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HYDROPOWER ECONOMICS*

by

Finn R. Førsund

Abstract: The key question in hydropower production is the time pattern of the use of the water in the reservoir. The water used to produce electricity today can alternatively be used tomorrow. The analysis of the operation of hydropower is therefore essentially a *dynamic* one. The paper introduces some basic models for social allocation of stored water over discrete time periods using non-linear programming assuming capacities of generation and transmission as given. Implications of constraints such as limited storage capacity and limited connector capacity for (international) trade are studied. Results are derived for water allocation and development of the electricity price over time. Graphical illustrations are provided in the two- period case and successive pairs of periods in a multi-period setting by means of the bathtub diagram. Thermal capacity is added to hydro and the optimal mix is studied. The walls of the hydro bathtub are extended endogenously by thermal capacities. Finally, the case of monopoly is studied. Different from standard monopoly behaviour of contracting output, if total available water is to be used, the strategy of a monopolist is to redistribute the use of water for electricity production over periods compared with the social optimal distribution.

Key words: Hydropower, electricity, reservoir, water value, monopoly

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1. Introduction

Electricity

Electricity is one of the key goods in a modern economy. The nature of electricity is such that supply and demand must be in a continuous physical equilibrium. The system breaks down in a relatively short time if demand exceeds supply and vice versa. The system equilibrium is therefore demand-driven. The spatial configuration of supply and demand is important for understanding the electricity system. A transmission network for transport of electricity connects generators and consumers. There is energy loss in the network. Physical laws govern the flows through the networks and the energy losses. The electricity is characterised by voltage (220 - 240 V) and the Hertz number (50 ± 2) for alternate current and measured as effect (kW), i.e. instantaneous energy, and energy (kWh), i.e. the amount of electricity during a time period (the integral of the effect over the time period in question). The capacity rating of the turbines of the generators is in effect units. The network, or grid, has capacity limits in effect units for a given spatial configuration of supply and demand nodes in the network.

The time period used in a study of the electricity system is of crucial importance for the detail by which the system is modelled. If the time resolution is one hour we can portray the demand by looking at the variation in energy use hour by hour during a day. The demand varies over the day with the lowest energy consumption during the night and peaks at breakfast time and the start of the working day, and again a peak round dinnertime. To see the need for effect capacity it is common to look at the demand for one year and sort the 8760 hours according to the highest demand and then decreasing. The hours with the highest energy demand are called the *peak load*, and the hours with the lowest demand are called the *base load*. In between we have the *shoulder*.

Hydropower

Electricity generators can use water, fossil fuels, bio-fuels, nuclear fuels, wind and geothermal energy as primary energy sources to run the turbines producing electricity. Hydropower is based on water driving the turbines. The primary energy is provided by gravity and the height the water falls down on to the turbine. We will assume the existence of a reservoir. The potential for electricity generation of one unit of water (a cubic meter) is usually expressed by the height from the dam level to the turbine level. The reservoir level will change when water is released and thus influence electricity production. Electricity production is also influenced by how processed water is transported away from the turbine allowing fresh water to enter the turbine. The turbine is constructed for an optimal flow of water. Lower or higher inflow of water will reduce electricity output per unit of water.

The key economic question in hydropower production is the time pattern of use of water in the reservoir. Given enough storage capacity the water used today can alternatively be used tomorrow. The analysis of hydropower is therefore essentially a *dynamic* one. This is in contrast with a fossil fuel (e.g. coal) generator. The question then is how to utilize the given production capacity for each time period. Assuming that the market for the primary energy source functions smoothly this is not a dynamic problem, but is a problem solved period by period (disregarding adjustment cost going from a "cold" state of not producing electricity to a "hot" state producing).

The economics of hydro production with reservoir was discussed early in operations research and economics literature (see Little (1955), Koopmans, 1957), but the topic is a typical engineering one.[1] In Norway a central production system was established after the Second World War based on an understanding of how the system was to be operated (see Hveding (1967), 1968). This approach has been refined and developed into a central model tool for Norway and later the NordPool area (see Haugstad et al. (1990), Gjelsvik et al., 1992). The highly simplified approach taken in this paper is based on Førsund (1994) (see also Bushnell (2003), Crampes and Moreaux (2001), Johnsen (2001), von der Fehr and Sandsbråten (1997), and Scott and Read, 1996).

The variables we are going to use are reservoir R_t, inflow of water w_t and electricity production, e_t^H, e_t^{Th}, from hydro and thermal capacities respectively. Flow variables in small

[1] In France there were early studies from the 40ies and 50ies, especially by people connected to Electricité de France, see Morlat (1964) for a translation into English of one of the papers.

letters are understood to refer to the period, while stock variables in capital letters refer to the end of the period, i.e. water inflow w_t takes place during period *t*, while the content of the reservoir R_t refers to the water at the end of period *t*. Release of water during period *t* is converted to electricity e_t^H measured in kWh according to a fixed transformation coefficient, reflecting the vertical height from the centre of gravity of the dam and to the turbines. Water reservoir and inflow can also be measured in kWh using the same conversion. The reduced electricity conversion efficiency due to a reduced height (head) the water falls as the reservoir is used is disregarded. For the Norwegian system, with high differences in elevations between dams and turbine stations of most of the dams and few river stations, this is an acceptable simplification at our level of aggregation.

The transformation of water into electricity can be captured in the simplest way by the production function

$$e_t^H \le \frac{1}{a} r_t \tag{1}$$

where r_t is the release of water from the reservoir during time period *t* and *a* is the *fabrication coefficient* for water. As mentioned above the coefficient may vary with the utilisation of the reservoir, and also with the release of water due to the construction of the turbine giving maximal productivity at a certain water flow. By assumption there are no other current costs. This is a very realistic assumption for hydropower. We will in the following assume that the production function (1) holds with equality and therefore we can drop this relation and measure water in electricity units.

The dynamics of water management is based on the filling and emptying of the reservoir:

$$R_t \le R_{t-1} + w_t - r_t,\ t = 1,..,T \tag{2}$$

Strict inequality means that there is overflow.

Some studies of hydropower at a high level of aggregation disregards the storage process and specifies directly the available water within a yearly weather cycle. The assumptions are then no spill of water and no binding upper reservoir constraint. The period concept may be as crude as two periods (summer and winter season based on difference in inflow and/or release profile), and anything from month, weeks, days and hours. A realistic modelling ("Samkjøringsmodellen", see Haugstad et al. (1990), Gjelsvik et al., 1992) may use a week as a period unit and involve 3 to 5 years. We will simplify and use a time horizon for water management problems following the yearly precipitation cycle. Therefore we will also disregard discounting. Although it is obvious that the world continues after one year, we will also simplify and not specify any terminal value of the reservoir or operate with a "scrap" value for the reservoir content of the last period.

2. The basic hydro model

Social optimum

The reservoir dynamics can be greatly simplified if we look at time periods where the bulk of inflow comes in one period and then there is a natural seasonal precipitation cycle with little inflow until one year later. In Norway the snow smelting during a few spring weeks fills the reservoirs with about 70 percent of the yearly total. The period with inflow will then naturally be the first period. The basic model is obtained by assuming that there is only inflow in the first period, and furthermore we assume that the production of electricity is efficient, i.e. we have equality in the production function (1). Finally there is unlimited transferability of water to the other periods of the given total amount of water available after the first period:

$$\sum_{t=1}^{T} r_t = w_1 \Rightarrow \sum_{t=1}^{T} a e_t^H = w_1 ,$$
$$\sum_{t=1}^{T} e_t^H = \frac{w_1}{a} = W \qquad (3)$$

where W is the total available inflows. The horizon, T, is assumed to be a seasonal cycle (one year) from spring to spring. In the first equation of (3) water is measured in m3, while in the second line of (3) "water", W, is measured in energy units, kWh, and no conversion from water to electricity is shown when using the variable W. We will still call W water.

The energy consumption in each period is evaluated by utility functions, which can be thought of as valid either for a representative consumer or constitute a welfare function. There is no discounting (the horizon is too short for discounting to be of practical significance). The utility functions representing the social value of electricity consumption are:

$$U_t(e_t^H) \ , \ U_t'(e_t^H) \geq 0 \ , \ U_t''(e_t^H) < 0 \ , \quad t = 1,..,T \tag{4}$$

The utility functions have the standard property of concavity. We will define the marginal utility U_t' measured in monetary units as the marginal willingness to pay, p_t, i.e. the *demand function* for electricity:

$$U_t'(e_t^H) = p_t(e_t^H)\,, \tag{5}$$

where p_t will also be referred to as the "price" of electricity for short below.

The social optimisation problem can be formulated as follows:

$$\begin{aligned} & Max \sum_{t=1}^{T} U_t(e_t^H) \\ & s.t. \\ & \sum_{t=1}^{T} e_t^H \leq W \end{aligned} \tag{6}$$

The Lagrangian function is:

$$L = \sum_{t=1}^{T} U_t(e_t^H) - \lambda(\sum_{t=1}^{T} e_t^H - W) \tag{7}$$

The horizon ends at T so there is no scrap value function in water handed over to period $T+1$. Necessary conditions for this problem where all the variables are non-negative are[2]:

$$\begin{aligned} & \frac{\partial L}{\partial e_t^H} = U_t'(e_t^H) - \lambda \leq 0 \perp e_t^H \geq 0 \ , t = 1,..,T \\ & \lambda \geq 0 \ (= 0 \text{ if} \sum_{t=1}^{T} e_t^H < W) \end{aligned} \tag{8}$$

From the Kuhn–Tucker conditions we know that the marginal utility of electricity consumption is equal to the shadow price on the resource constraint if we have an interior solution for the energy consumption for period t, i.e. $e_t^H > 0$. The shadow price on the resource constraint is zero

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[2] The use of " $\perp$ " is a shorthand notation for the conditions $\partial L / \partial e_t^H \leq 0$, $e_t^H \bullet \partial L / \partial e_t^H = 0$ (Sydsæter, Strøm and Berck (1999), p. 100) implying that $\partial L / \partial e_t^H = 0$ for $e_t^H > 0$.

if the constraint is not binding. In such a highly stylised model it is reasonable to assume that there is positive consumption of electricity in each period and that consumption is not satiated, i.e. that marginal utility is positive in all periods. It then follows that the shadow price on the resource constraint must be positive.

A sufficient condition for a solution to problem (6) is that the Lagrangian (7) is concave, which is satisfied under our assumptions.

The typical conclusion in this basic model with a given amount of resources is that the marginal utility of electricity is constant and equal for all periods. Measuring utility in money, marginal utility may be interpreted as marginal willingness to pay, i.e. as the demand function for electricity on price form. The result of the basic model can then be equivalently stated as the price of electricity being the same for all periods. This is *Hotelling's rule* for the resource price for our model. We do not discount, and by arbitrage the price must be the same for all periods. If prices were different then by the assumption of unlimited transferability of water between the periods welfare can be increased by transferring water to a high price period, etc., thus equalising the prices in the optimal solution.

The typical solution for both periods is illustrated in Figure 1 in the case of two periods by using a "bathtub" diagram. The two marginal-willingness-to-pay - functions are measured along one vertical axis each. Total available energy for the two periods corresponds to the horizontal length of the bathtub. The economic interpretation of the solution to the allocation problem is that energy should be allocated on the periods in such a way that the shadow price of energy (i.e. the increase in the objective function of a marginal increase in the given amount of total energy) is equal to the marginal utility of energy in each period, thus the marginal utilities become equal. In the illustration in Figure 1 if Period 1 is summer and Period 2 winter, the marginal utility should be equal. Although the marginal utility of energy consumption may be higher in winter than in summer for any level of consumption, marginal utility in the winter should not become greater than in summer.

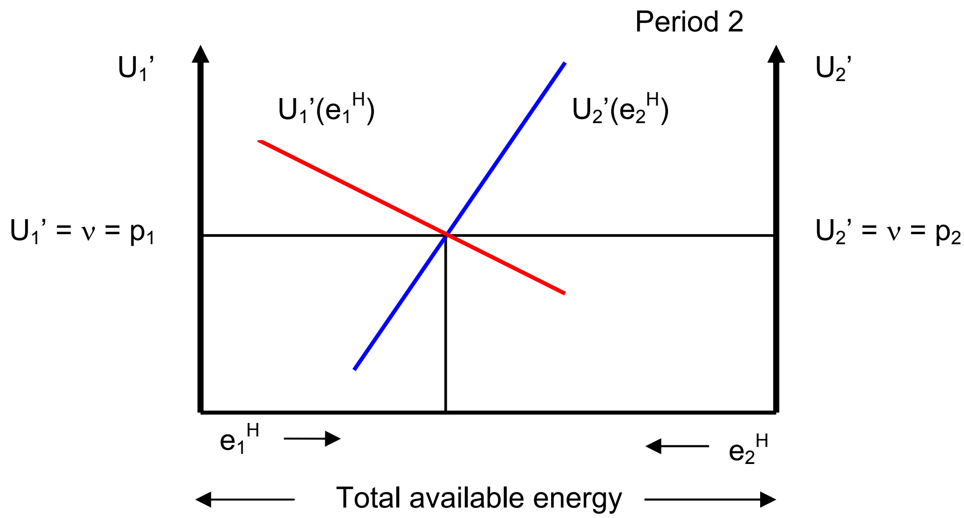

Figure 1. The bathtub illustration
Optimal allocation of energy on two periods

Constraints in hydropower modelling

There are many constraints on how to operate a dam. A fundamental constraint is the maximal amount of water that can be stored. This constraint will have a crucial importance for how the dam can be operated. Environmental concerns may impose a lower limit on how much the dam can be emptied. Empty dams create eyesores in the landscape, and can create bad smells from rotting organic material along the exposed shores. Fish may have problems surviving or spawning at too low water levels. The environmental lower constraint has a subscript for time, because the environmental problems may vary with season. In Norway, where the dams are covered by ice in the winter season, the lower level may be less then than in the summer.

