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Majority voting leads to unanimity*

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January 7, 2005

Abstract

We consider a situation where society decides, through majority voting in a secret ballot, between the alternatives of 'reform' and 'status quo'. Reform is assumed to create a minority of winners, while being efficient in the Kaldor-Hicks sense. We explore the consequences of allowing binding transfers between voters *conditional on the chosen alternative*. In particular, we establish conditions under which the winners wish to compensate *all* losers, thus leading to *unanimity for reform*, rather than compensating some losers to form a non-maximal majority. The analysis employs concepts from cooperative game theory.

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1 Introduction

Reforms create winners and losers. If the losers from reform have political influence, they may block the reform, even when the winners gain more than the losers lose so that reform is efficient in the Kaldor-Hicks sense. To make reform possible, the winners therefore have to give some losers compensation in exchange for accepting reform. But which of the losers will be compensated, and how large will the compensation be? In this paper, we consider these questions.

To maximize the winners' share, it may at first sight seem optimal for the winners to compensate as few losers as possible. Furthermore, it seems optimal to compensate losers having the smallest loss from reform, since they need the smallest compensation. In the literature on rational-choice theories of politics, this feature has appeared as a key prediction or as an essential assumption. Indeed, the size principle of Riker (1962) and the stationary equilibria of Baron and Ferejohn (1989) entail that minimal winning coalitions are likely to form.

This conclusion is not supported by empirical evidence. Rather, it seems to be an empirical regularity that minimal winning coalitions are rarely observed (Browne, 1993).[1] Theorists have responded to this evidence by providing reasons for the formation of non-minimal winning coalitions. One reason is that supermajorities are needed to ensure coalitions to be winning in the presence of uncertainty; another is that supermajorities are needed to establish stable coalitions that will be winning in different kinds of circumstances. A third reason is that *non-minimal winning coalitions may actually be cheaper than minimal winning coaltions.*[2] This explanation was suggested by Groseclose and Snyder (1996) (see also Banks, 2000; Groseclose

[1]Brams et al. (2003, Section 7) report on the distribution of U.S. House of Representatives majority coalitions. The distribution is bimodal, with modes at 50–60% and 90–100%. The latter, one may suspect, might comprise legislation that includes compensating measures, in line with the explanation for supermajorities that we will propose in the present paper.

[2]This list, which recapitulates parts of Groseclose and Snyder's (1996) brief survey, is not exhaustive: Axelrod (1970) argues that non-minimal winning coalitions are formed in order to minimize the conflict of interest among the coalition members. Brams and Fishburn (1995) and Fishburn and Brams (1996) present a combinatorial analysis of the size of majority coalitions. Volden and Carrubba (2004) offers a simultaneous test of five main theories of coalition formation.

and Snyder, 2000) and is the one that we will reexamine in this paper.

The following example illustrates why a supermajority may be cheaper. Consider a society (or a polity) with five members, where reform creates one winner and four losers. Decisions are made according to simple majority in a secret ballot, implying that two losers must be induced, through compensating transfers, to join the winner to form a minimal majority for reform. Utility measured in money is transferable between the members, and binding transfers can be made between voters conditional on the chosen alternative. The losers are equal and all lose 1, while the winner's gain exceeds 4. Hence, since the winner's gain is large enough to compensate all losers, reform is efficient in the Kaldor-Hicks sense, meaning that it constitutes a *potential* Pareto-improvement. Note, however, that a transfer of 1 to each of two losers need not be sufficient to ensure a robust majority for reform. The reason is that the two uncompensated losers are willing to induce one of the compensated losers to vote against reform by promising to pay a maximum of 2 if the status quo is retained. Hence, in a minimal robust majority for reform, the winner must transfer 3 to each of two losers to be immune against such a counter proposal. It is therefore cheaper to compensate all four losers since this costs only 4.

Groseclose and Snyder (1996) propose a dynamic non-cooperative game-theoretic model where two special-interest groups—one in favor of reform and one opposed to it—buy votes in a legislature. The special-interest group that prefers reform makes the first offer to the legislature, and the special-interest group with the opposite preference make the second and final offer. This last-mover advantage of the group supporting the status quo entails that a minimal winning coalition need not be robust against counterattack, as illustrated by the above example.

In the present paper we make different modeling choices. First, a key feature is that voters are allowed to play an active role—not by accepting or rejecting offers from vote-buyers, but by taking part in the forming of coalitions. In particular, we assume that voters can commit to transfers that are conditional on whether the reform is passed or not. One motivation for such conditional transfers concerns situations—like referenda—where in a literal sense there is a secret ballot and non-transferable voting rights, so that votes cannot be bought. Commitments must then be made conditional on the decision of the electorate. However, our modelling extends to situations—like legislature voting—where voting is not secret, but where vote buying is still ruled out by law or custom. In particular, "logrolling" (legal vote buying) may be limited by the extent to which politicians are willing to vote against their conviction or their promises in the electoral campaign. With limited scope for logrolling, compensation is necessary for reform to pass. In reality the transfers may not be as explicit as in our model. The transfers may consist of other policy changes that are linked to the main reform.[3] However, a distinctive feature of such "reform packages" is that transfers—i.e., policy changes other than the main reform—are conditioned on the decision made by the electorate.

A second difference, compared to Groseclose and Snyder (1996), is that we employ concepts from cooperative game theory and interpret core outcomes as having a robust majority for reform. This entails that the model is atemporal and therefore does not depend on specific procedure rules. In particular, we do not make the (somewhat arbitrary) assumption that the supporters of the status quo is endowed with a last-mover advantage.[4] Instead, we assume that conditional transfers once offered cannot be taken back; only further transfers can be made. We defend this assumption in two ways. One is substantive: If a group of voters has made a public concession to another group conditional on whether reform is passed or not, then it may be politically infeasible to withdraw this offer. The other is formal: If we were to assume that conditional transfers could be withdrawn, then core outcomes need not exist and our explanation for supermajorities could not be offered.

Within our model we establish quite general conditions under which it is actually least expensive for the reform winners to compensate *all* losers, so that the reform gets a unanimous vote. One of the conditions that are sufficient for unanimity under majority voting is that reform creates a minority of winners in the absence of com-

[3]Groseclose and Snyder (1996, p. 304) also mention this possibility: "If payments for votes on a bill are typically written into the bill itself (as special conditions, allowances, exemptions, transition rules, and so on) . . . ." For a discussion of compensations under reforms, see Haggard and Webb (1994, pp. 23–35), Angell and Graham (1995), Weyland (1998), and Edwards and Lederman (2002).

[4]As argued by Banks (2000), Grossman and Helpman (2001), and Baron (2001), the procedural rules are crucial in producing Groseclose and Snyder's result.

pensation. Thus—provided that the voters can make binding transfers conditional on whether or not reform is undertaken—our finding suggests that reforms with sufficiently large gains to few winners will lead to an *actual* Pareto-improvement if there is majority voting on the issue.

Our approach may be useful for other modeling purposes. For example, in political-economy models of economic reform it is often assumed that, if there is compensation, then all losers are compensated (see, e.g., Dewatripont and Roland, 1997; Claussen, 2002; Jain and Mukand, 2003). Although there is – as already mentioned – empirical support for wide compensation,[5] such an assumption may seem theoretically inconsistent with the modeling that such papers are based on. Our approach may be applied to furnish theoretical support for this assumption.

