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Finn R. Førsund

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Department of Economics
University of Oslo

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P. O.Box 1095 Blindern
N-0317 OSLO Norway
Telephone: + 47 22855127
Fax: + 47 22855035
Internet: http://www.oekonomi.uio.no
e-mail: econdep@econ.uio.no

In co-operation with
The Frisch Centre for Economic Research

Gaustadalleén 21
N-0371 OSLO Norway
Telephone: +47 22 95 88 20
Fax: +47 22 95 88 25
Internet: http://www.frisch.uio.no
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Good Modelling of Bad Outputs: Pollution and Multiple-Output Production[1]

by

Finn R. Førsund

Department of Economics, University of Oslo

Abstract: The materials balance principle points to the crucial role of material inputs in generating residuals in production processes. Pollution modelling must be of a multi-output nature. The most flexible transformation function in outputs and inputs used in textbooks is too general to make sense in pollution modelling. Specifying bads *as if* they are inputs, although may be defendable on a macro level as a reduced form, hides explicit considerations of various modification activities. Extending the non-parametric efficiency approach to cover bads as outputs, assuming weak disposability of the bads as the only change in the modelling of the technology, has serious weaknesses. A complete taxonomy of inputs as to the impact on both residuals and marketed products as joint outputs is derived, based on factorially determined multi-output production, thus providing information for choice of policy instruments.

Keywords: *multiple-output production, pollution, bads, purification*, *DEA*

JEL classification: *D62, Q50*

[1] The paper is an extended and improved version of Førsund (1998).

1. Introduction

Pollution is generically a problem with joint outputs in economic activities of production and consumption. The first law of thermodynamics tells us that matter cannot disappear. If we weigh the inputs employed in an activity, including non-paid factors like oxygen from the air, and weigh the products that are the conscious purpose of activities, the difference is the residuals that may turn out to be polluting the natural environment. Thus, the general feature of residuals is that they arise from use of inputs in a wide sense. Ayres and Kneese (1969) coined the phrase *materials balance* to underline the inevitability of residuals generation when employing material resources.

Although multiple outputs are the rule rather than exception at the micro level of production, economists usually specify single output only. The preoccupation with single output production in textbooks may be due to the technical complexities involved, or that economic issues often can be dealt with aggregating outputs to a single output index. This may be interpreted as a reflection of economics being concerned about principles of resource allocation, revealing economic mechanisms and devising policy instruments, etc. Operating with a representative firm and an output aggregate at the micro level will usually suffice.

But as pointed out above this is a difficult position to maintain when dealing with pollution. Therefore, some form of multiple outputs involving "ordinary" or intended outputs, and unintended residuals or pollutants, or generically "bads", have been used at least indirectly in the old externalities literature, and explicitly in the more recent environmental economics literature (see Mishan (1971) for a review of the externalities literature, and Fisher and Peterson (1976), Cropper and Oates (1992) for reviews of the literature covering the 70-ies and 80-ies decades). However, the choice in the literature of specifications of relationships between ordinary outputs and bads vary and is often not based on any explicit consideration of the most suitable model to pick from the field of multiple output production function theory. (But see Whitcomb (1972) for an outstanding exception.) Since policy conclusions that can be drawn based on results from environmental economics are basically concerned with choice among instruments for control of externalities, a firm grasp on the modelling of the multi-output nature is essential.

The modelling should make it possible to study the most important ways of reducing polluting residuals. Textbooks in environmental economics (see, e.g., Førsund and Strøm, 1988) usually list reduction of output, substitution between inputs, purification, recycling of waste within the activity, and technical change, and then one may in addition consider change in the product (e.g., using less phosphate in detergents) and relocation of the polluting activity.

The plan of the paper is as follows. Multiple output models are reviewed in Section 2. The most common way of including pollutants is studied in Section 3, and a comprehensive critique of standard practises in the literature, including the efficiency literature, is offered. A more suitable multi output structure is discussed in Section 4, and a new and comprehensive taxonomy for inputs based on the materials balance concept, is offered in Section 5. Section 6 concludes.

2. Multi-output production

At the micro level of a firm multiple outputs are the rule rather than the exception. At the time of the publication of the seminal paper by Ayres and Kneese (1969) (see also Kneese et al., 1970) modelling multiple output technologies was more or less out of fashion, at least at the textbook level.[2] Ayres and Kneese inspired a new generation of economist to model multiple output technologies encompassing both goods and bads. However, more often than not this modelling effort did not related to the existing production function literature in economics, but invented its own modelling style. The field of ecological economics in particular, when relevant, followed up the material balance approach based on the production theory of Georgescu-Roegen (1971) without exploring the standard neoclassical production theory that will be pursued below.

[2] However, as pointed out in Kurz (1986), classical and early neoclassical economists used just the generation of unwanted by-products as the main example of joint production.

When we want to model multiple outputs we should be aware of some main situations giving rise to multiple outputs treated in the economics literature (see e.g. Frisch (1965, Chapter 1d.) for a brief introduction). Inputs may be employed alternatively to produce different outputs, e.g., a piece of agricultural land may be used to produce potatoes or wheat, a wood cutting tool may be used to produce different types of furniture. There is freedom of choice in what outputs to produce. At the other end of the scale we may have multiple outputs due to jointness in production; sheep yield mutton as well as wool, cattle yield beef and hide, we get both wheat and straw, and coal can be converted to coke and gas, to use classical examples from Edgeworth and Marshall. As an extreme form of jointness we have that outputs are produced in fixed proportions, as the distillates of crude oil in a refinery. We will regard a firm as the unit, and not discuss issues concerning internal organisation such as parallel production of commodities or process chains of intermediate products, etc. (see e.g. Danø, 1966).

The standard multi-output representation

When employing a functional representation of multi-output production the common practice is to specify a transformation function as a continuously differentiable manifold in the m-dimensional output vector, y, and the n-dimensional input vector, x:

$$F(y,x) \le 0\,,\ y \in R_{+}^{m}\,,\ x \in R_{+}^{n}\,,\ \frac{\partial F(y,x)}{\partial y_i} \ge 0\,,\ \frac{\partial F(y,x)}{\partial x_j} \le 0\,,\ i = 1,..,m\,,\ j = 1,..,n \qquad (1)$$

Efficient utilisation of resources may be associated with equality sign and inefficient operations with the inequality sign after the transformation function. As a standard convention outputs have non-negative partial derivatives and inputs non-positive ones. The assumption about the signs of the partial derivatives is termed *free disposability* of inputs and outputs. This is a precise mathematical assumption, but is often interpreted as the production unit being able to physically discard both available inputs and produced outputs.

The mathematical condition for free disposability of inputs has a straightforward economic interpretation. Assuming that efficiency of the operation prevails, $F(y,x) = 0$, and that we have strict inequalities of the partial derivatives in (1), the standard concept of marginal productivity of input j in the production of output i is derived by differentiating (1):

$$\frac{\partial F(y,x)}{\partial y_i} dy_i + \frac{\partial F(y,x)}{\partial x_j} dx_j = 0 \Rightarrow$$
$$\frac{dy_i}{dx_j} = -\frac{\partial F(y,x)}{\partial x_j} / \frac{\partial F(y,x)}{\partial y_i} > 0 \ , \ i = 1,..,m \ , \ j = 1,..,n \tag{2}$$

This is the condition for being in the interior of the *substitution region* (Frisch, 1965), or the *economic region*. Free disposability in an economic sense means that the use of inputs will not be expanded to yield negative productivities. Substitution possibilities between inputs *i* and *j* for given outputs are shown by standard product *isoquants* (Frisch, 1965):

$$\frac{\partial F(y,x)}{\partial x_i} dx_i + \frac{\partial F(y,x)}{\partial x_j} dx_j = 0 \Rightarrow$$
$$\frac{dx_i}{dx_j} = -\frac{\partial F(y,x)}{\partial x_j} / \frac{\partial F(y,x)}{\partial x_i} < 0 \ , \ i, j = 1,..,n \tag{3}$$

The trade-offs between outputs for given inputs are shown by *factor isoquants* (Frisch, 1965). Differentiating (1) the trade-off between two outputs *i* and *s* are:

$$\frac{\partial F(y,x)}{\partial y_i} dy_i + \frac{\partial F(y,x)}{\partial y_s} dy_s = 0 \Rightarrow$$
$$\frac{dy_i}{dy_s} = -\frac{\partial F(y,x)}{\partial y_s} / \frac{\partial F(y,x)}{\partial y_i} < 0 \ , \ i, s = 1,..,m \tag{4}$$

The economic interpretation of free disposability of outputs when production is technically efficient is that outputs will be produced in such a combination that an increase in one output always is at the expense of (at least) one other output. To say that free disposability of outputs means that outputs can be thrown away at no cost has no good economic meaning.

