

MEMORANDUM

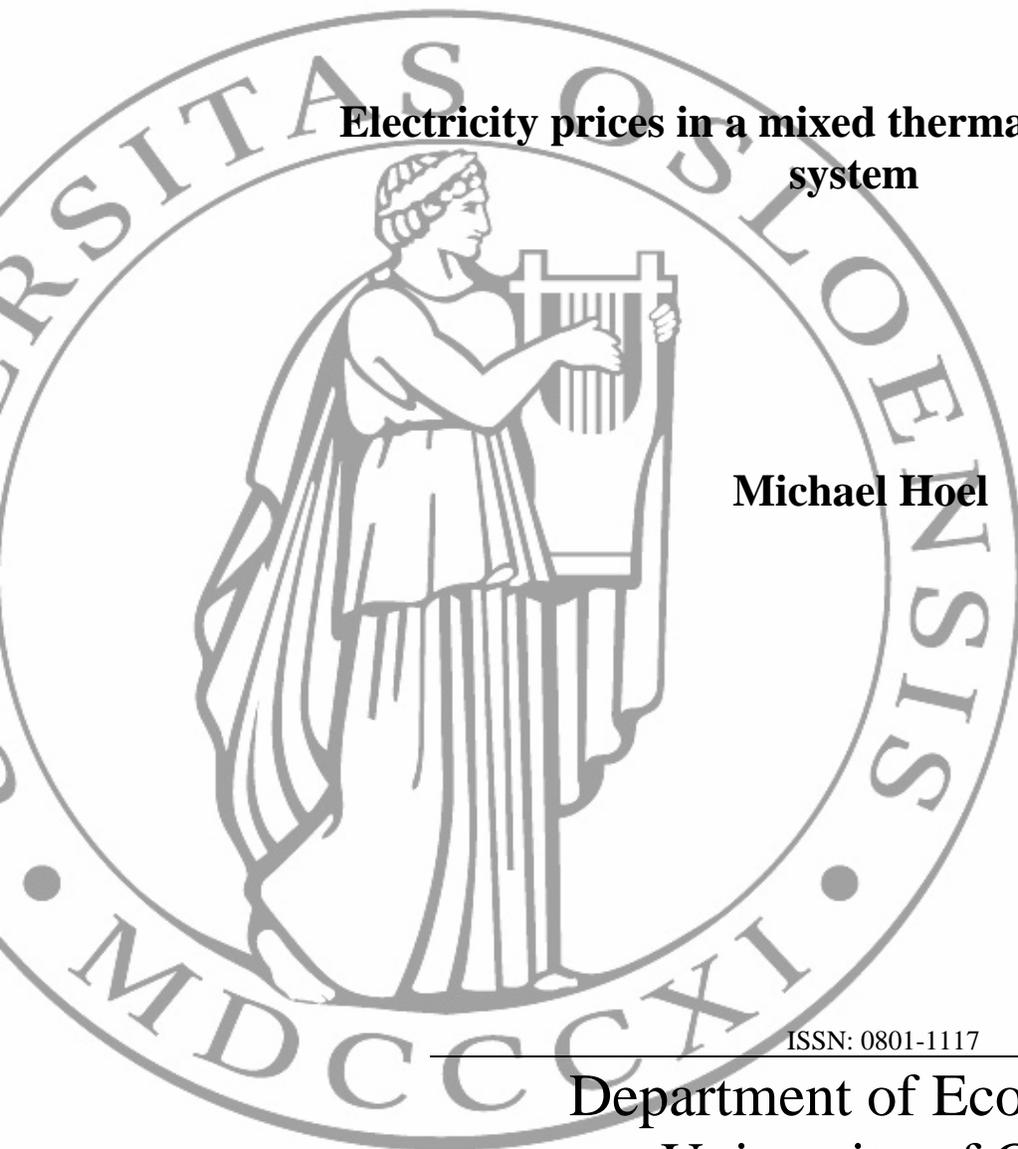
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Electricity prices in a mixed thermal and hydropower system

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November 29, 2004

Electricity prices in a mixed thermal and hydropower system

by

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Abstract

When a monopolistic hydro producer interacts with a competitive thermal fringe, the short-run revenue function of the hydro monopolist is non-concave. This implies that even if the demand function is stationary, equilibrium prices may fluctuate through the year. For given capacities, both hydro and thermal producers are better off under such an outcome than under the competitive outcome with constant prices, while consumers are worse off. Prices may fluctuate through the year also in the long-run equilibrium where capacities are endogenous. In such an equilibrium the hydropower monopoly will get a lower profit than it would have gotten had it been a price taker.

Keywords: *Electricity prices, Hydropower*

JEL classification: *L12, L13, L94, Q25*

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1 Introduction

Electricity markets have been deregulated in a number of countries. This has led to a considerable research, both theoretical and empirical, of how electricity markets perform.¹ However, there are relatively few studies that explicitly focus on markets where hydropower plays an important role. This is quite surprising, as 1/3 of all countries in the world depend on hydropower for over 50% of their electricity, see Edwards (2003). The share of hydropower is quite small in USA (about 6% in 2001, but considerably higher for some states²). For several other countries and regions the share of is considerably higher: In 2001 the proportion of electricity produced by hydropower was 58% in Canada, 67% in Central and South America, and 19% in Western Europe.³

Hydropower has some features making it quite different from thermal power. A thermal power plant will typically have a short-run capacity constraint that limits output at any time. Such a capacity constraint will also exist for a hydro plant, but will often be less important than the constraint given by the total amount of water reservoirs of the power plant. These reservoirs are replenished through a yearly cycle of precipitation, and the most important decision of the producer is how to allocate the water reservoirs through the year.