The effect capacity of a power station may be constrained by the installed turbines or the diameter of the pipe from the reservoir to the turbines. Such a constraint has no subscript for time period. The effect concept will follow the period definition. For example, if the period length is one hour the effect constraint is measured in kWh, by using the maximal kW for one hour. Using only energy as our variable the effect constraint is the same as a production constraint.

Table 1. Constraints in the hydropower model

Constraint type	Expression
Max Reservoir	$R_t \leq \overline{R}$
Environmental concerns, Min Reservoir	$R_t \geq \underline{R}_t$
Max Effect capacity	$e_t^H \leq \overline{e}^H$
Max Transmission capacity	$e_t^H \leq \overline{e}_t^H$
Water flows, environment	$\underline{r}_t \leq r_t \leq \overline{r}_t$
Ramping up	$0 < r_t - r_{t-1} \leq r_t^u$
Ramping down	$0 < r_{t-1} - r_t \leq r_t^d$

In aggregated analyses it is common not to specify the transmission system. But a constraint on transmission can be represented the same way as for effect capacity constraint, except that a time index may be used on the constraint to indicate that transmission capacity within some limits is an endogenous variable governed by physical laws of electrical flows of active and reactive power in a multi-link grid system between input and output nodes. The loss may also vary with temperature: resistance is higher in winter than summer time.

There may be environmental concerns about the size of the release from a reservoir. If the release occurs into a river system there may be concerns both about the lower and the higher amount of water that should be released due to impacts on the environment downstream. Impacts on fishing and recreational activity and pressure from tourism may be relevant. Erosion of riverbanks and temperature change for agricultural activity nearby may also count. Then there is concern about navigation and flood control. All these effects may also be present when releases change, so upper constraints may be introduced both on ramping up and ramping down. These constraints are most realistic for shorter time periods.

3. Hydro with reservoir constraints

Social optimum

In the older literature on hydropower referred to in the Introduction and in engineering literature the social objective function is often expressed as minimising the total costs of supplying a given amount of electricity. In economics a standard objective function in empirical studies is to maximise consumer plus producer surplus with the produced quantities as *endogenous* variables. The consumption side is conveniently summarised by using demand functions (defined in (5)) and the supply side by using cost functions. This is a partial equilibrium approach because no interaction with the rest of the economy is modelled. In the case of hydropower with zero operating market costs the social surplus is simplified to the area under the consumer demand function (since consumers' expenditure is identical to producers' profit).[3]

The social planning problem is:

$$\begin{aligned} & Max \sum_{t=1}^{T} \int_{z=0}^{e_t^H} p_t(z)dz \\ & s.t. \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad (9) \\ & R_t \le R_{t-1} + w_t - e_t^H,\ R_t \le \overline{R},\ t = 1,..,T \end{aligned}$$

All the water variables have been converted to energy units by dividing through the constraint with the fabrication coefficient, a, from (1), assuming equality to hold in (1) and then for notational convenience suppressed the coefficient a.

The optimisation problem (9) is a discrete time dynamic programming problem and special solution procedures have been developed for this class of problems (Sydsæter et al., 2005). However, due to the simple structure of the problem we shall treat it as a non-linear programming problem and use the Kuhn – Tucker conditions for discussing qualitative characterisations of the optimal solution. The Lagrangian is:

[3] We assume that there are no external costs involved in producing or consuming the hydropower.

$$L = \sum_{t=1}^{T} \int_{z=0}^{e_t^H} p_t(z)dz$$
$$-\sum_{t=1}^{T} \lambda_t (R_t - R_{t-1} - w_t + e_t^H) \qquad (10)$$
$$-\sum_{t=1}^{T} \gamma_t (R_t - \overline{R})$$

Necessary first order conditions for $t = 1,..,T$ are:

$$\frac{\partial L}{\partial e_t^H} = p_t(e_t^H) - \lambda_t \le 0 \perp e_t^H \ge 0 \qquad (11)$$
$$\frac{\partial L}{\partial R_t} = -\lambda_t + \lambda_{t+1} - \gamma_t \le 0 \perp R_t \ge 0$$
$$\lambda_t \ge 0 \, (= 0 \text{ for } R_t < R_{t-1} + w_t - e_t^H)$$
$$\gamma_t \ge 0 \, (= 0 \text{ for } R_t < \overline{R})$$

Now, our general objective is that the model should tell us something about optimal production/consumption of electricity that has real world interest. We will then limit the number of possible optimal solutions by making reasonable assumptions. One such assumption is that we require positive production in all periods yielding the conditions

$$p_t(e_t^H) = \lambda_t \; (e_t^H > 0),$$
$$-\lambda_t + \lambda_{t+1} - \gamma_t \le 0 \perp R_t \ge 0, t = 1,..,T \qquad (12)$$

The shadow price λ_t of the stored water may be termed the *water value*[4]. Note that we have not ruled out the possibility that the water value is zero. We see that the second equation in (11) is the essential one for the dynamics of our system. There are only two successive periods involved in the equation of motion. This means that a sequence of two period diagrams may capture the main features of the general solution.

According to Bellman's principle for solving dynamic programming problems with discrete time we start searching for the optimal solution by solving the optimisation problem for the last period and then work our way successively backwards towards the first period. The optimality conditions for the end period T are:

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[4] But remember that in our simplified model water is measured in electricity units. We should really measure water in m^3 to use the expression. This can easily be done by multiplying through with the fabrication coefficient *a*.

$$p_T(e_T^H) = \lambda_T \, (e_T^H > 0),$$
$$-\lambda_T - \gamma_T \leq 0 \perp R_T \geq 0 \tag{13a}$$

Our horizon ends at *T*, so the water value for the period *T* +1 does not exist. For period *T* we have two possibilities as to the utilisation of the water in the reservoir: either it is emptied, $R_T = 0$, or some water is remaining, $R_T > 0$. Since the water has no value from *T+1* on, the latter situation can only be optimal if the marginal utility of electricity becomes zero before the bottom of the reservoir is reached. We will adopt the alternative that the marginal utility of electricity remains positive to the last drop. This means that we will have a situation of *scarcity* in the last period *T* with $p_T(e_T^H) = \lambda_T > 0$. Scarcity in an economic sense means that the total available quantity of a good is consumed, and that there is a positive willingness to pay at the margin (i.e. a small decrease in price would have induced more consumption if more of the good was available). Scarcity gives economic value to the water in the last period. Since we cannot have a situation of scarcity at the same time as the upper limit on the reservoir is reached, the shadow price γ_T on the upper constraint is zero. The second relation in (13a) then implies $\lambda_T \geq 0$. This does not give us any new information as to the water value in period *T* (the shadow price may be zero although the expression in the water storage constraint is zero, as is our situation in period *T*), but by our assumption of no satiation in period *T* the value is positive.

The solution for period *T*-1 is obtained by solving the problem for period *T*-1 given that the solution for period *T* is known. The necessary conditions are:

$$p_{T-1}(e_{T-1}^H) = \lambda_{T-1} \, (e_{T-1}^H > 0),$$
$$-\lambda_{T-1} + \lambda_T - \gamma_{T-1} \leq 0 \perp R_{T-1} \geq 0 \tag{13b}$$

If we assume that the reservoir is not emptied in period *T*-1 then the last equation in (13b) holds with equality. Furthermore, if we assume that there is no threat of overflow, then the shadow price γ_{T-1} on the reservoir constraint is zero. We then have that the water value in period *T*-1 is the same as in period *T*, and consequently this is also the case for the prices.

Consider the situation that the upper constraint on the reservoir is not reached in any of the periods and that the reservoirs are never emptied. According to the last relation in (12), for all periods up to *T* we will then have the water values positive. The shadow price on the upper reservoir constraint remains zero, and the periods' reservoir amounts are positive implying $-\lambda_t + \lambda_{t+1} = 0$ for $t = 1,...,T-1$. The price of electricity remains positive and equal for all periods including the last period T if the reservoir is emptied only in the last period and the upper constraint on the reservoir is never reached. However, even a superficial knowledge of the electricity market tells us that electricity prices vary over seasons and even days. So we have to come up with mechanisms that generate price variations if our model is to be of help to understand actual electricity markets.

Bathtub illustrations of scarcity and threat of overflow

The basic price-determining events are periods of scarcity and periods with overflow or threat of overflow. Let us illustrate these two events using the bathtub diagram in the case of only two periods.

Scarcity

The length of the bathtub in Figure 2 is total inflow AC plus CD respectively in the two periods, and the storage possibility is measured to the left from C and is BC. The reservoir capacity is

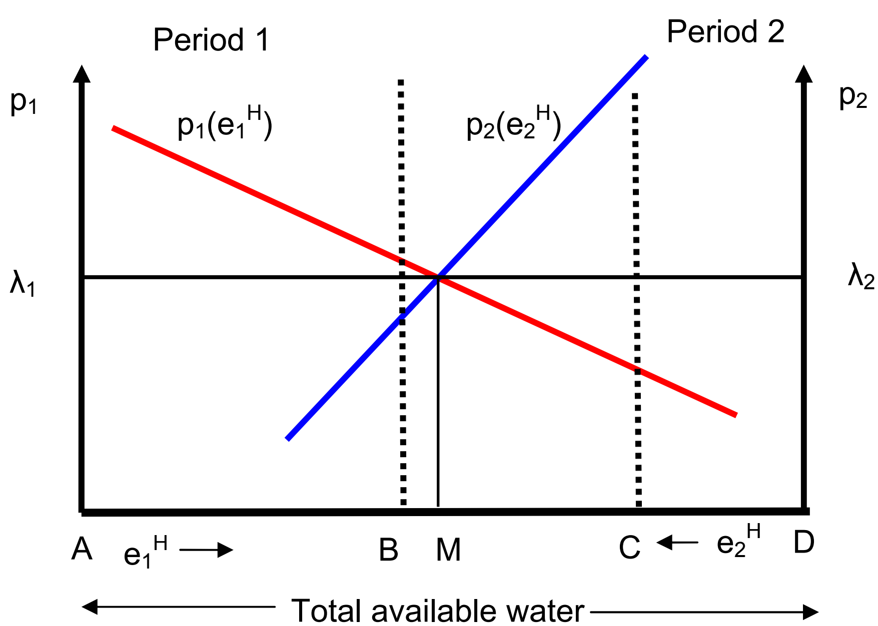

Figure 2. No overflow, but scarcity in period 2
Inflow AC in period 1, CD in period 2, storage BC

measured in this way because the decision problem is how much water to save in period 1 for use in period 2. Figure 2 illustrates the case when the reservoir limit is not reached, but there is scarcity in period 2 since all available water in that period is used up. The water values become the same and equal to the price for both periods. The amount of AM is consumed in period 1 and MC is saved and transferred to period 2, where MD is consumed.

Threat of overflow

The demand curves may intersect to the left of the vertical reservoir capacity line from B as illustrated in Figure 3. In the first period we have an inflow equal to AC, and in the second period an inflow equal to CD. The capacity of the reservoir is BC. The optimal allocation is to store the maximal amount BC in period 1 and consume what cannot be stored, AB, because the water value is higher in the second period. In this period the reservoir, containing BC from the first period and an inflow of CD coming in the period, is emptied. We go from a period of

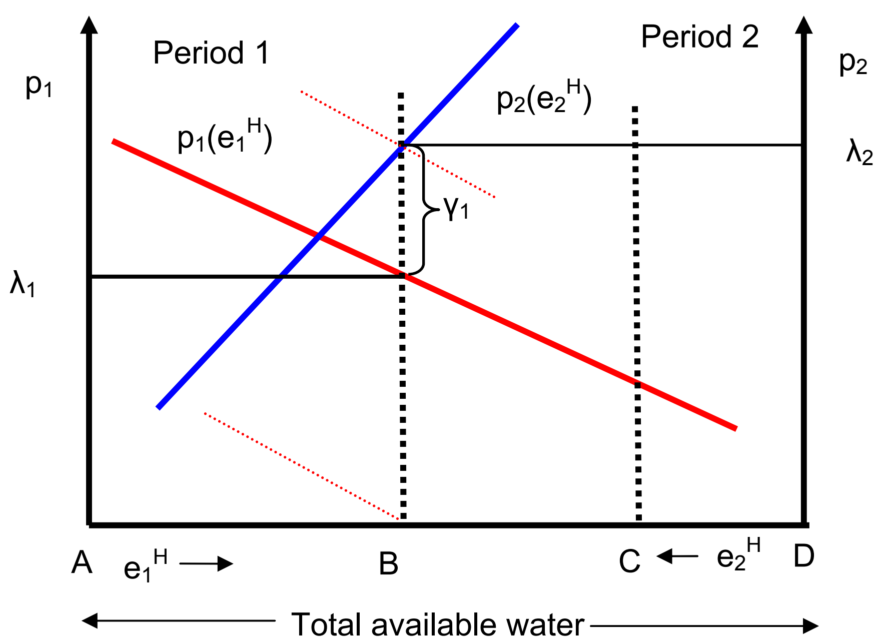

Figure 3. Social optimum with reservoir constraint binding

overflow to a period with scarcity. Using (12) above we have that $\lambda_1 = \lambda_2 - \gamma_1$. Notice that the water allocation will be the same for a wide range of period 1 demand curves keeping the same period 2 curve, or vice versa. (The period 1 curve can be shifted down to passing through B and shifted up to passing through the level indicated for period 2 water value, as indicated by the broken lines.) The price differences between the periods may correspondingly vary considerably.

