principles from physics. However, the contrast between economics and mechanics is striking. Whereas economic theory, with few exceptions, is unable to generate explicit functional form restrictions on quantitative empirical models, the situation in mechanics is totally the opposite. There the theory yields a complete characterization of the mathematical form of the laws, up to some unknown parameters.

Simon (1986, p. S213) has summarized the problem as follows: *"Contemporary neoclassical economics provides no theoretical basis for specifying the shape and content of the utility function, and this gap is very inadequately filled by empirical research using econometric techniques. The gap is important because many conclusions that have been drawn in the literature about the way in which the economy operates depend on assumptions about consumers' utility function."*

There have been many demonstrations of the sensitivity of estimates of structural models to assumptions about functional forms and distribution of unobservables: see, for example, the references in Heckman (2010, p. 357). Other examples are provided by the analyses of labor supply using a discrete choice framework by Dagsvik and Strøm (2006) and Dagsvik et al. (2011). They found that different specifications of the utility function resulted in more or less the same fit to the data but implied substantially different elasticities and counterfactual predictions.

Sometimes researchers refer to the research program of the Cowles Commission that Haavelmo and others developed in the mid-1940s in an attempt to achieve a scientific justification of model specifications. However, Haavelmo's approach only establishes a foundation for statistical inference restricted to a linear structural modeling framework. As both Haavelmo (1944) and others – see, for example, Heckman (1992) – have emphasized, there are seldom a priori arguments that limit the family of interesting relations to linear ones. Indeed, developments in recent years testify to the fact that non-linear model specifications are highly relevant, particularly in situations with limited dependent variables.

The general and abstract nature of theoretical economics as regards implications for quantitative structural relations has led to an unfortunate practice. On the one hand, there is research based on highly sophisticated structural models. These are sophisticated in the sense that super-rational agents (not to mention representative agent models) who behave according to stochastic dynamic programming and intricate games are postulated without any other evidence than an appeal to the principle of perfect rationality: see Elster (2009) and Chiappori (2009).[4] On the other hand, few theoretical principles are invoked to support the choice of the mathematical and statistical form of the corresponding empirical model.

Since the properties of the model are typically not robust under various mathematical

[4] Some progress has been made by means of experimental economics (laboratory-type experiments) as regards testing key qualitative implications from game theory and stochastic dynamic programming. For example, Güth et al. (1982) and Camerer et al. (1994), reviewed in Camerer (2003), carried out laboratory experiments that refuted backward induction. See also the discussion in Binmore (2007, 2010).

formulations (and theoretically equivalent ones), the structural approach has been discredited among some researchers as an unsustainable scientific strategy: see Angrist (2001), Angrist and Pischke (2010), Ginther (2010) and Heckman (2010).

There are notable examples of theories that actually generate explicit mathematical structures of the models. These include (i) von Neumann–Morgenstern expected utility theory (von Neumann and Morgenstern, 1944); (ii) Quiggin's theory of rank dependent utility (Quiggin, 1982); (iii) the Nash bargaining theory (Nash, 1950); and (iv) Luce's choice axiom (equivalent to the condition Independence from Irrelevant Alternatives), Luce (1959), McFadden (1973, 1981). Although these examples do not provide end specifications of empirical relations, they at least demonstrate how seemingly intuitive and "weak" qualitative principles can be translated into surprisingly strong functional form restrictions.

In several applications the results from the analysis may be fairly robust with respect to the empirical specifications. Also careful analyses based on conventional data may sometimes produce convincing predictions in counterfactual settings. A striking example of this is achieved for the TDFP (directed by McFadden), to be discussed in section 5 below. A second example is the analysis of Heckman and Hotz (1989). A study by LaLonde (1986) is generally interpreted as having demonstrated that the structural approach applied to non-experimental data cannot duplicate results obtained from a job training experiment. Heckman and Hotz (1989) showed that when specification tests are performed, the surviving structural models closely match the estimates produced from the experiment analyzed by LaLonde (1986).

3. Invariance principles as a strategy to generate functional forms

Measurement theory is concerned with what it means to measure and establish meaningful scientific laws: see Aczél and Moszer (1994), Aczél and Roberts (1989), Aczél et al.(1986), Krantz et al. (1971), Roberts (1979, 1985), Roberts and Rosenbaum (1986), Falmagne and Narens (1983), Luce (1996), Luce et al. (1990) and Narens (2002). These authors have discussed various concepts and issues that are fundamental for establishing scientific laws of scale representations of relations between physical stimuli and sensory responses. Psychologists and measurement theorists have, since the ground-breaking work of Fechner (1860/1966), been concerned with theoretical aspects of measuring sensory response to physical stimuli (psychophysics), and more generally with foundational aspects of measurement theory. Choice theory in economics can be viewed as a particular case within the field of psychophysics. Of particular importance in this literature is the application of invariance postulates. One of the most famous examples of the use of invariance principles appears in Einstein's special theory of relativity (Einstein, 1905).[5]

[5] One of Einstein's axioms states that the laws of physics are invariant (identical) in all inertial systems (non-accelerating frames of reference).

In statistics, the most famous invariance principles are associated with the asymptotic theory of limiting distributions for sums and maximum (minimum) of independent random variables. For example, if one requires that the probability distribution of any (suitably normalized) linear combination of independent and identically distributed random variables shall belong to the same class as each of the random variables, one obtains the class of stable distributions. This class is identical to the class of asymptotic distributions of (suitably normalized) sums of i.i.d. random variables. Similarly, the distribution of the maximum (suitably normalized) of i.i.d. random variables will have the same distribution as the distribution of the original variables if and only if the distribution belongs to the class of extreme value distribution: see, for example, Resnick (1987). There is a related literature on power laws in economics: see Mandelbrot (1997) and Gabaix (2016), and references therein.

