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The Fear of Exclusion: Individual Effort when Group Formation is Endogenous

Kjell Arne Brekke, Karine Nyborg and Mari Rege*

April 21, 2005

Abstract

To secure their membership in a popular group, individuals may contribute more to the group's local public good than they would if group formation were exogenous. Those in the most unpopular group do not have this incentive to contribute to their group. Substantial differences in individual efforts levels between groups may be the result. A principal may prefer either exogenous or endogenous group formation, depending on whether an increase in contributions to the local public good coincides with the principal's interests. We analyze two examples: Social interaction in schools, and multiple-task teamwork.

Keywords: Local public goods, opportunity costs, popularity, multiple-task principal-agent analysis.

JEL codes: C72, D11, D23, L24, Z13.

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1 Introduction

When interacting with others in a group, people care about who those others are and how they behave. Football players want their team-mates to be good players, team-workers want their colleagues be efficient and reliable, and most people prefer to have attentive friends. To secure group relationships which are valuable to them, people may be willing to make substantial efforts: For example, athletes exercise more to keep their place on the team, and people keep appointments with their friends even when they really want to be somewhere else. In short, fear of exclusion from a popular group can make individuals strive to make themselves more attractive as group members. This requires, of course, that the individual's membership in the group can in fact be influenced by his behaviour. If group membership is exogenous to individuals, people may still fear exclusion, but since there is nothing they can do to prevent it, this fear may not affect their behaviour.

This paper demonstrates that a principal can use the choice of exogenous versus endogenous team formation as an instrument to influence agent behaviour. We study a population which is to be divided into two equally large groups (school classes, work teams). Each agent shares his time between producing a private good (academic achievement, sales activity, writing papers) and contributing to a group-specific local public good (social activity, helping co-workers, administrative work). Agents have identical preferences for the private good, the local public good, and possibly their own contribution to the local public good (e.g. social activity may both be pleasant and provide benefits to other group members); but they differ with respect to their ability in both private and local public goods production.

All else given, everyone prefers to be a member of the group with highest local public good provision. In order to secure their membership in this popular group, agents may be willing to contribute more to the local public good – and hence also produce less of the private good – than they would if group formation were exogenous. We analyze endogenous group formation in a non-cooperative game. In equilibrium, the popular group consists of those who have a comparative advantage in producing the local public good; and in this group, many members – though not necessarily all – contribute more to the local public good than they would if groups were formed exogenously.

Our framework can be used to analyze a variety of phenomena, of which we focus on two: Social interaction and academic achievement in schools, and multiple-task teamwork in firms.

It is well known that academic excellence – although frequently envied by others – is not necessarily a trait that makes a student popular among her peers (Eder and Kinney 1995). Our model of endogenous group formation provides one possible explanation why academic ability appears to be one of the determinants for the segregation of students into social groups, such as "nerds" and "burnouts" (Coleman 1961, Eckert 1989, Akerlof and Kranton 2003), and why higher academic ability could in fact indirectly make a student less popular. The model suggests that the students with low academic ability are more popular because they have the comparative (although not absolute) advantage in socializing. This implies that low ability students can outbid the more able students, who have a higher alternative cost of socializing in terms of foregone benefit from academic achievements. Hence, the low ability students are more popular and have a higher social quality than the most able students. However, because of the fear of exclusion, many members of the popular group study less hard and spend more time socializing than they would if group membership were exogenous. Thus, if the teacher can use exogenous group formation, these students will increase their academic efforts, while all other students' behavior is unchanged.

Our next example is concerned with teamwork in firms, where team members share their time between group tasks and individual tasks. Holmstrom (1982) demonstrated that teamwork conditions may create a moral hazard problem with substantial free-riding. Further, Holmstrom and Milgrom (1991) showed that when agents have multiple tasks, and effort is not verifiable for some tasks, high-powered incentives may work badly, since it increases effort on observable tasks at the expense of the unobservable tasks. While much of the literature on these issues has focused on optimal payment schemes, we will show that endogenous group formation can reduce the free-riding problem; moreover, we show that the potential improvement is increasing in the heterogeneity of workers' relative abilities.

Our predictions fit nicely with the results reported in Hamilton et al. (2003), who analyzed data from a sewing factory which introduced voluntary team formation. They found that in spite of the opportunity to free-ride on others' effort, teams were, on average, more productive than individual workers. Further, the first teams to be formed yielded the highest gains.

These results are consistent with the model we propose.

Group formation has been studied extensively within club theory (see e.g. Tiebout's (1956) seminal paper on "voting with your feet", or the surveys of Schotchmer (2002) and Cornes and Sandler (1986)). This literature focuses on competition for members between a large number of endogenously sized clubs, such as electorates offering different local public goods and tax levels. In the present analysis, there are no local authorities that can coordinate and enforce members' contributions; indeed, individual contributions cannot be enforced through formal contracts at all. However, contributions may be enforced through informal sanctioning within the group. Under these assumptions, we study how a non-cooperative group formation game can change individuals' behavior, compared to the case of exogenous group formation, and how knowledge of this can be useful to a principal wishing to influence agent behavior.