Graphical analysis of multiple periods

The two-period nature of the dynamics of the system makes it possible to illustrate a sequence of optimal solutions using two period bathtub diagrams. Connecting figures like Figure 2 and 3 we must remember that the inflow AC in the first period now also contains what is stored in the period preceding the one we are studying. In the second period we will now see what is left for the next period. For ease of notation we keep the lettering from Figure 2 and 3. We will study typical events along the tine axis according to the presentation in Figure 4.

The terminal period

Applying backward induction we start with periods *T* and *T*-1. We will assume that this situation turns out identical to what is discussed above after Eq. (11) and portrayed in Figure 2. The common price for periods *T*-1 and *T* is termed p_T.

Neither overflow nor scarcity

Moving to the left on the time axis after period *T*-1 in Figure 4 we have a block of periods with neither threat of overflow nor emptying of reservoirs. From the necessary conditions (12) we

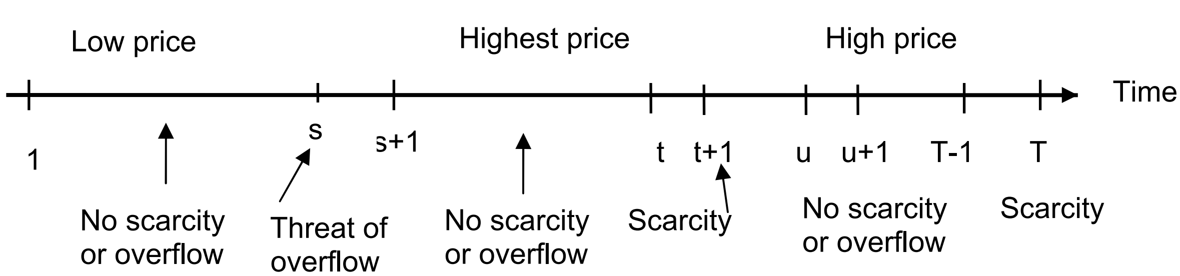

Figure 4. Main price-determining events

then know that the terminal period price p_T will prevail for all these periods. The way such periods can be illustrated is shown in Figure 5. AC is made up of inflows in period u plus what is remaining in the reservoir from period u-1. CD is the inflow in period u+1 and BC is the reservoir capacity. With the price level p_T given from the future this will be the price both in period u and u+1, and the amount of water indicated will be saved in period u+1 for period u+2. We have neither threat of overflow nor emptying of reservoirs in period u and u+1. For this to occur the future price p_T must be higher than p_T^{min}. If the future price is lower than p_T^{min} then all water will be used in period u and u+1 to a common price higher than the future price and there will be scarcity in period u+1. If the price p_T is higher than p_u^{max} then there will be threat of overflow in period u, and the maximal storage is passed to period u+1. If the future price is in between p_u^{max} and p_{u+1}^{max} then some water is transferred to period u+2 and the period u+1 price is p_T. If the future price is higher than p_{u+1}^{max} then the latter price will be the price in period u+1 and the maximal storage will be passed on to period u+2. We have assumed that $p_{u+1}^{max} > p_u^{max}$. If the reverse is the case p_u^{max} remains the critical price for period u, but what now happens in period u+1 is a little more complicated and is left to the reader.

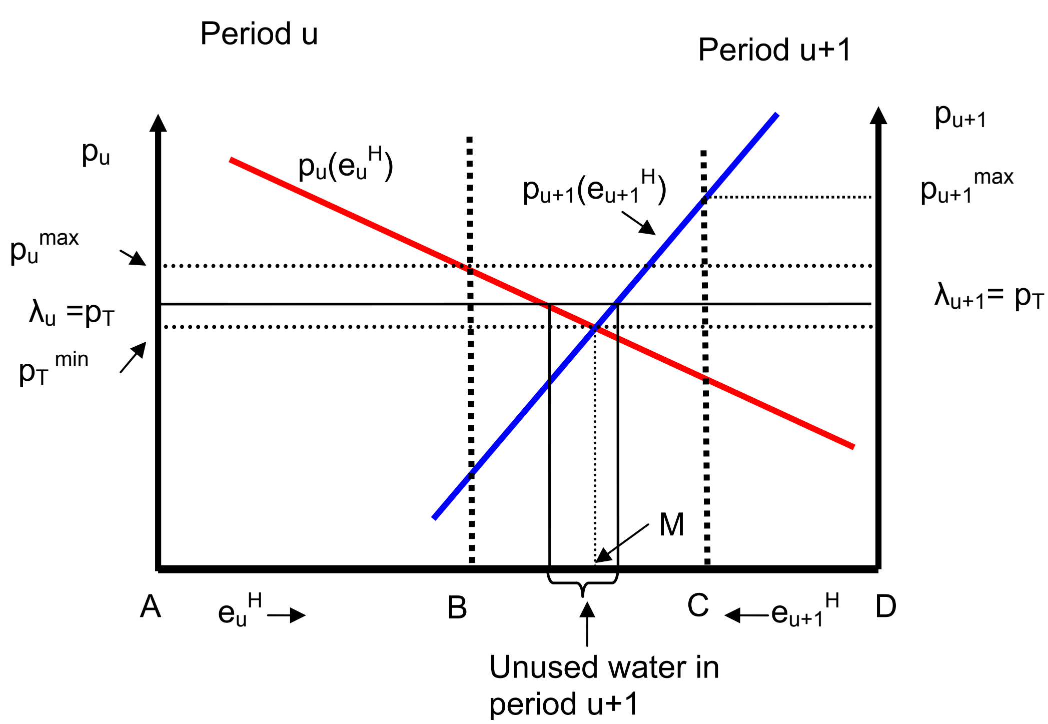

Figure 5. Neither threat of overflow nor scarcity

Scarcity in another period

We will now investigate what happens if the reservoir is emptied in other periods than the last one as indicated for period $t+1$ in Figure 4. We assume that there is no other scarcity period than the terminal period scarcity going to the left on the time axis, and that there are no overflow periods. Using condition (12) we have for the two periods:

$$\begin{aligned}
&p_t(e_t^H) = \lambda_t \ (e_t^H > 0)\\
&-\lambda_t + \lambda_{t+1} - \gamma_t \le 0 \perp R_t \ge 0\\
&p_{t+1}(e_{t+1}^H) = \lambda_{t+1} \ (e_{t+1}^H > 0)\\
&-\lambda_{t+1} + \lambda_{t+2} - \gamma_{t+1} \le 0 \perp R_{t+1} \ge 0
\end{aligned} \tag{14a}$$

The link with our optimal path story is that $\lambda_{t+2} = \lambda_T = p_T$. We will assume that there are no threat of overflow neither in period t nor period t+1 implying $\gamma_t = \gamma_{t+1} = 0$. Furthermore, by assumption $R_t > 0,\ R_{t+1} = 0$. We assume strictly positive prices for all periods. Combining conditions and assumptions yields

$$\begin{aligned}
&\lambda_t = \lambda_{t+1} > 0\\
&p_t(e_t^H) = p_{t+1}(e_{t+1}^H) > 0\\
&\lambda_{t+1} \ge \lambda_T > 0 \ (R_{t+1} = 0)
\end{aligned} \tag{14b}$$

The normal situation would be to have strict inequality in the last condition: $\lambda_{t+1} > \lambda_T$. We can use Figure 5 as an illustration (setting $t = u$) assuming that $0 < p_T < p_T^{\min}$. The water allocation on the two periods is then indicated by M, and the price will be the same in the two periods as indicated by the broken horizontal line through the intersection point of the two demand curves. All the water MD will be used up in period t+1 since the water value in period t+1 is higher than p_T. We note that the price in periods before the second scarcity period t+1 is higher (assuming neither overflow nor scarcity) than the price during the periods with neither overflow nor scarcity for the periods $t+2,\ldots, T$.

Overflow or threat of overflow

The last case we will investigate is overflow or threat of overflow (reservoir completely filled) for a period $s < t$, where $t+1$ is the first scarcity period after s. Using condition (12) we have the general conditions for the two periods:

$$\begin{aligned}
&p_s(e_s^H) = \lambda_s \ (e_s^H > 0) \\
&-\lambda_s + \lambda_{s+1} - \gamma_s \le 0 \perp R_s \ge 0 \\
&p_{s+1}(e_{s+1}^H) = \lambda_{s+1} \ (e_{s+1}^H > 0) \\
&-\lambda_{s+1} + \lambda_{s+2} - \gamma_{s+1} \le 0 \perp R_{s+1} \ge 0
\end{aligned} \tag{14c}$$

The link with our optimal path story is that $\lambda_{s+2} = \lambda_t > 0$. We assume that $R_s > 0, \gamma_s > 0$. These conditions yield

$$\begin{aligned}
&p_s(e_s^H) = \lambda_s \ (e_s^H > 0) \\
&\lambda_s = \lambda_{s+1} - \gamma_s \ (R_s > 0) \\
&p_{s+1}(e_{s+1}^H) = \lambda_{s+1} \ (e_{s+1}^H > 0) \\
&\lambda_{s+1} \ge \lambda_t - \gamma_{s+1} \ (R_{s+1} \ge 0)
\end{aligned} \tag{14d}$$

The second equality in (14d) follows from the Kuhn-Tucker condition in (12) when there is a positive amount of water in the reservoir. The shadow price on water λ_s is zero if there is actual overflow. This follows from the third condition (complementary slackness) in (11). If there is no spillage and the water is just maintained at the maximal level the water value λ_s will typically be positive. In any case the water value λ_s is smaller than the water value λ_{s+1} for the next period. We will adapt Figure 3 to illustrate the situation. Remember that the available water AC in period s includes the transfer of stored water from period s-1.

In Figure 6, adapted from Figure 3, overflow threatens in period s and the maximal reservoir filling BC is saved to the next period s+1, and AB is consumed in period s resulting in a price similar to the story in Figure 3 for period 1. However, if the price "inherited" from the future, p_t, lies in between the levels marked $p_t^{\min}, p_t^{\max}$ then this price will be the period s+1 price and water equal to BM will be saved to period s+2, and water MD will be consumed in period s+1. If we have $p_s < p_t < p_t^{\min}$ then the price in period s+1 will be $\lambda_{s+1} = p_{s+1} = p_t^{\min}$. In period s+2 the price will return to p_t. We get a *price spike* in period s+1 after the threat of overflow in period s because period s+1 becomes a scarcity period; all available water BD is used in period s+1.

If we have $p_t > p_t^{\max}$ then all stored water BC in period s is also stored in period s+1 (the storage possibility in period s+1 is started from the vertical line from C and to the left to B). But the

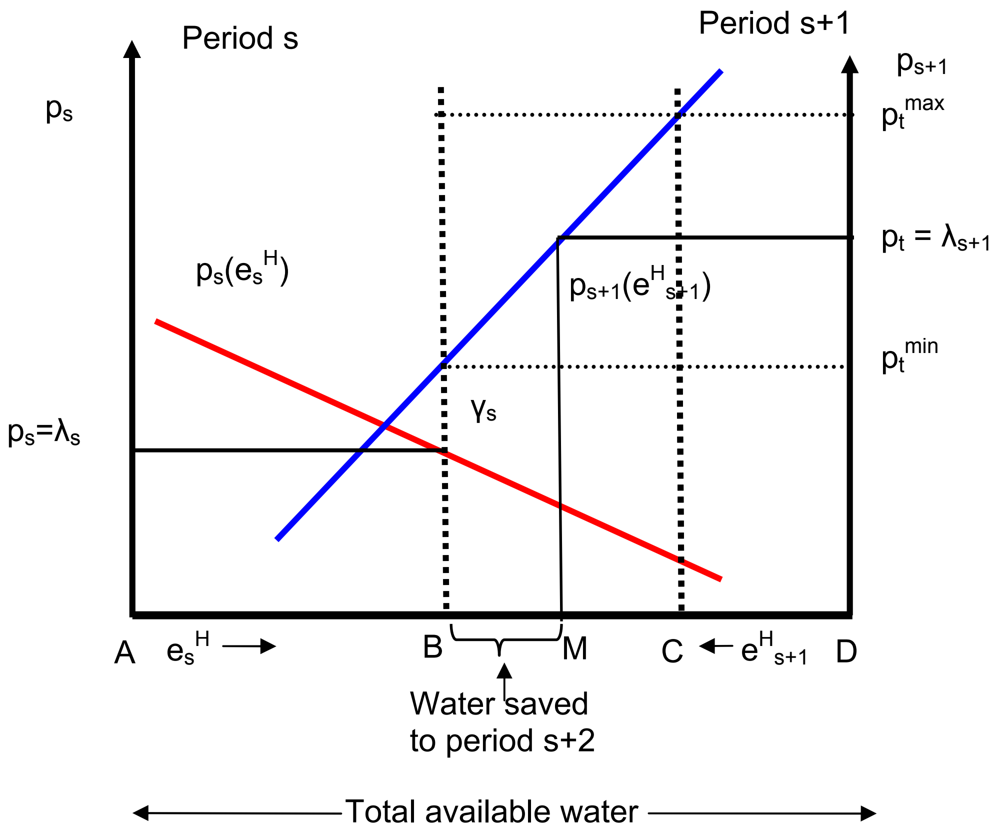

Figure 6. Threat of overflow

price in period *s*+1 has to be *lowered* to $p_t^{\max}$ in order to induce the consumers to by the amount CD to prevent overflow in period *s*+1. Therefore the shadow price on the reservoir constraint γ_{s+1} is subtracted from the price p_t in Eq. (14d). We now have threats of overflow in both periods. The shadow price on the reservoir constraint in period *s* depends on the level of p_t. The minimum shadow price when $p_s < p_t < p_t^{\min}$ is $\gamma_s = p_t^{\min} - p_s$, and the maximum is obtained when $p_t > p_t^{\max}$ as $\gamma_s = p_t^{\max} - p_s$.

