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Panel Data Dynamics and Measurement Errors: GMM Bias, IV Validity and Model Fit – A Monte Carlo Study

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PANEL DATA DYNAMICS AND MEASUREMENT ERRORS: GMM BIAS, IV VALIDITY AND MODEL FIT – A MONTE CARLO STUDY

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ABSTRACT: An autoregressive fixed effects panel data equation in error-ridden endogenous and exogenous variables, with finite memory of disturbances, latent regressors and measurement errors is considered. Finite sample properties of GMM estimators are explored by Monte Carlo (MC) simulations. Two kinds of estimators are compared with respect to bias, instrument (IV) validity and model fit: equation in differences/IVs levels, equation in levels/IVs in differences. We discuss the impact on estimators' bias and other properties of their distributions of changes in the signal-noise variance ratio, the length of the signal and noise memory, the strength of autocorrelation, the size of the IV set, and the panel length. Finally, some practical guidelines are provided.

KEYWORDS: Panel data, Measurement error, ARMA model, GMM, Signal-noise ratio,
Error memory, IV validity, Monte Carlo simulation, Finite sample bias

JEL CLASSIFICATION: C21, C23, C31, C33

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1 Introduction

In econometric literature, consistent moment-based estimation has been given substantial attention for autoregressive (AR) panel data models in the 'short panel' case. Properties of such methods are discussed in several contexts and while consistency of estimators in well-specified models may be ensured, it is known that non-negligible finite sample bias sometimes occurs. This is demonstrated through, *inter alia*, Monte Carlo (MC) simulations, see, *e.g.*, Kiviet (1995), Ziliak (1997) and Bun and Kiviet (2006). Consistent moment-based estimation has also been considered for static panel data regression models with measurement errors in the regressors. Not much is known about their finite sample bias in the static short panel case, however. In general, it is known that identification of coefficients in linear static equations with error-ridden regressors is impossible from *cross-section data* unless supplementary information, like 'extraneous' IVs, is called upon, and that the availability of panel data can improve the situation.[1] While the joint occurrence of AR mechanisms and EIV, and procedures to ensure consistency are given some attention in *time-series data* contexts.[2] For panel data situations the literature is sparse, especially when it comes to finite sample properties.

In this paper, finite sample properties of various Generalized Method of Moments (GMM) estimators for AR-EIV panel data models are explored. We do this by synthesizing a fairly large set of MC simulations from experiments with short panels. Feasible approaches to estimate this kind of models consistently while controlling for fixed heterogeneity are: (A) keep the equation in levels and use values of variables in differences as IVs, (B) transform the equation to differences and use values of variables in levels as IVs, or combine (A) and (B) – thus in all cases mixing levels and differences as a way of handling fixed effects. In our simulation setup, the errors are allowed to have memory. GMM estimators ensuring consistency in the case of white noise errors can be modified to account for finite memory of errors or disturbances. We then reduce the IV set to ensure that the memory of all *potential* IVs 'gets clear of' the memory of the errors, so that the IVs are uncorrelated with the errors elements, while being correlated with the variables for which they serve.

Allowing for measurement errors having memory can be motivated by a few examples: First, an equation may include a stock variable, *e.g.*, of finished goods or of fixed production capital constructed from cumulated flows, in which case errors tend to vary cyclically. Second, for flow variables like sales, investment and income, improper periodization of transactions, may create serial correlation – sometimes negative – between errors which are close in time. Third, a latent non-stationary variable in levels, say integrated of order P with a white noise measurement error, will after differencing P times become stationary and have an MA(P) error.

In this paper we specifically set out to explore finite sample bias of GMM estimators

[1] For the static errors-in-variables (EIV) case, see Griliches and Hausman (1986), Wansbeek and Koning (1991), Biørn (1992, 2000), Wansbeek and Meijer (2000, section 6.9), Wansbeek (2001), Biørn and Krishnakumar (2008, Section 10.2), and Xiao *et al.* (2007, 2010). For the AR-case, see Balestra and Nerlove (1966), Anderson and Hsiao (1981, 1982), Harris, Mátyás, and Sevestre (2008), Holtz-Eakin *et al.* (1988), Arellano and Bond (1991), Ahn and Schmidt (1995), Arellano and Bover (1995), and Blundell and Bond (1998).

[2] Grether and Maddala (1973), Pagano (1974), and Staudenmayer and Buonaccorsi (2005) discuss distributed lag models for pure time series combining errors in variables and serially correlated disturbances. Maravall and Aigner (1977), Maravall (1979) and Nowak (1993) discuss identification problems for such models.

which are consistent when the model is well-specified and the number of units goes to infinity. The existence of numerous consistent GMM estimators, combined with the difficulties in obtaining analytical expressions for the biases, call for an MC study. A main conclusion from our simulations is that estimation from the above mentioned (A) procedures in general perform better than estimation based on the (B) procedures. Among the issues explored are: (1) the impact on the bias of changes in the autoregressive coefficient; (1) the impact of changes in the relative variances and memory lengths of signal and noise, (2) possible gains obtained by supplementing IVs constructed from exogenous variables with IVs constructed from endogenous variables, and (3) the impact on the bias of the relative noise/signal variances and memory lengths.

In Section 2, we present the general model design, its IV set and identifying conditions as well as the relevant GMM estimators. The design of the simulation experiments, with the memory pattern parameterized by low-order MA processes, is described in detail in Section 3. In Section 4 the simulation results are presented, with focus on (a) bias and standard errors of the simulated estimates, (b) goodness of fit; (c) skewness and kurtosis of the estimates; and (d) the performance of IV validity tests and the impact on these results of changes in the IV set, in the variance of the measurement errors and in other noise elements, the degree of autocorrelation and the number of periods in the panel data set. At the end, some practical guidelines based on the simulation results are provided.

2 Model, moment conditions and estimators

Our model specifies N individuals, indexed by i, observed in T periods, indexed by t. It is first-order autoregressive, AR(1), has K strictly exogenous variables, and allows for fixed unstructured heterogeneity in the equation and measurement errors in all variables:

$$(1)\qquad \begin{aligned} \mu_{it} &= \alpha_i + \boldsymbol{\xi}_{it}\boldsymbol{\beta} + \mu_{i,t-1}\lambda + u_{it}, \qquad |\lambda|<1,\\ \boldsymbol{q}_{it} &= \boldsymbol{\xi}_{it} + \boldsymbol{\eta}_{it},\\ y_{it} &= \mu_{it} + \nu_{it},\\ \boldsymbol{\xi}_{it} &\perp \boldsymbol{\eta}_{it} \perp u_{it} \perp \nu_{it}. \end{aligned}$$

Here α_i is a latent individual effect, treated as fixed; $(\mu_{it}, \boldsymbol{\xi}_{it})$ are latent variables, $\boldsymbol{\xi}_{it}$ with memory equal to N_ξ; $(y_{it}, \boldsymbol{q}_{it})$ are their observable counterparts; $(\nu_{it}, \boldsymbol{\eta}_{it})$ are errors with zero means and memories (N_ν, N_η); $\boldsymbol{\xi}_{it}, \boldsymbol{\eta}_{it}$ are $(1 \times K)$-vectors; u_{it} is a disturbance with memory equal to N_u; $\boldsymbol{\beta}$ is a $(K \times 1)$ coefficient vector and λ is a scalar constant. For a sufficiently large potential IV set to exist N_ξ must be sufficiently large relative to N_ν, N_η, N_u; see Biørn (2012, Section 3) for elaboration.

Eliminating μ_{it} and $\boldsymbol{\xi}_{it}$, we obtain

$$(2)\qquad y_{it} = \alpha_i + \boldsymbol{q}_{it}\boldsymbol{\beta} + y_{i,t-1}\lambda + w_{it},$$

$$(3)\qquad \Delta y_{it} = \Delta\boldsymbol{q}_{it}\boldsymbol{\beta} + \Delta y_{i,t-1}\lambda + \Delta w_{it},$$

and hence

$$\begin{aligned} y_{it} &= (1-\lambda)^{-1}\alpha_i + \textstyle\sum_{s=0}^{\infty} \lambda^s[\boldsymbol{q}_{i,t-s}\boldsymbol{\beta} + w_{i,t-s}],\\ \Delta y_{it} &= \textstyle\sum_{s=0}^{\infty} \lambda^s[\Delta\boldsymbol{q}_{i,t-s}\boldsymbol{\beta} + \Delta w_{i,t-s}], \end{aligned}$$

where

$$w_{it} = u_{it} + \nu_{it} - \nu_{i,t-1}\lambda - \boldsymbol{\eta}_{it}\boldsymbol{\beta}, \tag{4}$$

so that $(y_{it}, \boldsymbol{q}_{i,t+\tau})$ and $w_{i,t+\theta}$ are correlated for some (τ, θ), uncorrelated for others. Let $N_q = \max[N_\xi, N_\eta]$ and $N_\omega = \max[N_u, N_\nu+1])$ be the memory of $\boldsymbol{q}_{it}$ and of $\omega_{it} = u_{it}+\nu_{it}-\nu_{i,t-1}\lambda$, respectively. Potential IVs are, as shown in Biørn (2012),

For (2): $\Delta\boldsymbol{q}_{i,t+\tau}$, $\tau \notin [-N_\eta, N_\eta+1]$, $\tau \in [-N_q, N_q+1]$; $\Delta y_{i,t+\tau}$, $\tau \in [-N_\xi, -(N_\omega+1)]$.
For (3): $\boldsymbol{q}_{i,t+\tau}$, $\tau \notin [-(N_\eta+1), N_\eta]$, $\tau \in [-(N_q+1), N_q]$; $y_{i,t+\tau}$, $\tau \in [-(N_\xi+1), -(N_\omega+2)]$.

Letting $\boldsymbol{x}_{it} = (\boldsymbol{q}_{it}, y_{i,t-1})$, $\boldsymbol{\gamma} = (\boldsymbol{\beta}', \lambda)'$, we write (2) and (3), after stacking by t, as

$$\begin{bmatrix} y_{i1} \\ y_{i2} \\ \vdots \\ y_{iT} \end{bmatrix} = \begin{bmatrix} \alpha_i \\ \alpha_i \\ \vdots \\ \alpha_i \end{bmatrix} + \begin{bmatrix} \boldsymbol{x}_{i1} \\ \boldsymbol{x}_{i2} \\ \vdots \\ \boldsymbol{x}_{iT} \end{bmatrix} \boldsymbol{\gamma} + \begin{bmatrix} w_{i1} \\ w_{i2} \\ \vdots \\ w_{iT} \end{bmatrix},$$

$$\begin{bmatrix} \Delta y_{i2} \\ \Delta y_{i3} \\ \vdots \\ \Delta y_{iT} \end{bmatrix} = \begin{bmatrix} \Delta\boldsymbol{x}_{i2} \\ \Delta\boldsymbol{x}_{i3} \\ \vdots \\ \Delta\boldsymbol{x}_{iT} \end{bmatrix} \boldsymbol{\gamma} + \begin{bmatrix} \Delta w_{i2} \\ \Delta w_{i3} \\ \vdots \\ \Delta w_{iT} \end{bmatrix},$$

or in compact notation, subscripts L and D denoting *Level* and *Difference*, as

$$\boldsymbol{y}_{Li} = \boldsymbol{\alpha}_i + \boldsymbol{X}_{Li}\boldsymbol{\gamma} + \boldsymbol{w}_{Li}, \tag{5}$$

$$\boldsymbol{y}_{Di} = \boldsymbol{X}_{Di}\boldsymbol{\gamma} + \boldsymbol{w}_{Di}. \tag{6}$$

Following the above prescriptions, let $\boldsymbol{Z}_{Di}$, obtained by selecting the relevant elements from $\boldsymbol{X}_{Di}$, be the IV matrix for $\boldsymbol{X}_{Li}$ in (5), and let $\boldsymbol{Z}_{Li}$, obtained by selecting the relevant elements from $\boldsymbol{X}_{Li}$, be the IV matrix for $\boldsymbol{X}_{Di}$ in (6). The 'step-two' GMM estimators for $\boldsymbol{\gamma}$ in (5) and (6) to be considered are, respectively,

$$\begin{aligned} \widetilde{\boldsymbol{\gamma}}_L = {} & \{[\textstyle\sum_i \boldsymbol{X}'_{Li}\boldsymbol{Z}_{Di}][\sum_i \boldsymbol{Z}'_{Di}\widehat{\boldsymbol{w}}_{Li}\widehat{\boldsymbol{w}}'_{Li}\boldsymbol{Z}_{Di}]^{-1}[\sum_i \boldsymbol{Z}'_{Di}\boldsymbol{X}_{Li}]\}^{-1} \\ & \times \{[\textstyle\sum_i \boldsymbol{X}'_{Li}\boldsymbol{Z}_{Di}][\sum_i \boldsymbol{Z}'_{Di}\widehat{\boldsymbol{w}}_{Li}\widehat{\boldsymbol{w}}'_{Li}\boldsymbol{Z}_{Di}]^{-1}[\sum_i \boldsymbol{Z}'_{Di}\boldsymbol{y}_{Li}]\}, \end{aligned} \tag{7}$$

$$\begin{aligned} \widetilde{\boldsymbol{\gamma}}_D = {} & \{[\textstyle\sum_i \boldsymbol{X}'_{Di}\boldsymbol{Z}_{Li}][\sum_i \boldsymbol{Z}'_{Li}\widehat{\boldsymbol{w}}_{Di}\widehat{\boldsymbol{w}}'_{Di}\boldsymbol{Z}_{Li}]^{-1}[\sum_i \boldsymbol{Z}'_{Li}\boldsymbol{X}_{Di}]\}^{-1} \\ & \times \{[\textstyle\sum_i \boldsymbol{X}'_{Di}\boldsymbol{Z}_{Li}][\sum_i \boldsymbol{Z}'_{Li}\widehat{\boldsymbol{w}}_{Di}\widehat{\boldsymbol{w}}'_{Di}\boldsymbol{Z}_{Li}]^{-1}[\sum_i \boldsymbol{Z}'_{Li}\boldsymbol{y}_{Di}]\}, \end{aligned} \tag{8}$$

where $\widehat{\boldsymbol{w}}_{Li}$ and $\widehat{\boldsymbol{w}}_{Di}$ are residual vectors from an introductory 'step-one' estimation; see Davidson and MacKinnon (2004, Sections 9.2–9.3).

3 Design of the Monte Carlo simulations. Diagnostic tests

Having defined the general model framework and the estimators, we specify the setup for the MC simulations, with $K = 2$ exogenous variables assumed. We first present the processes generating $(\boldsymbol{\xi}_{it}, \boldsymbol{\eta}_{it}, \nu_{it}, u_{it})$, next comment on the selection of IVs, and finally describe goodness of fit measures and diagnostic tests.