The remainder of the paper is organized as follows. In Section 2 we model the situation in question by adapting concepts from cooperative game theory. Although we focus on the case where decisions are made according to simple majority, we allow for the case where reform needs a qualified majority to be passed. We define and characterize the core and the stable set of the situation. By interpreting core outcomes as having a robust majority for reform, we provide in Section 3 conditions under which unanimity prevails. In Section 4 we argue that compensations beyond those needed to form a minimal winning coalition may be realized under even weaker conditions. In Section 5 we show that a non-empty core cannot be established if conditional transfers can be withdrawn. This analysis also shows that a simpler and more direct characteristic function formulation is unable to furnish an explanation for supermajorities. We conclude in Section 6, while all proofs are contained in an appendix.

[5] Empirical examples of full compensation include the two-tier labor market reforms in Spain in the 1980s. Under these reforms, firing restrictions on new labor contracts were relaxed while the regulations on existing contracts were unchanged, thereby implicitly compensating all losers for the effects of a full reform (see Saint-Paul, 1993). The dual-track reforms in China have the same features (see Lau et al., 2000).

2 The model

2.1 The alternatives

Consider a society (or polity), where N denotes the set of members and where $|N|$ is odd and greater than 1. Each member $i \in N$ will be referred to as an 'individual' or a 'voter', depending on the context. The society is to decide between two alternatives, 0 and 1, where later 0 will be associated with 'status quo' and 1 with 'reform'. Denote by R the set of alternatives; i.e., $R := \{0, 1\}$. We abstract from income effects and assume that utility measured in money is transferable. Denote by $v_i \in \mathbb{R}$ the added utility accruing to individual i as a direct result of alternative 1 being chosen instead of 0. We may w.l.o.g. assume that 1 is the efficient alternative.

Assumption 1 *Reform is efficient in the Kaldor-Hicks sense:*

$$\sum\nolimits_{i \in N} v_i > 0 \, .$$

The reform may, however, have both *winners*,

$$W := \{i \in N | v_i \geq 0\}$$

and *losers*,

$$L := \{i \in N | v_i < 0\} \, .$$

Note that $\{W, L\}$ is a partition of N. Since reform is efficient in the Kaldor-Hicks sense, there exist transfers such that moving from $r = 0$ to $r = 1$ represents a Pareto-improvement. It is a consequence of Assumption 1 that W is non-empty.

2.2 Collective decision-making

Decisions in society are made through a secret ballot, where all members of society are allowed to vote. Hence, votes cannot be purchased. However, each voter can make a binding conditional transfer to any other voter, to be paid conditional on a specified alternative being chosen. E.g., voter i can make a binding promise to pay $T_i(j, r)$ to voter j if alternative r is supported by a majority of the voters, where $i, j \in N$, $i \neq j$, and $r \in R$. Hence, a transfer schedule for individual i is defined as follows.

Definition 1 A *transfer schedule* for individual i, T_i, is a function that, for all other members of society and each alternative, specifies a conditional transfer from i:

$$T_i : N\backslash\{i\} \times R \to \mathbb{R}_+ \,.$$

That the conditional transfers are binding, means that a voter can never renege on the obligations of his transfer schedule. If, before the vote, he wants to make new conditional transfers, he must still honor the ones he has already promised to make.

The transfer schedule for the grand coalition N is given by $\mathbf{T} = (T_i)_{i\in N}$, while the transfer schedule for each subcoalition $S \in 2^N\backslash\{\emptyset\}$ is given by $\mathbf{T}_S = (T_i)_{i\in S}$.[6] Note that $\mathbf{T}_S$ is a matrix consisting of $|S| \cdot (|N| - 1) \cdot 2$ non-negative numbers, since any member of the coalition S specifies a non-negative number for each of the other members of society and for each of the two alternatives. Hence, for any two transfer schedules for coalition S, $\mathbf{T}_S$ and $\mathbf{T}'_S$, we write $\mathbf{T}_S \leq \mathbf{T}'_S$ if, for all $i \in S$, $j \in N\backslash\{i\}$, $r \in R$, it holds that $T_i(j, r) \leq T'_i(j, r)$. The utility of individual i if alternative r is chosen depends on the difference between payments received and payments made, plus v_i if $r = 1$. Hence, for all $(i, r) \in N \times R$,

$$u_i(\mathbf{T}, r) \;:=\; \sum\nolimits_{j\neq i}\big(T_j(i, r) - T_i(j, r)\big) \;+\; rv_i \,.$$

Although we are mainly concerned with the situation where decisions are made according to *simple majority*, we allow for the case where reform ($r = 1$) needs a *qualified majority* to be passed. In the latter case, a minority can ensure that the status quo ($r = 0$) remains. Hence, let $q \in \{0, 1, \ldots, (|N| - 1)/2\}$ be a parameter determining the voting rule. Denote by

$$\mathcal{M}^0 := \big\{S \in 2^N\backslash\{\emptyset\} \big|\; |S| > \tfrac{|N|}{2} - q\big\}$$

the collections of coalitions large enough to ensure the status quo, and denote by

$$\mathcal{M}^1 := \big\{S \in 2^N\backslash\{\emptyset\} \big|\; |S| > \tfrac{|N|}{2} + q\big\}$$

the collections of coalitions large enough to pass reform. Note that, for any coalition $S \in 2^N\backslash\{\emptyset\}$, $S \in \mathcal{M}^0$ if and only if $N\backslash S \notin \mathcal{M}^1$. Simple majority corresponds to $q = 0$, in which case $\mathcal{M}^0 = \mathcal{M}^1$.

---

[6] Likewise, write $\mathbf{0}$ and $\mathbf{0}_S$ for transfer schedules where all conditional transfers (from the grand coalition and the subcoalition S, respectively) are zero.

Given the transfer schedule $\mathbf{T}$, let $\rho(\mathbf{T})$ be the set of alternatives that is weakly supported by a large enough coalition. Hence,

$$0 \in \rho(\mathbf{T}) \quad \text{if} \quad \{i \in N | \, u_i(\mathbf{T}, 0) \geq u_i(\mathbf{T}, 1)\} \in \mathcal{M}^0 \, ,$$
$$1 \in \rho(\mathbf{T}) \quad \text{if} \quad \{i \in N | \, u_i(\mathbf{T}, 1) \geq u_i(\mathbf{T}, 0)\} \in \mathcal{M}^1 \, .$$

We can now define the set of *outcomes*, D, as the set of pairs $(\mathbf{T}, r)$ where r is weakly supported by a large enough coalition given the transfer schedule $\mathbf{T}$:

$$D := \{(\mathbf{T}, r) | \, r \in \rho(\mathbf{T})\} \, .$$

2.3 Blocking and domination

Say that coalition S blocks an outcome $(\mathbf{T}, r)$ if there exists a new transfer schedule for the coalition, $\mathbf{T}'_S$, where all payments are weakly greater, and an alternative r' weakly supported by a majority when S has changed its transfer schedule, so that all members of S are strictly better off. This is stated in the following definition.

