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# Welfare comparisons between societies with different population sizes and environmental characteristics

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# Welfare comparisons between societies with different population sizes and environmental characteristics

Geir B. Asheim*

July 18, 2005

**Abstract**

To provide a normative foundation for transfers between different societies, one needs information on the "per capita welfare" in different societies, having different population sizes and environmental characteristics. This paper reviews various methods for doing such comparisons. The main conclusion of the analysis is that there appears to be no practical alternative to applying real comprehensive per capita NNP. This is a per capita variant of Weitzman's stationary welfare equivalent of future utility. Welfare comparisons between different societies must be made in local real prices calculated according to "purchasing-power-parity", where non-traded environmental amenities may play an important role.

**Keywords and Phrases**: National accounting, Population, Dynamic welfare.
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# 1 Introduction

Transfers between different societies are sometimes motivated by redistribution: the worse-off societies should receive transfers from the better-off societies. Such redistribution may occur between regions of the same country, between countries of, e.g., the European Union, or between the rich and poor parts of the world. Also international negotiations on trade, debt relief, and climate control are concerned about the implicit transfers that various forms of agreements will result in.

In order to provide a normative foundation for transfers between different societies, one needs information on the "per capita welfare" in societies that differ in many respects, including having different population sizes and environmental characteristics. This paper reviews various methods for doing such comparisons.

Should we use

(a) real comprehensive per capita NNP,

(b) real per capita wealth, or

(c) an integral of the value of changes in per capita stocks?

Method (a) is a per capita variant of Weitzman's (1976) stationary welfare equivalent of future utility, method (b) seems to be a common proposal, while method (c) corresponds to what has been suggested in recent contributions (cf. Dasgupta and Mäler, 2000, Proposition 6). The present paper argues that method (a) appears to be the most practical alternative, judged from the sets of assumptions that are sufficient to obtain positive results.

After presenting the general model in Section 2, a variant of Weitzman's (1976) result is established in Section 3. The viability of methods (b) and (c) is considered in Sections 4 and 5. An example presented in Section 6 illustrates the analysis, showing that welfare comparisons between different societies must be made in local real prices calculated according to "purchasing-power-parity", where non-traded environmental amenities may play an important role, rather than in international prices calculated according to exchange rates.

## 2 Model

Consider a competitive world economy with different societies. I assume that, for any society, population is constant over time.[1] Use $N$ to denote population, and let $N^i$ represent the constant population of society $i$, where $i = a$, $b$, etc.

Denote by $\mathbf{C} = (C_1, \dots, C_m)$ the non-negative vector of goods that are consumed in a given society. To concentrate on the issue of distribution between different societies, I assume that goods and services consumed at any time are distributed equally among the population at that time. Thereby the instantaneous well-being for each individual may be associated with the *utility* $u(\mathbf{c})$ that is derived from the per capita vector of consumption flows, $\mathbf{c} := \mathbf{C}/N$.[2] Assume that $u$ is a time-invariant, increasing, concave, and differentiable function. That $u$ is time-invariant means that all variable determinants of current well-being are included in the vector of consumption flows. Labor supply is assumed to be constant and equal to population size.

Denote by $\mathbf{K} = (K_1, \dots, K_n)$ the non-negative vector of *capital* goods. This vector includes not only the usual kinds of man-made capital stocks, but also stocks of natural resources, environmental assets, human capital, and other durable productive assets, in the spirit of so-called "green" or comprehensive accounting. Corresponding to the stock of capital of type $j$, $K_j$, there is a net *investment* flow: $I_j := \dot{K}_j$. Hence, $\mathbf{I} = (I_1, \dots, I_n) = \dot{\mathbf{K}}$ denotes the vector of net investments.

The quadruple $(\mathbf{C}, \mathbf{I}, \mathbf{K}, N)$ is *attainable* in society $i$ at time $t$ if $(\mathbf{C}, \mathbf{I}, \mathbf{K}, N) \in \mathcal{C}^i(t)$, where $\mathcal{C}^i(t)$ is a convex and smooth set, with free disposal of consumption and net investment flows. In the setting of a competitive world economy, $\mathcal{C}^i(t)$ can also depend on the international prices that the society faces.

The set of attainable quadruples, $\mathcal{C}^i(t)$, is allowed to depend directly on time. This reflects that the technological level and terms-of-trade may change over time. To make accounting comprehensive, the value of the passage of time will be added to the value of consumption and investments, so that formally all variable determinants of current productive capacity are included.