Parametrization of distributions

(Vector) Moving Average ((V)MA) processes, normally distributed, are used to represent the various memory patterns. Unlike Xiao *et al.* (2007, 2010), who for a static model let $AR(1)$-processes generate the exogenous signal vector, we generate $\boldsymbol{\xi}_{it}$ as the sum of a time invariant vector $\boldsymbol{\chi}_i$ and a VMA process in the $(1 \times K)$-vector $\boldsymbol{\psi}_{it}$:

(9) $\boldsymbol{\xi}_{it} = \boldsymbol{\chi}_i + \sum_{s=0}^{N_\xi} \boldsymbol{\psi}_{i,t-s}\boldsymbol{A}_s,\quad \begin{array}{ll} \boldsymbol{\psi}_{it} \sim \mathsf{IIN}_K(\mathbf{0}, \boldsymbol{\Sigma}_{\boldsymbol{\psi}}),\ \boldsymbol{\Sigma}_{\boldsymbol{\psi}} \text{ diagonal}, & i=1,\dots,N, \\ \boldsymbol{\chi}_i \sim \mathsf{IIN}_K(\bar{\boldsymbol{\chi}}, \boldsymbol{\Sigma}_{\boldsymbol{\chi}}),\ \boldsymbol{\Sigma}_{\boldsymbol{\chi}} \text{ non-diagonal}, & t=1,\dots,T, \end{array}$

where the $\boldsymbol{A}_s$'s are diagonal matrices and the subscript on IIN indicates the dimension of the distribution. Measurement errors and disturbances are generated by

$$\begin{array}{lllll} & \boldsymbol{\eta}_{it} &=& \sum_{s=0}^{N_\eta} \boldsymbol{\epsilon}_{i,t-s}\boldsymbol{B}_s, & \boldsymbol{\epsilon}_{it} \sim \mathsf{IIN}_K(\mathbf{0}, \boldsymbol{\Sigma}_{\boldsymbol{\epsilon}}), \quad \boldsymbol{\Sigma}_{\boldsymbol{\epsilon}} \text{ diagonal}, \\ (10) & \nu_{it} &=& \sum_{s=0}^{N_\nu} \delta_{i,t-s} d_s, & \delta_{it} \sim \mathsf{IIN}_1(0, \sigma_\delta^2), \\ & u_{it} &=& \sum_{s=0}^{N_u} v_{i,t-s} c_s, & v_{it} \sim \mathsf{IIN}_1(0, \sigma_v^2), \quad i=1,\dots,N;\ t=1,\dots,T, \end{array}$$

where the $\boldsymbol{B}_s$'s are diagonal matrices. The intercept heterogeneity, assumed fixed in the general model setup, is generated by

$$\alpha_i \sim \mathsf{IIN}_1(0, \sigma_\alpha^2). \tag{11}$$

Combining (1) with (9) and (10) it follows that

$$\boldsymbol{q}_{it} = \boldsymbol{\xi}_{it} + \textstyle\sum_{s=0}^{N_\eta} \boldsymbol{\epsilon}_{i,t-s}\boldsymbol{B}_s = \boldsymbol{\chi}_i + \sum_{s=0}^{N_\xi} \boldsymbol{\psi}_{i,t-s}\boldsymbol{A}_s + \sum_{s=0}^{N_\eta} \boldsymbol{\epsilon}_{i,t-s}\boldsymbol{B}_s, \tag{12}$$

$$(1-\lambda\mathsf{L})\mu_{it} = \alpha_i + \boldsymbol{\xi}_{it}\boldsymbol{\beta} + \textstyle\sum_{s=0}^{N_u} v_{i,t-s}c_s, \tag{13}$$

$$\begin{aligned} (1-\lambda\mathsf{L})y_{it} &= \alpha_i + \boldsymbol{\xi}_{it}\boldsymbol{\beta} + \textstyle\sum_{s=0}^{N_u} v_{i,t-s}c_s + (1-\lambda\mathsf{L})\sum_{s=0}^{N_\nu}\delta_{i,t-s}d_s \\ &= \alpha_i + \boldsymbol{q}_{it}\boldsymbol{\beta} + \textstyle\sum_{s=0}^{N_u} v_{i,t-s}c_s + (1-\lambda\mathsf{L})\sum_{s=0}^{N_\nu}\delta_{i,t-s}d_s - [\sum_{s=0}^{N_\eta}\boldsymbol{\epsilon}_{i,t-s}\boldsymbol{B}_s]\boldsymbol{\beta}, \end{aligned} \tag{14}$$

with implied variance-covariance matrices of $\boldsymbol{\xi}_{it}$, $\boldsymbol{\eta}_{it}$ and $\boldsymbol{q}_{it}$[3]

$$\begin{aligned} \mathsf{V}(\boldsymbol{\xi}_{it}) &= \boldsymbol{\Sigma}_{\boldsymbol{\chi}} + \textstyle\sum_{s=0}^{N_\xi} \boldsymbol{A}_s'\boldsymbol{\Sigma}_{\boldsymbol{\psi}}\boldsymbol{A}_s, \\ \mathsf{V}(\boldsymbol{\eta}_{it}) &= \textstyle\sum_{s=0}^{N_\eta} \boldsymbol{B}_s'\boldsymbol{\Sigma}_{\boldsymbol{\epsilon}}\boldsymbol{B}_s, \\ \mathsf{V}(\boldsymbol{q}_{it}) &= \boldsymbol{\Sigma}_{\boldsymbol{\chi}} + \textstyle\sum_{s=0}^{N_\xi} \boldsymbol{A}_s'\boldsymbol{\Sigma}_{\boldsymbol{\psi}}\boldsymbol{A}_s + \sum_{s=0}^{N_\eta} \boldsymbol{B}_s'\boldsymbol{\Sigma}_{\boldsymbol{\epsilon}}\boldsymbol{B}_s. \end{aligned}$$

PARAMETER VALUES

The design parameters are $N = 100$, $T = 10$ and the processes are run with $R = 1000$ replications. Two sets of values for $(\boldsymbol{\beta}', \lambda) = (\beta_1, \beta_2, \lambda)$, with the same (β_1, β_2) and different λ, are considered, the latter representing 'sluggish' or 'fast' response of μ to $\boldsymbol{\xi}$. The latent exogenous variable vector and the disturbance have memories $N_\xi = 4$ and $N_u = 0$ throughout, while for the measurement errors we alternatively use $(N_\eta, N_\nu) = (0, 0)$ or $(1, 1)$. The disturbance variance is set to $\sigma_u^2 = \sigma_v^2 = 0.1$ throughout, while different variances are considered for the heterogeneity and the signal and noise elements – technically constructed by rescaling, from a baseline case with all variance ratios set to unity, the relative variances and covariances by factors 4 and $\frac{1}{4}$. Hence, the *spread* of all variables are rescaled proportionally, while $\mathsf{E}(\boldsymbol{q}_{it}) = \mathsf{E}(\boldsymbol{\xi}_{it}) = \bar{\boldsymbol{\chi}}$ is kept fixed.

The full parameter set, where the three values of the variance ratios are intended to represent 'baseline', 'high' and 'low', respectively, is:

Slope coefficients:

$$(\beta_1, \beta_2, \lambda) = \begin{cases} (0.6, 0.3, 0.8) & \text{(strong autocorrelation)}, \\ (0.6, 0.3, 0.2) & \text{(weak autocorrelation)}. \end{cases}$$

u_{it} process:

$N_u = 0,\ c_0 = 1,$
$\sigma_v^2 = \sigma_u^2 = 0.1,$

[3] The autocovariance matrices of $\boldsymbol{\xi}_{it}$ and $\boldsymbol{\eta}_{it}$ are: $C(\boldsymbol{\xi}_{it}, \boldsymbol{\xi}_{i,t+\tau}) = \boldsymbol{\Sigma}_{\boldsymbol{\chi}} + \sum_{s=0}^{N_\xi-\tau} \boldsymbol{A}_s'\boldsymbol{\Sigma}_{\boldsymbol{\psi}}\boldsymbol{A}_{s+\tau}$ $(\tau = 1, \dots, N_\xi)$ and $C(\boldsymbol{\eta}_{it}, \boldsymbol{\eta}_{i,t+\tau}) = \sum_{s=0}^{N_\eta-\tau} \boldsymbol{B}_s'\boldsymbol{\Sigma}_{\boldsymbol{\epsilon}}\boldsymbol{B}_{s+\tau}$ $(\tau = 1, \dots, N_\eta)$.

$\boldsymbol{\xi}_{it}$ *process:*

$N_\xi = 4,\ \boldsymbol{A}_s = a_s\boldsymbol{I}_2,\ (a_0,a_1,a_2,a_3,a_4)=(1,0.8,0.6,0.4,0.2) \implies \sum_{s=0}^{4} a_s^2 = 2.2,$

$\sigma_\psi^2/\sigma_v^2 = 1,4,\frac{1}{4}; \qquad \boldsymbol{\Sigma}_\psi = \sigma_\psi^2\boldsymbol{I}_2; \qquad \boldsymbol{I}_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.$

$\boldsymbol{\eta}_{it}$ *process:*

$N_\eta = 0,\ \boldsymbol{B}_0 = \boldsymbol{I}_2,$

$N_\eta = 1,\ \boldsymbol{B}_s = b_s\boldsymbol{I}_2,\ (b_0,b_1)=(1,0.5) \implies \sum_{s=0}^{1} b_s^2 = 1.25,$

$\sigma_\epsilon^2/\sigma_v^2 = 1,4,\frac{1}{4}; \qquad \boldsymbol{\Sigma}_\epsilon = \sigma_\epsilon^2\boldsymbol{I}_2.$

ν_{it} *process:*

$N_\nu = 0,\ d_0 = 1,$

$N_\nu = 1,\ d_0 = 1,\ d_1 = 0.5 \implies \sum_{s=0}^{1} d_s^2 = 1.25,$

$\sigma_\delta^2/\sigma_v^2 = 1,4,\frac{1}{4}.$

$\boldsymbol{\chi}_i$ *process:*

$(\bar{\chi}_1,\bar{\chi}_2)=(5,10).$

$\sigma_\chi^2/\sigma_v^2 = 1,4,\frac{1}{4}; \qquad \boldsymbol{\Sigma}_\chi = \sigma_\chi^2\boldsymbol{J}_2; \qquad \boldsymbol{J}_2 = \begin{bmatrix} 1 & \frac{1}{2} \\ \frac{1}{2} & 1 \end{bmatrix}.$

α_i *process:*

$\sigma_\alpha^2/\sigma_v^2 = 1,4,\frac{1}{4}.$

INITIALIZATION

The μ_{it} process is initialized in the simulations by using the 'long-run expected' value $\mu_{i0} = \mathsf{E}[\mu_{it}/(1-\lambda\mathsf{L})] = \bar{\boldsymbol{\chi}}\boldsymbol{\beta}/(1-\lambda)$ $(i = 1,\dots,N)$.

SELECTION OF IVS

The choice of the IVs, $\boldsymbol{Z}_{Di}$ and $\boldsymbol{Z}_{Li}$, was described in Section 2. The 'weak instrument' and 'instrument proliferation' problems for AR(1) equations in differences, discussed by Roodman (2009), motivate a 'curtailing' of the IV sets. To prevent an excessive number of potentially weak lagged y-IVs being included, we specifically select only the q- and the lagged y-values that satisfy jointly the IV-requirements.

GOODNESS OF FIT MEASURE AND DIAGNOSTIC TESTS:

R^2-measure: A goodness-of-fit R^2 measure for equations estimated by IVs, based on prediction errors, proposed by Pesaran and Smith (1994), is considered.

Relative Bias (RB), Standard Deviation (SD), and Relative Root Mean Squared Error (RRMSE): Let, for any parameter, θ denote the value assumed in the R replications, and $\widehat{\theta}_r$ is the r'th estimate, and $\bar{\widehat{\theta}} = \frac{1}{R}\sum_{r=1}^{R}\widehat{\theta}_r$. The three statistics are defined by

$$RB = \frac{\bar{\widehat{\theta}}-\theta}{\theta}, \tag{15}$$

$$SD^2 = \frac{1}{R}\sum_{r=1}^{R}(\widehat{\theta}_r - \bar{\widehat{\theta}})^2, \tag{16}$$

$$RRMSE^2 = \frac{1}{R}\sum_{r=1}^{R}\left(\frac{\widehat{\theta}_r-\theta}{\theta}\right)^2 = RB^2+(SD/\theta)^2. \tag{17}$$

Test of orthogonality conditions: Orthogonality conditions are, for (5) and (6), tested by, respectively; confer Hansen (1982), Newey (1985), and Arellano and Bond (1991),

$$\mathcal{J}_L = [\sum_i \widehat{\boldsymbol{w}}'_{Li}\boldsymbol{Z}_{Di}][\sum_i \boldsymbol{Z}'_{Di}\widehat{\boldsymbol{w}}_{Li}\widehat{\boldsymbol{w}}'_{Li}\boldsymbol{Z}_{Di}]^{-1}[\sum_i \boldsymbol{Z}'_{Di}\widehat{\boldsymbol{w}}_{Li}],$$
$$\mathcal{J}_D = [\sum_i \boldsymbol{w}'_{Di}\boldsymbol{Z}_{Li}][\sum_i \widehat{\boldsymbol{Z}}'_{Li}\widehat{\boldsymbol{w}}_{Di}\widehat{\boldsymbol{w}}'_{Di}\boldsymbol{Z}_{Li}]^{-1}[\sum_i \boldsymbol{Z}'_{Li}\widehat{\boldsymbol{w}}_{Di}].$$

Under the null, $\mathcal{J}_L$ and $\mathcal{J}_D$ are asymptotically distributed as χ^2 with a number of degrees of freedom equal to the number of overidentifying restrictions.[4]

F-test for 'IV-strength': A concentration parameter to measure IV strength for a panel data model with one endogenous regressor, proposed by Bun and Windmeijer (2010), is extended to suit our model with multi-endogenous regressors as follows: We write the prototype equation (5) or (6) as

$$\boldsymbol{y}_i = \boldsymbol{\alpha}_i + \boldsymbol{X}_i\boldsymbol{\gamma} + \boldsymbol{w}_i,$$

where $\boldsymbol{y}_i$, $\boldsymbol{w}_i$ and $\boldsymbol{\alpha}_i$ are $(T-1)\times 1$-vectors and $\boldsymbol{X}_i$ is a $(T-1)\times K_1$-matrix, let $\boldsymbol{Z}_i$ be a prototype $(T-1)\times K_2$ IV matrix for $\boldsymbol{X}_i$ $(K_2 \geq K_1)$, and check for a 'weak IV' problem by estimating the system of prototype auxiliary regression equations

$$\boldsymbol{X}_i = \boldsymbol{Z}_i\boldsymbol{\Pi} + \boldsymbol{E}_i,$$

where $\boldsymbol{\Pi}$ is a $K_2 \times K_1$ coefficient matrix and $\boldsymbol{E}_i$ is a $(T-1)\times K_1$ disturbance matrix with covariance matrix of each of its columns equal to $\boldsymbol{\Sigma}_{ii}$. Letting $\widehat{\boldsymbol{\Pi}}$ and $\widehat{\boldsymbol{\Sigma}}_{ii}$ be the estimates of $\boldsymbol{\Pi}$ and $\boldsymbol{\Sigma}_{ii}$, and following Staiger and Stock (1997), we construct the concentration measure, 'tr' denoting the trace operation,

$$\widehat{U} = \tfrac{1}{NK_1}\sum_{i=1}^{N}\mathrm{tr}\Big[\widehat{\boldsymbol{\Pi}}'\boldsymbol{Z}'_i\boldsymbol{Z}_i\widehat{\boldsymbol{\Pi}}\widehat{\boldsymbol{\Sigma}}_{ii}^{-1}\Big].$$

A Wald statistic for $H_0 : \boldsymbol{\Pi}=\boldsymbol{0}$ is $\widehat{U}/K_2$, which under H_0 is F-distributed with number of degrees of freedom equal to the total number of moments of the IVs (in numerator) and the number of observations minus the total number of moments of the IVs (in denominator). A low p-value for the test indicates that $\boldsymbol{Z}_i$ is a strong IV set for $\boldsymbol{X}_i$.

