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Retirement in Non-Cooperative and Cooperative Families

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Retirement in Non-Cooperative and Cooperative Families*

By

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Abstract

Models for non-cooperative as well as cooperative behavior of families are estimated on data from Norway from 1994 to 1998. The models aim at explaining labor supply behavior of married couples the first five months after the husband becomes eligible for early retirement, while the wife is not eligible. Estimates and predictions derived from the different models are compared. Econometric tests find that the Stackelberg model with the male as the leader is the best. Simulations with the estimated models show that taxing pension income the same way as labor income would reduce the propensity to retire early considerably.

Keywords: family labor supply, retirement, econometric models, policy simulations

JEL classification: D10, H55, J26

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1 Introduction

An increasing proportion of elderly persons in the population, falling labor force participation of older males and maturing of the public pension system all combine to threaten the financial stability of pay-as-you-go public pension systems in many industrialized countries. In Norway, problems have been exacerbated by the introduction of an early retirement program, hereafter called AFP (a Norwegian abbreviation). From a policy point of view, knowledge about how economic incentives affect workers' retirement, and to what extent they will respond to policy changes are therefore important.

Most of the literature on retirement behavior has focused on single individuals; see Lumsdaine and Mitchell (1999) for references. However, since a majority of older men and women are married or cohabitating, it is important to account for the fact that labor market behavior may be due to joint decisions by married couples. Among the relatively few empirical studies of retirement behavior in a household context, most have focused on patterns of family retirement, like "wife first", "joint retirement" and "husband first", see Henretta and O'Rand (1983) for an early contribution. In recent studies Gustman and Steinmeier (2000) find a tendency for spouses to retire together, which they attribute to correlation in preferences for (joint) retirement. Baker (2002) finds that the propensity to retire among males is around 5-10 percentage points higher when the wife is eligible for a supplementary pension. Blau (1997) finds "strong associations between the labor force transition probabilities of one spouse and the labor force status of the other spouse."

Lately, there have been retirement studies that explicitly model family behavior as the outcome of non-cooperative behavior. Hiedemann (1998) uses a Stackelberg model with male leadership to model the joint social security acceptance decisions and finds that it depends on several individual and household characteristics as well as financial incentives. But we have not seen much empirical evidence in the literature on how the decision process within the family really works. Do they cooperate in the sense that they share common interests and make the decision to maximize a 'family utility' function as if there is a benevolent dictator? Or do they maximize their own utility functions so that the family labor supply is just an outcome of a non-cooperative game?

In our analysis, we use the introduction of the AFP program as an opportunity to study the retirement decision of elderly, married couples and the responsiveness of that decision to the level of current earnings and potential pension benefits. The main purpose is to contrast different models for retirement behavior. We specify models for non-cooperative behavior as well as models for cooperative behavior within families. We follow Bresnahan and Reiss (1991) and Kooreman (1994) in calculating Nash and Stackelberg-equilibrium. In Kooreman (1994) reaction functions are derived from linear utility functions of the spouses, while we allow more general (flexible) functional forms of the deterministic part of the utility functions (nonlinear function of disposable income and leisure), with linear and Cobb-Douglas function as special cases. The models are estimated on Norwegian data from 1994-1998. Since the husband is usually older than the wife, on the average by three years, we restrict the sample to couples in which the husband becomes eligible, over a period when the eligibility age was at age 64, whereas the wife did not qualify. In contrast to the studies referred to above we observe the exact date of retirement and we also observe all details of the budgets sets, including pension benefits and taxes paid. The estimates of the different models are compared using econometric tests of how well the different models predict observed labor market attachments. We conclude that the Stackelberg model, with male leadership, performs best among the models we have studied, although they give quite similar parameter-estimates. The models are then employed to simulate the impacts on the labor supply of the families of replacing the rather generous taxation of pension benefits with the taxation of earnings for all kinds of income. It is shown that this policy change has a strong and negative impact on the propensity to retire early.

In section 2 we describe briefly the institutional setting in Norway. Section 3 presents the model and section 4 give a basic description of data sources and the sample used in the analysis. Estimates and policy simulation are given in section 5. Section 6 concludes.

2 Institutional settings

The institutional settings are described in detail in Hernæs et al (2000). Briefly summed up, an early retirement program (AFP) came into effect in Norway in 1989, as part of the national wage settlements of 1988. This program allows retirement before the standard retirement age 67, when ordinary old age pension can be received. The AFP age was 66 from 1 January 1989, 65 from 1 January 1990, 64 from 1 October 1993, 63 from October 1 1997 and 62 from March 1 1998.

The AFP program covers all government employees (of local and central government), and private sector employees of companies that have joined the program, in total about 60 per cent of the labor force. Participation is voluntary on the part of the private companies, and will usually be a part of the tariff agreement with the union. Self-employed and private employees of companies not participating are not covered. There are also individual requirements for being eligible for AFP, as only those are eligible who

- had been employed in the company the last 3 years or been employed in another company also operating the AFP scheme the last 5 year,
- had earnings at a level at least corresponding to the basic pension (G) when AFP is taken up,
- had earnings at least equal to the basic pension the year before,

- had an average proportion between earnings and the basic pension of at least 1 in the 10 best years after the age of 50 and
- had at least 10 years in which earnings were at least twice the basic pension.

Persons meeting individual criteria while working in companies covered by the program become eligible from the month after they turn the required age. With information on birth date, we are therefore able to identify exactly the date of eligibility.

Although the AFP program is a negotiated agreement, the benefits received are the same as in the ordinary public old age pension system. Private employees receive an AFP pension equal to the ordinary public old age pension, based on their actual earnings history and a projection of earnings from AFP take-up and up to age 67. This pension is also the pension they will receive from age 67, so that there is no penalty on early retirement. A detailed explanation of the how this pension is calculated is given in Hernæs and Strøm (2000). With pension level and exchange rate prevailing in the Autumn of 2001, it varies between 9 000 USD and 22 000 USD per year, Income above 68 000 USD does not count towards the pension. The system is therefore strongly re-distributive.

