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RESOURCE EXTRACTION AND UNCERTAIN TIPPING POINTS

by

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Abstract:
A global planning problem is analyzed for extracting an exhaustible resource like oil when resource extraction – the only source for current consumption – also generates additions to the stock of GHGs that influence the likelihood of hitting a threshold representing climate change. We derive conditions for optimal extraction when we take into account joint emissions that accumulate to a stock that is governing the planner's beliefs of facing a climate change that will involve a loss in the production capacity of the global economy.
Except for "annuity of the continuation payoff", which is the stationary rate of welfare after a climate change, the optimality conditions are very similar to the results found in Loury (1978) - where optimal extraction of a non-renewable resource of unknown size was analyzed. Not surprisingly we find that extraction has a cost ("environmental cost") beyond the standard opportunity cost ("resource rent"), implying a lower rate of extraction as long as no threshold has been hit, compared to the risk-free case. Such saving has an expected rate of return along an optimal strategy should be balanced against the standard required rate of return - the Keynes-Ramsey-Cass-Koopmans-condition.

(JEL classification: *C61, Q32, Q54*
Keywords: Resource extraction, tipping point uncertainty, climate change

1. Introduction

During the 1970's, there was a vivid activity related to the analysis of extraction of non-renewable resources, like oil, and a large number of contributions provided a lot of important insight. Among the topics that were under scrutiny was resource extraction under various aspects of risk or uncertainty, and how various kinds of uncertainty would affect the central equilibrium condition – the Hotelling Rule. Just to mention some important contributions: Dasgupta and Heal (1974), as well as Dasgupta and Stiglitz (1981) analyzed optimal depletion when a substitute was made available at an unknown future date. Kamien and Schwartz (1978), and Davison (1978), analyzed a similar problem but with endogenous R&D effort. Loury (1978) studied how unknown reserves would affect optimal extraction, whereas Hoel (1978) studied extraction when a future substitute had an uncertain cost. Dasgupta (1981) and Stiglitz and Dasgupta (1981, 1982) analyzed resource pricing and resource depletion under technological uncertainty for various imperfect market structures.

During the last two decades the role of oil or, more generally, fossil fuels, has received new attention as the fear of a climate change caused by the accumulation of GHGs in the atmosphere has increased.[1] Here we combine the relationship between economic activity (resource extraction/consumption) and the likelihood of a climate change being directly affected by the stock of GHGs, that will increase as fossil fuel is extracted. We neglect any kind of abatement, prevention or other policy measures against environmental catastrophes, and focus mainly on how the extraction path will be affected by taking the stock-probability effect into account.[2] Hence, in addition to derive optimal extraction paths, as in the "old" literature, one is now also interested in the impact of resource extraction, as an input in the generation of GHGs that may cause climatic change. Because we don't know how and when the stock of GHGs will trigger a change, we address the issue as one with a tipping point or a threshold. As emphasized by Dasgupta (2007; p.167), "…the Earth system is driven by interlocking non-linear processes running at different speeds and operating in spatial scales. Doing little about climate change would involve Earth crossing an unknown number of tipping points….." In this paper we look at this problem when accumulated emissions of GHGs exceed threshold whose position is not known, there is a "catastrophe" as

[1] The issue of stock pollution has a long history, as summarized by Xepapadeas (2005).

[2] A richer set of policy measures against environmental catastrophes has been discussed recently by Besley and Dixit (2017).

measured by a downgrading of the assets of the global economy. In the role as a global planner, we don't know exactly the location of a single threshold.[3] Instead there are some beliefs, represented by a probability distribution with an endogenous, and stock-dependent, hazard rate. Because we can adjust the rate of extraction, we should be able to affect the net additions to the stock of GHGs, and hence the likelihood for a catastrophe. The main question raised in this paper is therefore how optimal resource depletion will be affected by incorporating the possibility of having a climate catastrophe caused by the jointly produced emissions from oil extraction that add to the stock of GHGs, which again has an impact on the probability distribution for a climate change.

The paper is organized as follows: In section 2 we present a very skeleton-like partial resource-emission planning model for the global economy, having the standard properties of the "old-fashioned" resource extraction models. In section 3 we derive the main results and show the close similarity between the present model and the one studied by Loury (op.cit.). Section 4 concludes.