Below, we will begin by proposing a general framework for analyzing endogenous group formation; then, we turn to the cases of social exclusion in schools and teamwork in firms.

2 A Model of Team Formation

Consider a population that is to be divided into two equally sized groups[1]. Individuals are identical except for their abilities. Each individual i has to share his total available time (normalized to 1) between individual activities, r_i, and group activities, ℓ_i:

$$r_i + \ell_i = 1. \tag{1}$$

Time spent on individual activities produces a private good x_i,

$$x_i = w_i r_i, \tag{2}$$

where $w_i \geq 0$ denotes i's ability in production of the private good. Similarly, time spent on group activities produces a local public good, where the production function for individual i is given by

$$s_i = v_i \ell_i, \tag{3}$$

[1] This assumption is chosen for the purpose of simplification. Introducing more than two groups would not change the logic of the argument substantially, but allowing group size to vary endogenously would complicate our analysis considerably.

where $v_i \geq 1$ denotes i's ability in local public good production. Note that the assumption $v_i \geq 1$ implies that each person can produce at least one unit of the local public good.

We assume that i's contribution to the public good s_i is observable by fellow group members, but not by any observer external to the group, such as a principal. We further assume that abilities w_i and v_i are known to i himself, but cannot be observed by others.[2]

Let the average local public good production in i's group (i.e. average s_i in the group) be denoted S_i. Each individual i benefits from his production of the private good, x_i, and average contribution to the local public good in his group, S_i. In addition, we allow that the individual derives utility from his own contribution to the local public good, s_i. Let the preferences of each individual be represented by the following utility function[3]:

$$u_i = x_i + \rho f(s_i) + \gamma g(S_i) \tag{4}$$

where $\gamma > 0$, $\rho \in \{0, 1\}$ and f and g are strictly increasing and strictly concave. While the notion that individuals have preferences for their own contributions may be unfamiliar to economists, the inclusion of $\rho f(s_i)$ in the utility function is motivated by our focus on informal social interaction and the idea that contributing in a social interaction could be conceived as pleasant or interesting in its own right (for example going to a party).[4] If $\rho = 0$, individuals do not have preferences for their own contribution.

Inserting (1) - (3) in (4) yields

$$u_i = w_i(1 - \ell_i) + \rho f(v_i \ell_i) + \gamma g(S_i) \tag{5}$$

If group membership were exogenous, so that i's membership were independent of s_i, the individual would simply maximize utility (5) with respect to ℓ_i. We assume that the population is a continuum, where each individual has no mass, hence the individual will treat S_i as fixed in this maximization. In the following, we will refer to i's *unconstrained*

[2]This rules out inference of s_i via the individual's time budget. It also rules out direct allocation of group membership based on individual abilities, as well as any incentive mechanism requiring knowledge of individual abilities.

[3]Linear separability is assumed for the case of simplification.

[4]Note also that – although the context is different – this corresponds closely to Andreoni's (1990) "impure altruism".

contribution as that contribution to the local public good i would make if group membership were exogenous.

If $\rho = 0$, (5) is maximized when i spends no time at all on group activities. If $\rho = 1$, maximization of (5) yields the following first order condition for an interior solution:

$$f'(s_i) = \frac{w_i}{v_i} \equiv \Omega_i \tag{6}$$

Note that the fraction $\frac{w_i}{v_i} \equiv \Omega_i$ denotes how many units of the private good the individual must give up to produce one more unit of the local public good. Thus, in the following Ω_i will be referred to as i's *opportunity cost*.

In the following, we will assume that when $\rho = 1$, parameters are such that (6) has an interior solution. This simplifies the notation considerably without affecting the main insight of the paper. We now have the following lemma:

Lemma 1 *Individual i's unconstrained contribution to the local public good, $\hat{s}(\Omega_i)$, is given by*

$$\hat{s}(\Omega_i) = \begin{cases} 0 & \text{if} \quad \rho = 0 \\ f'^{-1}(\Omega_i) & \text{if} \quad \rho = 1 \end{cases}. \tag{7}$$

We will now assume that the opportunity cost, Ω_i, is uniformly distributed $[m - h, m + h]$, where m is the opportunity cost for the median person, while $h < m$ measures the heterogeneity of the population. People may have different opportunity cost because they differ in abilities for private good production, in abilities for local public good production, or in both.

Assume now that the population is partitioned into a popular and an unpopular group through the following group formation game:

The Group Formation Game

- Each individual makes a commitment, c_i, of how much he will contribute to local public good production if accepted into the popular group.