4 Results and discussion

The simulation results are collected in Tables 1 through 9. Tables 1–5 give means, RRMSE's, RB's, SD's, skewness and kurtosis statistics of the simulated estimates. Goodness of fit measures are reported in Table 6. Tables 7–9 contain results from tests for IV validity. A computer program constructed in the Gauss software code, version 7.0, cf. Gauss (2006), is used in the calculations.[5] Parallel results with both q-IVs and y-IVs utilized are reported for most variants, some are reported for the case with only q-IVs included. Results for the β_2 coefficient are omitted in these text tables.

The following discussion will concentrate on: [1] finite sample bias and its dependence on error, noise and signal variances, [2] differences in the bias pattern of coefficients of lagged dependent variables and of coefficients of exogenous variables, [3] the impact on the bias of changes in the degree of individual heterogeneity, and [4] the impact on the bias of changes in the error memory parameters.

[4] The number of orthogonality conditions minus the number of unrestricted coefficients under the null.

[5] The reported standard errors are calculated from the GMM formulae, as described in Biørn and Krishnakumar (2008, Section 10.2.5).

Table 1 contains means and Relative Root Mean Squared Errors (RRMSEs) of the simulated β_1 and λ estimates in the baseline version and five alternatives where, respectively, σ^2_ψ, σ^2_ϵ, σ^2_δ, σ^2_α, and σ^2_χ are increased by a factor of four. Supplementary results comparing the RRMSEs when these σ^2's are alternatively increased and reduced by a factor of four are given in Table 2. Table 3 contains, for estimators with only q-IVs included, corresponding relative biases (RB). By construction, RRMSE $>$ |RB|, but since their order of magnitude is not very different, the RB^2 component of RRMSE^2 dominates the $(\text{SD}/\theta)^2$ component. Standard deviations (SD) of the estimates are reported in Table 4. Full results for means and RRMSEs, including also β_2, are given in Tables A.1 and A.2.

For the *equation in levels*, a *negative bias (RB) in* β_1 *and a positive bias in* λ are indicated. In the baseline variant for the σ^2's ($\sigma^2_v = \sigma^2_\psi = \sigma^2_\epsilon = \sigma^2_\delta = \sigma^2_\alpha = \sigma^2_\chi = 0.1$), when using only q-IVs (in differences), the bias, measured by the RRMSEs, are about 10 % and 3 %, respectively. When y-IVs supplement the q-IVs, the RRMSE measure increases slightly, to about 11 % and 4 %, while the standard deviations of the simulated estimates (first panel of Table 4) are reduced. An increase in σ^2_ψ or σ^2_χ, both contributing to increased signal variance, reduces the RBs and the RRMSEs, while an increase in σ^2_ϵ or σ^2_δ, which increases the noise variances, tends to increase the bias and the RRMSE. An increase in σ^2_ψ also reduces the SDs, while an increase in σ^2_χ leaves them virtually unaffected. This conclusion is consistent with the general results in regression analysis that the larger the variation of exogenous variables, the smaller is the standard errors of their estimates, *cet. par.* From Table 4 we further see that an extension of the IV set tends to reduce the SDs. On the other hand, the RRMSEs may well increase (Table 1). Increasing the variance of the individual latent effect σ^2_α also tends to *increase* the bias and the RRMSE of the two coefficient estimates from the equation in levels.

For the *equation in differences*, we find a *negative bias (RB) in both the* β_1*- and the* λ*-estimates* (Table 3). Hence, the biases in the λ estimates have opposite signs for the equations in levels and in differences. In the baseline case for the σ^2s, the RRMSEs of the β_1 estimates are more than twice as large as for the equation in levels, but again – as was also the case for the equation in levels – the RRMSEs are smaller when only q-IVs are used than when also y-IVs are included. For the λ estimates, the negative bias in the baseline case is more than 40 % and may be even higher when the variances of the measurement errors are increased (see below).

For both versions of the equation, the absolute value of the bias increases when N_η, the error memory of the exogenous variables, increases from zero to one period. Again, the standard deviations (SD) are reduced (second panel of Table 4). An increase in σ^2_ψ, which increases the signal variance, gives a much lower RRMSE than in the baseline case: 8–10 % for β_1 and 13–15 % for λ.

The simulations give some interesting results regarding the impact of the *degree of individual heterogeneity* on the estimator distributions. This unit specific heterogeneity is represented by two parameters: the variances σ^2_α (degree of latent heterogeneity in the equation) and σ^2_χ (degree of heterogeneity in the exogenous variables). From Table 1 we find for the equation in differences, unlike the equation in levels, that an increase in σ^2_α also tends to *reduce* the bias and the RRMSE of the β_1 and the λ estimates. This contrast reflects that the variables in levels and in differences have 'opposite roles' in the two cases and that the 'roles' of the individual-specific latent variables $\boldsymbol{\chi}_i$ and α_i in the model are non-symmetric: First-differencing sweeps out $\boldsymbol{\chi}_i$ from both $\boldsymbol{q}_{it}$ and μ_{it}, so that an increase in σ^2_χ affects both of them, but not $\Delta\boldsymbol{q}_{it}$ and $\Delta\mu_{it}$. An increase in σ^2_α, on the other hand, changes μ_{it}, but leaves $\Delta\mu_{it}$, $\boldsymbol{q}_{it}$, and $\Delta\boldsymbol{q}_{it}$ unchanged. The difference in the impact on the RRMSE's of the estimates of an increase in the variances of $\boldsymbol{\epsilon}_{it}$ and ν_{it}, the white noise elements of the MA processes for the errors in the exogenous variables and the endogenous variable, respectively, is remarkable. For the equation in differences (Table 1, second panel, and Table 3), the bias may be as large as 80–90 %, even for an adequately specified model 'estimated' by N-consistent estimators.

A change in the *strength of the autocorrelation* also affects the estimators' precision, indicated by the RRMSEs. This can be seen by comparing Tables A.2 (for $\lambda=0.8$) and A.4 (for $\lambda=0.2$), from which an extract, confined to the baseline case, is:

Autocorrelation	Eq.	Coef.	$IV=q$, $(N_\xi,N_\eta,N_\nu)=$				$IV=q,y$, $(N_\xi,N_\eta,N_\nu)=$			
			(4,0,0)	(4,1,0)	(4,0,1)	(4,1,1)	(4,0,0)	(4,1,0)	(4,0,1)	(4,1,1)
Strong: $\lambda=0.8$	Level	β_1	0.0955	0.1104	0.0986	0.1185	0.1054	0.1183	0.1041	0.1254
	Level	λ	0.0285	0.0328	0.0285	0.0334	0.0429	0.0484	0.0377	0.0451
	Diff.	β_1	0.2154	0.2393	0.2280	0.2604	0.2342	0.2330	0.2299	0.2594
	Diff.	λ	0.4102	0.4523	0.4501	0.4953	0.4725	0.4471	0.4593	0.5001
Weak: $\lambda=0.2$	Level	β_1	0.0824	0.1044	0.0870	0.1022	0.0774	0.0967	0.0819	0.0973
	Level	λ	0.1595	0.4091	0.1627	0.4038	0.1657	0.3857	0.1653	0.4004
	Diff.	β_1	0.0972	0.1604	0.1034	0.1640	0.0975	0.1114	0.1012	0.1599
	Diff.	λ	0.3005	0.9147	0.3216	0.9671	0.3546	0.3182	0.3453	0.9700

The pattern is diverse. For the coefficient of the exogenous variable, β_1, the RRMSE of the level version is somewhat larger for $\lambda=0.8$ than for $\lambda=0.2$, while in the difference version, for the no error memory case, its value is 21.5 % and 9.7 %, respectively. For the coefficient of the lagged endogenous variable, λ, the RRMSE of the level version is markedly higher for $\lambda=0.2$ than for $\lambda=0.8$, in the no memory case 2.9 % versus 16.0 %. For the difference version the estimator precision is very sensitive to changes in the memory pattern. On the whole, weak autoregression and measurement error having memory is not a favourable constellation for ensuring estimator precision,

As regards *goodness of fit*, we see from Table 6 that, according to the *Pesaran-Smith R^2-indexes*, the fit is improved when y-IVs supplement the q-IVs. These fit indexes are, however, lower when the error of the exogenous vector $\boldsymbol{\eta}_{it}$ has a one-period memory ($N_\eta=1$) than when it is zero. On the whole, this index differs considerably from the standard R^2-index when the equation is in levels, while for the equations in differences, the values are more similar. Examples for the baseline case, with only q-IVs included, are:

	Eq. in levels. $(N_\xi,N_\eta,N_\nu)=$				*Eq. in differences.* $(N_\xi,N_\eta,N_\nu)=$			
	(4,0,0)	(4,1,0)	(4,0,1)	(4,1,1)	(4,0,0)	(4,1,0)	(4,0,1)	(4,1,1)
Standard R^2	0.0498	0.0330	0.0493	0.0334	0.1423	0.0932	0.1432	0.0926
Pesaran-Smith R^2	0.1003	0.0668	0.0991	0.0665	0.0870	0.0564	0.0829	0.0533

Table 5, giving the *skewness and the kurtosis of the coefficients*, illustrate how the shape of the distribution of the GMM estimators – in particular their departure from normality (skewness=0, kurtosis=3) – changes when increasing the σ^2-parameters, introducing memories in the various noises, and shortening the length of the time series. It seems that the coefficient distribution tends to depart more strongly from normality when

(a) the equation is taken to differences, (b) the times series are shortened and (c) the memories of the measurement errors $\boldsymbol{\eta}_{it}$ and ν_{it} are increased.[6]

Our setup for the simulations allows exploring how *changes in the memory of the various error processes* affect the finite sample properties of the GMM. Table 1 shows that *increasing the memory of the error* in the exogenous vector, $\boldsymbol{\eta}$, from $N_\eta = 0$ to 1, tends to reduce the RRMSE of both the β_1 and the λ estimates when basing estimation on the equation in levels and including only the q-IVs. However, the conclusions are modified both when we let y-IVs supplement the q-IVs and when taking the equation to differences.

The *orthogonality* tests come out with p-values that signalize 'IV-validity' for both the equations in levels and in differences and both when autocorrelation is strong ($\lambda = 0.8$) and weak ($\lambda = 0.2$) (Table 7).

Finally, the *F-test statistics* for IV strength come out with p-values that signalize 'IV-validity' in the case with strong autocorrelation ($\lambda = 0.8$) both when the equation considered is in levels and in differences, and both when only q-IVs are included and when they are supplemented with y-IVs (Table 9). In the weak autocorrelation case ($\lambda = 0.2$), we get acceptable p-values for the level version of the equation both when $N_\eta = 0$ and 1. For the difference version, however, the p-values are acceptable only when $N_\eta = 1$ (Table 8).

5 Concluding remarks

In this paper we have, for first-order autoregressive panel data models with measurement errors in the variables, compared, through Monte Carlo simulations, applications of consistent GMM estimators. The results illustrate that both the estimators' finite-sample properties and the performance of tests for instrument validity can differ widely. On the whole, coefficient estimators obtained from the equation in levels with instruments in differences perform better than estimators based on the reversed transformations. We also find that augmenting an instrument set consisting of values of exogenous variables by valid instruments based on endogenous variables will not invariably lead to improved small-sample properties and better performance of the tests. Also the relative size of the signal-noise variances, the memory length of the errors and the size of the autoregression coefficient in the equation are of importance.

Based on the simulation results, we would like to end by indicating a few 'rules of thumb' as a guidance for users of the models and methods. Our guidance type of rules have as purpose to provide an easy-to-use conclusion for employing the GMM in a dynamic panel data context where measurement errors in the variables are likely to occur, rather than providing a complete and accurate discussion:

1. With regard to coefficient estimation precision, working with the equation in levels and q-IVs in differences, in general – although there are exceptions – outperforms all the other alternatives.

2. As regards the goodness of fit, the Pesaran-Smith R^2 index, based on prediction errors

[6] Further experiments (not reported here) with shortening the panel length from $T = 6$ to $T = 5$, gave a dramatic increase in both the skewness and the excess kurtosis and moved the distribution of the estimates further away from normality.

for equations estimated by IVs, indicates that when the variance of the measurement errors is relatively large, the equation in difference with q- and y-IVs in levels comes out as the best choice. However, keeping the equation in levels and using q and y-IVs in difference may be a better choice for other variance-memory combinations.

3. With respect to memory pattern, both the coefficient estimation bias and the goodness of fit of the model, indicated by the RRMSE and the Pesaran-Smith R^2 index, are more sensitive to increased memory of the errors in the exogenous variables than to a corresponding increase for the endogenous variable. Therefore, when the data availability and the model specification allow, we recommend model specifications with memories of errors in exogenous variables to be avoided as much as possible, as this is a way to ensure efficiency in estimation along with acceptable goodness of fit.

4. Compared with the strong autocorrelation case, cases with weak autocorrelation tend to give less biased β estimates but more biased λ estimates.

5. To ensure that distributions of coefficient estimates do not depart too strongly from normality, the level version of the equation in combination with fairly long time series and no or short memory of errors in both exogenous and endogenous variables should be preferred.

6. As regards IV-validity, indicated by tests for over-identifying restrictions, the selection criteria we recommend seem robust across all combinations of autocorrelations and level/difference combinations.

7. Finally, with respect to IV-strength, our selection criteria seem, *cet. par*, to work better for the strong autocorrelation situation than under weak autocorrelation.

References

Ahn, S.C., and Schmidt, P. (1995): Efficient Estimation of Models for Dynamic Panel Data. *Journal of Econometrics* **68**, 5-27.

Anderson, T.W., and Hsiao, C. (1981): Estimation of Dynamic Models with Error Components. *Journal of the American Statistical Association* **76**, 598–606.

Anderson, T.W., and Hsiao, C. (1982): Formulation and Estimation of Dynamic Models Using Panel Data. *Journal of Econometrics* **18**, 47–82.

Arellano, M., and Bond, S. (1991): Some Tests of Specification for Panel Data: Monte Carlo Evidence and an Application to Employment Equations. *Review of Economic Studies* **58**, 277–297.

Arellano, M., and Bover, O. (1995): Another Look at the Instrumental Variable Estimation of Error-Components Models. *Journal of Econometrics* **68**, 29–51.

Balestra, P., and Nerlove, M. (1966): Pooling Cross Section and Time Series Data in the Estimation of a Dynamic Model: The Demand for Natural Gas. *Econometrica* **34**, 585–612.

Biørn, E. (1992): The Bias of Some Estimators for Panel Data Models with Measurement Errors. *Empirical Economics* **17**, 51–66.

Biørn, E. (2000): Panel Data with Measurement Errors. Instrumental Variables and GMM Estimators Combining Levels and Differences. *Econometric Reviews* **19**, 391–424.

Biørn, E. (2012): The Measurement Error Problem in Dynamic Panel Data Analysis: Modeling and GMM Estimation. Memorandum No. 02/2012. Department of Economics, University of Oslo.

Biørn, E., and Krishnakumar, J. (2008): Measurement Errors and Simultaneity. Chapter 10 in *The Econometrics of Panel Data. Fundamentals and Recent Developments in Theory and Practice*, ed. by L. Mátyás and P. Sevestre. Berlin: Springer-Verlag.

