# MEMORANDUM

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## Last 10 Memoranda

| | |
|---|---|
| No 13/07 | Knut Røed and Lars Westlie<br>*Unemployment Insurance in Welfare States: Soft Constraints and Mild Sanctions* |
| No 12/07 | Erik Hernæs and Weizhen Zhu<br>*Pension Entitlements and Wealth Accumulation* |
| No 11/07 | Øystein Kravdal and Ronald R. Rindfuss<br>*Changing relationships between education and fertility – a study of women and men born 1940-64* |
| No 10/07 | Tyra Ekhaugen<br>*Long-term Outcomes of Vocational Rehabilitation Programs: Labor Market Transitions and Job Durations for Immigrants* |
| No 09/07 | Di Tommaso M.L., Shima I., Strøm S. and Bettio F.<br>*As bad as it gets: Well being deprivation of sexually exploited trafficked women* |
| No 08/07 | Olav Bjerkholt<br>*Ragnar Frisch's contribution to business cycle analysis* |
| No 07/07 | Steinar Holden and Fredrik Wulfsberg<br>*Are real wages rigid downwards?* |
| No 06/07 | Morten Nordberg<br>*Employment behaviour of marginal workers* |
| No 05/07 | Morten Nordberg<br>*Total Tax on Labour Income* |
| No 04/07 | B. Gabriela Mundaca<br>*Monitoring, liquidity provision and financial crisis risk* |

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# Model selection for monetary policy analysis – How important is empirical validity?

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**Abstract**

We investigate the economic significance of trading off empirical validity of models against other desirable model properties, and the potential loss from 'overestimating' model uncertainty and basing monetary policy on a relatively robust model, or on a suite of models. We find that differences in model specification and even differences in estimates of key parameters across comparable models may entail widely different monetary policy and macroeconomic performance. Our results therefore caution against compromising the empirical validity of models when selecting a model for policy analysis. We also find that potential costs from basing monetary policies on the relatively robust model or on a suite of models, even when it contains the valid model by assumption, can be quite substantial. This suggests huge gains from efficient exploitation of available information sources to avoid overestimation of model uncertainty. Our investigation is based on three alternative econometric systems of wage and price inflation for Norway.

**Keywords:** *Model uncertainty; Economic significance; Econometric modelling.*

**JEL Codes:** *C52, E31, E52*

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# 1 Introduction

Macroeconomic models generally influence monetary policy. Often, policy makers have to choose among several models with different properties and different policy implications. There are, however, several criteria for selecting among models, even when they conform with one's views on important characteristics of an economy. Common criteria for choosing among models include consistency with some preferred economic theory and data. In addition, one may evaluate models on the basis of how easily they would enable one to communicate their mechanisms and functioning to a wider community, including users of models within the model-developing institution. Accordingly, one may use criteria such as the wider community's familiarity with a model, its similarity with models already in use, and its transparency. Parsimony of a model is another sought-after property, as it enhances a model's transparency and facilitates its updating.

However, model selection often involves a trade-off between desirable characteristics of a model since a model may perform well on one set of criteria and poorly on another set. For example, an economic theory may prefer a different model than available evidence suggests, while different statistical tests may favour different models; cf. Mankiw (1989) and Pagan (2003). One may also encounter cases where a model that is overwhelmingly supported by available evidence is more demanding to use and/or communicate than a model that is not fully data consistent.

Furthermore, several models may be available that appear almost equally attractive in light of economic theory and/or the empirical evidence considered. In the face of such model uncertainty, one may let policy be informed by the least fault-tolerant model, i.e. the model in which a deviation from the optimal policy has the most severe consequences, as suggested by the robust control approach towards designing monetary policy under model uncertainty; see e.g. Hansen and Sargent (2001). Alternatively, one may refrain from selecting one particular model and decide to base policy on a suite of models consistent with the Bayesian approach towards dealing with model uncertainty; see e.g. Levin *et al.* (1999). Such approaches may also be adopted, despite clear empirical evidence in favour of one particular model, if the policy maker has strong preferences for implementing monetary policy that is robust to model uncertainty; see Cogley and Sargent (2005).[1]

In this paper we investigate the economic significance of trading off empirical validity of models with other desirable model properties, and the potential loss from 'overestimating' model uncertainty and basing monetary policy on a relatively robust model, or on a suite of models. One can 'overestimate' model uncertainty by underutilizing available information or by leaving some infor-

[1] Cogley and Sargent (2005) have investigated the consequences of basing monetary policy on a suite of models for the inflation experience of the US since the 1960s. They ascribed the Fed's failure to curb the inflation of the 1970s to its preferences for implementing robust monetary policy in the face of perceived model uncertainty. Accordingly, the Fed placed too little weight on the natural rate model, even after accumulation of strong statistical evidence in its favour. The suite of models was assumed to contain three competing models of the Phillips curve, including a downward-sloping one, a vertical one in the long run, and one that did not allow any exploitable trade-off between inflation and unemployment at any horizon.

mation sources unexplored. The focus of our investigation is consistent with Granger (1992, 2001), who emphasizes that consequences of one's decision regarding model choice for policy analysis should be evaluated in terms of their economic significance. Accordingly, possible loss from choosing one model specification ahead of another should not only be assessed in terms of e.g. predictive power forsaken, or in terms of other desirable statistical properties, but also in terms of potential economic/welfare loss.

We find that differences in model specification and even differences in estimates of key parameters across similar models may entail widely different interest rate setting to counteract effects of a shock. Hence, we find that economic performance can suffer substantially if policy is based on a model other than the one that represents the closest approximation to the economy. We also find substantial costs of 'overestimating' model uncertainty and responding to such uncertainty by adopting robust monetary policies, either based on the least-fault tolerant model, or on a suite of models, even when it contains the valid model by assumption.

Our investigation is based on three alternative econometric systems of wage and price inflation for Norway. They represent three competing perspectives on modelling wage and price inflation in the Norwegian debate since the early 1980s; see Bårdsen *et al.* (2005). One of the systems is derived in the light of open economy models of imperfect competition in product markets and a wage-bargaining framework. The other two systems consist of open economy Phillips curves for prices and wages, where one of them specifies vertical Phillips curves for wage and price inflation through parameter restrictions.

The three systems may be considered alternative blocks of the supply side in a macroeconometric model for medium-term analysis. To close each of the three systems, we conduct our policy analysis by embedding, in turn, the three wage and price systems in a well specified macroeconometric model of Norway; see Bårdsen *et al.* (2003, 2005) and Akram *et al.* (2006). The macroeconometric model is part of the suite of models maintained by Norges Bank. A number of researchers have called for monetary policy analysis using models that are actually used in policy making institutions rather than simplified models used for illustrations; cf. Goodhart (2001). Our use of this model is motivated by this call.

We proceed as follows. Section 2 develops the three systems and examines the econometric properties. The three systems seem to explain wage and price inflation almost equally well, but possess significantly different econometric properties. A broad set of tests favours the system based on open economy models of imperfect competition in product markets and a wage-bargaining framework, consistent with the institutional features of the Norwegian economy. The two Phillips curve systems receive relatively weaker support and their statistical properties do not seem to differ much from each other.

Section 3 derives monetary policy response to demand and supply shocks based on each of the three systems. We assume that the central bank primarily targets the inflation rate but, without prejudice to this objective, also pursues output stability. The latter objective is pursued by choosing an appropriate horizon for achieving the inflation target, alternatively, an appropriate interest rate path, in line with the recently adopted practice of e.g. Norges Bank; see Qvigstad (2005). Previous studies assuming a hierarchy of monetary policy objectives include Smets (2003) and Driffill and Zeno (2004). This way of characterising monetary policy seems consistent with the actual practice of leading central banks; see e.g. Meyer (2004), Heikensten (2005) and Giavazzi and Mishkin (2006).[2] Subsection 3.1 briefly outlines the monetary policy objectives and interest rate rules.

Section 4 investigates the potential loss in terms of macroeconomic performance, viewed in the light of the central bank's objectives, from basing monetary policy on a different model than the one representing (the closest approximation to) the actual wage and price inflation process. This analysis helps to bring forward the potential macroeconomic loss when the econometric properties of a model are traded off against other desirable properties. It also sheds light on the potential loss from basing monetary policy on the least-fault tolerant model owing to concern for robustness in the face of perceived model uncertainty. To shed light on the potential loss that can result from basing monetary policy on a suite of models, we let monetary policy be equally informed by the three alternative econometric systems of wage and price inflation and compare the performance of such a policy with that based on the model representing the actual wage and price inflation process.

Finally, Section 5 presents our main conclusions. Appendix A contains precise definitions of the time series of the variables, while Appendix B sketches the derivation of interest rate rules and their implementation.

## 2 Alternative models of prices and wage dynamics

This section develops and evaluates three alternative empirical systems of Norwegian wage and price inflation. The three systems are derived from a common VAR model by imposing restrictions in the light of relevant economic theories and following a "general to specific" modelling strategy.[3]

[2]For the ECB "The Treaty establishes a clear hierarchy of objectives for the Eurosystem. It assigns overriding importance to price stability.", which is defined as "a year-on-year increase in the Harmonised Index of Consumer Prices (HICP) for the euro area of below 2%." and "Without prejudice to the objective of price stability" support the achievement of e.g. "high level of employment" and "sustainable and non-inflationary growth"; see www.ecb.int.
The Bank of England 1998 Act also expresses an hierarchical ordering between similar objectives; see http://www.bankofengland.co.uk/about/legislation/1998act.pdf.

[3]A Pc-Give batch file and a data file that can be used to replicate the results in detail can be downloaded from http://folk.uio.no/rnymoen/.

## 2.1 Time series of key variables

The models are based on quarterly time series for the mainland economy of Norway, i.e. exclusive of its offshore sector. They are estimated on a data set that covers the period 1972q4–2001q4. The data are seasonally non-adjusted and the time series are defined more precisely in Appendix A.

The variables of main interest to us include natural logs of average hourly wages ($w$), the consumer price index ($p$), productivity ($pr$), an import price index ($pi$) and the unemployment rate ($u$). In addition, payroll and indirect tax rates, $\tau 1$ and $\tau 3$, log of standard working hours ($h$) and an index of electricity prices ($pe$) appear as explanatory variables in the systems of wages and prices.

## 2.2 Wage-price error correction models (ECMs)

The following long-run relationships for wages and prices in Norway are consistent with open economy models of imperfect competition in product markets and a wage-bargaining framework:

$$w_t = p_t + \gamma_{w1} pr_t - \gamma_{w2} u_t + \gamma_{w0} + \varepsilon_{w,t}, \tag{1}$$

$$p_t = \gamma_{p1}(w_t + \tau 1_t - pr_t) + (1 - \gamma_{p1}) pi_t + \gamma_{p2} \tau 3_t + \gamma_{p0} + \varepsilon_{p,t}, \tag{2}$$

where the slope coefficients are non-negative and $\gamma_{w0}$ and $\gamma_{p0}$ are intercepts. A detailed rationalization is given in Bårdsen *et al.* (2005, Ch 5) based on the assumption that $w_t$, $p_t$, $pr_t$ and $pi_t$ are variables that are integrated of degree one, but cointegrated. The unemployment and tax rates are assumed to be without unit roots, while taxes may display deterministic non-stationarity due to shifts in their mean rate over time; i.e. discrete tax rate changes in the case of $\tau 3_t$. Therefore, $\varepsilon_{w,t}$ and $\varepsilon_{p,t}$ represent stationary deviations from the two long-run relationships. Bårdsen *et al.* (2003) show that (1) and (2) represent identified cointegrating relationships.[4]

Equation (1) is interpreted as a steady-state wage equation, which is implied by a bargaining framework where wages are determined by domestic prices and productivity, while the rate of unemployment affects the mean level of the implied wage share: $w_t - p_t - pr_t$. Equation (2) is interpreted as a steady-state price equation which incorporates both the effects of mark-up pricing behaviour (captured by the elasticity $\gamma_{p1}$), and a separate long-run elasticity of $(1-\gamma_{p1})$ for import prices. Since the price variable $p_t$ is (log of) the consumer price index (CPI), it is also affected by a measure of indirect taxes, $\tau 3$.