2. The model

We consider a global economy that has come to an international agreement as to how to manage the stock of exhaustible resources (like oil) and the quality of the global good we call the atmosphere. (Of course, such an approach hides a number of difficult issues, like how to come to an agreement and so on, as we have learnt since Kyoto.) But to see how we should act if we were so lucky to come to a global agreement, we choose to equip the planner with a preference function and a probability distribution for the location of a tipping point. Suppose we (the world population, fixed by assumption) consume an exhaustible resource ("fossil fuel") under the risk of hitting some future catastrophe (or "doomsday") caused by accumulated emissions from resource extraction. To fix ideas, we can think of fossil fuel being consumed and extracted from a known reserve, and that extraction generates emissions that add to a stock pollutant which again will affect the likelihood of hitting a threshold or tipping point of unknown location.

[3] Martin and Pindyck (2015) analyze the issue of multiple catastrophes.

The rate of consumption (equal to the rate of extraction) gives a utility rate $U(c(t))$, when the rate of extraction obeys a resource constraint at some point in time t, as given by $S(t) = S_0 - \int_0^t c(\tau)d\tau$, where $S(t)$ is the remaining reserve at *t*, while S_0 is the known initial reserve. The utility function has standard properties, with $U'(c) > 0, U'' < 0, \lim_{c \to 0} U'(c) = \infty$ and $\lim_{c \to \infty} U'(c) = 0$, with $U(0) = 0$.[4] Along with extracting the resource, there is, by proper choice of measurement, some addition to a stock pollutant, $Z(t) = \int_0^t c(\tau)d\tau$, where $Z(t)$ is the accumulated stock of GHGs. (We rule out any natural depreciation of this stock. This can be justified by pointing at the longevity of many GHGs in the atmosphere.) The term stock pollutant might be a bit confusing because we assume that there is no current cost or damage from this stock, but instead we assume that the size of the stock affects a probability distribution of hitting a threshold. Once such a threshold is hit, the economy will face some downgrading of its production capacity. Here the downgrading is represented by some expected reduction in the remaining reserve of the natural resource which is the sole asset in the model. (Also population is assumed to be fixed.)

If the economy hits a threshold after having a stock of GHGs, Z, there is an irreversible switching from one regime to another, characterized by some random downgrading of the economy's production capacity, as given by the remaining reserve of oil. We could imagine that the sea level rises to such a high level caused by ice melting, so that land, machinery and people will be lost or disappear. For ease of exposition we assume that the random downgrading is independent of when the tipping point is hit. What is of importance for current policy choice then, is the expected downgrading of the global economy's ability to consume after a climate change.

Let the location of the tipping point be characterized by a random variable Y. We can then define $\Pr(Y \geq Z(t)) = 1 - G(Z(t))$ as the unconditional probability that the threshold is above the size of the accumulated emissions at *t*. We have $G(0) = 0$ and we assume that $\lim_{Z \to \infty} G(Z) = 1$, implying that some threshold will be hit sooner or later. Then, because we have Z increasing as long as no threshold has been hit, we can

[4] We could have introduced a choke price so as to take into account some backstop or substitute technology. We choose not to do, because we want to keep track with standard models.

transform the probability distribution from the state space to the time space; as defining a new random variable related to *when* a threshold is hit. On defining this new random variable by $T := Z^{-1}(Y)$, we can derive a probability distribution, $\Omega(t)$, as given by $G(Z(t)) = \Pr(Y \leq Z(t)) = \Pr(Z^{-1}(Y) \leq t) = \Pr(T \leq t) := \Omega(t)$. Then the unconditional density for the event T to occur in in some interval $\left[t, t+dt\right]$, is $G'(Z)dZ = G'(Z(t))\dot{Z}(t)dt := \Omega'(t)dt$, which we assume being positive on $\left[0, \infty\right.$. We then have that $\dfrac{\Omega'(t)dt}{1-\Omega(t)} = \dfrac{G'(Z(t))\dot{Z}(t)dt}{1-G(Z(t))} := h(Z(t))c(t)dt$ is the conditional density for hitting a threshold in $\left[t, t+dt\right]$, or the hazard rate, given that no threshold has been hit before t.)

