MEMORANDUM

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Discrimination and Employment Protection

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Abstract

We study a search model with employment protection legislation. We show that if the output from the match is uncertain ex ante, there may exist a discriminatory equilibrium where workers with the same productive characteristics are subject to different hiring standards. If a bad match takes place, discriminated workers will use longer time to find another job, prolonging the costly period for the firm. This makes it less profitable for the firms to hire the discriminated workers, thus sustaining discrimination. In contrast to standard models, the existence of employers with a taste for discrimination may make it more profitable to discriminate also for firms without discriminatory preferences.

Key words: Discrimination, Employment Protection, Hiring Standards

JEL codes: J70, J60
1 Introduction

Can discrimination of a group of workers persist if there are profit maximizing employers with no "taste for discrimination"? In Becker’s (1957) model, discrimination is explained by prejudices or tastes among some employers. However, the existence of prejudices against one group of workers will lead to lower wages for this group, making them more profitable to hire for other employers, without prejudices. As argued by Arrow (1973), the employers without prejudices will profit from hiring the discriminated group at lower wages, and may ultimately drive the prejudiced employers out of the market.

The key mechanism of Becker’s argument is that discriminated workers become more attractive to hire for other, non-discriminatory firms. We argue that there is a mechanism working in the opposite direction, which in important cases makes discriminated workers less attractive for other firms. There will always be uncertainty as to the productivity of a new worker-firm match, and the worker may turn out to be less productive in the job than was expected in advance. This is a disadvantage for both parties, the firm receiving lower output, and the worker often receiving lower wages. Thus, if a bad match occurs, both parties will want to terminate the relationship so as to find a partner that suits better. However, if the worker is in a discriminated group, it may be difficult for him/her to find another job. If there in addition is employment protection legislation making it costly or difficult for the employer to lay off the worker, the worker may remain in the low productive match for a long time, leading to a loss for the employer. The upshot is that it is less profitable to hire an individual from a discriminated group, precisely because the individual is from this group.

The crucial assumption and source of discrimination in our model is the existence of Employment Protection Legislation (EPL) that is sufficiently strict that it constrains firms’ layoff decisions. Most OECD countries have extensive EPL, see overview in OECD (2004), and a large literature has documented that EPL affects important economic variables, such as unemployment, wages, hiring and firing rates, and investments in human capital (e.g., Addison and Teixeira, 2003; Bentolila and Bertola, 1990; Garibaldi and Violante, 2005; Hopenhayn and Roger-
son, 1993, Saint-Paul; 1996, OECD, 2004). Even in the US, which are among the OECD countries with least strict EPL, there is evidence that it affects employment rates of some groups in the labor market (Autor, Donahue and Schwab, 2006).

There is a considerable literature studying whether EPL aimed at protecting specific groups may have negative effects on the employment level of these groups. Behaghel, Crépon and Sédillot (2008) study the effect of the Delalande tax in France, upon which firms laying off workers aged 50 and above have to pay a tax to the unemployment insurance system. Behaghel et al show that the legislative change in 1992, which stated that the tax does not apply if workers are hired after the age 50, led to a significant increase in the hiring probability of the unemployed aged 50 and above, as compared to unemployed in the late 40s, consistent with the notion that the risk of paying the tax deterred firms from hiring unemployed workers in the late 40s. Several papers study the effect of the enforcement of the Americans with Disabilities Act (ADA), which among other things prohibit discriminatory discharge on the basis of disability, see. e.g. DeLeire (2000) and Acemoglu and Angrist (2001). Acemoglu and Angrist, using data from the Current Population Survey, show that there was a sharp drop in the employment of disabled individuals after the ADA went into effect. However, while our paper shares the idea that possible future firing costs may affect firms’ hiring decisions, a key difference is that we show that this mechanism may explain persistent discriminatory outcomes also for workers with the same productive characteristics as other workers.

Our model constitutes a novel explanation for the existence of persistent discrimination of a group of workers with identical productive characteristics, which applies in the case where it is costly or difficult for employers to lay off workers. We are however not the first to provide explanations for the existence of persistent discrimination of ex-ante identical groups, in the absence of preferences for discrimination. One strand of this literature is based on investment in human capital. The idea is that discrimination against one group of workers reduces their incentive to invest in human capital. This in turn makes the employers’ initial perception of productive differences self-fulfilling. (See e.g., Arrow, 1973; Lundberg and Startz, 1983; Coate and Loury, 1993; Mailath, Samuelson and Shaked, 2000). Another approach puts forward the idea that employers find it easier to assess or mentor
workers from their own group. (Cornell and Welch; 1996, Athey, Avery and Zemsky; 2000). In contrast to these papers, we obtain persistent discrimination of a group that has identical productive characteristics even at the hiring stage, and without any information asymmetries or differences between the groups. As in this paper, Rosén (1997) and Masters (2006) use a search framework and derive equilibrium discrimination as the result of the interaction between the firms’ hiring policies. Yet Rosén (1997) and Masters (2006) build on asymmetric information, while our explanation is based on symmetric information, and the combination of on-the-job search and firing costs is the source of a discriminatory outcome.¹

In the main part of our paper we analyse how discrimination can result even for a group that is identical to others in production sense, and without any inherent tastes or preferences against the discriminated group. Clearly, in this setting there also exists a neutral equilibrium, with no discrimination. Thus, our model does not predict that discrimination is inevitable. However, as there is evidence supporting the existence of taste-based discrimination (e.g. Charles and Guryan, 2008), we also explore the implications of this in our model. We show that if a sufficiently large share of the employers discriminate as a result of discriminatory preferences, it becomes unprofitable to hire the discriminated group, so that also pure profit-maximizing employers, without discriminatory preferences, will practice discriminatory hiring. The upshot is that the neutral equilibrium vanishes, while the discriminatory equilibrium still exists. Note also that if the economy is in a discriminatory equilibrium because a sufficiently large share of the employers have a taste for discrimination, the discriminatory equilibrium will prevail also if preferences change over time so that all employers become pure profit maximisers. To move from the discriminatory equilibrium to the neutral, some form of discrete change or concerted action is needed.

The aim of the paper is to make a theoretical point that discrimination is possible even for workers with identical productive characteristics. Thus, we do not try to accommodate the model to the differences in productive characteristics that describes most groups that are discriminated in the labor market. However, in

¹Lang et al. (2005) show within an urn-ball search model that wages and utility of the discriminated group are substantially lower with an arbitrary small taste for discrimination. Other papers that considers search and taste-based discrimination include Black (1995), Bowlus and Eckstein (2002) and Rosén (2003).
section 7 we nevertheless show that several predictions that can be drawn from the model are consistent with the labor market situation of immigrants in many European countries. Immigrants are much more likely than native-born to have temporary jobs, and they are more likely to exit from temporary help agencies into other sectors of activity, consistent with the idea that firms are reluctant to hire immigrants in permanent jobs without trying them first. In particular, the model may be relevant for Nordic labour markets. In spite of extensive measures to help immigrants entering the labour market, high unemployment rates and overqualification remain important problems for immigrants in the Nordic countries. In the Economic Survey of Sweden 2007, one of the key elements to combat exclusion is "to reduce the risk associated with hiring someone who turns out not to be the right person for the job" (OECD, 2007a), consistent with the key source of discrimination in our model.

Our model is related to Mortensen and Pissarides (1994), who also consider a sequential search model where the productivity of a specific match is uncertain. Furthermore, the model shares features with Saint-Paul (1995), notably the existence of multiple equilibria and a positive relationship between firms’ profit and the likelihood that the worker leaves in case of a bad match. However, neither of these papers mention possible implications for discrimination.

The paper is organized as follows: The model is presented in Section 2, while in Section 3, we consider the existence of a discriminatory equilibrium. Section 4 explores the effects of some employers having discriminatory preferences, alternatively follow a form of affirmative action to combat discrimination. In section 5, we consider the effect of an alternative modelling of the wage setting, where the wage outcome is also affected by outside opportunities. In addition, we report results from numerical simulations of the model. Section 6 explores the robustness of the discriminatory equilibrium. Among other things, we explore the possibility that workers in the discriminated group may "buy a job" by accepting a very low or even negative wage during an initial hiring stage. Section 7 compares the empirical implications to existing empirical literature. Section 8 concludes.
2 The model

We consider a sequential search model of the Diamond-Mortensen-Pissarides type with wages set by bargaining\textsuperscript{2}. There are two types of workers in the economy, \(n_G\) Greens and \(n_R\) Reds, where \(n_G + n_R = 1\). Workers are ex-ante equally productive, and there is no other difference between the types than some observable characteristic that determines the type, e.g. the color of the skin. There is free entry of jobs in the market, and firms may open a vacancy by incurring a cost \(K > 0\). These costs could e.g. be thought of as investment in relevant physical capital, and are once-for-all costs. As will become apparent below, these costs are important for ensuring that a low productivity match involves a loss for the firm, even in the case where the wage is determined by a sharing of the revenue from the match. The flow cost of maintaining a vacancy is \(c \geq 0\). All vacancies are identical. However, when a firm has hired a worker, a random draw determines whether the match is of high or low productivity, with output \(y^H > y^L > 0\), respectively.

There are three key assumptions in the paper. First, we assume that it is costly for firms to lay off a worker who wants to remain in the job. In many countries, such costs come in the form of Employment Protection Legislation (EPL). To simplify the exposition, we consider the extreme case where lay off costs are sufficiently high that firms will never profit from laying off a worker. In the numerical simulations we show that standard assumptions regarding the size of the firing costs may be sufficient for this to be the case.

Second, we assume that only some of the uncertainty about the match-specific productivity is revealed when the employer and worker meet. In the words of Pries and Rogerson (2005), match quality is both an inspection good and an experience good.\textsuperscript{3} Specifically, when a firm is matched with a worker, both parties observe a signal \(\gamma\) that corresponds to the probability that the match is of high productivity. The parameter \(\gamma\) is i.i.d. over matches and to keep the model transparent, may take only two values: \(\overline{\gamma}\) with probability \(\eta\) and \(\gamma < \overline{\gamma}\) with probability \(1 - \eta\).\textsuperscript{4}

\textsuperscript{2}Diamond (1982), Mortensen (1982), and Pissarides (1985).

\textsuperscript{3}The importance of match-specific uncertainty is supported by Nagypal (2007), who present evidence from French firm level data showing that the effect of learning about match quality dominates the effect of learning by doing at tenures above six months.

\textsuperscript{4}In a previous version of the paper, numerical simulations show the existence of a discrimi-
Having observed $\gamma$, the firm decides whether to offer the worker a job, and the worker decides whether to accept the offer. The idea is that when the firm and the worker meet, they will obtain information that makes it possible to assess the likelihood that the match will be of high quality, but they can not foresee perfectly how the worker will perform. There is considerable uncertainty as to both how well the worker fits in with the job requirements, and to how she/he fits in with the colleagues. Furthermore, the productivity of the match may also change over time. For example, the job requirements may change, implying that a previously well qualified worker becomes less productive. To keep the model as simple as possible, we assume that the match productivity is revealed immediately after the worker is hired, for then to be constant over time.\footnote{The same results could derived under the alternative assumption that all matches start out with high productivity, combined with a constant probability rate that productivity falls to a lower level. However, the analysis would be more cumbersome.} Crucially, employment protection is already at work at the time when match productivity is revealed, so that firms cannot fire costlessly upon discovering that productivity is low.\footnote{In many cases, worker turnover is costly to the firm, see e.g. Burdett and Mortensen (1998), which might make discriminated workers more attractive and go against our results. However, when exploring the effects of EPL, it seems reasonable to focus on the possibility of mismatch where firms want a separation to happen.}

Third, we assume that the employer and the employee share the benefits from the match being of high productivity. In the model, this is captured by the assumption that the wage is increasing in the output in the job, which is the standard assumption in search models.

