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Can NNP be used for welfare comparisons?

Geir B. Asheim*

September 29, 2005

Abstract

This paper contains a critical assessment of the claim that NNP can be used for welfare comparisons. The analysis assumes that national accounts are comprehensive (in particular, "greened" by taking into account environmental amenities and natural resource depletion), but does not assume optimal resource allocation. The general conclusion is that greater NNP does *not* correspond to welfare enhancement, unless the net investment flows are revalued. Real utility-NNP, and real measurable NNP made comparable across time by means of a consumer price index, allow for such revaluation, and thus indicate welfare improvement.

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Summary

This paper contains a critical assessment of the claim that comprehensive net national product (NNP) can be used for welfare comparisons. The assertion that NNP is endowed with welfare significance has been subject to controversy, from the seminal contributions by Samuelson and Weitzman to a number of more recent articles. Here I contribute to this debate in the following two ways:

1. I give an interpretation of the basic insights and results of welfare accounting in a general setting.
2. Building on these insights I warn against using NNP for measuring the welfare effects of capital perturbations, and derive the result that real NNP growth in variable consumption and net investment prices can be used to indicate welfare improvement.

The general conclusion is that greater NNP does *not* correspond to welfare enhancement, unless the net investment flows are revalued. Real utility-NNP, and real measurable NNP made comparable across time by means of a consumer price index, allow for such revaluation, and thus indicate welfare improvement. I reconcile my results with the findings presented in the relevant literature. I use the Dasgupta-Heal-Solow model to illustrate the analysis and results.

I invoke weak assumptions concerning how dynamic welfare is derived—by not necessarily assuming discounted utilitarianism—and how the economy functions—by not necessarily assuming an optimal resource allocation mechanism. Throughout I am concerned with local comparisons, either "small" perturbations, or local-in-time comparisons. I also assume that national accounts are comprehensive by including the effects of environmental amenities and natural resource depletion as well as technological progress.

Word count for main body of text, excluding abstract and summary, but including references: Approximately 7000 words.

1 Introduction

Can net national product (NNP) be used for welfare comparisons if national accounts are made comprehensive by including the effects of environmental amenities and natural resource depletion as well as technological progress?

In a perfectly competitive economy with comprehensive national accounting, NNP represents the maximized value of the flow of goods and services that are produced by the productive assets of an economy. If NNP increases, then the economy's capacity to produce has increased, and—one might think—the economy is better off. Although such an interpretation is often made in public debate, the assertion has been subject to controversy in the economic literature. While Samuelson (1961, p. 51) writes that "[o]ur rigorous search for a meaningful welfare concept has led to a rejection of all *current* income concepts ...", Weitzman (1976), in his seminal contribution, shows that greater NNP indicates higher welfare if

(a) dynamic welfare equals the sum of utilities discounted at a constant rate (i.e., discounted utilitarianism), and

(b) current utility equals the market value of the consumed goods and services (i.e., a linearly homogeneous utility function).

Weitzman's result is remarkable—as it means that changes in the stock of forward looking welfare can be picked up by changes in the flow of the value of current net product—but, unfortunately, strong assumptions are invoked. More recently, Asheim and Weitzman (2001) have established that assumption (b) can be relaxed when concerned with whether welfare is increasing locally in time: real NNP growth corresponds to welfare improvement even when current utility does *not* equal the market value of current consumption, as long as NNP is deflated by a consumer price index. Moreover, Asheim and Buchholz (2004) have shown that there are conditions under which even assumption (a) need not be invoked.

These findings are not, however, uncontroversial. Dasgupta and Mäler (2000) and Dasgupta (2001) warn against using NNP for welfare comparisons, while Weitzman (2001) and Li and Löfgren (2004) point out that there are other ways to deflate NNP (or argue that no NNP deflator is needed at all). Moreover, Heal and Kriström (2005) present a critical assessment of the usefulness of NNP for making welfare comparisons. Here I contribute to this debate in the following two ways:

1. In Section 2, I give an interpretation of the basic insights and results of welfare accounting, as developed by Samuelson (1961), Weitzman (1970, 1976), and

Dixit et al. (1980). In particular, Samuelson (1961) argues that welfare changes should be measured by the present value of future changes in consumption, an insight that Heal and Kriström have brought to our attention through various contributions during the last years (see, e.g., Heal and Kriström, 2005). Morover, Weitzman (1970, 1976) shows that there are conditions under which welfare changes can be measured by changes in utility-NNP, and through this, establishes the link between welfare improvement and a positive value of net investments (cf. Weitzman, 1976, equation above (14)). Finally, Dixit et al. (1980) demonstrate the relationship between a positive value of net investments, on the one hand, and a positive present value of future consumption growth, on the other.

2. In Sections 3 and 4, I build on these insights (a) to warn against using NNP for measuring the welfare effects of capital perturbations, and (b) to derive the result that real NNP growth in variable consumption and net investment prices can be used to indicate welfare improvement, as reported in Asheim and Weitzman (2001) and Asheim and Buchholz (2004). *I summarize the results in Section 5, where the following overall conclusion is stated:* NNP *can be used for welfare comparisons only if net investment flows are revalued.* There I argue for the relative merits of using a consumer price index as an NNP deflator, when compared to the alternative of measuring real NNP changes in fixed consumption and net investment prices. I reconcile my findings with Li and Löfgren's (2004) demonstration that welfare improvement can be related to real NNP growth, measured in fixed consumption and net investment prices. Throughout I use the Dasgupta-Heal-Solow model (Dasgupta and Heal, 1974, 1979; Solow, 1974) to illustrate the analysis and results.

In my analysis, I invoke weak assumptions concerning how dynamic welfare is derived—by not necessarily assuming discounted utilitarianism—and how the economy functions—by not necessarily assuming an optimal resource allocation mechanism. Instead, I assume throughout "sufficient differentiability" to derive my results.

Throughout I am concerned with local comparisons, either "small" perturbations, or local-in-time comparisons. I also assume that national accounts are comprehensive (i.e., they are "greened"). Global comparisons and non-comprehensive national accounting give raise to other issues that will not be addressed here.