- An initiator invites half the individuals to become members of the popular group[5]. The

[5] This is similar to Harstad (2005). Since the population is a continuum our analysis is independent of how the initiator is selected. However, it may be natural to think of the initiator as the individual who has given the highest commitment.

remaining individuals form the unpopular group.

- Each individual chooses s_i, how much to contribute to the local public good, subject to the constraint that $s_i = c_i$ if i is in the popular group. Payoff is then determined by (4).

In deriving our results, we will assume that there exists a mechanism making commitments c_i credible. The kind of mechanism we have in mind is that of social sanctions between group members. For example, members of the popular group can turn their backs to a fellow member breaking his promise, simply ignoring him; a sanction which may be considered costless to the sanctioner, while still sufficiently severe to make it optimal to keep promises. Below, we will focus on equilibria where individuals choose to use these sanctions if promises are broken.[6] Moreover, we will assume that side payments in terms of individual production x_i are not possible.

Since an individual's utility is increasing in the local public good, everyone prefers, *ceteris paribus,* to be in the group providing the highest S_i. Hence, the initiator will always invite the individuals with highest commitments to become members of the popular group. Let p denote the popular group and u the unpopular group, and let S^G denote the local public good supply in group $G \in \{p, u\}$. Then, $S^p \geq S^u$. This provides an incentive to commit more than one's unconstrained contribution $\hat{s}(\Omega_i)$ to be allowed into the popular group.

Any individual will now consider whether the benefit of popular group membership is high enough for her to be willing to provide the minimum required contribution. Let s be a requirement to achieve popular group membership. If $s \leq \hat{s}(\Omega_i)$, individual i satisfies the requirement simply by committing to her unconstrained contribution $c_i = \hat{s}(\Omega_i)$. If $s > \hat{s}(\Omega_i)$, however, the individual will have to contribute something extra to gain membership in the popular group. Individual i's utility given that she chooses to become a member of the popular group can be written as follows (noting that $x_i = w_i(1 - \ell_i) = w_i - \Omega_i s_i$):

$$U_{i,p} = w_i - \Omega_i s + \rho f(s) + \gamma g(S^p) \tag{8}$$

[6] Assuming existence of such sanctioning mechanisms seems reasonable in light of the substantial recent experimental evidence of reciprocal preferences (see e.g. Fehr and Falk 2002) and strictly positive willingnesses to pay to punish norm violators (Fehr and Fischbacher 2004).

If she instead seeks membership in the unpopular group, she will provide only her unconstrained contribution. This gives the following utility:

$$U_{i,u} = w_i - \Omega_i \hat{s}(\Omega_i) + \rho f\left(\hat{s}(\Omega_i)\right) + \gamma g\left(S^u\right) \tag{9}$$

Thus, for individuals with $\hat{s}(\Omega_i) < s$ the gain from being in the popular group is given by

$$\Delta U\left(\Omega_i; s\right) = -\Omega_i\left(s - \hat{s}(\Omega_i)\right) + \rho\left(f\left(s\right) - f\left(\hat{s}(\Omega_i)\right)\right) + \gamma\left(g\left(S^p\right) - g\left(S^u\right)\right) \tag{10}$$

Note that since $\Omega_i = f'(\hat{s}(\Omega_i))$, it follows that

$$\frac{\partial \Delta U}{\partial \Omega_i} = -\left(s - \hat{s}(\Omega_i)\right) < 0 \tag{11}$$

Thus we have the following Lemma:

Lemma 2 *A person's net benefit of entering the popular group is decreasing in Ω_i, her opportunity cost of local public good production.*

Note that this lemma holds irrespective of whether people have preferences for their own contributions ($\rho = 1$) or not ($\rho = 0$). Thus, if $S^p > S^u$ in equilibrium, it follows from Lemma 2 that those who have a comparative advantage in local public good production (i.e. $\Omega_i < m$) will be in the popular group, while those with a comparative advantage in private good production (i.e. $\Omega_i > m$) will be in the unpopular group.

Let the lowest contribution among the members of the popular group in equilibrium be denoted by $\bar{s}$. An individual's local public good production in equilibrium, $s^*\left(\Omega_i\right)$,is given by

$$s^*\left(\Omega_i\right) = \begin{cases} \hat{s}(\Omega_i) & \text{if } \Omega_i > m \\ \max\left\{\hat{s}(\Omega_i), \bar{s}\right\} & \text{if } \Omega_i \leq m \end{cases}. \tag{12}$$

Note that if $\rho = 0$, then $\hat{s}(\Omega_i) = 0$; hence $s^*(\Omega_i) = \bar{s}$ for all i such that $\Omega_i \leq m$.