The Frisch multi-output model

As pointed out in Tran and Smith (1983, p. 35) "a neoclassical production model is an approximate summary of the engineering features of the underlying technology." The multiple output model of Frisch is set up in this spirit, and can satisfy the demands raised in Krysiak and Krysiak (2003) when physical constraints such as material balance are taken into consideration.[3] The case of freedom in directing inputs into any output is termed *assorted* production in Frisch

[3] Lau (1972) introduces a terminology for the multiple output case that covers some of the concepts introduced by Frisch, but no reference to Frisch is given.

(1965). The core production function apparatus of Frisch (1965, Part four) is based on these concepts, and may be described by a set of transformation functions:[4]

$$F^i(y_1,.., y_m, x_1,..,x_n) \le 0\,,\ \frac{\partial F^i(y,x)}{\partial y_s} \ge 0\,, \frac{\partial F^i(y,x)}{\partial x_j} \le 0\,,\ i = 1,.., \mu\,, s = 1,.., m, j = 1,.., n \qquad (5)$$

The relation between number of outputs and equations is defined as the *degree of assortment*[5]: $\alpha = m - \mu$. A special case is $\mu = 1$. We are then back to the common textbook specification (1) of multiple outputs. The degree of assortment is then m - 1, or in the Frisch spirit m-1 - dimensional freedom of assortment, i.e., the maximal degree. The flexibility in combining outputs and inputs is maximal. A negative number means that there exist one or more *pure factor bands*, i.e. there are relationships as part of (5) between inputs independent of outputs:

$$F^b(x_1,..,x_n) = 0\,, b \in B \qquad (6)$$

where B is the set of pure factor bands.

In the case of specific technical relations between products Frisch talks about *couplings* between the outputs. The *product couplings* restrict the freedom of combining outputs. The degree of couplings[6], κ, is the number of relations between outputs, in the system (5), independent of inputs:

$$F^c(y_1,.., y_m) = 0\,,\ c \in C \qquad (7)$$

where C is the set of coupling relations. Note that the sign convention for partial derivatives does not apply to factor bands or product couplings. Obviously the direction of a coupling, e.g., $dy_{ic} / dy_{sc} = -F^c_{y_{sc}} / F^c_{y_{ic}}\ (i, s = 1,.., m,\ c \in C)$, is unrestricted in sign, as well as the corresponding direction within a factor band. Thus the sign convention applies to relations in (5) only with both outputs and inputs simultaneously present.

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[4] Whitcomb (1972) is the only one to my knowledge that explicitly discusses the appropriate form of multiple-outputs model to use. He sets up a system with each firm both receiving and generating externalities as abstract concepts, and uses electricity production based on fossil fuel as an example, thus illustrating the main purpose of doing multiple outputs in the present paper. Although he refers to Frisch in general he does not relate his system to Frisch. In fact, his specification is a special variant of (5).

[5] As Frisch puts it in characteristic style on p.270: "mnemonically α may be thought of as the first letter in the word 'assortment'."

[6] Frisch, p.270: "mnemonically κ may be thought of as the first letter in the word 'coupling'."

A special case of (2) is the case of *no assortment*. We will argue below that this case is of especial relevance for pollution modelling. Frisch calls this case *factorially determined* multi-output production:[7]

$$y_i = f^i(x_1,..,x_n)\,, \frac{\partial f^i}{\partial x_j} \geq 0\,,\ i = 1,..,m\,,\ j = 1,..,n \tag{8}$$

The degree of assortment is (m - m) = 0, meaning that for a given set of inputs, all the outputs are determined. However, this is not the same as output couplings. There are output isoquant maps for each function in (8). To the degree that these do not coincide, we will realise outputs in different proportions varying the inputs. In the terminology of Frisch, we have *product separation*. If the isoquant maps coincide completely we then have the case of couplings, i.e. the outputs cannot be separated. The coupling may be proportional at the simplest, or exhibiting more complex variability.

3. Pollution modelling

As stated in the Introduction, a fundamental observation as to modelling pollution generated by economic activity is that residuals are an inherent part of this activity, following the materials balance approach (Ayres and Kneese, 1969). Generation of residuals can therefore be regarded as joint outputs in economic activities.

Since Equation (1) is the standard representation of multi-output production possibilities it is natural first to extend this formulation also to represent pollutant generation, z (see Baumol and Oates (1988), p. 37):

$$F(y,z,x) = 0 \tag{9}$$

[7] As far as I am aware of, the first papers to use this specification for pollution generation modeling, letting residuals be part of the outputs in (8), was Førsund (1972a,b).

where z is the residuals vector of k elements. In accordance with the basic nature of the residuals as joint outputs, following the sign convention of the partial derivatives, F'_{z_s} , should be positive. However, we immediately get into problems with such a formulation, as shown below.

Our purpose of pollution modelling is to demonstrate the importance of multiple-output modelling. The framework is therefore as simple as possible and of a partial equilibrium nature. The social planning problem is to maximise consumer plus producer surplus, introducing demand functions on price form, $p_i(y_i)$, for each output, using fixed q_j's as social evaluation coefficients for inputs j, and evaluating pollutants through the monetised *damage function*

$$D = D(z_1,..,z_k)\,,\ \frac{\partial D}{\partial z_s} \geq 0\,, s = 1,...,k \tag{10}$$

We may also think of a firm selling products to fixed prices, p_i, and buying inputs to fixed prices, q_j, in competitive markets, and paying a non-linear pollution tax, $D(z)$. The maximisation problem for a single (representative) firm, characterised by some multiple-output technology, is then (cf. Martin, 1986):

$$\text{Max} = \sum_{i=1}^{m} \int_{w_i=0}^{y_i} p_i(w_i)dw_i \ - \sum_{j=1}^{n} q_j x_j - D(z_1,..,z_k) \tag{11}$$

s.t. a multiple-output technology.

When considering several firms demand functions have to be adjusted according to type of demand interactions, and it must be specified whether the damage functions are unique to each firm, or the nature of interactions if there are any. This simple model makes the fundamental trade-offs between "ordinary" goods, y_i, and "bads", z_s, transparent, allowing for Pareto-optimal allocation rules both for these two types of outputs and for inputs, and facilitates introduction of policy instruments implementing optimal solutions, etc. The explicit representation of pollutants and environmental damage distinguishes it from the models of the externalities-literature, where the transmission mechanisms of external effects are implicit.

Pollutants as outputs in the standard formulation

To see the problem with pollutants, z, as outputs, let us assume that the production possibilities are characterised by (5). The necessary first order conditions of the social planning problem (11) are:

$$p_i - \lambda F'_{y_i} \le 0 \ (=0 \text{ for } y_i > 0), i = 1,..,m$$
$$-q_j - \lambda F'_{x_j} \le 0 (=0 \text{ for } x_j > 0), j = 1,..,n \qquad (12)$$
$$-D'_{z_s} - \lambda F'_{z_s} \le 0 \ (=0 \text{ for } z_s > 0), s = 1,..,k$$

where λ is the non-negative shadow price on the production possibilities, expressing alternative costs. Considering interior solutions the first two conditions can be combined to give the textbook result that a factor should be employed so that the value of its marginal productivity equals factor cost:

$$p_i \frac{-F'_{x_j}}{F'_{y_i}} = q_j , i = 1,..,m , j = 1,..,n \qquad (13)$$

There is no explicit reflection here of pollutants generation, i.e. no "cost punishment" for neither inputs nor outputs possibly associated with generation of pollutants.

Proposition 1: Socially optimal level of bads is zero when the multi output technology including bads has maximal flexibility, i.e., the degree of assortment, $\alpha = m - \mu = m - 1$ ($\mu = 1$) in relation (9), is maximal.

Discussion: From the last condition in (12) it follows that the generation of the polluting residual should be set to zero since equality cannot be obtained in the regular case considered here. Since there is no explicit formalisation of the inevitability of residuals in the formulation (9), this is the logical result, pollutants being bads in the objective function. Therefore, marginal rates of transformation like in (13) will naturally not reflect any pollution generation. The maximal flexibility of the multiple-output modelling, implying that inputs can be directed to the production of any of the outputs, leads inevitably to intuitively nonsensical results that zero bads can be achieved at no costs; inputs are simply reallocated without costs to produce more of goods with a positive value and to produce zero of bads with negative values. Therefore, it does not have a good meaning to specify residuals as joint outputs as in (9).