2 Theory of welfare comparisons in a dynamic economy

Consider a dynamic economy where the instantaneous well-being of the economy is measured by a one-dimensional indicator U, which will be referred to as *utility*. If *dynamic welfare* is welfarist, forward-looking and numerically representable, then dynamic welfare, denoted V, is a function of the flow of future utilities:

$$V^*(t) = \mathcal{F}(\{U^*(s)\}_{s=t}^{\infty}) \,.$$

I assume throughout that the functional $\mathcal{F}$ is concave, time-invariant and smooth, and satisfies a condition of *independent future*: If $\{U'(t)\}_{t=0}^{\infty}$ and $\{U''(t)\}_{t=0}^{\infty}$ coincides during the interval $[0, \tau]$, then

$$\mathcal{F}(\{U'(t)\}_{t=0}^{\infty}) < \mathcal{F}(\{U''(t)\}_{t=0}^{\infty}) \quad \text{if and only if} \quad \mathcal{F}(\{U'(t)\}_{t=\tau}^{\infty}) < \mathcal{F}(\{U''(t)\}_{t=\tau}^{\infty}) \,.$$

Since $\mathcal{F}$ is smooth and satisfies independent future, there exists, for any path of utility flow $\{U^*(t)\}_{t=0}^{\infty}$, a path of supporting utility discount factors $\{\mu(t)\}_{t=0}^{\infty}$, unique up to a choice of numeraire, such that, for all t,

$$\lambda(t) dV^*(t) = \int_t^{\infty} \mu(s) dU^*(s) \tag{1}$$

for some $\lambda(t) > 0$. Since, in addition, $\mathcal{F}$ is time-invariant, local welfare comparisons across time for a given path of utility flow $\{U^*(t)\}_{t=0}^{\infty}$ depends on the present value of future growth in utility:

$$\lambda(t) \dot{V}^*(t) = \int_t^{\infty} \mu(s) \dot{U}^*(s) ds \,. \tag{2}$$

Assume now that the instantaneous well-being of the economy depends on a vector of non-negative *consumption flows* $\mathbf{C}$ that includes also environmental amenities. Let U be a increasing, concave, smooth, and time-invariant *utility function* that assigns utility $U(\mathbf{C})$ to any consumption vector. Here, $\mathbf{C}$ is comprehensive, containing all variable determinants of current instantaneous well-being. This implies that economy's instantaneous well-being is increased by moving from $\mathbf{C}'$ to $\mathbf{C}''$ if and only if $U(\mathbf{C}') < U(\mathbf{C}'')$.

By means of the time-invariant function U, dynamic welfare can be expressed as a function of the path of the vector of future consumption flows:

$$V^*(t) = \mathcal{G}(\{\mathbf{C}^*(s)\}_{s=t}^{\infty}) \,,$$

where $\mathcal{G}(\{\mathbf{C}^*(s)\}_{s=t}^{\infty}) = \mathcal{F}(\{U(\mathbf{C}^*(s))\}_{s=t}^{\infty})$. It follows from the assumptions on $\mathcal{F}$ and U that the functional $\mathcal{G}$ is concave, time-invariant and smooth, and satisfies a condition of independent future. Since $\mathcal{G}$ is smooth and satisfies independent future, there exists, for any path of the vector of consumption flows $\{\mathbf{C}^*(t)\}_{t=0}^{\infty}$, a path of supporting present value consumption prices $\{\mathbf{p}(t)\}_{t=0}^{\infty}$ satisfying, for all t,

$$\mu(t)\nabla U(\mathbf{C}^*(t)) = \mathbf{p}(t)\,, \tag{3}$$

where ∇ denotes a vector of partial derivatives. This means that

$$\lambda(t)dV^*(t) = \int_t^{\infty} \mathbf{p}(s)d\mathbf{C}(s)\,. \tag{4}$$

Furthermore, since $\mathcal{G}$ is time-invariant,

$$\lambda(t)\dot{V}^*(t) = \int_t^{\infty} \mathbf{p}(s)\dot{\mathbf{C}}(s)ds\,. \tag{5}$$

Let the instantaneous net productive capacity of the economy depend on a vector of non-negative *capital stocks* $\mathbf{K}$ that includes not only the usual kinds of man-made capital stocks, but also stocks of natural resources, environmental assets, human capital (like education and knowledge capital accumulated from R&D-like activities), and other durable productive assets. Moreover, let $\mathbf{I}$ $(= \dot{\mathbf{K}})$ stand for the corresponding vector of *net investment flows*. The net investment flow of a natural resource is negative if the extraction rate exceeds its natural growth.

Say that consumption-net investment pair $(\mathbf{C}, \mathbf{I})$ is *attainable* given $\mathbf{K}$ if and only if $(\mathbf{C}, \mathbf{I})$ is in $S(\mathbf{K})$, where $S(\mathbf{K})$ is a set that constitutes current *instantaneous net productive capacity*. Here, $\mathbf{K}$ is comprehensive, containing all variable determinants of current net productive capacity. This implies that society's productive capacity is changed by moving from $\mathbf{K}'$ to $\mathbf{K}''$ if and only if $S(\mathbf{K}') \neq S(\mathbf{K}'')$. Assume that the set of feasible triples

$$\{(\mathbf{C}, \mathbf{I}, \mathbf{K}) \mid (\mathbf{C}, \mathbf{I}) \in S(\mathbf{K})\}$$

is a convex, smooth, and time-invariant set, with free disposal of consumption and net investment flows.

Assume that the economy's actual decisions are taken according to a *resource allocation mechanism* (RAM) that assigns an attainable consumption-net investment pair to any vector of capital stocks $\mathbf{K}$. Hence, for any vector of capital stocks $\mathbf{K}$, the RAM determines the consumption and net investment flows. The net investment flows in turn maps out the development of the capital stocks. The resource allocation mechanism thereby implements a feasible path of consumption flows, net investment flows, and capital stocks, for any initial vector of capital stocks.

Since the set-valued function S is time-invariant, one can assume that the possibly inefficient RAM in the economy is Markovian and time-invariant. Hence, the RAM assigns to any vector of capital stocks $\mathbf{K}$ a consumption-net investment pair $(\mathbf{C}(\mathbf{K}), \mathbf{I}(\mathbf{K}))$ satisfying that $(\mathbf{C}(\mathbf{K}), \mathbf{I}(\mathbf{K})) \in S(\mathbf{K})$. I assume that there exists a unique solution $\{\mathbf{K}^*(t)\}_{t=0}^{\infty}$ to the differential equations $\dot{\mathbf{K}}^*(t) = \mathbf{I}(\mathbf{K}^*(t))$ that satisfies the initial condition $\mathbf{K}^*(0) = \mathbf{K}^0$, where $\mathbf{K}^0$ is given. Hence, $\{\mathbf{K}^*(t)\}_{t=0}^{\infty}$ is the capital path that the RAM implements. For all t, write $\mathbf{C}^*(t) := \mathbf{C}(\mathbf{K}^*(t))$ and $\mathbf{I}^*(t) := \mathbf{I}(\mathbf{K}^*(t))$.

As a consequence of $\mathcal{G}$ being time-invariant and the RAM being Markovian and time-invariant, the dynamic welfare of the implemented path

$$V^*(t) = V(\mathbf{K}^*(\mathbf{t}))\,,$$

is time-invariant and a function solely of the current vector of capital stocks $\mathbf{K}$. The *state valuation function* V satisfies $V(\mathbf{K}^*(t)) = \mathcal{G}(\{\mathbf{C}^*(s)\}_{s=t}^{\infty})$. Assume that, combined with a smooth $\mathcal{G}$, the RAM makes V differentiable. Hence, there exists a vector of net investment prices $\mathbf{q}(t)$ at time t satisfying

$$\lambda(t)\nabla V(\mathbf{K}^*(t)) = \mathbf{q}(t)\,. \tag{6}$$

This means that

$$\lambda(t) dV^*(t) = \mathbf{q}(t) d\mathbf{K}^*(t)\,. \tag{7}$$

Since V is time-invariant, local welfare comparisons across time for a given implemented path $\{\mathbf{C}^*(t), \mathbf{I}^*(t), \mathbf{K}^*(t)\}_{t=0}^{\infty}$ depends on the value of net investments:

$$\lambda(t)\dot{V}^*(t) = \mathbf{q}(t)\mathbf{I}^*(t)\,. \tag{8}$$

By comparing (1), (4) and (7) on the one hand, and (2), (5) and (8) on the other hand, we obtain the following result.