If the optimal path of hydropower production and reservoir levels involves an interwoven pattern of scarcity periods and periods with threat of overflow the price will cycle from higher values in periods after an overflow (or threat of overflow) episode to the next scarcity period and to a lower price after a scarcity period and until the next overflow (or threat of overflow) period. Price spikes may also be part of an optimal development. If we look to the left on the time axis in Figure 4 after a threat of overflow episode the connection to prices to the right on the time axis is completely broken. A succession of scarcity periods imply a building up of the price, being highest for the first scarcity period coming from the left on the time axis and then falling off after each scarcity period is passed until the last one. In this way our simple model may be able to generate a changing price pattern more in correspondence with what we observe.

An additional element is caused by uncertainty. Although we observe neither threat of overflow nor scarcity these situations may have been relevant for decision making if there is uncertainty about inflows or consumption (especially temperature–dependent consumption).

There is also another factor that may generate price changes over periods that is easy to incorporate in the model.

Run of the river

In most hydro systems power is also generated without having reservoirs that are relevant for the time unit of the analysis. This may be rivers where what flows in must be produced continuously or else the water is lost. In Norway power from plants without storage possibilities constitute about 30% of yearly production. The social planning problem including run of the river power generation is:

$$
\begin{aligned}
& Max \sum_{t=1}^{T} \int_{z=0}^{x_t} p_t(z)dz \\
& s.t. \qquad\qquad (15) \\
& x_t = e_t^H + e_t^R \\
& R_t \le R_{t-1} + w_t - e_t^H,\ R_t \le \overline{R},\ t = 1,..,T
\end{aligned}
$$

Here e_t^R is the electricity produced in period t by run of the river with assumed zero production-dependent operating costs. Energy is now supplied both based on using reservoirs and run of the river so the *energy balance* is entered as a new constraint. The river water has to be processed as it comes in order to avoid loosing the value. The reservoirs have to be used as buffers to absorb the river flow fluctuations. Since the energy balance has to hold as equality we can substitute for x_t in the optimisation problem and we get the following Lagrangian:

$$L = \sum_{t=1}^{T} \int_{z=0}^{e_t^H + e_t^R} p_t(z)dz$$
$$-\sum_{t=1}^{T} \lambda_t (R_t - R_{t-1} - w_t + e_t^H) \qquad (16)$$
$$-\sum_{t=1}^{T} \gamma_t (R_t - \overline{R})$$

The necessary first order conditions are exactly of the same form as (11) for problem (10). Our standard assumption is that electricity is produced every period (but now it may be more realistic that demand for electricity may be satiated). If hydropower from reservoirs is used, then the price is equal to the water value. If we assume that hydro from reservoirs is produced every period, then demand for electricity is not satiated and we have the same situation as described by Eq. (12) with $e_t^H + e_t^R$ as argument in the demand function in the first relation. Changes in the run of the river are the same as exogenous shifts in the demand functions[5].

This may be illustrated in a bathtub diagram by extending the "walls" with the run of the river and shifting the demand schedules accordingly, as shown in Figure 7 that is an adaptation of Figure 3 in the case of a river flow only in period 1. The river flow is added to the controllable hydro to the left and to the right of the old walls of the bathtub. The demand curve for period 1 now has to be anchored on the river-extended wall marked with the broken vertical line to the left of the vertical line from A, and the demand curve for period 2 is anchored to the vertical line to the right of D. There are horizontal shifts of the demand curves (from the broken lines to the solid ones) equal to the river flow for both periods. The river flow in period 2 is smaller than the river flow in period 1. The part of the demands satisfied using controllable hydro are the *residual* demand curves. In the situation with maximal storage from period 1 to period 2 we see that since no more water can be transferred to period 2 the electricity from river flow will be consumed in period 1, and in order to induce consumption of more water from controllable hydro the price in period 1 has to decrease (old price is λ_1). The price in period 2 decreases relatively more (price without river flow is λ_2) although the river flow in period 2 is smaller due to the demand in

[5] Wind power may be treated in the same way.

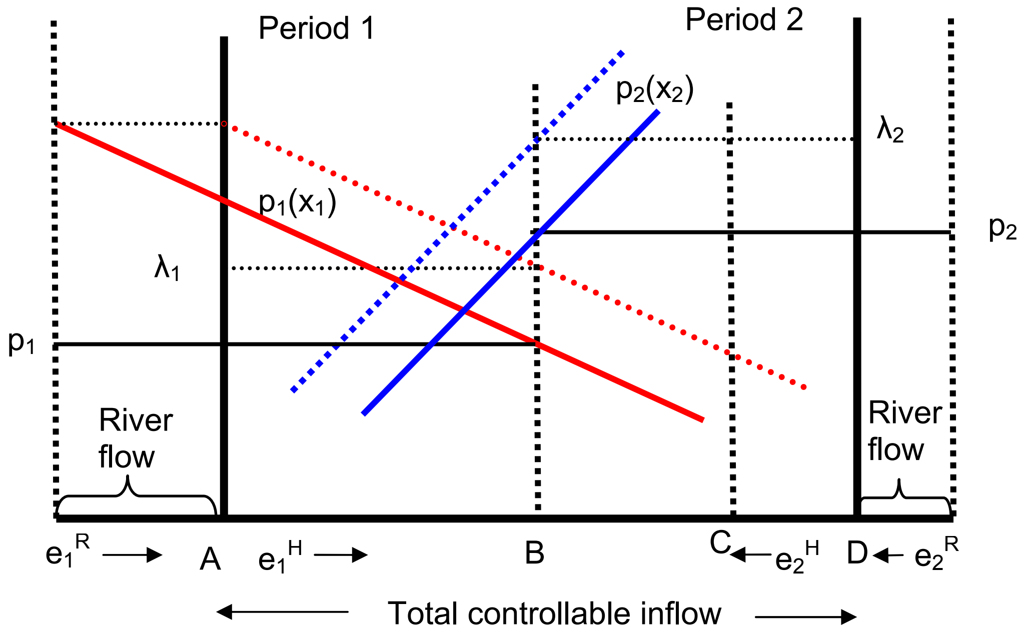

Figure 7. Run of the river

period 2 being more inelastic. Other configurations are easy to accommodate in the bathtub diagram.

Several producers

The reader may feel that assuming one hydro plant with one reservoir is limiting the realism of the model since there are over 600 hydropower producers in Norway, and a majority of them have reservoirs. We will therefore briefly study the implications of several producers for the optimal allocation of water. Each plant is assigned one reservoir. The planning problem is the same as (9), but now a subscript (*j*) for plant has to be introduced. We will also need a relation connecting the amount consumed to the total amount produced. This is popularly termed the energy balance. The social planning problem is:

$$\begin{aligned}
&Max \sum_{t=1}^{T} \int_{z=0}^{x_t} p_t(z)dz \\
&s.t. \\
&R_{jt} \le R_{j,t-1} + w_{jt} - e_{jt}^H, R_{jt} \le \overline{R}_j, j = 1,..,N \\
&x_t = \sum_{j=i}^{N} e_{jt}^H, t = 1,..,T
\end{aligned} \tag{17}$$

The energy balance for each period is the last restriction. It has to hold as equality. Electricity is a homogenous good so it does not matter to the consumer who supplies the electricity. Inserting the energy balance yields the Lagrangian:

$$\begin{aligned} L = & \sum_{t=1}^{T} \int_{z=0}^{\sum_{j=1}^{N} e_{jt}^{H}} p_t(z)dz \\ & -\sum_{t=1}^{T}\sum_{j=1}^{N} \lambda_{jt}(R_{jt} - R_{j,t-1} - w_{jt} + e_{jt}^{H}) \\ & -\sum_{t=1}^{T}\sum_{j=1}^{N} \gamma_{jt}(R_{jt} - \bar{R}_j) \end{aligned} \tag{18}$$

The first-order conditions (for t =1,..,T and j =1,..,N) are:

$$\begin{aligned} & \frac{\partial L}{\partial e_{jt}^{H}} = p_t(\sum_{j=i}^{N} e_{jt}^{H}) - \lambda_{jt} \leq 0 \perp e_{jt}^{H} \geq 0 \\ & \frac{\partial L}{\partial R_{jt}} = -\lambda_{jt} + \lambda_{j,t+1} - \gamma_{jt} \leq 0 \perp R_{jt} \geq 0 \\ & \lambda_{jt} \geq 0 (= 0 \text{ for } R_{jt} < R_{j,t-1} + w_{jt} - e_{jt}^{H}) \\ & \gamma_{jt} \geq 0 (= 0 \text{ for } R_{jt} < \bar{R}_j) \end{aligned} \tag{19}$$

We assume that electricity is consumed in all periods to positive prices. Using the backward-induction principle, assuming that demand is not satiated and all reservoirs are emptied in the terminal period T, we get:

$$-\lambda_{jT} - 0 \leq 0 \Rightarrow \lambda_{jT} = p_T(x_T) > 0 \tag{20}$$

The equality follows from the assumption that all units are producing electricity in the last period (at least the inflows w_{jT}) and that the market prices are positive. But the condition above is not specific to plant j, but applies to all plants. In the optimal solution all plants are assigned the same water value in the last period and the consumption of electricity is $\sum_{j=1}^{N} R_{jT}$.

For period T-1 the process is repeated. Without any overflow at any plant or any plant emptying its reservoir all plants are again facing the same water values and the price must be the same as for period T.

To investigate whether it could be part of an optimal plan that one plant has overflow in period *T-1* let us try with overflow for plant *j*. Since overflow is a loss we will make the reasonable assumption that the plant has positive production implying from the first condition in (19) that the water value is equal to the market price:

$$\lambda_{j,T-1} = p_{T-1}(x_{T-1}) \tag{21}$$

Assuming also positive production for all other plants they then have the same water value. But assuming overflow, or threat of overflow for unit *j* implies that $\gamma_{j,T-1} > 0$ and from the second condition in (19) we have $-\lambda_{j,T-1} + \lambda_{jT} - \gamma_{j,T-1} = 0$. But we then have a contradiction since the last relation implies $p_{T-1}(x_{T-1}) < p_T(x_T)$ for plant *j* and $p_{T-1}(x_{T-1}) = p_T(x_T)$ for other plants. We conclude that optimality requires all plants to have threat of overflow at the same time. Any loss of water is a social loss, so the optimal plan must imply manoeuvring to prevent such losses. The price in period *T-1* can only be lower than for period *T* if overflow threatens *all* plants.

The other extreme situation is that plant *j* empties its reservoir in period *T-1*, but not the other plants. The first condition in (19) again yields $\lambda_{j,T-1} = p_{T-1}(x_{T-1})$ since plant *j* has positive production. The second condition in (19) now yields $-\lambda_{j,T-1} + \lambda_{jT} \leq 0$. Assuming strict inequality we have that for plant *j* it is required that $p_{T-1}(x_{T-1}) > p_T(x_T)$, while the condition for the other plants yields $p_{T-1}(x_{T-1}) = p_T(x_T)$. Again we have a contradiction. We conclude that in the regular case all reservoirs have to be emptied at the same time for the plan to be optimal. But note that the inequality involved is not strict, so it may be optimal for plants to empty their reservoirs before others. But in that case the value of the social objective function remains the same. The reasoning above leads to the following result:

Hveding's conjecture: *In the case of many plants with one limited reservoir each the plants can be regarded as one plant and the reservoirs can be regarded as one reservoir in the social solution for operating the hydropower system* (see Hveding, 1967, 1968).

The individual reservoirs will all be utilised in the same fashion, *as if* there is only one reservoir. This is a result of important practical value since it may simplify greatly the modelling effort.

The results about price movement studied in the previous section for one plant and one reservoir are all valid also for the many plant case. But Hveding's conjecture may not hold strictly if more of the constraints entered in Table 1 are introduced. The constraints may be so demanding to fulfil, especially with a fine time resolution, that some reservoirs may experience overflow before others, and some may be emptied. But we may regard the conjecture as an approximation to a social optimal solution. This may hold even when extending the modelling to include stochastic inflows and demand.

4. Trade

Social optimum without constraints on transmission/trade

Introducing trade means that we introduce a second good, money, into the society in addition to electricity. We will simplify by just adding (subtracting) the export (import) sum to (from) the area under the demand curve for electricity, implying that in the background we assume utility functions separable in electricity and money (aggregate for all other goods). The objective function will then be the sum over the periods of consumer and producer surplus, which in our case for electricity will be the area under the demand curve since we have assumed zero production cost (only water value counts), and for money there is just the amount: positive for exports and negative for imports. There is no restriction to have balance of trade in electricity. The social planning problem is:

$$Max \sum_{t=1}^{T}[\int_{z=0}^{x_t} p_t(z)dz + p_t^{XI} e_t^{XI}]$$

s.t. (22)

$$\mathrm{x_t} = e_t^H - e_t^{XI}, \sum_{t=1}^{T} e_t^H \le W, \quad p_t^{XI} = \text{given}, \quad t = 1,..,T$$

The variable e_t^{XI} is net export and is positive if we have export and negative if we have imports. We assume that in one period we can only have either exports or imports, or both are zero.