It might be objected that residuals can be made necessary, in the sense of assuming that $F(y, 0, x) = 0 \Rightarrow y = 0$, i.e. a form of "null jointness". This may be due to a limit form of cross derivatives, either between pollutants and outputs becoming positive and very large as pollutants decrease, or between inputs and pollutants becoming very small as pollutants decrease, but this line of reasoning seems a little far fetched and leads logically to introducing lower limits for pollutants. Then these limits will replace the zeros in the solution for pollutants, and Proposition 1 correspondingly changed.

Pollutants as inputs

To avoid the problem above one possibility is to treat residuals generation *as if* they are inputs. This option is followed without any comment or explanation in the influential textbook by Baumol and Oates (1975, Table 4.1, p. 39). When a defence of the procedure is offered, the most satisfactory position is in a macro setting. It is then argued that good outputs increase when residuals generation increases because this means that less resources are used on pollution abatement, and these freed resources are then transferred to output production (see e.g. Cropper and Oates (1992); as well as Tahvonen and Kuuluvainen (1993); Barbera and McConnell, 1990) for using this explanation). In a macro world of only one good the alternative cost of pollution is correctly expressed in terms of this good. A single relation with residuals as inputs may be regarded as a reduced form of a larger system. But the same argument maintained at the micro level seems more awkward, since we obviously lose information as to the nature of the firm level purification activity. Considine and Larson (2006, p. 649) state that generators of residuals need services from Nature to take care of these residuals, and that such services can be measured by the volume of residuals. However, this argument cannot cover up for the need at the micro level to explicit model the generation of residuals. A partial increase in a residual as input cannot technically explain that a good output increases by reasoning that inputs are reallocated from abatement activity to the production of goods. By definition the inputs that are explicitly specified in this relation must be kept constant. Having sort of additional inputs behind the scene is not a very satisfactorily way of modelling. Pittman (1981, p. 3) estimates a translog production function for pulp and paper firms using residuals as inputs, arguing that increase in residuals free resources to produce more output.

Since the sign of F'_{z_s} now is *negative* in the last necessary condition in (12), the condition is formally identical to the condition for employing an input, leading to a positive level of pollutants generation in the case of an interior solution. The factor price in (13) is replaced with the marginal damage. There are, however, several weaknesses with this formulation.

Proposition 2: When the multi output technology has a maximal degree of assortment with bads formally as inputs, i.e., relation (9) describes the technology with $F'_{z_s} < 0$, we have:

i) the socially optimal level of bads is only determined as a trade-off with goods,

ii) no explicit purification activity in the form of inputs can be identified.

Discussion: Generation of pollutants acts as any other input, and thus has a positive impact on every output in general. The pollutants are thus associated with outputs, not with inputs. Using the conditions in (12) the trade-off between outputs and residuals generation is expressed by:

$$D'_{z_s} = p_i \frac{-F'_{z_s}}{F'_{y_i}} \Rightarrow p_i = D'_{z_s} \frac{F'_{y_i}}{-F'_{z_s}}, i = 1,..,m, s = 1,..,k \tag{14}$$

The first expression in (14) says that polluting residuals should be employed up to the point that the value of the resulting output *i* is equal to the marginal damage caused by the pollutant. The second, equivalent, expression shows that the marginal value of the output in question is equal to the damage evaluation of the marginal unit requirement of pollutants generation. There is thus a formal trade-off mechanism in place linking outputs and pollutants, so if this is the connection we want to model at the micro level, we have at least a *qualitative* representation, but not a correct quantitative one, since an active part of the resource base is left outside the explicit modelling. Most awkwardly, purification activity is swept under the carpet. So if one is interested in optimal abatement *per se* this is not the appropriate formulation. Abatement by input substitution cannot be captured neither, because there is no link between the "real" inputs, *x*, and the residuals generation, *z*, specified as input. The model specification allows low sulphur coal to be substituted for high sulphur coal when factor prices change, but this has no impact on generation of sulphur as a residual!

Introducing technology restrictions

It should be noted that reducing the degree of assortment in (9), i.e., when representing bads as outputs, by introducing more relations of the form (9) as in (5), does not help in general. We get the same propositions 1 and 2. This can intuitively be realised by inserting (5), with the bads, z, as part of the output vector, in the optimisation problem (11). As long as the partial derivatives $F^{i\,\prime}_{z_s}$ are non-negative zero values of the *z*'s will result. We must introduce explicitly either product couplings between y and z when both are interpreted as outputs, or factor bands between

x and z if z is interpreted as inputs.

Product coupling

As a formally more satisfying amendment to the general representation (9) let us introduce a product coupling following Frisch. The production relationship system is then:

$$F(y,z,x)=0\ ,F'_{y_i}>0,F'_{z_s}>0,F'_{x_j}>0$$
$$G(y,z)=0,\ a)\,G'_{y_i}>0,G'_{z_s}<0,\text{or}\ \ b)\,G'_{y_i}<0,\ G'_{z_s}>0 \qquad (15)$$
$$i=1,..,m\,,s=1,..,k\,,j=1,..,n$$

The second relationship is the coupling between outputs and pollution generation. In order to model the inevitability of generating bads together with goods, the signs of the partial derivatives must be opposite, i.e. case a) and b), yielding a positive relationship between a good and a bad (but the choice of which one to be positive and negative is arbitrary). Null jointness may also be assumed, i.e. $G(y, 0) = 0 \Rightarrow y = 0$. We will not pursue the issues of special limit conditions. The nature of the relationships between the three sets of variables can be illustrated in Figure 1.

Now we can capture the fact that residuals generation cannot be avoided, and that producing outputs entail an extra cost in terms of pollution generation. If we want to connect residuals generation to a limited number of outputs, this is, of course, straightforward. Necessary conditions of the social planning problem (11) with (15) as technology are:

$$p_i-\lambda F'_{y_i}-\gamma G'_{y_i}\le 0\ (=0\,\text{for}\,y_i>0)\,,i=1,..,m$$
$$-q_j-\lambda F'_{x_j}\le 0\,(=0\,\text{for}\,x_j>0)\,,\,j=1,..,n \qquad (16)$$
$$-D'_{z_s}-\lambda F'_{z_s}-\gamma G'_{z_s}\le 0\ (=0\,\text{for}\,z_s>0)\,,s=1,..,k$$

where λ is the shadow price on the first production relationship and γ on the coupling relationship.

Proposition 3: When the multi output technology has a maximal degree of assortment with bads formally as outputs, i.e. relation (5) describes the technology, and an output coupling is added, then both resource cost of a good and a bad, and the marginal coupling cost between them are evaluated determining the socially optimal level of the bad.

Discussion: Assuming interior solutions and eliminating the Lagrangian parameters yield:

$$p_i = q_j \frac{F'_{y_i}}{-F'_{x_j}} + q_j \frac{F'_{z_s}}{-F'_{x_j}} \frac{G'_{y_i}}{-G'_{z_s}} + D'_{z_s} \frac{G'_{y_i}}{-G'_{z_s}} = q_j \left(\frac{F'_{y_i}}{-F'_{x_j}} + \frac{F'_{z_s}}{-F'_{x_j}} \frac{G'_{y_i}}{-G'_{z_s}}\right) + D'_{z_s} \frac{G'_{y_i}}{-G'_{z_s}} \tag{17}$$

$i = 1,..,m$, $j = 1,..,n$, $s = 1,..,k$

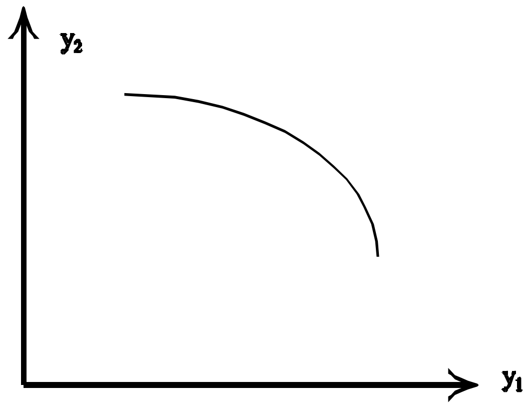
In addition to the first standard input cost term on the right hand side of the first equality sign, we have two new types of cost terms due to the coupling. From evaluating changes in the transformation functions in (15) we have that $F'_{y_i} / - F'_{x_j}$ is the marginal unit input requirement of input *j* producing one unit of output *i* (or marginal *fabrication coefficient* following Frisch), $F'_{z_s} / - F'_{x_j}$ is the marginal unit requirement of input *j* producing one unit of z_s, and $G'_{y_i} / - G'_{z_s}$ is the *marginal coupling effect* expressing the marginal increase in z_s for a unit increase in output *i*.