Von der Fehr and Sandsbråten (1997) give a good discussion of some of the properties of electricity markets where hydropower plays an important role. They explicitly focus on the technological complementarities between hydro and thermal systems, but they do not consider market power.⁴ Among the relatively few papers that explicitly consider market power in the context of hydropower are Crampes and Moreaux (2001), Garcia et al. (2001, 2004), and

¹ Important recent contributions include Amundsen and Bergman (2002), Borenstein (2002), Borenstein et al. (2002), Joskow (2001), Joskow and Kahn (2002), Newbery (2002), Spear (2003) and Wolak (2003).

² In the two large states California and New York the share of hydropower in total electricity in 2002 was about 17% in each.

³ Source: *International Energy Annual 2002*.

⁴ The interaction between hydropower and thermal power under competitive conditions is also briefly discussed in Borenstein et al. (2002, Appendix C).

Ambec and Doucet (2003). Crampes and Moreaux consider a model of two power producers, one of which is hydropower. They analyse the competitive case, the case of a monopoly controlling both firms, and the duopoly case. Garcia et. al consider the case of hydro producers engaged in dynamic Bertrand competition. Unlike the present study and the other studies referred to above, Garcia et al. allow for uncertainty regarding the replenishment of the water reservoirs. They also briefly consider the case where there are thermal producers in addition to the hydro producers. Ambec et al. consider various market forms, but only consider a pure system of hydropower.

The present paper studies the case of a monopolistic hydro producer in combination with a competitive thermal fringe. With this market structure, the short-run revenue function of the hydro monopolist is non-concave. This has important implications for the equilibrium. In particular, even if the demand function is stationary through the year, the equilibrium may have the property that prices fluctuate through the year. In a related paper, Førsund and Hoel (2004) treat the case of a pure hydro system, but with the possibility of electricity trade with neighbouring countries or regions. In this case the capacity limit of the transmission cables to neighbouring countries/regions plays the same role as the capacity limit of the thermal producers in the present case, leading to a non-concave revenue function in both cases. Unlike the present paper, Førsund and Hoel only consider the short-run equilibrium.

The rest of the paper is organized as follows. Section 2 gives a brief description of the short-run properties of a simple market with both thermal and hydro producers when all producers are price takers. Section 3 treats the case of a hydro monopoly, and derives the result mentioned above about fluctuating prices. Section 4 extends the results to the more realistic case in which the demand function differs across periods within year, e.g. due to climate fluctuations, and in Section 5 I show that the results derived do not depend on the specific aggregate supply function that is assumed in previous sections.

In Sections 6 - 7 I return to the case of a stationary demand function, and consider long-run equilibria. The competitive long-run equilibrium is treated in Section 6, and in Section 7 the long-run equilibrium for our dominant firm case is discussed. Section 8 concludes.

2 Perfect competition

Consider first the simplest possible model of a pure thermal system. Short-run unit operating costs are c , and in the short run there is a capacity constraint K . The short-run supply function thus has an inverse L shape as in Figure 1, where x and p are the output and price of electricity, respectively. In Section 5 it is shown that our qualitative results do not depend on this simplification.

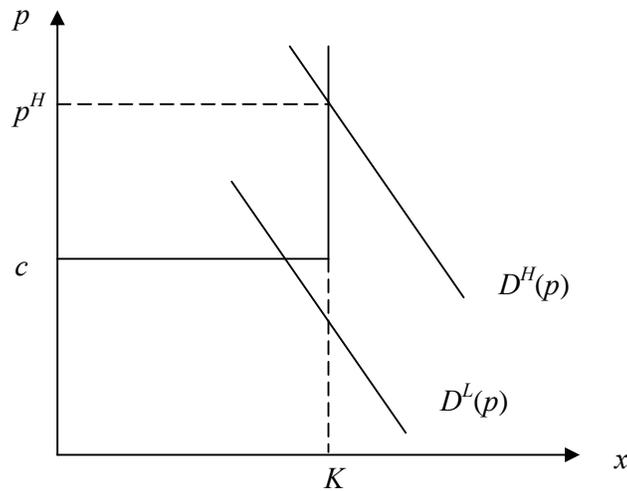


Figure 1

Demand varies across the year, both between different hours of the day and between days of the year. During any particular period, demand is a declining function of the electricity price. Each period thus has a particular market-

clearing price. In Figure 1, the market-clearing price is p^H when the demand curve is given by D^H , and c when the demand curve is given by D^L .⁵

A pure thermal system can thus be expected to give price fluctuations across the year. This is due to the fact that electricity cannot be stored from one period to another. Price fluctuations would only be avoided if capacity was so large that even the highest demand could be satisfied at the price c . However, since capacity is costly, it will not be optimal for producers to invest in such large capacity.

Let us now add a hydroelectric component to the supply side. Total production of electricity over the year from hydropower is determined by the available water reservoirs, which are given by an exogenous amount of yearly precipitation. I ignore the fact that in the beginning of the year this amount is uncertain. Moreover, I assume that there are no other constraints on the production of hydropower than the available water reservoirs for a year. Operating costs are also ignored,⁶ so that the surplus of the hydro producer is equal to its revenue.

Competitive suppliers of hydropower will obviously want to use the water reservoirs they have to produce electricity on the days when the electricity prices are highest. I assume that all producers have correct predictions of what the future price will be. The competitive outcome of a mixed thermal and hydro system will therefore be characterized as follows: There will exist a threshold price p^0 with the property that for periods where the demand at this price does not exceed K (such as $D^L(p)$ in Figure 2), there will be no production of hydropower. For these low-demand periods the equilibrium will be determined

⁵ Since the short-run demand curve in practice is very steep, small demand variations will give large price effects. This would be modified if we had a more realistic supply function where the capacity limit was not as absolute as in Figure 1, cf. also the discussion in Section 5.