We find support for the following estimates of the long-run elasticities: $\gamma_{w1} = 1$, $\gamma_{w2} = 0.15$, $\gamma_{p1} = 0.6$ and $\gamma_{p2} = 0.5$. The estimates of the intercept terms, $\gamma_{w0}$ and $\gamma_{p0}$, are close to the sample means of the cointegrating relationships defined by the elasticity estimates. These estimates are consistent with those found in a number of previous studies using data samples of different lengths,

[4]See the section on cointegration analysis in the provided PcGive batch file for documentation.

periods and level of aggregation; see e.g. Nymoen (1991), Johansen (1995), Bårdsen *et al.* (1998), Bårdsen and Nymoen (2003), Bårdsen *et al.* (2003), and Bårdsen *et al.* (2005, Ch 9).

We derive a system of structural error correction models (ECMs) of wages and prices, (3), to represent wage-price dynamics consistent with (1) and (2) being long-run cointegrating relationships. Here, we also condition on a number of explanatory variables that have been found relevant in earlier econometric models of Norwegian inflation. The following variables are conditioned on: First, changes in standard working hours, $\Delta h_t$, which capture wage compensation for reductions in the length of the working day; see Nymoen (1989). Second, the rate of change in aggregate demand, $\Delta y_t$, is included to represent directly short-term inflationary pressure in the product markets. Third, the rate of change in electricity prices, $\Delta pe_t$, is an important exogenous explanatory variable for *CPI*-inflation (due to Norway's hydroelectric-based energy system). Fourth, given that incomes policies and direct price regulations have been in operation on several occasions in the sample period, we control for their effects on wages and prices by employing the dummy variables $W_{d,t}$ and $P_{d.t}$, respectively.[5] Finally, the system of ECMs includes three seasonal dummies and intercept terms. The ECMs were estimated by the FIML method.[6]

$$
\begin{aligned}
\Delta w_t = &-\underset{(0.01)}{0.11}[w_{t-3} - p_{t-1} - pr_{t-1} + 0.15u_{t-2}] + \underset{(0.07)}{0.16}\Delta w_{t-1} \\
&+\underset{(0.02)}{0.06}\Delta pr_t - \underset{(0.12)}{0.54}\Delta h_t - \underset{(0.002)}{0.02}\, W_{d,t} \\
\Delta p_t = &-\underset{(0.01)}{0.06}[p_{t-3} - 0.6(w_{t-3} - pr_{t-1} + \tau 1_{t-1}) - 0.4pi_{t-1} + 0.5\tau 3_{t-1}] \\
&+\underset{(0.05)}{0.16}\Delta p_{t-2} + \underset{(0.03)}{0.21}\Delta w_t + \underset{(0.03)}{0.13}\Delta w_{t-1} + \underset{(0.02)}{0.04}\Delta_2 y_{t-1} \\
&-\underset{(0.01)}{0.01}\Delta pr_t + \underset{(0.01)}{0.03}\Delta pi_t + \underset{(0.01)}{0.06}\Delta pe_t - \underset{(0.001)}{0.01}\, P_{d,t}
\end{aligned}
\tag{3}
$$

The equation for wages in system (3) shows that nominal quarterly wage growth, $\Delta w_t$, adjusts to correct deviations from the steady-state relationship in (1).[7] The '$t$-value' associated with the equilibrium/error correction coefficient is $-11$, suggesting that one of the main implications of the underlying theoretical framework is strongly supported by the evidence. The remainder of the equation first contains a tendency of positive autocorrelation in the quarterly wage growth rate which arises mainly because most wages are adjusted in the second and third quarters. The

[5]The specification of *Wdum* and *Pdum* is given in Appendix A together with the definition and sources of the other variables. In addition, we refer to the downloadable PcGive batch and data for automatic and full documentation.

[6]The seasonal dummies and intercepts are suppressed from the equation. Coefficient standard errors are given below their respective coefficients.

[7]The 3-quarter lag in wages and the 2-quarter lag in unemployment only affect how the dynamics is parameterized, not the interpretation of $[w_{t-3} - p_{t-1} - pr_{t-1} + 0.15u_{t-2}]$, which is an equilibrium correction term consistent with (1).

last part of the wage equation contains short-run effects of productivity growth ($\Delta pr_t$) and wage compensation for changes in standard working hours ($\Delta h_t$).

The price equation in system (3) shows that also the hypothesized price equilibrium correction is supported by the data; the estimate of the equilibrium correction coefficient has a '$t$-value' of $-6$. The other variables also have straightforward interpretations: There is a statistically significant positive autoregressive coefficient consistent with commonly observed inflation persistence. Actually, the autoregressive coefficient would have been larger without the inclusion of the other explanatory variables in the model. Wage growth has a strong effect with an elasticity of 0.34 over two quarters. There is also a small positive effect of product demand growth if sustained over two quarters. The short-run effects of productivity and import prices are quite small when compared with the corresponding long-run elasticities in the equilibrium correction term. In contrast, the estimated elasticity of electricity prices ($\Delta pe_t$) and their effect are numerically significant, since they fluctuate widely; $\Delta pe_t$ varies in the range of $\pm 25\%$.

## 2.3 Phillips curve models

We derive the two systems of Phillips curves from the same information set, and embed then in the same VAR model as the system of wage-price error correction models (ECMs). Specifically, they originate from the same unrestricted reduced form, which corresponds to a VAR in levels with cointegration restrictions imposed.

The system of Phillips curves favoured by data is reported in (4):

$$\begin{aligned}\Delta w_t = & -\underset{(0.07)}{0.20}\Delta w_{t-1} + \underset{(0.13)}{0.27}\Delta p_{t-1} + \underset{(0.13)}{0.28}\Delta p_{t-2} \\ & - \underset{(0.004)}{0.01}\Delta u_t - \underset{(0.001)}{0.01}u_{t-1} - \underset{(0.002)}{0.016}W_{d,t} \\ \Delta p_t = & \ \underset{(0.07)}{0.10}\Delta p_{t-1} + \underset{(0.06)}{0.20}\Delta p_{t-2} + \underset{(0.05)}{0.31}\Delta w_t + \underset{(0.04)}{0.16}\Delta w_{t-1} + \underset{(0.01)}{0.05}\Delta_2 y_{t-1} \\ & + \underset{(0.01)}{0.03}\Delta pi_t + \underset{(0.01)}{0.07}\Delta pe_t - \underset{(0.001)}{0.01}P_{d,t}\end{aligned} \qquad (4)$$

This system is consistent with an open-economy triangular model of inflation, whereby inflation is determined by demand-pull, cost-push and expectations inherent in the wage-price spiral; see e.g. Calmfors (1977), Gordon (1997), Stock and Watson (1999) and Bårdsen *et al.* (2003). The long-run Phillips curve implied by this system is downward sloping since the maximum likelihood estimates of the coefficients of the price and wage inflation terms do not add up to homogeneity.[8]

[8] However, since we have an open economy model, the Phillips curve does not rationalize that there is an exploitable trade-off between steady-state inflation and unemployment. This is because, in the steady state of the complete macroeconometric model to be presented, the wage growth would be equal to foreign inflation plus productivity growth, while inflation would be determined by foreign inflation and nominal exchange rate depreciation consistent with PPP.

In the short run, effects of lagged inflation appear in both the wage equation and the price equation. This is in contrast to the system of wage-price ECMs, (3), but not unexpected since lagged inflation is correlated with both of the two equilibrium correction terms in (3), which by the definition of Phillips curves are omitted from the current system.

The Phillips curve model with a vertical long-run Phillips curve is obtained by imposing homogeneity restrictions on the combined effects of wages and prices in both of the equations in system (4):

$$\begin{aligned}\Delta w_t = &-\underset{(--)}{0.18}\Delta w_{t-1} + \underset{(0.12)}{0.58}\Delta p_{t-1} + \underset{(0.12)}{0.60}\Delta p_{t-2} \\ &- \underset{(0.006)}{0.01}\Delta u_t - \underset{(0.0013)}{0.003}u_{t-1} - \underset{(0.003)}{0.017}W_{d,t} \\ \Delta p_t = &\ \underset{(0.07)}{0.21}\Delta p_{t-1} + \underset{(0.06)}{0.26}\Delta p_{t-2} + \underset{(--)}{0.26}\Delta w_t + \underset{(0.03)}{0.16}\Delta w_{t-1} + \underset{(0.01)}{0.07}\Delta_2 y_{t-1} \\ &+ \underset{(0.01)}{0.04}\Delta pi_t + \underset{(0.01)}{0.07}\Delta pe_t - \underset{(0.001)}{0.01}P_{d,t}\end{aligned} \tag{5}$$

A notable difference between system (4) and system (5) is that the coefficient estimate of $u_{t-1}$ is much lower in the latter.[9] The price equation in (5), however, does not differ much from that in (4). This suggests that the homogeneity restrictions of the long-run Phillips curve model are largely data-consistent, especially in the price equation.

## 2.4 Model evaluation

In the following, we examine the econometric properties of the three systems. It appears that the choice between them may not seem obvious if one only consider their in-sample explanatory power, or just focus on few of their econometric properties. However, it becomes easier to choose between them if one uses information from a broad set of standard diagnostic tests.

The overall explanatory power of all of the models is fairly high and does not seem to differ much across models. In particular, both the systems of Phillips curves, (4) and (5), provide nearly the same level of fit to actual wage and price growth over the sample period; see Figure 1.

More precisely, Table 1 shows that the overall explanatory power, as measured by the standard deviations of the equation residuals, which are denoted $\hat{\sigma}_{\Delta w}$ and $\hat{\sigma}_{\Delta p}$, is less than 1 percent for (growth in) wages and less than 0.5 per cent for prices. This may be reckoned as quite satisfactory since the data are seasonally unadjusted. The explanatory power of the three models, especially that of the Phillips curves, is lower than that of the unrestricted VAR. For wages, the model with

[9] One can set this estimate at some preferred value and re-estimate the model. However, the econometric performance of the resulting model becomes inferior to that of system (5).

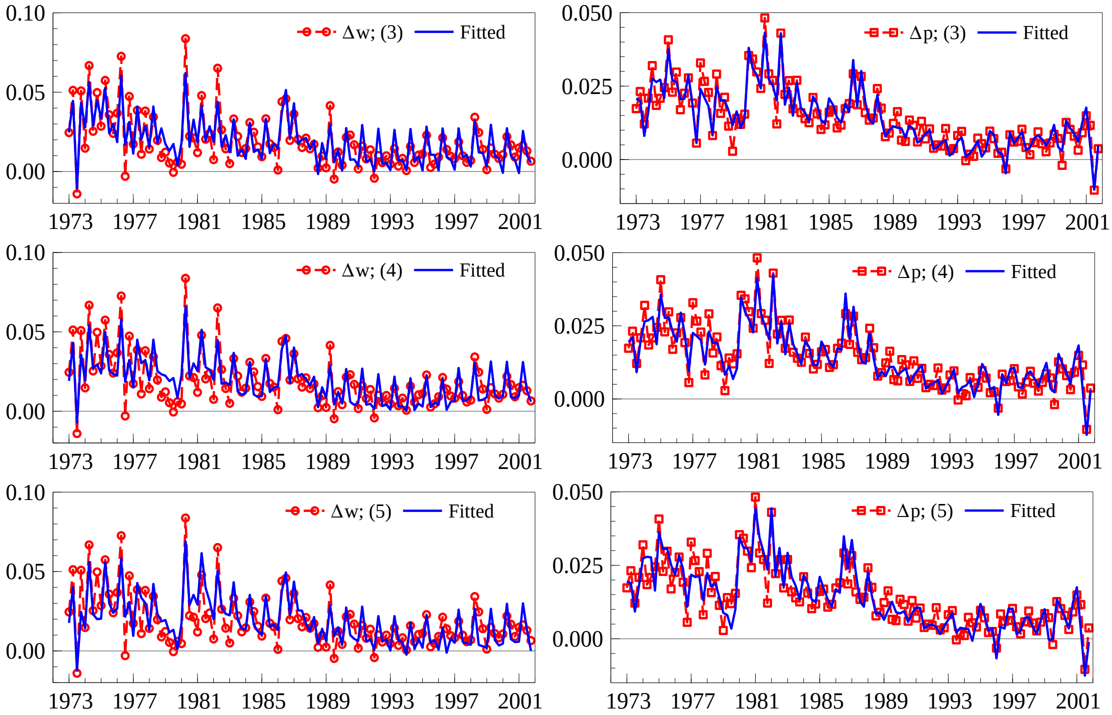


Figure 1: *Explanatory power of the three models (3)–(5) for (quarterly) growth in wages and prices over the sample period: 1972q4–2001q4. The left-hand column presents the explanatory power of the three models for growth in wages ($\Delta w$), while the right-hand column presents that for growth in prices. Dashed lines with circles represent actual values of growth in wages, while dashed lines with boxes represent the actual growth in prices. Solid lines represent the corresponding fitted values.*

the vertical Phillips curve, (5), has lower explanatory power than the model with the downward-sloping Phillips curve, (4), and the model of wage-price ECMs, (3). For prices, however, both Phillips curve models provide the same explanatory power, although lower than the ECM of prices.