The interpretation of the two additional cost terms is then for the second term on the right- hand side of (17) that producing output *i* we must also produce residual z_s, and this has a resource cost expressed in terms of input *j*; the marginal coupling coefficient is multiplied with the marginal input requirement producing z_s and then costed according to the price of input *j*. The third cost element is expressing the marginal coupling effect of producing output *i* on z_s, and then costed at marginal damage caused by z_s. The last equation shows the two basic cost terms, consisting of a resource cost term encompassing the unit input cost of output *i* and the corresponding residuals generation input cost.

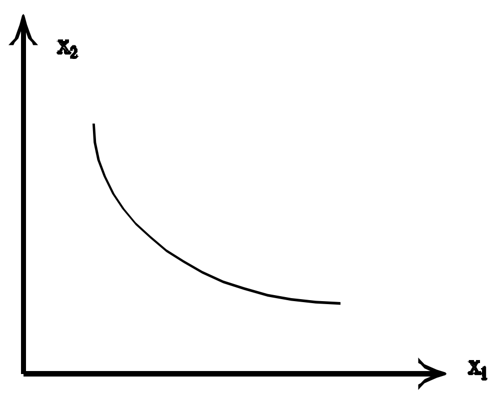
If we compare equation (17) with (14) we see the difference the introduction of an explicit coupling has made. Jointness in production of ordinary outputs and residuals generation is now represented formally with the extra resource cost term in (17), absent from (14). We have a formally satisfying exposition of how provision of outputs should be affected by the joint production of bads. However, the formulation is still too general to allow explicit insight into purification activities proper. The input-specific purification effects are expressed via different ratios of marginal productivities , $F'_{z_s} / - F'_{x_j}$, in bads-generation on marginal productivities in goods production, $F'_{y_i} / - F'_{x_j}$.

Factor bands

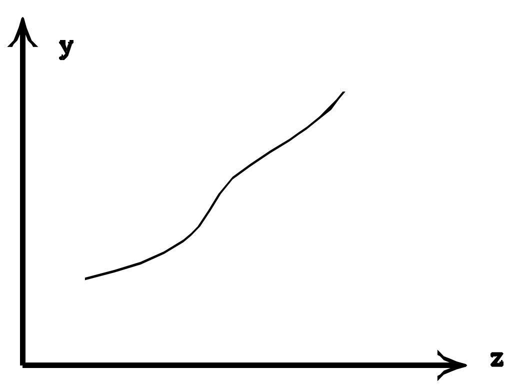
Since basically the pollutants are connected to the inputs used, it may also be logical to connect the bads, when interpreted as inputs, to ordinary inputs through factor bands. The model (9) will



Panel a) The factor isoquants



Panel b) The output isoquant



Panel c) The coupling curve

Figure 1 Multi-output production

then be:

$$F(y,z,x)=0\ ,F'_{y_i}>0,F'_{z_s}<0,F'_{x_j}>0$$
$$f(z,x)=0,a)f'_{z_s}>0,f'_{x_j}<0,\text{or}\,b)f'_{z_s}<0,f'_{x_j}<0 \tag{18}$$
$$i=1,..,m\,,s=1,..,k\,,j=1,..,n$$

In the factor band the two types of inputs may have opposite signs of partial derivatives, case a), or equal signs, case ii) (positive or negative do not matter). The former implies that the types runs parallel, while the latter implies a trade-off.

Necessary conditions of the social planning problem (11) with (18) as technology are:

$$p_i-\lambda F'_{y_i}\leq 0\ (=0\,\text{for}\,y_i>0),i=1,..,m$$
$$-q_j-\lambda F'_{x_j}-\gamma f'_{x_j}\leq 0(=0\,\text{for}\,x_j>0),\,j=1,..,n \tag{19}$$
$$-D'_{z_s}-\lambda F'_{z_s}-\gamma f'_{z_s}\leq 0\ (=0\,\text{for}\,z_s>0),s=1,..,k$$

where λ is the non-negative shadow price on the first production relationship and γ on the factor band.

Proposition 4: If factor bands are added to the multiple output relation (9) with the bads as inputs, then it is possible to identify purification activity.

Discussion: Assuming interior solutions and eliminating the Lagrangian parameters yield:

$$q_j=p_i\frac{-F'_{x_j}}{F'_{y_i}}+p_i\frac{-F'_{z_s}}{F'_{y_i}}\frac{-f'_{x_j}}{f'_{z_s}}-D'_{z_s}\frac{-f'_{x_j}}{f'_{z_s}}=p_i(\frac{-F'_{x_j}}{F'_{y_i}}+\frac{-F'_{z_s}}{F'_{y_i}}\frac{-f'_{x_j}}{f'_{z_s}})-D'_{z_s}\frac{-f'_{x_j}}{f'_{z_s}} \tag{20}$$
$$i=1,..,m\,,\,j=1,..,n\,,s=1,..,k$$

At the margin the costs and benefits of employing a factor should balance if this is to be used at a positive level. The benefit terms after the first equality sign in (20) start with the output price of output i times the marginal productivity of factor j in the production of this good. (The possibility of assortment is expressed by the fact that we do not sum over marginal effects on other outputs.) The next term is the value of good i created by the joint change in the bad as input due to the factor band relation, i.e., $-f'_{x_j}/f'_{z_s}$ expresses the amount of the bad z_s generated corresponding to one unit increase in the input x_j, the *marginal band effect*, and $-F'_{z_s}/F'_{y_i}$ is the productivity of the bad in the production of good y_i. The third term is the environmental cost term due to the marginal band effect. Note that if an ordinary input and a bad has a positive covariation, then increasing the input yields a positive output effect, but a negative environmental damage effect due to the factor band, while if the covariation is negative, then the two effects change sign, i.e., a negative output effect, but a positive environmental effect due to less damage. It is therefore natural to term the latter case corresponding to b) in (18), for a case with purification possibility.

Introducing the materials balance

Although the materials balance approach is firmly established in the economics literature on the environment it is not that often that one finds the materials balance condition explicitly stated. [8] Pethig (2003) and (2006) uses the materials balance to show that residuals cannot be modelled as inputs. (See also Martin (1986) for an extensive critique of this modelling approach based on the materials balance.) Taking the transformation function (9) as the point of departure, and assuming for simplicity that we have only a single output *y*, residual *z*, and material input *x*, the material balance is:

$$z = x - y \tag{21}$$

For simplicity the free inputs such as oxygen is included in the material input *x*, and all variables be measured in the same weight unit. Now, if the residual is assumed to be an input we get by differentiating (9):

$$F'_y dy + F'_z dz = 0 \Rightarrow \frac{dy}{dz} = \frac{-F'_z}{F'_y} > 0 \tag{22}$$

However, from the materials balance equation we have:

$$dz = -dy \Rightarrow \frac{dy}{dz} = -1 \quad , \tag{23}$$

contradicting the result in (22). It is not possible to reconcile the assumption of the residual as an input with the materials balance. The general conclusion is that the material balance matters for the modeling of joint good and bad.

The analysis of Pethig is extended to include abatement, observing the material balance principle that the total amount of residuals will actually increase if abatement is present. This simple point can be found in textbooks (Førsund and Strøm,1988), and lead to the rather obvious conclusion

[8] Papers referring to the materials balance principle, and where it is important for the understanding of environmental problems, may still not enter the material balance equation explicitly (Førsund, 1985; 2001).

that an integrated approach to residuals is needed, but actually introducing the material balance equations will also bring forward some limits on derivatives in the system of equations.[9]

4. The production set representation

Weak disposability of bads

When specifying multiple outputs one may use the production set representation (see e.g. Chambers, 1988). This specification makes it very straightforward to incorporate multiple outputs. The production possibility set, T is defined by:

$$T = \{ (y,z,x) : (y,z) \text{ can be produced by } x \} \tag{24}$$

This representation is equivalent to the functional representation (9) given some regularity conditions on the set T (see e.g. McFadden, 1978):

$$T = \{ (y,z,x) : (y,z) \text{ can be produced by } x \} \equiv \{ (y,z,x) : F(y,z,x) \leq 0 \} \tag{25}$$

The transformation-function representation of technology (9) is valid for the boundary of the production possibility set (25). Then also a technology set representation gives maximal flexibility in combining inputs into outputs, as expressed by (9), when no more restrictions are imposed. The definition of strong disposability at the boundary of the set T is the same as expressed by the signs of the partial derivatives of the transformation function (1) for "good" outputs and inputs. The implication for the set representation, encompassing inefficient operations, is that operations with less output, but fixed amounts of bads and inputs also belong to the set, and operations with more inputs but fixed amounts of goods and bads belong to the production possibility set.