[1] Under the assumption of constant-returns-to-scale (cf. Section 2), the case of exponential population growth can easily be accommodated, provided that only per capita consumption matters; see, e.g., Hamilton (2002). Contributions where population growth need not be exponential and where instantaneous well-being also depends on population size are emerging (cf., e.g., Arrow et al., 2003a; Asheim, 2004), but only for the purpose of doing local-in-time comparisons within the same society.

[2] This does not necessarily rule out (impure) collective goods. It is sufficient that per capita utility as a function of $\mathbf{C}$ and $N$, $\tilde{u}(\mathbf{C}, N)$, is homogenous of degree 0.

The set of attainable quadruples, $\mathcal{C}^i(t)$, is also allowed to depend on the society $i$. E.g., climate may influence the consumption and investment opportunities over and beyond the effect of the vector of capital stocks, $\mathbf{K}$, and time, $t$. If $\mathcal{C}^i(t)$ is a cone at each time $t$, then the technology exhibits constant-returns-to-scale. The assumption of constant-returns-to-scale will be imposed *only* in Sections 4 and 5.

Each society $i$, with constant population $N^i$, makes decisions according to a *resource allocation mechanism* that assigns to any vector of capital stocks $\mathbf{K}$ and time $t$, a consumption-investment pair $(\mathbf{C}(\mathbf{K}, t; i), \mathbf{I}(\mathbf{K}, t; i))$ satisfying that $(\mathbf{C}(\mathbf{K}, t; i), \mathbf{I}(\mathbf{K}, t; i), \mathbf{K}, N^i)$ is attainable at time $t$.[3] I assume that there exists a unique solution $\{\mathbf{K}^i(t)\}_{t=0}^{\infty}$ to the differential equations $\dot{\mathbf{K}}^i(t) = \mathbf{I}(\mathbf{K}^i(t), t; i)$ that satisfies the initial condition $\mathbf{K}^i(0) = \mathbf{K}^i_0$, where $\mathbf{K}^i_0$ is given. Hence, $\{\mathbf{K}^i(t)\}$ is the capital path that the resource allocation mechanism implements in society $i$. Write $\mathbf{C}^i(t) := \mathbf{C}(\mathbf{K}^i(t), t; i)$ and $\mathbf{I}^i(t) := \mathbf{I}(\mathbf{K}^i(t), t; i)$.

The program $\{\mathbf{C}^i(t), \mathbf{I}^i(t), \mathbf{K}^i(t)\}_{t=0}^{\infty}$ is *competitive* if, at each $t$,

1. $(\mathbf{C}^i(t), \mathbf{I}^i(t), \mathbf{K}^i(t), N^i)$ is attainable,

2. there exist present value prices of the flows of utility, consumption, labor input, and investment, $(\mu^i(t), \mathbf{p}^i(t), w^i(t), \mathbf{q}^i(t))$, with $\mu^i(t) > 0$ and $\mathbf{q}^i(t) \geq 0$, such that

   C1 $\mathbf{C}^i(t)$ maximizes $\mu^i(t) u(\mathbf{C}/N^i) - \mathbf{p}^i(t)\mathbf{C}/N^i$ over all $\mathbf{C}^i$,

   C2 $(\mathbf{C}^i(t), \mathbf{I}^i(t), \mathbf{K}^i(t), N^i)$ maximizes $\mathbf{p}^i(t)\mathbf{C} - w^i(t)N + \mathbf{q}^i(t)\mathbf{I} + \dot{\mathbf{q}}^i(t)\mathbf{K}$ over all $(\mathbf{C}, \mathbf{I}, \mathbf{K}, N) \in \mathcal{C}^i(t)$.

Here C1 corresponds to utility maximization, while C2 corresponds to intertemporal profit maximization. The present value price of utility at time $t$, $\mu^i(t)$, is a supporting utility discount factor.

Assume that the implemented program $\{\mathbf{C}^i(t), \mathbf{I}^i(t), \mathbf{K}^i(t)\}_{t=0}^{\infty}$ is competitive with finite utility and consumption values,

$$\int_0^{\infty} \mu^i(t) N^i u(\mathbf{C}^i(t)/N^i) dt \quad \text{and} \quad \int_0^{\infty} \mathbf{p}^i(t)\mathbf{C}^i(t) dt \text{ exist},$$

and that it satisfies a capital value transversality condition,

$$\lim_{t\to\infty} \mathbf{q}^i(t)\mathbf{K}^i(t) = 0\,. \tag{1}$$

[3] This is inspired by Dasgupta and Mäler (2000), Dasgupta (2001), and Arrow et al. (2003b).