Proposition 1 *Let (1) dynamic welfare be numerically representable by a welfarist, forward-looking and time-invariant function of the path of future utilities, satisfying a condition of independent future, (2) utility be a time-invariant function of the vector of consumption flows, and (3) the* RAM *be Markovian and time-invariant. Then, under the assumption of sufficient differentiability, there exist paths of discount factors*

$\{\mu(t)\}_{t=0}^{\infty}$, *present value consumption prices* $\{\mathbf{p}(t)\}_{t=0}^{\infty}$, *and present value net investment prices* $\{\mathbf{q}(t)\}_{t=0}^{\infty}$ *such that the implemented path* $\{\mathbf{C}^*(t), \mathbf{I}^*(t), \mathbf{K}^*(t)\}_{t=0}^{\infty}$ *satisfies:*

(a) A perturbation of the vector of capital stocks at time t increases welfare if and only if

$$\int_t^{\infty} \mu(s) dU^*(s) = \int_t^{\infty} \mathbf{p}(s) d\mathbf{C}^*(s) = \mathbf{q}(t) d\mathbf{K}^*(t) > 0\,.$$

(b) Welfare improves along the implemented path at time t if and only if

$$\int_t^{\infty} \mu(s)\dot{U}^*(s) ds = \int_t^{\infty} \mathbf{p}(s)\dot{\mathbf{C}}^*(s) ds = \mathbf{q}(t)\mathbf{I}^*(t) > 0\,.$$

While the discounted utilitarian welfare function is welfarist, forward-looking and time-invariant, and satisfies a condition of independent future, discounted utilitarianism is not implied by these properties. This motivates looking at discounted utilitarianism as one of two special cases.

2.1 Discounted utilitarian welfare

Let the welfare function be given as follows:

$$\mathcal{F}(\{U^*(s)\}_{s=t}^{\infty}) = \int_t^{\infty} e^{-\rho(s-t)} U^*(s) ds\,.$$

Then we obtain

$$\begin{aligned} \frac{d}{dt}\Big(\int_t^{\infty} e^{-\rho(s-t)} U^*(s) ds\Big) &= -U^*(t) + \rho \int_t^{\infty} e^{-\rho(s-t)} U^*(s) ds \\ &= e^{\rho t} \int_t^{\infty} e^{-\rho s} \dot{U}^*(s) ds\,, \end{aligned} \tag{9}$$

where the second equality follows by integrating by parts. This verifies (2) in the case of discounted utilitarianism by setting, for all t, $\lambda(t) = \mu(t) = e^{-\rho t}$. Equation (9) can be rewritten as

$$\nabla V(\mathbf{K}^*(t))\mathbf{I}^*(t) = -U(\mathbf{C}^*(t)) + \rho V(\mathbf{K}^*(t))$$

or

$$U(\mathbf{C}^*(t)) + \nabla V(\mathbf{K}^*(t))\mathbf{I}^*(t) = \rho V(\mathbf{K}^*(t))\,.$$

Differentiating once more w.r.t. time yields:

$$\nabla U(\mathbf{C}^*(t))\dot{\mathbf{C}}^*(t) + \frac{d\nabla V(\mathbf{K}^*(t))\mathbf{I}^*(t)}{dt} = \rho \nabla V(\mathbf{K}^*(t))\mathbf{I}^*(t)\,,$$

or equivalently, by (3) and (6),

$$\mu(t)\dot{U}^*(t) = \mathbf{p}(t)\dot{\mathbf{C}}^*(t) = -\frac{d\mathbf{q}(t)\mathbf{I}^*(t)}{dt} \tag{10}$$

as $d\nabla V(\mathbf{K}^*)\mathbf{I}^*/dt = d\big(\mathbf{q}\mathbf{I}^*/\lambda\big)/dt = \big(d\mathbf{q}\mathbf{I}^*/dt - \rho\mathbf{q}\mathbf{I}^*\big)/\lambda$. This means that the equalities in Proposition 1(b) follows through integration, provided that the following net investment value transversality condition holds:

$$\lim_{t\to\infty} \mathbf{q}(t)\mathbf{I}^*(t) = 0\,. \tag{11}$$

Note that the results reported in Proposition 1 and in the special case of discounted utilitarianism do not depend on the Markovian and time-invariant RAM implementing a welfare optimum. I turn now the case where the RAM implements a welfare optimum, but where dynamic welfare need not be discounted utilitarian.

2.2 Welfare optimum implemented by means of a competitive path

Say that the RAM $(\mathbf{C}(\cdot), \mathbf{I}(\cdot))$ implements a path $\{\mathbf{C}^*(t), \mathbf{I}^*(t), \mathbf{K}^*(t)\}_{t=0}^{\infty}$ that is *competitive* with respect to the path of discount factors $\{\mu(t)\}_{t=0}^{\infty}$ if there exists paths of present value consumption prices $\{\mathbf{p}(t)\}_{t=0}^{\infty}$ and present value net investment prices $\{\mathbf{q}(t)\}_{t=0}^{\infty}$ such that, for all t,

- C1 $\mathbf{C}^*(t)$ maximizes $\mu(t)u(\mathbf{C}) - \mathbf{p}(t)\mathbf{C}$ over all $\mathbf{C}$,
- C2 $(\mathbf{C}^*(t), \mathbf{I}^*(t), \mathbf{K}^*(t)) = (\mathbf{C}(\mathbf{K}^*(t)), \mathbf{I}(\mathbf{K}^*(t)), \mathbf{K}^*(t))$ maximizes $\mathbf{p}(t)\mathbf{C} + \mathbf{q}(t)\mathbf{I} + \dot{\mathbf{q}}(t)\mathbf{K}$ over all $(\mathbf{C}, \mathbf{I}, \mathbf{K})$ satisfying $(\mathbf{C}, \mathbf{I}) \in S(\mathbf{K})$.