Workers are assumed to leave the market at an exogenous rate $s$, and new workers enter as unemployed at the same rate. Assuming small, but strictly positive search costs, workers in high productivity matches will not search, since all high-productivity matches are equal. However, all unemployed workers will gain from searching, and so will workers in low productivity matches. For simplicity, search intensity is assumed to be exogenous and the same for employed and unemployed workers. The matching between vacancies and workers is random, independent of worker type, and described by a Cobb-Douglas matching function.
\[
X = A(u + \varepsilon(1 - u))^{\lambda} v^{1-\lambda}, \quad 0 < \lambda < 1, \quad (1)
\]

where \(X\) is the number of matches taking place as a function of the vacancy rate \(v\) and the rate of job applicants \(u + \varepsilon(1 - u)\), where \(u\) is the unemployment rate and \(\varepsilon\) is the fraction of employed workers who are in bad matches and thus search. The parameter \(A\) indicates the efficiency or speed of the matching process. The rate at which a vacancy is matched to a job seeker is \(q = X/v\), labour market tightness (the ratio of vacancies to job-seekers) is \(\theta = v/(u + \varepsilon(1 - u))\) and the rate at which a job-seeker is matched to a vacancy is \(\phi = \theta q\).

2.1 Value functions

Workers’ flow payoff is equal to their wage, \(w\), when employed and equal to \(z\) when unemployed. The asset value of an unemployed worker of type \(i\), \(i = G, R\) is:

\[
(r + s)U_i = z + \phi p_i (EW_i(\gamma) - U_i), \quad (2)
\]

where \(r\) is discount rate, \(\phi\) is the matching rate for a worker who search, \(p_i\) is the probability that a worker of type \(i\) is hired, conditional on being matched with a vacancy, \(E\) is the expectation operator, taken over the stochastic variable \(\gamma\), while \(W_i(\gamma)\) is the asset value of worker of type \(i\) who has just been hired, after observing \(\gamma\), but before observing whether the match is of high or low productivity. \(W_i(\gamma)\) is given by

\[
W_i(\gamma) = \gamma W_i^H + (1 - \gamma) W_i^L, \quad (3)
\]

where superscript indicate the productivity level of the match. As workers in a high productivity match do not search, their asset value is given by

\[
(r + s)W_i^H = w_i^H, \quad (4)
\]

where \(w_i^H\) is the wage in a high productive match for a worker of type \(i\). In contrast, a worker in a low productivity match, earning the wage \(w_i^L\), will continue to search. As noted above, we assume that due to EPL, the firm is not allowed, or find it
too costly, to lay off the worker even if the match is of low productivity. The asset value for a low productive worker is hence

\[(r + s)W^L_i = w^L_i + \phi p_i (E_W(\gamma) - W^L_i).\]  \tag{5}

Rewriting (5) gives

\[W^L_i = \frac{w^L_i + \phi p_i E_W(\gamma)}{r + s + \phi p_i}.\]  \tag{6}

Likewise, for the firms, the asset value of a job filled with a worker of type \(i\), in a match of high productivity, denoted \(J^H_i\), is given by

\[rJ^H_i = y^H - w^H_i + s(V - J^H_i),\]

or

\[J^H_i = \frac{y^H - w^H_i + sV}{r + s}.\]  \tag{7}

Correspondingly, the value of a match of low productivity, \(J^L_i\), is

\[rJ^L_i = y^L - w^L_i + (s + \phi p_i)(V - J^L_i),\]

or

\[J^L_i = \frac{y^L - w^L_i + (s + \phi p_i)V}{r + s + \phi p_i}.\]  \tag{8}

The value to a firm of hiring a worker from group \(i\) after having observed \(\gamma\) is \(\gamma J^H_i + (1 - \gamma)J^L_i\). Using (7) and (8), we find the value to the firm of hiring a worker from group \(i\) as a function of the probability that the match is of high productivity, \(\gamma\), and the probability that, in case of a low productivity match, the worker is hired when matched to another firm, \(p_i\):

\[J_i(\gamma, p_i) = \frac{\gamma}{r + s} (y^H - w^H_i + sV) + \frac{1 - \gamma}{r + s + \phi p_i} (y^L - w^L_i + (s + \phi p_i)V).\]  \tag{9}
As $J_i^H > J_i^L$, it follows that $J_i(\gamma, p_i)$ is strictly increasing in $\gamma$. As firms hire a worker if it is profitable, (if $J_i > V$) firms will choose a cut-off rule for each type $\gamma_i^C$, where they hire if $\gamma \geq \gamma_i^C$. Let $\mu(\gamma^C)$ denote the proportion of applicants that has a value $\gamma^C$ or higher. The value of a vacancy given the cut-off hiring rule is then

$$rV(\theta, \gamma_G^C, \gamma_R^C) = -c + q \left( \alpha_G \mu(\gamma_G^C)(E(J_G \mid \gamma \geq \gamma_G^C) - V) + \alpha_R \mu(\gamma_R^C)(E(J_R \mid \gamma \geq \gamma_R^C) - V) \right),$$

(10)

where $\alpha_i$ is the ratio of job seekers of type $i$ out of all job seekers. Rewriting (10) gives us

$$V(\theta, \gamma_G^C, \gamma_R^C) = \frac{-c + q \left( \alpha_G \mu(\gamma_G^C)E(J_G \mid \gamma \geq \gamma_G^C) + \alpha_R \mu(\gamma_R^C)E(J_R \mid \gamma \geq \gamma_R^C) \right)}{r + q (\alpha_G \mu(\gamma_G^C) + \alpha_R \mu(\gamma_R^C))}.$$

(11)

Then consider the wage setting, which takes place immediately after the match productivity is revealed. In the first part of the paper, we take literally the assumption that the EPL is sufficiently strict to prevent any involuntary layoffs, implying that the outside options are not the relevant threat points in a dispute over the wage. In line with Binmore, Rubinstein and Wolinsky (1986), the threats points should then reflect the players’ payoffs during a dispute in the bargaining. We assume that in this case no production takes place while the firm does not pay any wages to the worker; for simplicity the threat points of both players are then set to zero. According to Binmore et al (1986), the bargaining power of the worker, $\beta \in (0, 1)$ should reflect players’ time preferences, and these are assumed to be the same for both groups. Thus, the wage will be the same for both worker types. In a match with high productivity, the wage is

$$w^H = \beta y^H,$$

(12)

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7This follows the outside option principle of Binmore, Shaked and Sutton (1989), by which the outside options work as constraints on the bargaining outcome, and not as threat points. See Hall and Milgrom (2008) for a similar argument. In section 5 below, we consider the case where the wage is also affected by outside opportunities, implying that the discriminated group obtains lower wages in equilibrium.
and in a match of with low productivity it is

\[ w^L = \beta y^L. \] (13)

We want to explore a situation where EPL bites, in the sense that it prevents firms from laying off workers. To ensure this, we assume that firms want to get rid of workers in a low productivity match, i.e. that

\[ (1 - \beta) y^L < r V. \] (14)

The crucial assumption here is that if the match has low productivity, this is costly to the firm and the firm cannot avoid these costs by cutting the wage.

### 2.2 Equilibrium

We consider an equilibrium where all firms use the same hiring strategies; below, we will show that this is indeed the case.

**Definition 1** A steady-state equilibrium of the economy is a list \( \theta, J_i, W_i, U_i, V, w^H, w^L, \alpha_i, \varepsilon_i, u_i \), such that the following conditions hold:

First, free entry ensures that the value of a vacancy is equal to the cost of opening a vacancy:

\[ V = K, \]

with \( V \) given by (11). Second, firms hire a worker of type \( i \) iff it is profitable.

\[ J_i(\gamma, p_i) \geq V \text{ for } \gamma \geq \gamma^C_i \quad J_i(\gamma, p_i) < V \text{ for } \gamma < \gamma^C_i \]

with \( J_i(\gamma, p_i) \) given by (9). Third, workers’ behavior must be optimal. This implies that only unemployed or workers in low productivity matches search, and they accept all job offers. Finally, the steady state conditions (15), (16) and (17) must all be fulfilled.\(^8\)

\(^8\)See derivation in Appendix 1.
In steady state, the share of employed workers of type $i$ in jobs with low match quality, $\varepsilon_i$, is given by

$$\varepsilon_i = \frac{s(1 - E(\gamma | \gamma \geq \gamma_i^c))}{s + E(\gamma | \gamma \geq \gamma_i^c) \phi p_i}. \quad (15)$$

The unemployment rates for both groups are

$$u_i = \frac{s}{s + \phi p_i}. \quad (16)$$

The fraction of all job seekers, both unemployed and employed in low productivity matches, that are of type $i$, $\alpha_i$, is

$$\alpha_i = \frac{n_i(u_i + (1 - u_i)\varepsilon_i)}{n_G(u_G + (1 - u_G)\varepsilon_G) + n_R(u_R + (1 - u_R)\varepsilon_R)}. \quad (17)$$

### 3 Discriminatory equilibrium

In this section we establish the existence of at least one equilibrium in the model. Furthermore, we consider existence of a discriminatory equilibrium, where all firms hire Greens irrespective of the value of the signal $\gamma$, but where Reds are hired only if $\gamma = \overline{\gamma}$.

In a discriminatory equilibrium, all firms use the same hiring strategy where $\gamma_G^c = \underline{\gamma}$, and $\gamma_R^c = \overline{\gamma}$. We then have that $E(J_G | \gamma \geq \underline{\gamma}) = J_G(\gamma^M, 1)$ (where $\gamma^M$ is the expected value of $\gamma$), $E(J_R | \gamma \geq \overline{\gamma}) = J_R(\overline{\gamma}, \eta)$, $\mu(\gamma_G^c) = 1$, $\mu(\gamma_R^c) = \eta$, $p_G = 1$, and $p_R = \eta$. Using (11), we find the value of a vacancy as a function of labour market tightness

$$V(\theta) = \frac{-c + q(\theta) (\alpha_G J_G(\gamma^M, 1) + \alpha_R \eta J_R(\overline{\gamma}, \eta))}{r + q(\theta) (\alpha_G + \alpha_R \eta)}. \quad (18)$$

To rule out an outcome where no firm enters, we assume that the economy is productive, also with discriminatory hiring rules.

**Assumption 1:** $V(\theta) > K$ for $\theta = 0$, given $\gamma_G^c = \underline{\gamma}$, and $\gamma_R^c = \overline{\gamma}$.

Assumption 1 is satisfied as long as
\[ n_G(\gamma M(1-\beta)y^H + (1-\gamma M)(1-\beta)y^L) + n_R\eta(\gamma (1-\beta)y^H + (1-\gamma)(1-\beta)y^L) > r K \]  
\( (19) \)

(See Appendix 2). Note that for any \( \gamma \geq 0 \) there exist values of \( y^H \) such that (19) is satisfied. Assumption 1 says that the value of vacancy is greater than \( K \) when \( \theta = 0 \), given the hiring rules \( \gamma_G = \gamma \) and \( \gamma_R = \gamma \). However, there is no requirement that the hiring rules are optimal at \( \theta = 0 \).

In the typical search model the value of a vacancy is decreasing in \( \theta \), implying that if the economy is productive, then there also exist an equilibrium. In contrast, in our model the value of a vacancy, \( V \), may be increasing in \( \theta \) for some parameter values, as \( J_G \) and \( J_R \) are increasing in the workers’ matching rate \( \phi \). However, the following Lemma establishes that there always exists an interval \( [\theta^0, \theta^1] \) where \( V \) is decreasing in \( \theta \) and \( V < K \) for \( \theta > \theta^1 \).

**Lemma 1** There exists values \( \theta^0 \) and \( \theta^1 \) such that \( \frac{dV}{d\theta} < 0 \) for \( \theta \in (\theta^0, \theta^1] \), \( \frac{dV}{d\theta} \leq 0 \) for \( \theta = \theta^0 \), \( V(\theta^1) = 0 \) and \( V(\theta) < 0 \) for \( \theta > \theta^1 \).