It follows that the implemented program $\{\mathbf{C}^i(t), \mathbf{I}^i(t), \mathbf{K}^i(t)\}_{t=0}^{\infty}$ maximizes

$$\int_0^\infty \mu^i(t) u(\mathbf{C}/N^i) dt$$

over all programs that are attainable at all times and satisfies the initial condition. Moreover, writing $\mathbf{c}^i(t) := \mathbf{C}^i(t)/N^i$, it follows from C1 and C2 that

$$\mathbf{p}^i(t) = \mu^i(t) \nabla u(\mathbf{c}^i(t)) \,, \tag{2}$$

$$w^i(t) = \mathbf{p}^i(t) \frac{\partial \mathbf{C}(\mathbf{K}^i(t), N^i, t; a)}{\partial N} + \mathbf{q}^i(t) \frac{\partial \mathbf{I}(\mathbf{K}^i(t), N^i, t; a)}{\partial N} \,, \tag{3}$$

$$-\dot{\mathbf{q}}^i(t) = \mathbf{p}^i(t) \nabla_{\mathbf{K}} \mathbf{C}(\mathbf{K}^i(t), N^i, t; a) + \mathbf{q}^i(t) \nabla_{\mathbf{K}} \mathbf{I}(\mathbf{K}^i(t), N^i, t; a) \,, \tag{4}$$

where $\nabla$ denotes a vector of partial derivatives.

Denote by $\psi^i(t)$ the value of the passage of time measured in present value terms. Since $\psi^i(t)$ is measured in present value terms, the decrease of the value of the passage of time, $-\dot{\psi}^i(t)$, equals the marginal productivity of the passage of time:

$$-\dot{\psi}^i(t) = \mathbf{p}^i(t) \frac{\partial \mathbf{C}(\mathbf{K}^i(t), N^i, t; a)}{\partial t} + \mathbf{q}^i(t) \frac{\partial \mathbf{I}(\mathbf{K}^i(t), N^i, t; a)}{\partial t} \,. \tag{5}$$

For doing the welfare comparisons between societies, it will turn out to be a helpful intermediate result to value

$$\int_t^\infty \mu^i(s) \dot{u}^i(s) ds \,, \tag{6}$$

where, for all $s$, $u^i(s) = u(\mathbf{c}^i(s))$. By combining (2), (4), and (5), one obtains

$$\begin{aligned} \mu^i N^i \dot{u}^i &= \mu^i N^i \nabla u \cdot d(\mathbf{C}^i/N^i)/dt = \mu^i \nabla u \cdot \left( \nabla_{\mathbf{K}} \mathbf{C} \cdot \mathbf{I}^i + \tfrac{\partial \mathbf{C}}{\partial t} \right) \\ &= -\left( \dot{\mathbf{q}}^i \mathbf{I}^i + \mathbf{q}^i \dot{\mathbf{I}}^i + \dot{\psi}^i \right) = -\tfrac{d}{dt} \left( \mathbf{q}^i \mathbf{I}^i + \psi^i \right) . \end{aligned} \tag{7}$$

Assuming that

$$\lim_{t \to \infty} \left( \mathbf{q}^i(t) \mathbf{I}^i(t) + \psi^i(t) \right) = 0$$

holds as an investment value value transversality condition, one arrives at the following result by integrating (7).

**Lemma 1** *Under the assumptions of the present section*

$$\int_t^\infty \mu^i(s) \dot{u}^i(s) ds = (\mathbf{q}^i(t) \mathbf{I}^i(t) + \psi^i(t))/N^i \,.$$

## 3 Stationary welfare equivalent

Assume that, at time $t$, society $i$'s per capita *dynamic welfare* is given by discounted utilitarianism, in the sense that society ranks programs according to the sum of per capita utilities discounted at a constant rate $\rho$. Hence, the dynamic welfare of the implemented program at time $t$ is

$$\int_t^\infty e^{-\rho(s-t)} u(\mathbf{c}^i(s))ds\,.$$

Moreover, assume that the resource allocation mechanism implements an optimal program. Hence, $\{\mu^i(t)\}_{t=0}^\infty = \{\mu^i(0)\cdot e^{-\rho t}\}_{t=0}^\infty$, where $1/\mu^i(0)$ is the marginal utility of expenditures at time 0 measured in present value prices.