Another invariance principle, known as self-similarity, occurs in mathematical geometry and in the theory of stochastic processes: see Mandelbrot (1982, 1997). In the context of stochastic processes, self-similarity means that the distributional law of the process (normalized to have zero mean) is invariant under change of the time unit: for example, time units such as “year”, “month” or “week”. A common feature of some time series data is that they are (or can be interpreted as) temporal aggregates of data generated on a finer time scale (possibly in continuous time). For such processes Lamperti (1962) has proved that under mild regularity conditions the corresponding temporal aggregate process is approximately self-similar. A temporal aggregate process is understood as the process aggregated over time up to time t. In other words, if the aggregate process is self-similar it means that the aggregate process up to time t follows the same distributional law as the aggregate process up to time bt, for positive b, apart from a change of scale of the process. In this context of temporal aggregation Lamperti’s result on self-similarity plays a similar role as the central limit theorem in the context of aggregation of independent (or weakly dependent) random variables: see Beran (1994, pp. 48–50).

In measurement theory, the notion of dimensional invariance is crucial. To put it simply, a quantitative law is said to be dimensional invariant if its structure is invariant under admissible transformations of the input variables. For example, if the input and output variables of a law are measured on a ratio scale it might be reasonable in some contexts to postulate that the law should remain unchanged whenever the input variables are multiplied by positive constants, apart from a scale transformation of the output variables: see Falmagne and Narens (1983), Falmagne (1985), Luce (1996) and Narens (2002). An early and seminal contribution was made by Stevens (1946, 1951), who discussed the role of scale types in measurement theory. These scales are nominal (permits classification), ordinal (strictly increasing transformations), interval (positive affine transformations), log interval (positive power transformations) and ratio (multiplication of constants). Stevens’s argument was that laws expressing fundamental relations are only meaningful if they do not in any essential way depend on the relevant scales involved.

It turns out that assumptions about dimensional invariance have important bearings on the functional form of the model under investigation. In recent years considerable effort has been expended on developing a theoretical and philosophical foundation for, and interpretation of, dimensional invariance.[6] In some cases assertions about dimensional invariance have considerable intuitive appeal, whereas in others they may be controversial.

In physics dimensional analysis has been in use for a long time. It has been found particularly useful in very complicated physical settings where exact solutions using mathematical methods seem very difficult or even impossible. One example of the use of dimensional analysis is Einstein's approach to obtain an expression for the infrared characteristic frequency of solids (Einstein, 1911). A second example is the use of dimensional analysis to characterize the functional form of Kepler's third law. A third example is the use of dimensional analysis to characterize the functional form of the period of the pendulum (Sedov, 1959, Krantz et al. 1971, Narens, 2002).[7] The use of dimensional invariance in economics has been discussed by de Jong (1967) and Grudzewski and Rosanowska-Plichcinska (2013).

The approach we shall discuss in the next section builds on the work of Falmagne and Narens (1983) and demonstrates how selected versions of dimensional invariance, combined with probabilistic rationality postulates, can generate explicit functional form restrictions on the behavioral relations under study.

4. Application of dimensional invariance: an example

As we have already noted, conventional behavioral economic theory pretends to be quantitative but turns out to result in very few restrictions on the mathematical form of structural empirical model relations. In this section I shall demonstrate how in some cases it is possible to attain Frisch's ideal in the sense that the functional form of the empirical model follows from a set of qualitative axioms. More precisely, I shall apply the axiomatic approach to obtain a complete functional form characterization of a probabilistic binary choice models, apart from a set of unknown parameters. The theory of probabilistic choice originated in psychology: see Luce and Suppes (1965) and Suppes et al. (1989, Chapter 17). In this literature there are a number of results which may be characterized as testable properties in a non-parametric sense. They include Luce and Suppes (1965), Sattath and Tversky (1976), Falmagne (1978), Suppes et al. (1989) and references, Blavatskyy (2008), Dagsvik (1994, 2002, 2008, 2013, 2015), Dagsvik and Røine Hoff (2011) and Dagsvik et al. (2006). Remember that in probabilistic choice theory the individual agent is allowed to have uncertain preferences in the

[6] It is striking and somewhat mysterious that many laws in physics – Kepler's and Newton's laws, for example – are dimensionally invariant in the sense that their mathematical forms are unaffected by ratio scale transformations of the input variables.

[7] In contrast to many applications in social science and psychophysics the application of dimensional invariance principles in physics seems more obvious due to the fact that relevant scales are ratio scales and the physical laws are product of powers of quantities.

sense that he may make different choices in seemingly identical choice experiments. See Anand (1993) for a discussion on the foundation of intransitive preferences.