Suppose that if a threshold is hit at some point in time τ, the reserves left from the pre-catastrophic regime to the new, called the continuation regime, is given by some expected downgrading, according to $S(\tau^+) = aS(\tau^-)$, with $a \in \left[0,1\right]$ being a realization of the random downgrading, where $S(\tau^+)$ is the new initial state variable of the continuation regime. (In this new regime the stock of GHGs will no longer play any role; the world has adapted to the new situation, and we start a "new era". All the effects from a catastrophe will be captured by the continuation payoff, as being derived below.)

Then for any continuation regime, characterized by a pair (a, τ), starting at some given point in time τ, with a realized downgrading a of the pre-catastrophic resource reserve, we can define the continuation payoff, with r as a constant utility discount rate, as the value function of the following, standard optimization problem:

$$w(aS(\tau^-)) = Max_c \int_{\tau}^{\infty} e^{-r(t-\tau)} U(c(t))dt \text{ s.t. } S(\tau^+) = aS(\tau^-), \dot{S}(t) = -c(t), \text{ and } \lim_{t\to\infty} S(t) \geq 0$$

Because our assumptions imply that the resource constraint is binding, we have for any downgrading, a positive ex post shadow value of the resource, $w'(s) := \lambda(s)$. Ex ante, before we know the true or realized downgrading, we can define the expected continuation payoff, $W(S) := E_a w(aS)$, with an expected shadow value or resource rent as given by $W'(S)$. This value function is differentiable, increasing and concave in S, but, of course, independent of τ.

Ex ante, the planner will choose a consumption-extraction program so as to maximize expected welfare as given by the following standard control problem:

$$Max_c \int_0^{\infty} \Omega'(\tau)\left[\int_0^{\tau} e^{-rt}U(c(t))dt + e^{-r\tau}W(S)\right]d\tau$$

(1) s.t.

$$\dot{S}(t) = -c(t), S(0) = S_0, \lim_{t\to\infty} S(t) \geq 0\,,\; \dot{Z}(t) = c(t), Z(0) = 0, \lim_{t\to\infty} Z(t)\ is\ "free"$$

On using that $\Omega'(\tau)d\tau = G'(Z(\tau))\dot{Z}(\tau)d\tau$ in (1) and then integrating by parts, we can express the objective function in a standard and convenient way as:

(2) $$\int_0^{\infty} e^{-rt}\left[(1-G(Z(t)))U(c(t)) + G'(Z(t))c(t)W(S(t)\right]dt$$

The maximization of this integral subject to the differential equations for the state variables (S,Z) given in (1), requires that the following optimality condition for consumption to be obeyed, with p as a positive costate variable ("resource rent") associated with $\dot{S}(t) = -c(t)$, while m is a costate variable ("environmental cost") for $\dot{Z}(t) = c(t)$, and both costate variables interpreted as unconditional shadow prices:

(3) $$\frac{\partial H}{\partial c} = (1-G(Z))U'(c) + G'(Z)W(S) + m - p = 0$$

(4) $$\dot{p}(t) = rp(t) - \frac{\partial H}{\partial S} = rp(t) - G'(Z(t))c(t)W'(S(t))$$

(5) $$\dot{m}(t) = rm(t) - \frac{\partial H}{\partial Z} = rm(t) + G'(Z(t))U(c(t)) - G''(Z(t))c(t)W(S(t))$$

along with the following transversality conditions

(6) $\lim_{t\to\infty} S(t) \geq 0,\ \lim_{t\to\infty} e^{-rt}p(t) \geq 0$ and $\lim_{t\to\infty} e^{-rt}p(t)S(t) = 0$

(7) $\lim_{t\to\infty} Z(t)\ "free",$ with $\lim_{t\to\infty} e^{-rt}m(t) = 0$

when the current value Hamiltonian is given by[5]

(8) $H(c,S,Z,p,m,t)=(1-G(Z))U(c)+G'(Z)cW(S)+(m-p)c$

3. Some results

First we will derive a precise condition for balancing current and future extraction as long as no threshold has been hit. Using our optimality conditions above, we can characterize the optimal contingency plan as:[6]

$$(9)\qquad U'(c(t))=E_{\tau}\left[e^{-r(\tau-t)}\frac{U(c(\tau))-rW(S(\tau))}{c(\tau)}\middle|\tau\geq t\right]$$