Table 1: Explanatory power of the VAR and the three systems of ECMs

| **System** | *VAR* | (**3**) | (**4**) | (**5**) |
|---|---|---|---|---|
| $\hat{\sigma}_{\Delta w}$ | 0.85% | 0.88% | 0.93% | 0.98% |
| $\hat{\sigma}_{\Delta p}$ | 0.33% | 0.36% | 0.47% | 0.46% |

Notes: The three systems have been estimated by FIML on a sample for the period 1973q1–2001q4. The VAR column shows the diagnostics of the statistical model which has the three economic systems as special cases.

Yet, given the relatively high explanatory power of all three models, there may not seem to be any harm in selecting one of them to e.g. facilitate communication with the wider community, including financial markets, politicians, academics and the general public. On such grounds, one could select e.g. the system of the vertical Phillips curve for policy analysis instead of the other two systems: (3) and (4).

However, choosing the system with the vertical Phillips curve, or that of the downward-sloping one, may seem less obvious in the light of a further examination of their econometric properties. One may start by examining the validity of employing the FIML method for estimation since it rests on specific assumptions regarding residuals; see e.g. Andreou and Spanos (2003). Any violation of these assumptions on available data may signal model misspecification, such as omitted variables and/or wrong functional form. It is also of interest to formally test whether the explanatory power of the three models is comparable to that of the VAR model from which they originate.

Table 2 shows that the evidence is not favourable to the systems of Phillips curves, especially to that of the vertical Phillips curve, while it does not reject the validity of the system of wage-price error correction models. For all three systems, and the VAR model from which they originate, the null hypotheses of normally distributed errors are not rejected by the chi-square distributed test. The corresponding $p$-values are well above 10%. However, the hypotheses of no residual autocorrelation and heteroscedasticity are strongly rejected for the system with the vertical Phillips curve (5). The $F$-distributed tests of autocorrelation (up to order 5) and heteroscedasticity in the column for system (5) suggest that the two null hypotheses are rejected at even the 1% level of significance. This is not inconsistent with the support for the normally distributed errors, but indicates that misspecification tests have power in different directions. Moreover, the evidence for normally distributed errors also supports the validity of these tests as they rely on the normality assumption.

Table 2: Diagnostics for the VAR and the three systems of ECMs

| **System** | | *VAR* | (**3**) | (**4**) | (**5**) |
|---|---|---|---|---|---|
| *Autocorrelation* | $F$ | 0.71[0.80] | 1.12[0.33] | 1.41[0.12] | 2.17[0.00]** |
| *Heteroscedasticity* | $F$ | 1.08[0.32] | 0.88[0.76] | 1.32[0.05] | 1.82[0.00]** |
| *Normality* | $\chi^2$ | 2.98[0.56] | 6.23[0.18] | 0.95[0.92] | 3.02[0.55] |
| *Overidentification* | $\chi^2$ | | 34.3[0.10] | 69.1[0.00]** | 118.8[0.00]** |

Notes: The number in square brackets are $p$-values for the respective statistics, which are adjusted for degrees of freedom. References:
Autorcorrelation test: Godfrey (1978), Doornik (1996); Heteroscedasticity test: White (1980), Doornik (1996); Normality test: Doornik and Hansen (1994); and Overidentification test: Anderson and Rubin (1949, 1950) and Sargan (1988, pp 125).

In contrast to the results for system (5), the $F-$tests do not reject the null hypotheses of no-autocorrelation and heteroscedasticity at the 5% level of significance for systems (4) and (3). The corresponding tests for the VAR model suggest that it constitutes a valid foundation for deriving the three systems.

We now apply an encompassing test to examine whether the three systems originating from the VAR model explain the data nearly as well as the VAR model itself; cf. Hendry and Mizon (1993). This test supplements the impression gained by comparing the standard deviations of the residuals from the different systems in Table 1. Specifically, we test whether or not the sets of over-identifying restrictions distinguishing each of the three models from the VAR models are accepted by a chi-square distributed test. Table 2 shows that the set of (24) over-identifying restrictions specifying system (4) is strongly rejected; the insignificance of the misspecification tests notwithstanding. As one may expect, system (5) is also strongly rejected. In contrast, the set of (25) over-identifying restrictions specifying system (3) is not rejected at the 5% significance level; the $p$-value is 10%.

To test the empirical validity of system (4), and by implication that of system (5), we have also employed a likelihood-ratio test for the null hypothesis that the equilibrium correction terms (which appear in system (3)) are insignificant in system (4). This null hypothesis was strongly rejected by the test on the full sample and different subsamples. On the full sample the test-statistic was 36.2 ($=\chi^2(2)$), with a $p$-value of 0.00 per cent. On a subsample starting in 1981q1, the test-statistic was 12.5 ($=\chi^2(2)$), with a $p$-value of 0.2 per cent.

To summarize Section 2.4, one may say that competing dynamics wage and price equations can be formulated and estimated in such a way that requirements like a reasonable fit and correctly signed coefficients are met. If a model evaluation ends with checking such properties, one might argue that one faces model 'uncertainty' since available data are not sufficiently informative and do not strongly favour one model ahead of another, i.e. show little recalcitrance. However, if one extends model assessment and apply a number of standard misspecification tests, data may be quite recalcitrant, and be helpful in model selection; cf. Hoover (2001). Section 3 suggests that econometric differences between models may signal substantial differences in their policy implications.

## 2.5 Macroeconometric model

To investigate the policy implications, we embed the three alternative systems for wage and price inflation in a well specified and well documented macroeconometric model of Norway. This way we are able to close the wage and price system while conditioning on foreign prices.

The macroeconometric model is a version of the model developed in Bårdsen *et al.* (2003, 2005) which has been documented and employed in several studies, including Akram *et al.* (2006). The model is (log) linear and estimated on quarterly aggregate data for the period 1972–2001. In addition to a system of wages and prices, the model contains equations for aggregate demand, unemployment, import prices, labour productivity, credit demand, and three asset prices: house prices, domestic equity prices and the nominal exchange rate. Foreign variables and domestic government expenditures and electricity prices are treated as exogenous variables.

In particular, short-run fluctuations in aggregate demand are determined by the real exchange rate, real interest rate and wealth effects from house prices and equity prices. Thus, a change in consumer prices affects aggregate demand through its effect on the real exchange rate and the (ex-post) real interest rate. However, an increase in domestic prices mainly depresses aggregate demand because of the dominating effect of a subsequent appreciation of the real exchange rate. The unemployment rate follows growth in output in the short run, as in an Okun's law relationship. In addition, it exhibits reversion towards its equilibrium rate over time.

Monetary policy, represented by short-term interest rates has direct effects on the asset prices, credit and aggregate demand, but is neutral in the long run. The model may be considered a backward-looking model in the sense that it does not make the expectations formation processes explicit. The whole model may be considered econometrically well-specified, when system (3) is included, with apparently invariant parameters with respect to changes in monetary policy over the sample; see Bårdsen *et al.* (2003, 2005) for documentation. The lack of evidence for significant parameter instability in the face of shifts in monetary policy is in line with Ericsson and Irons (1995) and Rudebusch (1995). In the following, we assume that the model will remain invariant to the monetary policy decisions we consider.

## 3 Monetary policy implications of the models

We start by presenting the monetary policy objectives, which are pursued by following horizon-dependent interest rate rules in real time. Further details about the derivation of such rules are presented in Appendix B, which is based on Akram (2007a). In Section 3.2, we derive interest rate rules based on the three versions of the macroeconometric model and analyse the economic and policy implications of the three systems.

### 3.1 Monetary policy objectives and the interest rate rule

To devise an optimal response to an observable shock that occurs at time $\tau$, we assume that a forward-looking central bank minimizes the following loss function with respect to an interest rate path $i_\tau$, $i_{\tau+1}$, $i_{\tau+2}$,..$i_{\tau+H-1}$, $i_{\tau+H}$, $i_{\tau+H+1}$,...:

$$L = V_\tau(\pi - \pi^*) + \lambda V_\tau(y), \tag{6}$$

subject to the constraint that the (conditional and hence the unconditional) mean of inflation is equal to its constant target rate, $\pi^*$:

$$\mathsf{E}_\tau(\pi - \pi^*) = 0. \tag{7}$$

$V_\tau(\cdot)$ is a variance function while $\mathsf{E}_\tau$ is an expectation operator conditional on information at time $\tau$.[10] $(\pi - \pi^*)$ denotes deviation between actual and targeted inflation rate. $y$ denotes the output gap while $\lambda$ indicates the degree of concern for fluctuations in the output gap relative to that for fluctuations in inflation.[11]

[10]By ensuring $\mathsf{E}_\tau(\pi - \pi^*) = 0$, the unconditional expetation of the inflation gap would also become zero, i.e. $\mathsf{E}(\pi - \pi^*) = 0$, by the law of iterated expectations.

[11]Minimizing the loss function conditional on available information at each point in time, while ensuring that the conditional expectation of inflation is equal to a constant inflation target, is consistent with the common approach of minimizing a loss function based on unconditional variances of inflation gap and the output gap; see e.g. Taylor (1999). This can be shown by using the variance decomposition formula and additionally assuming that the conditional expectation of the output gap is constant, e.g. zero, which is a reasonable assumption.

This constrained optimization approach implies that the central bank does not compromise its inflation-targeting objective; cf. Smets (2003). Ensuring stability of inflation around its target is given less importance as the central bank is willing to trade off the (conditional) variance of inflation against that of the output gap in accordance with its value of $\lambda$. This approach is consistent with what Faust and Henderson (2004) regard as best-practice monetary policy.[12]

We use $H$ to represent the policy horizon, which we define as the number of periods during which the policy *interest rate* will deviate from its neutral value and stimulate or cool off the economy. $H$ can take on any discrete value from zero onwards. Thus, the precise policy horizon, when measured as the number of periods, would be $H+1$, because $H \geq 0$. The target horizon, i.e. the number of periods *inflation* will deviate from target, will generally be linked and be close to the policy horizon, but the exact relationship will be shock- and model-dependent.[13] Inflation will typically converge asymptotically to its target rate in a dynamic model. It is not unreasonable, however, to assume that the target is largely achieved at the policy horizon, i.e. $H$ periods after the shock in some period $\tau$; see Akram (2007b) for evidence.

We envision that in the face of a shock, the central bank derives a set of interest rate paths, each of them satisfying the constraint (7) for different policy horizons, i.e. $H$ values. Then, from this set of interest rate paths, it selects and implements the interest rate path, and the corresponding policy horizon, that would minimize the loss function (6).

However, there can be numerous interest rate paths that satisfy the constraint (7) for every possible value of $H$. By only considering interest rate paths that obey some reasonable pattern, however, the set of relevant interest rate paths can be limited to the number of policy horizons ($H$ values) considered by the central bank.

We need to assume that the central bank initiates changes in the interest rate when the shock occurs at time $\tau$ and thereafter allows the interest rate to return gradually towards its neutral rate, ($i_0$), as commonly observed in practice; see e.g. Sack and Wieland (2000) and Qvigstad (2005).[14,15] Then, if the model is stable and linear, an interest rate path corresponding to a specific policy horizon $H$ can be obtained from the following interest rate rule:

$$i_{\tau+m} = i_0 + (1-\varrho_H)\frac{\beta_\varepsilon}{(1-\phi)}\varepsilon_\tau + \varrho_H(i_{\tau+m-1} - i_0) \quad ; \; m = \; 0, \; 1, \; 2, ..., H, \; H+1, ... \qquad (8)$$

[12] Accordingly, "...best-practice policy can be summarized in terms of two goals: First, get mean inflation right; second, get the variance of inflation right.", but "...getting the mean right may be the goal of greatest importance"; see Faust and Henderson (2004, pp. 117–118).