To fix ideas I shall use the application of Fischer and Nagin (1981) to illustrate the approach. In this example, individuals in a SP survey were asked to make pairwise comparisons between car-parking alternatives, with each alternative being characterized by two attributes: namely, price per year and walking distance (measured in minutes). Each respondent was asked to consider 60 different pairs of hypothetical parking alternatives. Let $x_j = (w_j, d_j)$ denote the attribute vector of alternative *j*, where w_j and d_j represent price and walking distance respectively, and let *S* be the total set of possible attribute pairs.[8] Let $P(x_j; x_k)$ denote the binary choice probability of preferring alternative *j* over *k*, $x_j, x_k \in S$.[9]

Axiom 1 (product rule)

For any $x_j, x_k, x_n \in S$,

$$P(x_j;x_k)P(x_k;x_n)P(x_n;x_j) = P(x_j;x_n)P(x_n;x_k)P(x_k;x_j).$$

The product rule was proposed by Luce and Suppes (1965). The intuition behind this rule is as follows. Suppose that an individual is making binary choices from $\{x_j, x_k, x_n\}$, and suppose further that these choices are statistically independent. The left-hand side of the product rule in the equation above is the probability of the intransitive chain $x_j \succ x_k \succ x_n \succ x_j$, and the right-hand side is the probability of the intransitive chain $x_j \succ x_n \succ x_k \succ x_j$, where $\succ$ means "preferred to". The product rule can therefore be interpreted as the assertion that an intransitive chain in one direction is equally probable as an intransitive chain in the opposite direction. In other words, Axiom 1 captures the notion that departure from rationality is not systematic. It can therefore be viewed as a statement about probabilistic (imperfect) rationality. In the context of this paper the point is not only to what extent the product rule is *plausible*; it is also that this axiom has a clear and intuitive interpretation and can be tested empirically.

Luce and Suppes (1965, p. 350) have derived the following result:

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[8] Fischer and Nagin conducted their study at Duke University, NC, USA. At the time of the study the university was embroiled in a debate over procedures for allocating parking permits. Some participants in that debate suggested that a pricing mechanism be used, with higher prices being levied for parking spots closer to the centre of the campus. Fischer and Nagin selected 20 people from the faculty and administration of Duke University who were each asked to make binary choices in 60 experiments consisting of pairs of parking-lot alternatives.

[9] Here we assume that the price of parking is small relative to annual income, so that the respondents are perceived as comparing pairs of prices and walking distances instead of pairs of income minus prices and walking distances.

Theorem 1

Axiom 1 holds if and only if the binary choice probability has the form

$$P(x_j;x_k)=\frac{1}{1+\exp(v(x_k)-v(x_j))} \quad \textit{(4.1)}$$

where v is a scale function. This scale function is unique up to an additive constant.

In the probabilistic choice literature, the model in (4.1) is called the *strict utility model*. It is well known that there exists a random utility representation of the strict utility model: see McFadden (1973). This representation is given by $U(x_j)=v(x_j)+\varepsilon_j,$ where the random error terms $\{\varepsilon_j\}$ are independent with extreme value distribution $\exp(-\exp(-x))$ for real x. The strict utility model is a special case of the so-called Fechner model given by $P(x_j;x_k)=F(v(x_j)-v(x_k))$ where F is a c.d.f.: see Falmagne (1985). Note that the result of Theorem 1 is not restricted to cases with attribute vector x_j of dimension 2, but holds for attribute vectors of any dimension.

It now only remains to pin down the structure of the scale function $v(x).$ To address this problem, we consider a particular version of dimensional invariance analyzed in a more general setting by Falmagne and Narens (1983).

Axiom 2

For any $x_1,x_2,x_3,x_4\in S,$ *and any positive constants* λ *and* μ *such that whenever*

$$P((w_1,d_1);(w_2,d_2))\le P((w_3,d_3);(w_4,d_4)),$$

then

$$P((\lambda w_1,\mu d_1);(\lambda w_2,\mu d_2))\le P((\lambda w_3,\mu d_3);(\lambda w_4,\mu d_4)).$$

Axiom 2 asserts that if the fraction of individuals who prefer alternative 3 over 4 is greater than or equal to the fraction of individuals who prefer alternative 1 over 2, then the same inequality holds when all prices and distances are rescaled by the factors λ and μ respectively. The intuition is that whenever prices and distances change in such a way that the respective relative levels are kept constant, this may change the respective choice probabilities but not the average rank orderings of alternatives. Thus Axiom 2 does *not* assert that $P((\lambda w_1,\mu d_1);(\lambda w_2,\mu d_2))$ is independent of $(\lambda,\mu).$ It asserts only that if the fraction of individuals who prefer (w_3,d_3) to (w_4,d_4) is greater than the fraction of individuals who prefer (w_1,d_1) to $(w_2,d_2),$ this inequality remains true when price levels and distances are multiplied by the factors λ and μ respectively.

Theorem 2

Assume that $P((w_1,d_1);(w_2,d_2))$ *is continuous in* $(w_j,d_j)\in S,$ *j =1, 2. Then Axioms 1 and 2 hold if and only if (4.1) holds and*

$$v(w,d)=\frac{\kappa(w^{\alpha\gamma}d^{\beta\gamma}-1)}{\gamma}, \tag{4.2}$$

for $w_j>0,$ $d_j>0,$ *j = 1, 2, where* κ,α,β *and* γ *are constants.*

The result of Theorem 2 shows that Axioms 1 and 2 imply a complete characterization of the mathematical structure of the binary choice probabilities, apart from the constants $\alpha\gamma,\beta\gamma$ and $\kappa/\gamma.$ To the best of my knowledge the result in Theorem 2 is new. The proof of Theorem 2 is given in the appendix.[10]

The formula in (4.2) is expressed here in a compact way through the Box–Cox representation, in which it is understood that $v(x)$ is defined as $\kappa\beta\log w+\kappa\alpha\log d,$ when $\gamma=0.$ In this case the constant κ can of course be normalized to 1. The case with $\gamma=0$ yields a choice probability that is invariant under scale transformations of prices and walking distances.

Next, we shall consider an alternative and weaker set of axioms.