**Proof.** See Appendix 3.

Although it is possible that \( V \) is increasing in \( \theta \) for some parameters, we will only consider equilibria where \( \frac{dV}{d\theta} < 0 \), and the above Lemma establishes that such a region exists.

We now turn to the specific conditions for existence of discriminatory equilibria. In a discriminatory equilibrium a Red worker that is matched to a firm is only hired if \( \gamma = \gamma \), which happens with probability \( p_R = \eta \). This requires that the expected profits for the firm of hiring a Red worker with a low signal \( \gamma = \gamma \) is less or equal to the value of a vacancy \( V \), i.e. that

\[ J(\gamma, \eta) = \frac{\gamma}{r+s}((1-\beta)y^H + sV) + \frac{(1-\gamma)}{(r+s+\phi\eta)}((1-\beta)y^L + (s+\phi\eta)V) < V. \]
\( (20) \)

where we have used (9) and substituted out for the wage equations (12) and (13). Note that the expected profits from hiring a worker only depends on the
probability that the match has high output, $\gamma$, and the probability that the worker is hired when being matched with another firm. Thus, we may omit the subscript indicating worker type.

In addition, the existence of a discriminatory equilibrium requires that it is profitable to hire a Green with $\gamma = \gamma$, given that all other firms always hire Greens, implying that $p_G = 1$. Using (9) and the wage equations (12) and (13), this conditions reads

$$J(\gamma, 1) = \frac{\gamma}{r + s} ((1 - \beta)y^H + sV) + \frac{1 - \gamma}{r + s + \phi} ((1 - \beta)y^L + (s + \phi)V) > V. \quad (21)$$

As $V = K$ in equilibrium, a discriminatory equilibrium thus requires that $J(\gamma, \eta) < K < J(\gamma, 1)$.

Depending on the parameter values, the model may give rise to a several different trivial equilibria. For low enough values of $K$, it is profitable to hire both type of workers irrespective of the signal that is observed. Correspondingly, for $K$ being sufficiently high, firms will never hire workers with a bad signal, irrespective of type. Likewise, if the bad signal is not so bad, i.e. $\gamma$ quite high, it will be profitable to hire a worker with $\gamma = \gamma$ even when $p_R = \eta$, implying that discrimination cannot take place in equilibrium. Conversely, if the bad signal is really bad, i.e. $\gamma$ sufficiently low, it will not be profitable to hire a worker with $\gamma = \gamma$ even if $p_G = 1$.

To explore the possibility of a discriminatory equilibrium, we must consequently consider parameter values which allow firms to hire some but not necessarily all applicants. This requires that $\gamma$ and $K$ take "intermediate" values. Formally, we assume

**Assumption 2:** $\gamma \in [\gamma^0, \gamma^1]$ and $K \in [K^0, K^1]$.

The bounds $\gamma^0$, $\gamma^1$, $K^0$ and $K^1$ are defined in Appendix 4. Under these assumptions, a discriminatory equilibrium will exist

**Proposition 1** Given Assumptions 1 and 2 a discriminatory equilibrium exists.

**Proof.** See Appendix 4.

Thus, with $\gamma$ and $K$ taking intermediate values, there exist an equilibrium where firms hire Green workers irrespective of signal, while Red workers are only
hired when observing a good signal. As explained above, the reason why it is unprofitable to hire Red workers with a bad signal, is precisely because other firms do not hire Red workers with a bad signal. This makes it more difficult for Red workers to find a new job, making them stay longer in a low productive match, thus involving a cost to the firm.

If a discriminatory equilibrium exists, then there also exists a neutral equilibrium where \( \gamma_C^C = \gamma_R^C = \gamma \), i.e. where both type of workers are hired irrespective of the value of the signal \( \gamma \) that is observed.

**Lemma 2** Given Assumptions 1 and 2 a neutral equilibrium exists.

**Proof.** See Appendix 5

To understand the intuition, first observe that Lemma 2 would hold trivially if labor market tightness \( \theta \) were given. Clearly, if it is profitable to always hire a Green worker when all other firms do to same, then the same must be true with a Red workers, as Red and Green workers are identical apart from the type. However, \( \theta \) is not given, so we must take the effect of a change in \( \theta \) into account. In a neutral equilibrium, the probability that a badly matched worker finds another job is higher than in a discriminatory equilibrium. This raises the value of hiring a worker, thus raising the value of a vacancy, and consequently leading to an increase in \( \theta \) in equilibrium. This again leads to an increase \( \phi \) (in the rate at which a searching worker meets a firm), making it even more profitable to hire a worker with a bad signal. From the same argument it also follows directly that the neutral equilibrium Pareto dominates the discriminatory equilibrium.

Note that both equilibria are stable: if one firm deviates from the equilibrium hiring strategy, it will have "very small" impact on the probability that a worker of type \( i \) is hired, conditional on being matched, \( p_i \). Thus, the equilibrium strategy will still be optimal for other firms.
4 Taste for discrimination and affirmative action

In the model so far, there is a multiplicity of equilibria, due to the externality that the optimal hiring rule for profit maximizing firms depends on the hiring rules of other firms. The multiplicity implies that discrimination is not inevitable, as both a neutral and a discriminatory equilibrium exist. Thus, in some sense the model does not explain why there is discrimination, only that discrimination may take place. However, there are also circumstances which rule out one of the alternatives, giving rise to clearer predictions from the model.

One such circumstance is if some firms are not profit maximisers, but rather are influenced by other considerations, like discriminatory preferences or anti-discrimination in the form of affirmative action. Consider first the situation where some firms have a taste for discrimination, consistent with the evidence that some employers do indeed have prejudices against some groups of workers, see e.g. Charles and Guryan (2008). More specifically, assume that a proportion \( m \in (0, 1) \) of the vacancies for exogenous reasons always apply the discriminatory hiring rule: \( \gamma_{CG} = \gamma \) and \( \gamma_{CR} = \overline{\gamma} \). Thus, these firms are willing to hire Red workers if they observe a good signal, but not with a bad. Such behavior could arise from these employers receiving a certain disutility from hiring Red workers, as in Becker’s model. However, for simplicity, we do not include any such disutility explicitly in the model. Furthermore, while the free entry of profit maximising firms ensures that the value of a vacancy for these firms is zero in equilibrium, we have no similar restriction on the value of a vacancy for the discriminatory firms, as their existence is taken as exogenous. The focus is not the behavior of the discriminatory firms, but how the existence of the discriminatory firms affect the behaviour of the profit maximising firms. According to Becker’s argument, the existence of some discriminatory firms makes it profitable for non-discriminatory firms to hire the workers that are discriminated. We shall explore if this is really the case in our model.

If the profit maximising firms apply the neutral hiring rule \( \gamma_{CG} = \gamma_{CR} = \bar{\gamma} \), the expected profits from hiring a Red worker with a bad signal \( \bar{\gamma} \) is (using that \( V = K \)
in equilibrium)

\[ J_R(\gamma, p_R) = \frac{\gamma}{r + s}((1 - \beta)y^H + sK) + (1 - \gamma)\frac{(1 - \beta)y^L + (s + \phi p_R)K}{r + s + \phi p_R}, \]  

(22)

where \( p_R = 1 - m + m\eta \). We can show the following result.

**Proposition 2** There exists a critical value \( \tilde{m} \in (0, 1) \), given by \( J_R(\gamma, p_R) = K \) in (22), such that for \( m > \tilde{m} \) an equilibrium where profit-maximizing firms apply the neutral hiring rule \( \gamma_G^C = \gamma_R^C = \gamma \) does not exist, while the discriminatory equilibrium where they apply \( \gamma_G^C = \gamma, \gamma_R^C = \gamma \) does exist.

**Proof.** See Appendix 6. \[ \square \]

Thus, we observe that the existence of firms with taste for discrimination has the opposite effect of what it has in Becker’s model: neutral (non-discriminatory) behavior becomes less profitable. If there are only a few discriminatory firms, and the other firms have a neutral hiring strategy, the discriminatory firms earn a lower profit. However, if the share of discriminatory firms is above the critical value \( \tilde{m} \), then it is neutral hiring that is less profitable. Thus, in this case it is the non-discriminatory hiring that is driven out of the market, and profit maximising firms without any taste for discrimination will have the same discriminatory hiring rule as the firms with taste for discrimination.\(^9\) As both types of firms apply the same hiring rule, they also make the same profits.

Next we consider how the outcome is affected by affirmative action. In many countries, the authorities undertake various measures to reduce the extent of discrimination. One key measure is affirmative action in the form a hiring quotas, where firms are required to let a certain proportion of their hirings be from specific discriminatory groups, see a discussion of the US experience in Holzer (2007). This measure is controversial, and it is sometimes argued that it may be counterproductive. While hiring quotas work to reduce discrimination within the model of Athey et al (2000), it may have the opposite effect if discrimination is sustained via the discriminated group investing less in human capital due to the discrimination, as suggested by Coate and Loury (1993). The reason is that affirmative

\(^9\)In the numerical simulations below, we show the same qualitative result also when the wage depends on the outside option, implying that Red workers have lower wages than Green.
action may ensure that the discriminated group get jobs even with less investment in human capital, which may reduce their incentives to invest, thus amplifying the underinvestment. It is thus of interest to see how affirmative action works with our explanation of discrimination.

We capture the affirmative action by assuming that a proportion $a$ of vacancies always apply the neutral hiring rule: $\gamma^C_G = \gamma^C_R = \gamma$. One interpretation of this is that the affirmative action only applies to a part of the labor market, but here firms hire according to the share of each group among the applicants. With this specification, the affirmative action case is the mirror image of the case with taste for discrimination above. Thus the next Corollary follows immediately.

**Corollary 1** There exists a critical value $\tilde{a} \in (0,1)$, given by $J_R(\gamma, p_R) = K$ in (22), when $p_R = a + (1 - a)\eta$, such that for $a > \tilde{a}$ an equilibrium where profit-maximizing firms apply the discriminatory hiring rule $\gamma^C_G = \gamma, \gamma^C_R = \gamma$ does not exist, while the neutral equilibrium where they apply the neutral hiring rule $\gamma^C_G = \gamma^C_R = \gamma$ does exist.

Thus, we see that in our model, affirmative action in the form of minimum hiring quotas works to counteract discrimination. The underlying source of discrimination is that the discriminated group have a low job finding probability, and thus are less attractive given the uncertainty in match specific productivity. Affirmative action increases the job finding probability of the discriminated group, making them more attractive to hire. In other words, within this model affirmative action in some firms counteracts discriminatory behavior also in jobs that are not directly affected by the affirmative action rule.

The third point we will consider in this section is the effect of the recent history. Consider a labor market where discrimination has prevailed historically, for reasons of taste and power, and where discrimination was also a part of the legislation. Then assume that the discriminatory legislation is removed, and over time also the taste for discrimination gradually vanishes, in the sense that more and more firms become pure profit maximisers, without any preference for worker types. However, when the first firms loose their taste for discrimination, they still know that other firms have a discriminatory hiring strategy. Thus, it will still be optimal for the profit maximising firms to continue with the discriminatory hiring. Consequently,
the discriminatory equilibrium prevails, in spite of the removal of discriminatory legislation, and even if all firms lose their taste for discrimination. To get out of a discriminatory equilibrium, some sort of concerted action would be required, ensuring that firms hired discriminated workers even if it was not profitable, or ensuring changes so that it becomes profitable to hire discriminated workers. This line of argument is consistent with descriptions of the evolution of racial segregation in the US labor market given by Darity and Mason (1998), who argued that discriminatory practices were sustained long after any legal support was removed.