Inserting the energy balance the Lagrangian is:

$$L = \sum_{t=1}^{T} [\int_{z=0}^{e_t^H - e_t^{XI}} p_t(z)dz + p_t^{XI} e_t^{XI}] \\ -\lambda(\sum_{t=1}^{T} e_t^H - W) \tag{23}$$

The necessary first-order conditions for $t = 1,..,T$ are:

$$\frac{\partial L}{\partial e_t^H} = p_t(e_t^H - e_t^{XI}) - \lambda \le 0 \;\perp\; e_t^H \ge 0$$
$$\frac{\partial L}{\partial e_t^{XI}} = -p_t(e_t^H - e_t^{XI}) + p_t^{XI} = 0 \tag{24}$$
$$\lambda \ge 0 \; (= 0 \; for \; \sum_{t=1}^{T} e_t^H < W)$$

The second condition holds as an equality since there is no restriction on the sign of e_t^{XI}. It is quite reasonable to assume that $x_t > 0 \, \forall t = 1,..,T$. This means that in export periods hydro is used for home consumption and the first condition in (24) holds with equality. Now, since the shadow price on water is without period subscript we can only have *one* export period if we make the assumption that all the export/import prices are unique. With no restriction on transmission the foreign price regime will be adapted as the home country price regime. But notice that we do not necessarily use hydropower in all periods. If the price in the home market is less than the shadow price λ on water, no water shall be used for hydropower production in that period; we just import. Without any constraint on the possibility to store water the model is too extreme because we will only export in one period, the period with the highest export price, and import in all other periods. The shadow price on water will be set equal to this maximum price:

$$\lambda = \max_{t=1,..,T} \left\{ p_t^{XI} \right\} \tag{25}$$

The total export will be:

$$e_{t*}^{XI} = W - e_{t*}^H \quad [e_{t*}^H = p_{t*}^{-1}(p_{t*})] \tag{26}$$

where $t*$ is the period corresponding to the period with the maximal export price above. In all other periods we will only import to the going foreign price. Unlimited trade is therefore only of practical interest together with constraints on the possibility to store water (Førsund, 1994).

The social optimum with constraints on transmission/trade

We now introduce an upper constraint on export/import. The social planning problem is:

$$Max \ \sum_{t=1}^{T}[\int_{z=0}^{x_t} p_t(z)dz + p_t^{XI}e_t^{XI}]$$

s.t.

$$\begin{aligned} &\mathrm{x_t} = e_t^H - e_t^{XI}, \sum_{t=1}^{T} e_t^H \leq W, \\ &-\overline{e}^{XI} \leq e_t^{XI} \leq \overline{e}^{XI} \\ &p_t^{XI} = \text{given} > 0,\ t = 1,..,T \end{aligned} \tag{27}$$

The constraint on trade can be split up on export and import. The corresponding Lagrangian inserting the energy balance is

$$\begin{aligned} L = &\sum_{t=1}^{T}(\int_{z=0}^{e_t^H - e_t^{XI}} p_t(z)dz + p_t^{XI}e_t^{XI}) \\ &-\lambda(\sum_{t=1}^{T} e_t^H - W) \\ &-\sum_{t=1}^{T}\alpha_t(e_t^{XI} - \overline{e}^{XI}) \\ &-\sum_{t=1}^{T}\beta_t(-e_t^{XI} - \overline{e}^{XI}) \end{aligned} \tag{28}$$

The first-order conditions are:

$$\begin{aligned} &\frac{\partial L}{\partial e_t^H} = p_t(e_t^H - e_t^{XI}) - \lambda \leq 0 \ \perp \ e_t^H \geq 0 \\ &\frac{\partial L}{\partial e_t^{XI}} = -p_t(e_t^H -) + p_t^{XI} - \alpha_t + \beta_t = 0 \\ &\lambda \geq 0 \ (= 0 \text{ for} \sum_{t=1}^{T} e_t^H < W) \\ &\alpha_t \geq 0 \ (= 0 \text{ for } e_t^{XI} < \overline{e}^{XI}) \ (e_t^{XI} > 0) \\ &\beta_t \geq 0 \ (= 0 \ for - e_t^{XI} < \overline{e}^{XI}) \ (e_t^{XI} < 0) \end{aligned} \tag{29}$$

The second equation holds with equality since export/import can be both positive and negative. Only one of the shadow prices on maximal trade can be positive in the same period (both can be zero). We have that if both shadow prices are zero (import/export constraints are not biting), then the home price is equal to the export/import price.

We assume as before that $x_t > 0$ ($t = 1,..,T$). If there is export then $e_t^H > 0$ and the first equation in (29) holds with equality. Let us again assume that all the export/import prices are different. Then there can only be *one* export period for which the upper constraint is not binding. The reason is that the shadow value on water has no time subscripts and since the export prices are different we will have a contradiction with more than one such export period. Let us call the period for a *marginal export period*. If the constraint on export is binding then we may have that the export price is higher than the home price because:

$$p_t(x_t) = \lambda = p_t^{XI} - \alpha_t \ (e_t^{XI} > 0) \tag{30}$$

For import periods we may have $e_t^H = 0$ if the home price is less than the shadow price on water for zero hydro production. We have in general for import periods

$$p_t(x_t) = p_t^{XI} + \beta_t \ (e_t^{XI} < 0) \tag{31}$$

If we are at the upper constraint for imports with a positive shadow price β_t then the home price will typically be higher than the export/import price. Hydro can only be used in import periods if the transmission constraint is binding and the shadow price on the constraint is positive. The reason is that use of hydro with imports below the trade constraint implies equality in the first condition in (29), and since export/import prices are different we will again have a contradiction.

A feasible optimal solution is illustrated in the two-period case in Figure 8, adapting Figure 7. The hydro bathtub is extended with the imports in period 1, indicated by the broken vertical line as the new "wall" on the left. We assume that this is the full capacity imports. The shadow price on the import constraint is indicated as the difference between the export/import price in period 1 and 2. In addition to import some hydro will be used in period 1. The common water value is set equal to the highest trade price occurring in period 2. In this period we assume that the export is less than the transmission capacity. The home price is therefore equal to the export price in this period. Period 1 home price will also be the same since the alternative value of water in period 1 is to export in period 2 since there is capacity to do so. Without limit on trade we can use the broken demand curve for period 1 and find the intersection with the price line for the import price in period 1. The import will be more than the total available hydro, and no hydro will now be used in period 1. In period 2 the same quantity of hydro will be consumed at home, but the

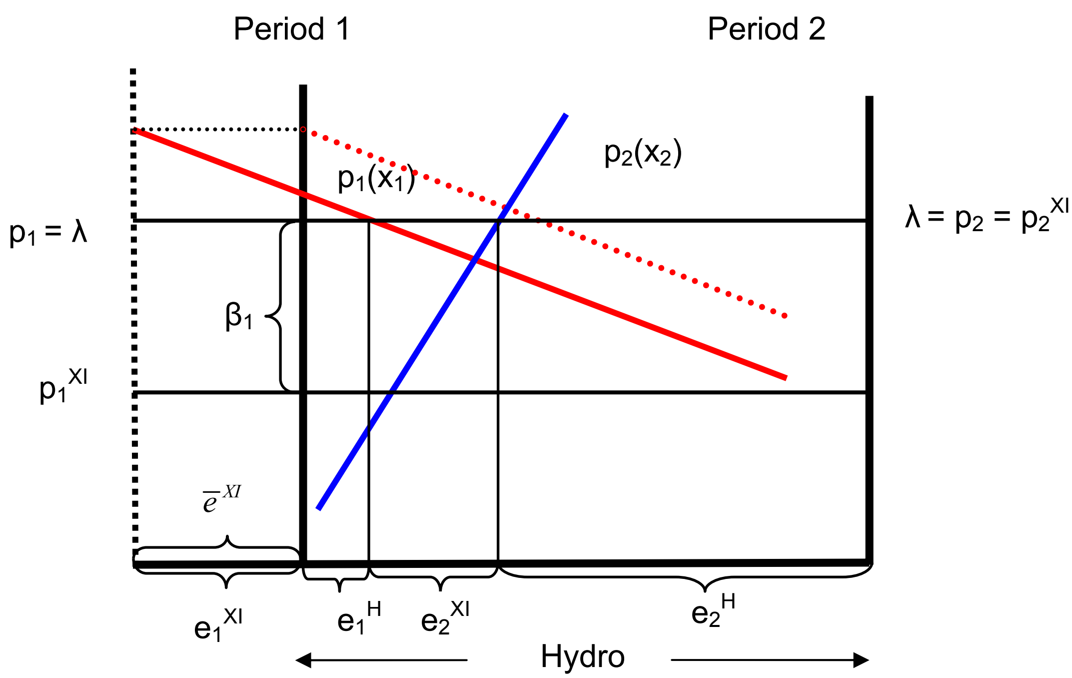

Figure 8. Hydro extended with trade. Limit on transmission capacity

export is extended with the amount of hydro used in the import period 1 with transmission constraint binding. Remember that we have no requirement of a trade balance in electricity.

Returning to the general multi-period case many different trade patterns may emerge. Let us simplify by sorting the export/import prices in descending order and assuming that they are all different so we have a unique ranking. With no constraints on the volume of trade we found that export will only take place in one period, the maximal price period, and there will be imports on all the other periods. We will also now have export in the highest export price period, but if it is assumed that the transmission constraint is binding, then there will be export in at least one more period, depending on the relationship between the total amount of water, water used in export and import periods and the constraint $\bar{e}^{XI}$. In the single export period where the constraint will typically not be binding (the case of all export periods hitting the constraint is arbitrary) the price for this period is the lowest among the set of prices for export periods. This price, $p_{t^*}^{XI-\min}$ for period t*, will then determine the shadow price λ on water. This is the marginal export period. If water is used in an import period it means that the import constraint is binding and that at this import price there is a positive residual home demand that can only be satisfied by using water.

Since the alternative use of water is to increase export in the marginal export period this means that the home price in an import period with the transmission constraint binding must be equal to the water value and equal to the export price in the marginal export period:

$$p_{t^*}^{XI-\min}{}_{[e_{t^*}^{XI}>0]} = \lambda = p_{t[e_t^{XI}<0, e_t^H>0]} = (p_t^{XI} + \beta_t)_{[e_t^{XI}<0, e_t^H>0]},\ t \in T^{H+imp} \tag{32}$$

The optimal water value must also satisfy the condition that the total available water, W, is just used up on home consumption and exports.

The number of export periods, t^{ex}, is determined by

$$t^{ex} = \frac{W - \sum_{t \in T^{H+imp}} e_t^H - e_{t^*}^H}{\bar{e}^{XI}} \tag{33}$$

where t^* is the single period when export is not hitting the upper constraint, and T^{H+imp} is the set of import periods when hydro is also used. The t^{ex} - numbers of highest prices will belong to the export periods, and the rest of the prices will belong to import periods. In the $t^{\exp} - 1$ number of periods with the highest prices the transmission constraint will be binding and typically the shadow price α_t is positive, driving a wedge between the lower home price and the export prices. As remarked above all the home prices are equal, so the shadow prices on the transmission constraint will all be different. In the period with the price ranked as number t^{ex} the export constraint is not binding and then the home price and the export price are equal and equal to the shadow price λ on water. In the periods with the price ranked $t^{\exp} + 1$ to T we will have imports and no use of hydro when the transmission constraint is not binding and use of hydro in addition when the transmission constraint is binding with positive shadow price.

5. Thermal plants

We introduce plant-specific variable cost functions for the generation of electricity based on thermal energy sources. Each plant has an upper capacity ($\bar{e}_{it}^{Th}$) for generation (e_{it}^{Th}) that can only be changed by investments. For simplicity the cost functions are not dated, but the cost function may change between periods due to different fuel prices (fuels may be more expensive in a high-demand season):

$$c_{it} = c_i(e_{it}^{Th}),\ c_i' > 0,\ e_{it}^{Th} \leq \overline{e}_i^{Th}, i = 1,..,N \tag{34}$$

The plant may be designed to have the smallest marginal cost at close to full capacity utilisation. We disregard costs of ramping up or down plants, and especially going from a cold to a spinning state. (Having a phase of declining marginal costs may capture a start-up effect.)

The set of individual thermal plants can be aggregated to a thermal sector by the following least-cost procedure satisfying a total generating requirement of e_t^{Th} for each period:

$$\begin{aligned} &Min \sum_{i=1}^{N} c_i(e_{it}^{Th}) \\ &s.t. \\ &\sum_{i=1}^{N} e_{it}^{Th} \geq e_t^{Th},\ e_{it}^{Th} \leq \overline{e}_i^{Th}, \end{aligned} \tag{35}$$

For each total generation level we get a set of plants producing positive output, and a set being idle according to the marginal cost levels. If the range of variation in the marginal costs for each plant is sufficiently small so that no interval is overlapping, all but one plant will be utilised to full capacity, and there will be a marginal unit partially utilised. We can perform a *merit order ranking* of the active units according to average costs at full capacity utilisation. Finally, the sequence of individual cost curves can be simplified or approximated by a smooth function:

$$c_t = c(e_t^{Th}), c' > 0, c'' > 0,\ e_t^{Th} \leq \sum_{i=1}^{N} \overline{e}_{it}^{Th} = \overline{e}^{Th} \tag{36}$$

Social solution of mixed hydro and thermal capacity

The basic hydro model (6) without constraints on reservoirs, but only on total availability of water, is adopted. Instead of using the utility function as in (6) the demand function for electricity is used making explicit the maximisation of consumer plus producer surplus. We assume that it does not matter how electricity is generated, i.e. the willingness to pay is the same for the two types of generation (no "green" preference). The optimisation problem faced by a system planner is:

$$Max \ \sum_{t=1}^{T}[\int_{z=0}^{x_t} p_t(z)dz - c(e_t^{Th})] \ s.t. \qquad (37)$$
$$x_t = e_t^H + e_t^{Th}, \ \sum_{t=1}^{T} e_t^H \leq W, \ e_t^{Th} \leq \overline{e}^{Th}$$

Inserting the energy balance the Lagrangian function is:

$$L = \sum_{t=1}^{T}[\int_{z=0}^{e_t^H + e_t^{Th}} p_t(z)dz - c(e_t^{Th})]$$
$$-\sum_{t=1}^{T} \theta_t (e_t^{Th} - \overline{e}^{Th}) \qquad (38)$$
$$-\lambda(\sum_{t=1}^{T} e_t^H - W)$$

The necessary conditions are:

$$\frac{\partial L}{\partial e_t^H} = p_t(e_t^H + e_t^{Th}) - \lambda \leq 0 \perp e_t^H \geq 0$$
$$\frac{\partial L}{\partial e_t^{Th}} = p_t(e_t^H + e_t^{Th}) - c'(e_t^{Th}) - \theta_t \leq 0 \perp e_t^T \geq 0 \qquad (39)$$
$$\lambda \geq 0 \ (= 0 \text{ for} \sum_{t=1}^{T} e_t^H < W)$$
$$\theta_t \geq 0 \ (= 0 \text{ for } e_t^{Th} < \overline{e}^{Th})$$

Assuming that electricity must be produced in all periods we must then in each period either activate hydro or thermal, or both. Thermal will not be used for periods where

$$c'(0) > \lambda \, . \qquad (40)$$

If the marginal cost curve starts at values greater than the water value, then thermal is not used. According to the last condition in (39) $\theta_t = 0$ when $e_t^{Th} = 0$.