If the bads are specified as inputs with non-positive partial derivatives of the transformation function (9), the bads are strongly disposable, but as shown in Section 3 this is in conflict with the material balance equation and is not tenable as a specification. Nevertheless, in the DEA

[9] Ebert and Welsch (2007) claim to have shown the equivalence of specifying residuals both as inputs and as outputs, but their result rests on inserting the material balance equation in (21) and eliminating material inputs as variables, and is thus not different from the conclusion in Pethig (2003), (2006) not specifying residuals as inputs.

literature there are papers that propose to treat bads as inputs (Hailu and Veeman, 2001; Scheel, 2001; Seiford and Zhu, 2002).

In order to capture the opportunity cost of reducing bad outputs Färe et al. (1986) were (to my knowledge) the first to propose to impose *weak disposability* on the production possibility set for the bads. This paper, and the follow-up paper Färe et al. (1989), has spawned a strand of research, applying the same assumptions, which may be termed the environmental performance indicator field.[10] A large number of papers have been published in a wide range of journals (Färe et al., 1996; Färe et al., 2001; Färe et al., 2004; Färe et al., 2005; Tyteca, 1997; Zhou et al., 2008; Zofio and Prieto, 2001). The technicalities of weak disposability (Shephard, 1970) are that if $(y,z,x) \in T$, then $(\theta y, \theta z, x) \in \mathrm{T}$ for $0 < \theta \leq 1$. Weak disposability implies a proportional reduction of the good when reducing the bad. Whereas strong disposability gives the maximal freedom of flexibility, i.e., there is a pure trade-off between outputs for given inputs, weak disposability introduce a restriction on the trade-off between outputs, i.e., it may be said that there is an opportunity cost of disposal.[11] In addition a condition of *null jointness* can be imposed, i.e. the good output cannot be produced without the bad, if the bad has zero value, so must the good. This case resembles a product coupling in the Frisch sense.

In view of the necessity of introducing additional restriction of couplings and/or factor bands to the flexible model (5) to make economic sense it may be of interest to have a closer look at the use of production set extended to bads by specifying weak disposability for the bad. Figure 2 illustrates the approach in Färe et al. (1986, 1989). Weak disposability means that the technology set in output space of one desirable good and one bad for given amounts of inputs is portrayed by OABCO in Färe et al. (1986), while strong disposability also for the bad means that the technology set is within OEBCD, i.e., the dotted horizontal line EB and the vertical dotted line CD to together with BC limits the production possibility set. Now, points on the border OAB correspond to a positive output coupling, case a) in (15) above. Continuing along the border to C, the coupling changes sign, we get case b). It may be difficult to find empirical examples, but may

[10] The use of the term environmental performance indicator within this approach seems to have introduced by Färe et al. (1996).

[11] The word "disposable" may be a little misleading here. In Färe et al. (1994, p.38) it is stated: "Weak disposability refers to the ability to dispose of an unwanted commodity at positive private cost." However, it is not the case that an output already produced is disposed of.

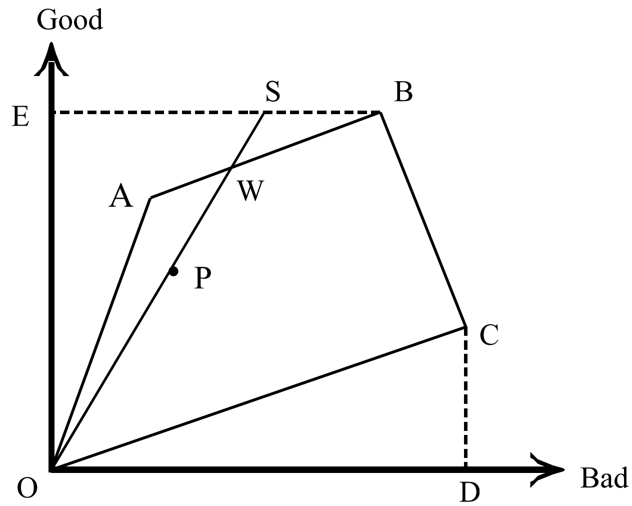


Figure 2. Strong and weak disposability
Source: Färe et al. (1986), (1989)

be not impossible. Then along CO we are back to case i) again. The null jointness is here symmetrical. In the next application of this model in Färe et al. (1989) a non symmetrical null jointness is illustrated, using the vertical line from C to D on the horizontal axis as the border forthe production possibility set when the combination strong disposability for the good and the input, and weak disposability for the bad is used. We then have that $(y, w) = (0,0)$ if $w = 0$, but $(y, w) = (0,w)$ if $y = 0$.

We should note some crucial differences with this model compared with the approach in the previous sections. It is not the case that reducing the bad is always costly in terms of reducing the good. For efficient units observed on the segment BC the bad can be reduced and the good increased. Indeed, this is potentially the case for any inefficient point inside the region OABCO or OABCD. (Remember that the input vector x is constant for the observation and is the amount required to be on any point the frontier boundary by definition.) It is only for efficient points on the segments OAB that we have costs in this sense. It is only this part of the technology set that corresponds to our model (15), case a).

The use of strong and weak disposability to calculate costs of regulations in Färe et al. (1986) and (1989) should be commented upon. The idea is that strong disposability is assumed to hold before the introduction of environmental regulations, and weak disposability after the introduction.

Considering an inefficient point (observation of a firm) P in Figure 2, the cost of regulation by applying a radial Farrell output-oriented efficiency calculation, is shown by calculating OW/OS, or 1 - OW/OS as the loss of goods.

This approach was refined in Färe et al. (1989) by using a hyperbolic efficiency measure, finding a comparison point on the frontier for an inefficient operation by simultaneously increasing the output of goods and decreasing the output of bads with the inverse factor, and also decreasing the inputs with the inverse factor, or keeping inputs fixed. The ratio of the hyperbolic efficiency measure assuming strong disposability and the hyperbolic efficiency measure assuming weak disposability for the bads was used to indicate "reduced productivity due to a forced departure from strong disposability of undesirable outputs." (Färe et al. (1989, p. 94). The problems with having a frontier like the line BC in Figure 2 is illustrated by the fact that 12 of 30 paper mills are located on this frontier, and thus do not have any opportunity cost in terms of reduced output of the good when reducing bads.

But this is a very peculiar way of seeing regulations. If the technology is of the strong type before environmental concern, certainly the optimal answer for firm P is to move straight to the vertical axis maintaining the same output (or even better to move to efficient goods point on the vertical axis corresponding to B). The same amount of resources is consumed, and no bads are produced. A regulator's dream! If strong disposability holds for every production unit before regulation, the only reason for observations outside the goods-axes must be inefficiency in using the costless opportunity of getting rid of all bads. The realistic situation corresponding to joint production of goods and bads seems to be that the weak disposability is the correct technology description, also before environmental concern. The most logical position seems to be that firms simply ignore the bads before regulation as mentioned above, and after regulation the complete technology description has to be taken into consideration. In order to calculate costs of regulation the regulation itself must be introduced. The rather huge number for regulation costs calculated in Färe et al. (1989), using a hyperbolic measures as explained above, is therefore of a very doubtful nature. At least in one reference (Tahvonen and Kuuluvainen, 1993, p.104) the number is taken at face value as the cost of regulation.

In Färe et al. (1996) the residual is removed from the model when calculating the efficiency score when forming the environmental performance index. This seems to be a better approach than above. However, the weak disposability formulation is still used as the only way of introducing joint production of goods and bads, so the weaknesses exposed above about this approach remain.

The change in the sign of the relationship between goods and bads as shown in Figure 2 is noted as problematic in Picazo-Tadeo and Prior (2005), and they state that the literature on the measurement of environmental performance runs into hundred of papers, but that few attempts have been made to explain this.[12] However, their solution seem to be to get rid of the negatively sloped parts by suggesting that errors of observation are causing these parts.[13,14]

Productivity measures with bads

Pittman (1983) addresses the question of how to credit production units for reduction of bads when measuring their productivity development. He uses the Caves et al. (1982a) multilateral productivity measure. The revenue share of the outputs is adjusted for bads by including the values of bads using the shadow prices from profit maximising with restrictions on the amount of bads.

The purpose of Färe et al. (1993) is to calculate shadow prices for bads based on output distance functions in order to adjust for bads in productivity measurement, like in Pittman (1983). This is done using duality theory "rather than constructing shadow prices from abatement costs,..." (Färe et al., 1993, p. 374) It is claimed that the shadow prices for the bads reflect the opportunity cost in terms of foregone revenue. However, the approach is based on making the assumption of weak disposability of the bads and null jointness. Therefore, this analysis has the same weaknesses as discussed above.