5 Outside opportunities affect bargaining

So far, we have assumed that the wage only depends on the productivity of the match, and not on the outside alternatives of the workers. We now consider an alternative wage setting mechanism, where the wage is also affected by the players’ outside options. The upshot is that the weaker outside alternatives of the Red workers lead them to have a lower wage. Specifically, we assume that the wage in a high productive match, when \( y = y^H \), maximizes the Nash product of each player’s asset values, i.e.

\[
(W_i^H - U_i)\beta (J^H - V)^{1-\beta}.
\]  

(23)

In line with the non-cooperative interpretation of Binmore et al (1986), (23) is the appropriate specification if there is a certain probability that the wage bargaining breaks down, leading the parties to separate so that both receive their outside options. As the worker in this case leaves voluntarily from the firm, no firing costs
has to be paid, implying that the firing costs do not enter the wage setting.\footnote{It is more common to include firing costs in the Nash Product, in which case it also affects the wage outcome, cf. e.g. Pissarides (2000). This would be appropriate for any costs incurred by the firm even in the case when the worker leaves voluntarily, which could be used as a threat by the worker to push up wages. However, most firing costs are not of this type. For example, for a professor with tenure, the firing costs could be very large, in the sense that it would be very costly for the university to lay off the professor. However, as is well known, these firing costs cannot be used by the professor to push up his or her wage. Note that a similar argument is also acknowledged by Pries and Rogerson (2005), who write that "In reality, dismissal costs are not incurred in the case of voluntary separations, ... In what follows, we shall abstract from this aspect and assume that all separations in the model lead the entrepreneur to incur the cost \( d \). In this sense, we are really analyzing a separation tax levied on employers, rather than a dismissal cost per se."} The solution to (23) is

\[ w_i^H = \beta y^H + (1 - \beta)(r + s)U_i - \beta r V, \]  
(24)

implying that the asset value of the worker is

\[ W_i^H = \frac{\beta y^H + (1 - \beta)(r + s)U_i - \beta r V}{r + s}. \]  
(25)

For workers in a low productive match, we now assume that there is a binding minimum wage \( w^L \) satisfying (14), i.e. ensuring that it is not profitable for the firm to continue the match. Without this assumption, a wage rule like (24) would ensure that even low productive matches were profitable for the firm. Thus, in this case EPL would be irrelevant, as the firm would never want to lay off a worker, implying trivially that no discriminatory equilibrium may exist.

We shall analyse whether the discriminatory equilibrium derived above, where all Greens are hired, while Reds are only hired if \( \gamma = \gamma^* \), still may exist. And, if so, under which circumstances.

There are now two opposing mechanisms at work in a discriminatory equilibrium. On the one hand, Red workers still have the disadvantage that they are less likely to find another job, implying that the expected duration of a low productive match is longer. On the other hand, the weaker outside option of Red workers imply that their wage is lower. If the latter effect is stronger, firms will prefer
Red workers to Green ones, conditional on a bad signal, and a discriminatory equilibrium of this type cannot exist. Thus, we must check if this is the case.

Conditional on the signal $\gamma$, the expected profits from hiring a Green worker in a discriminatory equilibrium is (note that we have substituted out for the wage equation (24), and that we now need a subscript indicating worker type as the wage differs between the types)

\[
J_G(\gamma, 1) = \frac{\gamma}{r + s} ((1 - \beta)(y^H - (r + s)U_G) + (\beta r + s)V) + \frac{(1 - \gamma)}{r + s + \phi} (y^L - w^L + (s + \phi)V) \tag{26}
\]

while for a Red worker it is

\[
J_R(\gamma, \eta) = \frac{\gamma}{r + s} ((1 - \beta)(y^H - (r + s)U_R) + (\beta r + s)V) + \frac{(1 - \gamma)}{(r + s + \phi \eta)} (y^L - w^L + (s + \phi \eta)V) \tag{27}
\]

where the assets values for unemployed workers are (see Appendix 7 for details)

\[
U_G = z(r + s + \gamma M \phi) + \phi(\gamma M \frac{\beta y^H - \beta r V (r + s + \phi)}{r + s}) + (1 - \gamma M)w^L \tag{28}
\]

\[
U_R = z(r + s + \gamma \phi \eta) + \phi \eta(\gamma M \frac{\beta y^H - \beta r V (r + s + \phi \eta)}{r + s}) + (1 - \gamma)w^L \tag{29}
\]

and the value of a vacancy is

\[
V = \frac{-c + q(\alpha G(\eta J_G(\gamma, 1) + (1 - \eta)J_G(\gamma, 1)) + \alpha R \eta J_R(\gamma, \eta))}{r + q(\alpha G + \alpha R \eta)} = K \tag{30}
\]

with $J_G(\gamma, 1)$ defined by (26), $J_R(\gamma, \eta)$ by (27), $U_G$ by (28) and $U_R$ by (29). For a discriminatory equilibrium to exist, we must have

\[
J_G(\gamma, 1) > K > J_R(\gamma, \eta)
\]

i.e. conditional on a signal $\gamma$, the expected profits from hiring a Green worker is
greater than the value of a vacancy, while the expected profits from hiring a Red worker is lower.

5.1 Numerical simulations

As the model involves opposing and non-linear effects, numerical illustrations are useful to explore the effects. Ideally, we would want to do a serious empirical analyses where all parameter values are based on empirical evidence. However, for many of the parameters, like the probability that an applicant with a good signal turns out to be of high productivity, $\gamma$, no such evidence exists. Furthermore, the model is in any case very stylized, including the assumption that all workers are ex ante identical, irrespective of type. Thus, our aim is more modest. First, we want to explore whether a discriminatory equilibrium is possible under plausible parameter values, using empirical estimates whenever possible. Second, we will analyse the effect on the discriminatory equilibrium of changes in key parameter values.

The simulations are based on the steady state equations, the matching function (1), as well as the equations (26) and (27) (both calculated twice, for $\gamma$ and $\tau$), (28), (29) and (30) (the latter also being two equations). We have chosen the following parameter values, where the period length is assumed to be one quarter. Output with high and low productivity are set to $y^H = 1.0$ and $y^L = 0.55$. The cost of opening a vacancy $K = 11$, which corresponds to a capital-output ratio in annual terms slightly below three. The interest rate $r = 0.012$, following Shimer (2005), corresponding to an annual rate of about five. The workers’ bargaining power $\beta = 0.5$. The wage in a low productivity match $w^L = 0.6$, and the value of being unemployed (leisure and unemployment benefits) $z = 0.5$. In equilibrium, this will induce an average replacement rate, calculated as $z$ divided by the average wage, of about 63%, which is about the level in many European countries. The weight on vacancies in the matching function, $1 - \lambda = 0.4$, see Petrongolo and Pissarides (2001), who suggest that it should be between 0.3 and 0.5. Flow into unemployment in European countries is between 0.3% and 2.8% on monthly basis in European countries, see OECD (1995), page 27-28, and we choose $s = 0.03$ on a quarterly basis. The probability that output takes a high value is $\tau = 0.7$ and
\( \gamma = 0.3 \), while \( \text{Prob}(\gamma = \overline{\gamma}) = \eta = 0.6 \). The cost of a vacancy \( c = 0.02 \); this is lower than what is often used, e.g. Shimer (2005) has \( c = 0.2 \), but this must be seen in connection with our also including fixed costs of opening a vacancy. The share of Green workers, \( n_G = 0.9 \), implying that the share of Red workers, \( n_R = 0.1 \). We set the parameter in the matching function, \( A = 0.3 \) which leads to equilibrium labour tightness \( \theta = 0.942 \), and the matching rate for job searchers \( \phi = 0.29 \). As this is on a quarterly basis, it is well within the range for the job finding rate for unemployed in European countries, which according to OECD (1995), page 27-28 is between 3\% and 21\% on monthly basis in European countries. The parameter values are summarized in Table 1.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( s )</td>
<td>0.03</td>
<td>( \lambda )</td>
<td>0.6</td>
</tr>
<tr>
<td>( r )</td>
<td>0.012</td>
<td>( c )</td>
<td>0.02</td>
</tr>
<tr>
<td>( y^H )</td>
<td>1.0</td>
<td>( K )</td>
<td>11</td>
</tr>
<tr>
<td>( y^L )</td>
<td>0.55</td>
<td>( \eta )</td>
<td>0.6</td>
</tr>
<tr>
<td>( \beta )</td>
<td>0.5</td>
<td>( \overline{\gamma} )</td>
<td>0.7</td>
</tr>
<tr>
<td>( w^L )</td>
<td>0.6</td>
<td>( \overline{\gamma} )</td>
<td>0.3</td>
</tr>
<tr>
<td>( z )</td>
<td>0.5</td>
<td>( n_G )</td>
<td>0.9</td>
</tr>
<tr>
<td>( A )</td>
<td>0.3</td>
<td>( n_R )</td>
<td>0.1</td>
</tr>
</tbody>
</table>

With these parameter values, we find that a discriminatory equilibrium exists. The values of the key variables are displayed in Table 2. We find that \( J_R(\gamma, \eta) = 10.913 < K = 11 < 11.026 = J_G(\gamma, 1) \). Thus, the advantage for the employer from Green workers being more attractive on the job market, making them more likely to quit if badly matched, dominates the effect of Red workers being paid less. In other words, it is profitable to hire a Green with \( \gamma = \overline{\gamma} \), while it is not profitable hire a Red with \( \gamma = \gamma \). The discriminatory behavior leads to a large difference in unemployment rates, which is 9.3\% for Green workers, and 14.6\% for Red workers. As Green workers are hired even with a bad signal, a slightly higher share of them are in low productivity matches, the respective shares are 7.3\% for Green workers and 5.9\% for Red workers. While workers in low productivity match receive a lower wage, the average wage for Green workers is still somewhat higher than that of
Red workers, 0.796 versus 0.787, because the lower unemployment rate for Green workers improves their disagreement point in the wage bargaining, giving Green workers higher wages than Red workers, conditional on a high productivity match.

Table 2: Simulation outcome

<table>
<thead>
<tr>
<th>Variable</th>
<th>Value</th>
<th>Variable</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\theta)</td>
<td>0.942</td>
<td>(u_G)</td>
<td>0.093</td>
</tr>
<tr>
<td>(\phi)</td>
<td>0.293</td>
<td>(u_R)</td>
<td>0.146</td>
</tr>
<tr>
<td>(J_G(\gamma, 1))</td>
<td>11.026</td>
<td>(\varepsilon_G)</td>
<td>0.073</td>
</tr>
<tr>
<td>(J_R(\gamma, \eta))</td>
<td>10.913</td>
<td>(\varepsilon_R)</td>
<td>0.059</td>
</tr>
<tr>
<td>(\bar{w}_G)</td>
<td>0.796</td>
<td>(w^H_G)</td>
<td>0.811</td>
</tr>
<tr>
<td>(\bar{w}_R)</td>
<td>0.787</td>
<td>(w^H_R)</td>
<td>0.798</td>
</tr>
</tbody>
</table>

\(\bar{w}_G\) and \(\bar{w}_R\) are the average wages for Green and Red workers, \(u_G\) and \(u_R\) are the respective unemployment rates. \(\varepsilon_G\) and \(\varepsilon_R\) are the share of employed workers in low productivity matches.

In the model we have assumed that the firing costs are prohibitive. However, it is of interest to see how large the firing costs would have to be for the firms to be willing to retain workers even in a low productive match, as is assumed in the model. If the firm can lay off a worker at a cost \(F > 0\), the firm will retain a worker of type \(i\) in a low productive match if

\[
J_i^L = \frac{y^L - w^L + (s + \phi p_i)V}{r + s + \phi p_i} > V - F
\]

or

\[
F - \frac{rV - y^L + w^L}{r + s + \phi p_i} > 0, \quad \text{where} \quad p_G = 1 \quad \text{and} \quad p_R = \eta
\]

With parameter values as in Table 1, this criterion is fulfilled for \(F_G = 0.543\) and \(F_R = 0.836\), i.e. less than the production for one quarter with average productivity, well within the parameter range of firing costs explored by Mortenson and Pissarides (1999), which is from zero to unity.