The AFP pension for (local and central) government employees is the same as for private employees up to age 65, when it becomes equal to the old age pension for public sector employees. Over the observation period, this latter pension equaled about 2/3 of income up to 45 000 USD and 2/9 of any part of the income between that level and the maximum level for accrual at 68 000 USD. Details can be found in Hernæs and Strøm (2000).

There are also special tax rules, which apply to retirement benefits. These are briefly described below, but all details are given in Haugen (2000). In the early retirement program a tax-free lump-sum amount is given to those who retire from a job in the private sector. In the government sector a higher, but taxed lump-sum amount is awarded.

Pensions for private employees are financed by a state subsidy of 40 per cent from age 64, and with the balance financed by the employers. In some industries the company of the incumbent pays 10 per cent of the pension whereas the rest is paid from pooled contributions levied according to the wage sum of the company. In other industries the contribution of the company equals the pensions of its (former) employees. Pensions for government employees are paid directly by the government.

3 Methodology

3.1 The models

We want to analyze the labor market decisions of elderly couples, when a new option (early retirement) becomes available to the husband. We assume that the decisions are results of either a two player non-cooperative game or more traditionally the maximization of a joint utility.

The available choices for the husband are:

$$y_m = \begin{cases} 1 & \text{if he decides to take early retirement} \\ 0 & \text{if he decides to continue to work} \end{cases}$$

Similarly, the wife's choices set is:

$$y_f = \begin{cases} 1 & \text{if she decides not to work} \\ 0 & \text{if she decides to work} \end{cases}$$

3.1.1 Non cooperative model: Separate utility functions for husband and wife

We first assume that the husband and the wife have his/her own utility function, and both of them try to maximize his/her own utility. As econometricians we do not know the preferences of the individuals and thus we have to deal with random utilities, although they may be assumed to be common knowledge within the household. Thus we assume the following random utility functions:

$$\begin{cases} U_m(y_m, y_f) = v_m(y_m, y_f) + \varepsilon_m(y_m) \\ U_f(y_m, y_f) = v_f(y_m, y_f) + \varepsilon_f(y_f) \end{cases}$$

where $v_k(.); k = f, m$ are the deterministic parts of the utility functions and $\varepsilon_k(.); k = f, m$ are the random parts. We recognize that it is actually a two-person discrete choice problem. One way to solve the problem is to use the multivariate qualitative model (see for example Maddala, 1983), which is an extension of univariate LOGIT or PROBIT. The choice then is determined by the following simultaneous equation system with discrete endogenous variables (endogenous dummy variables):

$$\begin{cases} y_m^* = v_m(1, y_f) - v_m(0, y_f) + e_m \quad \text{where } e_m = \varepsilon_m(1) - \varepsilon_m(0) \\ y_f^* = v_f(y_m, 1) - v_f(y_m, 0) + e_f \quad \text{where } e_f = \varepsilon_f(1) - \varepsilon_f(0) \\ y_l = 1 \qquad \text{if} \quad y_l^* > 0 \quad l = m, f \\ y_l = 0 \qquad \text{otherwise} \end{cases} \tag{1}$$

where we assume that e_m and e_f are logistic distributed with correlation ρ across the husband and wife. But as argued by Heckman (1978) and Maddala (1983), some coherency conditions are required for the equation system to be well defined. As a result of imposing these coherency conditions, the simultaneity, which is essential in our analysis, is unfortunately eliminated.

Bresnahan and Reiss (1991) model the multi-person discrete choice behavior as the result of a multi-player game, and use solution concepts such as Nash equilibrium or Stackelberg equilibrium, rather than the equation system (1). Kooreman (1994) discusses the estimation problem of the econometric models of discrete games.

In our analysis, we follow the approach developed in Kooreman (1994) to model the observed behavior. In the game discussed here, husband and wife can take one of two actions, working or not working. The pay-off is his/her utility function: $U_k(y_m, y_f)$; k=m,f;

The pay-off matrix of the game is given in Table 1.

Table 1: The pay-off matrix of the Game

Husband	Wife	
	Works, y_f=0	Home, y_f=1
Works, y_m=0	$U_m(0,0)$, $U_f(0,0)$	$U_m(0,1)$, $U_f(0,1)$
Retired, y_m=1	$U_m(1,0)$, $U_f(1,0)$	$U_m(1,1)$, $U_f(1,1)$

Two solution concepts of this one-shot game will be employed below.

Nash Equilibrium

Each player is assumed to maximize his/her utility function, given the action of the other player. Both players then adjust their actions until their decisions are mutually consistent. Or mathematically, choice (y_m, y_f) is a Nash-Equilibrium (NE) if

$$U_m(y_m, y_f) > U_m(1-y_m, y_f) \ \ and \ \ U_f(y_m, y_f) > U_f(y_m, 1-y_f); \ y_m, y_f = 0,1 \qquad (2)$$

A two-player game may have more than one NE or have no NE at all. Jia (2001) shows that the necessary and sufficient condition for (y_m, y_f) to be a NE for the above game is that it is a solution to the equation system (1). So the problem of equilibria non-uniqueness for the game is essentially the coherency problem for the simultaneous endogenous dummy model referred to above and vice versa.

There are several ways to solve the problems, as discussed both in Bresnahan and Reiss (1991) and Kooreman (1994).

We make the simplest assumptions following Kooreman (1994):

- If there is only one NE, the household will choose it.

- If there is more than one NE, we assume the household pick any one of them by random.
- If there is no NE, we assume each available choice is chosen with equal probability.

As shown in Table A.1 in Appendix 1, we can specify the NE corresponding to each of the sixteen possible combinations. Under the assumptions, we can calculate the probability of the household choosing (y_m, y_f) for $y_m, y_f=0,1$.