By standard arguments it follows from the concavity of $\mathcal{F}$, U, and $\{(\mathbf{C}, \mathbf{I}, \mathbf{K}) \mid (\mathbf{C}, \mathbf{I}) \in S(\mathbf{K})\}$ that the competitive path $(\mathbf{C}^*(t), \mathbf{I}^*(t), \mathbf{K}^*(t))$ implemented by the RAM $(\mathbf{C}(\cdot), \mathbf{I}(\cdot))$ is a welfare optimum if

(a) $\{\mu(t)\}_{t=0}^{\infty}$ supports $\{U(\mathbf{C}^*(t))\}_{t=0}^{\infty}$ under the welfare functional $\mathcal{F}$,

(b) $\int_0^{\infty} \mu(t)U(\mathbf{C}^*(t))dt$ exists, and

(c) the following capital value transversality condition holds:

$$\lim_{t\to\infty} \mathbf{q}(t)\mathbf{K}^*(t) = 0\,.$$

Furthermore, it follows from the smoothness of U and $\{(\mathbf{C},\mathbf{I},\mathbf{K}) \mid (\mathbf{C},\mathbf{I}) \in S(\mathbf{K})\}$ that, for all t

$$\mu(t)\nabla U(\mathbf{C}^*(t)) = \mathbf{p}(t)\,, \tag{12}$$

$$\mathbf{p}(t)\nabla_{\mathbf{K}}\mathbf{C}(\mathbf{K}^*(t)) + \mathbf{q}(t)\nabla_{\mathbf{K}}\mathbf{I}(\mathbf{K}^*(t)) = -\dot{\mathbf{q}}(t)\,. \tag{13}$$

Since (13) entails that $\mathbf{p}\dot{\mathbf{C}}^* = -\mathbf{q}\dot{\mathbf{I}}^* - \dot{\mathbf{q}}\mathbf{I}^*$, expression (10) is again obtained, showing again that the equation of Proposition 1(b) follows through integration, provided that the net investment value transversality condition (11) is satisfied.

In the second special case, where the path is competitive, the present value consumption and net investment prices may correspond to market prices in a perfect marked economy. In the general case of Proposition 1 (and also in the special case of discounted utilitarianism), the present value consumption and net investment prices are accounting prices, which are not necessarily directly observable.

2.3 Review of relevant literature

Samuelson (1961, pp. 51–52) states that—"in complete analogy with the static one-period case"—welfare comparisons should be made by comparing the present value of future changes in consumption, as stated in Proposition 1 of this section. In his notation, $\sum P^aQ^a$ and $\sum P^bQ^b$ are the present value of future consumption in two different situations A and B. Samuelson stresses that a comparison of $\sum P^bQ^b \gtreqless \sum P^aQ^a$ is meaningless; rather the comparisons should be of $\sum P^b(Q^b - Q^a) \gtreqless 0$ or $\sum P^a(Q^b - Q^a) \gtreqless 0$. Samuelson (1961, p. 52) states that "there is no meaning in comparing *money* wealth in one situation (i.e. time and place) with that of another situation", unless "we use the *same prices and interest rates* in the comparison". In the present notation this translates into the proposition that over-time comparisons should *not* be of

$$\frac{d}{dt}\left(\int_t^\infty \mathbf{p}(s)\mathbf{C}^*(s)ds\right) \gtreqless 0\,,$$

but rather of

$$\int_t^\infty \mathbf{p}(s)\dot{\mathbf{C}}^*(s)ds \gtreqless 0$$

as reported in Proposition 1. Samuelson (1961, p. 53) refers to the latter as comparisons of "wealth-like magnitudes". Recently, Samuelson's insights have been brought to our attention by the analysis that Heal and Kriström have presented in various contributions during the last years (see, e.g., Heal and Kriström, 2005), under the assumptions of discounted utilitarianism and an optimal RAM.

Any direct attempt to estimate the present value of future changes in consumption would seem futile. In the words of (Samuelson, 1961, p. 53): "We are left left with the pessimistic conclusion that there is so much 'futurity' in any welfare evaluation of any dynamic situation as to make it exceedingly difficult for the statistician to approximate to the proper wealth comparisons." Fortunately, Weitzman (1970, 1976) and later contributions show that Samuelson was overly pessimistic. By combining his (10) with the equation prior to his (14), one can see that Weitzman (1976) demonstrates, under the assumptions of discounted utilitarianism and an optimal RAM, that welfare is improving if and only if the value of net investments is positive, as reported in the present Proposition 1(b). In order to be able to compare Weitzman (1976) to the current analysis, one should identify Weitzman's one-dimensional composite consumption good $C^*(t)$ ("... any cardinal utility function", Weitzman, 1976, p.157) with the present indicator of instantaneous well-being $U^*(t)$.

Equation (10) above is shown by Dixit et al. (1980, proof of Theorem 1) in the case of a competitive path, but without assuming discounted utilitarianism. Hence, Dixit et al. thereby tie together the welfare results reported by Samuelson (1961) and Weitzman (1976), since—by integration—equation (10) implies that the present value of future consumption growth equals the value of net investments.

Dasgupta and Mäler (2000), Dasgupta (2001) and Arrow et al. (2003) have introduced the concept of a possibly inefficient RAM in the literature on welfare comparisons based on national accounting aggregates. Under the assumption that dynamic welfare is discounted utilitarian, but without assuming that the RAM implements the discounted utilitarian optimum, Arrow et al. (2003) report the results of Proposition 1 through their Theorems 2 and 4. I have followed Arrow et al. (2003) by assuming "sufficient differentiability", instead of establishing this property from more primitive assumptions. Through Proposition 1 I have generalized their results by not imposing that dynamic welfare is discounted utilitarian.

3 Can NNP measure the welfare effects of capital stock perturbations?

For a given implemented path $(\mathbf{C}^*(t), \mathbf{I}^*(t), \mathbf{K}^*(t))$, *utility*-NNP can be defined as

$$\mu(t)U(\mathbf{C}^*(t)) + \mathbf{q}(t)\mathbf{I}^*(t)\,,$$

and *measurable* NNP, y, can be defined as

$$y^*(t) = \mathbf{p}(t)\mathbf{C}^*(t) + \mathbf{q}(t)\mathbf{I}^*(t)\,.$$

In contrast to utility-NNP, y is a *linear* index of the produced goods and services. Utility-NNP does not have this property unless U is linearly homogeneous so that $\mu(t)U(\mathbf{C}^*(t)) = \mathbf{p}(t)\mathbf{C}^*(t)$. If the path is competitive, then it follows from C2 that $y^*(t)$ is the maximized value of the current net product, given the price vectors $\mathbf{p}(t)$ and $\mathbf{q}(t)$, and the set of attainable consumption-net investment vectors $S(K^*(t))$.

Let the assumptions of Proposition 1 be satisfied, and differentiate the expressions in part (a) of the proposition with respect to time. This yields

$$-\mu(t)dU^*(t) = -\mathbf{p}(t)d\mathbf{C}^*(t) = \mathbf{q}(t)d(d\mathbf{K}^*(t))/dt + \dot{\mathbf{q}}(t)d\mathbf{K}^*(t)\,.$$

Since $d(d\mathbf{K}^*(t))/dt = I(\mathbf{K}^*(t))d\mathbf{K}^*(t) = d\mathbf{I}^*(t)$, the following result is obtained:

$$\mu(t)dU^*(t) + \mathbf{q}(t)d\mathbf{I}^*(t) = \mathbf{p}(t)d\mathbf{C}^*(t) + \mathbf{q}(t)d\mathbf{I}^*(t) = -\dot{\mathbf{q}}(t)d\mathbf{K}^*(t)\,. \qquad (14)$$

Hence, the change in utility-NNP or measurable NNP in fixed consumption and net investment prices, as a result of perturbation of the vector of capital stocks, is equal to the value of the perturbation of the capital stocks using $-\dot{\mathbf{q}}(t)$ as relative prices. Compare this to Proposition 1(a) which states that a perturbation of the capital stocks is welfare enhancing if and only if the value of the perturbation is positive using $\mathbf{q}(t)$ as relative prices.