Axiom 3

For any $(w_1,d_1),(w_2,d_2),(w_3,d_1),(w_4,d_2)\in S,$ *and any positive* λ *such that whenever*

$$P((w_1,d_1);(w_2,d_2))\le P((w_3,d_1);(w_4,d_2)),$$

then

$$P((\lambda w_1,d_1);(\lambda w_2,d_2))\le P((\lambda w_3,d_1);(\lambda w_4,d_2)).$$

Axiom 3 asserts that if the fraction of individuals who prefer the alternative with attributes (w_1,d_1) over the alternative with attributes (w_2,d_2) is less than or equal to the fraction of individuals who prefer the alternative with attributes (w_3,d_1) over the alternative with attributes $(w_4,d_2),$ then the same is true when prices are scaled by a positive factor $\lambda.$ As with Axiom 2, Axiom 3 does *not* assert that $P((\lambda w_1,d_1);(\lambda w_2,d_2))$ is independent of $\lambda.$ It asserts only that if the fraction of individuals who prefer (w_3,d_1) to (w_4,d_2) is greater than the fraction of individuals who prefer (w_1,d_1) to $(w_2,d_2),$ this inequality remains true also when price levels are multiplied by the factor $\lambda.$

The next axiom is analogous to the previous one.

[10] One can in fact prove that the result of Theorem 2 holds when Axiom 2 is replaced by the assumption that $P(x_1,x_2)=F(v(x_1)-v(x_2))$, where F is a strictly increasing c.d.f. This is because the proof does not depend on the form of F.

Axiom 4

For any $(w_1,d_1),(w_2,d_2),(w_1,d_3),(w_2,d_4)\in S,$ *and any positive* μ *such that whenever*

$$P((w_1,d_1);(w_2,d_2))\le P((w_1,d_3);(w_2,d_4)),$$

then

$$P((w_1,\mu d_1);(w_2,\mu d_2))\le P((w_1,\mu d_3);(w_2,\mu d_4)).$$

We realize that Axiom 4 is analogous to Axiom 3 and the intuition is similar. We also realize immediately that both Axioms 3 and 4 are weaker than Axiom 2. Clearly, Axioms 2 to 4 are versions of dimensional invariance postulates.

Theorem 3

Assume that $P((w_1,d_1);(w_2,d_2))$ *is continuous in* $(w_j,d_j)\in S,$ j *= 1, 2. Then Axioms1, 3 and 4 hold if and only if (4.1) holds and*

(4.3)
$$v(w,d)=\beta_1\frac{(w^{\alpha_1}-1)}{\alpha_1}+\beta_2\frac{(d^{\alpha_2}-1)}{\alpha_2}+\beta_3\frac{(w^{\alpha_1}-1)(d^{\alpha_2}-1)}{\alpha_1\alpha_2},$$

for positive w and d, where $\alpha_1,\alpha_2,\beta_1,\beta_2$ *and* β_3 *are constants.*

The proof of Theorem 3 follows from Theorem 4 in Dagsvik and Røine Hoff (2011), replacing total consumption (minus subsistence consumption) by price and leisure by distance. Dagsvik and Strøm (2006), Dagsvik et al. (2011) and Dagsvik and Jia (2016) have applied suitable versions of Axioms 3 and 4 to justify the functional form of the utility of consumption and leisure in their analysis of labor supply. Thus, as with Theorem 2, when Axioms 1, 3 and 4 are combined, one achieves a full characterization of the functional form of the choice probabilities. Further restrictions on the constants $\alpha_1,\alpha_2,\beta_1,\beta_2$ and β_3 may be desirable, so as to ensure monotonicity and concavity.

Note that the result in Theorem 2 is a special case of Theorem 3. This is no surprise, because Axiom 2 evidently implies Axioms 3 and 4. To realize this, let $\beta_1=\beta_3/\alpha_2$ and $\beta_2=\beta_3/\alpha_1.$ Then the formula in (4.3) reduces to

$$\beta_3\frac{(w^{\alpha_1}d^{\alpha_2}-1)}{\alpha_1\alpha_2}+k$$

which is consistent with (4.2), where *k* is an irrelevant constant.

For the sake of emphasizing the key issue here it may be instructive to compare the approach discussed above with the literature on identification and estimation of models with binary response, see for example Manski (1988), Matzkin (1992) and Lewbel et al. (2012). Here, the problem is to identify and estimate models with dependent variable Y (say) given by $Y=1\{g(X)-u\ge 0\}$ where *u* is a random error term that is independent of *X*, with unknown distribution $F(y) = P(u \le y \mid X)$ and *g* is an unknown deterministic function of a vector *X* of exogenous covariates. From this setup it follows that $P(Y = 1 \mid X) = F(g(X))$. Evidently, without further assumptions one cannot separate *g* from *F* (apart from the property that *F* is a c.d.f.). Matzkin (1992) has demonstrated that both *g* and *F* can be non-parametrically identified and estimated under suitable conditions on *g* and *F*. When $g(X) = v_1(X_1) - v_2(X_2)$ where $X = (X_1, X_2)$, Matzkin (1992) provides alternative conditions for non-parametric identification of v_1 and v_2. In the case where *g* is assumed to be linear-in-parameter authors have developed sophisticated methods for estimating the unknown parameters of *g*, see Lewbel et al. (2012), and the references therein.