This condition says that the current gain from a higher consumption or extraction rate at some point in time *t*, given that no threshold yet has been hit, given by $U'(c(t))$, should, be equal to the conditional expected average net utility loss, discounted to t. As shown in the appendix, the RHS of (9) is given by

$$(10)\int_{t}^{\infty}e^{-r(\tau-t)}\frac{G'(Z(\tau))c(\tau)}{1-G(Z(t))}\frac{U(c(\tau))-rW(S(\tau))}{c(\tau)}d\tau:=E_{\tau}\left[e^{-r(\tau-t)}\frac{U(c(\tau))-rW(S(\tau))}{c(\tau)}\middle|\tau\geq t\right]$$

If on lowering consumption by one unit during a short interval of time, $\left[\tau-t\right]$,with a constant *rate* of consumption, $c(\tau)$, the point in time when the threshold is hit is pushed farther into the future. If we believed that we would meet the tipping point "just after t", this point is pushed $\tau-t=\dfrac{1}{c(\tau)}$ time units into the future. As we then avoid a loss in utility per unit of time equal to $U(c(\tau))-rW(S(\tau))$, (the difference between the pre-catastrophic utility rate and the annuity of a post-catastrophic regime), the discounted increase in net benefit to t, is $e^{-r(\tau-t)}\dfrac{U(c(\tau))-rW(S(\tau))}{c(\tau)}$. But, because $\tau\geq t$ is a random variable as seen from t, with a conditional distribution induced by the

[5] We assume that the maximized Hamiltonian is concave in the state variables, so that Arrow's sufficiency theorem is satisfied.

[6] A detailed derivation is found in the appendix. The condition in (9) is the same as derived by Loury (op.cit.)

extraction history, we have to weight each possible outcome with a conditional density for τ, as given by $\frac{G'(Z(\tau))\dot{Z}(\tau)}{1-G(Z(t))} = \frac{G'(Z(\tau))c(\tau)}{1-G(Z(t))}$ for any t in the pre-catastrophic regime, and then take expectation.[7] The RHS of (9) is therefore the true marginal cost of extraction (in units of utility) at some point in time t, without yet having hit the threshold.

Next, we want to derive properties of the conditional extraction path. On differentiating (3), when using (4) and (5), we get:[8]

$$(11-a) \quad -G'(Z)\dot{Z}U' + (1-G(Z))U''\dot{c} + G''(Z)\dot{Z}W(S) + G'(Z)W'(S)\dot{S} + \dot{m} - \dot{p} = 0$$

which can be written as

$$(11-b) \quad \begin{aligned} &-G'(Z)cU' + (1-G(Z))U''\dot{c} + G''(Z)cW(S) - G'(Z)W'(S)c \\ &+rm + G'U - G''cW - rp + G'cW' = 0 \end{aligned}$$

Dividing through by $(1-G(Z))$ and rearranging terms, while defining the absolute value of the elasticity of marginal utility with respect to consumption; $\hat{\omega}(c) = -\frac{cU''(c)}{U'(c)}$, yields:

$$(12) \quad \frac{\dot{c}(t)}{c(t)} = \frac{h(Z(t))c(t)\left[\frac{\frac{U(c(t)) - rW(S(t))}{c(t)} - U'(c(t))}{U'(c(t))}\right] - r}{\hat{\omega}(c(t))}$$

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[7] I think one gets a better understanding of what is the correct unit of measurement in (9), by writing the LHS as $U'(c(t)) \cdot 1$, showing the increased benefit from having a unit more consumption during a short interval of time without hitting a tipping point.

[8] See the appendix for details.

This is what we might call a "catastrophic-modified Keynes-Ramsey-Cass-Koopmans-Hotelling"– condition, which bears a strong resemblance with the one derived by Loury (op.cit.).[9]

In the absence of any threshold or no risk of facing a climate change, the first term in the numerator of (12) will vanish and we are left with the standard Hotelling-type condition, with the extraction path being declining over time, with remaining reserves approaching zero asymptotically. When facing the risk of a man-made or induced catastrophe, we have to modify the standard Hotelling-rule. The first term in the numerator will capture this random outcome. This term plays a similar role as "the real rate of return from investment" within the Keynes-Ramsey-Cass-Koopmans' framework of optimal growth, as the rate of return from deferring current extraction or consumption. The rate of return from deferring or delaying extraction must therefore be related to the gain, relative to the cost, from prolonging the duration of the period until a threshold is hit. This is similar to the Loury-effect on extraction from not knowing the true size of the initial resource reserves.