Blundell, R., and Bond, S. (1998): Initial Conditions and Moment Restrictions in Dynamic Panel Data Models. *Journal of Econometrics* **87**, 115–143.

Bun, M.J.G., and Kiviet, J. (2006): The Effects of Dynamic Feedbacks on LS and MM Estimator Accuracy in Panel Data Models. *Journal of Econometrics* **132**, 409–444.

Bun, M.J.G., and Windmeijer, F. (2010): The Weak Instrument Problem of the System GMM Estimator in Dynamic Panel Data Models. *Econometrics Journal* **13**, 95–126.

Davidson, R., and MacKinnon, J.G. (2004): *Econometric Theory and Methods.* Oxford: Oxford University Press.

Gauss (2006): Gauss^{TM}. Version 7.0. User's Guide. Maple Valley: Aptech Systems.

Grether, D.M., and Maddala, G.S. (1973): Errors in Variables and Serially Correlated Disturbances in Distributed Lag Models. *Econometrica* **41**, 255–262.

Griliches, Z., and Hausman, J.A. (1986): Errors in Variables in Panel Data. *Journal of Econometrics* **31**, 93–118.

Hansen, L.P. (1982): Large Sample Properties of Generalized Method of Moments Estimators. *Econometrica* **50**, 1029–1054.

Harris, M.N., Mátyás. L. and Sevestre, P. (2008): Dynamic Models for Short Panels. Chapter 8 in *The Econometrics of Panel Data. Fundamentals and Recent Developments in Theory and Practice*, ed. by L. Mátyás and P. Sevestre. Berlin: Springer-Verlag.

Holtz-Eakin, D., Newey, W. and Rosen, H.S. (1988): Estimating Vector Autoregressions with Panel Data. *Econometrica* **56**, 1371–1395.

Kiviet, J.F. (1995): On bias, inconsistency, and efficiency of various estimators in dynamic panel data models. *Journal of Econometrics* **68**, 53–78.

Maravall, A. (1979): *Identification in Dynamic Shock-Error Models.* New York: Springer-Verlag.

Maravall, A., and Aigner, D.J. (1977): Identification of the Dynamic Shock-Error Model: The Case of Dynamic Regression. In *Latent Variables in Socio-Economic Models*, ed. by D.J. Aigner and A.S. Goldberger. Amsterdam: North-Holland.

Newey, W.K. (1985): Generalized Method of Moments Specification Testing. *Journal of Econometrics* **29**, 229–256.

Nowak, E. (1993): The Identification of Multivariate Linear Dynamic Errors-in-Variables Models. *Journal of Econometrics* **59**, 213–227.

Pagano, M. (1974): Estimation of Models of Autoregressive Signal Plus White Noise. *Annals of Statistics* **2**, 99–108.

Pesaran, M.H., and Smith, R.J. (1994): A Generalized R^2 Criterion for Regression Models Estimated by the Instrumental Variables Method. *Econometrica* **62**, 705–710.

Roodman, D. (2009): A Note on the Theme of Too Many Instruments. *Oxford Bulletin of Economics and Statistics* **71**, 135–158.

Staiger, D., and Stock, J.H. (1997): Instrumental Variables Regression with Weak Instruments. *Econometrica* **65**, 557–586.

Staudenmayer, J., and Buonaccorsi, J.P. (2005): Measurement Error in Linear Autoregressive Models. *Journal of the American Statistical Association* **100**, 841–852.

Wansbeek, T.J. (2001): GMM Estimation in Panel Data Models with Measurement Error. *Journal of Econometrics* **104**, 259–268.

Wansbeek, T.J., and Koning, R.H. (1991): Measurement Error and Panel Data. *Statistica Neerlandica* **45**, 85–92.

Wansbeek, T.J., and Meijer, E. (2000): *Measurement Error and Latent Variables in Econometrics.* Amsterdam: Elsevier.

Xiao, Z., Shao, J., and Xu, R. (2007): Efficiency of GMM Estimation in Panel Data Models with Measurement Error. *Sankhya: The Indian Journal of Statistics* **69**, 101–118.

Xiao, Z., Shao, J., and Palta, M. (2010): Instrumental Variables and GMM Estimation for Panel Data with Measurement Errors. *Statistica Sinica* **20**, 1725–1747.

Ziliak, J.P. (1997): Efficient Estimation with Panel Data When Instruments Are Predetermined: An Empirical Comparison of Moment-Condition Estimators. *Journal of Business and Economic Statistics* **15**, 419–431.

Table 1: Mean and RRMSE of coefficient estimates.

$(N, T, R) = (100, 10, 1000)$. $(\beta_1, \beta_2, \lambda) = (0.6, 0.3, 0.8)$

Sensitivity to memory and σ^2 configuration

Equation in levels

	σ^2-parameter rescaling	$IV=q$, $(N_\xi, N_\eta, N_\nu) =$ (4,0,0)	(4,1,0)	(4,0,1)	(4,1,1)	$IV=q,y$, $(N_\xi, N_\eta, N_\nu) =$ (4,0,0)	(4,1,0)	(4,0,1)	(4,1,1)
Mean, β_1	Baseline	0.5635	0.5619	0.5657	0.5584	0.5521	0.5477	0.5570	0.5478
	$\sigma_\psi^2 \times 4$	0.5905	0.5879	0.5914	0.5901	0.5853	0.5823	0.5866	0.5852
	$\sigma_\epsilon^2 \times 4$	0.5080	0.4909	0.5074	0.4895	0.4999	0.4830	0.5016	0.4830
	$\sigma_\delta^2 \times 4$	0.5773	0.5749	0.5794	0.5780	0.5647	0.5612	0.5708	0.5682
	$\sigma_\alpha^2 \times 4$	0.5259	0.5191	0.5313	0.5194	0.4834	0.4672	0.4928	0.4719
	$\sigma_\chi^2 \times 4$	0.5699	0.5627	0.5676	0.5625	0.5725	0.5670	0.5708	0.5666
RRMSE, β_1	Baseline	0.0955	0.1104	0.0986	0.1185	0.1054	0.1183	0.1041	0.1254
	$\sigma_\psi^2 \times 4$	0.0415	0.0506	0.0415	0.0509	0.0429	0.0503	0.0436	0.0508
	$\sigma_\epsilon^2 \times 4$	0.1723	0.2063	0.1759	0.2111	0.1816	0.2123	0.1822	0.2178
	$\sigma_\delta^2 \times 4$	0.1131	0.1454	0.1268	0.1514	0.1168	0.1393	0.1252	0.1455
	$\sigma_\alpha^2 \times 4$	0.1597	0.1775	0.1498	0.1804	0.2169	0.2475	0.2033	0.2448
	$\sigma_\chi^2 \times 4$	0.0892	0.1097	0.0950	0.1144	0.0828	0.0958	0.0898	0.1068
Mean, λ	Baseline	0.8169	0.8180	0.8149	0.8178	0.8319	0.8355	0.8265	0.8318
	$\sigma_\psi^2 \times 4$	0.8048	0.8052	0.8043	0.8048	0.8100	0.8111	0.8088	0.8097
	$\sigma_\epsilon^2 \times 4$	0.8329	0.8393	0.8327	0.8387	0.8427	0.8492	0.8400	0.8467
	$\sigma_\delta^2 \times 4$	0.8048	0.8059	0.8008	0.8026	0.8188	0.8224	0.8107	0.8141
	$\sigma_\alpha^2 \times 4$	0.8512	0.8513	0.8487	0.8511	0.8991	0.9077	0.8903	0.8999
	$\sigma_\chi^2 \times 4$	0.8155	0.8180	0.8153	0.8173	0.8271	0.8316	0.8247	0.8289
RRMSE, λ	Baseline	0.0285	0.0328	0.0285	0.0334	0.0429	0.0484	0.0377	0.0451
	$\sigma_\psi^2 \times 4$	0.0120	0.0143	0.0118	0.0146	0.0152	0.0170	0.0141	0.0163
	$\sigma_\epsilon^2 \times 4$	0.0464	0.0552	0.0463	0.0556	0.0563	0.0647	0.0534	0.0627
	$\sigma_\delta^2 \times 4$	0.0284	0.0344	0.0312	0.0379	0.0329	0.0381	0.0307	0.0371
	$\sigma_\alpha^2 \times 4$	0.0701	0.0716	0.0676	0.0719	0.1256	0.1367	0.1152	0.1272
	$\sigma_\chi^2 \times 4$	0.0270	0.0327	0.0287	0.0328	0.0378	0.0442	0.0360	0.0421

Equation in differences

	σ^2-parameter rescaling	$IV=q$, $(N_\xi, N_\eta, N_\nu) =$ (4,0,0)	(4,1,0)	(4,0,1)	(4,1,1)	$IV=q,y$, $(N_\xi, N_\eta, N_\nu) =$ (4,0,0)	(4,1,0)	(4,0,1)	(4,1,1)
Mean, β_1	Baseline	0.4805	0.4703	0.4746	0.4583	0.4673	0.4702	0.4719	0.4562
	$\sigma_\psi^2 \times 4$	0.5646	0.5569	0.5603	0.5532	0.5573	0.5567	0.5597	0.5528
	$\sigma_\epsilon^2 \times 4$	0.4091	0.3819	0.3996	0.3758	0.3992	0.3860	0.3977	0.3746
	$\sigma_\delta^2 \times 4$	0.3954	0.3794	0.3741	0.3643	0.3726	0.3853	0.3700	0.3602
	$\sigma_\alpha^2 \times 4$	0.4909	0.4770	0.4816	0.4640	0.4905	0.4812	0.4866	0.4735
	$\sigma_\chi^2 \times 4$	0.4945	0.4816	0.4791	0.4717	0.4904	0.4878	0.4836	0.4776
RRMSE, β_1	Baseline	0.2154	0.2393	0.2280	0.2604	0.2342	0.2330	0.2299	0.2594
	$\sigma_\psi^2 \times 4$	0.0750	0.0922	0.0821	0.1004	0.0834	0.0866	0.0807	0.0979
	$\sigma_\epsilon^2 \times 4$	0.3294	0.3785	0.3461	0.3884	0.3434	0.3678	0.3477	0.3876
	$\sigma_\delta^2 \times 4$	0.3632	0.3969	0.3982	0.4215	0.3936	0.3797	0.4028	0.4220
	$\sigma_\alpha^2 \times 4$	0.2013	0.2317	0.2177	0.2520	0.1994	0.2191	0.2076	0.2330
	$\sigma_\chi^2 \times 4$	0.1954	0.2215	0.2207	0.2420	0.1984	0.2064	0.2116	0.2275
Mean, λ	Baseline	0.4803	0.4509	0.4484	0.4175	0.4280	0.4505	0.4401	0.4111
	$\sigma_\psi^2 \times 4$	0.7015	0.6876	0.6877	0.6726	0.6777	0.6856	0.6856	0.6713
	$\sigma_\epsilon^2 \times 4$	0.4093	0.3570	0.3713	0.3301	0.3691	0.3739	0.3633	0.3235
	$\sigma_\delta^2 \times 4$	0.1812	0.1401	0.1061	0.0699	0.1064	0.1605	0.0960	0.0589
	$\sigma_\alpha^2 \times 4$	0.5024	0.4710	0.4625	0.4375	0.5035	0.4871	0.4833	0.4707
	$\sigma_\chi^2 \times 4$	0.5133	0.4951	0.4742	0.4531	0.5017	0.5207	0.4879	0.4772
RRMSE, λ	Baseline	0.4102	0.4523	0.4501	0.4953	0.4725	0.4471	0.4593	0.5001
	$\sigma_\psi^2 \times 4$	0.1322	0.1542	0.1494	0.1732	0.1599	0.1513	0.1510	0.1726
	$\sigma_\epsilon^2 \times 4$	0.4993	0.5714	0.5461	0.6033	0.5467	0.5433	0.5550	0.6091
	$\sigma_\delta^2 \times 4$	0.7838	0.8402	0.8763	0.9259	0.8721	0.8067	0.8879	0.9373
	$\sigma_\alpha^2 \times 4$	0.3831	0.4283	0.4331	0.4698	0.3799	0.4031	0.4054	0.4251
	$\sigma_\chi^2 \times 4$	0.3696	0.3992	0.4183	0.4497	0.3814	0.3598	0.4002	0.4166

Table 2: RRMSE OF COEFFICIENT ESTIMATES

$(N,T,R) = (100,10,1000)$. $(\beta_1,\beta_2,\lambda) = (0.6,0.3,0.8)$. q-IVs ONLY

Sensitivity to memory configuration and increased/reduced σ^2's

	σ^2-parameter rescaling	*Equation in levels* $(N_\xi, N_\eta, N_\nu) =$				*Equation in differences* $(N_\xi, N_\eta, N_\nu) =$			
		(4,0,0)	(4,1,0)	(4,0,1)	(4,1,1)	(4,0,0)	(4,1,0)	(4,0,1)	(4,1,1)
β_1	$\sigma_\psi^2 \times 4$	0.0415	0.0506	0.0415	0.0509	0.0750	0.0922	0.0821	0.1004
	$\sigma_\epsilon^2 \times 4$	0.1723	0.2063	0.1759	0.2111	0.3294	0.3785	0.3461	0.3884
	$\sigma_\delta^2 \times 4$	0.1131	0.1454	0.1268	0.1514	0.3632	0.3969	0.3982	0.4215
	$\sigma_\alpha^2 \times 4$	0.1597	0.1775	0.1498	0.1804	0.2013	0.2317	0.2177	0.2520
	$\sigma_\chi^2 \times 4$	0.0892	0.1097	0.0950	0.1144	0.1954	0.2215	0.2207	0.2420
	Baseline	0.0955	0.1104	0.0986	0.1185	0.2154	0.2393	0.2280	0.2604
	$\sigma_\psi^2 \times \frac{1}{4}$	0.0398	0.0498	0.0441	0.0523	0.0738	0.0891	0.0859	0.0980
	$\sigma_\epsilon^2 \times \frac{1}{4}$	0.0789	0.0948	0.0827	0.0957	0.1787	0.2004	0.1938	0.2144
	$\sigma_\delta^2 \times \frac{1}{4}$	0.0876	0.1127	0.0900	0.1079	0.1445	0.1780	0.1534	0.1788
	$\sigma_\alpha^2 \times \frac{1}{4}$	0.0780	0.0956	0.0797	0.0978	0.2162	0.2429	0.2298	0.2619
	$\sigma_\chi^2 \times \frac{1}{4}$	0.0951	0.1120	0.0959	0.1189	0.2090	0.2400	0.2231	0.2583
λ	$\sigma_\psi^2 \times 4$	0.0120	0.0143	0.0118	0.0146	0.1322	0.1542	0.1494	0.1732
	$\sigma_\epsilon^2 \times 4$	0.0464	0.0552	0.0463	0.0556	0.4993	0.5714	0.5461	0.6033
	$\sigma_\delta^2 \times 4$	0.0284	0.0344	0.0312	0.0379	0.7838	0.8402	0.8763	0.9259
	$\sigma_\alpha^2 \times 4$	0.0701	0.0716	0.0676	0.0719	0.3831	0.4283	0.4331	0.4698
	$\sigma_\chi^2 \times 4$	0.0270	0.0327	0.0287	0.0328	0.3696	0.3992	0.4183	0.4497
	Baseline	0.0285	0.0328	0.0285	0.0334	0.4102	0.4523	0.4501	0.4953
	$\sigma_\psi^2 \times \frac{1}{4}$	0.0116	0.0145	0.0120	0.0146	0.1299	0.1511	0.1509	0.1740
	$\sigma_\epsilon^2 \times \frac{1}{4}$	0.0241	0.0275	0.0242	0.0283	0.3756	0.4186	0.4257	0.4569
	$\sigma_\delta^2 \times \frac{1}{4}$	0.0296	0.0344	0.0287	0.0338	0.2375	0.2869	0.2529	0.2952
	$\sigma_\alpha^2 \times \frac{1}{4}$	0.0195	0.0248	0.0193	0.0251	0.4151	0.4637	0.4524	0.5003
	$\sigma_\chi^2 \times \frac{1}{4}$	0.0287	0.0330	0.0280	0.0337	0.4012	0.4480	0.4432	0.4925