Although these results are interesting and useful they are of limited use for facing up to the challenge of predicting effects of counterfactual reforms, that is, they cannot be applied, without further assumptions, to hypothetical settings for which data are scarce. Consequently, parametric specifications that are valid beyond the domain covered by the existing data are called for. Note that the implication from Axiom 1 given in Theorem 1 does not hinge on any regularity conditions on the binary choice probabilities apart from the assumptions that, (i) the choice probabilities are different from zero and one, (ii) different choice experiments are independent. Similarly, in addition to the respective axioms, Theorems 2 and 3 only require choice probabilities to be continuous in the alternative-specific attributes in order to hold.

5. An example based on probabilistic rationality and natural experiment data

A famous example of the use of a combination of the axiomatic approach and data from a natural experiment is the TDFP directed by McFadden at UC Berkeley prior to the construction of the BART, (McFadden et al., 1977). BART is a fixed-rail rapid transit system built in the San Francisco Bay Area during the 1970s. McFadden's research group studied the impact of BART as a natural experiment to test and refine transportation choice models. They collected data on commuter behavior from a sample of individuals in 1972, prior to the introduction of BART, and estimated an empirical multinomial logit model, conditional on the actual available travel alternatives, that is, based on pre- BART commuter data.

Remember that the multinomial logit model follows from a random utility model that satisfies the well known Choice Axiom introduced by Luce (1959). In order to describe the choice axiom, let *S* denote the universe of choice alternatives and $C \subseteq S$ the choice set of the agent. Furthermore, let $J(C)$ denote choice function, that is, the index of the most preferred alternative in *C*. In a random utility model where U_j denotes the utility of alternative *j*, $j \in S$, the choice function is determined by

$U_{J(C)} = \max_{k \in C} U_k$. Since the utility functions are stochastic so will also be the choice function. Luce (1959) proposed the following axiom:

Axiom 5 (*Luce's choice axiom*)
Let j be an alternative and B and C choice sets such that $j \in B \subset C \subseteq S$. *Then*

$$P(J(C) = j \mid J(C) \in B) = P(J(B) = j).$$

Axiom 5 asserts that the probability that an agent shall choose alternative *j* from *C* given that his most preferred alternative belongs to the set *B* is equal to the probability that the agent chooses *j* from *B*. In other words, if the choice set is equal to *C* and it is known that the most preferred alternative belongs to $B \subset C$ then the aggregate choice from *C* equals the aggregate choice from *B*. Luce (1959) showed that Axiom 5 is equivalent to the multinomial logit choice model. As is well known, Axiom 5 can also be expressed in an equivalent way as the so-called constant-ratio rule given by

$$\frac{P_{\{j,k\}}(j)}{P_{\{j,k\}}(k)} = \frac{P_C(j)}{P_C(k)} \tag{5.1}$$

for $j, k \in C \subseteq S$, cf. Luce (1959). As is also well known, the relation in (5.1) can be viewed as a probabilistic analogue of Arrow's principle of independence from irrelevant alternatives (Luce and Raiffa, 1957).[11]

In McFadden (2000) a description of the empirical model and estimation results are provided. The model was then used to predict commuter behavior for the individuals in the sample selected in 1972 after BART began operation in 1975. Recall that in multinomial logit models it is possible to introduce a new alternative (in this case the BART option) and predict the share of individuals that will choose the respective alternatives after BART began operation. Table 1 in McFadden (2001) summarizes results for the journey-to-work. This table shows that the model predicts the actual commuter choices quite well. However, due to the small sample size the standard errors of the predictions are quite large.

Now, an interesting question is how the TDFP stands up to the requirement of being a scientific enterprise as understood in this paper. Recall that this entails (i) an axiomatic theory from which the behavioral model is derived, (ii) a corresponding empirical specification of the model, also derived from axioms, (iii) separate non-parametric testing of each of the axioms, or alternatively, tests

[11] In the concluding section of Luce (1977) he characterizes the choice axiom as follows: *"Perhaps the greatest strength of the choice axiom, and one reason it continues to be used, is as a canon of probabilistic rationality. It is a natural probabilistic formulation of K. J. Arrow's famed principle of the independence of irrelevant alternatives, and as such it is a possible underpinning for rational, probabilistic theories of social behavior. Thus, in the development of economic theory based on the assumption of probabilistic individual choice behavior, it can play a role analogous to the algebraic rationality postulates of the traditional theory*".

of the model performance in counterfactual settings. As regards (i) the modeling framework was derived from stochastic choice theory which allows for greater realism than in conventional textbooks by allowing for heterogeneous (stochastic) preferences. Furthermore, the model was derived from the probabilistic rationality postulate as expressed in Axiom 5. As a result, crucial restrictions on the functional form of the travel demand model follow, namely the multinomial logit framework. However, the functional form of the deterministic parts of the utility function entering the model was not derived from theoretical axioms. Nevertheless, the great advantage of this project is that data enabled testing of the prediction performance of the model in a key counterfactual setting, namely when BART had began operation. In other words, in this case the joint underlying assumptions of the model were tested by means of post-BART observations and the results in Table 1 of McFadden (2001) show that none of the predictions fall outside the respective confidence intervals and some of the point predictions are pretty accurate. Still, an open question is how this model would perform with essentially different attributes than the ones recorded in the data and with a different sample of individuals than the one used for estimation.