[13] In several studies including Batini and Nelson (2001) policy horizon is equated with target horizon, as defined here.

[14] It is quite common in the relevant literature to rule out interest rate paths that seem unreasonable. In contrast to our approach, this is typically obtained by including a measure of volatility in interest rates in the objective function of the central bank; see e.g. Smets (2003), Taylor (1999) and the references therein.

[15] By restricting movements of the interest rates, one looses some control over the movements of the inflation rate, however. Consequently, the inflation rate can e.g. fluctuate around its target rate before settling down to it instead of converging with it gradually in a geometric fashion. To make the inflation rate e.g. converge gradually with its target rate, the interest rate may need to move excessively around its neutral rate. This may seem at odds with stylized facts, though.

The response coefficient $\beta_{\varepsilon,H} \equiv (1-\varrho_H)\beta_\varepsilon/(1-\phi)$ determines how much the interest rate must deviate initially from the neutral rate to counteract inflationary effects of a shock $\varepsilon_\tau$. This initial deviation is thereafter eliminated gradually, depending on the value of an interest rate smoothing parameter $\varrho_H$. Both the response coefficient and the degree of smoothing depend on the policy horizon, as indicated by the subscript $H$.[16] $\phi$ denotes the degree of persistence in the shock and is assumed to be positive and less than one: $0 \leq \phi < 1$. It follows that a persistent shock requires a stronger initial response ($\beta_{\varepsilon,H}$) than a transitory shock (for which $\phi = 0$) for a given degree of interest rate smoothing ($\varrho_H$) and $\beta_\varepsilon$.

The value of $\beta_\varepsilon$ depends on the shock and the model. It is a derived parameter whose value increases with the inflationary effects of the shock over a specific period, but declines with the effectiveness of interest rates in checking inflation; see Appendix B. $\beta_\varepsilon$ can be considered a constant (shock- and model-specific) parameter, if the transmission mechanism of the shock and interest rate is super exogenous with respect to the policy changes considered; see Engle *et al.* (1983).

The policy horizon enters the interest rate rule through the interest rate smoothing parameter, $\varrho_H$. It is defined as $\delta^{1/(H+1)}$ and takes on a value in the range of $(0,1)$ depending on $H$ (for a chosen fraction $\delta$). Interest rates are considered converged with the neutral rate when just a fraction $\delta$ of the initial interest rate deviation (from the neutral rate remains. $\delta$ also determines how close inflation is to its target when monetary policy becomes neutral; cf. constraint (7)).

The degree of smoothing increases with the policy horizon in a concave fashion; since $\varrho_H = \delta^{1/(H+1)}$. In particular, $H = 0$ will lead to (almost) no interest rate smoothing ($\varrho_H = \delta$), while large values of $H$ will imply a high degree of interest rate smoothing since $\varrho_H = \delta^{1/(H+1)} \longrightarrow 1$ when $H \longrightarrow \infty$. The case $H = 0$ refers to the case when the policy-maker only allows interest rates to deviate from their reference rate in a single period at time $\tau$.

However, the value of the response coefficient $\beta_{\varepsilon,H}$ ($\equiv (1-\varrho_H)\beta_\varepsilon/(1-\phi)$) declines (in a geometric fashion) with the policy horizon or degree of interest rate smoothing. In particular, $(1-\varrho_H)\beta_\varepsilon/(1-\phi) \approx \beta_\varepsilon/(1-\phi)$ when $H = 0$, while $(1-\varrho_H)\beta_\varepsilon/(1-\phi) \longrightarrow 0$ when $H \longrightarrow \infty$; since $\varrho_H \longrightarrow 1$. This suggests that if a very long policy horizon is allowed, the interest rate needs to deviate only marginally from its reference value, but this deviation has to be quite persistent.

A long horizon would help subdue the required initial response to a relatively persistent shock. In particular, if persistence in a shock is matched by persistence in interest rates, i.e. $\varrho_H = \phi$, the response coefficient $\beta_{\varepsilon,H}$ becomes equal to $\beta_\varepsilon$. In contrast, a short horizon may imply a particularly large deviation from the neutral interest rate in the face of a persistent shock.

Clearly, the parameters characterising the interest rate rule depend on the policy horizon ($H$), ceteris paribus. By varying $H$, one can vary the interest rate rule and thus the complete interest rate path as well as the level of the loss, $L$.

[16] This rule resembles a Taylor-type rule with interest rate smoothing except that it is the determinant of (excess) inflation, i.e. $\varepsilon_\tau$, that enters the rule rather than inflation itself; see Taylor (1999) and the references therein.

It follows that once the rule (8) is implemented in the model, the optimal policy response to a shock can be found by minimizing the loss function (6) with respect to $H$. The optimal value of $H$ will then define the optimal interest rate change, $\beta_{\varepsilon,H^*}$, the optimal degree of smoothing, $\varrho_{H^*}$, as well as the optimal level of loss, $L$, conditional on a given (version of) the macroeconometric model, $\mathcal{M}$.

We are particularly interested in analyzing the effect of model choice on the loss and consequently the policy, represented by the policy horizon. We therefore express the loss function (6) as an explicit function of $H$ and $\mathcal{M}$:

$$L \equiv L(H, \mathcal{M}). \tag{9}$$

Optimal loss is defined by the optimal policy horizon, $H^*$, for a given model, $\mathcal{M}$. $H^*$ will depend on the degree of concern for fluctuations in the real economy ($\lambda$). Thus, $\beta_{\varepsilon,H^*}$ and $\varrho_{H^*}$, will also depend on $\lambda$.

## 3.2 Monetary policy response to shocks

For the sake of brevity, we refer to the three versions of the macroeconometric model, which would differ from each other only by the wage and price system included, as 'ECM', 'PCM' and 'PCMr', respectively. Specifically, ECM includes the system of wage-price error correction models, (3); PCM includes the system of Phillips curves (4) while PCMr includes the system with the vertical Phillips curve (5), which is a restricted version of (4).

The difference between the three versions of the macroeconometric model essentially consists of difference in restrictions on the overall equilibrium correction behaviour. The version with the wage-price error correction model has more equilibrium correction mechanisms than the version with the downward-sloping Phillips curve; which in turn is more equilibrium correcting than the version with a vertical Phillips curve system.

### 3.2.1 Interest rate rules in response to demand and supply shocks

The monetary policy response to a shock is characterised by rule (8), where the response coefficient ($\beta_\varepsilon$) is entirely shock- and model-dependent. In the following, $\beta_d$ and $\beta_s$, refer to the response coefficient in the case of a demand shock and a supply shock, respectively, conditional on a given model version, ECM, PCM or PCMr. Their estimates are derived for each of these model versions. A demand shock contributes to an initial growth of aggregate demand by one percentage point over four quarters, while a supply shock initially increases price inflation by one percentage point (over four quarters); see Appendix B.3 for details.

Values of $\varrho_H$ for different policy horizons are obtained from $\varrho_H = \delta^{1/(H+1)}$, where we set $\delta$ to say 0.1 to define convergence. That is, we would consider an interest rate deviation (of e.g. one

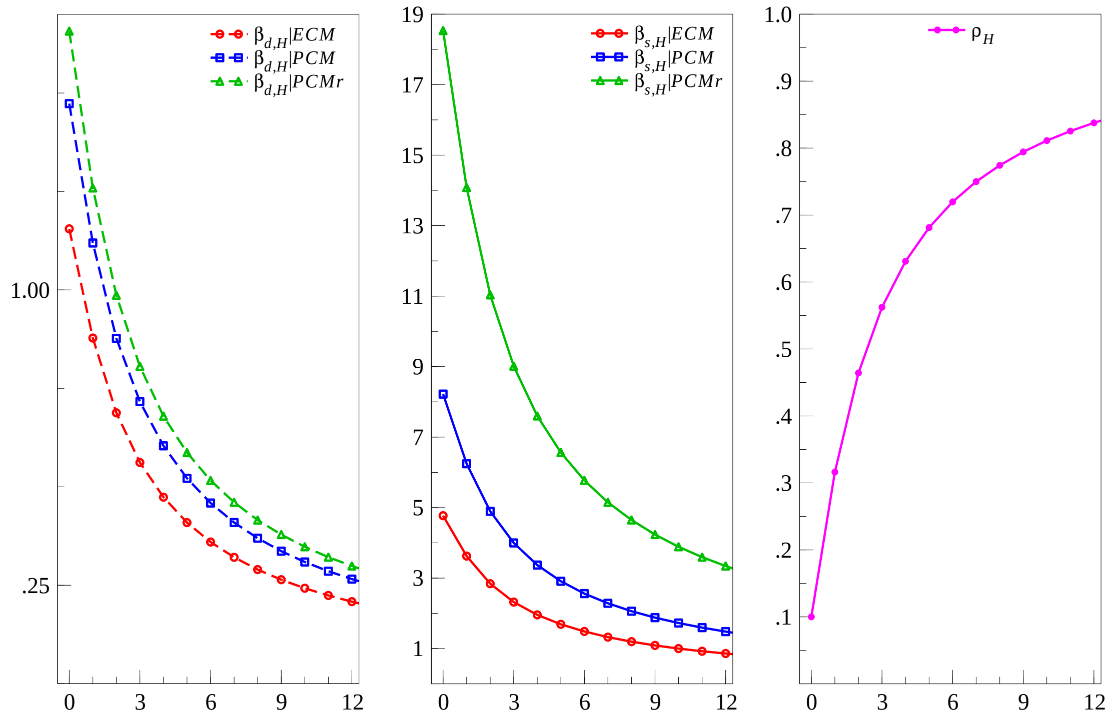


Figure 2: *Left: Initial interest rate responses to the demand shock (in percentage points) implied by different policy horizons (horizontal axes), $\beta_{d,H}$. They are suggested by the three versions of the macroeconometric model: ECM, PCM and PCMr. Middle: Initial interest rate responses to the supply shock implied by different policy horizons, $\beta_{s,H}$, suggested by the three model versions. Right: Interest rate smoothing, $\varrho_H$, associated with different policy horizons.*

percentage point) from the reference rate converged with the reference rate when it deviates not more than 1/10 of the initial deviation from the reference rate. Alternative values of $\delta$ do not bring about qualitatively different results.

Estimates of the horizon-specific response coefficients $\beta_{\varepsilon,H}$ for a given shock and model can be obtained from the formula: $(1-\varrho_H)\beta_\varepsilon/(1-\phi)$, for different degrees of persistence in the shock and interest rates, $\phi$ and $\varrho_H$, respectively. Obviously, $\varrho_H$ and thus $\beta_{\varepsilon,H}$ vary with the policy horizon.

The left and the middle frames of Figure 2 display values of the response coefficient for the (transitory, $\phi = 0$) demand shock and the supply shock, respectively, associated with the three model versions. The values of the response coefficients are presented for different policy horizons in the range 0–12 quarters. The right frame of Figure 2 depicts the degree of interest rate smoothing $\varrho_H$ implied by the different policy horizons. Before analyzing the results for each of the two shocks, we note the following general observations.

First, an increase in the policy horizon reduces the required initial interest rate response to a shock, but raises the degree of interest rate smoothing, ceteris paribus; see Figure 2. For example, the required initial interest rate response declines substantially if the policy horizon is increased from 0 to 8 quarters. This must, however, be accompanied by an increase in interest rate smoothing,

$\varrho_H$, from 0.1 to 0.77 (right frame). And second, an increase in the policy horizon from a low level leads to a larger reduction in the response coefficient than an increase in the policy horizon from a relatively high level. This is due to the concave relationship between the degree of interest rate smoothing and the policy horizon, since $\varrho_H = \delta^{1/(H+1)}$, which in turn leads to a convex relationship of geometric form between the response coefficient and the policy horizon. A linear relationship between the degree of interest rate smoothing and the policy horizon would have implied a linear relationship between the response coefficient and the policy horizon. However, the results presented to be presented would not have changed qualitatively.

#### 3.2.2 Economic and monetary policy implications

Figure 2 shows that both PCM and PCMr suggest a stronger response to both shocks than ECM, at all horizons. In particular, PCMr suggests a stronger response to both shocks than PCM and ECM. Figure 2 also reveals substantial differences between the monetary policy response to the two shocks across the three model versions.