For example:

$$\begin{aligned}
&\Pr(\text{husband retire, wife not work}) = \Pr(1,1) \\
&= \Pr(e_m > (v_m(0,1) - v_m(1,1)) \ \wedge e_f > (v_f(1,0) - v_f(1,1))) \\
&-\frac{1}{2}\Pr((v_m(0,0) - v_m(1,0)) > e_m > (v_m(0,1) - v_m(1,1))\wedge(v_f(0,0) - v_f(0,1)) > e_f > (v_f(1,0) - v_f(1,1))) \\
&+\frac{1}{4}\Pr((v_m(0,0) - v_m(1,0)) > e_m > (v_m(0,1) - v_m(1,1))\wedge(v_f(1,0) - v_f(1,1)) > e_f > (v_f(0,0) - v_f(0,1))) \\
&+\frac{1}{4}\Pr((v_m(0,1) - v_m(1,1)) > e_m > (v_m(0,0) - v_m(1,0))\wedge(v_f(0,0) - v_f(0,1)) > e_f > (v_f(1,0) - v_f(1,1)))
\end{aligned}$$

The likelihood function simply follows.

Stackelberg Equilibrium

Instead of the symmetric Nash-game we can assume that the roles of husband and wife are asymmetric, i.e. one of them is assumed to be the leader, the other acts as a follower. Then we have a Stackelberg-game. Note that the solution we get using this equilibrium concept is not the solution for the equation system (1).

It is easy to see that Stackelberg equilibrium always exists and that it is unique. Table A.2 in Appendix 1 shows the probability of the couple choosing state (y_m, y_f) for the case of male as the leader. Detailed deductions can be found in Hiedemann (1998). Similar to the case of Nash-Equilibrium, we can construct the likelihood function.

Notice that neither Nash-Equilibrium nor Stackelberg-Equilibrium is generally Pareto optimal. So the use of non-cooperative game is controversial. Kooreman (1994) tried to estimate a model implying Pareto-optimality of observed outcomes. With a very simple structure, i.e. linear reaction functions, he was not able to get convergence. Although he managed to succeed to estimate a mixed model of Pareto-optimality and Nash equilibrium, we have not tried to estimate a model along his line.

3.1.2 Joint utility for the couple; cooperative households

One possible way to account for cooperative behavior is to assume that the couple has one joint utility function. Or, that the decisions within the family are made in a cooperative setting. In the literature, there is an increasing interest in models of household behavior as the result of a cooperative game, particularly a Nash bargaining game. See for example Bourguignon and Chiappori (1992) for a review. But it turns out that the empirical estimation of such a model is very difficult, since we would like to estimate simultaneously the individual preferences of the spouses and the threat point. At the present stage, we are not able to do so. On the other hand no definite conclusion about which approach (joint utility versus Nash-bargaining) is better has been made yet. Kapteyn and Kooreman (1992) argued that more about the players' preferences should be known before one can discriminate between these two kinds of models empirically. We will therefore use the neoclassical joint utility for couples and assume the following random utility function:

$$U(y_m, y_f) = v(y_m, y_f) + \varepsilon(y_m, y_f) \qquad (3)$$

Under the assumption of $\varepsilon(y_m, y_f)$ being extreme value distributed with a location parameter η and a scale parameter σ, and the assumption of utility maximization, the probability that alternative (y_m, y_f) is chosen by the decision maker (household) is:

$$P(y_m, y_f) = \Pr(U(y_m, y_f) \geq U(k,s),\ \forall \quad (k,s) \in (1,0)\times(1,0))\,. \qquad (4)$$

Then we have

$$P(y_m, y_f) = \frac{e^{\sigma v(y_m, y_f)}}{\sum_k \sum_s e^{\sigma v(k,s)}}; \ y_m, y_f = 1, 0. \tag{5}$$

3.2 The utility function and the economic attributes in the alternatives

In the game theoretical models, we specify the deterministic part of the utility function as a Box-Cox transformation of household disposable income, his/her leisure and the leisure of the spouse. There are two points we need to clarify. First, we assume that there is some kind of income sharing within the household, and the sharing factor θ is absorbed into the parameter of income for male and female. So the household disposable income enters the utility instead of individual disposable income. Second, we assume that the preference is so called "altruistic" — one spouse's leisure enters the other member's utility function.

The utility functions for the husband and the wife are:

$$\begin{cases} U_m(i,j) = \alpha_m \dfrac{C_{ij}^{\lambda} - 1}{\lambda} + \beta_m \dfrac{L_{mi}^{\lambda} - 1}{\lambda} + \beta_{mf} \dfrac{L_{fj}^{\lambda} - 1}{\lambda} + \varepsilon_m(i) \\ U_f(i,j) = \alpha_f \dfrac{C_{ij}^{\lambda} - 1}{\lambda} + \beta_f \dfrac{L_{fj}^{\lambda} - 1}{\lambda} + \beta_{fm} \dfrac{L_{mi}^{\lambda} - 1}{\lambda} + \varepsilon_f(j) \end{cases} \tag{6}$$

where

- $U_k(i,j)$ = utility of spouse k, the husband is in state i and the wife in state j; i,j=0,1 and k=m,f,
- Disposable income C_{ij} and leisure L_{mi} and L_{fj} are defined below.
- $\alpha_k = \alpha_{k0} + \alpha_{k1}$ (Household wealth); k=m,f
- $\beta_m = \beta_{m0} + \beta_{m1}$(Age difference) $+\beta_{m2}$ (Sick history)+ $\beta_{m3} D_m$,
- $\beta_f = \beta_{f0} + \beta_{f1}(Age_f)$
- D_m =1 if the husband worked in the private sector before retirement, =0 otherwise,

- $\varepsilon_k(i)$ is an extreme value distributed random variable which may be correlated across spouses; k= m, f. Since only the difference e_k enter into the likelihood function, we simply assume $corr(e_m, e_f) = \rho_{mf}$ instead of directly assuming a correlation structure across $\varepsilon_k(i)$.

As can be seen from the specification of the utility function, we assume that the shape coefficient, λ, is the same for both spouses and all alternatives, while all scale coefficients are allowed to vary.

Disposable income, C_{ij}, is equal to annual after-tax income when the husband is in state i and the wife is in state j. Thus $C_{ij} = r_{Mi} + r_{Fj} - T(r_{Mi}, r_{Fj})$; $i, j = 0, 1$; where r_{Mi} is the gross income of the husband when he is in state i , and r_{Fj} is the gross income of the wife when she is in state j, and T(.) is the tax function. On average, pension income is taxed at lower rates than labor income. The unit of tax calculation is the couple, not the individual, which means that the taxes paid by the couple depends on the labor market states of both members of the household. The marginal tax rates are not uniformly increasing with income and therefore the tax rules imply non-convex budget sets. In the estimation of the model, all details of the tax structure, including the non-convexity of the budget sets, are accounted for.