6. Non-parametric testing of the axioms

Although some of the axioms discussed above have intuitive interpretations and may seem plausible, it is nevertheless of vital importance to be able to conduct empirical tests of the axioms. A major advantage of the approach discussed in this paper is that the postulated axioms can be tested non-parametrically, and independently of the functional form implications, by means of suitable SP survey data (Luce et al., 1990, section 21.8.4). Iverson and Falmagne (1985) have demonstrated how axioms such as Axioms 2 to 4 can be tested within the framework of binomial (or multinomial) models where the null hypotheses take the form of inequality restrictions on the probabilities. If the permissible space of multinomial models is defined by inequality constraints, then the maximum likelihood estimator may lie on the boundary of the parameter space. Under this condition, the asymptotic distribution of the likelihood ratio test is no longer a simple chi-square distribution. Iverson and Harp (1987), Shapiro (1988), Silvapulle and Sen (2005), Davis-Stober (2009) and Cavagnaro et al. (2014) have developed, discussed and applied appropriate statistical testing procedures to this end. I shall now give a brief outline of the approach. See also Luce (1977) for an overview of earlier work on testing of axioms that characterize stochastic choice models.

Recall that SP data allow researchers to collect several observations for each individual under alternative conditions. Also, one can specify conditions in the same way as in controlled laboratory experiments.[12] Accordingly, with this type of data one can avoid the problem of unobserved parameter heterogeneity.

[12] It is clear that one almost never obtains market data that are nearly so varied and detailed as data obtained from SP surveys. Clearly, unless one thinks that such data are worse than no data at all, researchers can fruitfully use them to establish support

I shall now briefly illustrate how the testing procedure of Axiom 2 can be conducted. Let $p_1 = P((w_1, d_1);(w_2, d_2))$, $q_1 = P((w_3, d_3);(w_4, d_4))$, $p_2 = P((\lambda w_1, \mu d_1);(\lambda w_2, \mu d_2))$ and $q_2 = P((\lambda w_3, \mu d_3);(\lambda w_4, \mu d_4))$, for a given set of prices and distances and for given values of the positive scale factors λ and μ. Let H_0 be the null hypothesis that Axiom 2 does not hold for these particular values of prices, distances and scale factors. Clearly, H_0 can be expressed as[13]

$$\{p_1 \le q_1, p_2 > q_2\} \text{ or } \{p_1 > q_1, p_2 \le q_2\}.$$

Thus rejection of H_0 provides support for Axiom 2. This approach is non-parametric since it does not rely on any a priori restrictions apart from the assumption that the data from the SP experiment are independently distributed. The advantage of this approach is that one avoids the controversial initial ad hoc stage of selecting a family of a priori functional forms within which conventional statistical testing is carried out. One can instead test the invariance assumptions proposed above without specific unjustified a priori assumptions about functional form. As mentioned above, testing of Axiom 1 can be done within the conventional likelihood ratio testing framework based on binomial experiments.

Finally, we shall discuss an example of non-parametric testing of Axiom 5. In this example the aim was to specify a model of who an individual would choose to turn to if help was needed. Data were obtained from the survey of time-use conducted by Statistics Norway, 1980-1981. In this survey respondents were asked who they would turn to if they needed help. The universe of alternatives *S* consists of five alternatives, namely, *S* ={Mother (1), father (2), brother (3), sister (4), and neighbor (5)}. However, the whole set *S* was not available to all the respondents. Specifically, there were 11 different choice sets, $C_1, C_2, ..., C_{11}$, where only $C_{11} = S$. We only consider the subsample of individuals less than 45 years of age. The data and choice sets are given in Table 2 in Appendix B. The data can thus be viewed as outcomes from 11 different multinomial choice models with altogether 35 probabilities. Since the probabilities for each multinomial model add up to 1there are 24 "free" probabilities. Our null hypothesis is that the true model is a multinomial logit model (conditional on the choice set), equivalent to Axiom 5, with all the respondents having the same parameters, against the set of multinomial models with different probabilities for each of the 11 choice sets. Under the null hypothesis the multinomial model contains only 4 free parameters. In this case testing can be conducted by means of conventional likelihood ratio tests.[14] The value of twice the loglikelihood ratio turns out to be equal to 38.2.The corresponding critical value of the Chi square distribution (with 19 degrees of freedom) at 5 per cent significance level equals 31.4 which implies that the null hypothesis is rejected by the data. More information about estimates is given in Table 1 in Appendix B.

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for particular functional form properties. In recent years researchers have studied the validity of SP data and have concluded that in many circumstances they seem to be reliable: see, for example, Louviere et al. (2000).

[13] Alternatively, an equivalently assertion is given by $\{p_1 \ge q_1, p_2 < q_2\}$ or $\{p_1 < q_1, p_2 \ge q_2\}$.

[14] See Hausman and McFadden (1984) and McFadden (1987) have considered testing of Axiom 5 in the context of structurally specified choice models

Our second null hypothesis is that Axiom 5 only holds conditional on the "family" alternatives, that is, when alternative 5 is excluded from the choice set. The intuition is that alternative 5 ("neighbor") differs from the family alternatives in the sense that the family alternatives depend on a latent variable capturing latent "family aspects", that represents "family closeness". As a consequence, the family alternatives might have correlated utilities. To allow for this effect we assume as our null hypothesis that the model is a nested logit model (McFadden, 1984) with correlated utilities across family alternatives but independent of alternative 5. This model thus has 5 free parameters of which one represents the correlation between utilities and it contains the multinomial logit model as a special case. In this case the critical level of the Chi square distribution (with 20 degrees of freedom) equals 30.1. It turns out that twice the loglikelihood ratio in this case equals 17.6 which means that the nested logit hypothesis passes the test. Thus, we have demonstrated that a choice model with only 5 parameters can rationalize the data which initially were outcomes from multinomial models with 24 parameters. Furthermore, the model conditional on choice sets where the neighbor alternative is excluded satisfies Axiom 5. From Table 1 in Appendix B we note that the parameter estimates of the nested logit model are much more precise that the corresponding multinomial logit parameter estimates. Prediction results for the Multinomial logit-and the Nested logit models are reported in Table 2 in Appendix B.