Under discounted utilitarianism, the change in dynamic welfare is given by

$$\frac{d}{dt}\Big(\int_t^\infty e^{-\rho(s-t)} u^i(s)ds\Big) = -u^i(t) + \rho\int_t^\infty e^{-\rho(s-t)} u^i(s)ds = e^{\rho t}\int_t^\infty e^{-\rho s}\dot{u}^i(s)ds\,,$$

where the second equality follows by integrating by parts. By applying Lemma 1, the following result is obtained:

$$\rho\int_t^\infty e^{-\rho(s-t)} u^i(s)ds = u^i(t) + \frac{e^{\rho t}}{\mu^i(0)}\big(\mathbf{q}^i(t)\mathbf{I}^i(t) + \psi^i(t)\big)/N^i\,. \tag{8}$$

This is Weitzman's (1976) seminal result in the current setting, showing that utility-NNP (the r.h.s. of eq. (8)) is a stationary per capita welfare equivalent of future utility.

To be able to express the stationary welfare equivalent of future utility in terms of current prices and quantities, a stronger assumption must be invoked. If the utility function $u$ is homothetic, then a Divisia *consumer price index* is path independent, so that real prices can be determined globally.[4] Moreover, if $u$ is assumed to be linearly homogeneous, then these real prices are measured in a numeraire that is in a fixed proportion to utils. W.l.o.g. we may set the factor of proportionality equal to one,

$$\mathbf{P}^i(t) = \nabla u(\mathbf{c}^i(t)) = \frac{\mathbf{p}^i(t)}{\mu^i(t)} = \mathbf{p}^i(t)\frac{e^{\rho t}}{\mu^i(0)}$$

$$\mathbf{Q}^i(t) = \frac{\mathbf{q}^i(t)}{\mu^i(t)} = \mathbf{q}^i(t)\frac{e^{\rho t}}{\mu^i(0)}$$

$$\Psi^i(t) = \frac{\psi^i(t)}{\mu^i(t)} = \psi^i(t)\frac{e^{\rho t}}{\mu^i(0)}\,,$$

[4] Cf. Hulten (1987) for a discussion of the properties of Divisia indices.

so that it follows from the linear homogeneity of $u$ that

$$u^i(t) = u(\mathbf{c}^i(t)) = \nabla u(\mathbf{c}^i(t)) \cdot \mathbf{c}^i(t) = \mathbf{P}^i(t)\mathbf{c}^i(t)\,. \tag{9}$$

Thus, by combining the assumption of a linearly homogeneous utility function with (8), the following variant of Weitzman's (1976) result is established.

**Proposition 1** *Under discounted utilitarianism and a linearly homogeneous utility function, real comprehensive per capita NNP in money terms,*

$$\big(\mathbf{P}^i(t)\mathbf{C}^i(t) + \mathbf{Q}^i(t)\mathbf{I}^i(t) + \Psi^i(t)\big)/N^i\,,$$

*is the stationary per capita welfare equivalent of future utility.*

Consider now the case with two societies, $a$ and $b$. Societies $a$ and $b$ may, at any time $t$, have a different set of attainable quadruples. Hence, $\mathcal{C}^a(t)$ may differ from $\mathcal{C}^b(t)$, due, e.g., to different climatic conditions. However, both societies follow a competitive program that maximizes dynamic welfare. Moreover, assume that the utility function is identical in societies $a$ and $b$, and that both societies adhere to discounted utilitarianism with the same discount rate $\rho$. It now follows from Proposition 1 that per capita welfare is higher in society $a$ than in society $b$ if and only if

$$\big(\mathbf{P}^a(t)\mathbf{C}^a(t) + \mathbf{Q}^a(t)\mathbf{I}^a(t) + \Psi^a(t)\big)/N^a > \big(\mathbf{P}^b(t)\mathbf{C}^b(t) + \mathbf{Q}^b(t)\mathbf{I}^b(t) + \Psi^b(t)\big)/N^b\,.$$

The assumption of a linearly homogeneous utility function is a very strong assumption, which, however, cannot be relaxed for the alternative measures for across space welfare comparison discussed in the two next sections. As I indicate in Asheim (2003, Section 6), Weitzman (2001) provides methods for satisfying the informational demands that this assumption involves (see also Li and Löfgren, 2002).