We note that a potential climate change, induced by the extraction-emission policy, will lead to a higher growth rate in the conditional extraction path, which should motivate the planner to implement resource saving in the initial years. This should not come as a surprise.

We also note that if we define the Armageddon or "Doomsday", by $rW(S) \equiv 0$, we get full equivalence with the result derived by Loury. The lower is the continuation annuity, $rW(S)$, the higher is the expected return from delaying extraction with a corresponding lower extraction rate.

An alternative way of expressing our result is to state the equality between the required rate of return from delaying extraction (saving) and the expected real rate of return from investment or deferring extraction at some point in time *t* as long as no threshold has been hit so far, as given by a well-known balancing condition, which is just another way of determining the social rate of discount:

[9] See equation (18) in Loury (op.cit.)

$$(13) \qquad r+\hat{\omega}(c(t))\frac{\dot{c}(t)}{c(t)}=h(Z(t))c(t)\left[\frac{\frac{U(c(t))-rW(S(t))}{c(t)}-U'(c(t))}{U'(c(t))}\right]$$

The LHS of (13) is the required rate of return from delaying extraction at some point in time *t*, conditional on not having experienced a climate change, whereas the RHS is the conditional expected real rate of return from resource saving.

The correct spot price per unit of the resource at some point in time as long as no threshold has been hit, is given by the RHS of (9). It is easy to see that the conditional spot price has to change over time at a rate $r-h(Z(t))c(t)\left[\frac{\frac{U(c(t))-rW(S(t))}{c(t)}-U'(c(t))}{U'(c(t))}\right]$, which is below the utility rate of discount. In this case with a kind of dynamic externality caused by current extraction, the global planner should impose a tax on extraction so as to delay extraction or motivate to more resource saving in the early stages of the planning period than what would be the case with no taxation.

4. Some conclusions

Within a highly partial framework and for a global planner, we have derived optimal extraction of a non-renewable resource, like oil or fossil fuel, when resource extraction being an input for current consumption, also adds to a stock of GHGs that influence the likelihood of hitting a threshold representing a climate change. We have derived the conditions for optimal extraction when we also take into account joint emissions that accumulate to a stock that is governing the planner's beliefs of facing a climate change. A climate change is modelled as an expected loss in the production capacity of the global economy.

Except for "the annuity of the continuation payoff", which is the stationary rate of welfare after a climate change, the conditions derived in the paper are identical to the conditions derived in Loury's very fine paper from 1978, where optimal extraction of a non-renewable resource of unknown size was analyzed. Not surprisingly we find that when extraction has a cost (or a stochastic dynamic externality) beyond the standard opportunity cost ("resource rent"), the rate of extraction should become less excessive, with a strong motive for resource saving. Such saving has an expected rate of return which along the optimal contingency plan should be balanced against the standard required rate of return that is found in the well-known Keynes-Ramsey-Cass-Koopmans-framework of optimal growth. It is also shown that this rate of return is higher the more severe is the expected climate change.

Appendix

Derivation of (10):

From (5) and (7) we have

$$\dot{m}(t) - rm(t) = G'(Z(t))U(c(t)) - G''(Z(t))c(t)W(S(t)) \text{ and } \lim_{t\to\infty} e^{-rt}m(t) = 0.$$

On solving this differential equation when using the transversality condition as an end-point constraint, we get:

$$(a-1) \quad m(t) = m(0)e^{rt} + \int_0^t e^{r(t-\tau)}\Big[G'(Z(\tau))U(c(\tau)) - G''(Z(\tau))c(\tau)W(S(\tau))\Big]d\tau$$

with

$$(a-2) \quad e^{-rt}m(t) = m(0) + \int_0^t e^{-r\tau}\Big[G'(Z(\tau))U(c(\tau)) - G''(Z(\tau))c(\tau)W(S(\tau))\Big]d\tau$$