Another way to consider reduction of bads when measuring productivity is based on using a

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[12] To my knowledge this is a correct observation, because the only papers I know pointing out this problem is Førsund (1998) and Murty and Russell (2002) (neither papers are referred to in Picazo-Tadeo and Prior, 2005).

[13] Discussing this problem, based on Picazo-Tadeo and Prior (2005), Thanassoulis et al. (2008, p. 306) state: "However, as yet there is no definitive resolution of this issue in the literature." Førsund (1998) and Murty and Russell (2002) providing a resolution are not referred to.

[14] It is interesting to note that in Fried et al. (2008, p. 51) Figure 2 is reproduced, while in Färe et al. (2005, Fig. 2, p. 474) the part with the wrong slope is drawn conspicuously small, and in the preceding figure there is a vertical line closing the production possibility set.

directional distance function approach (Chung et al., 1997; Färe et al., 2008; Kumar, 2006). The resulting productivity index is called the Luenberger productivity indicator in this literature.. However, again the key point of departure is to make the assumption of weak disposability of bads and null jointness. Again, the weaknesses of these assumptions also apply here.

Färe et al. (2004) calculate an environmental performance index by using Malmquist output indices (Caves et al., 1982b) for the goods, respectively bads, and then forming the ratio of the change in the goods volume index over the change in the bads volume index, in a way resembling a productivity measure. The comparison may be between two different units, or for the same unit for two different time periods. However, when calculating the volume index for the bads the assumption of weak disposability is used, and thus the problems with this specification elaborated upon above are also present here.

Representing explicitly the generation of bads

The ways bads are generated are not shown explicitly in the weak disposability model above as done with product couplings and factor bands in Section 3. But in order to represent the materials balance a better approach than operating with output couplings and factor bands is to introduce the way residuals are generated, namely by using material inputs. This is done in Murty and Russell (2002).[15] The two relations specified are

$$\begin{aligned} & F(y,z,x) \le 0 \ , F'_{y_i} > 0, F'_{z_s} > 0, F'_{x_j} > 0 \\ & z \ge g(x), g' > 0 \end{aligned} \tag{26}$$

Two production possibility sets are specified:

$$\begin{aligned} & T_1 = \{ (y,z,x): F(y,z,x) \le 0 \} \\ & T_2 = \{ (y,z,x): z \ge g(x) \} \end{aligned} \tag{27}$$

The overall technology set is then the intersection of these two sets; $T = T_1 \cap T_2$. Elaborating on these sets it is shown in Murty and Russell (2002) that as a "reduced form" relation it is a non-negative correlation between a good output and a bad.

Coelli et al. (2007) make direct use of the materials balance approach and point out that the weak

[15] Note that the material balance equation (21) is not specified explicitly, but rather how the generation of residuals is related to the use of material inputs.

disposability formulation to cover bads is not consistent with the materials balance concept (and neither is a bad specified as an input). If (21) applies, then there cannot be any interior point (i.e., inefficient point) in the production technology, as assumed in the illustration of the technology in, e.g., Färe et al. (1989).[16] Consider that (21) applies to a point in the interior of the production possibility set portrayed in Figure 2. Looking for a hyperbolic measure of a common factor, λ, of increase in the good and a decrease of the bad and the input, we have:

$$z / \lambda = x / \lambda - y\lambda \qquad (28)$$

The only solution is $\lambda = 1$, implying that the point must be a frontier, or boundary point. This will also be the case if inputs, respectively goods, are kept constant. The point is that the materials balance is absolute and in itself does not allow inefficiency.

The approach of Coelli et al. (2007), in a more general setting than (21), assuming multiple inputs and multiple goods that relate to the measuring unit of the pollutant through fixed coefficients, is to focus on the combination of inputs that results in the lowest possible quantity of pollution for a specified amount of outputs. The environmental efficiency measure is thus defined as the minimum of the input that generates pollution over the observed use. The measure can be decomposed multiplicatively into a technical efficiency measure calculated in the standard way in the DEA literature of a relative comparison of inputs containing the pollutant at the frontier, and the observed level, and an allocative measure comparing the optimal with the observed input mix point projected to the frontier.

Abatement of bads

In the approach of specifying weak disposability for the bads the basic idea is that reduction of output is the only possibility of abating pollution for efficient units.[17] However, as pointed out in the Introduction, reduced level of production is only one of several ways of abatement. Purification of residuals is the most common option to include in standard textbooks in environmental economics. Using good output reduction to reduce generation of residuals may be the most costly way of abating pollution, missing out both on input substitution and purification. In the environmental performance indicator literature the position is somewhat ambivalent on the

[16] However, as pointed out in Coelli et al. (2007), if the output has zero material content, then this problem does not exist. This is the case for electricity production that is studied in Färe et al. (1986), (1996), (2005).

[17] In Fried et al, (2008, p. 50) it is clearly stated that reducing the good output is the only way to reduce the bad.

issue of purification. In several papers, just as for papers referred in Section 3, it is indicated that an explanation for the proportional reduction of goods and bads is that in order for bads to be reduced, resources are taken away from the production of goods to abate pollution.[18] However, this possibility is outside what is actually modelled. The input vector is fixed, so reallocation of inputs from producing goods to abating bads is not allowed. Ideally, purification in real sense should influence the *shape* of the technology set; bads are reduced for given level of goods when increasing use of inputs. The technology set in Figure 2 should shift North - West.

In Murty and Russell (2002) abatement is explicitly introduced as an activity, one variant is that abatement output is purchased, and another variant is that abatement is produced within the production unit. Abatement is viewed as an intermediate input of production. Introducing y_a as the intermediate abatement output and x^1 as material inputs and x^2 as "pollution-free" inputs the following extensions are:

$$\begin{aligned} & y_a = h(z, x^2)\ ,\ z \geq g(y_a, x^2) \Rightarrow \\ & F(y, h(z, x^2), x^1, x^2) = F(y, z, x^1, x^2) \leq 0 \end{aligned} \tag{29}$$

The reduced-form trade-off between goods and bads will now capture both reduction of goods and abatement efforts.

5. Factorially determined multi-output production

We will now return to the Frisch multi-output production system, and concentrate on efficient operations. This is not to say that efficiency issues are not of interest concerning pollution, but that to clarify modelling issues assuming technical efficiency is demanding enough, and also corresponds to the style of analysis in textbooks in environmental economics. As mentioned previously, Frisch was inspired by engineering in his approach to production theory (Førsund, 1999). It is therefore not surprising that his approach covers the need for including residuals as part of the production process. Neoclassical production theory is criticised in the literature for not incorporating residuals (Tran and Smith, 1983), but it is unfortunate that the critiques are not

[18] In Färe et al. (2001) it is stated. "…, abatement uses resources that otherwise could have been used to expand production of the good output.", and Färe et al. (2008,p. 561) state: "..disposal of bad outputs is costly – at the margin, it requires diversion of inputs to 'clean up' bad outputs…".

familiar with Frisch (1965). However, it turns out that several authors formulate relationships that correspond directly to the Frisch approach of factorially determined multi-output production without being aware of this (van den Bergh, 1999; Mäler, 1974; Martin, 1986).[19]

Residuals arise from use of inputs in a wide sense. Any co-variation with outputs is incidental and not expressing any natural law or engineering relationship. Sulphur as a bads output in electricity generation stems from the sulphur content of the fossil fuel used, may it be coal, oil, peat, etc. and is not linked to electricity output as such, but because *both* outputs result from applying certain inputs. Therefore, it would seem desirable to establish a multi-output structure reflecting this fact. Pollutants cannot be inputs in an engineering sense. According to the materials balance principle they are linked to inputs, just as marketed products, but cannot be varied partially in an engineering sense to influence those outputs, as ordinary inputs. One may say that a firm is using “renovation services” of Nature as inputs when discharging residuals, and that the latter serve as measures of such inputs. However, it is still meaningless in an engineering sense to assume that partial increases in the use of these services can increase marketed outputs directly. In addition to modelling the bads as outputs, we would like to see explicitly the purification possibilities. Remember that the main purpose of the exercise is to provide information as to choice of instruments to reduce pollution.