In the case of the demand shock, the interest rate response is relatively low varying in the range of 0.25–1.75 percentage points across the three models. The differences across the three models are relatively small. This reflects that the interplay of the wage and price system with the rest of the model, particularly with aggregate demand and unemployment, is not that different across the three wage and price systems. A demand shock has relatively larger inflationary effects, while a monetary policy shock has relatively stronger deflationary effects in PCM and especially in PCMr, relative to ECM. This is also reflected in the response coefficients, but to a smaller extent since the two effects partly outweigh each other.

Figure 3 sets out the economic performance of the policies in the face of the demand shock suggested by the three models. The economic performance associated with every policy horizon is measured by the standard deviations of the output gap and inflation; see the left column. A policy horizon fully describes the interest rate rule for given values of the response coefficient, $\beta_d$; see Section 3.1. Hence, the optimal policy is found by minimizing the loss function (6) with respect to the policy horizon. We assume that the parameter reflecting concern for output gap fluctuations, $\lambda$, is equal to 0.5. The right column presents values of the loss functions under different policy horizons relative to their value under the optimal policy horizon ($H^*$) for a given model version ($\mathcal{M}$).

We define the relative loss, $\Delta L(H;\mathcal{M})$, as:

$$\Delta L(H;\mathcal{M}) \equiv \frac{L(H;\mathcal{M}) - L(H^*;\mathcal{M})}{L(H^*;\mathcal{M})}. \tag{10}$$

Here, $L(H;\mathcal{M})$ denotes the level of loss by choosing $H$ conditional on a specific model (version)

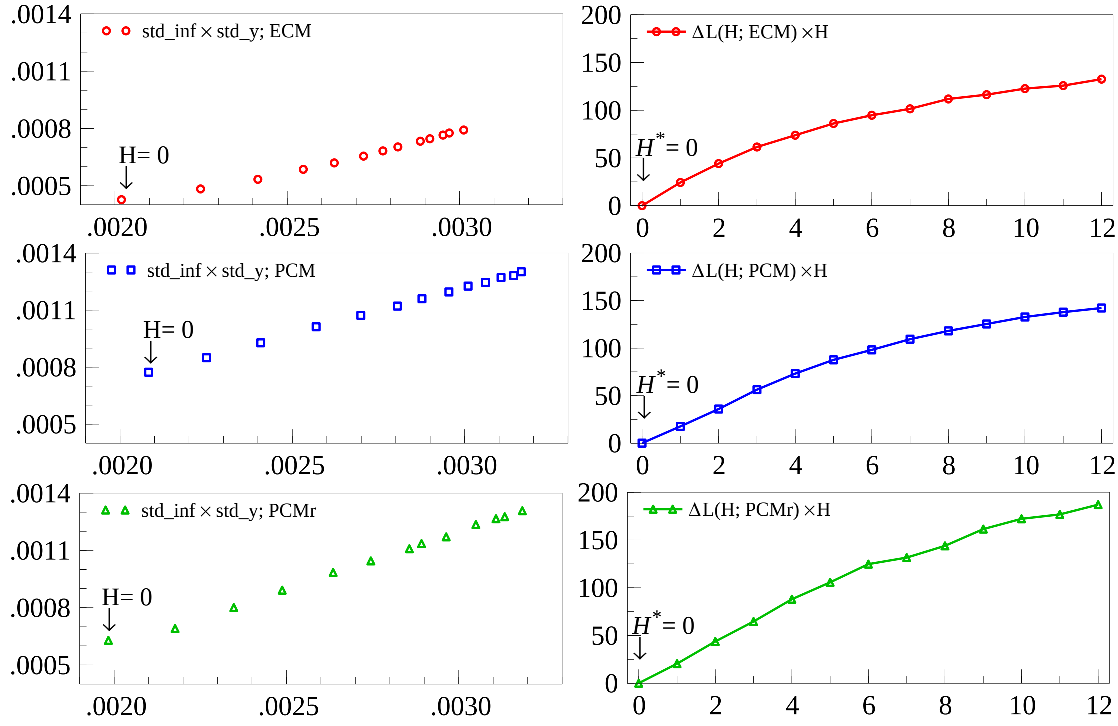


Figure 3: *Economic performance and optimal policy suggested by three models in the face of the demand shock. Left column: Trade-offs between standard deviations of inflation gap and output gap (horizontal axis) associated with different (policy) horizon-specific rules in response to the demand shock. The trade-offs are plotted for rules associated with policy horizons ($H$) in the range of 0–12 quarters. Here and elsewhere, the trade-offs associated with different horizons follow each other, where that one for $H$=0 is indicated. Right column: Values of the relative loss function (in %), defined by equation (10), at the different policy horizons (horizontal axis).*

$\mathcal{M}$, while $L(H^*; \mathcal{M})$ expresses the loss under optimal policy horizon conditional on model $\mathcal{M}$. It follows that $\Delta L(H; \mathcal{M}) > 0$ for $H \neq H^*$ while $\Delta L(H; \mathcal{M}) = 0$ when $H = H^*$, assuming the loss function is continuous in $H$ and there is a unique optimum.

As expected, there is no conflict between the objectives of price stabilization and output stabilization in the case of the demand shock; see Figure 3, left column. Moreover, it appears that both objectives can be promoted by reducing the policy horizon as much as possible. Hence, a policy horizon of zero appears as the most efficient one. The values of the relative loss functions are zero, i.e. at their optimal level, for $H$= 0; see right column. Hence, the optimal policy horizon would be zero irrespective of which wage and price system we implement in the model. This finding is consistent with the bulk of studies suggesting that demand shocks should be counteracted as aggressively as possible, since inflation can be stabilized jointly with output. However, even though the three systems imply the same optimal policy horizon, they imply different interest rate response to the demand shock and macroeconomic performance; see Figure 2. It may therefore not be immaterial which system is adopted to derive and implement the optimal policy response to the demand shock, as suggested by the different levels of standard deviations of inflation gap

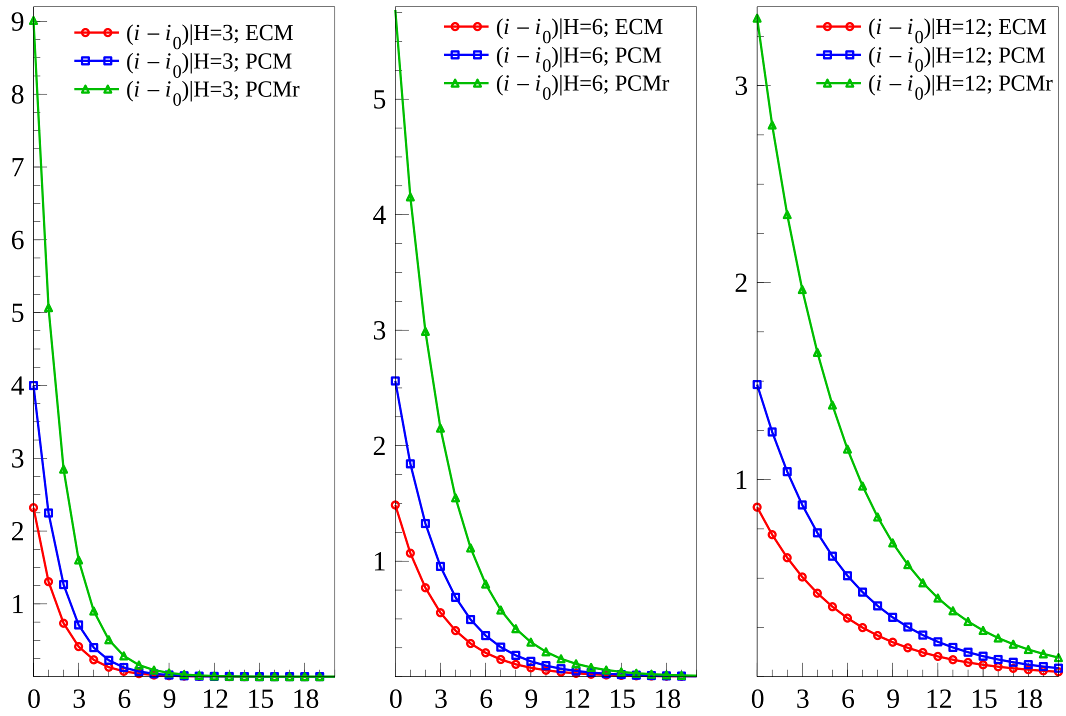


Figure 4: *Interest rate paths over time suggested by three models in the face of the supply shock. The three frames shows interest rate paths associated with the policy horizons of 3, 6 and 12 quarters, respectively. The interest rates are measured as deviation from the reference interest rate in percentage points, while the horizontal axes depict periods in quarters.*

and output gaps implied by the different model versions; see the left column for e.g. $H = 0$.

In the case of the supply shock, the implied monetary policy response is much stronger and differs widely across the three models; see Figure 2, middle frame. Figure 4 depicts the interest rate paths implied by the three models for three different policy horizons: 3, 6 and 12 quarters. These paths exhibit clearly the differences in monetary policy response implied by the three models. Moreover, the gap in policy implications of PCM and PCMr is wider than the gap between those of PCM and ECM. Notably, if there was an exogenously provided fixed policy horizon, the three wage and price systems, particularly the two systems of Phillips curves, would have suggested substantially different monetary policy responses to the supply shock. One may therefore say that seemingly minor parameter restrictions can alter the policy implications of a model fundamentally.

The large differences in the response coefficients across the three models can be ascribed to the associated wage and price systems, specifically to differences in the autoregressive coefficients across the three systems and to the effect of the unemployment term. The autoregressive coefficients largely determine the degree of persistence in the inflationary effects of the supply shock, i.e. how fast the inflationary effects of the transitory supply shock are exhausted. The larger the persistence, the more lasting the inflationary effects and the stronger the required interest rate response will be. Due to the relatively weak effect of unemployment on the wage growth in system (5), relative to those in systems (3) and (4), the monetary policy becomes less effective in PCMr, than in ECM and PCM. Hence, a relatively larger change in the interest rate is required in the case of PCMr than in the cases of ECM and PCM.

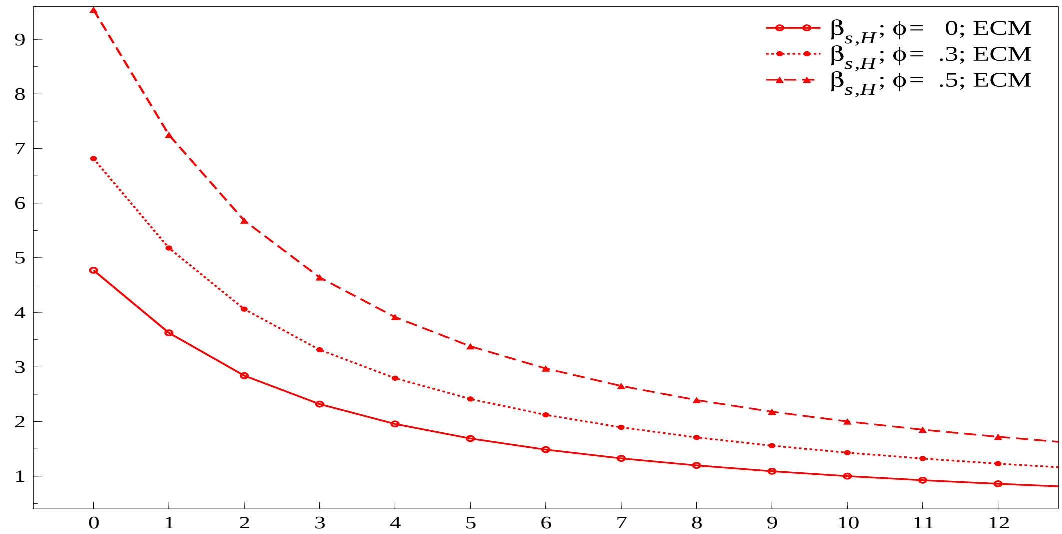


Figure 5: *Initial interest rate response suggested by ECM to supply shocks with different degrees of persistence, $\phi$. The initial interest rate response is implied by policy horizons in the range 0–12 quarters (horizontal axis).*

For example, the degree of persistence implied by the lagged and contemporaneous terms of wages and prices in system (5) is higher than that implied by system (4), which itself implies higher persistence than system (3). Consequently, the inflationary effects of the transitory supply shock are more lasting in the case of PCMr than in the case of PCM, which itself implies more lasting effects than ECM. Accordingly, the required interest rate response is higher in the case of PCMr than in the case of PCM and relatively low in the case of ECM.