Definition 2 Coalition S $(\in 2^N \backslash \{\emptyset\})$ *blocks* an outcome $(\mathbf{T}, r) \in D$ by means of another outcome $(\mathbf{T}', r') \in D$ if

1. $\mathbf{T}_S \leq \mathbf{T}'_S$ and $\mathbf{T}_{N\backslash S} = \mathbf{T}'_{N\backslash S}$,
2. $\forall i \in S, u_i(\mathbf{T}, r) < u_i(\mathbf{T}', r')$.

The condition $\mathbf{T}_S \leq \mathbf{T}'_S$ signifies that the transfers to be paid conditional on a specified alternative being chosen are assumed to be binding. Hence, a coalition S cannot default on the previous obligations inherent in $\mathbf{T}$, but may engage in new ones to form $\mathbf{T}' = (\mathbf{T}'_S, \mathbf{T}_{N\backslash S})$.[7]

Let $(\mathbf{T}, r)$ and $(\mathbf{T}', r')$ be two outcomes. Write

$$(\mathbf{T}, r) \prec (\mathbf{T}', r')$$

and say that $(\mathbf{T}, r)$ is *dominated* by $(\mathbf{T}', r')$ if there exists $S \in 2^N \backslash \{\emptyset\}$ such that S blocks $(\mathbf{T}, r)$ by means of $(\mathbf{T}', r')$.

[7] In Section 5 we consider the case with non-binding transfers, so that the condition $\mathbf{T}_S \leq \mathbf{T}'_S$ of Definition 2 is replaced by $\mathbf{0}_S \leq \mathbf{T}'_S$. Although this allows for a standard characteristic function formulation, it does not ensure a non-empty core, and such modeling cannot be used to establish a new explanation for non-minimal winning coalitions.

2.4 Defining and characterizing solution concepts

Above, we have described a set of outcomes and a dominance relation on this set. On this basis, we can follow Greenberg (1990, Ch. 4), who in turn builds on von Neumann and Morgenstern (1947), and define the concept of a system and two solution concepts: the core and the stable set.

Definition 3 The *system* $(D, \prec)$ consists of the set of outcomes D and the dominance relation $\prec$ on D.

Definition 4 The *core* of the system $(D, \prec)$ is a set $C \subseteq D$ such that $(\mathbf{T}, r) \in D \backslash C$ if and only if there exists $(\mathbf{T}', r') \in D$ such that $(\mathbf{T}, r) \prec (\mathbf{T}', r')$.

Definition 5 A *stable set* for the system $(D, \prec)$ is a set $G \subseteq D$ such that $(\mathbf{T}, r) \in D \backslash G$ if and only if there exists $(\mathbf{T}', r') \in G$ such that $(\mathbf{T}, r) \prec (\mathbf{T}', r')$.

While the core rules out any outcome that is dominated by another outcome, independently of whether the dominating outcome itself is in the core, a stable set rules out an outcome if and only if the dominating outcome itself is in the stable set. Hence, a stable set G satisfies $G \supseteq C$. A stable set G satisfies both *internal* (IS) and *external* (ES) stability in the following sense:

IS If $(\mathbf{T}, r) \in G$, then there does not exist $(\mathbf{T}', r') \in G$ such that $(\mathbf{T}, r) \prec (\mathbf{T}', r')$,

ES If $(\mathbf{T}, r) \in B$, then there exists $(\mathbf{T}', r') \in G$ such that $(\mathbf{T}, r) \prec (\mathbf{T}', r')$,

where the "good" set (G) and the "bad" set ($B := D \backslash G$) partition the set of outcomes. While it follows from the definition that the core is unique, there may in principle be multiple stable sets (although for the system $(D, \prec)$ there is a unique stable set as established in Proposition 2 below).

The following are our main characterization results. An outcome is in the core if and only if reform is chosen and each coalition large enough to ensure the status quo unanimously supports reform after a suitable redistribution among its members.

Proposition 1 *The core, C, of the system $(D, \prec)$ is given by:*

$$C = \left\{ (\mathbf{T}, r) \in D \,\middle|\, r = 1 \text{ and, } \forall S \in \mathcal{M}^0, \sum\nolimits_{i \in S} u_i(\mathbf{T}, 1) \geq \sum\nolimits_{i \in S} u_i(\mathbf{T}, 0) \right\}.$$

On the other hand, an outcome is in the unique stable set if and only if reform is chosen, independently of how the gains from reform are distributed.

Proposition 2 *The unique stable set, G, for the system $(D, \prec)$ is given by:*

$$G = \{(\mathbf{T}, r) \in D |\ r = 1\} .$$

Hence, both the core and the stable set admit only outcomes where the efficient alternative ("reform") is chosen. However, while the stable set equals the set of all outcomes leading to reform, the core is a proper subset of that set.

In the following two sections we will investigate the consequences of choosing the core as our solution concept and interpreting core outcomes as having a robust majority for reform. To supplement these findings, we will also discuss how our results would change if instead the stable set was applied as the solution concept.

3 Reaching unanimity

Suppose that the winners from an efficient reform does not constitute a majority large enough to pass reform in the absence of any binding conditional transfers; this means that the losers are sufficient numerous to ensure the status quo. Envision the following situation: The winners get together and discuss how best to achieve reform, they agree among themselves on a set of transfers that their members will make and offer these to the rest of the society. The reform gets passed and the transfers are implemented. Within the formal structure of our model, this corresponds to the winners blocking the status quo by means of an outcome in the core where losers are induced through compensation to join a majority for reform.

Proposition 3 below considers such a situation and shows that it is least expensive for the winners to compensate *all* losers to join the majority. Hence, even though decisions are made according to majority vote, winners might as well compensate all losers, so that reform is chosen unanimously after the conditional transfers.

For the statement of Proposition 3, we need the following definition.

Definition 6 An outcome $(\mathbf{T}, r) \in D$ is *unanimous* if

$$\{i \in N |\ u_i(\mathbf{T}, r) \geq u_i(\mathbf{T}, r')\} = N ,$$

where $r' \neq r$.

Proposition 3 *Assume $L \in \mathcal{M}^0$. Consider the set, E, of outcomes in the core that dominate the status quo with no conditional transfers, and where the blocking coalition is a subset of W:*

$$E := \{(\mathbf{T}, 1) \in C | \, \exists S \in 2^W \backslash \{\emptyset\} \text{ blocking } (\mathbf{0}, 0) \text{ by means of } (\mathbf{T}, 1)\} .$$

The set E is non-empty, and there exists a unanimous outcome in

$$\arg\max_{(\mathbf{T},1) \in E} \sum_{i \in W} u_i(\mathbf{T}, 1)) .$$

Proposition 3 is illustrated by the example of the introduction, where $N = \{1, 2, 3, 4, 5\}$, $v_1 > 4$ and, for $i \in \{2, 3, 4, 5\}$, $v_i = -1$. Hence, $\sum_{i \in N} v_i > 0$ so that Assumption 1 is satisfied, and $W = \{1\}$ and $L = \{2, 3, 4, 5\}$, so that $L \in \mathcal{M}^0$ (where we assume that simple majority is needed to pass reform, so that $q = 0$ and $\mathcal{M}^0 = \mathcal{M}^1$ consists of coalitions with 3 or more members). While it is sufficient for the winner to compensate only two losers to form a minimal majority, one can verify—by using the core condition of Proposition 1—that a core outcome requires a total compensation of $2 \cdot 3 = 6$ if the winner compensates two losers, a total compensation of $3 \cdot 1.5 = 4.5$ if the winner compensates three losers, and a total compensation of $4 \cdot 1 = 4$ if the winner compensates all four losers.