We then want to explore the effects of labour market institutions. Which types of labour market institutions are conducive to the existence of a discriminatory equilibrium?
Lower unemployment benefits, $z$.

If $z$ is reduced, workers receive lower wages, and in particular this happens for Red workers, who have a larger probability of being unemployed. This makes Red workers more attractive, implying that a discriminatory equilibrium becomes less likely. For the parameter values displayed in Table 1, except for $z$, a discriminatory equilibrium exists as long as $z \geq 0.43$. For $z < 0.43$, the wages of Red workers have fallen sufficiently so that it is now profitable to hire them even conditional on a bad signal, implying that a discriminatory equilibrium of the mentioned type does not exist.

Lower minimum wages $w^L$.

A reduction in minimum wages $w^L$ reduces the loss for firms in the case of a bad match. This makes Red workers more attractive, making a discriminatory equilibrium less likely. For the parameter values in Table 1, except $w^L$, a discriminatory equilibrium exists as long as $w^L \geq 0.58$. For $w^L < 0.58$, it is again profitable to hire Red workers even conditional on a bad signal, and a discriminatory equilibrium is not possible.

Higher cost of opening a vacancy, $K$

In what types of jobs should we expect to find discriminating behavior? In our model, all jobs are identical. However, we could think of the economy consisting of many segmented labour markets, where the equilibrium of the model describes each segmented market, and different parameter values capture the characteristics of each labour market. Higher costs of a vacancy would correspond to jobs with higher capital intensity. Higher costs of opening a vacancy increases the loss from a bad match, which increases the possible costs of hiring a Red worker. The upshot is to make a discriminatory equilibrium more likely. This can be illustrated by increasing $K$ from 11 to $K = 14$, and then redo the exercises above. We find that the critical value for the minimum wage is reduced, so that a discriminatory equilibrium now exists for $w^L \geq 0.53$, down from 0.58 for $K = 11$. Thus, in jobs with high capital requirement, a discriminatory equilibrium is more likely, and it can happen even for lower minimum wages.
Numerical simulations with taste for discrimination  We then consider the situation where a share of all vacancies also are affected by other motives than profit maximisation. Specifically, we assume that a proportion \( m \) of vacancies have a taste for discrimination in the sense that they exogenously use a discriminatory hiring rule. As above, we do not include any direct effect on utility or wages of this taste for discrimination, the only effect is via the hiring rule.

Let superscript \( j \) denote type of firm, with \( j = N \) for a neutral firms (a firm that applies \( \gamma^C_G = \gamma \) and \( \gamma^C_R = \gamma \)) and with \( j = D \) for a discriminatory firm (a firm that applies \( \gamma^C_G = \gamma \) and \( \gamma^C_R = \gamma \)). The wage for a high productivity worker of color \( i \) in a \( j \)-type firm is determined by

\[
\hat{w}^{Hj}_i = \beta y^H + (1 - \beta)(r + s)U_i - \beta r V^j.
\]  

Since wages for high productivity workers now depend on the type of firm they are employed in, we cannot a priori exclude on-the-job search for workers in high productivity matches who are in a firm paying low wages. However, with the parameter values we use, only workers in low productivity matches want to change firms, consistent with the previous formulation. For a full description of the model see Appendix 8.

Using the parameter values from the basis simulation above, we find that the critical value for \( m \), defined in Proposition 2, is \( \hat{m} = 0.35 \). Thus, in this case if more than 35\% of the vacancies have taste for discrimination, then it becomes profitable for all vacancies to discriminate. As emphasized above, this works in the opposite direction of the effect in Becker’s model. Here, discriminated workers become less attractive for employers in spite of being paid less, implying that the existence of employers with a taste for discrimination may indeed induce other employers also to discriminate. Correspondingly, from Corollary 1, if more that \( \hat{\alpha} = 1 - \hat{m} = 0.65 \) of the firms for exogenous reasons apply a neutral hiring rule, any remaining profit maximising firm will also adopt a neutral hiring rule, implying that the discriminatory equilibrium vanishes.
6 Robustness

Here we discuss the robustness of the discriminatory equilibrium to different modelling assumptions.

6.1 Efficient hiring

So far, we have implicitly assumed that the wage is given by ex post bargaining, without any possibility of negotiations at the hiring stage. One implication of this is that it rules out the possibility that a discriminated worker accepts an initial period with a low wage, so as to make it profitable for the employer to hire him/her. In a discriminatory equilibrium, Red workers would be willing to do so, as they are strictly disadvantaged by not having the opportunity to see whether output turns out to be high even with a bad signal. Now, we shall relax this assumption. More specifically, we assume that there is an additional hiring stage, where the players bargain over the surplus of the match, after having observed the signal $\gamma$. At this stage, output is $y_0 > 0$, there is no on-the-job search and no employment protection legislation. The wage is flexible, and hiring will take place if the joint surplus from the match is positive. If the worker is hired, the match transits to the second stage at a rate $\kappa > 0$. At this second stage, employment protection with firing costs is invoked, and the stochastic output is realised, being equal to $y^H$ or $y^L$. This two-stage framework follows e.g. Mortensen and Pissarides (1999).

At the second stage the wage is renegotiated, reflecting the change in bargaining situations. With low productivity, the minimum wage $w^L$ binds, while with high productivity, the wage is determined by maximizing$^{12}$

$$\beta(J^H_i - V)_{1-\beta},$$

(32)

From (32), the wage at the second stage solves

$$W^H_i - U_i = \beta(J^H_i + W^H_i - V - U_i).$$

$^{12}$In line with the arguments in footnote 9 above, we have not included firing costs $F$ in (32), in contrast to most of the literature. However, as we assume flexible wages in stage 1, including firing costs in (32) would not affect the hiring decision made by the firm.
As noted above, the firing-cost is not present at the first stage. Define $J^0$ and $W^0$ as the values to the firm and the worker at the first stage. The wage at the first stage is determined by maximizing

$$(W_i^0 - O_i)^\beta (J_i^0 - V)^{1-\beta}.$$ 

where $O_i = U_i$ if the worker is unemployed and $O_i = W_i^L$ if the worker is employed. Denote the joint surplus at stage one by $S^0$. The outcome of the Nash bargain implies that

$$W_i^O - O_i = \beta S^0 = \beta (J_i^O + W_i^O - V - O_i).$$

$$J_i^0 - V = (1 - \beta) S^0 = (1 - \beta) (J_i^O + W_i^O - V - O_i).$$

Note that the players at the first stage will take into consideration their expected payoffs at the second stage. Thus, we allow for the possibility that a Red worker accepts to work for a low or even negative wage in the first period, to compensate for the possibility of coming to the more rewarding second stage. While one can discuss the realism of the wage being negative in an initial period, our aim is to explore if a discriminatory equilibrium may exist even under circumstances that come a long way towards ensuring efficiency.

Asset values at stage 1 are

$$(r + s + \kappa) W_i^0(\gamma) = w_i^0(\gamma) + \kappa (\gamma W_i^H + (1 - \gamma) W_i^L),$$

$$(r + s + \kappa) J_i^0(\gamma) = y^0 - w_i^0(\gamma) + sV + \kappa (\gamma J_i^H + (1 - \gamma) J_i^L).$$

Hiring will be efficient and take place iff $S^0$ is positive. Thus, for a discriminatory equilibrium to exist we need that, conditional on a bad signal, the surplus of hiring a Green worker is positive, while it is negative for a Red worker:

$$S^0_G(\gamma) > 0 > S^0_R(\gamma), \text{ when } \gamma_C^G = \gamma \text{ and } \gamma_C^R = \overline{\gamma}. \quad (33)$$

Given these hiring strategies the value of a vacancy is
\[ V = -c + q(1 - \beta)(\alpha_G \eta S_G^0(\gamma) + \alpha_G (1 - \eta) S_G^0(\tau) + \eta \alpha_R S_R^0(\gamma)), \]

and in equilibrium we still have \( V = K \).

In appendix 9, we show that for certain parameter values, (33) holds, implying that there exists a discriminatory equilibrium also allowing for efficient hiring.\(^{13}\)

The intuition can be explained as follows. A discriminatory equilibrium involves an efficiency loss arising from not hiring Red workers conditional on a bad signal, as this involves loosing the possibility of a high productivity match. However, the costs to the firm of having a worker in a low productive match may be so high that it outweighs this efficiency loss. In this case the joint surplus from hiring a Red worker conditional on a bad signal is negative, implying that the worker is not willing to reduce the first period wage sufficiently that the firm profits from hiring him/her. Note however that the conditions under which a discriminatory equilibrium exists are much more restrictive with efficient hiring than under the previous specifications, and the simulations we report in appendix 8 involve some implausible parameter values.

### 6.2 Workers fired

To simplify the analysis, we have assumed that the firing costs are prohibitive so that workers are never laid off. The numerical simulations above have shown that the firing costs need not be higher than what is often assumed in the literature. However, qualitatively the same results may be derived also under the weaker assumption that workers sometimes are laid off, but this is costly to the firm. The key feature of the model that would be required is that in case the match is low productive, the firm is better off with a Green worker that can be expected to find a new job, so no firing costs are incurred, than a discriminated Red worker who may have difficulty finding a job. Whether the firing costs are so high that the firm

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\(^{13}\)To simplify we assume that \( w_L = z \) which implies that that \( O = W^L_i = U_i \). Assuming \( w_L > z \) would complicate the analysis considerably, as it would then be necessary to include four types of applicants, depending both on type and on being unemployed or in bad match. The simplification \( w_L = z \) is however likely to work against our finding a discriminatory equilibrium: As illustrated in the numerical simulations in section 5, increasing the minimum wage makes a discriminatory equilibrium more likely.
prefers not to lay off a Red workers (as illustrated in the numerical simulations above), or whether the firm profits from laying off the worker and incurring the costs, is not important, as long as both alternatives are considerably more costly than letting a Green worker leaving voluntarily when being offered another job.

Similarly, it would be straightforward to extend our model to incorporate a third level of productivity; \( y^{Min} \) with \( y^{Min} < y^L \) at which firms always would fire, while firms wait for voluntary transition at the intermediate level \( y^L \).

6.3 Workers’ search intensity

We have assumed that the search intensity is exogenous and identical for off- and on-the-job search. Having different exogenous search intensities would not affect our result as long as workers in matches with \( y = y^L \) prefer employment to unemployment. Rather, in a setting with endogenous search intensity discriminatory outcomes seem more likely. Green workers would presumably choose a higher search intensity than Red workers, as their return to search is higher. As a consequence, Green workers would leave low productivity matches even faster.

7 Empirical evidence and implications:

In this section we discuss how our model fares when contrasted with the actual labor market situation of two groups for whom there is evidence of discrimination in the labor market, namely immigrants and older workers. We start the discussion with a huge caveat: the main aim of our model is to show a theoretical point, that discrimination can prevail even for identical workers. In practice, groups that are discriminated against are seldom identical on other accounts. Immigrants, for example, have generally lower language and cultural skills than the native population, and often also weaker formal qualifications. Such differences are likely to be reflected in the labor market outcomes. Thus we focus on the key empirical implications, rather than trying to match empirical data.

Generally, unemployment is considerably higher among immigrants than among natives. Comparing unweighted averages across OECD countries, the unemployment rate is almost 40 percent higher for foreign-born than for native-born (OECD
Some evidence suggests that when comparable measures of skill exists, immigrants with levels comparable to the native-born seem to find work commensurate with their formal qualifications (OECD, 2007b, page 50). However, other types of evidence suggest that discriminatory practices exist. One piece of evidence is based on experimental tests of hiring procedures, where applications for the same job are submitted from two fictitious candidates with essentially the same qualifications, but where different name indicates different nationality. In all countries where this test has been undertaken, the results show that employers favor the native applicants at all stages of the hiring process. On average, the net additional elimination of immigrant applicants is about one third, varying somewhat across countries (OECD, 2007b, page 53). Further suggestive evidence of discrimination is the fact that also second-generation immigrants generally have weaker labour market outcomes than native-borns (OECD, 2007b, page 59). While some of the difference reflects lower educational levels, part of the gap remains even if one controls for lower education (OECD 2007b, page 64). Even after long stays in the host country, immigrants are also much more likely than native-borns to be overqualified, i.e. to have a job that require lower education than the person has (OECD, 2006, page 145).