⁶ Introducing a constant unit operating cost would not change our results, as long as this cost is below the unit operating cost of the thermal power producers (which typically is the case).

by the intersection of the inverse L supply curve for thermal energy and the demand curve, see Figure 2. This gives an equilibrium price somewhere in the range $[c, p^0]$. For periods with $D(p^0) > K$ (such as $D^H(p)$ in Figure 2), the production of hydropower will be positive, and equal to $K - D(p^0)$. The equilibrium price will thus be equal to p^0 in all such high-demand periods.

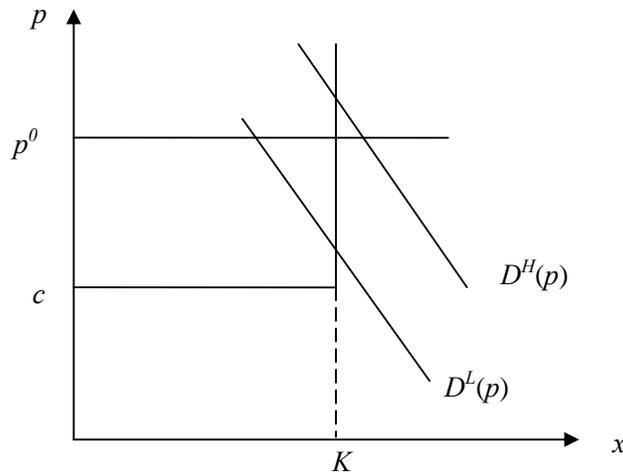


Figure 2

The threshold price p^0 will be determined by the demand functions for the different periods of the year, the capacity limit on thermal production (K in Figure 1), and the amount of water reservoirs. In particular, p^0 will be lower the higher is the size of the water reservoirs, since higher reservoirs allow higher electricity production during high demand periods. For any given size of water reservoirs, p^0 will be lower the higher is the capacity limit on thermal production. In a long-run equilibrium, the size of this capacity will be determined such that $p^0 > c$, and larger the larger is the capacity cost. We shall return to this long-run equilibrium in Sections 5 and 6.

From the discussion above it is clear that in a competitive equilibrium, price fluctuations through the year will be smaller under a mixed thermal and hydro system than under a pure thermal system. In the next Section we shall show

that this is not necessarily the case if producers of hydropower are not price takers.

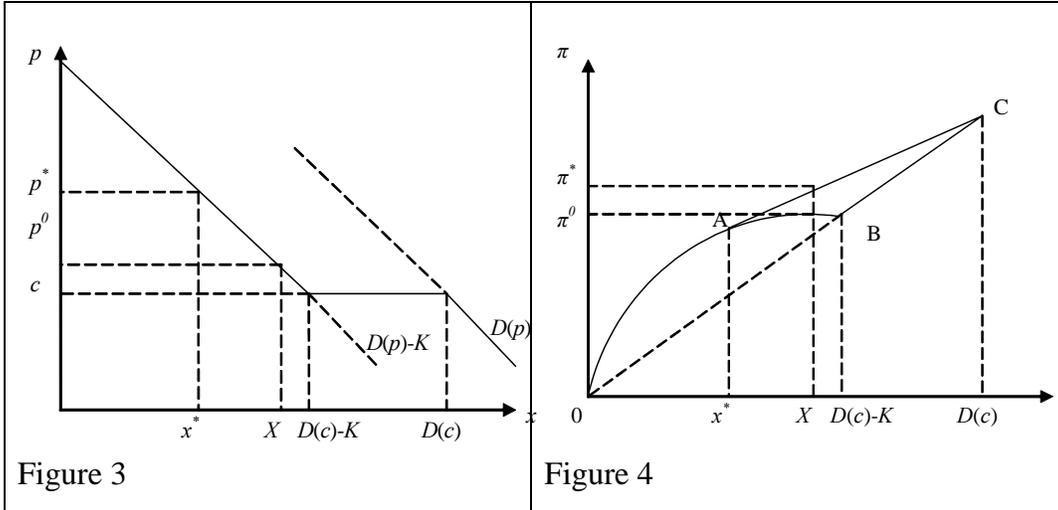
3 Hydropower monopoly

Consider the case where a monopolist controls all of the production of hydropower. As before, the producers of thermal power are assumed to be price takers.

It is useful first to consider the case in which the demand function is stable throughout the year. In the competitive equilibrium, discussed in the previous section, this would give a constant price equal to p^0 , with constant production of hydropower throughout the year and thermal power always produced at the capacity limit K . Figure 2 illustrates this case, with $D^H(p)$ as the stationary demand function.

The demand function facing the producer of hydropower is illustrated in Figure 3. At prices below c , this demand is $D(p)$, while the demand is $D(p)-K$ at prices above c . The corresponding revenue function is given by OABC in Figure 4 (the location of the point A is explained below)⁷. Notice that this revenue function is not concave, which is an important feature of such a mixed thermal and hydro system.

⁷ To keep the discussion as simple as possible, I assume that the elasticity of the function $D(p)$ is lower than one (measured positively) at prices below c . This means that the revenue function in Figure 3 has its maximum at the point C.