If the economy is in a steady state, so that the rate of decline of the present value prices is constant, then $-\dot{\mathbf{q}}(t)$ is proportional to $\mathbf{q}(t)$, with a constant and positive real interest rate being the proportionality factor. In this case, a positive change in utility-NNP or measurable NNP in fixed consumption and net investment prices as a result of a perturbation of the vector of capital stocks indicates that the perturbation is welfare enhancing.

This conclusion does not hold in general. While $-\dot{\mathbf{q}}(t)$ measures the *instantaneous* net marginal productivity of the vector of capital components *as stocks*, $\mathbf{q}(t)$ measures the present value of the *future* marginal contributions that the capital components make, *both as stocks and flows*. Both $-\dot{\mathbf{q}}(t)$ and $\mathbf{q}(t)$ are measured relative to the possibly inefficient RAM. The case of a non-renewable resource is a prime example of a capital component where instantaneous net marginal productivity as a stock need *not* correspond to the present value of future contributions both as a stock and a flow.

Therefore, to show that $-\dot{\mathbf{q}}(t)$ need not be proportional to $\mathbf{q}(t)$, consider the Dasgupta-Heal-Solow (DHS) model (Dasgupta and Heal, 1974, 1979; Solow, 1974). In this model, the consumption flow is one-dimensional, and the net investment flows and capital stocks are two-dimensional, having both a manmade (M) and a natural (N) component. The latter is a non-renewable resource which is not productive as a stock. For positive stocks of manmade and natural capital, the set of attainable consumption-net investment pairs is given as

$$S(K_M, K_N) = \{(C, I_M, I_N) \mid C + I_M \leq K_M^{\alpha}(-I_N)^{\beta}; C \geq 0; I_N \leq 0\} .$$

It is worth noting $S(K_M, K_N)$ does not depend on K_N, as long as K_N is positive. Therefore, along a competitive path it follows from (13) and the envelope theorem that $-\dot{q}_N(t) = 0$. This reflects that natural capital (the non-renewable resource) has zero net productivity as a stock; only the flow of resource extraction is productive. That $q_N(t)$ is constant in present value terms is of course the Hotelling rule.

In the DHS model, $q_N(t) > 0$; thus, a perturbation of the stock of natural capital has an effect on welfare. However, as shown above, along a competitive path such a perturbation does not change utility-NNP or measurable NNP in fixed consumption and net investment prices. Since manmade capital is productive as a stock, so that $-\dot{q}_M(t) > 0$, one can easily construct examples of a welfare enhancing perturbation of capital stocks, with a relatively small negative dK_M^* and a relatively large positive dK_N^*, that decreases utility-NNP and measurable NNP in fixed consumption and net investment prices.

4 Can NNP growth measure welfare improvement?

Turn now to the question of measuring local-in-time changes in welfare along the implemented path. Let, as before, the assumptions of Proposition 1 be satisfied, and differentiate the expressions in part (b) of the proposition with respect to time. This yields

$$-\mu(t)\dot{U}^*(t) = -\mathbf{p}(t)\dot{\mathbf{C}}^*(t) = \mathbf{q}(t)\dot{\mathbf{I}}^*(t) + \dot{\mathbf{q}}(t)\mathbf{I}^*(t) ,$$

and leads to the following result:

$$\mu(t)\dot{U}^*(t) + \mathbf{q}(t)\dot{\mathbf{I}}^*(t) = \mathbf{p}(t)\dot{\mathbf{C}}^*(t) + \mathbf{q}(t)\dot{\mathbf{I}}^*(t) = -\dot{\mathbf{q}}(t)\mathbf{I}^*(t) . \tag{15}$$

Hence, the change in utility-NNP or measurable NNP in fixed consumption and net investment prices along the implemented path is equal to the value of the net investments in the capital stocks using $-\dot{\mathbf{q}}(t)$ as relative prices. Compare this to

Proposition 1(b) which states that welfare improves along the implemented path if and only if the value of the net investments is positive using $\mathbf{q}(t)$ as relative prices.

Thus, we obtain the same conclusion as when we were considering a perturbation of the capital stocks in the previous section: since $-\dot{\mathbf{q}}(t)$ need not be proportional to $\mathbf{q}(t)$, the growth in utility-NNP or measurable NNP in fixed consumption and net investment prices along the implemented path does not, in general, indicate welfare improvement.

Again we can use the DHS model to illustrate this negative result. Consider discounted utilitarian optimum that is implemented by means of a competitive path. Any such path has an eventual phase with decreasing consumption. It follows from Proposition 1(b) that welfare and the value of net investments (using $(q_M(t), q_N(t))$ as relative prices) is negative in this eventual phase. One can, however, choose the parameters of the model in such a way that this eventual phase is proceeded by an initial phase in which welfare and the value of net investments are positive (cf. Pezzey and Withagen, 1998).

Hence, initially $q_M I_M^* + q_N I_N^* > 0$, while later $q_M I_M^* + q_N I_N^* < 0$, since the path will reach the eventual phase with decreasing consumption. Since all variables develop in a continuous manner and $(q_M, q_N) \gg 0$ and $I_N^* < 0$ throughout, there exists some interval of time—in the beginning of the eventual phase with decreasing consumption—in which welfare is decreasing, while $I_M^* > 0$. In this interval, $-\big(\dot{q}_M I_M^* + \dot{q}_N I_N^*\big) > 0$, since $-\dot{q}_M > 0$ and $-\dot{q}_N = 0$ throughout. Hence, in the beginning of the eventual phase with decreasing consumption, NNP in fixed consumption and net investment prices is still growing, while welfare has started to decrease.

Hence, in order to get further, we must consider NNP in variable prices. Moreover, for local-in-time comparisons to be meaningful, NNP must be measured in *real* (not *nominal*) prices. The present-value prices $\{\mathbf{p}(t), \mathbf{q}(t)\}_{t=0}^{\infty}$ considered so far can be turned into real prices $\{\mathbf{P}(t), \mathbf{Q}(t)\}_{t=0}^{\infty}$ by using a price index $\{\pi(t)\}_{t=0}^{\infty}$ to make the following transformations at each t:

$$\mathbf{P}(t) = \tfrac{\mathbf{p}(t)}{\pi(t)}$$
$$\mathbf{Q}(t) = \tfrac{\mathbf{q}(t)}{\pi(t)} \,,$$

implying that the *real* interest rate, $R(t)$, at time t is given by

$$R(t) = -\tfrac{\dot{\pi}(t)}{\pi(t)} \,,$$

since the nominal prices $\{\mathbf{p}(t), \mathbf{q}(t)\}_{t=0}^{\infty}$ are present-value prices. However, what kind of price index will entail that real NNP growth indicates welfare improvement?