This result is subject to two caveats. First of all, Proposition 3 considers only core outcomes. In the example just mentioned, the core rules out the outcome where the winner forms a minimal majority for reform by transferring 1 to each of two losers. The resulting outcome is not in the core since it can, e.g., be blocked by a coalition consisting of one of the compensated losers and the two uncompensated losers. Through internal conditional transfers, this coalition can form a majority for the status quo, implying by Proposition 1 that the dominating outcome is *not* in the core and thus itself subject to counter proposals. In contrast, a stable set is a solution concept that rules out an outcome if *and only if* the dominating outcome itself is admitted by the solution concept. As shown in Proposition 2, any reform outcome—including the one that results when the winner forms a minimal majority for reform by transferring 1 to each of two losers—is in the unique stable set.

The other caveat is the following: Proposition 3 does not hold if the coalitions blocking the status quo with no conditional transfers are allowed to consist of both winners and losers. The following example illustrates this. Let $N = \{1, 2, 3\}$, where $v_1 = 6$ and, for $i \in \{2, 3\}$, $v_i = -2$. Hence, $\sum_{i \in N} v_i = 2$ so that Assumption 1 is satisfied, and $W = \{1\}$ and $L = \{2, 3\}$, so that $L \in \mathcal{M}^0$ (where we assume that simple majority is needed to pass reform, so that $q = 0$ and $\mathcal{M}^0 = \mathcal{M}^1$ consists of coalitions with 2 or more members). If 1 compensates both losers, then his utility is $6 - (2 + 2) = 2$ after reform when the sufficient conditional transfers have been paid. However, consider a blocking by $S = \{1, 2\}$ of $(\mathbf{0}, 0)$ by means of $(\mathbf{T}, 1)$, where $\mathbf{T}$ is determined as follows:

$$
\begin{array}{lll}
T_1(2,0) = 0 & T_2(3,0) = 0 & T_3(1,0) = 0 \\
T_1(3,0) = 0 & T_2(1,0) = 1 & T_3(2,0) = 0 \\
T_1(2,1) = 3 & T_2(3,1) = 0 & T_3(1,1) = 0 \\
T_1(3,1) = 0 & T_2(1,1) = 0 & T_3(2,1) = 0
\end{array}
$$

Then

$$
\begin{array}{lll}
u_1(\mathbf{0},0) = 0 & u_2(\mathbf{0},0) = 0 & u_3(\mathbf{0},0) = 0 \\
u_1(\mathbf{T},0) = 1 & u_2(\mathbf{T},0) = -1 & u_3(\mathbf{T},0) = 0 \\
u_1(\mathbf{0},1) = 6 & u_2(\mathbf{0},1) = -2 & u_3(\mathbf{0},1) = -2 \\
u_1(\mathbf{T},1) = 3 & u_2(\mathbf{T},1) = 1 & u_3(\mathbf{T},1) = -2
\end{array}
$$

Inspection shows that the condition of Proposition 1 is satisfied, so that $(\mathbf{T}, 1)$ is in the core. Moreover, according to Definition 2, $S = \{1, 2\}$ blocks $(\mathbf{0}, 0)$ by means of $(\mathbf{T}, 1)$. Since $u_1(\mathbf{T}, 1) = 3$, individual 1's utility is higher than it would have been if individual 1 had blocked the status quo on his own by compensating both losers. The trick is that individual 2, who is one of the two losers, by being part of the blocking coalition S can make a binding commitment to pay individual 1, the winner, a transfer if reform is not undertaken. This makes it harder for the uncompensated loser (individual 3) to block the reform, since it becomes more expensive to induce the compensated loser (individual 2) to vote against the reform.

4 An argument for wide compensations

Suppose now, as an alternative to the situation considered by Proposition 3 and as a follow-up to the second caveat above, that the winners from reform invite also some would-be losers into a coalition whose members collectively agree on a set of transfers that will make all of them better off. Allowing in this way also coalitions consisting of both winners and losers to block the status quo does not remove all reasons for supermajorities, as Proposition 4 below shows: When the direct gains of reform for the winners are small compared to the direct losses for the losers, there is (perhaps counter-intuitively) a pressure to spread the gains more evenly by compensating many losers.

The argument of Proposition 4 can be illustrated by the example considered at the end of Section 3. In the blocking outcome of that example, two of the three core constraints of Proposition 1 are satisfied with equality:

$$\sum_{i\in\{1,3\}} u_i(\mathbf{T},1) = 3-2 = 1+0 = \sum_{i\in\{1,3\}} u_i(\mathbf{T},0)\,,$$

$$\sum_{i\in\{2,3\}} u_i(\mathbf{T},1) = 1-2 = -1+0 = \sum_{i\in\{2,3\}} u_i(\mathbf{T},0)\,.$$

Furthermore, inspection verifies that all three constraints can *not* be satisfied if the direct gain of individual 1, v_1, is reduced below 6, *unless also individual* 3 *is compensated*. If $4 < v_1 < 6$, then reform is efficient in the Kaldor-Hicks sense. Still, the status quo (with no conditional transfers) can be blocked by a core outcome only if both losers are compensated. A generalized version of this insight is formally established through the following result. It states that there can be uncompensated losers in a core outcome dominating the no-transfer status quo, only if the direct gains to the winners exceed the direct losses of the losers with a sufficient margin.

Proposition 4 *Consider the set, F, of outcomes in the core that dominate the status quo with no conditional transfers:*

$$F := \{(\mathbf{T},1)\in C|\,\exists S\in 2^N\backslash\{\emptyset\} \text{ blocking } (\mathbf{0},0) \text{ by means of } (\mathbf{T},1)\}\,.$$

Let $(\mathbf{T},1)\in F$. Then

$$\sum_{i\in N} v_i \;\geq\; \frac{\frac{|N|-1}{2}+q}{\frac{|N|+1}{2}-q-|U|}\Big(-\sum_{i\in U} v_i\Big)\,,$$

where $U := \{i\in L|u_i(\mathbf{T},1)=v_i\}$ denotes the set of uncompensated losers.

In contrast to Proposition 3, Proposition 4 assumes neither that the blocking coalition consists only of winners nor that the losers can ensure the status quo. It still provides a necessary condition for an outcome $(\mathbf{T}, 1)$ blocking $(\mathbf{0}, 0)$ to be in the core: As long as there are uncompensated losers, $(\mathbf{T}, 1)$ is in the core only if $\sum_{i \in N} v_i$ is sufficiently positive; i.e., more than efficiency in the Kaldor-Hicks sense is required. This necessary condition becomes more demanding when

- an additional individual is included in the set of uncompensated losers, both because $\left(-\sum_{i \in U} v_i\right)$ increases, and because the larger $|U|$ increases the factor with which $\left(-\sum_{i \in U} v_i\right)$ is multiplied,
- a larger qualified majority is needed to pass reform, because a larger q increases the factor with which $\left(-\sum_{i \in U} v_i\right)$ is multiplied.