Hydro will not be used in periods where

$$p_t(x_t) = c'(e_t^{Th}) + \theta_t < \lambda \qquad (41)$$

If the market price is less than the water value then the water is saved to a period with a higher price.

For periods where both hydro and thermal is used we have:

$$p_t(x_t) = \lambda = c'(e_t^{Th}) + \theta_t \tag{42}$$

In a situation with no reservoir constraints and assuming that hydro will be used in every period the price will be constant for all periods.

Regarding the concepts base load and peak load it has been stated that in a mixed system thermal capacity would serve as peak load. However, without reservoir constraints and assuming that thermal capacity will not be exhausted, thermal capacity may be regarded as base load because it will be used at constant capacity for all periods, while the use of hydro will follow any shift of the demand curve over the periods. Rearranging (42) yields:

$$c'(e_t^{Th}) + \theta_t = \lambda \Rightarrow e_t^{Th} = c'^{-1}(\lambda - \theta_t). \tag{43}$$

If the shadow price $\theta_t = 0$ then from (43) thermal production is constant. But thermal capacity may reach the capacity constraint in one or more periods, and then thermal may also be termed peak load. However, thermal output will only have two values; full capacity or constant output less than full capacity. Hydro output may vary continuously over periods.

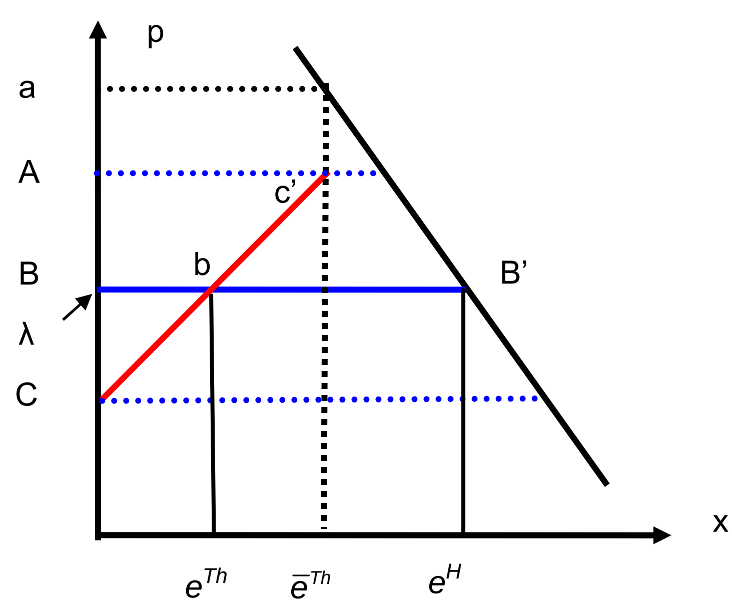

Figure 9. Hydro and thermal. Social optimum

An illustration for one period is shown in Figure 9. The marginal cost curve, c', for thermal capacity starts at C and ends at the full capacity value, $\overline{e}^{Th}$. Assuming bB' to be the available water the optimal solution is the price at level B equal to the shadow price of water, and a thermal contribution of Bb = e^{Th} and a hydro contribution of bB' = e^{H}.

If we assume that the figure is representing just one of many periods it is meaningful to introduce two alternative water values by the dotted lines at C and A. For water values from levels A to a the full capacity of thermal units will be utilised. For water values higher than at level a only thermal capacity will be used. For water values lower than at level C no thermal capacity will be used. In a multi-period setting with identical demand functions and average availability of water being bB' the one period solution shown in the figure will be repeated each period.

For two periods we may use the bathtub diagram to illustrate the allocation of the two types of power on the two periods. In Figure 10 the length of the bathtub AD is extended (analogous to

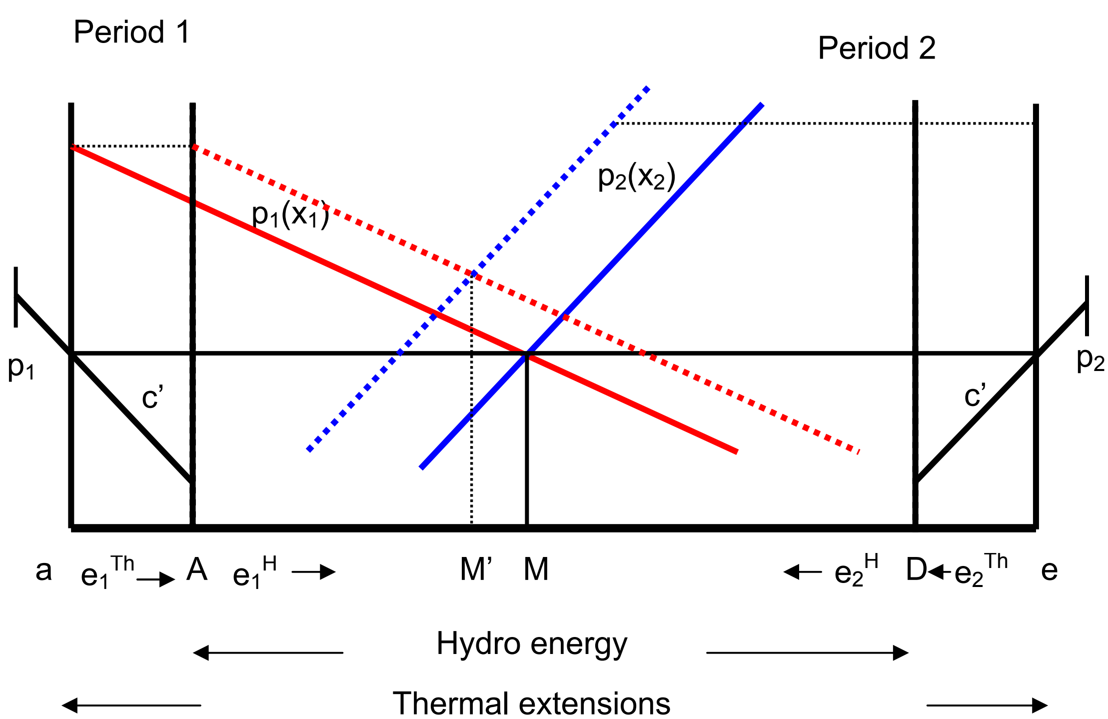

Figure 10. Energy bathtub with thermal-extended walls of the hydro bathtub

the procedure in Figure 7) at each end with the thermal capacity. The demand curves without thermal capacity are indicated by broken lines. The demand curves after introduction of thermal capacity are anchored at the thermal "walls", i.e. horizontal shifts to the left respectively right for period 1 and 2. The marginal cost curve of thermal capacity is anchored in the hydro wall at c'(0) to the left for period 1 and to the right for period 2. We assume the same cost curve for the two periods. The capacity limit is indicated by the short vertical line at the end of the cost curves. Using the result (43) we have that the thermal extension of the bathtub is equal at each end; with aA in period 1 and De in period 2 and aA = De. The equilibrium allocation is at point M, resulting in an allocation of aA thermal and AM hydro in period 1, and MD hydro and De thermal in period 2 to the same market price. In our example the allocation with thermal capacity results in more hydro used in period 2 indicated by the allocation point M' for the situation without thermal capacity. The reason is that the demand in period 2 is more inelastic than for period 1. Removing thermal capacity in the demand functions in the first equation of (39) the price in period 1 increases less than for period 2 leading to a decreased share of water to period 1 and a higher shadow price for water.

Introducing a reservoir constraint

Introducing a reservoir constraint into problem (37) yields the following Lagrangian function:

$$
\begin{aligned}
L = & \sum_{t=1}^{T}[\int_{z=0}^{e_t^H + e_t^{Th}} p_t(z)dz - c(e_t^{Th})] \\
& -\sum_{t=1}^{T}\theta_t(e_t^{Th} - \bar{e}^{Th}) \\
& -\sum_{t=1}^{T}\lambda_t(R_t - R_{t-1} - w_t + e_t^H) \\
& -\sum_{t=1}^{T}\gamma_t(R_t - \bar{R})
\end{aligned}
\qquad (44)
$$

The total hydro supply condition in (38) is replaced with the two last conditions in (44) showing the dynamics of water storage and the upper constraint on total storage. The necessary first order conditions are:

$$
\begin{aligned}
&\frac{\partial L}{\partial e_t^H} = p_t(e_t^H + e_t^{Th}) - \lambda_t \le 0 \perp e_t^H \ge 0 \\
&\frac{\partial L}{\partial R_t} = -\lambda_t + \lambda_{t+1} - \gamma_t \le 0 \perp R_t \ge 0, t \in T \\
&\frac{\partial L}{\partial e_t^{Th}} = p_t(e_t^H + e_t^{Th}) - c'(e_t^{Th}) - \theta_t \le 0 \perp e_t^{Th} \ge 0 \\
&\lambda_t \ge 0 (= 0 \text{ for } R_t < R_{t-1} + w_t - e_t^H) \\
&\gamma_t \ge 0 (= 0 \text{ for } R_t < \bar{R}) \\
&\theta_t \ge 0 (= 0 \text{ for } e_t^{Th} < \bar{e}^{Th})
\end{aligned} \tag{45}
$$

Regarding combining hydro and thermal we will now have as a general rule that the water value is period specific in the first condition, implying that thermal capacity may vary between periods when both hydro and thermal capacities are used. A possible situation is illustrated in Figure 11. The figure is built up in the same way as Figure 10. The total hydro capacity is AD with inflow AC in period 1 and CD in period 2 and storage capacity is BC. In period 1 the maximal amount is stored for use in period 2. Since thermal capacity is not utilised to its maximum in any of the two periods the period water value should be set equal to the marginal thermal costs. This implies that less thermal capacity, aA, is used in period 1 with the lowest water value, and more

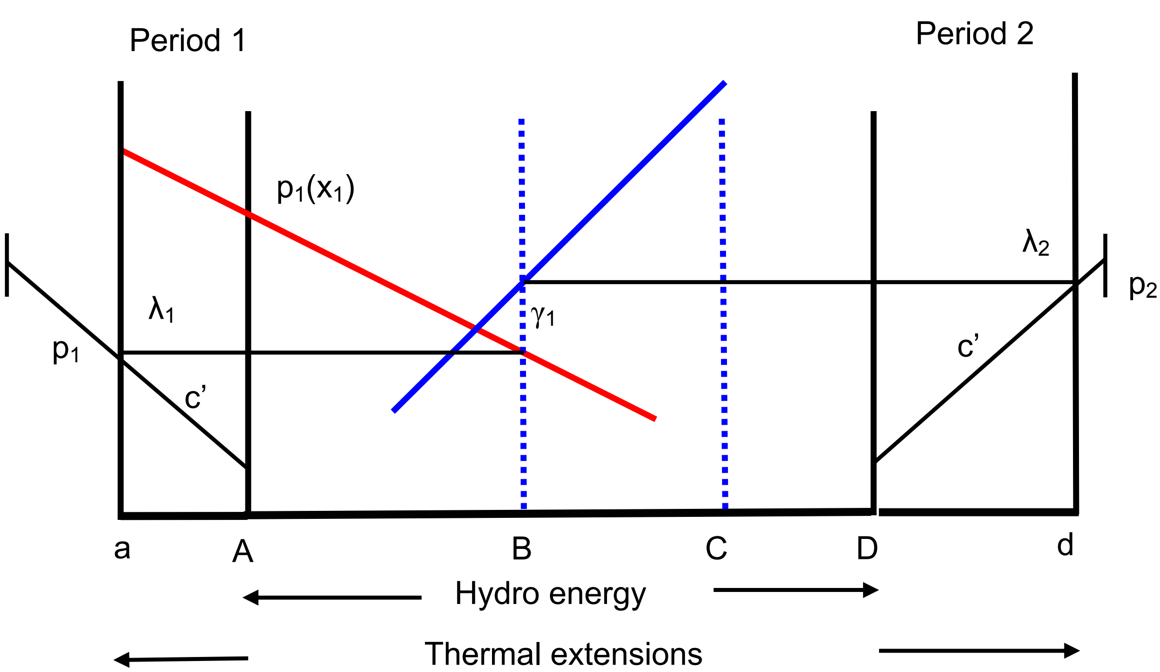

Figure 11.Thermal and hydro with reservoir constraint

thermal capacity, Dd, is taken into use in the second period. Other possible configurations of the optimal social solution follow the discussion in Section 3.

From (45) we have that the use of thermal capacity when it is positive is determined by equalisation of marginal costs and market price. The market price is equal to the water value for the period in question if hydro is used also. When the market price varies due to reservoir constraints being binding, then the use of thermal will vary and the peak-load role follows.