7. Concluding remarks

Whereas classical physics, and particularly mechanics, have attained a superior scientific status, other fields, such as social sciences and economics, are still struggling with understanding and discovering first principles on which a rigorous quantitative research program can be founded. The ideal is an *empirical* scientific theory based on an axiomatic approach. Although existing theories in mathematical economics are axiomatic they provide at best crude representations of the real phenomena they are supposed to explain. Moreover, theories are typically rather general, with a lot of "unknowns" left for the econometricians to determine by means of statistical inference.

In this paper I have argued that the challenge of establishing the precise structure of quantitative causal laws by means of statistical analysis using conventional revealed preference data and existing economic theories alone is simply too demanding to be productive as a scientific strategy in economics. I have further argued that, with the typical data sets available, it is hard to establish rigorous structural relations because data are too limited to allow validation of the model's ability to produce reliable counterfactual predictions. Moreover, the current practice of testing theoretical model assumptions jointly with ad hoc auxiliary functional form specifications is unsatisfactory because it does not identify which of the maintained assumptions are critical (the Quine-Duhem problem): see Hausman (1992, p. 306). Clearly, theoretical and empirical approaches that will allow us to proceed beyond the current state of affairs are called for.

In this paper I have proposed an alternative approach illustrated by means of an example. This example demonstrates that in some cases it is possible to supplement existing theories with additional testable postulates that allow researchers to derive specific functional forms. The approach depends on the ability of the researcher to represent crucial features of the phenomenon under study by qualitative axioms, such as, for example, the versions of dimensional invariance given above. It is important that the axioms have a clear and intuitive interpretation, and to some extent seem reasonable a priori, because empirical tests can only be carried out for a limited set of values of the input variables.

The application in section 4 represents a fairly simple type of choice settings. It is therefore largely an open question how to proceed in both more general and more complicated cases. In many instances it will probably not be possible to follow such a rigorous approach as the one discussed in this paper.

Nevertheless, regardless of how far it is possible to make progress following the axiomatic approach, it is important to realize that data from so-called natural experiments or laboratory - and SP surveys, are essential in the validation of structural models. In this regard, it is clear that Frisch was ahead of his times: see Bjerkholt and Dupont (2009).

Appendix A

Proof of Theorem 2

Let $\tilde{v}(w,d)=\exp(v(w,d))$. By assumption $P((w,d);(w_0,d_0))$ is continuous in (w,d). Hence it follows from (4.1) that $\tilde{v}(w,d)$ is also continuous. When Axiom 1 holds, it follows from Theorem 1 that (4.1) holds and therefore Axiom 2 is equivalent to the assertion that whenever

(A.1) $$\frac{\tilde{v}(w_1,d_1)}{\tilde{v}(w_2,d_2)} \le \frac{\tilde{v}(w_3,d_3)}{\tilde{v}(w_4,d_4)}, \quad \text{then} \quad \frac{\tilde{v}(\lambda w_1,\mu d_1)}{\tilde{v}(\lambda w_2,\mu d_2)} \le \frac{\tilde{v}(\lambda w_3,\mu d_3)}{\tilde{v}(\lambda w_4,\mu d_4)},$$

for any positive scale factors λ and μ where $w_j > 0$, j = 1, 2,3,4. In particular, with $w_2 = w_4$ and $d_2 = d_4$, it follows from (A.1) and Axiom 2 that whenever

(A.2) $$\tilde{v}(w_1,d_1) \le \tilde{v}(w_3,d_3), \text{ then } \tilde{v}(\lambda w_1,\mu d_1) \le \tilde{v}(\lambda w_3,\mu d_3).$$

Now apply the Corollary to Theorem 5 in Falmagne and Narens (1983) (or alternatively Theorem 14.17 in Falmagne, 1985, p. 337), which gives

(A.3) $$\tilde{v}(w,d) = H(w^{\alpha} d^{\beta}),$$

where H is a positive, strictly increasing continuous function and α and β are constants. Let $d = d_0$, where d_0 is fixed, and let $h(w) = H(w^{\alpha} d_0^{\beta})$ and $R(w_1,w_2) = h(w_1)/h(w_2)$. In the terminology of Falmagne and Narens (1983), the function $R(w_1,w_2)$ has a multiplicative representation. Moreover, from (A.1) with $d_j = d_0$ and $\mu = 1$, it follows that whenever

(A.4) $$\frac{h(w_1)}{h(w_2)} \le \frac{h(w_3)}{h(w_4)}, \quad \text{then} \quad \frac{h(\lambda w_1)}{h(\lambda w_2)} \le \frac{h(\lambda w_3)}{h(\lambda w_4)}$$

for any positive λ. We can now apply Theorem 8 in Falmagne and Narens (1983) (or alternatively, Theorem 14.19 in Falmagne, 1985, p. 138) which gives

(A.5) $$\frac{h(w_1)}{h(w_2)} = Q\left(\frac{\kappa(w_1^\gamma - 1) + \delta(w_2^\gamma - 1)}{\gamma}\right),$$

where κ, δ and γ are constants and Q is a positive, strictly increasing continuous function and where, consistent with the usual convention, we define

$$\frac{w^0 - 1}{0} = \log w.$$

When $w_1 = w_2$ it follows that the left-hand side of (A.5) is a constant. This can be achieved only if $\delta = -\kappa$. Hence (A.5) reduces to