# 4 Real per capita wealth

Add now the assumption that society $i$'s set of attainable quadruples, $\mathcal{C}^i(t)$, is a cone, so that the technology exhibits constant-returns-to-scale. Then it follows directly from C2 that, at each t,

$$\mathbf{p}^i(t)\mathbf{C}^i(t) - w^i(t)N^i + \mathbf{q}^i(t)\mathbf{I}^i(t) + \dot{\mathbf{q}}^i(t)\mathbf{K}^i(t) = 0\,,$$

or, equivalently,

$$-\frac{d\big(\mathbf{q}^i(t)\mathbf{K}^i(t)\big)}{dt} = \mathbf{p}^i(t)\mathbf{C}^i(t) - w^i(t)N^i\,.$$

This means that the present value of future consumption equals the value of capital plus the present value of future wages,

$$\int_t^\infty \mathbf{p}^i(s)\mathbf{C}^i(s)ds = \mathbf{q}^i(t)\mathbf{K}^i(t) + \int_t^\infty w^i(t)N^i ds\,, \tag{10}$$

provided that the appropriate transversality condition holds. Assume now that $u$ is linearly homogeneous, and let

$$Q_N^i(t) = \frac{e^{\rho t}}{\mu^i(0)} \cdot \int_t^\infty w^i(s)ds$$

denote the real capitalized per capita value of labor. Then it follows from (9) that (10) can be rewritten as

$$\int_t^\infty e^{-\rho(s-t)}u^i(s)ds = \mathbf{Q}^i(t)\mathbf{K}^i(t)/N^i + Q_N^i(t)\,.$$

Thus, the following result has been established.

**Proposition 2** *Under discounted utilitarianism, a linearly homogeneous utility function, and constant-returns-to-scale, real per capita wealth including the real capitalized per capita value of labor,*

$$\mathbf{Q}^i(t)\mathbf{K}^i(t)/N^i + Q_N^i(t)\,,$$

*equals per capita dynamic welfare.*

The assumption of constant-returns-to-scale imposes strong informational demands in the sense that it entails that, not only variable determinants, but also fixed determinants of current productive capacity are included. It means that all flows of future earnings can be treated as currently existing capital. For this reason, no term involving the effects of technological progress need be explicitly taken into account. However, in the spirit of Lindahl (1933, pp. 401–402) and (Samuelson, 1961, p. 53), the result requires that the real capitalized per capita value of labor is included.

Note that, when comparing the welfare of two economies by means of Proposition 2, per capita wealth must be made comparable through the application of a Divisia consumer price index. This will be illustrated in the example of Section 6.

## 5 Value of changes in per capita stocks

Assume, in addition to the earlier assumptions, that society $i$'s set of attainable quadruples at time $t$, $\mathcal{C}^i(t)$, does not depend on the characteristics of this particular society, and hence, can be written without superscript $i$: $\mathcal{C}(t)$. This means that, e.g., the effects of a geographically differentiated climate on the consumption and investment opportunities are captured by the vector of capital stocks, $\mathbf{K}$.

By combining this with the assumption of a linearly homogeneous utility function — so that $Nu(\mathbf{C}/N)$ does not depend on $N$, but only on $\mathbf{C}$ — the sum of discounted total utilities at time $t$ can be written as society-independent function $V$ of the vector of capital stocks $\mathbf{K}^i(t)$ and time $t$:

$$V(\mathbf{K}^i(t), t) = \int_t^\infty e^{-\rho(s-t)} N^i u(\mathbf{C}^i(s)/N^i) ds \,. \tag{11}$$

Assuming that $V$ is differentiable, it follows, since $\mathbf{Q}^i(t)$ are investment prices in terms of utility, that

$$\nabla_{\mathbf{K}} V(\mathbf{K}^i(t), t) = \mathbf{Q}^i(t) \,. \tag{12}$$

Finally, by invoking the assumption that the technology exhibits constant-returns-to-scale, it follows from (11) that

$$V(\mathbf{k}^i(t), t) = \int_t^\infty e^{-\rho(s-t)} u(\mathbf{c}^i(s)) ds \,, \tag{13}$$

where $\mathbf{k} := \mathbf{K}/N$ denotes the per capita vector of capital stocks.