6. Market organisations

Perfect competition

In Section 3 we investigated the consequence for social planning of many hydropower producers, and found that the system could be treated as one aggregate unit (Hveding's conjecture). We now assume that we are studying one among several suppliers selling electricity in a spot market for every period. There is no uncertainty, so the period prices p_t are known. Given the capacity of each producer and the size of his reservoir he will in the situation of no (active) constraints on his reservoir obviously choose to deliver all his electricity in the period with the highest price in order to maximise profits. Therefore, in order to have positive total supply in all periods, prices must be equal over periods in market equilibrium. The allocation over periods is then completely demand driven, and since producers are indifferent about when to produce some additional rule has to be introduced.

In the case of a constraint on the reservoirs the profit maximisation problem of a producer (j) is:

$$\begin{aligned} & Max \sum_{t=1}^{T} p_t e_{jt}^H \\ & s.t. \\ & R_{jt} \le R_{j,t-1} + w_{jt} - e_{jt}^H, R_{jt} \le \overline{R}_j,\ t = 1,..,T \end{aligned} \qquad (46)$$

The Lagrangian for the problem is:

$$
\begin{aligned}
L &= \sum_{t=1}^{T} p_t e_{jt}^H \\
&-\sum_{t=1}^{T} \lambda_{jt}(R_{jt} - R_{j,t-1} - w_{jt} + e_{jt}^H) \\
&-\sum_{t=1}^{T} \gamma_{jt}(R_{jt} - \overline{R}_j)
\end{aligned} \tag{47}
$$

For notational ease we have used the same symbols for shadow prices as in the social planning case with a single producer. The shadow prices are plant specific. The necessary conditions are:

$$
\begin{aligned}
&\frac{\partial L}{\partial e_{jt}^H} = p_t - \lambda_{jt} \le 0 \perp e_{jt}^H \ge 0 \\
&\frac{\partial L}{\partial R_{jt}} = -\lambda_{jt} + \lambda_{j,t+1} - \gamma_{jt} \le 0 \perp R_{jt} \ge 0 \\
&\lambda_{jt} \ge 0 \ (= 0 \text{ for } R_{jt} < R_{j,t-1} + w_{jt} - e_{jt}^H) \\
&\gamma_{jt} \ge 0 \ (= 0 \text{ for } R_{jt} < \overline{R}_j),\ t = 1,..,T
\end{aligned} \tag{48}
$$

Let us assume that there is a positive market price in every period. The producer will not supply any electricity if the water value is higher than the market price. For the periods he will supply a positive amount the market price has to be equal to his water value. In general the producer will strive to sell all his energy at the period with the highest price, but he is prevented from doing this by the upper constraint on his reservoir. When overflow threatens his water value will be adjusted downwards for that period. He is willing to sell at a lower price now than a higher price in a later period to prevent overflow. But to the right price he may sell in an even earlier period and prevent an overflow situation happening.

Comparing the private conditions (48) with the social conditions (19) we have that if the prices faced by the producers are the same as in the social solution, and provided the planning horizon is the same for all plants and equal to the social planning horizon, then a competitive market will sustain the social solution. This is in accordance with the textbook welfare theorems in economics. But remember the pitfalls (external effects, etc.), and notice that we have not shown *how* such prices may be formed in private markets. The reasoning of a hydropower producer determining when to process his water will follow the discussion set out in Section 3.

Monopoly without binding reservoir constraints

We now turn to the case of all hydro producers being part of a monopoly and simplify further by considering the monopolist as a single production unit (i.e. the coordination problem expressed by Hveding's conjecture is solved by the monopolist). We assume that the monopolist faces the demand functions $p_t = p_t(e_t^H)$, $t = 1,..,T$. The optimisation problem of the monopolist in the basic case of a single water availability constraint is:

$$
\begin{aligned}
& Max \sum_{t=1}^{T} p_t(e_t^H) \cdot e_t^H \\
& s.t. \\
& \sum_{t=1}^{T} e_t^H \leq W
\end{aligned}
\tag{49}
$$

The Lagrangian is:

$$\sum_{t=1}^{T} p_t(e_t^H) \cdot e_t^H - \lambda(\sum_{t=1}^{T} e_t^H - W) \tag{50}$$

The necessary first order conditions are:

$$
\begin{aligned}
& \frac{\partial L}{\partial e_t^H} = p_t'(e_t^H)e_t^H + p_t(e_t^H) - \lambda \leq 0 \perp e_t^H \geq 0,\ t = 1,..,T \\
& \lambda \geq 0 \ (= 0 \text{ for } \sum_{t=1}^{T} e_t^H < W)
\end{aligned}
\tag{51}
$$

Assuming that the monopolist will produce electricity in all periods the conditions may be written:

$$p_t(e_t^H)(1+\breve{\eta}_t) = p_{t'}(e_{t'}^H)(1+\breve{\eta}_{t'}),\ \ t,t' = 1,..,T \tag{52}$$

In the expression for the marginal revenue we have introduced the *demand flexibility*, $\breve{\eta}_t = p_t' e_t^H / p_t$, which is negative (the inverse of the demand elasticity). The condition is that the marginal revenue should be equal for all the periods and equal to the shadow price on stored water. The absolute value of the demand flexibilities must be less than (or equal to) one.

An illustration in the case of two periods is provided in Figure 12. The broken lines are the marginal revenue curves. We see that in our case (the same demand curves as in Figure 1) the marginal revenue curves intersect for a positive value, i.e. it will not be optimal for the monopolist to spill any water. This value is the shadow value on water. But this result depends on the form of the demand functions. If we have spillage as an optimal solution, then the shadow water value is zero. We see that the water value in general is smaller than the shadow value for water in the social optimal case in Figure 1. Going up to the demand curves gives us the monopoly prices for the two periods. An important general result is that in the case of monopoly the market prices become different for the periods in contrast to the constant price in the social optimal solution. For the period with the most inelastic demand the price becomes larger than the social optimal price, and for the most elastic period the price becomes smaller. Thus we have a general *shifting* in the utilisation of water from periods with relative inelastic demand to periods with relative elastic demand.

Monopoly and reservoir constraints:

The profit maximisation problem is:

$$
\begin{aligned}
& Max \sum_{t=1}^{T} p_t(e_t^H) \cdot e_t^H \\
& s.t. \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad (53) \\
& R_t \le R_{t-1} + w_t - e_t^H, R_t \le \overline{R},\ t = 1,..,T
\end{aligned}
$$

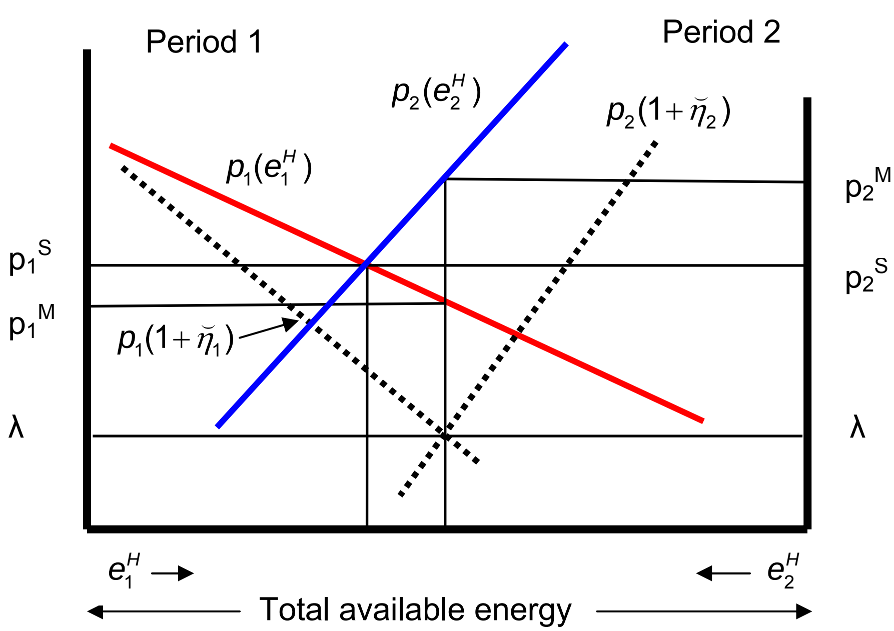

Figure 12. The basic monopoly case

The Lagrangian is:

$$L = \sum_{t=1}^{T} p_t(e_t^H)e_t^H - \sum_{t=1}^{T} \lambda_t (R_t - R_{t-1} - w_t + e_t^H) - \sum_{t=1}^{T} \gamma_t (R_t - \overline{R}) \tag{54}$$

The necessary first order conditions for $t = 1,..,T$ are:

$$\begin{aligned} &\frac{\partial L}{\partial e_t^H} = p_t'(e_t^H)e_t^H + p_t(e_t^H) - \lambda_t \leq 0 \perp e_t^H \geq 0 \\ &\frac{\partial L}{\partial R_t} = -\lambda_t + \lambda_{t+1} - \gamma_t \leq 0 \perp R_t \geq 0 \\ &\lambda_t \geq 0 \ (0 = \text{for } R_t < R_{t-1} + w_t - e_t^H) \\ &\gamma_t \geq 0 \ (0 = \text{for } R_t < \overline{R}) \end{aligned} \tag{55}$$

Assuming electricity is always supplied and introducing the demand flexibility $\breve{\eta}_t = p_t' e_t^H / p_t$:

$$\begin{aligned} &p_t'(e_t^H)e_t^H + p_t(e_t^H) - \lambda_t = p_t(e_t^H)(1 + \breve{\eta}_t) - \lambda_t = 0 \\ &-\lambda_t + \lambda_{t+1} - \gamma_t \leq \perp R_t \geq 0 \ \forall t \in T \end{aligned} \tag{56}$$

The marginal willingness to pay (the price) is substituted with the marginal revenue. The discussion of the use of water is parallel to the social optimum case. But will a monopolist choose the same time profile for the same inflows, demand functions, etc.?

Let us first assume that the monopolist will not find it profitable to spill any water. The constraint on the reservoir capacity will in general lead to the monopoly prices being closer to the prices in the social solution. If it is optimal for a monopolist to have the upper constraint on the reservoir binding in a period, then this means that he must charge the market price given by the intersection of the demand curve and the vertical reservoir constraint. If the same amount of water is available as in the social case then the monopoly price must be equal to the price in social optimum. The shadow value of water must adjust downwards for this to be possible. But when the monopolist follows the general strategy of using more water in elastic periods and having less water for the more inelastic periods there will be a tendency to reduce the number of periods with binding constraints. The point is that due to the lower shadow price on water maximal storing may become more seldom the optimal strategy for a monopolist.

Spilling water is profitable if marginal revenue becomes zero before water becomes scarce. Notice that the monopolist does not have to empty the reservoir to create value, just demand a positive price, it is only overflow that will break the attempt to increase price in all periods.

In the illustration in Figure 13 the reservoir constraint is not binding in the monopoly case, but was binding in the social optimal solution, as indicated by the horizontal price lines to the vertical reservoir constraint through B, and we have no spillage. We get the same type of solution as in Figure 2 without a binding reservoir constraint. But we note that the monopoly price in period 1 with the relatively most elastic demand becomes lower than the social optimal price with a binding reservoir constraint, and the monopoly price in period 2 with relatively inelastic demand becomes higher than in the social optimal case. This is the general effect of shifting of water from periods with relative inelastic demand to periods with relatively elastic demand in the case of market power.

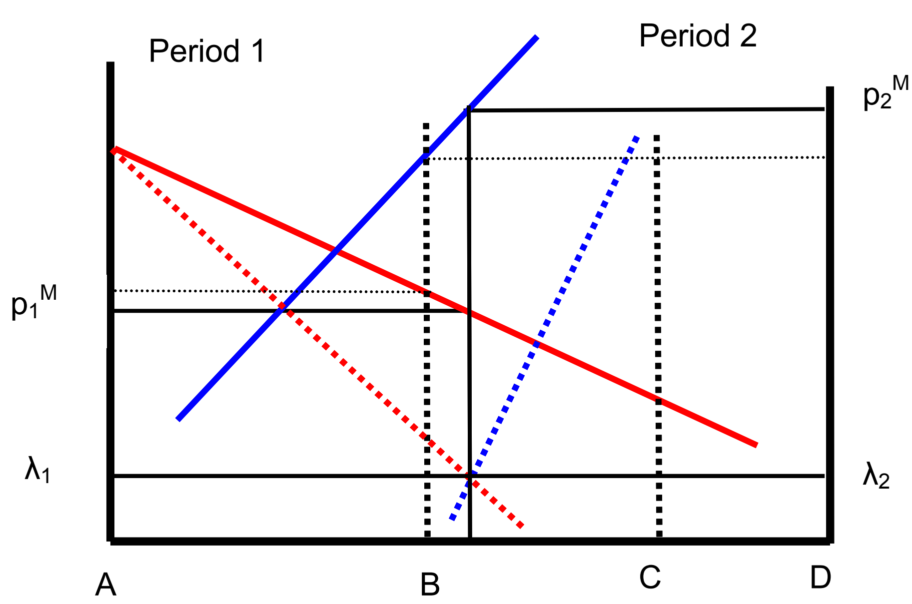

Figure 13. Monopoly with reservoir constraint

But a monopolist may also experience a binding reservoir constraint if the intersection of marginal revenue curves is to the left of the vertical through B representing the reservoir constraint. In this case if the monopolist tries to shift more water from inelastic periods to elastic periods he will not maximise profits. In a two-period case with the same availability of water in the first period with the binding reservoir constraint the monopolist cannot do better than adopt the social solution although the demand in period 1 is more elastic.

