MEMORANDUM

No 28/2008

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ISSN: 0809-8786

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This series is published by the
University of Oslo
Department of Economics

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Climate Change, Catastrophic Risk and the Relative Unimportance of Discounting

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Abstract

Discounting in the presence of catastrophic risk is a hotly debated issue, in particular with respect to climate change. Many scientists and laymen concerned with potentially catastrophic impacts feel that if an increase in the discount rate drastically increases the likelihood of catastrophic outcomes, this discredits economic cost-benefit calculations. This paper argues that this intuition is sound and that if cost-benefit calculations are done within a model that encompasses the type of catastrophic risk that these scientists worry about, the resulting stabilization target will only be slightly influenced by the discount rate. This is shown within a stylized model of a risk neutral decision maker facing a problem with a catastrophic threshold with unknown location.

Key words: climate change, discounting, catastrophic risk, optimal control.

Acknowledgements: We thank Geir Asheim, Michael Hoel, Michael Oppenheimer, and in particular, Atle Seierstad for useful comments.

Introduction

Discourse on policy responses to climate change has a tendency to become a debate on the appropriate method of discounting. The Stern Review was severely criticized by influential economists such as William Nordhaus and Martin Weitzman who claimed that much of the results where artificially driven by low discount rates [12], [7], [15]. Indeed, much of the discussion about these models boils down to the appropriate choice of numerical value of discount rates and measures of income inequality aversion, [3]. To the extent that discount rates actually matter for climate policy this is a fruitful debate, but it is not clear that the discount rate is of paramount importance if climate change induces catastrophic risk which must be managed. It has for some time been recognized that climate change carries with it the risk of catastrophes when certain boundaries, termed thresholds or tipping points are crossed. Examples are of possible catastrophic scenarios include coral bleaching, marine ice sheet instability, methane hydrate destabilization and disruption of the thermohaline circulation (Gulf Stream),[10], [5], [1], [6]. There is unfortunately a disconnect between scientists concerned with potentially catastrophic threshold effects and economists who do not include them in their models. This has led to an unfortunate breakdown of communication between the scientists who feel that the intelligent management of catastrophic risk should not be very sensitive to discounting while economists armed with results from integrated assessment models claim the discount rate is a crucial parameter in climate policy.

The economic analysis of problems with threshold risk is obviously confounded by the lack of precise knowledge about the location of these thresholds. Partha Dasgupta has even suggested that the existence of such tipping points may severely restrict the usefulness of cost-benefit analysis, [4]. It is therefore all the more worrisome that threshold risk is not an integral part of current economic models of climate change. Further, if threshold effects are an important part of the possible damages induced by climate change, one may argue that economic discussion of the role of discounting is premature until the role of discounting in dynamic models with threshold risk is properly understood.

There is very little formal economic analysis of threshold effects with unknown threshold location, and what there is pays very little attention to the role of the interest rate, [8], [9], [13], [14]. Here we present a stylized model showing that if catastrophic risk of crossing a crucial climate threshold is incorporated into an economic decision model, the rate of discounting is of little importance for the question of what level to stabilize atmospheric CO_2. The model is solved analytically and contains a number of simplifying assumptions in order to clarify the role of the discount rate in the control of catastrophic climate risk. We assume risk-neutrality and standard exponential discounting and frame the consequence of a catastrophe as a fixed cost which does not entail the possibility of the marginal utility becoming infinite. The model is aimed to capture the rational deliberations of a standard economic decision maker who faces the possibility of a severe catastrophe, which does not however entail an outcome where the human race is pushed below a minimum subsistence level.

A Simple Model of Carbon Emissions and Catastrophic Risk

Consider the following stylized model of catastrophic climate change. Let the stock of atmospheric CO_2 above pre-industrial levels be determined by the following differential equation:

$$\frac{dx(t)}{dt} = u(t) - \delta x(t), \;\; x(0) \;\text{given.} \tag{1}$$

Here x is the stock of atmospheric carbon above pre-industrial levels, u is the flow of CO_2 emissions and δ is the inverse of the mean atmospheric lifetime of CO_2. Assume further that there is a threshold $\bar{x}$ such that if $x = \bar{x}$ then an irreversible catastrophic event is triggered. The threshold location $\bar{x}$ is a random variable with a distribution $f(X)$. Denote the hazard rate of $f(X)$ as $\lambda_f(X)$. As x is a function of t then for any given function $u(t)$ the point in time τ such that $x(\tau) = \bar{x}$ is a random variable. One can show that along an optimal path, the distribution of τ is given by $f\left(x\left(t\right)\right) \times \max\left(0, \frac{dx(t)}{dt}\right)$ over $[0, \infty)$. The corresponding hazard rate as a function of t is given by:[1]

$$\lambda_\tau\left(t\right) = \lim_{dt \to 0} \frac{\Pr\left(\tau \in \left[t, t + \mathrm{d}t\right] \mid \tau > t\right)}{dt} = \lambda_f\left(x\left(t\right)\right) \times \max\left(0, \frac{dx(t)}{dt}\right) \tag{2}$$

[1] This expression assumes that $x(t)$ has at most one local maximum. See [9] for a detailed discussion.