In Section 3 the Frisch concepts of product bands product couplings were explored. However, the special system Frisch called *factorially determined multi-output production* seems best suited for modelling bads and incorporating the materials balance approach. The general model (5) is then replaced with:

[19] Both Anderson (1987) and van den Bergh (1999) use production function concepts from Georgescu-Roegen (1971), which hardly offer any additional insights to that of Frisch (1965).

$$y_i = f^i(x_1,..,x_n), \frac{\partial f^i}{\partial x_j} \geq 0,\ i = 1,..,m,\ j = 1,..,n$$

$$z_s = g^s(x_1,..,x_n), \frac{\partial g^s}{\partial x_j} \gtrless 0,\ j = 1,..,m, s = 1,..,k \qquad (30)$$

$$\sum_{s=1}^{k} z_s = \sum_{s=1}^{k}\sum_{j=1}^{n} a_j^s x_j - \sum_{s=1}^{k}\sum_{i=1}^{m} b_i^s y_i$$

Both the outputs and residuals generation are functions of the same set of inputs. Choosing input levels, the production of bads follow. The formulation is suited for grouping the normal goods on one hand, and the residuals on the other. The crucial feature for pollution modelling is the second set of relations. The third type of relation is expressing the material balance principle (using equality implies that all free inputs from Nature, like oxygen and water, are included.). For simplicity the residuals are measured in the same weight unit, and then unit coefficients a_j^s and b_i^s translate from the input, respectively output unit of measurement to units of weight of the different types of residuals (Coelli et al., 2007). Typically, many of the coefficients will be zero.

Note that the marginal productivity of some inputs may be zero in the first relationship, given that the list of *n* inputs is exhaustive. Such inputs will be unique to purification activities. The marginal productivities may be positive, zero or negative in the residuals generation function. Positive productivities correspond to ordinary inputs with positive productivities also in the output relations. These inputs will typically be materials and energy. Inputs with zero productivities in the residuals generation function may have positive productivities in the output relations, but do not generate residuals (e.g., capital equipment, labour). Inputs with negative productivities in the residuals generation function are either purification inputs proper, i.e., they then have zero productivities in the output relations, or they may be both purification inputs and production inputs with positive productivities in the output relations, e.g., labour that operates processes, and the more labour the less waste of materials due to more careful handling and consequently more output for given amounts of materials, and using more capital to automate the handling of raw materials making the process more resource efficient and thus reducing waste and increasing production. Such effects may also be connected with recycling or materials-recovery activities. Waste heat may be recaptured by applying more capital in the form of heat exchangers and reduce the amount of residuals for constant primary energy, and thus increase production (Martin, 1986). Such efforts cannot then be identified as a separate purification activity, but such inputs may be applied with also the residuals generation effect in mind. It is, in fact, quite realistic that it may be impossible to identify a purification activity within a plant. In the case of "end of pipe" purification or a separate "purification division" we have the case of inputs with zero productivity in the production relations, and negative productivities in the residuals generation function.

In order to cover as many options as possible the list of n inputs should be general, as mentioned above, and also cover inputs not in current use. Consider the example of input substitution that did not work within the framework of the general formulations (1) or (9). The use of various sources of energy may be chosen to be only heavy oil in the production relations in (30) without any environmental concern. If the price of this input is increased, sooner or later the firm will switch to light oil, and then the generation of sulphur will, quite correctly, decrease according to the residuals generation function (keeping the same caloric value of primary energy).

We have above identified five types of inputs. We will term them *dirty*, *clean*, *green*, *pure purification*, and *integrated purification*. The classification of the first three is according to the total impact on environmental damages. Corresponding to the materials balance principle we have opened for keeping track of the complete picture of residuals generation, i.e., when employing an input, residuals may increase as well as decrease. The total environmental impact is measured by the damage function. In addition, we will add the characterisation strong and weak to the dirty, clean and green terminology, according to the nature of the residuals generation.

The general social planning problem (11) will now take the following form:

$$\text{Max } \sum_{i=1}^{m} \int_{w_i=0}^{y_i} p_i(w_i)dw_i - \sum_{j=1}^{n} q_j x_j - D(z_1,..,z_k)$$

$$\text{s.t.} \qquad (31)$$

$$y_i = f^i(x_1,..,x_n),\ z_s = g^s(x_1,..,x_n),\ i = 1,..,m,\ s = 1,..,k$$

The materials balance equation in (30) is dropped for simplicity, because the relation will not influence the type of trade-offs that we are looking for. But the material balance equation will put bounds on derivatives in empirical modelling, as shown in Pethig (2003), (2006), see (23). Inserting the relations for goods and bads the necessary first order conditions are:

$$\sum_{i=1}^{m} p_i f_{x_j}^{i}{}' - q_j - \sum_{s=1}^{k} D'_{z_s} g_{x_j}^{s}{}' \leq 0 \ (= 0 \text{ for } x_j > 0), \ j = 1,..,n \tag{32}$$

The rules for whether or not an input should be used at all, or a product supplied, follows from the first-order conditions in the standard way of the Kuhn -Tucker problem, e.g., it may be economic only to use one type of energy source, etc.

Proposition 5: Specifying the multi output production to be factorially determined as in (30) allows:

i) an exhaustive classification of the effects of inputs

ii) reveal all technical possibilities of changing residuals generation.

Discussion: Assuming interior solutions and rearranging yield:

$$q_j + \sum_{s=1}^{k} D'_{z_s} g_{x_j}^{s}{}' = \sum_{i=1}^{m} p_i f_{x_j}^{i}{}' \ , \ j = 1,..,n \tag{33}$$

Using these relations the different cases of inputs are set out in Table 1 below. The horizontal line in the middle distinguishes between application of inputs without any environmental concern (top part) and environmentally conscious applications (bottom part), based on the damages impact. In a dynamic setting consequences of technical change may be identified by changes in the $f^i(.)$ and $g^s(.)$ functions in (31). We have technical progress if $f^{i*}(x) > f^i(x)$, and $g^{s*}(x) < g^s(x)$, where the most recent technologies are marked with "*".

Let us try to give examples of the classification in Table 1. A strongly dirty input means that it has a positive productivity in the production of marketed goods, but that the use of this input also increases the generation of a subset of residuals, and no component of this subset is reduced, therefore the term strong. The input may be fossil fuel in electricity production. Residuals such as SO_2, NO_x, CO_2, soot, particles, etc. may all increase by increased use of the fuel. The total marginal damage increases, therefore the term dirty. Weakly dirty means that while some residuals are increased, one or more residuals are also decreased, but the total marginal damage is still increasing. One example may be from cement production; increasing the input of clinker increases the amount of calcium particles, but reduces the amount of sulphur due to chemical reactions.

A clean input means that there is no net environmental damage according to the damage function. Strongly clean means that no residuals are generated. For this to be true we must have stock inputs like capital and labour. If the use of capital is increased, e.g. substituting labour at no increased use of energy, no extra residuals are generated. Weakly clean means that some residuals may increase, other decrease, but the total marginal damage remains constant. This is a limiting case of weakly dirty. A material input must be involved. In the corresponding cement example above it may be the case that the increased damage from particles is exactly balanced by the reduction in sulphur emissions. An input is strongly green if all residuals that are generated decrease, so that the total marginal damage is reduced, and weakly green if some increase, some decrease, but still reducing total marginal damage.[20] An example of the former may be labour engaged in better supervision of the production process and succeeding in reducing various forms of waste of raw materials. A weakly green input may be capital, increasing the use of energy and then generating associated residuals, but making better use of, e.g., a fibre inputs in a pulp and paper process. The reduction of fibres is weighing more in damages reduction than the increase in energy- related residuals.

The separable purification is the standard end-of-pipe purification activity, e.g., a waste water treatment plant connected to a firm using process water. The inputs used in the end-of-pipe plant have no impacts on the production of marketed goods, other than if the total amount of inputs are considered given and thus production is reduced if resources are diverted to purification (Martin, 1986), and this plant may increase generation of some residuals, and necessarily reducing the generation of others. To capture this materials balance principle, *modification* should be used as the terminology instead of purification (Russell and Spofford, 1972). Organic waste may be removed from the waste water, but sludge is created instead that is solid waste. In addition some chemicals may be used to deal with bacteria, so we have a plus and minus situation here to. But the total activity may be termed purification because total marginal damage is reduced.

Finally, some technological ways of dealing with residuals are integrated within the production process of the marketed goods. Increasing inputs to reduce damage impact may then divert

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[20] Cf. Martin (1986, p. 349): "…one can distinguish 'production' inputs from 'abatement' inputs on the basis of the sign of their externality marginal products."

resources from production proper, resulting in partial decrease of marketable products, but production may also be increased to recycling of waste as inputs. Although some residuals obviously are decreased, some may also increase. An example may be that some key process machinery is improved in order to contain residuals, but this feature reduces the production capacity, another example may be redesign of equipment to reduce some emissions like particles, but needing more energy, with accompanying residuals increases. An example of both increasing production of marketed goods and reducing residuals may be recovering of waste heat integrated in the production equipment, e.g. a closed aluminium oven. This will increase production and reduce energy residuals as well as other gases and particles.