(A.6) $$\frac{h(w_1)}{h(w_2)} = Q\left(\frac{\kappa(w_1^\gamma - 1) - \kappa(w_2^\gamma - 1)}{\gamma}\right).$$

Let

$$u = \frac{a^\gamma - 1 - (w_2^\gamma - 1)}{\gamma}, \quad \text{and} \quad z = \frac{w_1^\gamma - 1 - (a^\gamma - 1)}{\gamma},$$

where a is a suitable positive constant. From (A.6) it follows that

(A.7) $$\frac{h(a)}{h(w_2)} = Q(u), \quad \frac{h(w_1)}{h(a)} = Q(z) \quad \text{and} \quad \frac{h(w_1)}{h(w_2)} = Q(u+z)$$

which implies that

(A.8) $$Q(u)Q(z) = Q(u+z).$$

Eq. (A.8) is a Cauchy equation and it has a unique solution that is the exponential function, $Q(u) = \exp(ku)$, where k is a positive constant: see for example Falmagne (1985). From (A.6) we therefore get, with $w_2 = 1$, that

(A.9) $$\log h(w) = \log H(w^\alpha d_0^\beta) = \frac{\kappa(w^\gamma - 1)}{\gamma},$$

where the constant k is absorbed in κ. Now replace w^α by $w^\alpha (d/d_0)^\beta$ in (A.9), which gives

$$v(w,d) = \log H(w^\alpha d^\beta) = \frac{\kappa(d_0^{-\beta\gamma/\alpha}(w^\alpha d^\beta)^{\gamma/\alpha} - 1}{\gamma} = \frac{\tilde{\kappa}((w^\alpha d^\beta)^{\tilde{\gamma}} - 1)}{\tilde{\gamma}} + c,$$

where

$$\tilde{\kappa} = \frac{\kappa d^{-\beta\gamma/\alpha}}{\alpha} \quad \text{and} \quad c = \frac{\kappa(d^{-\beta\gamma/\alpha} - 1)}{\gamma}.$$

The constant c is irrelevant because it is cancelled out in utility comparisons and we therefore conclude that we can write $v(x)$ in the form stated in Theorem 2.

Q.E.D.

Appendix B

Table 1. Parameter estimates of the Multinomial logit-and the Nested logit model

	Multinomial logit model		Nested logit model	
Parameters	Estimates	t-values	Estimates	t-values
v_1	2.119	18.9	1.932	31.8
v_2	-0.519	0.7	0.654	5.5
v_3	0.099	0.2	0.801	8.3
v_4	0.725	4.8	1.242	16.8
θ			0.455	15.0
loglikelihood	-424.9		-416.1	
− Twice loglikelihood ratio	38.2		17.6	

Corr$(U_j, U_k) = 1 - \theta^2 = 0.79$, for $j \neq k, j < 5, k < 5, v_j = EU_j$, $v_5 = 0$, sample size: 526 individuals

Table 2. Data and prediction results for the Multinomial logit- and Nested logit model

Choice sets			Alternatives 1 Mother	2 Father	3 Brother	4 Sister	5 Neighbor	# observations
	Observed		30	NF	NF	NF	6	36
C_1	Predicted	MN Logit	32.1	NF	NF	NF	3.9	
	Predicted	Nested logit	31.4	NF	NF	NF	4.6	
	Observed		NF	NF	36	NF	20	56
C_2	Predicted	MN Logit	NF	NF	29.4	NF	26.6	
	Predicted	Nested logit	NF	NF	38.6	NF	17.3	
	Observed		21	NF	2	NF	1	24
C_3	Predicted	MN Logit	19.2	NF	2.5	NF	2.3	
	Predicted	Nested logit	19.4	NF	1.5	NF	2.9	
	Observed		NF	NF	9	21	2	32
C_4	Predicted	MN Logit	NF	NF	8.5	15.8	7.7	
	Predicted	Nested logit	NF	NF	7.0	18.6	6.4	
	Observed		NF	5	NF	NF	2	7
C_5	Predicted	MN Logit	NF	2.6	NF	NF	4.4	
	Predicted	Nested logit	NF	4.6	NF	NF	2.4	
	Observed		65	3	NF	NF	10	78
C_6	Predicted	MN Logit	65.4	4.7	NF	NF	7.9	
	Predicted	Nested logit	64.5	3.9	NF	NF	9.6	
	Observed		50	4	4	NF	6	64
C_7	Predicted	MN Logit	48.3	3.5	6.4	NF	5.8	
	Predicted	Nested logit	49.2	3.0	4.1	NF	7.7	
	Observed		23	NF	NF	7	8	38
C_8	Predicted	MN Logit	27.8	NF	NF	6.9	3.3	
	Predicted	Nested logit	27.5	NF	NF	6.0	4.4	
	Observed		45	2	NF	5	8	60

C_9	Predicted	MN Logit	41.7	3.0	NF	10.3	5	
	Predicted	Nested logit	41.5	2.5	NF	9.1	6.8	
	Observed		21	NF	2	6	8	37
C_{10}	Predicted	MN Logit	24.7	NF	3.3	6.1	3.0	
	Predicted	Nested logit	25.2	NF	2.1	5.5	4.2	
	Observed		64	4	5	15	6	94
C_{11}	Predicted	MN Logit	60.0	4.3	7.9	14.8	7.2	
	Predicted	Nested logit	61.3	3.7	5.1	13.4	10.5	

NF = Not feasible

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