It is assumed for simplicity that λ_f is the hazard rate of the exponential distribution for the threshold with intensity λ, distributed over $[x(0), \infty)$. The catastrophic event that occurs when $x(\tau) = \overline{x}$ is that society incurs a constant loss of utility flow given by G per unit of time.[2] Formally we define a state-variable $\gamma(\tau)$, with $\gamma(0) = 0$, so that $\forall t \neq \tau$, $\frac{d\gamma(t)}{dt} = 0$ and having a jump at the unknown τ, as given by $\gamma(\tau^+) - \gamma(\tau^-) = -G$. Finally, assume that the cost of emission reduction is given by:

$$C\left(u\right) = \frac{c}{2}\left(u^0 - u\right)^2 \tag{3}$$

Here u^0 denotes the business as usual emission levels. In order to focus on the role of catastrophic risk, no other damages from CO_2 emissions are included in the model. Applying the principles of conventional economic analysis leads to the following decision or planning problem:

$$\max_{u(t)} E\left(\int_0^\infty \left(\gamma(t) - \frac{c}{2}\left(u^0 - u\right)^2\right) e^{-rt} dt\right) \tag{4}$$

subject to (1), (2), and (5) below, with $x(0)$ given, the control $u \in \left[0, u^0\right]$ for any t, and with r as the much maligned discount rate. Note that the model employs traditional exponential discounting and that there is no risk aversion. The state variable γ satisfies

$$\dot{\gamma}(t) = 0 \ \forall t \neq \tau, \gamma\left(0\right) = 0, \ \gamma\left(\tau^+\right) - \gamma\left(\tau^-\right) = -G \tag{5}$$

Optimality conditions will be phrased in terms of the risk-adjusted Hamiltonian:[3]

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[2] The assumption that the disaster gives rise to a constant flow of disutility is not crucial as it is always possible to replace the integral for net present value of actual damages with an annuity of damages which is equivalent. Furthermore, the modelling implies a rather "optimistic" view as to the occurrence of the catastrophe. A more realistic approach would require the hazard rate, $\lambda_f(x)$, to increase with x.

[3] See [8],[9] for details of the mathematics of controlling threshold risk, and [11] for a thorough presentation of piecewise deterministic control problems .

$$H = \gamma - \frac{c}{2}\left(u^0 - u\right)^2 + \mu(u - \delta x) + \lambda\left(u - \delta x\right)\left(-\frac{G}{r} - z\left(t\right)\right) \tag{6}$$

where $\frac{G}{r}$ is the permanent "annual" cost ("annuity") of a catastrophic event, whereas $z(t)$ is the value function of the program at t conditional that no catastrophe so far has occurred, formally defined by $\max\limits_{u(s)} E\left(\int_t^\infty \left(\gamma\left(s\right) - \frac{c}{2}\left(u^0 - u\right)^2\right) e^{-rs} ds\right)$, subject to the same constraints as in (4). Applying the appropriate Pontryagin Maximum Principle to this problem gives the following optimality conditions:[4]

$$u = \arg\max H = u^0 + \frac{\mu}{c} + \frac{\lambda}{c}\left(\frac{-G}{r} - z\right) \tag{7}$$

$$\dot{\mu} = r\mu - \frac{\partial H}{\partial x} = [r + \delta + \lambda(u - \delta x)]\mu + \lambda\delta\left[-\frac{G}{r} - z\right] \tag{8}$$

$$\dot{z} = rz + \frac{c}{2}\left(u^0 - u\right)^2 - \lambda\left(u - \delta x\right)\left(-\frac{G}{r} - z\left(t\right)\right) \tag{9}$$

In (8) we have used that $\frac{\partial z}{\partial x} = \mu$. Paths satisfying these conditions, along with (1) and (5), as well as standard transversality conditions are optimal for all $t \in [0, \tau]$. Along an optimal path, u and x will converge towards the steady state expressions stated below in (10) and (11).

The maximization problem in (4) is performed subject to (1), (2) and the consequence of the catastrophe given by: $\gamma(\tau^+) - \gamma(\tau^-) = -G$, along with no further restrictions on the state variables x and γ. Optimal emission levels and stocks of atmospheric CO_2 will converge to:

$$u_{ss} = \lim_{t\to\infty} u\left(t\right) = u^0 + \frac{1}{\lambda}\left(\left(r + \delta\right) - \sqrt{\left(r + \delta\right)^2 + 2G\frac{\lambda^2}{c}}\right) \tag{10}$$

$$x_{ss} = \lim_{t\to\infty} x\left(t\right) = \frac{u^0}{\delta} + \frac{1}{\delta\lambda}\left(\left(r + \delta\right) - \sqrt{\left(r + \delta\right)^2 + 2G\frac{\lambda^2}{c}}\right) \tag{11}$$

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[4] A co-state variable may also be calculated for the state variable γ. The value of this variable will however have no impact on the solution.