If $S^p > S^u$ in equilibrium, then the average contribution in each group is given by

$$\begin{aligned} S^u &= \int_m^{m+h} s^*\left(\Omega_i\right) d\Omega_i \\ S^p &= \int_{m-h}^{m} s^*\left(\Omega_i\right) d\Omega_i \end{aligned} \tag{13}$$

In equilibrium, a marginal person ($\Omega_i = m$) must be indifferent between the two groups. Thus, the minimum contribution required to obtain membership in the popular group in equilibrium, $\bar{s}$, is determined by

$$\Delta U\left(m; \bar{s}\right) = 0 \tag{14}$$

Below, we will focus on Nash equilibria in which $\bar{s} \leq 1$, implying that membership in the popular group is feasible for everybody.[7]

The following Lemma is proven in Appendix A:

Lemma 3 *Assume that $\Delta U(m, 1) < 0$. Then, if $\rho = 1$ and/or $\gamma g'(0) > m$ then there must be at least one solution $\bar{s} \in (0, 1)$ to equation (14).*

Let $c\left(\Omega_i\right)$ denote the equilibrium commitment of person i. Clearly, if $\Omega_i \leq m$, then $c\left(\Omega_i\right) = s^*\left(\Omega_i\right) \geq \bar{s}$; while if $\Omega_i > m$, $c\left(\Omega_i\right) \leq \bar{s}$. Any member of the popular group promising strictly more than her unconstrained contribution $\hat{s}(\Omega_i)$ would have preferred to lower her commitment, had she not by that lost her popular group membership. Thus, in equilibrium, some individuals in the unpopular group must have commitments arbitrarily close to the minimum requirement $\bar{s}$. Let $\bar{c}\left(\Omega_i\right)$ denote the commitment that makes person i indifferent between membership in the popular and the unpopular group, i.e the solution to $\Delta U\left(\Omega_i; \bar{c}\right) = 0$. Thus, $\bar{c}\left(\Omega_i\right)$ is i's maximum willingness to contribute. Then, individual i's commitment is given by

$$c\left(\Omega_i\right) = \begin{cases} s^*\left(\Omega_i\right) & \text{if } \Omega_i \leq m \\ \bar{c}\left(\Omega_i\right) & \text{if } \Omega_i > m \end{cases} \tag{15}$$

Note that by definition $\bar{c}(m) = \bar{s}$. By continuity, $\bar{c}(\Omega_i) \to \bar{s}$ as $\Omega_i \downarrow m$. Thus, when every individual's commitment is as specified by equation (15), no-one has an incentive to change her commitment (recall that the commitment is conditional on being allowed into the popular group).

Thus, we have the following theorem:

Theorem 1 *Assume that $\Delta U(m, 1) < 0$, and that either $\rho = 1$ and/or $\gamma g'(0) > m$. Then there exists a Nash equilibrium in which individuals with a comparative advantage in local*

[7] This simplifies the analysis considerably. A discussion of the case when this assumption is relaxed is given in the Appendix.

public good production (i.e. all i such that $\Omega_i < m$) are in the popular group, and those with a comparative advantage in private good production (i.e. all i such that $\Omega_i \geq m$) are in the unpopular group. Each person's actual contribution to local public good production is determined by equations (12) - (14). Each person's commitment is given by equation (15).

The requirement that $\Delta U(m, 1) < 0$ ensures that $\bar{s} < 1$, so that the minimum requirement $\bar{s}$ is feasible even for individuals with low social abilities ($v_i = 1$). This assumption simplifies the analysis, but is not essential. A discussion of the case when the minimum requirement may be unattainable to some individuals can be found in Appendix B.

Note that when $\rho = 0$, i.e. people have no preference for their own contribution, then $\bar{s} = 0$ constitutes an equilibrium: In this case, $S^p = S^u = 0$, yielding no benefits of membership in one group as opposed to the other. However, the condition $\gamma g'(0) > m$ ensures that the local public good is sufficiently important to allow for another equilibrium, in which $\bar{s} > 0$. If people care for their own contribution ($\rho = 1$), then their unconstrained contributions will be strictly positive; this implies $S^p > S^u$, so popular group membership does secure a strictly higher level of the local public good. In this case, every equilibrium will sort individuals perfectly into groups based on their opportunity cost.

Hence, endogeneous group formation can be regarded as a screening device: Although neither absolute nor relative abilities are observable as such, the population is separated according to their relative ability, i.e. their opportunity cost, in equilibrium. One implication of Theorem 1 is thus that a principal external to the groups can use exogenous versus endogenous group formation as a tool to influence agents' behavior.

With endogenous group formation, marginal individuals will provide an effort strictly above their unconstrained supply, and by continuity the same applies to all near marginal individuals. Hence, endogenous group formation can induce some individuals to increase their local public good production. Conversely, disallowing endogenous group formation can decrease local public good production. From the principal's point of view, the optimal choice of group formation mechanism depends, of course, on whether increased production of the local public good is beneficial or detrimental to the principal's interests.