4.1 NNP price index

One possibility—a seemingly natural method, which appears to be employed in practise—is to make measurable NNP at different times comparable by means of a (Divisia) NNP *price index* $\{\pi_n(t)\}_{t=0}^{\infty}$. The NNP price index $\{\pi_n(t)\}_{t=0}^{\infty}$ lets price changes be weighted by consumption and net investment flows and satisfies, for all t,

$$\frac{\dot{\pi}_n(t)}{\pi_n(t)} = \frac{\dot{\mathbf{p}}(t)\mathbf{C}^*(t) + \dot{\mathbf{q}}(t)\mathbf{I}^*(t)}{\mathbf{p}(t)\mathbf{C}^*(t) + \mathbf{q}(t)\mathbf{I}^*(t)} .$$

It turns $\{\mathbf{p}(t), \mathbf{q}(t)\}_{t=0}^{\infty}$ into real prices $\{\mathbf{P}_n(t), \mathbf{Q}_n(t)\}_{t=0}^{\infty}$ satisfying $\dot{\mathbf{P}}_n\mathbf{C}^* + \dot{\mathbf{Q}}_n\mathbf{I}^* = 0$. Let *real measurable* NNP *deflated by means of an* NNP *price index* be defined as follows:

$$Y_n(t) := \mathbf{P}_n(t)\mathbf{C}^*(t) + \mathbf{Q}_n(t)\mathbf{I}^*(t) .$$

Then—since $\dot{\mathbf{P}}_n\mathbf{C}^* + \dot{\mathbf{Q}}_n\mathbf{I}^* = 0$ by construction of the NNP price index—it follows from (15) that growth in real NNP in variable prices is given by

$$\dot{Y}_n(t) = \mathbf{P}_n(t)\dot{\mathbf{C}}^*(t) + \mathbf{Q}_n(t)\dot{\mathbf{I}}^*(t) = -\tfrac{\dot{\mathbf{q}}(t)}{\pi_n(t)}\mathbf{I}^*(t) .$$

Hence, $\dot{Y}_n(t)$ is positive if and only if NNP in fixed prices is increasing. This implies that the use of an NNP price index does *not* lead to the result that growth in real measurable NNP in variable prices indicates welfare improvement.

4.2 Consumer price index

Another possibility is to make measurable NNP at different times comparable by means of a (Divisia) *consumer price index* $\{\pi_c(t)\}_{t=0}^{\infty}$. The consumer price index $\{\pi_c(t)\}_{t=0}^{\infty}$ lets price changes be weighted by consumption flows only and satisfies, for all t,

$$\frac{\dot{\pi}_c(t)}{\pi_c(t)} = \frac{\dot{\mathbf{p}}(t)\mathbf{C}^*(t)}{\mathbf{p}(t)\mathbf{C}^*(t)} .$$

It turns $\{\mathbf{p}(t), \mathbf{q}(t)\}_{t=0}^{\infty}$ into real prices $\{\mathbf{P}_c(t), \mathbf{Q}_c(t)\}_{t=0}^{\infty}$ satisfying $\dot{\mathbf{P}}_c\mathbf{C}^* = 0$. By (3) this entails that instantaneous well-being is increasing if and only if the real value of consumption $\mathbf{P}_c\mathbf{C}^*$ is increasing:

$$\mu\dot{U}^* = \mu\nabla U(\mathbf{C}^*)\dot{\mathbf{C}}^* = \mathbf{p}\dot{\mathbf{C}}^* = \pi\mathbf{P}_c\dot{\mathbf{C}}^* = \pi\left(\dot{\mathbf{P}}_c\mathbf{C}^* + \mathbf{P}_c\dot{\mathbf{C}}^*\right) = \pi\tfrac{d}{dt}\left(\mathbf{P}_c\mathbf{C}^*\right) .$$

Let *real measurable* NNP *deflated by means of a consumer price index* be defined as follows:

$$Y_c(t) := \mathbf{P}_c(t)\mathbf{C}^*(t) + \mathbf{Q}_c(t)\mathbf{I}^*(t)\,.$$

Then—since $\dot{\mathbf{P}}_c\mathbf{C}^* = 0$ by construction of the consumer price index—it follows from (15) that growth in real NNP in variable prices is given by

$$\begin{aligned}\dot{Y}_c(t) &= \mathbf{P}_c(t)\dot{\mathbf{C}}^*(t) + \mathbf{Q}_c(t)\dot{\mathbf{I}}^*(t) + \dot{\mathbf{Q}}_c(t)\mathbf{I}^*(t) \\ &= \left(-\tfrac{\dot{\mathbf{q}}(t)}{\pi_c(t)} - \tfrac{\dot{\pi}_c(t)}{\pi_c(t)}\tfrac{\mathbf{q}(t)}{\pi_c(t)} + \tfrac{\dot{\mathbf{q}}(t)}{\pi_c(t)}\right)\mathbf{I}^*(t) = R_c(t)\mathbf{Q}_c(t)\mathbf{I}^*(t)\,.\end{aligned}$$

Hence, $\dot{Y}_c(t)$ is positive if and only if $\mathbf{Q}_c(t)\mathbf{I}^*(t)$ is positive, provided that the real consumption interest rate $R_c(t)$ is positive. This implies that the use of an consumer price index leads to the result that growth in real measurable NNP in variable prices indicates welfare improvement, under the provision that the real consumption interest rate is positive.

Note that in the DHS model, the real consumption interest equals the net marginal productivity of manmade capital, which is positive throughout. Therefore, in the DHS model, growth in real measurable NNP deflated by means of a consumer price index indicates welfare improvement.

Note also that consumer price index $\{\pi_c(t)\}_{t=0}^{\infty}$ can be calculated from observable consumer prices and quantities. Hence, welfare improvement can be indicated from the change in an observable linear index of the produced goods and services, namely real measurable NNP deflated by means of a consumer price index.

4.3 Utility price index

A third possibility is to make utility-NNP at different times comparable by means of a *utility price index* $\{\pi_c(t)\}_{t=0}^{\infty}$. The utility price index $\{\pi_u(t)\}_{t=0}^{\infty}$ satisfies, for all t,

$$\pi_u(t) = \mu(t)\,,$$

where $\{\mu(t)\}_{t=0}^{\infty}$ is the path of supporting utility discount factors introduced in Section 2. A utility price index turns $\{\mathbf{p}(t), \mathbf{q}(t)\}_{t=0}^{\infty}$ into real prices $\{\mathbf{P}_u(t), \mathbf{Q}_u(t)\}_{t=0}^{\infty}$ measured in terms of utility. Let *real utility*-NNP be defined as follows:

$$Y_u(t) := U(\mathbf{C}^*(t)) + \mathbf{Q}_u(t)\mathbf{I}^*(t)\,.$$

Then, since $\nabla U(\mathbf{C}^*) = \mathbf{P}_u$ by invoking (3), it follows from (15) that growth in real utility-NNP in variable net investment prices is given by

$$\begin{aligned}\dot{Y}_u(t) &= \mathbf{P}_u(t)\dot{\mathbf{C}}^*(t) + \mathbf{Q}_u(t)\dot{\mathbf{I}}^*(t) + \dot{\mathbf{Q}}_u(t)\mathbf{I}^*(t) \\ &= \left(-\frac{\dot{\mathbf{q}}(t)}{\pi_u(t)} - \frac{\dot{\pi}_u(t)}{\pi_u(t)}\frac{\mathbf{q}(t)}{\pi_u(t)} + \frac{\dot{\mathbf{q}}(t)}{\pi_u(t)}\right)\mathbf{I}^*(t) = R_u(t)\mathbf{Q}_u(t)\mathbf{I}^*(t)\,.\end{aligned}$$