The systems of Phillips curves, (4) and (5), which have relatively stronger autoregressive effects than the system of wage-price ECMs, (3), effectively make the transitory supply shock a more persistent one than the system of ECMs. In the system of ECMs, persistence is to a large extent modelled by lagged wage and price growth variables *and* equilibrium correction terms in the levels of variables. In terms of a VAR in levels, this entails that some of the characteristic roots are on the unit circle, while others are on the stable side of the unit circle. The system of the downward-sloping Phillips curve (4), however, implies a reduction in the number of stable roots since the direct equilibrium correction in wage and price setting is omitted, and as a consequence, an increase in the degree of persistence of any shock. The system of vertical Phillips curve system (5), has even more in-built persistence, because extra unit-roots are implied by the homogeneity restrictions; cf. Bårdsen and Nymoen (2006).

The analytical expression for the required interest rate response suggests that an increase in the degree of persistence in a shock increases the required interest rate response; see equation (8). Figure 5 suggests that the required interest rate responses in the case of ECM can become comparable to those implied by PCM and PCMr if we raise the persistence in the supply shock; cf. Figure 4.

Figure 6 presents the economic performance of (optimal and suboptimal) policies employed in response to the supply shock. The left column of the figure shows that there is a trade-off between price and output stabilization for different ranges of policy horizons. Specifically, in the case of ECM and PCM there is a trade-off in the range of 0 to 8 quarters. Policy horizons that are longer than 8 quarters appear inefficient as both price and output stabilization can be improved by shortening the policy horizon. The opposite is the case for PCMr. In this case, the trade-off curve is associated with policy horizons that are longer than 6 quarters, while policy horizons shorter than 6 seem inefficient.

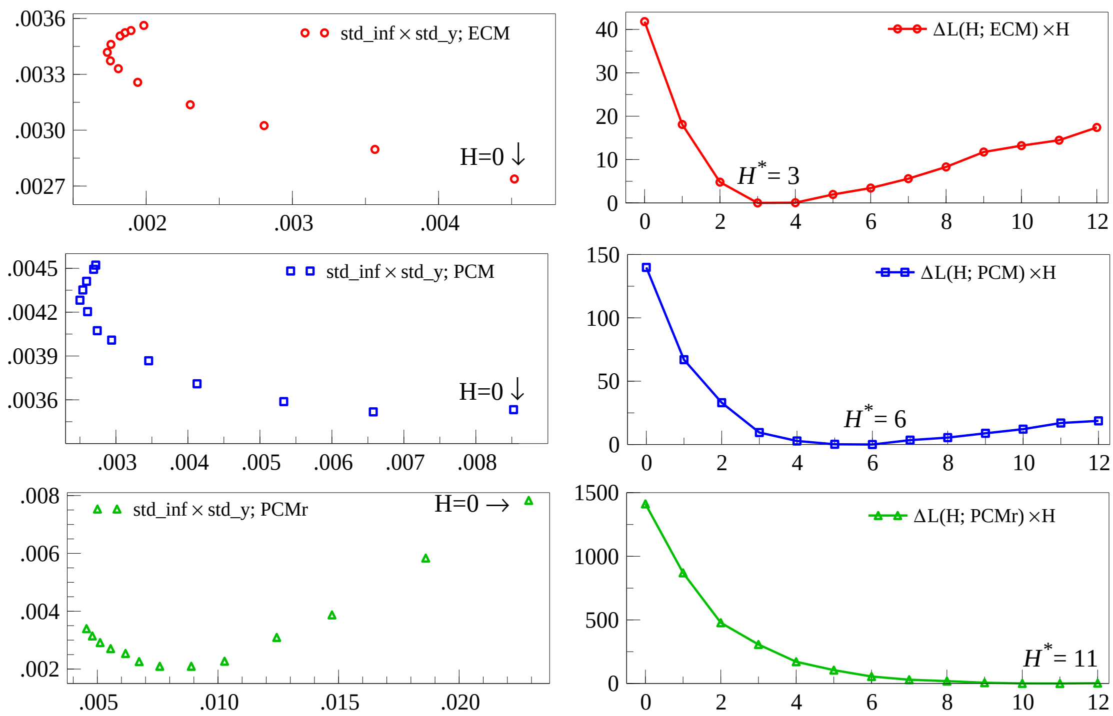


Figure 6: *Economic performance and optimal policy suggested by three models in the face of the supply shock. Left column: Trade-offs between standard deviations of inflation gap and output gap (horizontal axis) associated with different (policy) horizon-specific rules in response to the supply shock. The trade-offs are plotted for rules associated with policy horizons ($H$) in the range of 0–12 quarter, where that for $H = 0$ is indicated. Right column: Values of the relative loss function (in %), defined by equation (10), at the different policy horizons (horizontal axis).*

Figure 6 shows, in right column, that the three models recommend quite different policy horizons. Even though the efficiency frontiers for ECM and PCM are defined by almost the same policy horizon, the optimal horizon is 3 quarters in the case of ECM, but 6 quarters in the case of PCM. In the case of PCMr the policy horizon is 11 quarters. (An increase in the value of $\lambda$ from 0.5

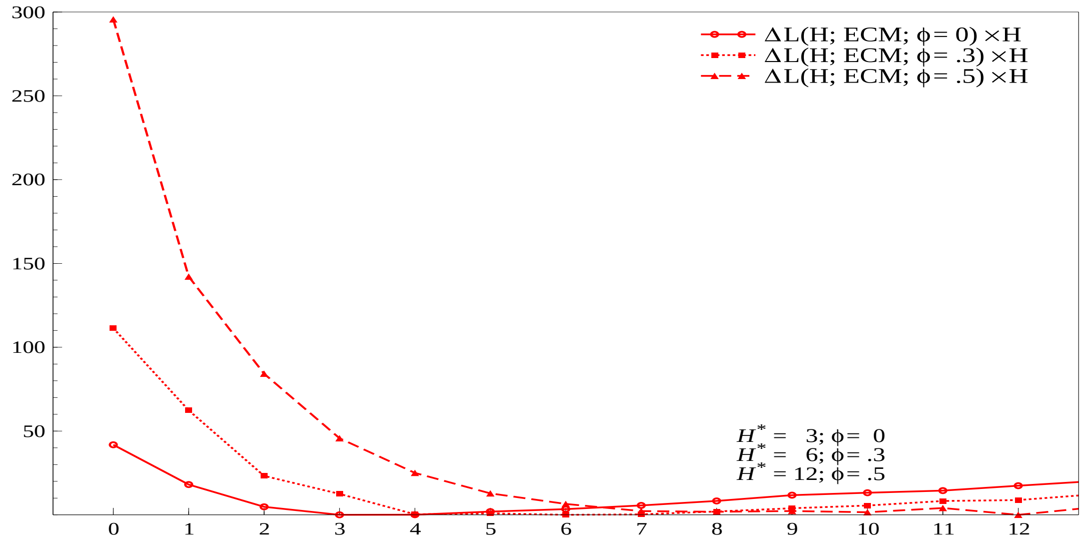


Figure 7: *Relative losses at different policy horizons, 0–12, when monetary policy responds to supply shocks with different degrees of persistence. The relative losses (in %), defined by (10), are based on* ECM.

would have increased the optimal policy horizons in all three models.)

The largely different optimal policy horizons imply widely different interest rate paths. They can be seen from Figure 4, where the interest rate path favoured by ECM appears in the left frame, that by PCM in the middle frame, while that favoured by PCMr would be close to that for $H=$ 12 in the right frame. Both ECM and PCM suggest about the same initial interest rate increase for $H = 3$ and $H = 6$, respectively; 2.25 and 2.5. However, the degree of interest rate smoothing associated with $H = 6$ is relatively higher, i.e. 0.72, which makes monetary policy contractionary over a relatively longer period than when $H= 3$. PCMr suggests an initial interest rate increase of about 4 for $H= 11$, while the implied interest rate smoothing is 0.83. Hence, PCMr suggests a more aggressive as well as a more prolonged contractionary monetary policy stance than the other two models.

In sum, the more persistent the inflationary effects are in a model, the longer is the preferred policy horizon. Both a higher degree of persistence in the inflationary effects of the shocks and the implied policy response, which also increases with the degree of persistence, contribute to relatively large economic fluctuations, i.e. high standard deviation of prices and the output gap. This is especially the case at especially short policy horizons. A relatively long horizon leads to a less aggressive policy response and a more prolonged contractionary policy. This helps to achieve a better synchronization between the destabilizing effect of the persistent inflationary effects with the stabilizing effect of monetary policy. Monetary policy thereby becomes more effective in stabilizing the economy.

The above results underscore the importance of imposing valid coefficient estimates when conducting monetary policy analysis. As demonstrated, restrictions on parameter values can have a more profound effect on monetary policy than alterations in model specification. The differences in suggested policy horizons can be mainly ascribed to the alteration in the degree of persistence by the different model specifications and to the estimated effects of unemployment on wages. For example, Figure 7 shows that ECM would have produced similar results if the shock had been more persistent. If the persistence in the shock had been 0.3, ECM would have suggested an optimal policy horizon of 6 quarters, and of 12 quarters if the persistence had been 0.5. This suggests that imposing seemingly weak restrictions to make the model e.g. more presentable may not be an innocuous act.

# 4 Costs of basing monetary policy on an invalid model

In the following, we investigate potential costs of basing monetary policy on a model that turns out to be invalid, or on a suite of models containing the valid model. For brevity, we only focus on monetary policy response to the supply shock and only consider monetary policy rules that are optimal in their respective models. Moreover, we only present results where ECM is assumed to represent the actual economy.[17]

## 4.1 Potential costs when the model selected turns out to be invalid

The following analysis sheds light on the potential loss when the econometric properties of a model are traded-off against other desirable properties, and when monetary policy is based on the least-fault tolerant model owing to concern for robustness in the face of perceived model uncertainty.

Figure 8 suggests the potential costs of choosing rules that are optimal in PCM and in PCMr in response to the supply shock when ECM is by assumption the valid model; the former rules are referred to as the PCM-rule and the PCMr-rule, respectively. The upper frame of Figure 8 depicts the outcomes in terms of standard deviations of inflation and the output gap when the PCM-rule and PCMr-rule are implemented in ECM. For comparison, the outcomes under (suboptimal and optimal) rules based on ECM itself, referred to as ECM-rules, are also plotted. The lower frame of the figure presents the relative losses under the ECM-rules as well as under the (optimal) PCM-rule and PCMr-rule.

The losses are measured relative to the level under the optimal ECM-rule: $L(\text{ECM-rule}; \mathsf{ECM})$, defined by $H = 3$. For example, the relative loss under the PCMr-rule is defined as:

$$\Delta L(\text{PCMr-rule}) \equiv \frac{L(\text{PCMr-rule}; \mathsf{ECM}) - L(\text{ECM-rule}; \mathsf{ECM})}{L(\text{ECM-rule}; \mathsf{ECM})}, \qquad (11)$$

where $L(\text{PCMr-rule; ECM})$ expresses that the value of the loss function (6) has been obtained by

[17] The results for the cases where PCM and in PCMr are assumed to be valid models do not affect our conclusions.