The most popular case of abatement in environmental economics textbooks is separable purification activity or end-of-pipe. According to the signs of partial derivatives in Table 1 we may then write the generation of emissions on an additive form, assuming just one type of residual:

$$z = g(x_1,..,x_{n_y}) - a(x_1,...,x_{n_a}), \frac{\partial g}{\partial x_j} \gtrless 0, \frac{\partial a}{\partial x_i} > 0 , j = 1,..,n_y , i = 1,..,n_a \qquad (34)$$

Only two input types, production of goods and in the production of abatement, are considered for simplicity. The amount abated is the net amount. However, the materials balance approach teach us that the purification processes themselves may also generate residuals, as pointed out above, and not only of the type they are set up for purify, but residuals that are modifications of the primary residuals. Catching particles emitted to the air by electrostatic precipitators leaves us with solid waste, etc.

A more general formulation of end-of-pipe may use the idea of relating outputs from activities, in the case of purification intermediate outputs, to the inputs like in (30). However, for each purification activity where end-of-pipe is relevant, the set of secondary residuals that are generated also has to be modelled. The direct abatement is then to be subtracted from the primary generation covered by the $g^s(.)$ functions in (30), but the generation of the type of residual in question generated in the other abatement activities has to be added. Each purification activity may generate several types of residuals, and also types of residuals that are not generated in the main production process. This generalisation is left to the interested reader.

Table 1. Classification of inputs
(note the slight change in the symbols for partial derivatives; the slashes are dropped for convenience)

Input type	**Production impact**	**Residuals impact**	**Damage impact**	**Optimality condition**
Strongly dirty	$f_{xj}^{i} > 0, \forall i$	$g_{xj}^{s} \geq 0, \forall s$	$\Sigma_s D_{zs} g_{xj}^{s} > 0$	$q_j + \Sigma D_{zs} g_{xj}^{s} = \Sigma p_i f_{xj}^{i}$
Weakly dirty	$f_{xj}^{i} > 0, \forall i$	$g_{xj}^{s} \gtrless 0, \forall s$	$\Sigma_s D_{zs} g_{xj}^{s} > 0$	$q_j + \Sigma D_{zs} g_{xj}^{s} = \Sigma p_i f_{xj}^{i}$
Strongly clean	$f_{xj}^{i} > 0, \forall i$	$g_{xj}^{s} = 0, \forall s$	$\Sigma_s D_{zs} g_{xj}^{s} = 0$	$q_j = \Sigma p_i f_{xj}^{i}$
Weakly clean	$f_{xj}^{i} > 0, \forall i$	$g_{xj}^{s} \gtrless 0, \forall s$	$\Sigma_s D_{zs} g_{xj}^{s} = 0$	$q_j = \Sigma p_i f_{xj}^{i}$
Strongly green	$f_{xj}^{i} > 0, \forall i$	$g_{xj}^{s} < 0, \forall s$	$\Sigma_s D_{zs} g_{xj}^{s} < 0$	$q_j = \Sigma p_i f_{xj}^{i} + \Sigma D_{zs}(-g_{xj}^{s})$
Weakly green	$f_{xj}^{i} > 0, \forall i$	$g_{xj}^{s} \gtrless 0, \forall s$	$\Sigma_s D_{zs} g_{xj}^{s} < 0$	$q_j = \Sigma p_i f_{xj}^{i} + \Sigma D_{zs}(-g_{xj}^{s})$
Separable purification	$f_{xj}^{i} = 0, \forall i$	$g_{xj}^{s} \gtrless 0, \forall s$	$\Sigma_s D_{zs} g_{xj}^{s} < 0$	$q_j = \Sigma_s D_{zs}(-g_{xj}^{s})$
Integrated purification	$f_{xj}^{i} \gtrless 0, \forall i$	$g_{xj}^{s} \gtrless 0, \forall s$	$\Sigma_s D_{zs} g_{xj}^{s} < 0$	$q_j + \Sigma p_i(-f_{xj}^{i}) = \Sigma D_{zs}(-g_{xj}^{s})$

Implementing the optimality conditions

The classical policy instrument for implementing the optimal solution is pollution charges. Considering only one firm, and for simplicity considering only one output, and assuming that the firm operates in competitive markets for the product and the inputs, a fixed tax, t_s, can be levied on each residual. The profit maximising problem of the firm in our static setting is:

$$\text{Max } \pi = py - \sum_{j=1}^{n} q_j x_j - \sum_{s=1}^{k} t_s z_s$$

s.t. (35)

$$y = f(x_1,..,x_n)\,,\, z_s = g^s(x_1,..,x_n)$$

Inserting the production relations yields the following first-order conditions for inputs:

$$pf'_{x_j} = q_j + \sum_{s=1}^{k} t_s g^{s\,\prime}_{x_j}\,,\, j = 1,..,n \tag{36}$$

For the inputs the firm decides to use in positive quantities we have the condition that the value of the marginal products of an input is set equal to the input price plus the tax paid on the emission of residuals. Comparing the social optimality condition (33) we see that such a system of taxes can in principle realise the optimal solution in competitive markets if taxes are set equal to the marginal damages of residuals; $t_s = D'_{z_s}$.

Comparing the relative use of inputs with the extreme classifications; a strongly green input *i* and a strongly dirty input *j*, we have:

$$\frac{f'_{x_i}}{f'_{x_j}} = \frac{q_i + \sum_{s=1}^{k} t_s g^{s\,\prime}_{x_i}}{q_j + \sum_{s=1}^{k} t_s g^{s\,\prime}_{x_j}},\; i, j = 1,..,n \tag{37}$$

With the environmental policy instrument in place the relative cost of the strongly dirty input increases, since the productivities in the generation function $g^s(.)$ are non-negative and thus the input cost becomes higher than the market price of the input, while the opposite is the case for the strongly green input with negative productivities. A strongly green input gets an "environmental discount." However, the environmental discount cannot exceed the input price in the optimal solution. The relative use of the strongly dirty input goes down compared with the situation before introducing the charges on residuals, as can be seen from the new values of the marginal rate of substitution on the left-hand side of (37), assuming positive and decreasing marginal productivities.

The various cases for inputs with different classification as set out in Table 1 can be worked out straightforwardly. Notice especially separable purification. The abatement activity will typically generate a separate set or overlapping set of residuals. But a condition for using resources on such abatement is that total damage from the activity goes down. This means that the abatement inputs also will have environmental discounts. The increase in residuals generation due to abatement activities is usually not modeled in environmental economics. The condition in Table 1 is that the total impact on the environment of reductions of primary generation of residuals and increase in secondary generation must have a negative impact on total environmental damages, at the margin. The environmental cost of disposing of solid waste from electrostatic precipitators must be less than the environmental cost of emitting particles to the air.

The output reaction can be found in principle by working out all the optimal solutions for the inputs and inserting in the relation for output. However, it suffices here to point out that output will decrease according to (36) when a tax regime is introduced for residuals because either the input prices remain the same, or they increase, and increase more the dirtier the input. Thus , our model encompasses both input substitution, abatement and output reduction.

5. Conclusions

The materials balance principle points to the crucial role of material inputs in generating residuals in production processes. Thus, material-based production must also produce "bads" as joint outputs to marketed goods. Pollution modelling therefore must be of a multi-output nature. It has been demonstrated that the most flexible transformation function in outputs and inputs used in textbooks is too general to make sense in pollution modelling. The standard approach has therefore been to specify the bads *as if* they are inputs. It is demonstrated that this practice, although defendable on a macro level, hides explicit considerations of various modification activities. This is most awkward on a micro level, since a main purpose of environmental economics is to come up with results helping making choices as to the most efficient policy instruments to apply in order to reduce pollution.

In recent years joint production of bads have been studied intensively in the efficiency literature using a non-parametric approach, one main motivation being that firms should be credited for reducing bads when calculating productivity measures. However, the key assumption used for modelling joint production of bads has been to assume weak disposability of the bads within a production possibility set formulation. Serious shortcomings with this approach have been exposed.

The main result of the paper is to come up with a complete taxonomy of inputs as to the impact on both residuals and marketed products as joint outputs, thus providing the information for choice of instruments. The multi-output model making such a taxonomy possible, making explicit how bads are generated and encompassing abatement, is what Frisch (1965) termed *factorially determined multi output production*.

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