As argued above, the competitive equilibrium is characterized by the production of hydropower being constant throughout the year. This output level, denoted by X , is equal to size of the water reservoirs divided by the number of periods in a year (e.g. 365 days). This gives the electricity producer a revenue equal to π^0 per day. It is clear from Figure 4 that the hydropower monopolist can do better than having a constant production equal to X : By alternating between the production levels x^* and $D(c)$ in Figures 3 and 4, keeping average production equal to X , its average revenue per period will be given by $\pi^* > \pi^0$.

Formally, the equilibrium above may be derived as follows. Average revenue per period for the monopolist is

$$(1) \quad \pi = \theta xp(x+K) + (1-\theta)cD(c)$$

where $p(x+K)=D^{-1}(x+K)$ and θ is the share of periods where the thermal producers produce at full capacity. This revenue is maximized with respect to x and θ subject to the constraint

$$(2) \quad \theta x + (1-\theta)D(c) \leq X$$

The Lagrangian to the problem above is

$$(3) \quad L = \theta xp(x+K) + (1-\theta)cD(c) + \lambda[X - \theta x - (1-\theta)D(c)]$$

and the first order conditions for an interior solution are

$$(4) \quad p(x+K) + xp'(x+K) = \lambda$$

$$(5) \quad \frac{cD(c) - p(x+K)x}{D(c) - x} = \lambda$$

The l.h.s. of (4) is the slope of the revenue function OAB, i.e. the marginal revenue corresponding to the residual demand function $D(p)-K$ (equal to total demand minus the supply from the producers of thermal energy). The l.h.s. of (5) is the slope of a line from this revenue function OAB to the point C in Figure 4. Together, these equations tell us that the optimal price is given at the point A where these two slopes are equal. From now on this price is denoted p^* , and the corresponding output level x^* . Once the optimal price for the high price period is found, the optimal value of θ , denoted θ^* , follows from the constraint in (2) with an equality sign.

If X is sufficiently small (smaller than x^* in Figure 4), we get the corner solution where $\theta^*=1$, which is identical to the competitive outcome. If X is sufficiently large (larger than $D(c)$ in Figure 4), we get $\theta^*=0$, and the monopolist supplies all of the market in all periods at the price c . In this case actual production of hydropower will be lower than what is possible given the water reservoirs (i.e. $D(c) < X$)⁸.

⁸ This follows from our assumption on the demand function, see footnote 7. For the borderline case of $X=D(c)$ we also have $\theta^*=1$.

The reasoning above demonstrates that unlike for the competitive equilibrium for a stationary demand function, the electricity price may fluctuate over the year in the case of hydro monopoly. It is also clear that the average electricity price received by the hydro producer, given by π^*/X in Figure 3, is higher than the competitive equilibrium price π^0/X . The thermal producers will also get a higher surplus with price fluctuations than without. This follows from $\pi^* > \pi^0$, which may be written as (using (1))

$$(6) \quad \theta^* p^* x^* + (1 - \theta^*) cD > p^0 X$$

where $p^0 \equiv \pi^0/X$ is the competitive price. Using (2) with equality this may be rewritten as

$$(7) \quad \theta^* (p^* - c)x^* > (p^0 - c)X$$

Since $x^* < X$ it follows that

$$(8) \quad \theta^* (p^* - c) > (p^0 - c)$$

The r.h.s. of (8) is the surplus of the fringe per unit of capacity in the competitive case, while the l.h.s. is the same surplus under monopoly (i.e. when prices fluctuate).

Since both the hydro producer and the thermal producers get higher surpluses under hydro monopoly than in the competitive equilibrium, consumers are worse off under hydro monopoly than they are in the competitive equilibrium.

The comparisons above are only true when the capacity of the thermal producers and the size of the hydro reservoirs are the same in the two cases. In the long run both these capacities are endogenous, and will be different when

the hydro producer is a monopolist than when it is a price taker. The long-run equilibria will be treated in more detail in Sections 5-7.

4 Demand fluctuations

Before considering long-run equilibria, I briefly consider the hydro monopoly case when demand fluctuates during a year. Let the year be divided into n periods, during each of these periods j demand is stationary given by $D_j(p)$. In each period we thus have an average revenue function corresponding to OAC in Figure 4. Denote this average revenue function by $\pi_j(x_j)$. The monopolist's problem is to allocate its total water reservoirs across periods so that its total revenue during the year is maximized. Denote the fraction of the year we are in period j by α_j (with $\sum_j \alpha_j = 1$). Total yearly revenue is given by

$$(9) \quad \Pi = \sum_j \alpha_j \pi_j(x_j)$$

which is maximized subject to the reservoir constraint

$$(10) \quad \sum_j \alpha_j x_j \leq X$$

The Lagrangian to this problem is

$$(11) \quad L = \sum_j \alpha_j \pi_j(x_j) + \gamma \left(X - \sum_j \alpha_j x_j \right)$$

The solution to this maximization problem is (remembering that $\pi_j(x_j)$ is maximized for $x_j = D_j(c)$, see footnote 7)

$$(12) \quad \begin{aligned} \pi_j'(x_j) &\geq \gamma \\ x_j &= D_j(c) \text{ for } \pi_j'(x_j) > \gamma \end{aligned}$$

To interpret this, remember that $\pi_j'(x_j)$ is the slope of the average revenue functions corresponding to OAC in Figure 4. The water reservoirs should thus be allocated across periods so that the slope of these OAC-curves should be equal across periods, or, for periods where the slope OAC is “high” for all values of x_j , the output should be equal to $D_j(c)$.

The size of the shadow price γ depends on the size of the reservoirs. The most interesting case is when the slopes corresponding to AC in Figure 4 are lower than γ for some periods, but higher for other periods. In the Appendix, I have argued that it is not unreasonable to expect the slope of AC to be smaller the larger the demand is, although such a relationship doesn’t hold generally independent of the specifications of the demand functions. I shall therefore call periods with the slope of AC larger than γ low-demand periods, and periods with the slope of AC smaller than γ high-demand periods.