Table 3: RELATIVE BIAS (RB) OF COEFFICIENT ESTIMATES

$(N,T,R) = (100,10,1000)$. $(\beta_1,\beta_2,\lambda) = (0.6,0.3,0.8)$. q-IVs ONLY

Sensitivity to memory and σ^2 configuration

	σ^2-parameter rescaling	*Equation in levels* $(N_\xi, N_\eta, N_\nu) =$				*Equation in differences* $(N_\xi, N_\eta, N_\nu) =$			
		(4,0,0)	(4,1,0)	(4,0,1)	(4,1,1)	(4,0,0)	(4,1,0)	(4,0,1)	(4,1,1)
β_1	Baseline	-0.0608	-0.0635	-0.0572	-0.0694	-0.1992	-0.2162	-0.2090	-0.2362
	$\sigma_\psi^2 \times 4$	-0.0159	-0.0202	-0.0144	-0.0166	-0.0589	-0.0719	-0.0662	-0.0781
	$\sigma_\epsilon^2 \times 4$	-0.1533	-0.1818	-0.1544	-0.1841	-0.3182	-0.3635	-0.3341	-0.3736
	$\sigma_\delta^2 \times 4$	-0.0378	-0.0419	-0.0344	-0.0367	-0.3410	-0.3676	-0.3765	-0.3929
	$\sigma_\alpha^2 \times 4$	-0.1235	-0.1349	-0.1145	-0.1344	-0.1818	-0.2050	-0.1974	-0.2267
	$\sigma_\chi^2 \times 4$	-0.0502	-0.0622	-0.0540	-0.0625	-0.1758	-0.1974	-0.2015	-0.2138
λ	Baseline	0.0211	0.0225	0.0186	0.0222	-0.3996	-0.4364	-0.4395	-0.4782
	$\sigma_\psi^2 \times 4$	0.0061	0.0065	0.0054	0.0060	-0.1231	-0.1405	-0.1404	-0.1593
	$\sigma_\epsilon^2 \times 4$	0.0411	0.0491	0.0408	0.0484	-0.4884	-0.5537	-0.5359	-0.5874
	$\sigma_\delta^2 \times 4$	0.0060	0.0074	0.0009	0.0032	-0.7736	-0.8249	-0.8674	-0.9126
	$\sigma_\alpha^2 \times 4$	0.0640	0.0642	0.0608	0.0639	-0.3720	-0.4113	-0.4219	-0.4531
	$\sigma_\chi^2 \times 4$	0.0193	0.0224	0.0191	0.0216	-0.3584	-0.3812	-0.4072	-0.4336

Table 4: STANDARD DEVIATION (SD) OF COEFFICIENT ESTIMATES

$(N,T,R) = (100, 10, 1000)$. $(\beta_1, \beta_2, \lambda) = (0.6, 0.3, 0.8)$

Sensitivity to memory and σ^2 configuration

Equation in levels

	σ^2-parameter rescaling	$IV = q$, $(N_\xi, N_\eta, N_\nu) =$ (4,0,0)	(4,1,0)	(4,0,1)	(4,1,1)	$IV = q, y$, $(N_\xi, N_\eta, N_\nu) =$ (4,0,0)	(4,1,0)	(4,0,1)	(4,1,1)
β_1	Baseline	0.0442	0.0542	0.0482	0.0576	0.0413	0.0481	0.0454	0.0542
	$\sigma_\psi^2 \times 4$	0.0230	0.0278	0.0234	0.0289	0.0211	0.0244	0.0224	0.0266
	$\sigma_\epsilon^2 \times 4$	0.0471	0.0585	0.0507	0.0620	0.0431	0.0504	0.0476	0.0581
	$\sigma_\delta^2 \times 4$	0.0639	0.0835	0.0732	0.0882	0.0606	0.0740	0.0692	0.0813
	$\sigma_\alpha^2 \times 4$	0.0608	0.0692	0.0579	0.0722	0.0578	0.0665	0.0582	0.0719
	$\sigma_\chi^2 \times 4$	0.0442	0.0542	0.0469	0.0575	0.0413	0.0471	0.0453	0.0547
λ	Baseline	0.0153	0.0191	0.0173	0.0199	0.0125	0.0155	0.0145	0.0170
	$\sigma_\psi^2 \times 4$	0.0083	0.0102	0.0084	0.0106	0.0070	0.0079	0.0070	0.0087
	$\sigma_\epsilon^2 \times 4$	0.0172	0.0203	0.0175	0.0218	0.0144	0.0161	0.0150	0.0183
	$\sigma_\delta^2 \times 4$	0.0222	0.0269	0.0250	0.0302	0.0184	0.0207	0.0221	0.0261
	$\sigma_\alpha^2 \times 4$	0.0229	0.0255	0.0235	0.0264	0.0165	0.0189	0.0183	0.0196
	$\sigma_\chi^2 \times 4$	0.0151	0.0190	0.0171	0.0198	0.0134	0.0159	0.0149	0.0173

Equation in differences

	σ^2-parameter rescaling	$IV = q$, $(N_\xi, N_\eta, N_\nu) =$ (4,0,0)	(4,1,0)	(4,0,1)	(4,1,1)	$IV = q, y$, $(N_\xi, N_\eta, N_\nu) =$ (4,0,0)	(4,1,0)	(4,0,1)	(4,1,1)
β_1	Baseline	0.0492	0.0615	0.0546	0.0658	0.0460	0.0520	0.0513	0.0597
	$\sigma_\psi^2 \times 4$	0.0278	0.0347	0.0291	0.0378	0.0261	0.0288	0.0268	0.0349
	$\sigma_\epsilon^2 \times 4$	0.0510	0.0633	0.0544	0.0638	0.0464	0.0537	0.0508	0.0574
	$\sigma_\delta^2 \times 4$	0.0752	0.0897	0.0778	0.0916	0.0637	0.0764	0.0744	0.0814
	$\sigma_\alpha^2 \times 4$	0.0518	0.0648	0.0551	0.0660	0.0482	0.0564	0.0516	0.0595
	$\sigma_\chi^2 \times 4$	0.0511	0.0603	0.0539	0.0680	0.0464	0.0525	0.0506	0.0604
λ	Baseline	0.0740	0.0951	0.0777	0.1034	0.0673	0.0758	0.0741	0.0938
	$\sigma_\psi^2 \times 4$	0.0387	0.0510	0.0408	0.0544	0.0374	0.0394	0.0388	0.0500
	$\sigma_\epsilon^2 \times 4$	0.0829	0.1130	0.0838	0.1101	0.0750	0.0859	0.0803	0.1021
	$\sigma_\delta^2 \times 4$	0.1010	0.1278	0.0992	0.1252	0.0750	0.0867	0.0949	0.1139
	$\sigma_\alpha^2 \times 4$	0.0732	0.0957	0.0783	0.0995	0.0669	0.0780	0.0700	0.0850
	$\sigma_\chi^2 \times 4$	0.0722	0.0949	0.0766	0.0953	0.0640	0.0695	0.0714	0.0828

Table 5: Kurtosis and skewness of coefficient estimates

$(N, R) = (100, 1000)$. $(\beta_1, \beta_2, \lambda) = (0.6, 0.3, 0.8)$. q-IVs only

Sensitivity to panel length, memory, and σ^2 configuration

Standard panel length: $T = 10$

	σ^2-parameter rescaling	*Equation in levels* $(N_\xi, N_\eta, N_\nu) =$				*Equation in differences* $(N_\xi, N_\eta, N_\nu) =$			
		(4,0,0)	(4,1,0)	(4,0,1)	(4,1,1)	(4,0,0)	(4,1,0)	(4,0,1)	(4,1,1)
kurt, β_1	Baseline	3.1835	2.8669	2.8542	3.2685	3.3448	3.1029	2.7574	3.1658
	$\sigma_\psi^2 \times 4$	2.9977	2.9013	3.5762	3.0913	3.1076	3.2087	3.2561	4.0214
	$\sigma_\epsilon^2 \times 4$	2.8840	3.1894	3.1524	3.2017	3.1952	3.1178	2.9951	2.9536
	$\sigma_\delta^2 \times 4$	2.9797	3.1841	2.8085	3.1226	3.5469	3.3227	2.9528	3.1865
	$\sigma_\varrho^2 \times 4$	3.2709	3.1134	3.3007	3.0753	2.7283	3.4608	2.9186	3.0294
	$\sigma_\chi^2 \times 4$	3.0304	3.1497	3.0653	3.4181	2.7119	2.9218	2.9709	3.0299
skew, β_1	Baseline	-0.0341	0.0377	-0.0135	-0.0487	-0.0516	-0.2113	-0.0127	0.0407
	$\sigma_\psi^2 \times 4$	0.0515	-0.1358	-0.0249	-0.0539	-0.2217	-0.1770	0.0144	0.1386
	$\sigma_\epsilon^2 \times 4$	-0.0733	0.0445	-0.0802	-0.0366	-0.1320	-0.0125	0.0348	0.0696
	$\sigma_\delta^2 \times 4$	-0.0253	-0.0262	0.0381	0.1403	0.1444	0.0965	0.0025	-0.0382
	$\sigma_\varrho^2 \times 4$	-0.2353	-0.1299	-0.1270	-0.1233	0.0124	-0.0350	-0.0990	-0.1214
	$\sigma_\chi^2 \times 4$	0.1341	-0.0793	-0.0056	-0.1160	-0.0197	-0.0181	-0.1414	0.0886
kurt, λ	Baseline	2.9014	3.1190	2.8380	3.1390	3.1876	3.0982	3.2122	3.0466
	$\sigma_\psi^2 \times 4$	2.9786	3.2700	3.1537	3.1653	3.2414	3.4303	3.0629	3.1123
	$\sigma_\epsilon^2 \times 4$	2.8622	3.1647	3.1514	2.9979	3.4757	3.2467	3.3983	2.9054
	$\sigma_\delta^2 \times 4$	3.2335	3.0213	3.2549	3.0204	2.9272	2.9166	3.0237	2.8448
	$\sigma_\varrho^2 \times 4$	3.0637	3.0071	2.9861	2.9979	3.1275	2.9076	2.9963	3.2060
	$\sigma_\chi^2 \times 4$	3.0371	3.0818	2.9874	2.8678	3.3063	2.9172	3.1245	3.1848
skew, λ	Baseline	0.0420	0.0342	-0.0827	-0.0323	-0.2471	-0.1641	-0.2544	-0.2036
	$\sigma_\psi^2 \times 4$	0.0999	-0.1008	-0.0021	0.0077	-0.1811	-0.3059	-0.1584	-0.3128
	$\sigma_\epsilon^2 \times 4$	0.0070	0.2187	-0.0183	0.1226	-0.3128	-0.1536	-0.2454	0.0190
	$\sigma_\delta^2 \times 4$	-0.1967	-0.0805	0.1040	-0.0833	0.0162	0.0123	-0.1503	-0.0076
	$\sigma_\varrho^2 \times 4$	0.0956	0.1882	0.1248	0.1892	-0.2428	-0.2364	-0.2858	-0.1763
	$\sigma_\chi^2 \times 4$	0.0492	0.1224	0.0202	0.0680	-0.3339	-0.2310	-0.1923	-0.2026

Reduced panel length: $T = 6$

	σ^2-parameter rescaling	*Equation in levels* $(N_\xi, N_\eta, N_\nu) =$				*Equation in differences* $(N_\xi, N_\eta, N_\nu) =$			
		(4,0,0)	(4,1,0)	(4,0,1)	(4,1,1)	(4,0,0)	(4,1,0)	(4,0,1)	(4,1,1)
kurt. β_1	Baseline	3.7312	3.7190	3.2467	3.6232	3.4941	4.0589	3.2504	3.9656
	$\sigma_\psi^2 \times 4$	3.9523	3.7372	3.4363	3.8348	3.5514	4.7611	3.4032	4.5384
	$\sigma_\epsilon^2 \times 4$	3.1838	4.0265	3.0053	3.1345	3.1996	4.2097	3.1345	3.2785
	$\sigma_\delta^2 \times 4$	3.4907	4.4097	3.1126	6.3953	2.9035	3.4072	3.4367	3.5426
	$\sigma_\varrho^2 \times 4$	3.7374	3.9091	3.5000	3.6821	3.4785	3.9006	2.8735	3.5890
	$\sigma_\chi^2 \times 4$	3.4811	4.3997	3.2035	3.8791	3.3902	3.5724	3.1344	3.4237
skew. β_1	Baseline	-0.1499	0.0282	0.0990	0.1518	-0.1199	0.2508	0.0677	-0.0286
	$\sigma_\psi^2 \times 4$	-0.1565	-0.2425	-0.1284	-0.1472	-0.0988	-0.2021	0.0132	0.1336
	$\sigma_\epsilon^2 \times 4$	-0.0120	0.1455	0.0162	0.0082	-0.0083	-0.0810	0.0395	-0.0220
	$\sigma_\delta^2 \times 4$	-0.0179	-0.0607	-0.0069	-0.4165	0.0532	-0.0266	-0.1180	0.0771
	$\sigma_\varrho^2 \times 4$	-0.3426	-0.0494	-0.1291	0.0356	0.1786	-0.2175	-0.0688	0.2020
	$\sigma_\chi^2 \times 4$	-0.1626	0.2361	0.0152	-0.1098	-0.0025	0.0617	-0.1016	-0.2078
kurt. λ	Baseline	3.1216	5.6171	3.1685	4.5750	3.0147	3.8882	3.6882	4.1246
	$\sigma_\psi^2 \times 4$	3.0715	4.7136	2.9242	4.8713	3.1237	5.2533	3.7129	5.8797
	$\sigma_\epsilon^2 \times 4$	3.3842	3.6634	3.1615	3.7897	3.2097	3.6584	3.1045	3.3906
	$\sigma_\delta^2 \times 4$	2.9535	3.4612	3.4024	5.5302	2.9384	3.2606	2.9736	3.6990
	$\sigma_\varrho^2 \times 4$	3.1489	3.6206	3.4042	3.4096	3.3554	3.9122	4.3656	3.7974
	$\sigma_\chi^2 \times 4$	3.6027	4.3810	3.0331	3.3108	3.3210	3.9105	3.4307	3.6814
skew. λ	Baseline	-0.0169	-0.3005	0.0315	0.2617	-0.2397	-0.1732	-0.0749	-0.3289
	$\sigma_\psi^2 \times 4$	-0.0396	0.0836	-0.0305	-0.2444	-0.1270	-0.4579	-0.2551	0.2722
	$\sigma_\epsilon^2 \times 4$	0.0408	0.0593	0.0467	0.0269	-0.2806	-0.2284	-0.3255	-0.1831
	$\sigma_\delta^2 \times 4$	0.0525	-0.0241	0.0144	0.1369	-0.0776	0.0156	-0.1421	-0.0293
	$\sigma_\varrho^2 \times 4$	0.2346	0.1213	0.1843	0.2764	-0.1031	-0.4231	-0.5517	-0.1991
	$\sigma_\chi^2 \times 4$	-0.0565	-0.0176	0.0919	0.1546	-0.2493	-0.3699	-0.3263	-0.2669