Our hypothesis suggests that part of the discrimination of immigrants is linked to the existence of EPL. This hypothesis is consistent with the results of Kahn (2007), who explores the effect of EPL on joblessness and temporary employment by demographic groups on micro data from the International Adult Literacy Survey for Canada, the US and five European countries. Kahn finds that strict EPL has a significant negative effect on employment levels for immigrants, as predicted by the current model, and it also raises the incidence of temporary employment for immigrant women.

Sá (2008) on the other hand, using the EU Labour Force Study, finds that strict EPL favours immigrants, as strict EPL reduces employment for natives with less or no effect on immigrant employment. However, Sá interprets this finding as arising from immigrants generally being less informed about their rights, and thus in practice being less protected by EPL. One support for this interpretation is that

\[14\] Bertrand and Mullainathan (2004) obtain about the same extent of discrimination for African-Americans in the US, using essentially the same method.
unionization rates are generally much lower for immigrants than for natives (Sá, 2008), and union membership is often a key way of getting information and help in exercising the rights.

One should however be fairly cautious when interpreting evidence on the link between immigrant unemployment and EPL. Immigrant unemployment across countries is also strongly influenced by vast differences in immigrant population. OECD countries with large job immigration, like Spain or the US, or rather selective immigration, like Australia and Canada, have fairly low immigrant unemployment, as compared to total unemployment rates. In contrast, unemployment among immigrants is much higher as compared to native unemployment in countries like Denmark, Norway and Sweden, where many immigrants are refugees or family reunification. Furthermore, employment protection legislation is a complex issue and the effective job protection varies strongly across various parts of the labor market. Thus, in a country with strict employment protection in parts of the labor market, immigrants may find employment in other parts where legislation is less strict. Finally, even if immigrants are discriminated against when comes to permanent jobs, they may accept temporary jobs or become self-employed (see below), preventing any effect on unemployment rates.\(^\text{15}\)

A way to avoid these problems is to look at the effect of differences in employment protection within countries instead of across countries. Using data from the European Community Household Panel (ECHP), Clark and Postel-Vinay (2009) find that workers feel most secure in permanent public sector jobs, and least secure in temporary jobs.\(^\text{16}\) Incidentally, in almost all OECD countries (Belgium being the only exception) immigrants are under represented in public sector jobs, as compared to native-borns (OECD, 2007c, page 73). Indeed, even children of

\(^\text{15}\)The differences in wage dispersion across countries is also a factor that makes it more difficult to find a link between cross-country difference in EPL and differences in unemployment. Causa and Jean (2007) find empirical support for stronger "EPL dualism" (defined as the relative level of EPL for permanent vs. temporary contracts) widens the wage gap but reduces the employment/activity gap between immigrants and natives. This is explained as being caused by immigrants more frequently holding temporary jobs, so that EPL in temporary jobs is more relevant for them.

\(^\text{16}\)Surprisingly, Clark and Postel-Vinay (2009) find that perceived job security is lower in countries with strict EPL, suggesting that there important aspects not captured by the OECD index. The exception to this negative relationship is for employees in permanent public jobs, who feel secure everywhere.
immigrants tend to be under represented in the public sector (OECD, 2007c, page 85). The underrepresentation of immigrants in the public sector is consistent with public sector employers being reluctant to hiring immigrants, being aware of the additional difficulty in the public sector of getting rid of employees who do not fit to the job requirements. However, there are also other possible explanations for underrepresentation of immigrants in the public sector, for instance that requirements as to language or societal knowledge may be higher in parts of the public sector.

Also other labour market features are consistent with the notion that some immigrants face discrimination when trying to obtain a permanent job in the regular labor market. First, in almost all OECD countries, immigrants are much more likely to have temporary jobs than are native-born (OECD, 2007c, page 75). This would follow if employment protection applies only on permanent jobs, and that they are less relevant for temporary jobs. Second, evidence from Sweden shows that immigrants are more likely than native-borns to exit from temporary help agencies into other sectors of activity (Anderson and Wadensjö, 2004a). This may suggest that temporary work assignments may work as a screening device which may overcome possible reluctance among employers in hiring immigrants. This interpretation is consistent with evidence from Denmark, showing that wage subsidies and subsidised employer-based training are more effective at getting immigrants into employment than native born persons (OECD, 2007b, page 51). These programs enable employers to experience how well immigrants perform on the job, implying that hiring decisions are made on more certain information, reducing the risk of a low productive and unprofitable match. Anderson and Wadensjö (2004b) show that immigrants in the labor market in many cases also use self employment as a way of escaping marginalization in the labor market.

Another salient aspect of immigrant unemployment is that the unemployment gap between immigrants and natives widen as the education level rises in all OECD countries (OECD, 2008, page 114). This is as predicted by the model in the current paper, conditional on the reasonable assumption that the loss associated with a low productive match are higher for higher levels of education. One reason to assume that costs of a low productive match are higher for higher levels of education is the empirical finding that capital is complementary with the use of skilled labor, see
e.g. Hornstein, Krusell and Violante (2005). Capital being complementary with high skilled labor suggests that jobs with employees with high education should be associated with higher costs of opening a vacancy, \( K \), which in our model makes a discriminatory equilibrium more likely.

A different prediction of our model is that employers are less reluctant to discriminate a group of workers if a weaker labor market situation has a strong downward effect on wages. This prediction would suggest that foreign unemployment will be high relative to native unemployment in countries with small wage compression. This is consistent with evidence across 16 OECD countries, where we find a strong negative relationship between the ratio of foreign to native unemployment and wage dispersion, as measured by the ratio of the fifth to the first percentile (this measure of wage dispersion captures dispersion in the lower part of the wage distribution, which is the wage dispersion of relevance in the theoretical model).\(^{17}\)

Another group that is subject to discrimination in the labor market is older workers (OECD, 2006). Clearly, as group, older workers differ from younger ones in several ways, including job experience, expected future working years, and formal qualifications. However, in contrast to for immigrants, there are no issues of race or cultural or languages skills. Yet in all OECD countries, hiring rates of workers decline significantly after the age of 50, although more sharply in some countries than others (OECD, 2006, page 132). While we clearly cannot say why this is so, the fact that hiring rates fall will within our model have negative feedback effects on the employability of older workers, as firms expect older workers to have a harder time finding another job in case the current one does not work out. Furthermore, our model is also consistent with the findings of Behaghel, Crépon and Sédillot (2008) mentioned in the Introduction. They find that a tax on firms’ laying off workers aged 50 and above had a negative effect on firms’ hiring of workers in the relevant age groups, consistent with the mechanism proposed in this paper. Autor, Donohue and Schwab (2006) found that the introduction of wrongful-discharge laws have reduced state employment rates, and the long-term effects are greater for older workers than for younger.

\(^{17}\)The evidence is available on request.
8 Conclusions

We offer a novel argument for discriminatory outcomes of equally productive groups, in labor markets where it is costly for firms to layoff workers. The key mechanism is that workers’ with low job-finding rates are risky to hire since they might stay for long also in case of a bad match. Discrimination is self-enforcing, as it is precisely because a group is discriminated against that it has lower job-finding rates, and thus become less attractive to hire.

In our model, groups are identical except for the color and discrimination may in principle hit any group. Furthermore, there also exists an equilibrium without any discrimination. However, we show that if a sufficiently large proportion of the employers have a taste for discrimination, then the unique equilibrium outcome is a situation with discrimination of this group. Thus, also employers without a taste for discrimination will apply discriminatory hiring strategies, because hiring the discriminated group is less profitable. We also show that affirmative action is potentially an efficient measure against discrimination. If a sufficiently large share of the employers hire according to the population shares of each group, the discriminatory equilibrium vanishes. This effect of affirmative action is different from the effect of affirmative action if discrimination is the result of underinvestment in human capital by the discriminated group.

In the core model outside options only serve as constraints. When outside options are the threat points in the bargaining, workers from the discriminated group receive lower wages, for a given productivity, making discriminatory outcomes less likely. However, a discriminatory equilibrium still exists. Numerical simulations suggests that a discriminatory outcome is more likely the higher the unemployment benefits and the higher the minimum wages.

While the main aim of the paper is to make a theoretical point, several of the empirical predictions of the model are consistent with salient features of the labor market situation of immigrants to OECD countries. In almost all OECD countries, immigrants are much more likely to hold temporary jobs than are native-borns, consistent with employers being more reluctant to hire immigrants in permanent jobs with employment protection. In the OECD Economic Survey of Sweden 2007, one of the key elements to combat exclusion is "to reduce the risk associated with
hiring someone who turns out not to be the right person for the job". In spite of extensive measures to help immigrants entering the labor market, high unemployment rates and overqualification remain an important problem for immigrants in the Nordic countries. This is consistent with the model, in view of the high welfare level and small wage dispersion of the Nordic countries, as compared to most other OECD countries. The model is also consistent with the strong tendency across OECD countries that the unemployment gap between immigrants and natives widen as the education level increases.

9 Appendix

Appendix 1: Steady state conditions

Let $\varepsilon_i$ denote the share of employed workers of type $i$ in jobs with low match quality. For both types $i$, the outflow from jobs with bad matches must equal the inflow to jobs with bad matches

$$n_i\varepsilon_i(s + \phi p_i)(1 - u_i) = vq\mu(\gamma_i^c)(1 - E(\gamma | \gamma \geq \gamma_i^c))\alpha_i.$$  

Similarly, the outflow from good matches equals inflow to good matches

$$n_i(1 - \varepsilon_i)s(1 - u_i) = vq\mu(\gamma_i^c)E(\gamma | \gamma \geq \gamma_i^c)\alpha_i.$$  

Using the above two equations gives us

$$E(\gamma | \gamma \geq \gamma_i^c)\varepsilon_i(s + \phi p_i) = s(1 - E(\gamma | \gamma \geq \gamma_i^c))(1 - \varepsilon_i),$$

or

$$\varepsilon_i = \frac{s(1 - E(\gamma | \gamma \geq \gamma_i^c))}{s + E(\gamma | \gamma \geq \gamma_i^c) \phi p_i}.$$  \hspace{1cm} (34)

As the outflow from unemployment equals inflow to unemployment, we must have

$$n_iu_i(s + \phi p_i) = sn_i.$$
Solving for the rate of unemployment

\[ u_i = \frac{s}{s + \phi p_i} \]  \hspace{1cm} (35)

The job seekers consist of the unemployed as well as the employed in bad matches. The fraction of all job seekers that are of type \( i \), \( \alpha_i \), is defined by

\[ \alpha_i = \frac{n_i(u_i + (1-u_i)\varepsilon_i)}{n_G(u_G + (1-u_G)\varepsilon_G) + n_R(u_R + (1-u_R)\varepsilon_R)}. \]  \hspace{1cm} (36)

**Appendix 2: Assumption 1 holds if equation (19) holds**

From equation (18) we have that

\[ V(\theta) = -\frac{c}{q(\theta)} + \frac{\alpha_G J_G(\gamma^M, 1) + \alpha_R \eta J_R(\gamma, \eta)}{r/c(\theta) + \alpha_G + \alpha_R \eta}. \]

Using (9) and that \( \phi = 0 \) and \( \alpha_i = n_i \) when \( \theta = 0 \) gives

\[ V(0) = n_G(1-\beta)g^H + (1-\gamma^M)(1-\gamma^L) \]
\[ + n_R \eta (1-\gamma^H + (1-\gamma^L)(1-\beta)y^L). \]

or

\[ rV(0) = n_G(1-\beta)g^H + (1-\gamma^M)(1-\gamma^L) \]
\[ + n_R \eta (1-\gamma^H + (1-\gamma^L)(1-\beta)y^L). \]

Thus \( V > K \iff \)

\[ n_G(1-\beta)g^H + (1-\gamma^M)(1-\gamma^L) + n_R \eta (1-\gamma^H + (1-\gamma^L)(1-\beta)y^L) > rK. \]

**Appendix 3: Proof of Lemma 1**

\( V \) is continuous in \( \theta \), \( V(0) > K \), and \( \lim_{\theta \to \infty} V = -c/r \). Hence, there exists values
and $\theta^1$ such that $\frac{dV}{d\theta} < 0$ for $\theta \in (\tilde{\theta}^0, \theta^1]$, $\frac{dV}{d\theta} \leq 0$ for $\theta = \tilde{\theta}^0$, $V(\theta^1) = K$ and $V(\theta) < 0$ for $\theta > \theta^1$. Define $\theta^0$ to be the smallest non-negative value of $\tilde{\theta}^0$.