For low-demand periods (i.e. periods when the slope AC is higher than γ), the electricity price is c , and hydro production covers all of the demand at this price. For these periods thermal production is thus zero. For high-demand periods (i.e. periods with the slope of AC smaller than γ), the hydro producer charges an electricity price above c , and its output is lower than x^* in Figures 3 and 4. Thermal producers produce at full capacity all the time during these periods. In addition to these types of periods, there may be one (or possibly several) periods where the slope of AC is equal to γ . For such a period we get the situation as described in the previous section: some days (or hours) during this period we get a situation similar to what we get during low-demand periods, and some days (or hours) during this period we get a situation similar to what we get during high-demand periods.

It is interesting to note the difference between the present case and the competitive case when demand varies through the year. In the competitive

case, the price was the same for all high-demand periods, while it varied across low-demand periods. Moreover, hydropower was only produced in the high-demand periods. In the present case, prices vary across high-demand periods, but are constant (equal to c) across low-demand periods. Moreover, hydro production is highest in low-demand periods.

5. A more general supply function for thermal producers

The inverse L supply function for the thermal producers that we have used till now is obviously only a rough approximation to a real aggregate supply function. An inverse L supply function will be valid for each individual thermal producer if they have constant unit operating costs up to a capacity limit. However, if unit operating costs vary across producers, we will have a stepwise aggregate supply function instead of the inverse supply L supply function assumed till now. If there are many thermal producers, such a stepwise supply function can be approximated by a standard upward sloping continuous supply function. The properties of such a supply function will depend on properties of the distribution of unit costs and capacities across producers. For the special case where all producers have the same unit costs, we get an inverse L supply function of the type assumed till now. If instead all producers have identical capacity limits but unit costs differ and are uniformly distributed across producers, the supply function will be linear and upward sloping up to the production level corresponding to full capacity utilization for all producers, when it becomes vertical. Most electricity markets (and many other markets) seem to be characterized by a large fraction of the producers having relatively similar unit costs, with more cost dispersion for the remaining producers. If this is the case, the supply function will be upward sloping and relatively flat up to a point not very much below the total capacity limit. Then the steepness of the supply curve will increase rapidly, and become vertical when we have full capacity utilization for all producers. More precisely, denote the aggregate supply function $S(p)$, and let p^K be the unit cost of the producer with the highest unit cost. We thus have $S(p)=K$ for $p \geq p^K$ and $S(p)<K$ for $p < p^K$. Moreover,

for some range of prices below but close to p^K we have S'' negative and “large” in absolute value.

The residual demand facing the hydro producer for this general case is $E(p) = D(p) - S(p)$, which is a declining function. The revenue of the hydro producer is $x E^{-1}(x)$. It is straightforward to verify that the marginal revenue will be increasing in x if the second derivative of $E^{-1}(x)$ is positive and sufficiently large. This in turn will be the case if $D''(p) - S''(p)$ is positive and sufficiently large, which will occur whenever $-S''(p)$ is positive and sufficiently large and the demand function is linear or convex (i.e. $D''(p) \geq 0$). As argued above, we expect $-S''(p)$ to be positive and “large” for some range of prices below but close to p^K . In other words, it is not unreasonable to expect the marginal revenue of the hydro producer to be increasing over some range of its output. But whenever this is the case, we get a situation similar to the one described in Figure 4 and discussed in Section 3: The revenue function for the production per day will be non-concave, and there will be a range of levels for average production per day such that it is optimal for the hydro producer to alternate between high production days and low production days. The results for the previous sections will thus be valid also for a general supply function for the fringe producers, provided the steepness of this function rises rapidly as we approach the aggregate capacity limit for the thermal producers.

6 The long-run competitive equilibrium

We again turn to the case of a stationary demand function. Assume that the cost of thermal capacity is given by the constant unit capacity cost b . In a long-run equilibrium, there must be zero profit in the thermal sector. The short-run surplus $(p-c)K$ must therefore be equal to the capacity cost bK , the competitive equilibrium price p^0 is therefore given by

$$(13) \quad p^0 = c + b$$

For the hydro sector, an assumption of constant unit costs of capacity (i.e. of water reservoirs) is not particularly reasonable. Potential sites for hydropower will have different natural characteristics, with corresponding cost differences. Also, for any particular site, investments in dams beyond some limit will increase costs more than proportionally to the increased reservoirs. Ignoring possible increasing returns at the site level for small capacities, it is therefore reasonable to assume that the cost function for reservoirs is increasing and strictly convex. We denote this cost function by $B(X)$, and assume $B' > 0$ and $B'' > 0$.

The reservoir capacity is in the competitive case determined by the maximization of $p^0 X - B(X)$. Denoting the optimal reservoir capacity by X^0 , this gives (since $p^0 = c + b$)

$$(14) \quad B'(X^0) = c + b$$

7 The long-run equilibrium with hydro monopoly

I shall only consider long-run equilibria where there are price fluctuations through each year even when the demand function is stationary. In other words, I am assuming that cost and demand functions are such that none of the following three situations occur in the long-run equilibrium: (i) zero thermal capacity, (ii) zero hydro capacity, (iii) such a low hydro capacity that $X \leq x^*$ where x^* as before is given by (4) and (5). Appendix II gives a discussion of the properties cost and demand functions must have for prices to be fluctuating through the year in the long-run equilibrium.