Table 6: PESARAN-SMITH R^2, MEAN VALUES

$(N, T, R) = (100, 10, 1000)$. $(\beta_1, \beta_2, \lambda) = (0.6, 0.3, 0.8)$

Equation in levels

σ^2-parameter rescaling	$IV = q$, $(N_\xi, N_\eta, N_\nu) =$ (4,0,0)	(4,1,0)	(4,0,1)	(4,1,1)	$IV = q, y$, $(N_\xi, N_\eta, N_\nu) =$ (4,0,0)	(4,1,0)	(4,0,1)	(4,1,1)
Baseline	0.1003	0.0668	0.0991	0.0665	0.1182	0.0842	0.1101	0.0771
$\sigma_\psi^2 \times 4$	0.0984	0.0667	0.0974	0.0677	0.1182	0.0848	0.1096	0.0789
$\sigma_\epsilon^2 \times 4$	0.0992	0.0655	0.0977	0.0677	0.1174	0.0829	0.1086	0.0784
$\sigma_\delta^2 \times 4$	0.0986	0.0664	0.0974	0.0667	0.1165	0.0844	0.1082	0.0770
$\sigma_\alpha^2 \times 4$	0.0991	0.0678	0.0992	0.0666	0.1188	0.0873	0.1114	0.0788
$\sigma_\chi^2 \times 4$	0.0976	0.0682	0.0966	0.0661	0.1187	0.0879	0.1093	0.0784

Equation in differences

σ^2-parameter rescaling	$IV = q$, $(N_\xi, N_\eta, N_\nu) =$ (4,0,0)	(4,1,0)	(4,0,1)	(4,1,1)	$IV = q, y$, $(N_\xi, N_\eta, N_\nu) =$ (4,0,0)	(4,1,0)	(4,0,1)	(4,1,1)
Baseline	0.0870	0.0564	0.0829	0.0533	0.1000	0.0763	0.0914	0.0622
$\sigma_\psi^2 \times 4$	0.1298	0.0831	0.1282	0.0819	0.1525	0.1176	0.1432	0.0968
$\sigma_\epsilon^2 \times 4$	0.0740	0.0463	0.0688	0.0446	0.0861	0.0632	0.0760	0.0518
$\sigma_\delta^2 \times 4$	0.0493	0.0326	0.0426	0.0298	0.0561	0.0439	0.0472	0.0345
$\sigma_\alpha^2 \times 4$	0.0882	0.0576	0.0834	0.0529	0.1096	0.0807	0.0978	0.0668
$\sigma_\chi^2 \times 4$	0.0913	0.0595	0.0835	0.0554	0.1138	0.0873	0.0969	0.0689

Table 7: ORTHOGONALITY TESTS. DIFFERENT STRENGTH OF AUTOCORRELATION

MEAN OF p-VALUES FOR J-STATISTIC.

$(N, T, R) = (100, 10, 1000)$. q-IVS ONLY

Strong autocorrelation: $(\beta_1, \beta_2, \lambda) = (0.6, 0.3, 0.8)$

σ^2-parameter rescaling	*Equation in levels* $(N_\xi, N_\eta, N_\nu) =$ (4,0,0)	(4,1,0)	(4,0,1)	(4,1,1)	*Equation in differences* $(N_\xi, N_\eta, N_\nu) =$ (4,0,0)	(4,1,0)	(4,0,1)	(4,1,1)
Baseline	0.4087	0.4452	0.4139	0.4555	0.3687	0.3704	0.3623	0.3867
$\sigma_\psi^2 \times 4$	0.4083	0.4527	0.4071	0.4515	0.3829	0.4070	0.3787	0.3945
$\sigma_\epsilon^2 \times 4$	0.4085	0.4453	0.4061	0.4408	0.3794	0.3833	0.3744	0.3761
$\sigma_\delta^2 \times 4$	0.4134	0.4571	0.4096	0.4460	0.3567	0.3567	0.3509	0.3465
$\sigma_\alpha^2 \times 4$	0.4102	0.4278	0.4114	0.4443	0.3716	0.3837	0.3685	0.3755
$\sigma_\chi^2 \times 4$	0.4129	0.4512	0.4088	0.4378	0.3791	0.3790	0.3700	0.3740

Weak autocorrelation: $(\beta_1, \beta_2, \lambda) = (0.6, 0.3, 0.2)$

σ^2-parameter rescaling	*Equation in levels* $(N_\xi, N_\eta, N_\nu) =$ (4,0,0)	(4,1,0)	(4,0,1)	(4,1,1)	*Equation in differences* $(N_\xi, N_\eta, N_\nu) =$ (4,0,0)	(4,1,0)	(4,0,1)	(4,1,1)
Baseline	0.4060	0.4423	0.4116	0.4509	0.3899	0.4183	0.4022	0.4241
$\sigma_\psi^2 \times 4$	0.4104	0.4474	0.4049	0.4357	0.4064	0.4332	0.4049	0.4392
$\sigma_\epsilon^2 \times 4$	0.4042	0.4403	0.4031	0.4432	0.3923	0.4369	0.4007	0.4170
$\sigma_\delta^2 \times 4$	0.4176	0.4477	0.4121	0.4417	0.3904	0.4223	0.3883	0.4194
$\sigma_\alpha^2 \times 4$	0.3924	0.4305	0.3966	0.4321	0.3962	0.4215	0.3988	0.4255
$\sigma_\chi^2 \times 4$	0.4018	0.4467	0.4050	0.4413	0.3971	0.4303	0.3938	0.4276

Table 8: Tests for IV strength. Different strength of autocorrelation
Mean of p-values for F-statistic.
$(N, T, R) = (100, 10, 1000)$. q-IVs only

Strong autocorrelation: $(\beta_1, \beta_2, \lambda) = (0.6, 0.3, 0.8)$

σ^2-parameter rescaling	*Equation in levels* $(N_\xi, N_\eta, N_\nu) =$				*Equation in differences* $(N_\xi, N_\eta, N_\nu) =$			
	(4,0,0)	(4,1,0)	(4,0,1)	(4,1,1)	(4,0,0)	(4,1,0)	(4,0,1)	(4,1,1)
Baseline	0.0000	0.0000	0.0000	0.0001	0.0018	0.0020	0.0012	0.0025
$\sigma_\psi^2 \times 4$	0.0000	0.0001	0.0000	0.0001	0.0048	0.0051	0.0046	0.0043
$\sigma_\epsilon^2 \times 4$	0.0000	0.0001	0.0000	0.0002	0.0010	0.0018	0.0009	0.0020
$\sigma_\delta^2 \times 4$	0.0000	0.0001	0.0000	0.0001	0.0007	0.0017	0.0007	0.0013
$\sigma_\alpha^2 \times 4$	0.0000	0.0001	0.0000	0.0001	0.0010	0.0022	0.0013	0.0017
$\sigma_\chi^2 \times 4$	0.0001	0.0001	0.0000	0.0001	0.0019	0.0025	0.0015	0.0026

Weak autocorrelation: $(\beta_1, \beta_2, \lambda) = (0.6, 0.3, 0.2)$

σ^2-parameter rescaling	*Equation in levels* $(N_\xi, N_\eta, N_\nu) =$				*Equation in differences* $(N_\xi, N_\eta, N_\nu) =$			
	(4,0,0)	(4,1,0)	(4,0,1)	(4,1,1)	(4,0,0)	(4,1,0)	(4,0,1)	(4,1,1)
Baseline	0.1285	0.0002	0.1176	0.0001	0.8472	0.0018	0.8086	0.0016
$\sigma_\psi^2 \times 4$	0.4416	0.0001	0.4321	0.0001	0.9984	0.0025	0.9977	0.0025
$\sigma_\epsilon^2 \times 4$	0.0551	0.0001	0.0482	0.0001	0.6244	0.0018	0.5652	0.0019
$\sigma_\delta^2 \times 4$	0.0525	0.0001	0.0437	0.0002	0.3587	0.0017	0.2544	0.0016
$\sigma_\alpha^2 \times 4$	0.0372	0.0001	0.0381	0.0001	0.8463	0.0022	0.8020	0.0018
$\sigma_\chi^2 \times 4$	0.0852	0.0001	0.0791	0.0001	0.7551	0.0018	0.7044	0.0019

Table 9: Tests for IV strength. Supplementing q-IVs with y-IVs
Mean of p-values for F-statistic.
$(N, T, R) = (100, 10, 1000)$. $(\beta_1, \beta_2, \lambda) = (0.6, 0.3, 0.8)$

σ^2-parameter rescaling	*Equation in levels* $IV = q,\ (N_\xi, N_\eta, N_\nu) =$				*Equation in levels* $IV = q, y,\ (N_\xi, N_\eta, N_\nu) =$			
	(4,0,0)	(4,1,0)	(4,0,1)	(4,1,1)	(4,0,0)	(4,1,0)	(4,0,1)	(4,1,1)
Baseline	0.0000	0.0000	0.0000	0.0001	0.0000	0.0001	0.0000	0.0001
$\sigma_\psi^2 \times 4$	0.0000	0.0001	0.0000	0.0001	0.0001	0.0001	0.0000	0.0002
$\sigma_\epsilon^2 \times 4$	0.0000	0.0001	0.0000	0.0002	0.0000	0.0000	0.0000	0.0002
$\sigma_\delta^2 \times 4$	0.0000	0.0001	0.0000	0.0001	0.0000	0.0001	0.0000	0.0001
$\sigma_\alpha^2 \times 4$	0.0000	0.0001	0.0000	0.0001	0.0005	0.0017	0.0003	0.0009
$\sigma_\chi^2 \times 4$	0.0001	0.0001	0.0000	0.0001	0.0002	0.0001	0.0000	0.0001

σ^2-parameter rescaling	*Equation in differences* $IV = q,\ (N_\xi, N_\eta, N_\nu) =$				*Equation in differences* $IV = q, y,\ (N_\xi, N_\eta, N_\nu) =$			
	(4,0,0)	(4,1,0)	(4,0,1)	(4,1,1)	(4,0,0)	(4,1,0)	(4,0,1)	(4,1,1)
Baseline	0.0018	0.0020	0.0012	0.0025	0.0022	0.0034	0.0009	0.0014
$\sigma_\psi^2 \times 4$	0.0048	0.0051	0.0046	0.0043	0.0044	0.0096	0.0041	0.0028
$\sigma_\epsilon^2 \times 4$	0.0010	0.0018	0.0009	0.0020	0.0013	0.0026	0.0006	0.0009
$\sigma_\delta^2 \times 4$	0.0007	0.0017	0.0007	0.0013	0.0128	0.0147	0.0005	0.0007
$\sigma_\alpha^2 \times 4$	0.0010	0.0022	0.0013	0.0017	0.0017	0.0040	0.0017	0.0017
$\sigma_\chi^2 \times 4$	0.0019	0.0025	0.0015	0.0026	0.0056	0.0066	0.0018	0.0023

Appendix Tables

Table A.1: Mean of simulated estimates. Detailed results

$(N, T, R) = (100, 10, 1000)$. $(\beta_1, \beta_2, \lambda) = (0.6, 0.3, 0.8)$

Sensitivity to memory and σ^2 configuration

Equation in levels

	σ^2-parameter	$IV = q$, $(N_\xi, N_\eta, N_\nu) =$				$IV = q, y$, $(N_\xi, N_\eta, N_\nu) =$			
	rescaling	(4,0,0)	(4,1,0)	(4,0,1)	(4,1,1)	(4,0,0)	(4,1,0)	(4,0,1)	(4,1,1)
β_1	Baseline	0.5635	0.5619	0.5657	0.5584	0.5521	0.5477	0.5570	0.5478
	$\sigma_\psi^2 \times 4$	0.5905	0.5879	0.5914	0.5901	0.5853	0.5823	0.5866	0.5852
	$\sigma_\epsilon^2 \times 4$	0.5080	0.4909	0.5074	0.4895	0.4999	0.4830	0.5016	0.4830
	$\sigma_\delta^2 \times 4$	0.5773	0.5749	0.5794	0.5780	0.5647	0.5612	0.5708	0.5682
	$\sigma_\alpha^2 \times 4$	0.5259	0.5191	0.5313	0.5194	0.4834	0.4672	0.4928	0.4719
	$\sigma_\chi^2 \times 4$	0.5699	0.5627	0.5676	0.5625	0.5725	0.5670	0.5708	0.5666
	$\sigma_\psi^2 \times \frac{1}{4}$	0.5915	0.5890	0.5897	0.5897	0.5862	0.5826	0.5850	0.5848
	$\sigma_\epsilon^2 \times \frac{1}{4}$	0.5821	0.5795	0.5829	0.5851	0.5682	0.5655	0.5723	0.5723
	$\sigma_\delta^2 \times \frac{1}{4}$	0.5608	0.5522	0.5608	0.5561	0.5489	0.5393	0.5504	0.5444
	$\sigma_\alpha^2 \times \frac{1}{4}$	0.5749	0.5710	0.5773	0.5724	0.5728	0.5685	0.5758	0.5694
	$\sigma_\chi^2 \times \frac{1}{4}$	0.5640	0.5588	0.5664	0.5571	0.5464	0.5398	0.5538	0.5416
β_2	Baseline	0.2683	0.2654	0.2730	0.2685	0.2302	0.2212	0.2435	0.2329
	$\sigma_\psi^2 \times 4$	0.2909	0.2908	0.2915	0.2912	0.2795	0.2774	0.2815	0.2800
	$\sigma_\epsilon^2 \times 4$	0.2474	0.2372	0.2486	0.2396	0.2231	0.2121	0.2303	0.2196
	$\sigma_\delta^2 \times 4$	0.2960	0.2939	0.3074	0.3024	0.2619	0.2529	0.2826	0.2737
	$\sigma_\alpha^2 \times 4$	0.1856	0.1885	0.1914	0.1905	0.0667	0.0499	0.0886	0.0714
	$\sigma_\chi^2 \times 4$	0.2692	0.2652	0.2709	0.2676	0.2348	0.2242	0.2424	0.2324
	$\sigma_\psi^2 \times \frac{1}{4}$	0.2910	0.2912	0.2933	0.2908	0.2795	0.2782	0.2830	0.2799
	$\sigma_\epsilon^2 \times \frac{1}{4}$	0.2744	0.2741	0.2760	0.2724	0.2338	0.2249	0.2447	0.2330
	$\sigma_\delta^2 \times \frac{1}{4}$	0.2612	0.2570	0.2639	0.2592	0.2224	0.2145	0.2318	0.2222
	$\sigma_\alpha^2 \times \frac{1}{4}$	0.2919	0.2898	0.2940	0.2902	0.2834	0.2787	0.2872	0.2818
	$\sigma_\chi^2 \times \frac{1}{4}$	0.2686	0.2677	0.2711	0.2672	0.2298	0.2242	0.2415	0.2325
λ	Baseline	0.8169	0.8180	0.8149	0.8178	0.8319	0.8355	0.8265	0.8318
	$\sigma_\psi^2 \times 4$	0.8048	0.8052	0.8043	0.8048	0.8100	0.8111	0.8088	0.8097
	$\sigma_\epsilon^2 \times 4$	0.8329	0.8393	0.8327	0.8387	0.8427	0.8492	0.8400	0.8467
	$\sigma_\delta^2 \times 4$	0.8048	0.8059	0.8008	0.8026	0.8188	0.8224	0.8107	0.8141
	$\sigma_\alpha^2 \times 4$	0.8512	0.8513	0.8487	0.8511	0.8991	0.9077	0.8903	0.8999
	$\sigma_\chi^2 \times 4$	0.8155	0.8180	0.8153	0.8173	0.8271	0.8316	0.8247	0.8289
	$\sigma_\psi^2 \times \frac{1}{4}$	0.8045	0.8047	0.8042	0.8050	0.8097	0.8107	0.8088	0.8098
	$\sigma_\epsilon^2 \times \frac{1}{4}$	0.8117	0.8123	0.8110	0.8115	0.8280	0.8316	0.8235	0.8272
	$\sigma_\delta^2 \times \frac{1}{4}$	0.8196	0.8224	0.8188	0.8212	0.8350	0.8392	0.8316	0.8358
	$\sigma_\alpha^2 \times \frac{1}{4}$	0.8069	0.8083	0.8058	0.8078	0.8102	0.8125	0.8083	0.8112
	$\sigma_\chi^2 \times \frac{1}{4}$	0.8163	0.8177	0.8154	0.8182	0.8326	0.8357	0.8276	0.8326