Appendix 4: Proof of Proposition 1

First we do some definitions. Using (9) we can define $J(\gamma, p)$ as a function of $\theta$; $\tilde{J}(\theta, \gamma, p)$, where

$$ \tilde{J}(\theta, \gamma, p) = \frac{\gamma}{r+s}(y^H - w^H + sV(\theta)) + \frac{(1-\gamma)}{r+s + \phi(\theta)p}(y^L - w^L + (s + \phi(\theta)p)V(\theta)). $$

(37)

Denote $\tilde{J}(\theta, \gamma, 1)$ by $\tilde{J}_G(\theta, \gamma)$ and denote $\tilde{J}(\theta, \gamma, \eta)$ by $\tilde{J}_R(\theta, \gamma)$.

Using (9) we can define $E(J_G | \gamma \geq \gamma)$ as a function of $\gamma$ and $\theta$; $\tilde{J}_G(\theta, \gamma)$ where,

$$ \tilde{J}_G(\theta, \gamma) = \frac{\eta \gamma + (1-\eta)\gamma}{r+s}(1-\beta)y^H + sV(\theta)) + \frac{1-\gamma}{r+s + \phi(\theta)\eta}((1-\beta)y^L + (s + \phi(\theta))V(\theta)) $$

(38)

Noting that $E(J_R | \gamma \geq \gamma)$ is independent of $\gamma$, we can similarly define $E(J_R | \gamma \geq \gamma)$ as a function of $\theta$ only; $\tilde{J}_R(\theta)$ where

$$ \tilde{J}_R(\theta) = \frac{\gamma}{r+s}((1-\beta)y^H + sV(\theta)) + \frac{(1-\gamma)}{r+s + \phi(\theta)\eta}((1-\beta)y^L + (s + \phi(\theta))V(\theta)) $$

Using (11) we can define $V$ as a function of $\theta$, $\tilde{J}_G(\theta, \gamma)$ and $\tilde{J}_R(\theta)$; $\tilde{V}(\theta, \tilde{J}_G(\theta, \gamma), \tilde{J}_R(\theta))$ where

$$ \tilde{V}(\theta, \tilde{J}_G(\theta, \gamma), \tilde{J}_R(\theta)) = \frac{-c + q(\theta) \left( \alpha_G \tilde{J}_G(\theta, \gamma) + \alpha_R \eta \tilde{J}_R(\theta) \right)}{r + q(\theta) (\alpha_G + \alpha_R \eta)} $$

(39)

and finally we define

$$ \hat{V}(\theta, \gamma) = \tilde{V}(\theta, \tilde{J}_G(\theta, \gamma), \tilde{J}_R(\theta)). $$

As part of the existence proof we want to show that there exist values $\bar{\theta}$ and $\underline{\theta}$, such that when $\underline{\theta} \leq \theta < \bar{\theta}$ it is profitable to hire Green workers, i.e. $J(\gamma, 1) - \hat{V} \geq 0$ and at the same time not profitable to hire Red, i.e. $J(\gamma, \eta) - \hat{V} < 0$. As a first
step we show that the profitability of a hiring a worker is increasing in \( \theta \), and \( \gamma \) on the interval where \( V \) is decreasing in \( \theta \).

**Lemma 3**

a) \( \hat{J}_i(\theta, \gamma) - \hat{V}(\theta, \gamma) \) is strictly increasing in \( \theta \) on \([\theta^0, \theta^1]\), \( i = G, R \).

b) \( \hat{J}_i(\theta, \gamma) - \hat{V}(\theta, \gamma) \) is strictly increasing in \( \gamma \) on \([\theta^0, \theta^1]\), \( i = G, R \).

**Proof.** Define \( f_i(\theta, \gamma) = \hat{J}_i(\theta, \gamma) - \hat{V}(\theta, \gamma) \).

Part a) Taking the derivative of \( f_i \) w.r.t. \( \theta \) gives

\[
\frac{\partial f_i(\theta, \gamma)}{\partial \theta} = \frac{\partial \hat{J}_i}{\partial \theta} \frac{\partial \phi}{\partial \phi} + \frac{\partial \hat{J}_i}{\partial \gamma} \frac{\partial V}{\partial \theta} - \frac{\partial \hat{V}(\theta)}{\partial \theta}.
\]

From (37) we have that

\[
\frac{\partial \hat{J}_i}{\partial \phi} = \frac{p_i(1 - \gamma)(rV - (y^L - w^L))}{(r + s + \phi p_i)^2} > 0,
\]

and

\[
\frac{\partial \hat{J}_i}{\partial V} = \frac{\gamma}{r + s} + \frac{(1 - \gamma)}{(r + s + \phi p_i)(\phi p_i + s)} < 1.
\]

Since \( \frac{\partial \phi}{\partial \theta} > 0 \), \( \frac{\partial \hat{J}_i}{\partial \phi} > 0 \), \( \frac{\partial \hat{J}_i}{\partial \gamma} < 1 \) and \( \frac{\partial V}{\partial \theta} \leq 0 \) when \( \theta \in [\theta^0, \theta^1] \) it follows that \( \frac{\partial f_i(\theta, \gamma)}{\partial \theta} > 0 \).

b) Taking partial derivative of \( f_i \) w.r.t \( \gamma \) gives

\[
\frac{\partial f_i(\theta, \gamma)}{\partial \gamma} = \frac{\partial \hat{J}_i}{\partial \gamma} \frac{\partial V}{\partial \gamma} - \frac{\partial \hat{V}(\theta)}{\partial \gamma} = \frac{\partial \hat{J}_i}{\partial \gamma} \frac{\partial \hat{V}(\theta)}{\partial \hat{J}_G} \frac{\partial \hat{J}_G}{\partial \gamma}.
\]

From (37) and (38) follows that \( \frac{\partial \hat{J}_i}{\partial \gamma} \frac{\partial \hat{V}(\theta)}{\partial \hat{J}_G} \frac{\partial \hat{J}_G}{\partial \gamma} > 0 \). From (39) it follows that \( 0 < \frac{\partial V}{\partial \hat{J}_G} < 1 \). Hence, \( \frac{\partial f_i(\theta, \gamma)}{\partial \gamma} > 0 \). From (39) it follows that \( \frac{\partial \hat{J}_i}{\partial \gamma} > 0 \). Hence, \( \frac{\partial \hat{J}_i}{\partial \gamma} > 0 \). From (39) it follows that \( \frac{\partial \hat{J}_i}{\partial \gamma} > 0 \). Hence, \( \frac{\partial \hat{J}_i}{\partial \gamma} > 0 \). From (39) it follows that \( \frac{\partial \hat{J}_i}{\partial \gamma} > 0 \). Hence, \( \frac{\partial \hat{J}_i}{\partial \gamma} > 0 \). From (39) it follows that \( \frac{\partial \hat{J}_i}{\partial \gamma} > 0 \). Hence, \( \frac{\partial \hat{J}_i}{\partial \gamma} > 0 \).

**Lemma 4**

a) There exist a unique \( \gamma^0 \), for which \( \hat{J}_G(\theta^1, \gamma^0) - \hat{V}(\theta^1, \gamma^0) = 0 \) and a unique \( \gamma^1 \), for which \( \hat{J}_G(\theta^0, \gamma^1) - \hat{V}(\theta^0, \gamma^1) = 0 \) where \( 0 < \gamma^0 < \gamma^1 < 1 \).
**P proof.** Recall that $f_G(\theta, \gamma) = \hat{J}_G(\theta, \gamma) - \hat{V}(\theta, \gamma)$. Since we are considering the case when firms want workers with $y = y^L$ to leave we know that $f_G < 0$ for $\gamma = 0$. Furthermore, as $\hat{J}_G(\theta, \gamma)$ is the highest possible expected value at the hiring stage we have that $\hat{J}_G(\theta, \gamma) > \hat{V}(\theta)$ for any $\theta$.

Since $f_G(\theta, 0) < 0$, $f_G(\theta, \gamma) > 0$ for any $\theta \in [\theta^0, \theta^1]$ and $f_G(\theta, \gamma)$ is decreasing and continuous in both its arguments there exists a value $\gamma^0$, for which $f_G(\theta^1, \gamma^0) = 0$ and a value $\gamma^1$ for which $f_G(\theta^0, \gamma^1) = 0$. Since $\theta^1 > \theta^0$, $\hat{J}_G - \hat{V}$ is increasing in both $\theta$ and $\gamma$, we have that $\gamma^0 < \gamma^1$.

Next we want to show that for intermediate values of gamma there exist a value $\theta \in [\theta^0, \theta^1]$ defined by $\hat{J}_G(\theta, \gamma^0) - \hat{V}(\theta, \gamma^0) = 0$ such that for $\theta < \theta$ it is unprofitable to hire a Green worker, while for $\theta > \theta$ it is profitable. ■

**Lemma 5** For $\gamma \in (\gamma^0, \gamma^1)$, there exists a $\theta \in [\theta^0, \theta^1]$ such that $\hat{J}_G(\theta, \gamma) - \hat{V}(\theta, \gamma) \geq 0$ for $\theta \geq \theta$.

**P proof.** Consider any $\gamma' \in (\gamma^0, \gamma^1)$. Since $\hat{J}_G - \hat{V}$ is continuous and strictly increasing in $\theta$ it is sufficient to show that $\hat{J}_G(\theta^0, \gamma') - \hat{V}(\theta^0, \gamma') > 0$ and $\hat{J}_G(\theta^1, \gamma') - \hat{V}(\theta^1, \gamma') < 0$. This, however, follows directly from the definitions of $\gamma^0$, and $\gamma^1$ and that $\hat{J}_G(\theta, \gamma) - \hat{V}(\theta, \gamma)$ is strictly increasing in $\gamma$ for $\theta \in [\theta^0, \theta^1]$. ■

**Definition:** Define $\theta$ by $\hat{J}_G(\theta, \gamma) - \hat{V}(\theta, \gamma) = 0$ and define $\theta$ by $\theta = \arg \min \left\{ \hat{J}_R(\theta, \gamma) - \hat{V}(\theta, \gamma) = 0, V(\theta) = K \right\}$. Furthermore, define $K$ and $\overline{K}$ by $K = V(\theta)$ and $\overline{K} = V(\theta)$.

Thus, $\theta$ is the lowest value of $\theta$ for which it is profitable to hire a Green worker with $\gamma = \gamma$ and $\overline{K}$ the lowest value of $\theta$ for which it is profitable to hire a Red worker with $\gamma = \gamma$ if this value of $\theta$ implies that $V > K$, otherwise $\theta$ is defined by $V(\theta) = K$. Note the $\theta^0 < \theta < \theta^1$, where the first inequality holds from Lemma 5 and the last inequality holds from the definition of $\theta$.