This factor equals the number of individuals in a maximal coalition not large enough to pass reform divided by the number of individuals that must join U to form a minimal coalition large enough to ensure the status quo. The factor equals 1 if $|U| = 1$ and $q = 0$.

Thus, when reform is only marginally efficient, there will be an upper bound on the number of losers that can remain uncompensated in a core outcome dominating the no-transfer status quo. *Proposition 4 thereby yields an argument for supermajorities, or at least, for compensations that extend beyond those needed to form a minimal majority for reform.* The result can be illustrated by revisiting the example of the introduction, where $N = \{1, 2, 3, 4, 5\}$, $v_1 > 4$ and, for $i \in \{2, 3, 4, 5\}$, $v_i = -1$, and simple majority is needed to pass reform. By applying Proposition 4, we find that, in any core outcome dominating the no-transfer status quo, there can be no uncompensated loser if $v_1 < 5$, and at most one uncompensated loser if $5 \leq v_1 < 8$.

5 Alternative modeling choices

The analysis that has been presented in this paper may be subjected to two kinds of fundamental criticism. According to one kind of criticism the analysis contains unnecessary formalism by describing in detail the transfer schedules for all individuals. According to another kind of criticism, it is a strong assumption — made in the formulation of dominance relation $\prec$ — to impose that conditional transfers once offered cannot be taken back; only further transfers can be made.

In this section we consider alternative modeling choices designed to take these kinds of criticism into account. In particular, we consider

- a standard cooperative formulation based on a characteristic function, and
- an alternative to the modeling chosen in this paper, but where conditional transfers can be withdrawn.

While these alternative formulations are certainly feasible, they do not, however, provide an explanation for non-minimal majorities. The reason is that existence of a non-empty core cannot be established; even in simple examples the core becomes empty.

5.1 A characteristic function formulation

A standard way to model the kind of coalitional bargaining considered in this paper, going all the way back to von Neumann and Morgenstern (1947), is to model only the end outcome. The transfers are not explicitly modeled, as this paper does, only what each individual ends up with. This leads to a cooperative game described by a characteristic function $v : 2^N \backslash \{\emptyset\} \to \mathbb{R}$, where $v(S)$ is the surplus that coalition S can guarantee itself; this surplus is transferable freely among its member. In the setting of qualified majority voting, this yields the following characteristic function:

$$\begin{aligned} v(S) &= \max\left\{0, \sum\nolimits_{i\in S} v_i\right\} && \text{if } S \in \mathcal{M}^1\,, \\ v(S) &= 0 && \text{if } S \in \mathcal{M}^0 \backslash \mathcal{M}^1\,, \\ v(S) &= \min\left\{0, \sum\nolimits_{i\in S} v_i\right\} && \text{if } S \notin \mathcal{M}^0\,. \end{aligned}$$

Hence, while coalitions that constitute a (qualified) majority can ensure a most preferred alternative, coalitions that cannot even ensure the status quo are by the characteristic function assigned a least preferred alternative. Constructing the characteristic function by assigning to coalitions the worst outcomes that they cannot avoid is the usual convention in the theory of cooperative games. This procedure maximizes the possibility of a non-empty core. In the case where $q > 0$, so that a majority for reform must be qualified and $\mathcal{M}^1 \subset \mathcal{M}^0$, coalitions large enough to ensure the status quo but not large enough to constitute a winning coalition for reform are by the characteristic function assigned the status quo.

The construction of the characteristic function can be illustrated by the 3 voter example introduced in Section 3. In this example we have

$$v(N) = 2$$

$$v(\{1,2\}) = 4 \qquad v(\{1,3\}) = 4 \qquad v(\{2,3\}) = 0$$

$$v(\{1\}) = 0 \qquad v(\{2\}) = -2 \qquad v(\{3\}) = -2$$

An *imputation* $\mathbf{x} = (x_i)_{i \in N}$ is in the core if $\sum_{i \in N} x_i = v(N)$ and, for each subcoalition S, $\sum_{i \in S} x_i \geq v(S)$. This standard characteristic function formulation leads to an empty core in the simple example above, as the restrictions $x_1 + x_2 + x_3 = 2$, $x_1 + x_2 \geq 4$, $x_1 + x_3 \geq 4$, and $x_2 + x_3 \geq 0$ cannot be simultaneously satisfied.

5.2 Withdrawable conditional transfers

Conditional transfers that can be withdrawn can be modeled by replacing Definition 2 with the following alternative:

Definition 7 Coalition S ($\in 2^N \backslash \{\emptyset\}$) *blocks* an outcome $(\mathbf{T}, r) \in D$ by means of another outcome $(\mathbf{T}', r') \in D$ if

1. $\mathbf{0}_S \leq \mathbf{T}'_S$ and $\mathbf{T}_{N \backslash S} = \mathbf{T}'_{N \backslash S}$,
2. $\forall i \in S, u_i(\mathbf{T}, r) < u_i(\mathbf{T}', r')$.

Hence, the condition $\mathbf{T}_S \leq \mathbf{T}'_S$ of Definition 2 is replaced by $\mathbf{0}_S \leq \mathbf{T}'_S$, reflecting that its previous commitments are not binding when the blocking coalition proposes a new transfer schedule. Otherwise the set-up of Section 2 is kept unchanged.

However, by combining the following result with the observation just made for the characteristic function formulation, it follows that core existence fails also for this modeling alternative.

Proposition 5 *If* $\sum_{i \in S} u_i(\mathbf{T}, r) < v(S)$, *then there exists* $(\mathbf{T}', r') \in D$ *such that* S *blocks (in the sense of Definition 7)* $(\mathbf{T}, r)$ *by means of* $(\mathbf{T}', r')$.

5.3 Discussion

Proposition 5 means that the simpler characteristic function formulation where individual transfers are not explicitly modeled corresponds to a situation where conditional transfers can be withdrawn. Such alternative modeling, however, does not ensure core existence and, therefore, cannot replace the modeling and results of the main parts of the present paper as an explanation for non-minimal majorities. We can summarize this as follows: On the one hand, when conditional transfers can be withdrawn, we do not obtain an explanation for non-minimal majorities. On the other hand, when conditional transfers cannot be withdrawn, the situation cannot be modeled by the simpler characteristic function formulation.

The fact that cooperative games need not have a non-empty core (unless restrictions are imposed on the characteristic function) has been a main motivating force behind the development of alternative cooperative solution concepts (see, e.g., Aumann, 1987). Although such solution concepts can be applied to the two alternative models considered in this section, we consider this to be outside the scope of the present paper. Instead we have concentrated our attention on the phenomenon that we want to explain: the frequent occurrence of non-minimal majorities. This focus has led us to study a different underlying situation, by assuming that conditional transfers cannot be withdrawn, while keeping the core as our solution concept and interpreting core outcomes as having a robust majority for reform.