Monopoly with hydro and thermal plants

Let us assume that a monopolist has full control over both hydro and thermal capacity. The demand functions are $p_t(x_t)$. The optimisation problem is:

$$\begin{aligned} &Max \ \sum_{t=1}^{T}[(p_t(x_t)x_t - c(e_t^{Th})] \ s.t. \\ &x_t = e_t^H + e_t^{Th}, \ \sum_{t=1}^{T} e_t^H \leq W, \ e_t^{Th} \leq \overline{e}^{Th} \end{aligned} \tag{57}$$

Substituting for total energy the Lagrangian is

$$\begin{aligned} L = &\sum_{t=1}^{T}[p_t(e_t^H + e_t^{Th})(e_t^H + e_t^{Th}) - c(e_t^{Th})] \\ &-\sum_{t=1}^{T}\theta_t(e_t^{Th} - \overline{e}^{Th}) \\ &-\lambda(\sum_{t=1}^{T} e_t^H - W) \end{aligned} \tag{58}$$

The necessary conditions are:

$$\begin{aligned} &p_t'(e_t^H + e_t^{Th})(e_t^H + e_t^{Th}) + p_t(e_t^H + e_t^{Th}) - \lambda \leq 0 \perp e_t^H \geq 0 \\ &p_t'(e_t^H + e_t^{Th})(e_t^H + e_t^{Th}) + p_t(e_t^H + e_t^{Th}) - c'(e_t^{Th}) - \theta_t \leq 0 \perp e_t^{Th} \geq 0 \\ &\lambda \geq 0 \ (= 0 \text{ for } \sum_{t=1}^{T} e_t^H < W) \\ &\theta_t \geq 0 \ (= 0 \text{ for } e_t^{Th} < \overline{e}^{Th}) \end{aligned} \tag{59}$$

Concentrating on periods where both hydro and thermal are used the general result is that marginal revenue substitutes for the marginal willingness to pay in the social optimal solution:

$$p_t(x_t)(1 + \breve{\eta}_t) = \lambda = c'(e_t^{Th}) + \theta_t \tag{60}$$

The monopoly solution is illustrated in Figure 14. If the monopolist's water value is OB in a period both thermal and hydro capacity will be used according to the marginal revenue condition (59). The thermal capacity will be Oe^{Th} and the hydro capacity (Oe^{H} - Oe^{Th}).

For two periods we may again use the bathtub diagram to illustrate the allocation of the two types of power on the two periods. In Figure 15 the length of the hydro bathtub (bd) is extended at each end with the thermal capacity. Using the result (60), with the shadow price on the thermal capacity being zero, we have that the thermal extension of the bathtub is equal at each end; with (ab) in period 1 and (de) in period 2 and (ab) = (de). The equilibrium allocation is at point c, resulting in an allocation of (ab) thermal and (bc) hydro in period 1, and (cd) hydro and (de) thermal in period 2. Introducing a reservoir constraint as in Figure 11 will not change the solution for the case of an intersection of the marginal revenue curves within the area delimited with the lines from B and C in that figure showing the storage possibilities. A monopolist will equate the water value with the marginal cost of thermal, and not the market price. The use of thermal capacity may be reduced in all periods and will be base load unless a hydro reservoir constraint is binding. For such periods thermal capacity will also be used as peak.

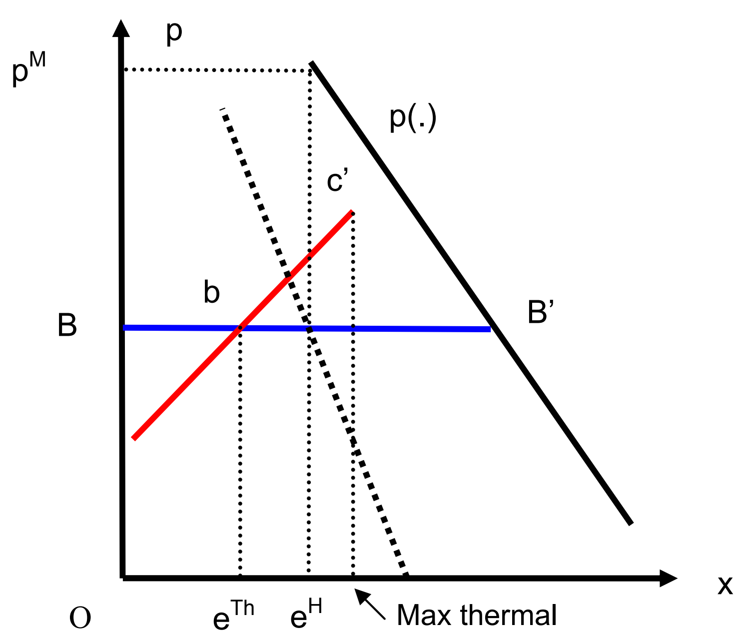

Figure 14. Monopoly. Hydro and thermal capacity

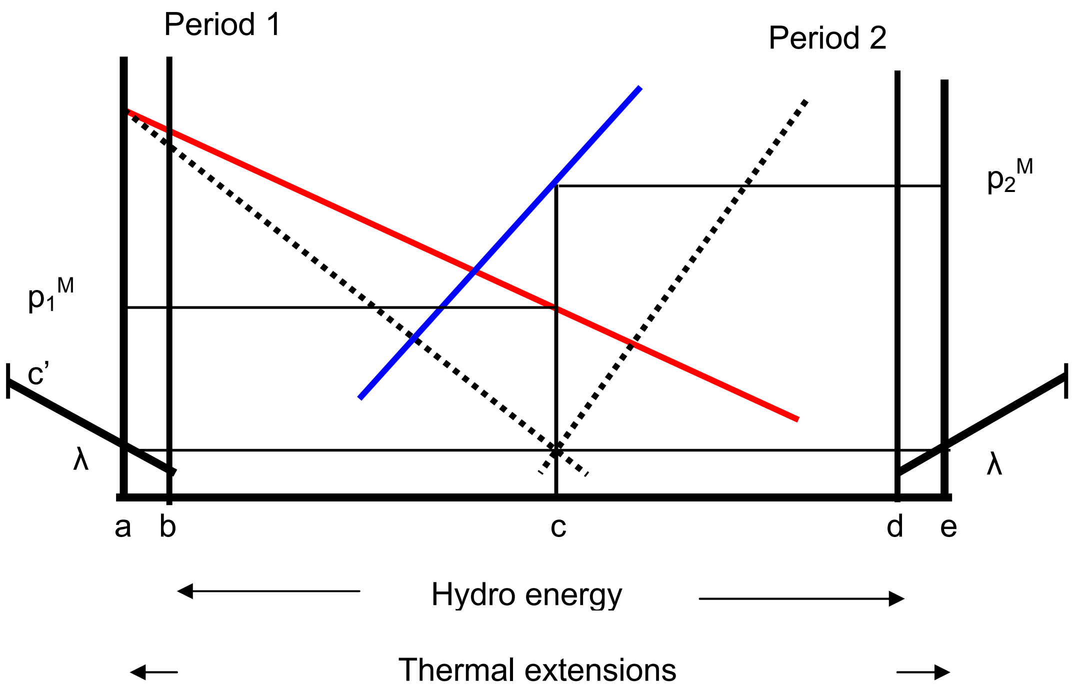

Figure 15. Two periods and monopoly, hydro and thermal

7. Further topics

There are, of course, many more interesting aspects of hydropower for economists than the topics covered above. We will give an indication of the nature of some of the aspects.

Transmission

Production and consumption of electricity takes place within a network. The economic choice of type and dimensions of a connector from production node to a consumption node is a classic within economics, and is an example of an engineering production function (see Førsund, 1999). Transmission lines have upper limits on how much electricity can be transferred, and losses are incurred as a function of loads. The flow in networks follows basic physical laws. Economists should note that there may be external effects in networks of significance for practical policy (Borenstein et al. (2000), Hogan, 1997). There is an interesting trade off between increase in generation capacity and increase in transmission capacity, especially when the latter concerns international connectors. (See Førsund (1994) for a general modelling of transmission within the framework used in this paper.)

Investments

Physical capacities of generation and transmission have been assumed given in the presentation. We have focussed on decisions of operation. A first tentative investment analysis can be done within that framework by inspecting the shadow prices on the capacity constraints (Førsund, 1994). However, this procedure is only valid for marginal investments. Large-scale investments must be evaluated by simulating on the total system within a long-term time horizon and then comparing values of the social objective function including investment costs.

Uncertainty

For a realistic modelling of hydropower there is no way to escape the introduction of stochastic variables. Inflows to the reservoirs are fundamentally stochastic variables. The household (general) demand for electricity in Norway is also dependent on outside temperature, especially due to the high share of electricity heating of dwellings. This relationship also makes part of demand stochastic. The objective functions used in the paper have to be reformulated to expected values. However, there are some technical difficulties of mathematical nature to get qualitative insights in the case of constraints, because the constraints have to be obeyed in a physical sense. It is not so interesting for policy analysis to require that a reservoir should not be emptied in an expected sense: emptying is an absolute event. The mathematical tool that can be used is stochastic dynamic programming (see Wallace et al., 2002).

Market power

The features of zero operating costs, extremely quick regulation of production and storage of huge amount of water compared with what is currently used make hydropower an especially interesting case for use of market power. Although the share of hydropower was small in California in the crisis experienced in 2000-2001, hydropower producers was blamed for use of market power (see Borenstein et al. (1999), Borenstein et al. (2002), Joskow and Kahn, 2002). This is also a "hot" topic in the Nordic countries (see Report from the Nordic competition authorities, 2003), especially after the experience in Norway in 2002-2003 (see Bye et al., 2003).

The role of transmission is also interesting for market power in the sector (see Borenstein et al. (2000), Bushnell (1999), Hogan (1997), Johnsen, 2001).

References

Borenstein, S., Bushnell, J. B. and Wolak, F. A. (2002): "Measuring market inefficiencies in California's restructured wholesale electricity market," *American Economic Review* 92(5), 1376-1405.

Borenstein, S., Bushnell, J. B. and Knittel, C. R. (1999): "Market power in electricity markets: beyond concentration measures," *Energy Journal* 20(4), 65-88.

Borenstein, S., Bushnell, J. B., and Stoft, S. (2000): "The competitive effects of transmission capacity in a deregulated electricity market," *Rand Journal of Economics* 31(2), 294-325.

Bushnell, J. B. (1999): "Transmission rights and market power," *Electricity Journal* 12(8), 77-85.

Bushnell, J. (2003): A mixed complementarity model of hydrothermal electricity competition in the Western United States, *Operations Research* 2003, 51(1) January-February.

Bye, T., Mørch von der Fehr, N-H., Riis, C. og Sørgard, L. (2003): "Kraft og makt. En analyse av konkurranseforholdene i kraftmarkedet", rapport fra Arbeids- og administrasjonsdepartementet, Oslo-Bergen, November 2003.

Crampes, C. and Moreaux, M. (2001): "Water resource and power generation," *International Journal of Industrial Organization* 19, 975-997.

Førsund, F. R. (1994): "Driftsoptimalisering i vannkraftsystemet", SNF - rapport 29/94, SNF-Oslo.

Førsund, F. R. (1999): "On the contribution of Ragnar Frisch to production theory," *Rivista Internazionale di Scienze Economiche e Commerciali (International Review of Economics and Business)* XLVI, 1-34.

Gjelsvik, A., Røtting, T. A. and Røynstrand, J. (1992): "Long-term scheduling of hydro-thermal power systems," in Broch and Lysne (eds): *Hydropower 92*, Rotterdam: Balkema, 539-546.

Haugstad, A., Reinertsen, T. Green, T. Johannesen, A. og Botnen, O. J. (1990): "Samkjøringsmodellen. Et verktøy for drifts- og utbyggingsplanlegging", EFI TR A3678.

Hogan, W. W. (1997): "A market power model with strategic interaction in electricity networks," *Energy Journal* 18(4), 107-141.

Hveding, V. (1967): "Driftssimulering av et kraftproduksjonssystem", *Elektroteknisk Tidsskrift* 80 (25), 517-525.

Hveding, V. (1968): "Digital simulation techniques in power system planning," *Economics of Planning* 8(2), 118-139.

Johnsen, T. A. (2001): "Hydropower generation and storage, transmission constraints and market power," *Utilities Policy* 10, 63-73.

Joskow, P. L. and Kahn, E. (2002): "A quantitative analysis of pricing behavior in California's wholesale electricity market during summer 2000," *Energy Journal* 23(4), 1-35.

Koopmans, T. C. (1957): "Water storage policy in a simplified hydroelectric system," *Proceedings of the First International Conference on Operational Research*, Oxford, September, 1-35.

Little, J. D. C. (1955): "The use of storage water in a hydroelectric system," *Journal of the Operations Research Society of America* 3, 187-197

Morlat, G. (1964): "On instructions for the optimum management of seasonal reservoirs," in Nelson, J. R. (ed.) : *Marginal cost pricing in practice*, Englewood Cliffs: Prentice-Hall Inc., 159-179.

Report from the Nordic competition authorities (2003): "A powerful competition policy," Report No. 1/2003

Scott, T. J. and Read, E. G. (1996): Modelling hydro reservoir operation in a deregulated electricity market," *International Transactions in Operational Research* 3(3/4) , 243-253.

Sydsæter, K., Strøm, A. and Berck, P. (1999): "Economists' mathematical manual", Berlin, Heidelberg, New York: Springer Verlag.

Sydsæter, K., Hammond, P., Seierstad, A. and Strøm, A. (2005): *Further mathematics for economic analysis*, Harlow: Pearson Education Limited.

von der Fehr, N.-H. M. and Sandsbråten, L. (1997): "Water on fire: gains from electricity trade," *Scandinavian Journal of Economics* 99, 281-297.

Wallace, S. W. and Fleten, S.-E. (2002): "Stochastic programming models in energy," in: A. Ruszczynski and A. Shapiro, eds., *Handbooks in Operations Research and Management Science* 10, North-Holland.