Table A.1: (cont.) Mean of simulated estimates. Detailed results
$(N, T, R) = (100, 10, 1000)$. $(\beta_1, \beta_2, \lambda) = (0.6, 0.3, 0.8)$
Sensitivity to memory and σ^2 configuration

Equation in differences

	σ^2-parameter rescaling	$IV = q$, $(N_\xi, N_\eta, N_\nu) =$ (4,0,0)	(4,1,0)	(4,0,1)	(4,1,1)	$IV = q, y$, $(N_\xi, N_\eta, N_\nu) =$ (4,0,0)	(4,1,0)	(4,0,1)	(4,1,1)
β_1	Baseline	0.4805	0.4703	0.4746	0.4583	0.4673	0.4702	0.4719	0.4562
	$\sigma^2_\psi \times 4$	0.5646	0.5569	0.5603	0.5532	0.5573	0.5567	0.5597	0.5528
	$\sigma^2_\epsilon \times 4$	0.4091	0.3819	0.3996	0.3758	0.3992	0.3860	0.3977	0.3746
	$\sigma^2_\delta \times 4$	0.3954	0.3794	0.3741	0.3643	0.3726	0.3853	0.3700	0.3602
	$\sigma^2_\alpha \times 4$	0.4909	0.4770	0.4816	0.4640	0.4905	0.4812	0.4866	0.4735
	$\sigma^2_\chi \times 4$	0.4945	0.4816	0.4791	0.4717	0.4904	0.4878	0.4836	0.4776
	$\sigma^2_\psi \times \frac{1}{4}$	0.5647	0.5593	0.5585	0.5543	0.5577	0.5587	0.5585	0.5541
	$\sigma^2_\epsilon \times \frac{1}{4}$	0.5053	0.4964	0.4956	0.4891	0.4880	0.4949	0.4937	0.4888
	$\sigma^2_\delta \times \frac{1}{4}$	0.5218	0.5051	0.5170	0.5059	0.5184	0.5071	0.5152	0.5047
	$\sigma^2_\alpha \times \frac{1}{4}$	0.4813	0.4672	0.4725	0.4578	0.4646	0.4672	0.4696	0.4546
	$\sigma^2_\chi \times \frac{1}{4}$	0.4850	0.4699	0.4772	0.4592	0.4710	0.4707	0.4771	0.4606
β_2	Baseline	0.2414	0.2336	0.2367	0.2307	0.2327	0.2330	0.2345	0.2304
	$\sigma^2_\psi \times 4$	0.2821	0.2796	0.2795	0.2773	0.2786	0.2802	0.2799	0.2772
	$\sigma^2_\epsilon \times 4$	0.2055	0.1893	0.1990	0.1858	0.2008	0.1910	0.1979	0.1845
	$\sigma^2_\delta \times 4$	0.2018	0.1916	0.1888	0.1827	0.1898	0.1945	0.1878	0.1809
	$\sigma^2_\alpha \times 4$	0.2444	0.2418	0.2395	0.2346	0.2432	0.2442	0.2423	0.2393
	$\sigma^2_\chi \times 4$	0.2465	0.2387	0.2409	0.2338	0.2448	0.2441	0.2423	0.2370
	$\sigma^2_\psi \times \frac{1}{4}$	0.2830	0.2799	0.2801	0.2769	0.2792	0.2793	0.2799	0.2765
	$\sigma^2_\epsilon \times \frac{1}{4}$	0.2555	0.2500	0.2446	0.2441	0.2459	0.2490	0.2438	0.2436
	$\sigma^2_\delta \times \frac{1}{4}$	0.2608	0.2530	0.2595	0.2526	0.2584	0.2541	0.2588	0.2520
	$\sigma^2_\alpha \times \frac{1}{4}$	0.2416	0.2334	0.2355	0.2247	0.2335	0.2337	0.2339	0.2234
	$\sigma^2_\chi \times \frac{1}{4}$	0.2403	0.2352	0.2380	0.2277	0.2333	0.2359	0.2391	0.2287
λ	Baseline	0.4803	0.4509	0.4484	0.4175	0.4280	0.4505	0.4401	0.4111
	$\sigma^2_\psi \times 4$	0.7015	0.6876	0.6877	0.6726	0.6777	0.6856	0.6856	0.6713
	$\sigma^2_\epsilon \times 4$	0.4093	0.3570	0.3713	0.3301	0.3691	0.3739	0.3633	0.3235
	$\sigma^2_\delta \times 4$	0.1812	0.1401	0.1061	0.0699	0.1064	0.1605	0.0960	0.0589
	$\sigma^2_\alpha \times 4$	0.5024	0.4710	0.4625	0.4375	0.5035	0.4871	0.4833	0.4707
	$\sigma^2_\chi \times 4$	0.5133	0.4951	0.4742	0.4531	0.5017	0.5207	0.4879	0.4772
	$\sigma^2_\psi \times \frac{1}{4}$	0.7033	0.6914	0.6859	0.6722	0.6797	0.6893	0.6854	0.6712
	$\sigma^2_\epsilon \times \frac{1}{4}$	0.5079	0.4783	0.4671	0.4483	0.4513	0.4729	0.4613	0.4454
	$\sigma^2_\delta \times \frac{1}{4}$	0.6183	0.5844	0.6060	0.5766	0.6052	0.5927	0.6009	0.5721
	$\sigma^2_\alpha \times \frac{1}{4}$	0.4770	0.4428	0.4468	0.4126	0.4151	0.4488	0.4362	0.4023
	$\sigma^2_\chi \times \frac{1}{4}$	0.4874	0.4555	0.4535	0.4191	0.4388	0.4611	0.4542	0.4224

Table A.2: RRMSE of simulated estimates. Detailed results
$(N, T, R) = (100, 10, 1000)$. $(\beta_1, \beta_2, \lambda) = (0.6, 0.3, 0.8)$
Sensitivity to memory and σ^2 configuration

Equation in levels

	σ^2-parameter rescaling	$IV = q$, $(N_\xi, N_\eta, N_\nu) =$				$IV = q, y$, $(N_\xi, N_\eta, N_\nu) =$			
		(4,0,0)	(4,1,0)	(4,0,1)	(4,1,1)	(4,0,0)	(4,1,0)	(4,0,1)	(4,1,1)
β_1	Baseline	0.0955	0.1104	0.0986	0.1185	0.1054	0.1183	0.1041	0.1254
	$\sigma_\psi^2 \times 4$	0.0415	0.0506	0.0415	0.0509	0.0429	0.0503	0.0436	0.0508
	$\sigma_\epsilon^2 \times 4$	0.1723	0.2063	0.1759	0.2111	0.1816	0.2123	0.1822	0.2178
	$\sigma_\delta^2 \times 4$	0.1131	0.1454	0.1268	0.1514	0.1168	0.1393	0.1252	0.1455
	$\sigma_\alpha^2 \times 4$	0.1597	0.1775	0.1498	0.1804	0.2169	0.2475	0.2033	0.2448
	$\sigma_\chi^2 \times 4$	0.0892	0.1097	0.0950	0.1144	0.0828	0.0958	0.0898	0.1068
	$\sigma_\psi^2 \times \frac{1}{4}$	0.0398	0.0498	0.0441	0.0523	0.0419	0.0501	0.0450	0.0523
	$\sigma_\epsilon^2 \times \frac{1}{4}$	0.0789	0.0948	0.0827	0.0957	0.0860	0.0988	0.0864	0.0993
	$\sigma_\delta^2 \times \frac{1}{4}$	0.0876	0.1127	0.0900	0.1079	0.1012	0.1250	0.1014	0.1190
	$\sigma_\alpha^2 \times \frac{1}{4}$	0.0780	0.0956	0.0797	0.0978	0.0760	0.0881	0.0791	0.0946
	$\sigma_\chi^2 \times \frac{1}{4}$	0.0951	0.1120	0.0959	0.1189	0.1128	0.1274	0.1064	0.1316
β_2	Baseline	0.1746	0.2064	0.1822	0.2081	0.2603	0.2999	0.2342	0.2743
	$\sigma_\psi^2 \times 4$	0.0806	0.0953	0.0800	0.0992	0.0947	0.1065	0.0910	0.1044
	$\sigma_\epsilon^2 \times 4$	0.2339	0.2812	0.2329	0.2809	0.2879	0.3296	0.2709	0.3145
	$\sigma_\delta^2 \times 4$	0.2060	0.2421	0.2280	0.2821	0.2150	0.2506	0.2113	0.2586
	$\sigma_\alpha^2 \times 4$	0.4293	0.4342	0.4191	0.4370	0.7926	0.8529	0.7261	0.7859
	$\sigma_\chi^2 \times 4$	0.1754	0.2074	0.1842	0.2093	0.2508	0.2903	0.2348	0.2734
	$\sigma_\psi^2 \times \frac{1}{4}$	0.0781	0.0965	0.0783	0.1000	0.0929	0.1054	0.0877	0.1070
	$\sigma_\epsilon^2 \times \frac{1}{4}$	0.1637	0.1869	0.1680	0.2024	0.2514	0.2859	0.2271	0.2706
	$\sigma_\delta^2 \times \frac{1}{4}$	0.1790	0.2030	0.1715	0.2055	0.2809	0.3115	0.2523	0.2920
	$\sigma_\alpha^2 \times \frac{1}{4}$	0.1315	0.1623	0.1327	0.1682	0.1196	0.1450	0.1219	0.1543
	$\sigma_\chi^2 \times \frac{1}{4}$	0.1821	0.2083	0.1746	0.2121	0.2640	0.2921	0.2327	0.2756
λ	Baseline	0.0285	0.0328	0.0285	0.0334	0.0429	0.0484	0.0377	0.0451
	$\sigma_\psi^2 \times 4$	0.0120	0.0143	0.0118	0.0146	0.0152	0.0170	0.0141	0.0163
	$\sigma_\epsilon^2 \times 4$	0.0464	0.0552	0.0463	0.0556	0.0563	0.0647	0.0534	0.0627
	$\sigma_\delta^2 \times 4$	0.0284	0.0344	0.0312	0.0379	0.0329	0.0381	0.0307	0.0371
	$\sigma_\alpha^2 \times 4$	0.0701	0.0716	0.0676	0.0719	0.1256	0.1367	0.1152	0.1272
	$\sigma_\chi^2 \times 4$	0.0270	0.0327	0.0287	0.0328	0.0378	0.0442	0.0360	0.0421
	$\sigma_\psi^2 \times \frac{1}{4}$	0.0116	0.0145	0.0120	0.0146	0.0148	0.0168	0.0142	0.0166
	$\sigma_\epsilon^2 \times \frac{1}{4}$	0.0241	0.0275	0.0242	0.0283	0.0384	0.0435	0.0342	0.0396
	$\sigma_\delta^2 \times \frac{1}{4}$	0.0296	0.0344	0.0287	0.0338	0.0461	0.0520	0.0420	0.0482
	$\sigma_\alpha^2 \times \frac{1}{4}$	0.0195	0.0248	0.0193	0.0251	0.0191	0.0232	0.0188	0.0239
	$\sigma_\chi^2 \times \frac{1}{4}$	0.0287	0.0330	0.0280	0.0337	0.0438	0.0486	0.0385	0.0459