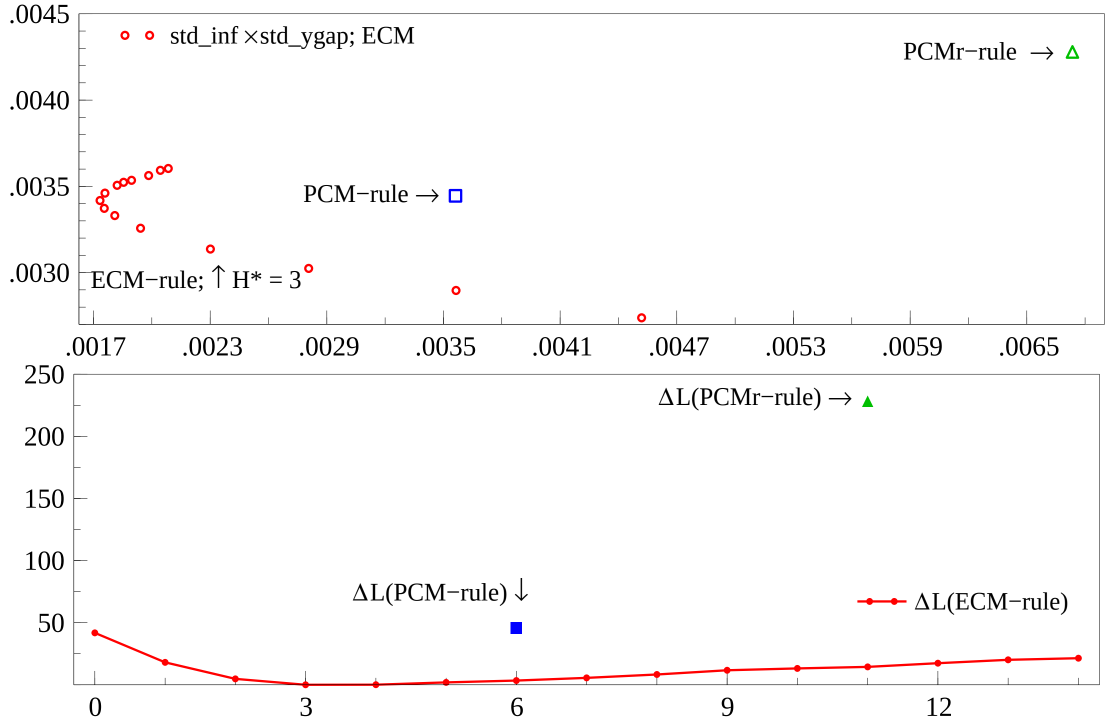


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Figure 8: *Economic performance and relative losses (in %) under different rules in response to the supply shock when* ECM *is the valid model. In the upper column, we plot outcomes in terms of standard deviations of the inflation gap (vertical axis) and output gap conditional on rules that would have been optimal if* PCM *or* PCMr *was the valid model. For comparison, we also reproduce the outcomes associated with (policy) horizon-specific rules based on* ECM *itself; cf. Figure 6. The policy horizon is varied in the range of 0-14; thus outcomes under 15 rules based on the valid model itself are reported. In the lower frame, we report values of the relative loss function (in %) under the optimal rules based on* PCM *and* PCMr*. The relative losses are calculated relative to the optimal loss if the optimal rule based on* ECM *itself was implemented; cf. (11) for a definition. The lower frame also reproduces values of the relative loss for different policy horizons under* ECM *itself; cf. Figure 6.*

implementing the rule that is optimal in PCMr, in ECM. As shown above, $H^* = 11$ defines the optimal rule conditional on PCMr, while $H^* = 3$ defines the optimal rule conditional on ECM. As above, we calculate the losses assuming $\lambda = 0.5$.

The relative losses summarizes the loss due to both bias in achieving the inflation target and excess variances of inflation and output gap. Clearly, when we implement the interest rate path minimizing the loss function (6) while satisfying the constraint (7) conditional on a different model than the valid model, the constraint (7) will not be satisfied while the linear combination of variances around the inflation target and that of output gap will not be minimized.

Figure 8 shows that *if* ECM is the valid model, while the policy rules are based on PCM and PCMr, both inflation and the output gap will become relatively more unstable, especially if the PCMr-rule is implemented. We note that under the PCM-rule, the loss would be 46% higher relative to that under the optimal ECM-rule. Under the vertical PCM-rule, however, the relative loss would be much higher: 228%.

It also seems that choosing the valid model and the corresponding rule is much more important than choosing the optimal policy horizon, and thereby the corresponding interest rate path. The lower frame of the figure shows that the relative losses under both the PCM-rule and particularly under the PCMr-rule are higher than under rules that are based on ECM but are defined by policy horizons that differ from the optimal policy horizon, 3. It can be shown that, at least for policy horizons within the range of 0–20 quarters, the relative loss under such rules never exceeds 42%, which is for $H = 0$.

A decomposition of the relative losses to derive possible bias in achieving the inflation target over the simulation horizon suggests that both the PCM-rule and the PCMr-rule imply a downward bias in average inflation. Precisely, in the case of the transitory supply shock considered here, the average inflation is about 1/10 of a percentage percentage point and 1/4 of a percentage point lower than the inflation target under the PCM-rule and the PCMr-rule, respectively. The downward seem reasonable as both of these rules imply stronger monetary policy response to the supply shock than the ECM-rule. It can be shown that the downward biases in the case of a rather persistent supply shock would be much higher.[18]

## 4.2 Potential costs when policy is based on a suite of models

We now assume that the economy is adequately characterised by one of the three models considered. However, we do not distinguish between the models and consider them equally probable, and hence also the associated monetary policy rules as equally relevant. Therefore, instead of implementing one specific monetary policy rule in the face of a given shock, we implement an 'average-rule'. Specifically, we define the required interest rate response, $\beta_s$, in the rule (8) as 1/3 of the sum of the $\beta_s$s implied by the three models and then determine the response for different horizons by (8), as above, for a transitory shock $\phi = 0$. This is consistent with a Bayesian approach to formulating a robust policy in the face of model uncertainty.

Figure 9 presents the outcome of the average-rule *if* the economy is characterised by ECM. For comparison, we also present the performance under the "valid rule", i.e. the rule implied by ECM. The upper frame depicts the curves presenting the trade-off between stability in inflation and output gap, while the lower frame depicts the loss under both rules relative to the loss under the optimal ECM-rule, defined in (11).

It appears that the average-rule will lead to much higher variation in both inflation and the output gap than the ECM-rule, irrespective of the policy horizon; in the figure this is shown only for policy horizons in the range of 0–14 quarters. Under average-rules, which can be defined by

[18] A topic for further research could be to analyse whether and to what extent the persistent undershooting of Norwegian core inflation relative to its target since the year 2002 can be ascribed to possible influence on Norwegian monetary policy of vertical Phillips curve models, comparable to that implemented in PCMr; see Chart 1.3 in Norges Bank (2007).

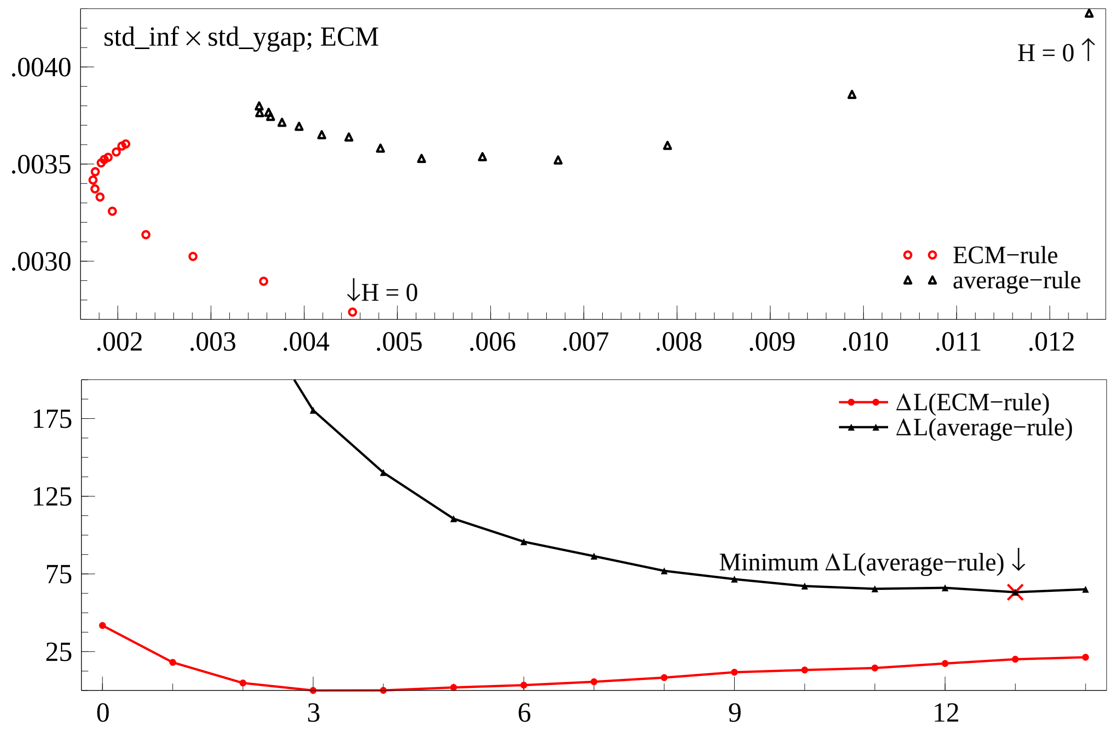


Figure 9: *Economic performance and relative losses (in %) under the 'average-rule' in response to the supply shock when* ECM *is the valid model. In the upper column, we plot outcomes in terms of standard deviations of the inflation gap (vertical axis) and output gap conditional on the average-rule for different policy horizons (horizontal axis). For comparison, we also reproduce the outcomes associated with (policy) horizon-specific rules based on* ECM *itself; cf. Figure 6. The policy horizon is varied in the range of 0–14; thus outcomes under 15 rules based on the average-rule as well as* ECM *itself are reported. In the lower frame, we report values of the relative loss function under the average- rule as well as the* ECM *itself. The relative losses are calculated relative to the loss if the optimal rule based on* ECM *itself was implemented; cf. equation (11) for a definition.*

different policy horizons, there is a trade-off between price and output stability for policy horizons above 4 quarters. The poor performance of average-rules is mainly because they suggest a much stronger policy response than favoured by the valid model, by assumption, ECM. Relatively short policy horizons under an average-rule are especially destabilizing because they suggest relatively strong immediate interest rate hikes; cf. Figure 2. A comparison of the trade off curves in Figure 9 suggests that the performance of an average-rule will be considered inferior to that of an ECM-rule, irrespective of policy horizon.

The lower frame of Figure 9 shows relative values of the loss function under both (suboptimal and optimal) ECM-rules and average-rules. The difference between relative losses indicates the costs of implementing the average-rule relative to that of implementing ECM-rules. It appears that the loss will be higher, the lower is the policy horizon under an average-rule. Notably, if we implement an average-rule, the loss when ECM is valid will tend to decrease with the policy horizon. Thus, even though we choose the policy horizon for which the loss under an average-rule will be at a minimum, which is at $H = 13$, the relative loss under the average-rule will be ca. 63%

higher than that under the optimal ECM-rule, defined by $H = 3$. It should also be noted that possible costs of basing policy on a suite of models may become higher if the suite does not include the valid model.

In addition, if an average-rule consistent with one's preferences is implemented, by choosing one of the policy horizons considered, it will generally make inflation deviate from its target rate. In the case of transitory shock considered here, the average inflation is below target by 1/10 of a percentage point over the simulation horizon. The bias is downward as the average rule is more contractionary relative to the ECM-rule.

As observed above, it also seems more important to choose the rule consistent with the valid model than choosing the optimal policy horizon. We note that under an ECM-rule defined by $H \neq H^* = 3$, the relative loss does not exceed that under an average-rule even when the relative loss under an average-rule is at its minimum, i.e. when $H = 13$.

# 5 Conclusions

We have investigated the economic significance of trading off the empirical validity of models against other desirable model properties, and the potential loss from 'overestimating' model uncertainty and basing monetary policy on a relatively robust model, or on a suite of models. We have based our investigation on three alternative econometric systems for wage and price inflation for Norway that have been embedded as the supply side in a well specified macroeconometric model for medium-term analyses.

Our results substantiate the view that a model for policy analysis should be empirically valid and caution against compromising this property for other desirable model properties.[19] We find that differences in model specification and even differences in estimates of key parameters across similar models may entail widely different monetary policy and macroeconomic performance. Interestingly, it appears that imposing a set of parameter restrictions may have stronger influence on policy implications than choosing a different functional form of the model.

Our results also suggest huge gains from extracting as much information from available information sources as possible to avoid 'overestimating' model uncertainty and obtain as narrow set of empirically valid models as possible, if not a single model, for policy analysis, rather than including a large number of models in the suite. We find substantial costs from 'overestimating' model uncertainty and responding to such uncertainty by adopting robust monetary policies, either based on the least-fault tolerant model, or on a suite of models, even when it contains the valid model by assumption. Our results imply that reliance on robust policies is more justifiable in cases of severe model uncertainty than when ample empirical evidence clearly favours one specific model.

[19]This is consistent with e.g. Granger (1992) who states that "...it should be generally agreed that a model that does not generate many properties of actual data cannot be claimed to have any 'policy implications'...".