On using (7), we get

$$\lim_{t\to\infty} e^{-rt}m(t) = 0 = m(0) + \int_0^\infty e^{-r\tau}\Big[G'(Z(\tau))U(c(\tau)) - G''(Z(\tau))c(\tau)W(S(\tau))\Big]d\tau \text{, so that}$$

$$-e^{-rt}m(t) = \int_t^\infty e^{-r\tau}\Big[G'(Z(\tau))U(c(\tau)) - G''(Z(\tau))c(\tau)W(S(\tau))\Big]d\tau$$

Hence we get the *unconditional* shadow cost of emission, as given by:

$$(a-3) \quad -m(t) = \int_t^\infty e^{-r(\tau-t)}\Big[G'(Z(\tau))U(c(\tau)) - G''(Z(\tau))c(\tau)W(S(\tau))\Big]d\tau$$

From (4) while using a stronger transversality condition $\lim_{t\to\infty} e^{-rt}p(t)=0$, we get:

$$(a-4) \quad e^{-rt}p(t)=p(0)-\int_0^t e^{-r\tau}G'(Z(\tau))c(\tau)W'(S(\tau))d\tau \text{, so that}$$

$$(a-5) \quad p(t)=\int_t^\infty e^{-r(\tau-t)}G'(Z(\tau))c(\tau)W'(S(\tau))d\tau$$

Using (a-4) and (a-5) in (3), we find:

$$(1-G(Z(t))U'(c(t))=\int_t^\infty e^{-r(\tau-t)}G'(Z(\tau))c(\tau)W'(S(\tau))d\tau$$

$$+\int_t^\infty e^{-r(\tau-t)}\Big[G'(Z(\tau))U(c(\tau))-G''(Z(\tau))c(\tau)W(S(\tau))\Big]d\tau -G'(Z(t))W(S(t))$$

$$=\int_t^\infty e^{-r(\tau-t)}\Big[G'(Z(\tau))c(\tau)W'(S(\tau))+G'(Z(\tau))U(c(\tau))-G''(Z(\tau))c(\tau)W(S(\tau))\Big]d\tau-G'(Z(t))W(S(t))$$

On using Leibniz's formula to write:

$$\frac{d}{dt}\{\int_t^\infty e^{-r(\tau-t)}G'(Z(\tau))W(S(\tau))d\tau\}=-G'(Z(t))W(S(t))+r\int_t^\infty e^{-r(\tau-t)}G'(Z(\tau))W(S(\tau))d\tau$$

and inserting this into the reformulated version of (3) above, we get:

$$\begin{aligned}&(1-G(Z(t))U'(c(t))\\&=\int_t^\infty e^{-r(\tau-t)}\Big[G'(Z(\tau))c(\tau)W'(S(\tau))+G'(Z(\tau))U(c(\tau))-G''(Z(\tau))c(\tau)W(S(\tau))\Big]d\tau\\&+\frac{d}{dt}\{\int_t^\infty e^{-r(\tau-t)}G'(Z(\tau))W(S(\tau))d\tau\}-r\int_t^\infty e^{-r(\tau-t)}G'(Z(\tau))W(S(\tau))d\tau\end{aligned}$$

$$\begin{aligned}&=\int_t^\infty e^{-r(\tau-t)}G'(Z(\tau))U(c(\tau))d\tau+\int_t^\infty e^{-r(\tau-t)}\Big[G'(Z(\tau))c(\tau)W'(S(\tau))-G''(Z(\tau))c(\tau)W(S(\tau))\Big]d\tau\\&+\frac{d}{dt}\{\int_t^\infty e^{-r(\tau-t)}G'(Z(\tau))W(S(\tau))d\tau\}-r\int_t^\infty e^{-r(\tau-t)}G'(Z(\tau))W(S(\tau))d\tau\end{aligned}$$