Below, we will apply the results of Theorem 1 to two quite different cases. The first is a case of social exclusion in schools; the second is a teamwork situation.

3 Nerds and Burnouts

Sociologists have long been aware that in schools, students categorize themselves into social groups such as "nerds", "jocks" and "burnouts", each group with its separate requirements of appearance and behavior (Coleman 1961, Eckert 1989, Akerlof and Kranton 2003). The requirements of such social categories seem to affect students' school effort, and may hence also indirectly affect the productivity of resources allocated to schools. If the peers of a "nerd" find high effort at school acceptable, while the same behavior is socially unacceptable among "burnouts", individuals in the latter group will be more reluctant to do their best at school. Academic excellence – although frequently envied by others – is not necessarily a trait that makes a student popular among her peers (Eder and Kinney 1995). Our analysis provides one possible explanation why academic ability appears to be one of the determinants for the segregation of students into social groups, and why higher ability could in fact indirectly make a student *less* popular. Below we will show that endogenous group formation in an educational setting, such as a school class, can negatively affect such students' learning environment.

Each student is assumed to have preferences for private payoff achieved through school work, x_i, and for social quality, consisting of benefits arising from one's own social activity $(f(s_i))$ and from the social activity of others $(\gamma g(S_i))$, as specified in equation (4) above. Private benefits from studying can for example be college entry, parent approval, or higher future earnings. Equation (2) captures that private benefits from studying is increasing in student ability w_i and time spent studying r_i. Note that a low w_i may be caused either by low academic talent or by factors external to the individual affecting the rewards he will obtain from his studies: For example, if i belongs to an ethnic or social minority subject to discrimination in the labor market, and j does not, then even if i and j have the same academic talent, the skills produced from j's effort may yield higher earnings than an equivalent effort from i, implying that $w_i < w_j$.

If students are allowed to form groups endogenously, those who contribute most to the group's social quality will be the preferred group members. This gives students an incentive to contribute more social time, and consequently study less, than they would have done if their group affiliation were exogenous. Theorem 1 above shows that there exist an equilibrium in which a student's popularity will depend on her opportunity cost $\Omega_i = \frac{w_i}{v_i}$. The students with a comparative advantage in socializing (i.e. $\Omega_i < m$), are in the popular group, whereas the students with a comparative advantage in school work (i.e. $\Omega_i > m$) are in the unpopular group.

To demonstrate the argument in the simplest possible fashion, assume that $v_i = 1$ for all i, i.e. every student is equally productive in contributing to the group's social activity. Referring to the model above, this implies that $s_i = \ell_i$, and $\Omega_i = w_i$. Moreover, assume that spending time on social activities is enjoyable *per se*, i.e. $\rho = 1$. Then, in equilibrium, students will be perfectly separated according to their academic ability w_i, such that the *least* able students are in the popular group, while the most able students are in the unpopular group. Using Theorem 1, it follows that students' commitments and actual levels of social activity can be illustrated as in Figure 1.

[**Figure 1 about here.**]

The higher a student's academic ability, the lower her unconstrained supply of social time, since able students have a high alternative cost in terms of foregone benefits derived from academic achievements. Further, some students with $w_i < m$ will make commitments which are substantially higher than their unconstrained supply. They do this because if they promised less, somebody else would take their place in the popular group. For those with very low ability, however, unconstrained social activity levels exceed the minimum requirement to become accepted. Hence, these students will contribute even more social time than required by the group – and their popularity is unthreatened. It is those whose ability level is in the middle range, but below the median, who will change their behavior in order to gain popularity.

The result that students with low academic ability become the most popular may seem counter-intuitive. Recall, however, that students are, by assumption, identical except from academic ability, so their different opportunity costs drives the result. More able students will end up as less popular, and with a lower social quality; but their *utility*, including the benefits of academic achievement, will be higher than the utility of less able, but more popular students.

If group membership were exogenous to students, they would not have to worry about social exclusion, and every student would contribute her unconstrained supply of social time. The academic achievement of students whose ability level is in the middle range, but below the median, would then increase, while every other student's academic achievement would be unaffected. This suggest that if a choice is available, teachers will prefer exogenous to endogenous group formation. Note, however, that even if exogenous group formation increases school effort, it will decrease students' average social quality.

Furthermore, note that there are conflicting interests among students concerning the group formation mechanism. In the present model, exogenous group formation amounts to drawing groups at random. For the least able students, whose unconstrained contribution exceeds $\bar{s}$, behavior is independent of the mechanism used; nevertheless, exogenous group formation reduces their social quality. A similar argument holds for the "nerds"; their behavior is unaffected, but exogenous group formation will increase their social quality. These conflicting interests make unambiguous conclusions on welfare effects difficult to draw.

4 Teamwork

Let us now turn to the case of teamwork. Assume that individuals work together in one of two equally large teams, governed by a common principal. We will study the effects of letting team formation be endogenous, rather than determined exogenously by the principal.