Hence, $\dot{Y}_u(t)$ is positive if and only if $\mathbf{Q}_u(t)\mathbf{I}^*(t)$ is positive, provided that the real utility interest rate $R_u(t) = -\dot{\mu}(t)/\mu(t)$ (= supporting utility discount *rate*) is positive. This implies that, by measuring net investment prices in terms of utility, growth in real utility-NNP in variable net investment prices indicates welfare improvement, under the provision that the real utility interest rate is positive.

Note that along a discounted utilitarian path in the DHS model, the real utility interest rate equals the constant utility discount rate ρ, which is positive throughout. Therefore, along a discounted utilitarian path in the DHS model, real utility-NNP growth indicates welfare improvement.

Note also that local-in-time comparisons by means of real utility-NNP requires that changes in utility are measurable.

5 Summary of results and relevant literature

The observations of Sections 3 and 4 can be summarized in the following proposition.

Proposition 2 *Under the assumptions of Proposition 1, the following holds:*

(a) Change in NNP *in fixed consumption and net investment prices cannot be used to measure the welfare effects of capital stock perturbations.*

(b) Growth in NNP *in fixed consumption and net investment prices cannot be used to measure welfare improvement along the implemented path.*

(c) Growth in real measurable NNP *(deflated by means of an* NNP *price index) in variable consumption and net investment prices cannot be used to measure welfare improvement along the implemented path.*

(d) Provided that the real consumption interest rate is positive, growth in real measurable NNP *(deflated by means of a consumer price index) in variable consumption and net investment prices can be used to measure welfare improvement along the implemented path.*

(e) *Provided that the real utility interest rate (= supporting utility discount rate) is positive and real utility-*NNP *is measurable, growth in real utility-*NNP *in variable net investment prices can be used to measure welfare improvement along the implemented path.*

Part (e) of Proposition 2 is shown by Weitzman (1970, 1976) under the assumption that the RAM implements a discounted utilitarian optimum. Then real utility-NNP is in welfare terms the stationary equivalent of the path of future utilities. Hence, when the welfare derived from the path of future utilities increases, so does utility-NNP. When Weitzman (1970, 1976) makes comparisons of utility-NNP, he considers utility-NNP in *variable* net investment prices, made comparable by using utility (or his "composite consumption good" Weitzman, 1976, p. 156) as numeraire. He has not claimed that changes in NNP in fixed consumption and net investment prices has welfare significance.

The problem associated with measuring the change in utility-NNP, which is *not* a linear index of the produced goods and services, can be solved by calculating the change in "consumers' surplus" $U(\mathbf{C}^*(t)) - \nabla U(\mathbf{C}^*(t))\mathbf{C}^*(t)$ since

$$\dot{U}^*(t) = \tfrac{d}{dt}\big(U(\mathbf{C}^*(t)) - \nabla U(\mathbf{C}^*(t))\mathbf{C}^*(t)\big) + \dot{\mathbf{P}}_u(t)\mathbf{C}^*(t) + \mathbf{P}_u(t)\dot{\mathbf{C}}^*(t)\,.$$

Then the utility price index can be determined by setting

$$\tfrac{d}{dt}\big(U(\mathbf{C}^*(t)) - \nabla U(\mathbf{C}^*(t))\mathbf{C}^*(t)\big) + \dot{\mathbf{P}}_u(t)\mathbf{C}^*(t) = 0\,,$$

since this ensures that $\nabla U(\mathbf{C}^*) = \mathbf{P}_u$ throughout. Weitzman (2001) argues that the change in such "consumers' surplus" is in principle observable. His analysis has been further developed by Li and Löfgren (2002).

Part (d) of Proposition 2 is shown by Asheim and Weitzman (2001) under the assumption that the RAM implements a discounted utilitarian optimum, thereby showing how the problem of measuring changes in utility can be circumvented. The result is generalized by Asheim and Buchholz (2004), who do not assume that dynamic welfare is discounted utilitarian. In Proposition 2(d) the result has been generalized even further—provided that there is "sufficient differentiability"—by showing that the RAM need not be optimal.

The result underlying part (b) of Proposition 2 is demonstrated by Li and Löfgren (2004) under the assumption that the RAM implements a discounted utilitarian optimum—this an assumption that is not made in the present analysis. They, however, interpret this result in a different manner than what I have done here. They rewrite the second equation of (15) as follows:

$$\mathbf{p}(s)\dot{\mathbf{C}}^*(s) + \mathbf{q}(t)\dot{\mathbf{I}}^*(t) = -\frac{\dot{\mathbf{q}}(t)\mathbf{I}^*(t)}{\mathbf{q}(t)\mathbf{I}^*(t)}\,\mathbf{q}(t)\mathbf{I}^*(t)\,,$$

thereby establishing the result that growth in NNP if fixed consumption and net investment prices indicates welfare improvement, provided that $-\dot{\mathbf{q}}(t)\mathbf{I}^*(t)/\mathbf{q}(t)\mathbf{I}^*(t)$ is positive. They refer to $-\dot{\mathbf{q}}(t)\mathbf{I}^*(t)/\mathbf{q}(t)\mathbf{I}^*(t)$ "the overall (average) rate of return on investment" (Li and Löfgren, 2004, p. 11). This terminology may not be appropriate, as $-\dot{\mathbf{q}}(t)\mathbf{I}^*(t)$ captures only the instantaneous net marginal productivity of the capital components as stocks, where for each such component j, $-\dot{q}_j/q_j$ is the component's *own rate of interest*. The rate of return on investment for each capital component does not depend only on its own rate of interest, but also on its anticipated capital gains, reflecting the future contributions that the capital component makes both as a stock and a flow. This is the essence of the no-arbitrage equation, which holds along an efficient path. Moreover, as we have already seen in the DHS model, $-\dot{\mathbf{q}}(t)\mathbf{I}^*(t)/\mathbf{q}(t)\mathbf{I}^*(t)$ need not positive. Note that, in the DHS model in a discounted utilitarian optimum, the rate of return on investment in each of the two capital components is positive, both in terms of consumption and utility. Of course, for the natural capital component (the non-renewable resource), the positive returns are solely in terms of capital gains, which are not captured by its zero own rate of interest.

The problem of indicating welfare improvement by means of NNP growth in fixed consumption and net investment prices has also been observed and discussed by Dasgupta and Mäler (2000) and Dasgupta (2001).