Consider now again the case with two societies, $a$ and $b$. By (12) and (13), the difference in dynamic welfare at time $t$ between these societies can be written as:

$$V(\mathbf{k}^b(t), t) - V(\mathbf{k}^a(t), t) = \int_{\mathbf{k}^a(t)}^{\mathbf{k}^b(t)} \nabla_{\mathbf{K}} V(\mathbf{k}^i(t), t) d\mathbf{k}^i(t) = \int_{\mathbf{k}^a(t)}^{\mathbf{k}^b(t)} \mathbf{Q}^i(t) d\mathbf{k}^i(t) \,,$$

where the integral is independent of the path, $\{\mathbf{k}^i(t)\}$, between $\mathbf{k}^a(t)$ and $\mathbf{k}^b(t)$. However, the path of investment prices in utility terms, $\{\mathbf{Q}^i(t)\}$, must be calculated in utility terms along the path of imaginary intermediate societies determined by $\{\mathbf{k}^i(t)\}$.

The following result has been shown.

**Proposition 3** *Under discounted utilitarianism, a linearly homogeneous utility function, constant-returns-to-scale, and society-independent technology, society b's per capita dynamic welfare exceeds that of society a if and only if* $\int_{\mathbf{k}^a(t)}^{\mathbf{k}^b(t)} \mathbf{Q}^i(t) d\mathbf{k}^i(t) > 0$.

Hence, across space welfare comparisons can be made by means of an integral of the value of changes in per capita stocks. However, to establish this result, I have, in addition to a linearly homogeneous utility function and constant-returns-to-scale, also invoked the assumptions that

- the set of attainable quadruples in the two societies in question, $\mathcal{C}^a(t)$ and $\mathcal{C}^b(t)$, coincide at each point in time, and
- investment prices in utility terms, $\{\mathbf{Q}^i(t)\}$, can be calculated in utility terms along a path of imaginary societies that lie between societies $a$ and $b$.

These assumptions appear to be very strong, and they are added to those used to establish Propositions 1 and 2.

## 6 An example

To illustrate how Proposition 1 can be used to compare the per capita welfare in two societies, it is useful to consider the following example. The example is intended to highlight the following observations:

1. The real prices in each society depend on the domestic consumption vector.
2. Non-traded environmental amenities are not only important to make NNP comprehensive, but also to calculate the real prices in each economy.
3. Alternatives to Proposition 1, involving comparison of real per capita wealth (cf. Proposition 2) or the integral of the real value per capita stock changes (cf. Proposition 3), are difficult to apply for the purpose of comparing the per capita welfare of different societies.

Consider a competitive world economy consisting of two societies, $a$ and $b$, where $N^a = 1$ and $N^b = 2$ are the sizes of populations that cannot migrate between the societies. The vector of capital goods consists of a stock of a reproducible capital good that can be used in either society independent of ownership, where $K^a = 0$ and $K^b = 3$ (i.e., all capital-owners live in society $b$), and stocks of an immobile environmental amenity good that can be thought of as space, where $E^a = 1$ and $E^b = 1$. Hence, there is less space per capita in society $b$, but to compensate for this society $b$ has the ownership to the whole stock of reproducible capital. Assume that there is zero net investment in reproducible capital, and note that the available stocks of the environmental amenity good cannot change over time.

Production of a freely traded material consumption good is governed by a constant-returns-to-scale production function,

$$K^{0.4}L^{0.6},$$

leading to a total production of 3, where the production in society $a$ equals 1 and production in society $b$ equals 2, but where the remuneration to the factor owners implies that 0.6 is allocated to society $a$ and 2.4 to society $b$. The investment in reproducible capital is zero in both societies, entailing that material consumption is given by $C^a = 0.6$ and $C^b = 2.4$ and per capita consumption by $c^a = 0.6$ and $c^b = 1.2$.

Environmental amenities constitute the other consumption good and equal per capita space: $e^a = 1$ and $e^b = 0.5$. In each society, the linearly homogeneous utility function is given by[5]

$$u(c, e) = \tfrac{20}{3} \cdot c^{0.5} \cdot e^{0.5} \,,$$

leading to the following real prices in the two societies (using a Divisia consumer price index, which is path independent due to the linearly homogeneous utility function),

$$P_c^a = 2 \qquad\qquad P_c^b = 1$$
$$P_e^a = 1.2 \qquad\qquad P_e^b = 2.4 \,,$$

entailing that material consumption and environmental amenities yield the same utility in both societies,

$$P_c^a c^a + P_e^a e^a = 2.4 \qquad\qquad P_c^b c^b + P_e^b e^b = 2.4 \,,$$

something that can be checked directly from the utility function. Since there is zero investment in both societies, this is the real comprehensive per capita NNP in money terms, which by Proposition 1 is the stationary per capita welfare equivalent of future utility. Hence, per capita welfare is the same in both societies.