Each worker i has to choose how to share his time between two types of tasks. Let r_i denote the time spent by i on individual tasks, such as sales activities, writing papers, seeing patients or clients, while ℓ_i is the time spent on group tasks, for example marketing on behalf of the entire team, administrative work, supervision of students, or taking part in discussions of others' patient or client cases. Each individual produces a verifiable output y_i, which is increasing in private time r_i, in ability w_i, and in the average contribution to group tasks from members of his team, S_i:[8]

[8] The type of teamwork discussed here is thus slightly different from that in Holmstrom (1982), who assumes that individual production is not verifiable by the principal. Our focus is rather on the problem of allocating time between multiple tasks (as in Holmstrom and Milgrom 1991).

$$y_i = w_i r_i + \gamma g(S_i) \tag{16}$$

The worker's contribution to group tasks s_i, which we may think of as helping activities, is produced according to (3), i.e. $s_i = v_i \ell_i$. Contributions are observable to other team members, but cannot be verified by the principal. The principal, however, observes y_i, and rewards each person with an exogenously given fraction α of this individual production, keeping a profit per worker of $(1-\alpha)y_i$.

In contrast to the school application discussed above, we will now assume that the individual cares *only* about his monetary payoff, so that utility U_i can be written as

$$U_i = \alpha y_i. \tag{17}$$

Inserting (16) in (17), and using (2), yields an expression which is a monotone transformation of the utility function (4) of the general model presented above, but with $\rho = 0$. Lemma 1 then implies that with exogenous group formation, no-one will contribute anything to group tasks: Without the threat of exclusion, each team member spends all his time on r_i, individual tasks. This is a standard public good problem; everybody would prefer a situation in which all team members contributed, but no-one has an individual incentive to do so.

Assume now that group formation is endogenous. Then, it follows from Theorem 1 (provided its conditions are satisfied) that there exists an equilibrium in which the popular group consists of those workers who have a comparative advantage in helping activities (i.e. $\Omega_i < m$), while the unpopular group consists of those with a comparative advantage in individual production[9]. In the popular group, people help each other by contributing $\bar{s}$ to group tasks. In the unpopular group, no help is provided to others.

The principal is interested in promoting the groups' total production, and thus prefers that workers share their time efficiently between the two tasks. However, since he can observe neither abilities, opportunity costs, nor contributions, few instruments are available to reduce the free-rider problem discussed above. Nevertheless, the principal can use the fact that although *he himself* does not observe contributions, fellow teamworkers do: By allowing

[9] There also exists an equilibrium with no production of the local public good, i.e. $S_p = S_u = \bar{s} = 0$.

endogenous team formation, the principal can indirectly benefit from the the existence of an informal sanctioning mechanism among workers, and thus let the fear of exclusion from the popular team be the incentive to help others. Since $\bar{s} > 0$, endogenous team formation will induce every individual in the popular group to contribute more to group tasks than he would with exogenous team formation, while the behavior of members of the unpopular group is unaffected.

Note that $\bar{s}$ can be either higher or lower than that level of s which would have maximized total production in the popular group. The productivity gain from endogenous group formation for an average worker in the popular group, as compared to the exogenous groups case, is in fact proportional to the level of heterogeneity in the population: Compared to the case with $s = 0$, the productivity gain of an average worker equals $\gamma g(\bar{s}) - (m - h/2)\bar{s} = h\bar{s}/2$, where the latter equality is derived from the equilibrium condition $\gamma g(\bar{s}) = m\,\bar{s}$. While the gain from endogenous group formation can be substantial when workers have very different relative abilities, it can thus be negligible if workers are almost identical. The reason is that in the latter case, the competition to get into the popular group becomes so fierce as to produce inefficiently high contributions.

A principal might perhaps wonder whether S^p thus could become too large, i.e. whether the desire to be accepted in the popular team could induce workers to divert so much attention to group tasks, at the expense of individual tasks, that it would be more profitable to let team formation be exogenous after all. This can never be the case, however: The principal will always benefit from endogenous team formation. Endogenous team formation makes everybody in the popular group earn more (otherwise they would not have preferred the popular group); and since the payoff of workers in the unpopular group is unaffected, and the principal receives a fraction of every worker's payoff, the principal's payoff must be higher in the endogenous groups case.

5 Conclusions

The impact on individual behavior of threats of exclusion may be an important consideration for teachers, firm managers and others who deal with teams. In general, whether a principal should choose endogenous or exogenous group formation varies from case to case. The key question is whether individual contributions to group-specific, local public goods is beneficial or harmful to the principal. In the case of schools, the teacher presumably wants students to study hard, not spending too much time socializing with each other; the teacher's goal is then best served by giving students as little influence as possible on their group affiliations. In the teamwork case, however, the opposite result obtains: in that case, the principal wants team members to help each other, and if team formation is endogenous, some workers will contribute more to common tasks in order to keep their membership of a popular team.