Part (c) of Proposition 2 is a restatement of part (b), designed to make the point that—even though result of part (b) does not depend on a particular choice of price index—NNP growth in fixed consumption and net investment prices is equivalent to NNP growth in variable consumption and net investment prices using an NNP price index. A comparison of the negative result of part (c) with the positive result of part (d) yields a theory for deflating NNP: In order for real measurable NNP to have local-in-time welfare significance, NNP must be deflated by a consumer price index.

Part (a) of Proposition 2 is reported in Asheim (2001) under the assumption that the RAM implements a discounted utilitarian optimum. This has been a key result in the critical assessment of the usefulness of NNP for making welfare comparisons that Heal and Kriström have presented through various contributions during the last years (see, e.g., Heal and Kriström, 2005).

Let me end by some concluding remarks.

- I have shown under week assumptions that a change in NNP in fixed consumption and net investment prices equals the value of net investments in the capital stocks, using their instantaneous net marginal productivities as weights. Welfare enhancement is, however, measured by the the value of net investment, using the net investment prices as weights. The net investment prices reflect the present value of the future marginal contributions that the capital components make, both as stocks and flows. Outside a steady state, the two kinds of weights need not be proportional, implying that changes in NNP in fixed consumption and net investment prices do not have welfare significance.

- Under competitive conditions, NNP is the maximized value of the economy's instantaneous net productive capacity. Depleting a stock of a non-renewable resource does not change the economy's instantaneous net productive capacity. The welfare-decreasing effects of such depletion can therefore *not* be captured by changes in NNP in fixed consumption and net investment prices. Rather, the effect is captured by the property that—in accordance with the Hotelling rule—the price of resource inputs is increasing (in terms of consumption or utility). This positive change in the net investment price of the non-renewable resource *cet. par.* decreases the maximized value of the economy's instantaneous net productive capacity, since the net investment flow of the resource is negative.

- Changes in NNP in fixed consumption and net investment prices do not allow for such revaluation of net investment flows and, hence, welfare improvement is not properly indicated. This same holds for growth in real measurable NNP in variable consumption and net investment prices, when NNP is deflated by means of an NNP price index. When instead NNP is deflated by means of a consumer price index, the net investment flows are appropriately revalued, leading to the conclusion that growth in real measurable NNP in variable consumption and net investment prices indicates welfare improvement, as long as the real consumption interest rate is positive.

- Hence, *a consumer price index—rather than an* NNP *price index—endows real measurable* NNP *in variable consumption and net investment prices with welfare significance. This yields a theory for deflating* NNP. When applying a consumer price index in models with environmental amenities, it is important to take the relative price changes of such amenities into account.

- Growth in real utility-NNP in variable net investment prices also allows for the revaluation of net investment flows, implying that it indicates welfare improvement, as long as the supporting utility discount rate is positive. This indicator requires that changes in utility-NNP can be measured.

- The real consumption interest rate is positive in the growth models that economists analyze. The only interesting exception is the *cake-eating* model (where the consumption interest rate is zero), which however should be considered as a pedagogical tool rather than a model of empirical interest. The supporting utility discount rate is positive and constant under discounted utilitarianism. If the utility function is strictly concave, then it is a general result—not being dependent on a discounted utilitarian welfare function—that the consumption interest rates exceeds the supporting utility discount rate when utility is increasing, and vice versa; this is the Ramsey rule.

- In the present paper, I have only been concerned with local comparisons—i.e., small perturbations of the capital stocks and local-in-time comparisons. Global comparisons raise other issues, some of which are analyzed in Asheim (2003, 2005).

- In the present paper, I have only considered comprehensive national accounting. There are obvious problems associated with applying the theory of welfare measurement by national accounting aggregates, as presented here, if the changes in some consumption and capital components cannot be measured or valued.

References

Arrow, K., Dasgupta, P.S., and Mäler, K.-G. (2003), Evaluating projects and assessing sustainable development in imperfect economies. *Environmental and Resource Economics 26*, 647–685.

Asheim, G.B. (2001), Welfare effects of small perturbations in optimal multi-sector growth, mimeo, Department of Economics, University of Oslo.

Asheim, G.B. (2003), Green national accounting for welfare and sustainability: A taxonomy of assumptions and results, *Scottish Journal of Political Economy 50*, 113–130.

Asheim, G.B. (2005), Welfare comparisons between societies with different population sizes and environmental characteristics, mimeo, Department of Economics, University of Oslo.

Asheim, G.B. and Buchholz, W. (2004), A general approach to welfare measurement through national income accounting, *Scandinavian Journal of Economics 106*, 361–384.

Asheim, G.B. and Weitzman, M.L. (2001), Does NNP growth indicate welfare improvement? *Economics Letters 73*, 233–239.

Dasgupta, P.S. (2001), Valuing objects and evaluating policies in imperfect economies, *Economic Journal 111*, C1–C29.

Dasgupta, P.S. and Heal, G.M. (1974), The optimal depletion of exhaustible resources, *Review of Economic Studies* (Symposium), 3–28.

Dasgupta, P.S. and Heal, G.M. (1979), *Economic Theory and Exhaustible Resources*, Cambridge University Press, Cambridge, UK.

Dasgupta, P.S. and Mäler, K.-G. (2000), Net national product, wealth, and social well-being, *Environment and Development Economics 5*, 69–93.

Dixit, A., Hammond, P. and Hoel, M. (1980), On Hartwick's rule for regular maximin paths of capital accumulation and resource depletion, *Review of Economic Studies 47*, 551–556.

Heal, G. and Kriström, B. (2005), Income, wealth and welfare in representative-agent economies, mimeo, Columbia Business School and SLU Umeå.

Li, C.-Z. and Löfgren, K.-G. (2002), On the Choice of Metrics in Dynamic Welfare Analysis: Utility versus Money Measures, Umeå Economic Studies, No 590, University of Umeå.

Li, C.-Z. and Löfgren, K.-G. (2004), NNP growth, welfare improvement and the overall rate of return on investment, Umeå Economic Studies, No 640, University of Umeå.

Pezzey, J.C.V. and Withagen, C.A. (1998). The rise, fall and sustainability of capital-resource economies, *Scandinavian Journal of Economics, 100*, 513–527.

Samuelson, P. (1961), The evaluation of 'social income': Capital formation and wealth, in F.A. Lutz and D.C. Hague (eds.), *The Theory of Capital*, St. Martin's Press, New York, 32–57.

Solow, R.M. (1974), Intergenerational equity and exhaustible resources, *Review of Economic Studies* (Symposium), 29–45.

Weitzman, M.L. (1970), Aggregation and disaggregation in the pure theory of capital and growth: A new parable, Cowles Foundation Discussion Paper 292.

Weitzman, M.L. (1976), On the welfare significance of national product in a dynamic economy, *Quarterly Journal of Economics 90*, 156–162.

Weitzman, M.L. (2001), A contribution to the theory of welfare accounting, *Scandinavian Journal of Economics 103*, 1–23.