Leisure, L_k, k=F,M, is defined as one minus the ratio of hours of work to total annual hours. Thus, when the husband is retired or the wife is not working, L_k=1, when husband works full time, L_m=1-(37.5*46)/8760.

Because the individual can be observed in one state only, we can observe the gross income of the individual only in that state. In order to model different possible outcomes, we need to impute or simulate the gross income also in those states in which the individual is not observed. We have done the following:

- If the husband or the wife is observed working in the current period or in the year prior to the date of the husband's eligibility, then working are characterized by their observed earnings and leisure. A justification for this assumption is that at the age of the individuals considered here there is some rigidity in the labor market attachments.

- If the wife is observed to be out of the labor force the current and the previous period, then working is characterized by predicted earnings based on a log earnings function estimated on earnings data among those women working full time. Leisure is predicted as leisure consistent with the working load related to the earnings that are assigned to the women. The estimated log earnings function is given in Appendix 2.
- For the husband, potential pension following eligibility is calculated according to rules applied to his earnings history, which is observed. Details about pension rules are set out in Haugen (2000).

Household wealth is defined as financial wealth and we expect that the marginal utility of income of both spouses (evaluated by the deterministic part of the utility function) will decrease with wealth. As alluded to in the next section, all males are 64 years old and thus it makes no sense to let the marginal utility of male leisure depend on the age of male. However, the age difference, defined as husband's age minus wife's age, may have an impact on the marginal utility of male leisure. We expect that the larger this difference is, the less is the marginal utility of male leisure. Sick-history is measured as the ratio of sick leave to working hours in the 15 months prior to AFP-eligibility. We expect that the marginal utility of male leisure is increasing in the sick history of the male. For the males belonging to the cohorts studied here, working in the private sector may have been more strenuous than working in the public sector. Thus we expect that the marginal utility of leisure is higher among private sector employees than among those working in the public sector. The age of the wife may vary across the sample and we therefore let the marginal utility of female leisure depend on her age. The higher age is, the higher we expect the marginal utility of leisure to be. Similarly, we define the joint utility function as following:

$$U(i,j)=\alpha\frac{C_{ij}^{\lambda}-1}{\lambda}+\beta_m\frac{L_{mi}^{\lambda}-1}{\lambda}+\beta_f\frac{L_{fj}^{\lambda}-1}{\lambda}+\varepsilon_{ij}$$

where

- $\alpha = \alpha_0 + \alpha_1$ (Household wealth),
- $\beta_m = \beta_{m0} + \beta_{m1}$(Age-difference) $+\beta_{m2}$ (Sick-history)$+ \beta_{m3}D_m$,
- $\beta_f = \beta_{f0}+\beta_{f1}(Age_f)$

To some extent, we can regard the joint utility function as a weighted sum of the two members' utility function. The discussion about the expected property of the coefficient estimates should hold also in the joint utility case.

3.3 Identification of the parameters

One key factor when examining the identification problem in a discrete choice setting is that only the difference in utility counts. When taking the difference, the common factor in utilities of different alternatives is eliminated and we will not be able to identify the parameters that only appear in these factors. For instance, given our structure of the utility function, both the "altruistic " parameters β_{mf} and β_{fm} cannot be recovered in the Nash setting. The reason is as following. In the Nash settings, both husband and wife take the others' action as given when they make their own decision, i.e. they compare either the state pair $(1, y_f)$ and $(0, y_f)$ or $(y_m,1)$ and $(y_m,0)$. Since the "altruistic " parts depend only on the leisure of the other member, those parts become common factors in the utility function comparison, and cancel out in the likelihood function. So both β_{mf} and β_{fm} are not identified. It is the same reason that β_{fm} in the Stackelberg setting is not identified. However, husband's "altruistic " parameter β_{mf} can be identified, since he has to make comparison between the state pair where the wife is in different state, such as $(1,1)$ and $(0,0)$.

Another important issue is the scale of the estimated parameters. It is well known that we are not able to identify the parameters that enter the utility function linearly, because the variance of the disturbance σ is absorbed in these scale coefficients. However, the shape parameter of the utility function, λ, is identified.

4 Data

The empirical basis for the analysis is register files held by Statistics Norway. The files are all based on an encrypted personal identification number that allows linking of files with different kinds of information and covering different periods in time. Details about the data sources can be found in Hernæs and Strøm (2000).

For the present study, we used register files covering the entire population and spanning the period 1993-98. The data sets give detailed information on employment spells (including identification of the employer), earnings (based on tax reports, implying that all earnings are included, possibly from more than one employer) and benefits of various types (including pension income), wealth (from tax reports) gender, age (including birth date), marital status, educational attainment, sick-history and place of residence.

Eligibility for the AFP is determined in two steps. In the first step we identify all persons employed in companies in which some employees have previously taken out AFP. In the second step we use information on current and previous employment to identify those persons who meet the individual requirements. Then, we include information about the month in which the retirement option becomes available and the month in which it is taken out.

During the observation period, 50 per cent of earnings in excess of the basic amount in the public pension system (USD 5 600) when retired were deducted from the pension. With a marginal tax rate on earnings and pension at say 40 per cent, the effective tax rate on earnings was 70 per cent. We have therefore disregarded the option of combining earnings and early retirement (partly retired).

The earnings history is available from 1967 in the form of accrued rights in the public sector pension system, via year-by-year total pension-accruing income and pension points in the public pension system. This is the basis for predicting potential public pension and thus also the potential pension in the AFP program.