6 Concluding remark

We have presented a novel approach to voting theory in general and to the puzzle of non-minimal majorities in particular. The approach is motivated by the fact that, with a secret ballot and non-transferable voting rights, votes cannot be bought. Commitments must instead be made conditional on the decision of the electorate.

We show that the application of such conditional commitments to a situation where voters decide, through majority voting in a secret ballot, between the alternatives of "reform" and "status quo" leads to outcomes where a minority of winners from reform compensate all losers from reform. With such compensation, there is unanimity for reform. The winners find it optimal to compensate all losers since this is actually a least expensive way for them to ensure a robust majority for reform. Thus, a situation with binding conditional transfers entails that reform will be undertaken in a manner that benefits all individuals. It thereby constitutes an actual Pareto-improvement, leading to a welfare improvement.

7 Appendix: Proofs

The following is an immediate, but useful, observation.

Lemma 1 *If* $(\mathbf{T}, r)$ *is dominated by* $(\mathbf{T}', r')$*, then* $r \neq r'$*.*

Proof. Assume that $(\mathbf{T}, r) \prec (\mathbf{T}', r')$. Hence, there exists some S that blocks $(\mathbf{T}, r)$ by means of $(\mathbf{T}', r')$. By the first condition of Definition 2, we have that $\mathbf{T}_S \leq \mathbf{T}'_S$ and $\mathbf{T}_{N \setminus S} = \mathbf{T}'_{N \setminus S}$, and it follows that $\sum_{i \in S} u_i(\mathbf{T}, 0) \geq \sum_{i \in S} u_i(\mathbf{T}', 0)$ and $\sum_{i \in S} u_i(\mathbf{T}, 1) \geq \sum_{i \in S} u_i(\mathbf{T}', 1)$. Therefore, the second condition of Definition 2 can only be satisfied if $r \neq r'$. ■

Propositions 1 and 2 will be proven by means of Lemma 2, which uses the concept of a *unanimous* outcome, defined in Definition 6 of Section 3.

Lemma 2 *(i) The set of unanimous outcomes is non-empty.*

(ii) If $(\mathbf{T}, r)$ *is unanimous, then* $r = 1$*.*

(iii) If $(\mathbf{T}, r)$ *is unanimous, then* $(\mathbf{T}, r) \in C$*.*

(iv) For any $(\mathbf{T}', r') \in D$ *with* $r' = 0$*, there exists a unanimous outcome* $(\mathbf{T}, r)$ *such that* $(\mathbf{T}', r') \prec (\mathbf{T}, r)$*.*

Proof. *Part (i).* It follows from Assumption 1 that there exists $\mathbf{T}$ with $T_i(j, r) > 0$ only if $i \in W$, $j \in L$, and $r = 1$, such that $u_i(\mathbf{T}, 1) = -\sum_{j \in L} T_i(j, 1) + v_i \geq 0$ if $i \in W$ and $u_j(\mathbf{T}, 1) = \sum_{i \in W} T_i(j, 1) + v_j \geq 0$ if $j \in L$. Then, for all $i \in N$, $u_i(\mathbf{T}, 1) \geq 0 = u_i(\mathbf{T}, 0)$, implying that $(\mathbf{T}, 1)$ is unanimous.

Part (ii). This follows directly from Assumption 1.

Part (iii). Suppose that $(\mathbf{T}, r)$ is unanimous and there exists $(\mathbf{T}', r')$ such that $(\mathbf{T}, r) \prec (\mathbf{T}', r')$. Then $r = 1$ by part (ii) of this lemma, and $r' = 0$ by Lemma 1.

Hence, there exists S such that S blocks $(\mathbf{T}, 1)$ by means of $(\mathbf{T}', 0)$. By Definition 2, we have that $\mathbf{T}_S \leq \mathbf{T}'_S$ and $\mathbf{T}_{N\setminus S} = \mathbf{T}'_{N\setminus S}$, and it follows from the definition of unanimity that $\sum_{i\in S} u_i(\mathbf{T}, 1) \geq \sum_{i\in S} u_i(\mathbf{T}, 0) \geq \sum_{i\in S} u_i(\mathbf{T}', 0)$. Therefore, the second condition of Definition 2 cannot be satisfied. Hence, blocking $(\mathbf{T}, r)$ is impossible if $(\mathbf{T}, r)$ is unanimous, implying that any unanimous outcome is in the core.

Part (iv). Define, for all $i \in N$, $v'_i := u_i(\mathbf{T}', 1) - u_i(\mathbf{T}', 0)$, and determine W' and L' as follows: $W' := \{i \in N | \, v'_i \geq 0\}$ and $L' := \{i \in N | \, v'_i < 0\}$. Since $\sum_{i\in N} v'_i = \sum_{i\in N} v_i$, Assumption 1 entails that W' is non-empty and there exists $\mathbf{T} \geq \mathbf{T}'$ with $T_i(j, r) > T'_i(j, r)$ only if $i \in W'$, $j \in L'$, and $r = 1$, such that $u_i(\mathbf{T}, 1) - u_i(\mathbf{T}', 0) = -\sum_{j\in L} \left(T_i(j, 1) - T'_i(j, 1)\right) + v_i > 0$ if $i \in W'$ and $u_j(\mathbf{T}, 1) - u_j(\mathbf{T}', 0) = \sum_{i\in W} \left(T_i(j, 1) - T'_i(j, 1)\right) + v_j > 0$ if $j \in L'$. Then, for all $i \in N$, $u_i(\mathbf{T}, 1) > u_i(\mathbf{T}', 0) = u_i(\mathbf{T}, 0)$, implying that $(\mathbf{T}, 1)$ is unanimous and $(\mathbf{T}', 0) \prec (\mathbf{T}, 1)$. ■

Proof of Proposition 1. We establish the equality in Proposition 1 by showing $\subseteq$ in part 1 and $\supseteq$ in part 2.

Part 1: $(\mathbf{T}, r) \in C$ only if $r = 1$ and, $\forall S \in \mathcal{M}^0$, $\sum_{i\in S} u_i(\mathbf{T}, 1) \geq \sum_{i\in S} u_i(\mathbf{T}, 0)$.

Consider any $(\mathbf{T}, r) \in D$. It is sufficient to show that $(\mathbf{T}, r) \in D\setminus C$ if (i) $r = 0$ or (ii) $r = 1$ and $\exists S \in \mathcal{M}^0$ s.t. $\sum_{i\in S} u_i(\mathbf{T}, 1) < \sum_{i\in S} u_i(\mathbf{T}, 0)$.

(i) Suppose $r = 0$. Then $(\mathbf{T}, 0) \in D\setminus C$ by Definition 4 and Lemma 2(iv).