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Appendix

A. Proof of Lemma 3

If $\Delta U(m, \varepsilon) > 0$ for some $\varepsilon > 0$, then, since $\Delta U(m, 1) \leq 0$ and $\Delta U(m, s)$ is continuous in s, it follows that there is an $\bar{s} \in (\varepsilon, 1]$ such that $\Delta U(m, \bar{s}) = 0$. We claim that with $\rho = 1$ or $\gamma g'(0) > m$ there does exist a (small) $\varepsilon \geq 0$ such $\Delta U(m; \varepsilon) > 0$.

Consider first the case $\rho = 1$, and consider a minimum requirement $s = \varepsilon = 0$. In this case, $s^*(\Omega_i) = \hat{s}(\Omega_i)$, and since $\hat{s}(\Omega_i)$ is declining in Ω_i, it follows that $S^p > S^u$. Thus

$$\Delta U(m; 0) = \gamma \left(g\left(S^p\right) - g\left(S^u\right)\right) > 0.$$

Next, when $\rho = 0$, $\hat{s}(\Omega_i) = 0$ for all i. Hence $S^p = s$, while $S^u = 0$. Now

$$\Delta U(m; s)\left.\right|_{s=0} = \gamma \left(g\left(S^p\right) - g\left(S^u\right)\right) = \gamma \left(g\left(0\right) - g\left(0\right)\right) = 0$$

Using (10), $S^p = s$ and $S^u = 0$,

$$\frac{\partial \Delta U(m; s)}{\partial s} = \gamma g'(s) - m > 0$$

Hence, there exists an $\varepsilon > 0$ such that $\Delta U(m; \varepsilon) > 0$.

B. Infeasible requirements

To simplify the discussion above we invoked the assumption that $\Delta U(m, 1) < 0$. If we relax this assumption, there still exists an equilibrium $\bar{s} > 0$, provided that $\rho = 1$ or $g'(0) > m$ (as in Lemma 3). The complication which arise, however, is that the equilibrium requirement $\bar{s}$ could exceed 1, in which case it would be infeasible to some individuals, since $s_i = v_i \ell_i$ and $\ell_i \leq 1$. If $v_i < \bar{s}$, the requirement is infeasible for individual i. Hence, when we allow $\Delta U(m, 1) \geq 0$, group membership in equilibrium may depend not only on opportunity costs Ω_i, but also on the absolute ability level v_i.

In this situation, individuals may be excluded from the popular group either because their opportunity cost is so high that they are not *willing to* contribute the requirement, as before, or because the requirement is *infeasible.* Individuals with low Ω_i, *and* low v_i, who would otherwise be in the popular group in equilibrium, will now be unable to join this group.

The equilibrium level $\bar{s}$ will thus be such that exactly half the population both *can* and *want to* join the popular group. However, as less than half the population satisfy the joint requirement $v_i \geq s$ and $\Omega_i \leq m$, an individual with $\Omega_i = m$ is no longer a marginal individual. Hence, define a function $\tilde{m}(s)$ such that exactly half the population is included in

$$\{i : v_i \geq s \text{ and } \Omega_i \leq \tilde{m}(s)\}.$$

Now, a marginal individual will satisfy $\Omega_i = \tilde{m}(s)$, and the equilibrium condition generalizes to

$$\Delta U\left(\tilde{m}(\bar{s});\bar{s}\right) = 0. \tag{18}$$

The following Lemma summarizes the generalized result.

Lemma 4 *If* $\rho = 1$ or $\gamma g'(0) > m$, *and* (v_i, w_i) is continuously distributed in R^2, *then there is at least one solution* $\bar{s} > 0$ *to equation (18).*

Proof. As above, $\rho = 1$ or $\gamma g'(0) > m$ ensures that there exist a (small) $\varepsilon \geq 0$ such $\Delta U\left(m;\varepsilon\right) > 0$. If $\Delta U(m,1) \leq 0$ there is an $\bar{s} \in (\varepsilon, 1]$ such that $\Delta U(m,\bar{s}) = 0$, as above. Hence we focus on the case where $\Delta U(m,1) > 0$, and we have to consider requirements $s > 1$.