Starting with eligible persons, we restrict the sample in this study to comprise all married couples in which the husband qualified during the period from 1 October 1994 until 31 December 1996. Since the eligibility age was 64 from 1 October 1993 until 1 October 1997, the couples in the sample then knew at least one year in advance that retirement would become possible, and could plan retirement. Previous studies (Røgeberg, 2000) have shown that a sudden change in the eligibility age entails a lagged response. We then restrict the data to couples in which the wife did not qualify and in which the wife is younger than the husband. These restrictions are imposed in order to make sure that the options postulated for the two spouses are reasonable. The restrictions reduce the sample from 12475 couples in which the husband qualifies down to 8210 that fulfill all the criteria. Some descriptive statistics are given in Table 2.

Table 2. Descriptive statistics

Variable	Average value	Minimum value	Maximum value
Household disposable income, when both are working (100,000 NOK)	3.0642	1.1425	29.5826
Household disposable income, when husband is working but wife is not	1.8474	0.5052	27.7983
Household disposable income, when wife is working but husband is not	2.5060	1.1072	7.9971
Household disposable income, when husband takes early retirement and wife is not working	1.2892	0.7056	1.6440
Wealth (100,000 NOK)	5.6966	0	1930.93
Age of wives	58.8996	33	63
Sick history (proportion of previous 15 months on sick leave)	0.0231	0	0.8667
Private sector dummy (=1, if works in private sector)	0.4534	0	1

5 The estimations and policy simulation

5.1 The game theoretical model

We would like to estimate the shape parameter λ together with other parameters using maximum likelihood method. However, the log-likelihood functions for both Nash and Stackelberg case are not differentiable w.r.t. λ.[2] This means that although consistency can still be guaranteed, asymptotic normality is questionable, thus we will not be able to do the conventional inferences[3]. This problem calls for a new strategy of estimation.

[2] **See Appendix 1 for the explanation.**

[3] Discussions of some general results on asymptotic distribution theory for estimators derived from nonsmooth objective function can be found in Newey and McFadden(1999). Unfortunately, we were not able to derive asymptotic normality for our case based on their results.

Note that for any given λ, the likelihood function is well behaved, so we will be able to avoid the non-smooth problem if we assume λ to be a constant as Kooreman (1994) and Hiedemann (1998) did. However, there is no obvious theoretical argument favoring any particular value. In the literature, linearity or log-linearity is often assumed, but it is mainly because of the computational convenience. In our case, we think that we should let the data decide. So we do the estimation in two steps. First, we obtain a consistent estimate λ^* for λ. Note that fact that MLE are consistent despite of non-differentiability, we simply maximize the log-likelihood function w.r.t. all unknown parameter of the model to obtain λ^*. Then we estimate the model using MLE based on the assumption that $\lambda = \lambda^*$. The estimation results for the game theoretic models are given in Table 3.

Table 3. Estimates of Nash and Stackelberg Model

		Nash		Stackelberg (husband leader)	
	The shape parameter	λ =0.5690		λ =0.5522	
Coefficient	Variable	Estimate	Asy t-value	Estimate	Asy t-value
	Wife's utility function				
α_{f0}	Household disposable income: constant	5.3268	31.5004	5.3372	31.4340
α_{f1}	Household disposable income: linear in wealth	-0.0015	-1.2315	-0.0014	-1.2839
β_{f0}	Female leisure: Constant	-0.7550	-0.3569	-1.0900	-0.5135
β_{f1}	Female leisure: Linear in age	0.4228	12.8205	0.4192	12.6995
	Husband's utility function				
α_{m0}	Household disposable income: constant	1.3340	12.0470	1.3349	10.1122
α_{m1}	Household disposable income: linear in wealth	-0.0028	-1.4396	-0.0027	-1.1829
β_{m0}	Male leisure: Constant	-2.1609	-7.3752	-1.9968	-6.7869
β_{m1}	Male leisure: Linear in age difference	-0.1240	-3.8367	-0.1285	-3.9568
β_{m2}	Male leisure: Linear in sick history	13.6448	8.9227	13.6734	8.8327
β_{m3}	Male leisure: Private sector	4.2346	18.0343	4.3160	18.1141
β_{mf}	Female leisure	NA	NA	9.0977	4.8238
	In both utility functions				
ρ_{mf}	Correlation	0.1668	9.2157	0.1655	9.1949
	Observations	8210		8210	
	Log-likelihood	-9837.61		-9826.17	
	ρ^2 [4]	0.1356		0.1367	
	$\bar{\rho}^2$	0.1346		0.1355	

[4] ρ^2 and $\bar{\rho}^2$ are both informal goodness of fit measures, defined as $\rho^2 = 1 - \frac{\ell(\hat{\beta})}{\ell(0)}$ and $\bar{\rho}^2 = 1 - \frac{\ell(\hat{\beta}) - K}{\ell(0)}$ respectively, which are used in a fashion similar to R^2 in regression analysis. K is the number of parameters.

We observe that the estimates of these two game models are quite similar. Because these two models are estimated on the same data set, one simple way to tell if one is better than the other is using the goodness of fit criteria. In our case, according to the log-likelihood values, the Stackelberg model, with male as the leader, performs slightly better. There are some tests available to test non-nested hypothesis as well as to be used in model selections. Ben-Akiva and Swait (1984) shows that under the null hypothesis that model A is the true specification, the following holds asymptotically,

$$\Pr(\bar{\rho}_B^2 - \bar{\rho}_A^2 > z) \leq \Phi\{-[-2z\ell(0) + (K_B - K_A)]^{1/2}\}, \quad z > 0 \tag{7}$$

where

$\bar{\rho}_l^2$ = the adjusted likelihood ratio index for model l, $l=A,B$

K_l = the number of parameters in model l.

Φ = the standard normal cumulative distribution function.

$l(0)$ = is the log-likelihood when the number of parameters are set equal to zero

If we think the model with the greater $\bar{\rho}^2$ is the right one, the probability of erroneously choosing the incorrect model is less than the expressions to the right in (7). Alternatively we can perform the likelihood ratio test developed by Vuong (1989) to test the hypothesis that these two models are equivalent against the hypothesis that one is better than the other. Details of the test are given in Vuong (1989).