(ii) Suppose $r = 1$ and $\exists S \in \mathcal{M}^0$ s.t.

$$\sum_{i\in S} u_i(\mathbf{T}, 1) < \sum_{i\in S} u_i(\mathbf{T}, 0)\,. \tag{1}$$

Define, for all $i \in S$, $v'_i := u_i(\mathbf{T}, 1) - u_i(\mathbf{T}, 0)$, and determine W' and L' as follows: $W' := \{i \in S | \, v'_i \geq 0\}$ and $L' := \{i \in S | \, v'_i < 0\}$. Since $\sum_{i\in S} v'_i = \sum_{i\in S} \big(u_i(\mathbf{T}, 1) - u_i(\mathbf{T}, 0)\big)$, it is a consequence of (1) that L' is non-empty. Hence, there exists some $\mathbf{T}'$ derived from $\mathbf{T}$ by specifying additional transfers from L' to W' conditional on the status quo ($r = 0$), such that S blocks $(\mathbf{T}, 1)$ by means of $(\mathbf{T}', 0)$: $\exists \mathbf{T}'$ ($\geq \mathbf{T}$) with $T'_i(j, r) > T_i(j, r)$ only if $i \in L'$, $j \in W'$ and $r = 0$, such that $-\sum_{j\in W'} \big(T'_i(j, 0) - T_i(j, 0)\big) - v'_i > 0$ if $i \in L'$ and $\sum_{i\in L'} \big(T'_i(j, 0) - T_i(j, 0)\big) - v'_j > 0$ if $j \in W'$. Then,

$$u_i(\mathbf{T}, 1) = u_i(\mathbf{T}', 1) < u_i(\mathbf{T}', 0) \quad \text{if} \quad i \in S\,, \tag{2}$$

implying that $(\mathbf{T}', 0) \in D$ (since $S \in \mathcal{M}^0$ combined with (2) entail $0 \in \rho(\mathbf{T}')$) and $(\mathbf{T}, 1) \prec (\mathbf{T}', 0)$ (by (2) since $\mathbf{T}_{N\setminus S} = \mathbf{T}'_{N\setminus S}$). Hence, $(\mathbf{T}, 1) \in D\setminus C$ by Definition 4.

This completes part 1.

Part 2: $(\mathbf{T}, r) \in C$ if $r = 1$ and, $\forall S \in \mathcal{M}^0$, $\sum_{i\in S} u_i(\mathbf{T}, 1) \geq \sum_{i\in S} u_i(\mathbf{T}, 0)$.

Suppose $(\mathbf{T}, 1) \in D\backslash C$. It is sufficient to show that $\exists S \in \mathcal{M}^0$ s.t. $\sum_{i\in S} u_i(\mathbf{T}, 1) < \sum_{i\in S} u_i(\mathbf{T}, 0)$.

By Definition 4 and Lemma 1, there exist S' and $(\mathbf{T}', 0) \in D$ such that S' blocks $(\mathbf{T}, 1)$ by means of $(\mathbf{T}', 0)$. We may w.l.o.g. set $T_i'(j, 1) = T_i(j, 1)$ for all $i \in S'$ and $j \in N$. The reason is that this is permitted by part 1 of Definition 2, it does not affect part 2 of Definition 2, and, if anything, it expands

$$S := \{i \in N |\, u_i(\mathbf{T}', 1) \leq u_i(\mathbf{T}', 0)\}\,, \tag{3}$$

i.e., the set of voters that weakly supports $r = 0$ under $\mathbf{T}'$, since it means that

$$u_i(\mathbf{T}', 1) = u_i(\mathbf{T}, 1) < u_i(\mathbf{T}', 0) \quad \text{if} \quad i \in S'\,,$$

so that $S' \subseteq S$, and it makes $(\mathbf{T}', 0)$ no less attractive as compared to $(\mathbf{T}', 1)$ for any $i \in N\backslash S'$, keeping in mind that $\mathbf{T}_{N\backslash S'} = \mathbf{T}'_{N\backslash S'}$.

It remains to show that S as defined in (3) satisfies $S \in \mathcal{M}^0$ and $\sum_{i\in S} u_i(\mathbf{T}, 1) < \sum_{i\in S} u_i(\mathbf{T}, 0)$. Since $(\mathbf{T}', 0) \in D$ (so that $0 \in \rho(\mathbf{T}')$), the set of voters that weakly supports $r = 0$ under $\mathbf{T}'$ is large enough: $S \in \mathcal{M}^0$. By $\mathbf{T}_{N\backslash S'} = \mathbf{T}'_{N\backslash S'}$, we get

$$\sum\nolimits_{i\in S} u_i(\mathbf{T}', 0) \leq \sum\nolimits_{i\in S} u_i(\mathbf{T}, 0)\,, \tag{4}$$

since no $i \in N\backslash S \subseteq N\backslash S'$ increases conditional transfers to others when moving from $\mathbf{T}$ to $\mathbf{T}'$. Moreover, since $T_i(j, 1) = T_i'(j, 1)$ for all $i, j \in N$,

$$\begin{aligned} u_i(\mathbf{T}, 1) = u_i(\mathbf{T}', 1) \leq u_i(\mathbf{T}', 0) \quad &\text{if} \quad i \in S\backslash S'\,, \\ u_i(\mathbf{T}, 1) < u_i(\mathbf{T}', 0) \quad &\text{if} \quad i \in S'\,. \end{aligned} \tag{5}$$

Combining (4) and (5) yields

$$\sum\nolimits_{i\in S} u_i(\mathbf{T}, 1) < \sum\nolimits_{i\in S} u_i(\mathbf{T}, 0)\,,$$

where $S \in \mathcal{M}^0$. Thus, $\exists S \in \mathcal{M}^0$ s.t. $\sum_{i\in S} u_i(\mathbf{T}, 1) < \sum_{i\in S} u_i(\mathbf{T}, 0)$ if $(\mathbf{T}, 1) \in D\backslash C$.

This completes the proof of part 2. ■

Proof of Proposition 2. The following holds for any stable set G: By (ES) and Lemma 2(iii), any unanimous outcome is an element of G. Moreover, by (IS) and

Lemma 2(iv), $(\mathbf{T}', r') \in D\backslash G$ if $r' = 0$. Finally, by (ES) and Lemma 1, $(\mathbf{T}, r) \in G$ if $r = 1$. Hence, $\{(\mathbf{T}, r) \in D | \, r = 1\}$ is the only candidate for a stable set. It follows from Lemma 1 that $\{(\mathbf{T}, r) \in D | \, r = 1\}$ satisfies (IS), and it follows from Lemma 2(ii)&(iv) that $\{(\mathbf{T}, r) \in D | \, r = 1\}$ satisfies (ES). ■

Proof of Proposition 3. Following the proof of Lemma 2(i) and recalling part 2 of Definition 2, there exists a *unanimous* outcome $(\mathbf{T}, 1)$ in E that (i) maximizes $\sum_{i \in W} u_i(\mathbf{T}'', 1)$ over all *unanimous* outcomes $(\mathbf{T}'', 1)$ in E and (ii) satisfies $u_i(\mathbf{T}, 1) > 0$ if $i \in S \subseteq W$ and $u_i(\mathbf{T}, 1) = 0$ if $i \in N\backslash S \supseteq L$. Moreover, we have that

$$\sum\nolimits_{i \in W} u_i(\mathbf{T}, 1) = \sum\nolimits_{i \in W} v_i - \Big(\sum\nolimits_{i \in L} (-v_i) \Big) = \sum\nolimits_{i \in N} v_i \, ,$$

$$\sum\nolimits_{i \in L} u_i(\mathbf{T}, 1) = \sum\nolimits_{i \in L} v_i + \Big(\sum\nolimits_{i \in L} (-v_i) \Big) = 0 \, .$$