We are now ready to prove Proposition 1: Since $\hat{J}_G(\theta, \gamma) - \hat{V}(\theta, \gamma)$ is strictly increasing over the interval $[\theta^0, \theta^1]$, it is also strictly increasing over the interval $[\theta, \overline{\theta}]$. (As $\theta > \theta^0$ and $\overline{\theta} \leq \theta^1$). From the definition of $K$ and $\overline{K} \overline{K}$ we have that $K \in (K, \overline{K}) \Rightarrow \theta \in (\theta, \overline{\theta})$ and hence that $\hat{J}_G(\theta, \gamma) - \hat{V}(\theta, \gamma) \geq 0$ and $\hat{J}_R(\theta, \gamma) - \hat{V}(\theta, \gamma) < 0$ for $K \in (K, \overline{K})$. Thus, conditions (20) and (21) are satisfied and the proposition follows.
Appendix 5  Proof of Lemma 2

Proof. Let superscript $NE$ denote the neutral equilibrium where $\gamma_G^C = \gamma_R^C = \gamma$ and superscript $DE$ denote the discriminatory equilibrium where $\gamma_G^C = \gamma_R^C = \gamma$. Assume $\gamma_G^C = \gamma_R^C = \gamma$. Then $J_{NE}^R(\gamma, 1) = J_{NE}^G(\gamma, 1)$. For $\gamma_G^C = \gamma_R^C = \gamma$ to be an equilibrium we need to show that $J_{NE}^G(\gamma, 1) > K$. It is sufficient to show that $J_{NE}^G(\gamma, 1) > J_{DE}^G(\gamma, 1)$ since $J_{DE}^G(\gamma, 1) > K$ by assumption (discriminatory equilibria exists).

Profit from hiring a Green worker when $p_G = 1$ is

$$J_i(\gamma, 1) = \frac{\gamma}{r + s}((1 - \beta)y^H + sV) + \frac{(1 - \gamma)}{r + s + \phi}(y^L - w^L + (s + \phi)V)$$

It is profitable to hire a Green worker with $\gamma = \gamma$ if $J_G(\gamma, 1) > V$. In equilibrium $V = K$. $J_G(\gamma, 1)$ is increasing in $\phi$. Thus if $\phi_{NE}^G > \phi_{DE}^G$ we know that $J_{NE}^G(\gamma, 1) > J_{DE}^G(\gamma, 1)$. Since $V$ is increasing in $p_R$ it follows that when $p_G = p_R = 1$ we have that $V > K$ at $\phi_{DE}$. Thus $\phi_{NE} > \phi_{DE}$.

Appendix 6: Proof of Proposition 2.

From (20) we know that $J_R(\gamma, p_R)$ (given by (22)) > $K$ for $m = 1$, and from (21) that $J_R(\gamma, p_R) < K$ for $m = 0$. As $J_R(\gamma, p_R)$ is continuous in $m$, it then suffices to show that $J_R(\gamma, p_R)$ is decreasingly monotonically in $m$, as there then must exist a critical value for $m$ above which a neutral equilibrium will not exist.

$$\frac{dJ_R(\gamma, p_R)}{dm} = \frac{\partial J_R(\gamma, p_R)}{\partial m} + \frac{\partial J_R(\gamma, p_R)}{\partial \phi} \frac{\partial \phi}{\partial m}$$

We see from (22) that $\frac{\partial J_R(\gamma, p_R)}{\partial m} < 0$, and that $\frac{\partial J_R(\gamma, p_R)}{\partial \phi}$. Consider $\frac{\partial \phi}{\partial m}$, for a given $\phi$, which also implies a given $\theta$, a higher $m$ leads to a lower $J_R(\gamma, p_R)$, implying a lower $V$. A lower $V$ will reduce entry of firms, thus reducing labor market tightness $\theta$, implying that $\phi$ will also be lower. Hence, $\frac{\partial \phi}{\partial m} < 0$ and thus $\frac{dJ_R(\gamma, p_R)}{dm} < 0$.

Appendix 7: Equations (28) and (29)
The asset value of a Green, unemployed worker is (from (2))

\[(r + s + \phi)U_G = z + \phi EW_G(\gamma) \tag{40}\]

Using (3), (4), and (6) gives

\[EW_G(\gamma) = \frac{\gamma^M u_H^G (r+s+\phi)}{r+s+\gamma^M \phi} + (1 - \gamma^M)w^L.\]

Inserting for \(w_H^G\) given by (24) yields

\[EW_G(\gamma) = \frac{\gamma^M (\beta y^H (r+s+\phi)U_G - \beta rV)(r+s+\phi)}{r+s+\gamma^M \phi} + (1 - \gamma^M)w^L \tag{41}\]

Using (40) and (41) gives

\[(r + s + \phi)U_G = z + \phi \frac{\gamma^M (\beta y^H (r+s+\phi)U_G - \beta rV)(r+s+\phi)}{r+s+\gamma^M \phi} + (1 - \gamma^M)w^L\]

or

\[(r + s + \gamma^M \phi)(r + s + \phi)U_G = z(r + s + \gamma^M \phi) + \phi(\gamma^M \frac{(\beta y^H (r+s+\phi)U_G - \beta rV)(r+s+\phi)}{r+s} + (1 - \gamma^M)w^L)\]

or

\[(r + s + \gamma^M \phi)(r + s + \phi)U_G - \phi(1 - \beta)\gamma^M (r + s + \phi)U_G = z(r + s + \gamma^M \phi) + \phi(\gamma^M \frac{(\beta y^H - \beta rV)(r+s+\phi)}{r+s} + (1 - \gamma^M)w^L)\]

or

\[U_G = \frac{z(r + s + \gamma^M \phi) + \phi(\gamma^M \frac{(\beta y^H - \beta rV)(r+s+\phi)}{r+s} + (1 - \gamma^M)w^L)}{(r + s + \beta \gamma^M \phi)(r + s + \phi)}\]

Likewise for unemployed Red workers

\[U_R = \frac{z(r + s + \gamma \phi \eta) + \phi \eta(\gamma \frac{(\beta y^H - \beta rV)(r+s+\phi)}{r+s} + (1 - \gamma)w^L)}{(r + s + \beta \gamma \phi \eta)(r + s + \phi \eta)}\]
Appendix 8: Numerical simulations with a taste for discrimination

The analogue to (26) and (27) is

\[ J_i^j(\gamma, p) = \frac{\gamma}{r + s} ((1 - \beta)(y^H - (r + s)U_i) + (\beta r + s)V^j) + \frac{(1 - \gamma)}{r + s + \phi p_i} (y^L - w^L + (s + \phi p_i)V^j) \quad j = N, D. \quad (42) \]

Let \( W_i^j(\gamma) \) denote the expected value of employment for a color \( i \) worker in a type \( j \) firm given signal equals \( \gamma \)

\[ W_i^j(\gamma) = \frac{w_i^{Hj}}{r + s} + (1 - \gamma) \frac{w_L + \phi p_i E W_i}{r + s + \phi p_i} \quad (43) \]

where \( w_i^{Hj} \) is determined by (31). The value of a \( D \)–vacancy is

\[ V^D = \frac{-c + q(\alpha_G(\eta J^D_G(\gamma, 1) + (1 - \eta) J^D_G(\gamma, 1)) + \alpha_R(\eta J^D_R(\gamma, 1)))}{r + q(\alpha_G + \alpha_R\eta)} \quad (44) \]

and the value of a \( N \)–vacancy is

\[ V^N = \frac{-c + q(\alpha_G(\eta J^N_G(\gamma, 1) + (1 - \eta) J^N_G(\gamma, 1)) + \alpha_R(\eta J^N_R(\gamma, 1) + (1 - \eta) J^N_R(\gamma, 1)))}{r + q(\alpha_G + \alpha_R\eta)} \quad (45) \]

The hiring probabilities are given by

\[ p_G = 1 \quad (46) \]

\[ p_R = (1 - m) + m\eta \quad (47) \]

As before let \( \varepsilon_i \) denote the share of employed workers of type \( i \) in jobs with low match quality. For both types \( i \), the outflow from jobs with bad matches must equal the inflow to jobs with bad matches. For Red workers we have

\[ n_R\varepsilon_R(s + p_R\phi)(1 - u_R) = vq((1 - m)(1 - \gamma^M) + m\eta(1 - \gamma^L))\alpha_R. \]
Similarly, the outflow from good matches equals inflow to good matches

\[ n_R(1 - \varepsilon_R)s(1 - u_R) = vq((1 - m)\gamma^M + m\eta\gamma)\alpha_G. \]

Using the above two equations gives us

\[ \varepsilon_R = \frac{s(1 - \frac{(1-m)\gamma^M + m\eta\gamma}{1-m+mn})}{s + \frac{(1-m)\gamma^M + m\eta\gamma}{1-m+mn} \phi p_R}. \]  

(48)

Similarly we find that

\[ \varepsilon_G = \frac{s(1 - \frac{\gamma^M}{1+\gamma^M})}{s + \gamma^M \phi}. \]  

(49)

The expressions for \( U_i \) is given by (2), \( u_i \) given by (16) \( \alpha_i \) given by (17) and \( \theta \) by \( V^N = K \).

**Appendix 9: Numerical simulations with efficient hiring**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( s )</td>
<td>0.03</td>
<td>( \lambda )</td>
<td>0.6</td>
</tr>
<tr>
<td>( r )</td>
<td>0.012</td>
<td>( c )</td>
<td>0.02</td>
</tr>
<tr>
<td>( y^H )</td>
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<td>( K )</td>
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</tr>
<tr>
<td>( y^L )</td>
<td>0</td>
<td>( \eta )</td>
<td>0.65</td>
</tr>
<tr>
<td>( y^0 )</td>
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<td>( A )</td>
<td>0.3</td>
</tr>
<tr>
<td>( \kappa )</td>
<td>1</td>
<td>( \gamma )</td>
<td>0.7</td>
</tr>
<tr>
<td>( \beta )</td>
<td>0.5</td>
<td>( \gamma )</td>
<td>0.3</td>
</tr>
<tr>
<td>( w^L )</td>
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<td>( n_G )</td>
<td>0.9</td>
</tr>
<tr>
<td>( z )</td>
<td>0.9</td>
<td>( n_R )</td>
<td>0.1</td>
</tr>
</tbody>
</table>

With these parameter values, we find that a discriminatory equilibrium exists. The values of the key variables are displayed in Table 2. We find that \( S_G^0(\gamma) > 0 > S_R^0(\gamma) \), implying that, conditional on a bad signal, it is profitable to hire Green workers but not profitable to hire Red workers. Thus, again the advantage for the employer from Green workers being more attractive on the job market,
making them more likely to quit if badly matched, dominates the effect of Red workers being paid less. The discriminatory behavior leads to a large difference in unemployment rates, which is 24.6% for Green workers, and 33.4% for Red workers. As Green workers are hired even with a bad signal, a slightly higher share of them are in low productivity matches, the respective shares are 16.2% for Green workers and 12.5% for Red workers. Note that both types of workers must accept a large, negative wage in the first period, -3.67, to make it profitable for the firm to hire them.

Table 2: Simulation outcome

<table>
<thead>
<tr>
<th>Variable</th>
<th>Value</th>
<th>Variable</th>
<th>Value</th>
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</thead>
<tbody>
<tr>
<td>$\theta$</td>
<td>0.0523</td>
<td>$u_G$</td>
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<tr>
<td>$\phi$</td>
<td>0.092</td>
<td>$u_R$</td>
<td>0.334</td>
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<td>$S_G^0(\gamma)$</td>
<td>0.0004</td>
<td>$\varepsilon_G$</td>
<td>0.162</td>
</tr>
<tr>
<td>$S_R^0(\gamma)$</td>
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<td>$\varepsilon_R$</td>
<td>0.125</td>
</tr>
<tr>
<td>$w_G^0$</td>
<td>-3.67</td>
<td>$w_R^0$</td>
<td>-3.67</td>
</tr>
</tbody>
</table>
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