Table A.2: (cont.) RRMSE of simulated estimates. Detailed results

$(N, T, R) = (100, 10, 1000)$. $(\beta_1, \beta_2, \lambda) = (0.6, 0.3, 0.8)$

Sensitivity to memory and σ^2 configuration

Equation in differences

	σ^2-parameter	$IV = q$, $(N_\xi, N_\eta, N_\nu) =$				$IV = q, y$, $(N_\xi, N_\eta, N_\nu) =$			
	rescaling	(4,0,0)	(4,1,0)	(4,0,1)	(4,1,1)	(4,0,0)	(4,1,0)	(4,0,1)	(4,1,1)
β_1	Baseline	0.2154	0.2393	0.2280	0.2604	0.2342	0.2330	0.2299	0.2594
	$\sigma^2_\psi \times 4$	0.0750	0.0922	0.0821	0.1004	0.0834	0.0866	0.0807	0.0979
	$\sigma^2_\epsilon \times 4$	0.3294	0.3785	0.3461	0.3884	0.3434	0.3678	0.3477	0.3876
	$\sigma^2_\delta \times 4$	0.3632	0.3969	0.3982	0.4215	0.3936	0.3797	0.4028	0.4220
	$\sigma^2_\alpha \times 4$	0.2013	0.2317	0.2177	0.2520	0.1994	0.2191	0.2076	0.2330
	$\sigma^2_\chi \times 4$	0.1954	0.2215	0.2207	0.2420	0.1984	0.2064	0.2116	0.2275
	$\sigma^2_\psi \times \frac{1}{4}$	0.0738	0.0891	0.0859	0.0980	0.0815	0.0841	0.0837	0.0959
	$\sigma^2_\epsilon \times \frac{1}{4}$	0.1787	0.2004	0.1938	0.2144	0.2017	0.1953	0.1941	0.2100
	$\sigma^2_\delta \times \frac{1}{4}$	0.1445	0.1780	0.1534	0.1788	0.1477	0.1698	0.1547	0.1761
	$\sigma^2_\alpha \times \frac{1}{4}$	0.2162	0.2429	0.2298	0.2619	0.2389	0.2377	0.2322	0.2616
	$\sigma^2_\chi \times \frac{1}{4}$	0.2090	0.2400	0.2231	0.2583	0.2283	0.2320	0.2216	0.2525
β_2	Baseline	0.2534	0.2926	0.2725	0.3054	0.2666	0.2753	0.2704	0.2954
	$\sigma^2_\psi \times 4$	0.1070	0.1322	0.1145	0.1372	0.1093	0.1184	0.1086	0.1275
	$\sigma^2_\epsilon \times 4$	0.3544	0.4162	0.3765	0.4326	0.3614	0.3992	0.3729	0.4281
	$\sigma^2_\delta \times 4$	0.4048	0.4471	0.4439	0.4898	0.4215	0.4194	0.4370	0.4734
	$\sigma^2_\alpha \times 4$	0.2428	0.2761	0.2655	0.2986	0.2365	0.2519	0.2515	0.2739
	$\sigma^2_\chi \times 4$	0.2408	0.2788	0.2593	0.2954	0.2350	0.2519	0.2490	0.2788
	$\sigma^2_\psi \times \frac{1}{4}$	0.1026	0.1281	0.1134	0.1405	0.1067	0.1153	0.1084	0.1329
	$\sigma^2_\epsilon \times \frac{1}{4}$	0.2147	0.2497	0.2481	0.2764	0.2281	0.2324	0.2423	0.2636
	$\sigma^2_\delta \times \frac{1}{4}$	0.1784	0.2130	0.1813	0.2204	0.1785	0.1968	0.1790	0.2115
	$\sigma^2_\alpha \times \frac{1}{4}$	0.2523	0.2938	0.2703	0.3227	0.2640	0.2780	0.2678	0.3134
	$\sigma^2_\chi \times \frac{1}{4}$	0.2576	0.2919	0.2662	0.3181	0.2649	0.2711	0.2556	0.3006
λ	Baseline	0.4102	0.4523	0.4501	0.4953	0.4725	0.4471	0.4593	0.5001
	$\sigma^2_\psi \times 4$	0.1322	0.1542	0.1494	0.1732	0.1599	0.1513	0.1510	0.1726
	$\sigma^2_\epsilon \times 4$	0.4993	0.5714	0.5461	0.6033	0.5467	0.5433	0.5550	0.6091
	$\sigma^2_\delta \times 4$	0.7838	0.8402	0.8763	0.9259	0.8721	0.8067	0.8879	0.9373
	$\sigma^2_\alpha \times 4$	0.3831	0.4283	0.4331	0.4698	0.3799	0.4031	0.4054	0.4251
	$\sigma^2_\chi \times 4$	0.3696	0.3992	0.4183	0.4497	0.3814	0.3598	0.4002	0.4166
	$\sigma^2_\psi \times \frac{1}{4}$	0.1299	0.1511	0.1509	0.1740	0.1570	0.1473	0.1508	0.1738
	$\sigma^2_\epsilon \times \frac{1}{4}$	0.3756	0.4186	0.4257	0.4569	0.4435	0.4187	0.4321	0.4582
	$\sigma^2_\delta \times \frac{1}{4}$	0.2375	0.2869	0.2529	0.2952	0.2527	0.2711	0.2583	0.2985
	$\sigma^2_\alpha \times \frac{1}{4}$	0.4151	0.4637	0.4524	0.5003	0.4880	0.4482	0.4642	0.5104
	$\sigma^2_\chi \times \frac{1}{4}$	0.4012	0.4480	0.4432	0.4925	0.4590	0.4336	0.4412	0.4865

Table A.3: MEAN OF SIMULATED ESTIMATES. WEAK AUTOCORRELATION. DETAILED RESULTS
$(N, T, R) = (100, 10, 1000)$. $(\beta_1, \beta_2, \lambda) = (0.6, 0.3, 0.2)$
Sensitivity to memory and σ^2 configuration

Equation in levels

	σ^2-parameter rescaling	$IV = q$, $(N_\xi, N_\eta, N_\nu) =$ (4,0,0)	(4,1,0)	(4,0,1)	(4,1,1)	$IV = q, y$, $(N_\xi, N_\eta, N_\nu) =$ (4,0,0)	(4,1,0)	(4,0,1)	(4,1,1)
β_1	Baseline	0.5742	0.5646	0.5740	0.5680	0.5742	0.5642	0.5745	0.5677
	$\sigma_\psi^2 \times 4$	0.5944	0.5926	0.5928	0.5902	0.5943	0.5923	0.5929	0.5907
	$\sigma_\epsilon^2 \times 4$	0.5139	0.4919	0.5123	0.4928	0.5145	0.4913	0.5126	0.4934
	$\sigma_\delta^2 \times 4$	0.5776	0.5666	0.5731	0.5652	0.5780	0.5673	0.5729	0.5653
	$\sigma_\alpha^2 \times 4$	0.5710	0.5598	0.5711	0.5564	0.5711	0.5583	0.5713	0.5575
	$\sigma_\chi^2 \times 4$	0.5788	0.5661	0.5768	0.5667	0.5784	0.5658	0.5771	0.5658
β_2	Baseline	0.2940	0.2784	0.2944	0.2797	0.2926	0.2782	0.2936	0.2789
	$\sigma_\psi^2 \times 4$	0.2987	0.2941	0.2988	0.2940	0.2984	0.2935	0.2986	0.2940
	$\sigma_\epsilon^2 \times 4$	0.2938	0.2704	0.2942	0.2735	0.2921	0.2701	0.2931	0.2727
	$\sigma_\delta^2 \times 4$	0.2985	0.2920	0.3002	0.2956	0.2979	0.2911	0.2998	0.2953
	$\sigma_\alpha^2 \times 4$	0.2743	0.2267	0.2742	0.2286	0.2696	0.2238	0.2715	0.2263
	$\sigma_\chi^2 \times 4$	0.2941	0.2794	0.2938	0.2806	0.2928	0.2785	0.2930	0.2798
λ	Baseline	0.2181	0.2557	0.2171	0.2536	0.2213	0.2568	0.2189	0.2549
	$\sigma_\psi^2 \times 4$	0.2041	0.2154	0.2045	0.2153	0.2049	0.2170	0.2050	0.2155
	$\sigma_\epsilon^2 \times 4$	0.2258	0.2833	0.2246	0.2766	0.2300	0.2840	0.2274	0.2788
	$\sigma_\delta^2 \times 4$	0.2058	0.2231	0.2039	0.2134	0.2074	0.2257	0.2046	0.2134
	$\sigma_\alpha^2 \times 4$	0.2646	0.3825	0.2657	0.3795	0.2755	0.3899	0.2725	0.3842
	$\sigma_\chi^2 \times 4$	0.2166	0.2539	0.2170	0.2502	0.2198	0.2561	0.2187	0.2525

Equation in differences

	σ^2-parameter rescaling	$IV = q$, $(N_\xi, N_\eta, N_\nu) =$ (4,0,0)	(4,10)	(4,0,1)	(4,1,1)	$IV = q, y$, $(N_\xi, N_\eta, N_\nu) =$ (4,0,0)	(4,1,0)	(4,0,1)	(4,1,1)
β_1	Baseline	0.5611	0.5192	0.5582	0.5182	0.5569	0.5510	0.5574	0.5177
	$\sigma_\psi^2 \times 4$	0.5903	0.5772	0.5890	0.5750	0.5884	0.5875	0.5885	0.5749
	$\sigma_\epsilon^2 \times 4$	0.4907	0.4327	0.4878	0.4331	0.4881	0.4701	0.4859	0.4323
	$\sigma_\delta^2 \times 4$	0.5424	0.4830	0.5346	0.4731	0.5366	0.5327	0.5303	0.4719
	$\sigma_\alpha^2 \times 4$	0.5581	0.5210	0.5568	0.5177	0.5553	0.5497	0.5555	0.5174
	$\sigma_\chi^2 \times 4$	0.5623	0.5208	0.5580	0.5181	0.5576	0.5506	0.5572	0.5160
β_2	Baseline	0.2784	0.2599	0.2821	0.2583	0.2760	0.2763	0.2805	0.2573
	$\sigma_\psi^2 \times 4$	0.2948	0.2881	0.2934	0.2882	0.2937	0.2930	0.2936	0.2879
	$\sigma_\epsilon^2 \times 4$	0.2443	0.2198	0.2425	0.2126	0.2438	0.2371	0.2407	0.2132
	$\sigma_\delta^2 \times 4$	0.2674	0.2411	0.2714	0.2373	0.2645	0.2641	0.2681	0.2357
	$\sigma_\alpha^2 \times 4$	0.2804	0.2600	0.2780	0.2570	0.2778	0.2737	0.2776	0.2569
	$\sigma_\chi^2 \times 4$	0.2793	0.2585	0.2778	0.2565	0.2768	0.2757	0.2778	0.2572
λ	Baseline	0.1483	0.0321	0.1444	0.0235	0.1350	0.1442	0.1388	0.0201
	$\sigma_\psi^2 \times 4$	0.1875	0.1496	0.1865	0.1454	0.1821	0.1842	0.1848	0.1440
	$\sigma_\epsilon^2 \times 4$	0.1248	-0.0316	0.1189	-0.0404	0.1163	0.1249	0.1103	-0.0439
	$\sigma_\delta^2 \times 4$	0.0854	-0.0996	0.0692	-0.1282	0.0619	0.0789	0.0573	-0.1329
	$\sigma_\alpha^2 \times 4$	0.1475	0.0362	0.1422	0.0222	0.1341	0.1404	0.1364	0.0204
	$\sigma_\chi^2 \times 4$	0.1466	0.0298	0.1431	0.0199	0.1329	0.1410	0.1370	0.0167

Table A.4: RRMSE OF SIMULATED ESTIMATES. WEAK AUTOCORRELATION. DETAILED RESULTS
$(N, T, R) = (100, 10, 1000)$. $(\beta_1, \beta_2, \lambda) = (0.6, 0.3, 0.2)$
Sensitivity to memory and σ^2 configuration

Equation in levels

	σ^2-parameter rescaling	$IV = q$, $(N_\xi, N_\eta, N_\nu) =$ (4,0,0)	(4,1,0)	(4,0,1)	(4,1,1)	$IV = q, y$, $(N_\xi, N_\eta, N_\nu) =$ (4,0,0)	(4,1,0)	(4,0,1)	(4,1,1)
β_1	Baseline	0.0824	0.1044	0.0870	0.1022	0.0774	0.0967	0.0819	0.0973
	$\sigma_\psi^2 \times 4$	0.0368	0.0475	0.0409	0.0476	0.0342	0.0425	0.0384	0.0441
	$\sigma_\epsilon^2 \times 4$	0.1629	0.2038	0.1669	0.2025	0.1595	0.1994	0.1646	0.1987
	$\sigma_\delta^2 \times 4$	0.1030	0.1295	0.1116	0.1354	0.0965	0.1179	0.1069	0.1276
	$\sigma_\alpha^2 \times 4$	0.1094	0.1346	0.1041	0.1317	0.1005	0.1223	0.0994	0.1256
	$\sigma_\chi^2 \times 4$	0.0801	0.1029	0.0796	0.1003	0.0743	0.0943	0.0756	0.0958
β_2	Baseline	0.0533	0.1142	0.0542	0.1152	0.0519	0.1058	0.0531	0.1132
	$\sigma_\psi^2 \times 4$	0.0351	0.0564	0.0373	0.0563	0.0317	0.0513	0.0352	0.0522
	$\sigma_\epsilon^2 \times 4$	0.0578	0.1414	0.0619	0.1338	0.0573	0.1318	0.0616	0.1302
	$\sigma_\delta^2 \times 4$	0.0623	0.1135	0.0629	0.1151	0.0583	0.1043	0.0617	0.1076
	$\sigma_\alpha^2 \times 4$	0.1138	0.2733	0.1166	0.2683	0.1224	0.2759	0.1208	0.2705
	$\sigma_\chi^2 \times 4$	0.0537	0.1096	0.0564	0.1103	0.0526	0.1047	0.0557	0.1058
λ	Baseline	0.1595	0.4091	0.1627	0.4038	0.1657	0.3857	0.1653	0.4004
	$\sigma_\psi^2 \times 4$	0.0678	0.1733	0.0683	0.1762	0.0677	0.1638	0.0686	0.1655
	$\sigma_\epsilon^2 \times 4$	0.2009	0.5373	0.2022	0.5124	0.2111	0.5117	0.2093	0.5076
	$\sigma_\delta^2 \times 4$	0.1789	0.3889	0.1840	0.3895	0.1759	0.3574	0.1820	0.3677
	$\sigma_\alpha^2 \times 4$	0.3772	0.9970	0.3797	0.9848	0.4211	1.0134	0.4072	0.9930
	$\sigma_\chi^2 \times 4$	0.1546	0.3942	0.1591	0.3880	0.1618	0.3815	0.1632	0.3789

Equation in differences

	σ^2-parameter rescaling	$IV = q$, $(N_\xi, N_\eta, N_\nu) =$ (4,0,0)	(4,1,0)	(4,0,1)	(4,1,1)	$IV = q, y$, $(N_\xi, N_\eta, N_\nu) =$ (4,0,0)	(4,1,0)	(4,0,1)	(4,1,1)
β_1	Baseline	0.0972	0.1604	0.1034	0.1640	0.0975	0.1114	0.1012	0.1599
	$\sigma_\psi^2 \times 4$	0.0398	0.0606	0.0451	0.0640	0.0383	0.0434	0.0422	0.0604
	$\sigma_\epsilon^2 \times 4$	0.1997	0.2958	0.2035	0.2956	0.2005	0.2323	0.2046	0.2933
	$\sigma_\delta^2 \times 4$	0.1438	0.2326	0.1629	0.2546	0.1441	0.1569	0.1612	0.2474
	$\sigma_\alpha^2 \times 4$	0.1016	0.1615	0.1027	0.1636	0.0990	0.1150	0.1011	0.1603
	$\sigma_\chi^2 \times 4$	0.0967	0.1579	0.1025	0.1623	0.0963	0.1106	0.1003	0.1615
β_2	Baseline	0.1618	0.2119	0.1615	0.2264	0.1534	0.1648	0.1543	0.2188
	$\sigma_\psi^2 \times 4$	0.0757	0.1022	0.0790	0.0996	0.0708	0.0844	0.0740	0.0939
	$\sigma_\epsilon^2 \times 4$	0.2451	0.3257	0.2490	0.3446	0.2353	0.2689	0.2461	0.3335
	$\sigma_\delta^2 \times 4$	0.2484	0.3065	0.2513	0.3392	0.2329	0.2490	0.2400	0.3219
	$\sigma_\alpha^2 \times 4$	0.1587	0.2211	0.1683	0.2311	0.1508	0.1754	0.1582	0.2140
	$\sigma_\chi^2 \times 4$	0.1549	0.2177	0.1679	0.2265	0.1469	0.1692	0.1588	0.2139
λ	Baseline	0.3005	0.9147	0.3216	0.9671	0.3546	0.3182	0.3453	0.9700
	$\sigma_\psi^2 \times 4$	0.1006	0.3236	0.1076	0.3459	0.1185	0.1147	0.1127	0.3406
	$\sigma_\epsilon^2 \times 4$	0.4165	1.2382	0.4512	1.2853	0.4476	0.4196	0.4894	1.2857
	$\sigma_\delta^2 \times 4$	0.6202	1.5726	0.7011	1.7142	0.7171	0.6334	0.7552	1.7261
	$\sigma_\alpha^2 \times 4$	0.3055	0.8992	0.3340	0.9702	0.3607	0.3361	0.3590	0.9628
	$\sigma_\chi^2 \times 4$	0.3150	0.9296	0.3339	0.9789	0.3675	0.3332	0.3596	0.9841