Let $(\mathbf{T}', 1)$ be any outcome in E. To establish

$$(\mathbf{T}, 1) \in \arg \max_{(\mathbf{T}, 1) \in E} \sum\nolimits_{i \in W} u_i(\mathbf{T}, 1)) \, ,$$

we must show that $\sum_{i \in W} u_i(\mathbf{T}', 1) \leq \sum_{i \in N} v_i$ or, equivalently,

$$\sum\nolimits_{i \in L} u_i(\mathbf{T}', 1) \geq 0 \, . \tag{6}$$

Since $(\mathbf{T}', 1) \in E \subseteq C$ and $L \in \mathcal{M}^0$, it follows from Proposition 1 that

$$\sum\nolimits_{i \in L} u_i(\mathbf{T}', 1) \geq \sum\nolimits_{i \in L} u_i(\mathbf{T}', 0) \geq 0 \, ,$$

since, for any $i \in L \subseteq N\backslash S$, i is not a member of the blocking coalition and, thus, makes no conditional transfers to others under $\mathbf{T}'$. Hence, (6) is satisfied. ■

Proof of Proposition 4. If $i \in U$, then $u_i(\mathbf{T}, 1) = v_i < 0 = u_i(\mathbf{0}, 0)$, and it follows from Definition 2 that (i) i is not a member of the blocking coalition and (ii) for all $j \in N\backslash\{i\}$, it holds that $T_i(j, 0) = T_i(j, 1) = 0$, $T_j(i, 0) \geq 0$, and $T_j(i, 1) = 0$. This implies, for all $i \in U$, $u_i(\mathbf{T}, 1) = v_i < 0 \leq u_i(\mathbf{T}, 0)$, so that $U \notin \mathcal{M}^0$ since $1 \in \rho(\mathbf{T})$. Moreover,

$$\sum\nolimits_{i \in M} u_i(\mathbf{T}, 1) = \sum\nolimits_{i \in M} v_i \quad \text{and} \quad \sum\nolimits_{i \in M} u_i(\mathbf{T}, 0) = - \sum\nolimits_{i \in U} u_i(\mathbf{T}, 0) \leq 0 \, , \tag{7}$$

since transfers among the members of $M := N\backslash U$ $(\in \mathcal{M}^1)$ cancel out.

Denote by $k := (|N|+1)/2 - q - |U|$ the number of individuals from M that must join U to form a coalition large enough to ensure the status quo. Furthermore, let

$$\mathcal{K} := \{K \in 2^M \setminus \{\emptyset\} \mid |K| = k\}$$

be the collection of minimal subsets K of M such that $K \cup U$ can ensure the status quo. Since $(\mathbf{T}, 1) \in F \subseteq C$, it follows from Proposition 1 that, for all $K \in \mathcal{K}$,

$$\sum\nolimits_{i \in K} \big(u_i(\mathbf{T}, 1) - u_i(\mathbf{T}, 0)\big) \;\geq\; \sum\nolimits_{i \in U} \big(-v_i + u_i(\mathbf{T}, 0)\big)\,.$$

By summing the l.h.s. and r.h.s. over all $K \in \mathcal{K}$, and noting that each member of M appears in a fraction $k/|M|$ of these inequalities, (7) implies

$$\begin{aligned}\sum\nolimits_{i \in M} v_i &= \sum\nolimits_{i \in M} \big(u_i(\mathbf{T}, 1) - u_i(\mathbf{T}, 0)\big) - \sum\nolimits_{i \in U} u_i(\mathbf{T}, 0) \\ &\geq \sum\nolimits_{i \in M} \big(u_i(\mathbf{T}, 1) - u_i(\mathbf{T}, 0)\big) - \frac{|M|}{k} \sum\nolimits_{i \in U} u_i(\mathbf{T}, 0) \geq -\frac{|M|}{k} \sum\nolimits_{i \in U} v_i\,,\end{aligned}$$

since $|M|/k \geq 1$ and $\sum_{i \in U} u_i(\mathbf{T}, 0) \geq 0$. By adding $\sum_{i \in U} v_i$ on both sides, and recalling that M and U partition N, we obtain

$$\sum\nolimits_{i \in N} v_i \;\geq\; -\frac{|M| - k}{k} \sum\nolimits_{i \in U} v_i\,.$$

The result follows by the definition of k since $|M| - k = |N| - |U| - \big((|N|+1)/2 - q - |U|\big) = (|N|-1)/2 + q$. ■

Proof of Proposition 5. Let

$$u(S; \mathbf{T}_{N \setminus S}) := \max\left\{\sum_{i \in S} \sum_{j \in N \setminus S} T_j(i, 0), \sum_{i \in S} \left(\sum_{j \in N \setminus S} T_j(i, 1) + v_i\right)\right\} \;\text{ if }\; S \in \mathcal{M}^1\,,$$

$$u(S; \mathbf{T}_{N \setminus S}) := \sum_{i \in S} \sum_{j \in N \setminus S} T_j(i, 0) \;\text{ if }\; S \in \mathcal{M}^0 \setminus \mathcal{M}^1\,,$$

$$u(S; \mathbf{T}_{N \setminus S}) := \min\left\{\sum_{i \in S} \sum_{j \in N \setminus S} T_j(i, 0), \sum_{i \in S} \left(\sum_{j \in N \setminus S} T_j(i, 1) + v_i\right)\right\} \;\text{ if }\; S \notin \mathcal{M}^0\,.$$

Clearly, $u(S; \mathbf{T}_{N \setminus S}) \geq v(S)$ since $u(S; \mathbf{T}_{N \setminus S})$ includes non-negative transfers from individuals outside S. Suppose $\sum_{i \in S} u_i(\mathbf{T}, r) < u(S; \mathbf{T}_{N \setminus S})$. Then there exists $(\mathbf{T}', r') \in D$ satisfying $\mathbf{0}_S \leq \mathbf{T}'_S$ and $\mathbf{T}_{N \setminus S} = \mathbf{T}'_{N \setminus S}$ such that

$$\begin{aligned}&\Big(\sum\nolimits_{i \in S} u_i(\mathbf{T}, r) <\Big)\; u(S; \mathbf{T}_{N \setminus S}) \leq \sum\nolimits_{i \in S} u_i(\mathbf{T}', r')\,, \\ &\forall i \in S, \;\; u_i(\mathbf{T}, r) < u_i(\mathbf{T}', r')\,,\end{aligned}$$

by letting $T'_i(j,0) = T'_i(j,1) = 0$ if $i \in S$ and $j \in N \backslash S$ and choosing $T'_i(j,0)$ and $T'_i(j,1)$ appropriately if i, $j \in S$. Hence, S blocks $(\mathbf{T}, r)$ by means of $(\mathbf{T}', r')$. Since $\sum_{i \in S} u_i(\mathbf{T}, r) < v(S)$ implies $\sum_{i \in S} u_i(\mathbf{T}, r) < u(S; \mathbf{T}_{N \backslash S})$, the result follows. ■

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