When we performed these two tests, both tests rejected the Nash model in favor of Stackelberg model at very low level of significance (<0.001), even though the log-likelihood is quite close. Even so we cannot then be sure that Stackelberg is the right model while the Nash is not in the household decision making process. It only means that the Stackelberg model may be a better description of the data used in the present study. It may just be a special phenomenon for the age cohorts studied here. (The males in this study were born between 1930-1935.)

The shape coefficients, the λ-s, are very close to 0.5. This is a value, which has been found in psychophysical experiments, see Stevens (1975).

From the estimate of the deterministic part of the utility function we observe that

- the marginal utilities of disposable income is positive and significantly different from zero; the effect of wealth on the marginal utility of disposable income is not significant,
- the marginal utility of female leisure is positive for all relevant age levels and it is increasing with age, which is in line with our expectations,
- the marginal utility of male leisure is positive for all relevant sick-history, it is higher if working in the private sector, and it increases with sick-history. It decreases with the age difference, which suggests that the older the husband is relative to the wife the more likely it is that the husband delays his retirement — it can be interpreted as an appreciation of so called 'joint leisure'. Hurd (1997) and Hiedemann (1998) have found a similar effect..
- the marginal utility of wife's leisure for males is significant and positive. It may suggest that the husband does care about his wife's well being. This result fits well the finding of Gustman and Steinmeier (2000), who found that the wife's retirement appears to have a larger effect on the husband's propensity to retire than vice versa, although they found only the joint effect to be significant.
- the unobserved variables affecting the utility levels of the spouses are positively correlated. It can be explained by common taste, either due to why they got married in the first place or it had been formed during the long years of adjustments and compromises from both parties. Hiedemann (1998) reported similar results also, but with much higher magnitude. But since she used grid search on the correlation instead of estimating it

together with other parameters using maximum likelihood method, we do have reason to question her estimates.

5.2 Joint utility model

The estimation results of the joint utility model are given in Table 4

Table 4. Estimates of the joint utility model

Coefficent	Variable	Estimate	Asy t-value
α_0	Income female constant	2.9780	23.9090
α_1	Income female, linear in wealth	-0.0020	-1.3239
β_{f0}	Female leisure: Constant	-14.4226	-7.2166
β_{f1}	Female leisure: Linear in age	0.4590	13.8711
β_{m0}	Male leisure: Constant	0.4089	1.5335
β_{m1}	Male leisure: Linear in age difference	-0.1954	-6.1077
β_{m2}	Male leisure: Linear in sick history	12.8957	8.4550
β_{m3}	Male leisure: Private sector	4.8458	20.5799
λ	Shape parameter	0.5315	13.9872
	Observations	8210	
	Log-likelihood	-10041.3	
	ρ^2	0.1178	
	$\bar{\rho}^2$	0.1170	

From Table 4, we notice that the log-likelihood and the goodness of fit criteria $\bar{\rho}^2$ are well below both game theoretical models. If we perform the two model selection tests on the joint model against the Stackelberg model, the same results are obtained: the joint model is rejected. But the joint model did recover the shape parameter very well. The sign of the coefficients entering the marginal utility of disposable income and leisure are as expected. We note that the shape parameter λ is very sharply determined. The estimate is almost identical to the estimates we obtained in the game case above! We can reject both log-linear utility function ($\lambda = 0$) and linear utility function ($\lambda = 1$).

5.3 Observed versus predicted proportion

Based on the estimates of the three models, we can calculate the average probability of choosing each state across the couples. Table 5 shows the observed proportions as well as the predicted average probabilities and average marginal probabilities.

Table 5. The observed proportions versus predicted probabilities

	Observed	Nash	Stackelberg (husband leader)	Joint
State (1,1)	0.1454	0.1556	0.1557	0.1396
State (1,0)	0.2115	0.2046	0.2085	0.2038
State (0,1)	0.2451	0.2794	0.2769	0.3053
State (0,0)	0.3981	0.3604	0.3590	0.3513
Male retire	0.3569	0.3602	0.3642	0.3434
Male work	0.6431	0.6398	0.6358	0.6566
Female not work	0.3905	0.4349	0.4326	0.4449
Female work	0.6095	0.5651	0.5674	0.5551

State (i,j) means male in state i and female in state j; i,j=1=not work; i,j=0=work

Of most interest here is the marginal probability of male retirement. We notice that 35,69% of the males has decided to retire at the eligibility date. All three models give almost similar predictions that are very close to the observed fractions.

We notice that we predict the labor market situation of the wife less well than the labour market situation of the husband. This may be because for males we are modeling the adjustment right after a new option has become available. For the wife, we are modeling the labor market affiliation that may follow from choices related to the life cycle. The economic incentives incorporated are primarily related to the current situation, and may therefore be insufficient to explain the wife's labor market situation.

5.4 Policy simulation

In order to illustrate the magnitude of the estimated relationship and the corresponding impact of potential policy changes, we have performed a policy simulation based on the estimated models. In the simulation, pension benefits are taxed the same way as labor earnings.

Table 6 below shows how the average choice probabilities across the sample are affected by the policy changes and how the marginal probabilities of work and leisure across gender are affected.

Table 6. Choice probabilities in policy simulations

	Nash		Stackelberg (husband leader)		Joint	
	Model	Policy	Model	Policy	Model	Policy
State (1,1)	0.1556	0.1001	0.1557	0.0997	0.1396	0.0748
State (1,0)	0.2046	0.1943	0.2085	0.2191	0.2038	0.1399
State (0,1)	0.2794	0.3157	0.2769	0.2941	0.3053	0.3651
State (0,0)	0.3604	0.3900	0.3590	0.3872	0.3513	0.4202
Male retire	0.3602	0.2943	0.3642	0.3187	0.3434	0.2147
Male work	0.6398	0.7057	0.6358	0.6813	0.6566	0.7853
Female not work	0.4349	0.4158	0.4326	0.3938	0.4449	0.4399
Female work	0.5651	0.5842	0.5674	0.6062	0.5551	0.5601

As seen from Table 6, the tax system favors retirement. Therefore, making the taxation of pension benefits less generous, (equal to the taxation of labor income) reduces early retirement. We also observe that although the three models had almost the same prediction of within-sample frequencies, the joint utility model differs considerably from the two game models with regard to the prediction of a change in policy rules. Based on the joint utility model the predicted reduction in the marginal probability of male retirement averages around 13 percentage points, while in the case of game models the average reduction amounts to 5-7 percentage points. We probably should pay more attention to the predictions of the Stackelberg model, for the two tests we performed are in favor of it. According to the test, the joint model is the worst among the three, so to some extent it may be misleading to rely on the policy simulations in this case.