Estimates can always be contested in economics. Thus, further research using alternative models and alternative ways of characterising monetary policy would be useful in assessing the robustness of our results. Nevertheless, it seems reasonable to conclude that there may be huge gains from utilizing empirical evidence efficiently to select the valid model and parameter estimates. In this endeavour, there may also be large gains from improving data quality and timely availability of data as well as further research on improving tools for the efficient use of available information.

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# A Appendix: Data definitions

Unless another source is given, the time series have been extracted from databases maintained by Norges Bank (the central bank of Norway). The variables are precisely defined in Rikmodnotat 140, Norges Bank. The variables as named in the RIMINI database are noted in hard brackets [.] below. Where relevant, the base year is 1991 and the unit of measurement is mill. NOK.

*Mainland economy* is defined as the total Norwegian economy excluding oil and gas production and international shipping.

Impulse dummies are denoted as *iyyq***x**. For example, *i80q2* is 1 in the second quarter of 1980 and 0 in all other quarters.

### Definitions

$H$ Normal working hours per week; [NH]

$P$ Consumer price index; [CPI].

$PI$ Deflator of total imports; [PB].

$Y$ Total value added at market prices in the mainland economy; [YF].

$PR$ Mainland economy value added per man-hour at factor costs; [ZYF].

$RS$ 3 month Euro-krone interest rate; [RS].

$\tau 1$ Employers' tax rate; [$T1$]

$\tau 3$ Indirect tax rate; [T3].

$U$ Unemployment rate; [UTOT].

$W$ Mainland economy hourly nominal wages; constructed from several time series in the RIMINI database.

$Wdum$ Composite dummy for wage freeze: 1 in 1979q1, 1979q2, 1988q2 and 1988q3.

$Pdum$ Composite dummy for introduction and removed of direct price regulations. 1 in 1971q1, 1971q2, 1976q4, 1979q1; -1 in 1975q1, 1980q1, 1981q1, 1982q1; and zero otherwise.

# B The interest rate rule

In the following, we present the assumptions behind rule (8) and sketch how it can be derived using a simple model of inflation gap (deviation between actual and desired inflation rate). This approach is particularly useful when employing large scale macroeconometric models; see Akram (2007a) for details.

## B.1 Assumptions

1.A Assume the following linear model of inflation $\pi$:

$$\pi_t - \pi^* = \sum_{l=0}^{\widetilde{T}_0} \alpha_l \Delta z_{t-l} - \sum_{m=0}^{T_0} \gamma_m \Delta i_{t-m}, \tag{12}$$

where $\pi^*$ is the inflation target, $z$ is an exogenous shock variable, while $i$ is the nominal interest rate. Here, we use $\Delta$ to denote deviation from the neutral or steady state level. Thus, e.g. $\Delta i_t$ denotes an interest rate deviation relative to the neutral level, and hence the extent of a non-neutral monetary policy stance. Inflation gap at time $t$, $\pi_t - \pi^*$, depends on finite lagged and contemporaneous effects of shocks and nominal interest rates. Thus, effects of a transitory shock will eventually die out, even when conditioned on the equilibrium nominal interest rate path, i.e. $\Delta i_{t-m} \equiv i_{t-m} - i_0 = 0$ for all $m$. The partial effects, $\alpha$s and $\gamma$s, are assumed to be constant and their sums are assumed to be strictly positive: $\sum_{l=0}^{\widetilde{T}_0} \alpha_l > 0$, while $\sum_{m=0}^{T_0} \gamma_m > 0$, where $\widetilde{T}_0$ and $T_0$ are finite values.

2.A A shock $\Delta z_t$ with persistence $\phi \in [0,\, 1)$ is characterised as an AR(1) process:

$$\Delta z_t = \phi \Delta z_{t-1} + \varepsilon_t, \tag{13}$$

where $\varepsilon_t$ is an exogenous shock term whose expected value is zero. Any change in $\Delta z_t$ from zero will die out asymptotically, but we expect that it becomes negligible after $N+1$ periods, i.e. $\Delta z_{t+N+1} = \phi^{N+1} \Delta z_t \approx 0$.

Monetary policy is operationalised as follows, for simplicity.

3.A The overriding objective of monetary policy is $\mathsf{E}\pi = \pi^*$, which is obtained by ensuring that $\mathsf{E}_t \pi = \pi^* = 0$; see (7). Operationally, interest rate paths considered in response to a given shock must ensure that inflation is equal to its target on average:

$$\overline{\pi} = \pi^*. \tag{14}$$

4.A In response to the shock, the interest rate is shifted abruptly to a non-neutral level but is thereafter brought more or less gradually to its neutral level over time. Monetary policy response to a shock is initiated when the shock occurs; assuming away possible observation and decision lags.[20]

[20] Under such simplifying assumptions as 3.A–4.A, inflation rate targeting become effectively equal to price path targeting, however. This is because, a potential base drift in prices, which may arise under inflation targeting because of observation and decision lags in actual monetary policy making do not arise here. Price path targeting with a positive drift in prices differs from inflation targeting by not letting "bygones be bygones"; price level targeting is a special case of price path targeting.

Such an interest rate behaviour in response to shock at time $t$ can be characterised as:

$$\Delta i_{t+m} = \beta \Delta z_\tau + \varrho \Delta i_{t+m-1};\ m = 0, 1,\ 2, ..H, H+1, ... \tag{15}$$

where $0 < \varrho < 1$. The interest rate is brought gradually towards $i_0$, in principle over infinite periods.

However, interest rates are considered converged with the neutral rate after a finite number of periods. Specifically, in period $t + H + 1$ just a small fraction, say $\delta$, of the initial interest rate deviation is assumed to remain:

$$\Delta i_{t+H+1} = \delta \Delta i_\tau. \tag{16}$$

where $0 < \delta < 1$ is a sufficiently small ratio (of choice). Equations (15) and (16) imply that $\varrho^{H+1} = \delta$ and hence, $\varrho = \delta^{1/(H+1)}$, where $H \geq 0$.

## B.2 Derivation

We now derive the value of $\beta$ and $\varrho$ (or $H$) that will define an interest rate path consistent with the overriding monetary policy objective, as expressed by 3.A, for the given specification of the interest rate rule, the shock process and the inflation model.

Assume that a state of equilibrium is disturbed in period $\tau$ by a shock $\Delta z_\tau$ with persistence $\phi$. That is, $\Delta z_\tau = \varepsilon_\tau$, while $\Delta z_t = 0$ for $t < \tau$ and $\varepsilon_t = 0$ for $t \neq \tau$.

3.A implies that the interest rate in response to the shock is set such that:

$$\sum_{l=0}^{T} (\pi_{\tau+l} - \pi^*) = 0. \tag{17}$$

This would be satisfied if the accumulated effects on inflation of current and future interest rate deviations (from $i_0$) outweigh the accumulated effects of the shock on inflation over time.

The model given by 1.A implies that a transitory shock in period $\tau$, $\Delta z_\tau \neq 0$, will affect inflation contemporaneously and in $\widetilde{T}_0$ future periods. Under 2.A, the accumulated effects of a persistent shock $\Delta z_\tau$, which are neglected after $N+1$ periods, will become:

$$\left[\frac{1-\phi^{N+1}}{1-\phi}\right] \left[\sum_{l=0}^{\widetilde{T}_0} \alpha_l\right] \Delta z_\tau. \tag{18}$$

The model also implies that an interest rate deviation in period $\tau$, $\Delta i_\tau \neq 0$ , will affect inflation contemporaneously and in $T_0$ future periods. Under 4.A, the accumulated effects of an interest rate deviation $\Delta i_\tau$, which is eliminated gradually over $H+1$ periods, will amount to:

$$\left[\frac{1-\varrho^{H+1}}{1-\varrho}\right]\left[\sum_{m=0}^{T_0}\gamma_m\right]\Delta i_\tau. \tag{19}$$

Consistency with 3.A (and (17)) would require that:[21]

$$\left[\frac{1-\phi^{N+1}}{1-\phi}\right]\left[\sum_{l=0}^{\widetilde{T}_0}\alpha_l\right]\Delta z_\tau - \left[\frac{1-\varrho^{H+1}}{1-\varrho}\right]\left[\sum_{m=0}^{T_0}\gamma_m\right]\Delta i_\tau = 0. \tag{20}$$

4.A implies that $\Delta i_\tau = \beta\Delta z_\tau$ if the interest rate is initially at the neutral rate, i.e. $\Delta i_{\tau-1} = 0$. Thus, $\beta$ will be:

$$\beta = \left[\frac{1-\varrho}{1-\varrho^{H+1}}\right]\left[\frac{1-\phi^{N+1}}{1-\phi}\right]\left[\frac{{}^{\widetilde{T}_0}_{l=0}\alpha_l}{{}^{T_0}_{m=0}\gamma_m}\right]. \tag{21}$$

Hence, (15) can be formulated as:

$$i_{\tau+m} = i_0 + \left[\frac{1-\varrho}{1-\varrho^{H+1}}\right]\left[\frac{1-\phi^{N+1}}{1-\phi}\right]\beta_\varepsilon\varepsilon_\tau + \varrho(i_{\tau+m-1}-i_0);\ m = 0, 1,\ 2, ..H, H+1, ... \tag{22}$$

where $\beta_\varepsilon \equiv \sum_{l=0}^{\widetilde{T}_0}\alpha_l / \sum_{m=0}^{T_0}\gamma_m$; $\Delta z_\tau = \varepsilon_\tau$ and e.g. $\Delta i_t \equiv i_t - i_0$. Different values of $\varrho$, or $H$ since $\varrho = \delta^{1/(H+1)}$ where $\delta$ is a given fraction, will specify different interest rate paths satisfying 3.A, i.e. (14) and (17).

The interest rate rule (22) can be closely approximated by the rule (8) as $\varrho^{H+1} = \delta$, which is close to zero by assumption (4.A) and $\phi^{N+1} \approx 0$ can be reasonable when $N$ is sufficiently large. In rule (8), we use $\varrho_H \equiv \varrho$ to indicate explicitly that degree of smoothing depends on the policy horizon.

## B.3 Implementation

To implement the interest rate rule (8) in response to a specific shock $\varepsilon$ with a given persistence $0 \leq \phi < 1$, we need to derive estimates of $\varrho_H$ and $\beta_\varepsilon$.[22] Values of interest rate smoothing for different $H$, $\varrho_H$, can then be obtained from the relationship $\varrho_H = \delta^{1/(H+1)}$.

We assume that $\delta = 0.1$, but our empirical results are quite invariant to the choice of $\delta$ values as long as they are relatively small.

To estimate $\beta_\varepsilon$, we derive impulse responses of inflation to a transitory shock $\varepsilon$, conditional on a given nominal interest rate path, and impulse responses of inflation to a transitory increase in interest rates. Then, in line with its definition, $\left(\beta_\varepsilon \equiv \sum_{l=0}^{\widetilde{T}_0}\alpha_l / \sum_{m=0}^{T_0}\gamma_m\right)$, the estimate of $\beta_\varepsilon$ is

[21] $T$ in (17) would be equal to $\sup\{\widetilde{T}_0 + N,\ T_0 + H\}$.

[22] The results are almost the same if we employ the exact rule (22) rather than its approximation (8).

obtained as the ratio of accumulated impulse responses of inflation to shock $\varepsilon$ to the accumulated impulse responses of inflation to the interest rate increase.

To provide a transitory demand shock ($\varepsilon_d$) to a given version of the model (ECM, PCM or PCMr), we raise the residual of the aggregate demand equation ($y$-equation) over the period 1995q1–1995q4 such that the growth in aggregate demand initially increases by one percentage point over the year 1995. Thereafter, we set the residual of the demand equation to zero and let the dynamic model adjust on its own over the simulation period: 1995q1–2000q4. This is a sufficiently long period to let the partial effects of the demand shock (as well as those of the supply shock and an interest rate change) work out. We let the policy interest rates remain invariant to the shock and the subsequent adjustment by letting them follow their reference path.

We follow the same procedure in the case of the supply shock ($\varepsilon_s$), but raise the residual of the *cpi*-equation over the period 1995q1–1995q4 such that inflation increases by one percentage point over the year 1995.

To estimate the partial effects of a transitory increase in interest rates, we raise the interest rate by one percentage point relative to its reference path over the period 1995q1–1995q4, ceteris paribus, and thereafter set it back to its reference path.