Because

$$\int_t^\infty e^{-r(\tau-t)}\Big[G'(Z(\tau))c(\tau)W'(S(\tau)) - G''(Z(\tau))c(\tau)W(S(\tau))\Big]d\tau = -\int_t^\infty e^{-r(\tau-t)}\frac{d}{d\tau}\Big[G'(Z(\tau))W(S(\tau))\Big]d\tau$$

we have the following:

$$(1-G(Z(t))U'(c(t)) = \int_t^\infty e^{-r(\tau-t)}G'(Z(\tau))U(c(\tau))d\tau - r\int_t^\infty e^{-r(\tau-t)}G'(Z(\tau))W(S(\tau))d\tau$$

$$+\frac{d}{dt}\{\int_t^\infty e^{-r(\tau-t)}G'(Z(\tau))W(S(\tau))d\tau\} - \int_t^\infty e^{-r(\tau-t)}\frac{d}{d\tau}\Big[G'(Z(\tau))W(S(\tau))\Big]d\tau$$

Define then $G'(Z(\tau))W(S(\tau)) \coloneqq F(\tau)$, and our balancing condition can be written as:

$$(1-G(Z(t))U'(c(t)) = \int_t^\infty e^{-r(\tau-t)}G'(Z(\tau))U(c(\tau))d\tau$$

$$+\frac{d}{dt}\int_t^\infty e^{-r(\tau-t)}F(\tau)d\tau - \int_t^\infty e^{-r(\tau-t)}\{rF(\tau)+F'(\tau)\}d\tau$$

$$= \int_t^\infty e^{-r(\tau-t)}G'(Z(\tau))U(c(\tau))d\tau - F(t) + r\int_t^\infty e^{-r(\tau-t)}F(\tau)d\tau - r\int_t^\infty e^{-r(\tau-t)}F(\tau)d\tau - \int_t^\infty e^{-r(\tau-t)}F'(\tau)d\tau$$

$$= \int_t^\infty e^{-r(\tau-t)}G'(Z(\tau))U(c(\tau))d\tau - F(t) - \int_t^\infty e^{-r(\tau-t)}F'(\tau)d\tau$$

$$= \int_t^\infty e^{-r(\tau-t)}G'(Z(\tau))U(c(\tau))d\tau - F(t) - \Big[e^{-r(\tau-t)}F(\tau)\Big]_t^\infty - r\int_t^\infty e^{-r(\tau-t)}F(\tau)d\tau$$

$$= \int_t^\infty e^{-r(\tau-t)}G'(Z(\tau))U(c(\tau))d\tau - F(t) + F(t) - r\int_t^\infty e^{-r(\tau-t)}G'(Z(\tau))W(S(\tau))d\tau$$

Hence we get our final result, as given in (9) and in (10):

$$U'(c(t)) = \int_t^\infty e^{-r(\tau-t)}\frac{G'(Z(\tau))c(\tau)}{1-G(Z(t))}\frac{U(c(\tau))-rW(S(\tau))}{c(\tau)}d\tau$$

Derivation of (12):

On differentiating (3) in the text while using (4) – (5), we get:

$$(a-6) \quad -G'(Z)\dot{Z}U' + (1-G(Z))U''\dot{c} + G''(Z)\dot{Z}W(S) + G'(Z)W'(S)\dot{S} + \dot{m} - \dot{p} = 0$$

which can be written as:

$$(a-7) \quad \begin{array}{l} -G'(Z)cU' + (1-G(Z))U''\dot{c} + G''(Z)cW(S) - G'(Z)W'(S)c + rm + G'U - G''cW \\ -rp + G'cW' = 0 \end{array}$$

Rearranging terms and dividing through by $(1-G(Z))$, while using the definition of the hazard rate in the Z-space, $h(Z)$, we get from $(a-7)$:

$$(a-8) \quad -h(Z)cU' + U''\dot{c} + r\frac{m-p}{1-G(Z)} + h(Z)U = 0$$

From (3) we have $\dfrac{p-m}{1-G(Z))} = U'(c) + h(Z)W(S)$

Use this in $(a-8)$ to get:

$$-U''\dot{c} = h(Z)c\left[\frac{U}{c} - U'\right] - r[U'(c) + h(Z)W(S)]$$

Hence, on defining the absolute value of the elasticity of the marginal utility (or the intertemporal elasticity of substitution), $\hat{\omega}(c) := -\dfrac{U''(c)c}{U'(c)}$, and making the necessary manipulations, we get (12) and (13) in the text.

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