But anyway, these results indicate that the current tax system favors retirement and that the change in the tax rules described above may have a large and positive impact on male labor supply among those males who are eligible for early retirement.

In our simulations, female labor supply does not change much due to the shift in policy. If anything, a slight increase in labor supply is predicted. This is the same across models.

Thus, the considered change in the taxation of pension incomes clearly increases labor supply among the elderly men eligible for early retirement, with a modest but positive impact on their wives' labor supply. Thus, the considered change in tax rules is a good policy candidate, if one wants to counteract the negative effects on labor supply implied by the early retirement programs.

6 Conclusions

The paper makes a first attempt to compare non-cooperative game-theoretic and joint utility models of early retirement and labor force participation for married couples, using detailed Norwegian micro data. Although the estimates indicate that the marginal utility of leisure and the shape coefficient is rather similar across models, based on some model selection tests, both the joint utility model and the Nash model are rejected against the Stackelberg model with male as a leader.

We are not yet able to estimate a cooperative game model such as a Nash-bargaining model, which is at the focus in the literature on household behavior analysis (see for example McElroy and Horney (1981), (1990) and Chiappori (1988), (1991)). Thus we have not been able to compare the Stackelberg model with a Nash-bargaining model. This we leave for future research.

The three models do not differ to any great extent with regards to how within-sample fractions are predicted. However, they vary more with respect to the prediction of choice probabilities generated by a change in taxation. All simulations indicate that the lenient taxation of pension income favors early retirement. Taxing pension income by the rules of earning reduces on average the marginal probability of male retirement by 5-7 percentage points in the game models and by as much as 13 percentage points in the joint utility model. In all three models female labor supply is predicted to increase slightly.

It should be noted that the results in this paper are based only on observations of couples in which only the husband qualifies for early retirement. Another topic for further research will be to estimate the models on observations of couples over a period in which both spouses qualify. The indication of a positive correlation in retirement behavior is found in previous research, for instance Blau (1997), Zweimüller, Winter-Ebmer and Falkinger (1996) and Hiedemann (1998).

References

Baker, M. (2002), The Retirement Behavior of Married Couples: Evidence from Spouse's Allowance. *Journal of Human Resources,* **37**(1), 1-34

Ben-Akiva, M. and Swait, J. (1984), The Akaike Likelihood Ratio Index, *Working paper*, *Department of civil engineering, MIT,* Cambridge, Mass, USA.

Blau, D. H. (1997), Social Security and the Labor Supply of Older Married Couples, *Labour Economics* **4**, 373-418.

Bresnahan, T. F and Reiss, P.C. (1991), Empirical Models of discrete games, *Journal of Econometrics,* **48**, 57-81.

Bourguignon, F. and Chiappori, P.A. (1992), "Collective Models of Household Behavior, an Introduction, *European Economic Review* 36, 355-364.

Chiappori, P.A. (1988), Nash-bargained Household Decisions: A Comment, *International Economic Review* 29, 791-796.

Chiappori, P.A. (1991), Nash-Bargained Household Decisions: A Rejoinder, *International Economic Review* 32, 761-62.

Gustman, A.L. and Steinmeier, T. (2000), Retirement in Dual-Career Families: A Structural Model, *Journal of Labor Economics*, **18**/3, 503-45.

Haugen, F. (2000), Insentivvirkninger av skatte- og pensjonsregler, Master Thesis, Department of Economics, University of Oslo (in Norwegian only).

Heckman, J. J. (1978), Dummy Endogenous Variables in a Simultaneous equation system, *Econometrica*, **46**/7, 931-59.

Henretta, J.C. and O'Rand, A.M. (1983), Joint Retirement in the Dual Worker Family, *Social Forces* **62**, 504-520.

Hernæs, E., Sollie, M. and Strøm, S. (2000), Early Retirement and Economic Incentives, *Scandinavian Journal of Economics*, Vol 102, No 3.

Hernæs, E. and Strøm, S. (2000), Family labor supply when the husband is eligible for early retirement, *Memorandum from Department of Economics*, University of Oslo, No. 13.

Hiedemann, B. (1998), A Stackelberg Model of Social Security Acceptance Decisions in Dual-Career Households, *Journal of Economic Behavior & Organization* 34, 263-78.

Hurd, M.D. (1997), The Joint Retirement Decisions of Husbands and Wives, In *Issues in the Economics of Aging*, edited by David A. Wise, pp. 231-254 Chicago: University of Chicago Press.

Jia, Z. (2001), *A Note on Discrete Choice Model of Two Agents*, Manuscript, University of Oslo.

Kapteyn, A. and Kooreman, P. (1992), Household Labor Supply: What Kind of Data Can Tell Us How Many Decision Makers There Are? *European Economic Review* 36, 365-371.

Kooreman, P. (1994), Estimation of Econometric Models of Some Discrete Games, *Journal of Applied Econometrics* 9, 255-68.

Lumsdaine, R. L. and Mitchell, O.S. (1999), New Developments in the Economic Analysis of Retirement, in Ashenfelter, O. and Card, D. (eds) *Handbook of Labor Economics*.Volume 3, Elsevier Science B. V.

Maddala, G.S. (1983), *Limited-Dependent and Qualitative Variables in Econometrics,* Econometric Society Monographs No.3, Cambridge University Press.

McElroy, M.B. and Horney, J.M .(1981), Nash Bargained Decisions: Towards a Generalisation of the Theory of the Demand, *International Economic Review* 22, 333-349.

McElroy , M.B. and Horney, J.M. (1990), Nash Bargained Decisions, A Reply, *International Economic Review* 31, 237-42.