Once both capacities are given, we have the situation discussed in Section 3 and illustrated by Figure 4. In this equilibrium, the electricity price is p^* ($> c$) during a fraction θ^* of the year, and c the rest of the year. Total surplus over

the year for the thermal sector is thus $\theta^*(p^*-c)K$, and in the long-run equilibrium this surplus must be equal to the capacity cost bK , giving

$$(15) \quad p^* = c + \frac{b}{\theta^*}$$

The average price over the year is $\theta^*p^*+(1-\theta^*)c$, and it is straightforward to see from (15) that this average price must be equal to $c+b$. The average price over the year is thus the same as in the competitive equilibrium (cf. (13)). This is as expected, since the competitive thermal producers have a horizontal long-run supply curve. Although the focus of this paper is not on consumers, one observation is worth mentioning. Provided consumers face a perfect credit market through the year, consumers are better off with a fluctuating electricity price through the year than a constant price, provided the average price is the same in both cases (and ignoring differences between the two cases in income from the electricity sector to the consumers). To see this, denote utility maximizing electricity consumption per period by y^0 for the case when the price is p^0 in each period. If the price fluctuates, the consumers still have the possibility of buying y^0 in each period provided the average price is p^0 . However, consumers can do better by consuming less when the price exceeds p^0 and more when the price is lower than p^0 .

We saw above that the average price over the year to consumers was the same for the monopoly case as for the competitive case. However, the average price for the hydro producer on its output is lower under monopoly than in the competitive equilibrium: This average price, denoted \hat{p} , follows from (1), (2) and (15)

$$(16) \quad \hat{p} = \frac{\pi}{X} = \frac{\theta^* p^* x^* + (1-\theta^*)cD(c)}{X} = c + b \frac{x^*}{X} < c + b = p^0$$

Notice that this inequality holds for any pair (K, X) that satisfies the equilibrium condition (15). The profit of the monopolist must therefore satisfy

$$(17) \quad \pi - B(X) = \hat{p}X - B(X) < \max_x [p^0 X - B(X)]$$

Since p^0 is the constant electricity price in the competitive equilibrium, this implies that the profit of the hydro producer is lower under monopoly than under the competitive equilibrium. Although this might seem surprising at first, it follows directly from the fact that consumers are better off under monopoly than under competition, provided their income is the same. If profits were higher under monopoly the first fundamental theorem in welfare economics would be violated.

An obvious question is why cannot the monopolist simply mimic competitive producers, and thereby get the same profit as under competition? The answer is that the monopolist cannot commit itself to behave as competitive producers once capacities are given. As shown in Sections 2 and 3, once capacities are given, competitive producers will have a constant output through the year, while a monopolist will let its output fluctuate, thereby increasing its profit compared with the competitive case *given both capacities K and X* . Competitive thermal producers foresee this behaviour when they make their price predictions in order to make a best possible capacity decision. The capacity K chosen by these producers is therefore different under hydropower monopoly than under competition in both electricity-producing sectors.⁹

The maximized average revenue per year of the hydro producer depends on X as well as K , cf. (1) and (2). We thus denote this maximized revenue by $\pi(X, K)$. Applying the envelope theorem to (1) - (3) gives¹⁰

⁹ It is well known that there are many cases in which the inability to make future commitments can make an increase in market power disadvantageous, see e.g. Karp (1996), Maskin and Newbery (1990), Ulph and Ulph (1989) and Salant et al. (1983).

¹⁰ From (4) and (5) it is clear that x^* and λ depend only on K , and not on X .

$$(18) \quad \pi_X(X, K) = \lambda(K) > 0$$

Moreover, when (2) holds with equality it follows from (1) - (5) that

$$(19) \quad \pi(X, K) = cD(c) - \lambda(K)(D(c) - X)$$

implying

$$(20) \quad \pi_K(X, K) = (X - D(c))\lambda'(K)$$

Applying the envelope theorem to (1) gives

$$(21) \quad \pi_K(X, K) = \theta^* x^* p'(x^* + K) < 0$$

Combining (20) and (21) gives

$$(22) \quad \lambda'(K) = \frac{-\theta^* x^* p'(x^* + K)}{D(c) - X} > 0$$

While the monopolist's revenue thus is lower the higher is K , its *marginal* revenue is higher the higher is K . This last feature can also easily be derived by examining Figures 3 and 4: From these figures it is clear that the larger K is, the lower lies the curve OAB in Figure 4 (and the further to the left is the point B). The point C in Figure 4 is however independent of K . A higher value of K must therefore make the slope of AC higher. But the slope of AC (equal to λ) is the marginal revenue of the monopolist, which therefore must be higher the higher is K .¹¹

In the rest of this section I shall distinguish between two cases. First I consider the case in which the competitive thermal producers choose their total capacity

¹¹ The relationship between K and the marginal revenue of the monopolist is reversed when X is sufficiently small: For X smaller than x^* in Figure 4, marginal revenue is simply $p(X+K) + Xp'(X+K)$, which is declining in K for $p'(X+K) < 0$. This inequality must always hold for sufficiently small values of X .

K simultaneously with the hydro monopoly's decision of reservoir capacity X . In the second case considered the hydropower capacity is determined before the thermal power capacity.

Simultaneous capacity decisions.

When K and X are determined simultaneously, K is regarded as given when the hydro monopolist chooses X . The monopolist chooses X to maximize $\pi(X, K) - B(X)$, taking K as given, which gives

$$(23) \quad \Pi_X(X, K) = B'(X)$$

Using (18), we can rewrite this as

$$(24) \quad B'(X) = \lambda(K)$$

This equation gives the optimal value of X whatever K is. Since B'' and λ' are both positive, it follows from (24) that X must be higher the higher is K .