To complete the derivation of real prices, it follows from the value of marginal products that real wages are given by

$$W^a = 1.2 \qquad\qquad W^b = 0.6 \,,$$

and that the real interest rate is given by

$$R = 0.4$$

in both societies. The latter result—combined with the observation that investment in reproducible capital is zero—means that, in either society, the dynamic discounted

[5]The linearly homogenous utility function can been found by integrating from a demand system that is consistent with observed quantities and prices, noting that the expenditure share for each good in either country is 0.5.

utilitarian welfare of the implemented program at time $t$ is

$$\int_t^\infty e^{-0.4(s-t)} u(c,e) ds \, .$$

It is important to note that, although material consumption, $c$, is a freely traded good, the real price of material consumption for the purpose of comparative welfare analysis differs in the two societies. The comparative welfare analysis is *not* made in international prices calculated according to exchange rates. Rather, *the comparative welfare analysis is made in local real prices calculated according to "purchasing-power-parity", on the basis of the consumption goods $c$ and $e$, and where not only material consumption $c$ but also non-traded environmental amenities $e$ play an important role.* It can be seen that, in international prices calculated according to exchange rates, the value of consumption in society $b$ is twice as big as the value of consumption in society $a$; this does *not* reflect the fact that $c$ and $e$ yield the same utility in both societies.

Since the production function exhibits constant-returns-to-scale, per capita wealth can also be used for welfare comparisons. However, to be able to invoke Proposition 2 for such a comparison, per capita wealth in each society must be calculated in local real prices and includes the present value of future wages. The capitalized real per unit value of reproducible capital, environmental amenities, and labor is given by

$$\begin{aligned} Q_K^a &= 2 & \qquad Q_K^b &= 1 \\ Q_E^a &= 3 & \qquad Q_E^b &= 6 \\ Q_N^a &= 3 & \qquad Q_N^b &= 1.5 \, , \end{aligned}$$

entailing that real per capita wealth, including the present value of future wages are the same in both societies,

$$\big(Q_K^a K^a + Q_E^a E^a\big)/N^a + Q_N^a = 6 \qquad \big(Q_K^b K^b + Q_E^b E^b\big)/N^b + Q_N^b = 6 \, .$$

Note, however, that a comparison of per capita wealth excluding the present value of future wages and measured in international prices calculated according to exchange rates gives a very different result. Such a comparison would produce the result that per capita wealth in society $b$ is three times the per capita wealth in society $a$, a result that lacks welfare significance. This shows the importance of the Divisia consumer price index when calculating real prices for the purpose of per capita wealth comparison.

Since both societies have the same set of attainable quadruples, we may also consider integrating the value of per capita stock changes when going from society $a$ to society

$b$ (cf. Proposition 3). The per capita stocks in the two societies are given by

$$K^a/N^a = 0 \qquad K^b/N^b = 1.5$$
$$E^a/N^a = 1 \qquad E^b/N^b = 0.5 \ ,$$

However, for such integration to have welfare significance, the relative price of the environmental amenity stock in terms of reproducible capital, $Q_E/Q_K$, must increase from $Q_E^a/Q_K^a = 1.5$ in society $a$ to $Q_E^b/Q_K^b = 6$ in society $b$, in a manner that depends on the real stock prices in the imaginary intermediate societies that the integration passes through. This indicates that such a method is of little practical usefulness.

# 7 Conclusion

This paper has shown that welfare comparisons between societies with different population sizes and environmental characteristics can be made according to real comprehensive per capita NNP, provided that the societies maximize discounted utilitarian welfare, and the utility function is linearly homogeneous.

Comparisons based on per capita wealth require in addition that the technologies exhibit constant-returns-to-scale. This assumption imposes strong informational demands in the sense that, not only variable determinants, but also fixed determinants of current productive capacity must be included.

Comparisons based on the value of changes in per capita stocks require even stronger assumptions: society-independent technology and an ability to determine real stock prices in the imaginary societies that lie between the societies that are compared.

The main conclusion of the analysis is therefore that there appears to be no practical alternative to applying real comprehensive per capita NNP. This is a per capita variant of Weitzman's (1976) stationary welfare equivalent of future utility.

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