Here u_{ss} is the non-negative steady state level of emissions and x_{ss} is the upper bound on optimal stock of atmospheric CO_2.[5] Both expressions have intuitive properties. E.g.:

$$\lim_{\lambda \to 0} u_{ss} = \lim_{G \to 0} u_{ss} = u^0, \quad \lim_{\lambda \to 0} x_{ss} = \lim_{G \to 0} x_{ss} = \frac{u^0}{\delta} \tag{12}$$

If there is almost no risk or the cost of crossing the tipping point is close to zero, then steady state emissions and stock of atmospheric CO_2 reverts to unregulated levels. Evidently, the steady-state solutions in (10) and (11) depend on the discount rate. However, a closer examination shows that the role of the discount rate is not very large. To see this, examine the terms within the parenthesis:

$$\left((r+\delta) - \sqrt{(r+\delta)^2 + 2G\frac{\lambda^2}{c}} \right) \tag{13}$$

Defining $r + \delta$ to be A and $2G\lambda^2 c^{-1}$ to be B, the non-positive expression in (13), may be written:

$$A - \sqrt{A^2 + B} \tag{14}$$

Let the unit of time be "one year". The annual discount rate is then typically lower than 0.07. $1/\delta$ is the average lifetime of a CO_2 molecule in the atmosphere. This number was popularly believed to be on the order of a few hundred years, but recent work indicates that it may be considerably higher, which implies that δ is less than or equal to 1/200, see [1]. In any event, the number A is of the order of magnitude 10^{-1}. The number B depends on the ratio of the cost of catastrophe G and, roughly speaking, the cost of emission reduction c. If the catastrophe has consequences that are truly serious so that the number B is of an order of magnitude, say 10^6 or more,

[5] If the cost from a catastrophe is large, then, as seen from (10), the optimal steady-state emissions might be zero. The steady-state solutions are derived easily from (7) – (9) with $\dot{x} = \dot{\mu} = \dot{z} = 0$. These equations will be quadratic and in fact have two solutions. One of these solutions will imply emission levels exceeding the unregulated level u^0, and is clearly not optimal.

then the number B will clearly dominate the expression in (13). Indeed, the expression has A minus the root of the square of A plus something and will tend to disappear. (The elasticity of $-[A-\sqrt{A^2-B}]$ w.r.t. A can easily be seen to be positive but less than one. An increase in the discount rate by one per cent will increase the absolute value of the expression in (13), but by less than one per cent.)

It should be clear from this discussion that the discount rate does not matter much for what level one should stabilize atmospheric CO_2. As the probability of crossing the threshold is given by the integral $\int_{x(0)}^{x_{ss}} f(x)dx$, this probability is not very dependent on the discount rate either. Any fruitful scientific and economic discussion about this topic should therefore focus on the magnitude of the parameters G, c and λ. This does not imply that the interest rate is completely insignificant. The path of emissions and atmospheric CO_2 leading up to the stabilized levels in (10) and (11) will in general be sensitive to changes in interest rates, but for the determination of the actual stabilization targets, the discount rate plays a minor role. Also, the discount rate is important in determining the magnitude of the shadow price and the objective functions whose steady state values are given by:[6]

$$\begin{aligned} \mu &= \frac{-c\delta\left((r+\delta)-\sqrt{(r+\delta)^2+2\frac{G}{c}\lambda^2}\right)}{r\lambda} > 0 \\ z &= \frac{-c\left(\frac{G\lambda^2}{c}+(r+\delta)\left[r+\delta-\sqrt{(r+\delta)^2+2\frac{G}{c}\lambda^2}\right]\right)}{r\lambda^2} < 0 \end{aligned} \tag{15}$$

The steady-state solution for z is found from the quadratic equation in (9), with two negative real roots. Note that $z \in (-\frac{G}{r}, 0)$, and also that $\lim \mu = 0 = \lim z$ when $G \to 0$, as expected. Clearly, these expressions are highly dependent on the interest rate. Indeed, one would expect that if the stabilization target for atmospheric CO_2, as given by the steady state level of x in (11), is not very dependent on the discount

[6] It may seem peculiar that the shadow price on x is positive. This shadow price here has the interpretation as the value of a risk free marginal increase in x.

rate, there is all the more reason to expect that the cost of abiding by this target will be affected.

Summary

The debate between proponents of conventional discounting and sceptics concerned about catastrophic risk is somewhat misplaced as the role of discounting in catastrophic risk is minor if the threshold nature of the risk structure is accounted for. To the extent that threshold effects are important in climate change, this should be incorporated into integrated assessment models and thereby conciliate the results of these models with the concerns of climate scientists.

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