The equilibrium combination of (K, X) must in addition to (24) satisfy the equilibrium condition (15). We shall denote this equilibrium pair by (K^*, X^*) .

Since $\lambda(K)$ is equal to the slope of AC in Figure 4, it follows from Figure 4 that $\lambda(K) < c$. Comparing (24) with (14), it is therefore clear that investments in reservoir capacity are lower in the present monopoly case than in the competitive equilibrium.

Capacity of hydro producer determined prior to thermal capacity

An increase of hydropower capacity can be of two types. The simplest is an upgrading of existing power plants through the installation of new and more efficient turbines, making it possible to get more electricity out of a given water

reservoir. For larger capacity expansions, such investments will not suffice, and one must instead rely on the second type of investments, namely building new reservoir capacity through new dams or expanding existing dams. This type of investment typically takes quite a long time from the construction starts until the capacity can be used. If such investments take a considerably longer time to complete than expansion of thermal power capacity, a static modelling of these investments ought to let the hydropower capacity be determined before the thermal power capacity. I therefore consider this case in the present section.

The monopolist's profit must be at least as large in the present case as in the case where both capacities were determined simultaneously. The reason for this is that the monopolist has the option of setting its capacity exactly equal to what it would have in the case of simultaneous capacity determination. In this case the thermal capacity will also be equal to what it was with simultaneous capacity determination, and the monopolist's profit will therefore also be the same. However, the monopolist can usually do better by choosing a different capacity, thereby inducing thermal producers also to choose a different capacity than the capacity they would have chosen if capacity determinations were simultaneous.

Formally, the optimal hydro capacity in the present case is found by maximizing $\Pi(X, K(X)) - B(X)$, where the function $K(X)$ is defined as the K -value making the equilibrium condition (15) hold (in particular it must thus be true that $K(X^*) = K^*$). The difference between the present case and the case with simultaneous capacity decisions is illustrated in Figure 5. In this Figure, the curved lines are iso-profit lines for the hydro monopolist. It follows from (21) that the profit $\pi(X, K) - B(X)$ is lower the higher is K , so that the iso-profit lines have lower values the further to the right in the diagram we are. The line $X(K)$ between the vertical tangents of the iso-profit curves is increasing for (X, K) pairs implying that we have fluctuating prices, cf. the discussion after (24). The line $K(X)$ in Figure 5 is defined by the equilibrium condition (15),

where it follows from the discussion in Section 3 that p^* depends on K , and that θ^* depends on X and K . The equilibrium pair (X^*, K^*) derived in the previous section is given by the intersection between the lines $X(K)$ and $K(X)$. However, if the hydro monopolist can determine its capacity before the capacity decision of the thermal fringe, it will choose a capacity level making the fringe capacity smaller than K^* . If $K(X)$ is a declining function, as in Figure 5, this gives an optimal capacity X^{**} that is larger than X^* .

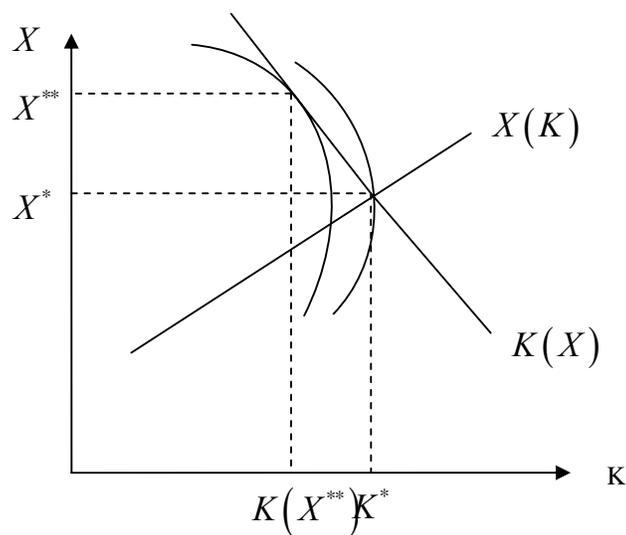


Figure 5

8 Concluding remarks

The paper has demonstrated that even in the hypothetical case of the electricity demand function being stationary, electricity prices may fluctuate through the year if a producer of hydro electricity has market power. In the simple model used in this paper the electricity price would be constant through the year in a competitive equilibrium even if demand fluctuated, provided the demand fluctuations were so small that there was positive production of hydropower throughout the year. In a more realistic model of the electricity market, electricity prices would fluctuate also in a competitive equilibrium. One reason

for this is that there in reality will be other constraints on hydropower production in addition to the constraint given by the total reservoir capacity (see e.g. von der Fehr and Sandsbråten, and Ambec and Doucet). A second important reason for price fluctuations is that the total amount of precipitation through a year in reality is uncertain in the beginning of the year. In the model used here this variable (denoted X) was important for the equilibrium price both under competition and monopoly. When this variable is uncertain at the beginning of the year, the hydro producer(s) will continuously be getting more information about its true value as time passes. We would thus expect prices to be continuously changing over time. Finally, even if the profit function facing a hydro monopoly is concave, the optimal monopoly price will usually fluctuate over time as the demand function fluctuates.¹²

It is clear from the discussion above that we typically will observe fluctuating electricity prices also when hydropower is an important component of the total electricity supply, even if each producer is a price taker. An interesting and challenging topic for future work would be investigate to what extent observed price fluctuations are caused by a large hydro producer behaving as explained in Section 3, and illustrated by Figure 4.

¹² The optimal policy for the hydro monopolist in this case will be to equalize its marginal profit across periods, which in the absence of short-run production costs will give a constant price only if the difference in demand functions across periods is multiplicative.