There is an $\bar{S}$ such that exactly half the population is in $\left\{i : v_i \geq \bar{S}\right\}$, and hence $\tilde{m}(s)$ is well defined over the interval $s \in [0, \bar{S}]$. For $s < 1$, $\tilde{m}(s) = m$ and hence $\Delta U\left(\tilde{m}(s);s\right) > 0$ for s small. If $\Delta U\left(\tilde{m}(\bar{S});\bar{S}\right) < 0$, a solution exists by continuity. It remains to prove the existence of an equilibrium when $\Delta U\left(\tilde{m}(\bar{S});\bar{S}\right) \geq 0$. In this case, however, all individuals for whom $\bar{S}$ is feasible will prefer the popular group, and since this is exactly half the population, that constitutes an equilibrium. ■

Recall that when $\rho = 1$, we have assumed that there is an interior solution to each individual's time allocation problem; hence, that $\hat{s}(\Omega_i) > 0$ for all i. This is used to prove the claim that $\Delta U\left(m;0\right) > 0$. In fact, it is sufficient that $\hat{s}(\Omega_i) > 0$ for *some* individuals, since this ensures $S^p > S^u$ when the requirement is $s = 0$.

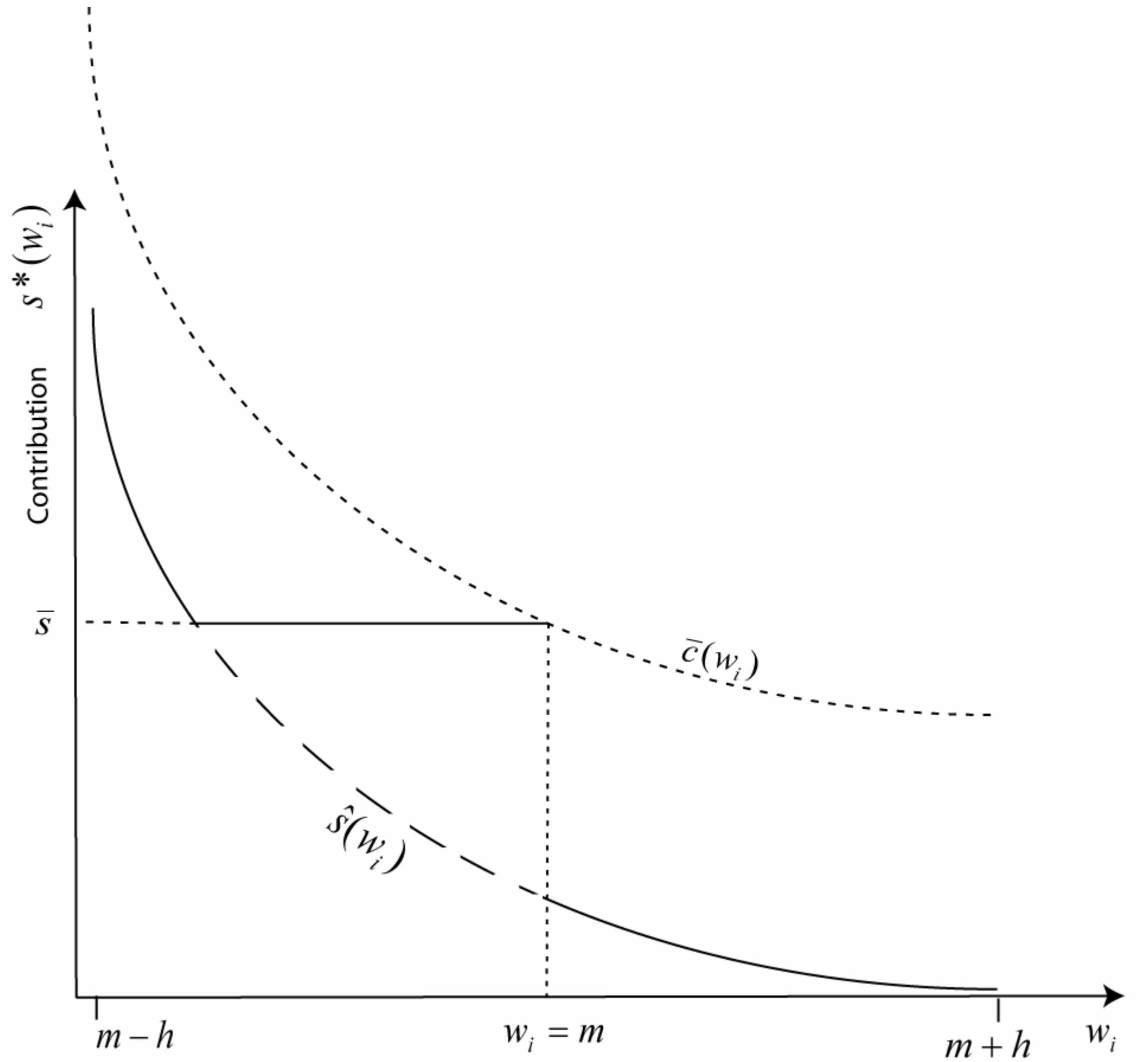

Figure 1: Contribution $s^*(w_i)$ to the local public good (solid line). Note that $\Omega_i = w_i$. $\hat{s}(w_i)$ is the unconstrained contribution, $\bar{s}$ is the minimum requirement in equilibrium, and $\bar{c}(w_i)$ is the maximum willingness to contribute.