Newey, W.K. and McFadden, D.L. (1994), Large Sample Estimation and Hypothesis Testing, In *Handbook of Econometrics*, volume 4, edit by Engle, R.F. and McFadden, D.L. pp. 2113-2148, Elsevier.

Røgeberg, Ole, J. (1999), *Married Men and Early Retirement Under the AFP Scheme*, Master Thesis, Department of Economics, University of Oslo.

Stevens, S.S. (1975), *Psychophysics: Introduction to its Perceptual Neural and Social Prospects*, Wiley, New York.

Vuong Q.H (1989), Likelihood Ratio tests for Model Selection and Non-Nested Hypotheses, *Econometrica,* 57(2), 307-333.

Zweimüller, Josef, Winter-Ebmer, Rudolf and Falkinger, Josef (1996), Retirement of spouses and social security reform, *European Economic Review* 40, 449-472.

Appendix 1. Nash and Stackelberg equilibrium

Table A.1 Nash equilibrium (NE)

	$U_m(1,1)-U_m(0,1)>0$ $U_m(1,0)-U_m(0,0)>0$	$U_m(1,1)-U_m(0,1)>0$ $U_m(1,0)-U_m(0,0)<0$	$U_m(1,1)-U_m(0,1)<0$ $U_m(1,0)-U_m(0,0)>0$	$U_m(1,1)-U_m(0,1)<0$ $U_m(1,0)-U_m(0,0)<0$
$U_f(1,1)-U_f(1,0)>0$ $U_f(0,1)-U_f(0,0)>0$	(1,1)	(1,1)	(0,1)	(0,1)
$U_f(1,1)-U_f(1,0)>0$ $U_f(0,1)-U_f(0,0)<0$	(1,1)	(1,1) or (0,0)	No pure NE	(0,0)
$U_f(1,1)-U_f(1,0)<0$ $U_f(0,1)-U_f(0,0)>0$	(1,0)	No pure NE	(1,0) or (0,1)	(0,1)
$U_f(1,1)-U_f(1,0)<0$ $U_f(0,1)-U_f(0,0)<0$	(1,0)	(0,0)	(1,0)	(0,0)

Table A.2 Stackelberg equlibrium (SE) (male as leader)

$y_f(1)=1$ $y_f(0)=1$	$e_f> \max[v_f(1,0)-v_f(1,1), v_f(0,0)-v_f(0,1)]$	$e_m>v_m(0,1)-v_m(1,1)$ $e_m<v_m(0,1)-v_m(1,1)$	(1,1) is SE (0,1) is SE
$y_f(1)=1$ $y_f(0)=0$	$v_f(0,0)-v_f(0,1) > e_f > v_f(1,0)-v_f(1,1)$	$e_m>v_m(0,0)-v_m(1,1)$ $e_m<v_m(0,0)-v_m(1,1)$	(1,1) is SE (0,0) is SE
$y_f(1)=0$ $y_f(0)=1$	$v_f(1,0)-v_f(1,1)>e_f > v_f(0,0)-v_f(0,1)$	$e_m>v_m(0,1)-v_m(1,0)$ $e_m<v_m(0,1)-v_m(1,0)$	(1,0) is SE (0,1) is SE
$y_f(1)=0$ $y_f(0)=0$	$e_f< \min[v_f(0,0)-v_f(0,1), v_f(1,0)-v_f(1,1)]$	$e_m>v_m(0,0)-v_m(1,0)$ $e_m<v_m(0,0)-v_m(1,0)$	(1,0) is SE (0,0) is SE

The non-differentiability of the likelihood functions:

Note that in our probability formula for both Nash and Stackelberg case, the likelihood functions involve the terms similar to:

$$\Pr(v_f(1,0)-v_f(1,1)<e_f<v_f(0,0)-v_f(0,1) \wedge e_m < v_m(0,0)-v_m(1,1))$$
$$=\Pr(\alpha_f \frac{C_{10}^{\lambda}-C_{11}^{\lambda}}{\lambda}+\beta_f \frac{L_{f0}^{\lambda}-L_{f1}^{\lambda}}{\lambda}<e_f<\alpha_f \frac{C_{00}^{\lambda}-C_{01}^{\lambda}}{\lambda}+\beta_f \frac{L_{f0}^{\lambda}-L_{f1}^{\lambda}}{\lambda} \wedge e_m < v_m(0,0)-v_m(1,1)))$$

Let $b = v_m(0,0)-v_m(1,1))$

$a = \beta_f \frac{L_{f0}^{\lambda}-L_{f1}^{\lambda}}{\lambda}$

and let $F(x,y,\rho)$ be the CDF for (e_m, e_f)

then the above term equals to

$$\begin{cases} F(b,\alpha_f \frac{C_{00}^{\lambda}-C_{01}^{\lambda}}{\lambda}+a,\rho)-F(b,\alpha_f \frac{C_{10}^{\lambda}-C_{11}^{\lambda}}{\lambda}+a,\rho) & \text{if } \frac{C_{10}^{\lambda}-C_{11}^{\lambda}}{\lambda}<\frac{C_{00}^{\lambda}-C_{01}^{\lambda}}{\lambda} \\ 0 & \text{other wise} \end{cases}$$

So we see immediately that this term is not differentiable w.r.t. λ. Neither is the log-likelihood function.

Appendix 2. Female earnings function

If the wife is observed to be out of the labor force the current and the previous period, then gross annual labour income, w , is predicted from the estimated annual income function given below:

$$\ln w = X\lambda + \tau$$

where τ is a normal distributed error term. The covariates entering the X-vector are:

1) Constant term ,
2) Age,
3) Education, number of years in schooling,
4) Dummy for work between 20 to 29 hours,
5) Dummy for work more than 30 hours.

The estimates are given in the following table:

Table A.4 Estimates of wage regression

	Estimate	Std.dev	t-value
1) C	11.2727	0.0456	247.1100
2) Age	-0.0069	0.0007	-9.3600
3) Education in years	0.0455	0.0011	40.2000
4) Dummy for work between 20 and 29 hours per week	0.1417	0.0086	16.4100
5) Dummy for work more than 30 hours	0.4783	0.0079	60.9300
R square	30.5%		
Adjusted R square	30.3%		