Appendix I: The relationship between the demand function and the slope of AC in Figure 4

It is not unreasonable for the difference in demand functions across periods to be such that the relative difference in demand (the highest demand relative to the lowest demand) is non-declining in price. Notice that both a multiplicative difference in demand and a constant positive difference in demand have this property. To see what happens when demand differs in this way, consider first the hypothetical case of a multiplicative increase both in demand and in thermal capacity K . Clearly, this would simply blow up all curves in Figure 4 proportionally, leaving all slopes unchanged. In particular, the slope AC would remain unchanged. The actual demand difference we are considering differs from this hypothetical change in two ways. First, K remains unchanged. But this means that AC must be flatter than it was for the hypothetical change. Second, if demand increases relatively more for high than for low prices (i.e. more the further to the left in Figure 4 we are), the derivative of the revenue function OAB must be smaller at any given value of x than if the demand change was proportional. This will make the line AC even flatter. It is thus clear that a demand difference of the type assumed must make the line AC in Figure 4 flatter the larger the demand is.

Appendix II: Properties of cost and demand functions for the long-run equilibrium to have fluctuating prices

Consider equations (4) and (5), which hold in an equilibrium with fluctuating prices. These give the short-run equilibrium values p^* , x^* and λ as functions of K , we thus denote these values as $p^*(K)$, $x^*(K)$ and $\lambda(K)$. To have fluctuating prices we must have $0 < \theta^* < 1$. Together with the long-run equilibrium condition (15) this implies

$$(25) \quad p^*(K) > c + b$$

In order for a long-run equilibrium with fluctuating prices to exist, there must exist values of K satisfying (25). For any given value of K , the value of $p^*(K)$ is determined

by the unit operating cost c and the demand function D . Given c and D , there will exist values of K satisfying (25) provided capital costs b are sufficiently low.

Denote the set of K -values satisfying (25) by Ω . For any $K \in \Omega$, it follows from the long-run equilibrium condition (15) that

$$(26) \quad \theta^* = \frac{b}{p^*(K) - c}$$

In a long-run equilibrium (2) must hold with an equality sign, from (26) it therefore follows that

$$(27) \quad X = \frac{b}{p^*(K) - c} x^* + \left(1 - \frac{b}{p^*(K) - c}\right) D(c)$$

It therefore follows from (24) that we will have a long-run equilibrium with fluctuating prices if

$$(28) \quad B \left(\frac{b}{p^*(K) - c} x^*(K) + \left(1 - \frac{b}{p^*(K) - c}\right) D(c) \right) = \lambda(K) \text{ for some } K \in \Omega.$$

It is useful to consider a simple numerical example. Without loss of generality we normalize units so that $c = D(c) = 1$. Let the demand function be $D(p) = p^{-1}$, i.e.

$$(29) \quad p(x + K) = \frac{1}{x + K}$$

From (4) and (5) straightforward calculations reveal that

$$(30) \quad x^*(K) = \frac{1 - K}{2}$$

$$(31) \quad p^*(K) = \frac{2}{1+K}$$

$$(32) \quad \lambda(K) = \frac{4K}{(1+K)^2}$$

so that the condition (25) is

$$(33) \quad \frac{2}{1+K} > c+b$$

which may be rewritten as

$$(34) \quad K < \frac{1-b}{1+b}$$

From (34) it is clear that $b < 1$ is necessary in order to have an equilibrium with fluctuating prices. For any given value of $b < 1$, the set Ω is simply $\left(0, \frac{1-b}{1+b}\right)$.

For a positive K satisfying (34), equations (26) and (27) in the present case are

$$(35) \quad \theta^* = b \frac{1+K}{1-K}$$

$$(36) \quad X = 1 - \frac{b(1+K)^2}{2(1-K)}$$

so that the condition (28) becomes

$$(37) \quad B \left(1 - \frac{b(1+K)^2}{2(1-K)} \right) = \frac{4K}{(1+K)^2} \text{ for some } K \in \left(0, \frac{1-b}{1+b} \right)$$

Whether or not (37) will hold will of course depend on the function hydro capacity cost function $B(X)$. Consider e.g. the case of

$$(38) \quad B(X) = h \frac{X}{1-X}$$

implying

$$(39) \quad B'(X) = \frac{h}{(1-X)^2}$$

Moreover, let $b=h=1/3$. In this case the competitive equilibrium (given by (14)) is $X=0.5$. The corresponding value of K (from (13), i.e. $p(X+K)=b+c$) is 0.25. From (2), (4) and (5) it follows that for these capacities, the monopolist would like to $x=0.375$ and $p=1.6$ for 55.6% of the year, and $x=p=1$ for the rest of the year. In table 1, the variables in parentheses in the column *competitive equilibrium* represent short-run equilibrium values for the monopoly case when capacities are given at the competitive equilibrium levels.

The long-run competitive equilibrium capacities ($X=0.5$ and $K=0.25$) are not the equilibrium outcome in the monopoly case. The equilibrium capacities in the monopoly case follow from inserting (32), (36) and (39) into (24). This gives $X=0.375$ and $K=0.444$. All other variables follow from our equations (30)-(35), see the column denoted *monopoly I* in table 1. Notice in particular that the total capacity $X+K$ is larger in the monopoly case than in the competitive case, although the hydro producer's capacity is smaller.

In table 1 the column denoted *monopoly II* is the case when the hydro monopolist chooses its capacity before the competitive fringe determines the thermal capacity. To find the values of X and K in this case we combine (19) and (32), which together with (38) gives

$$(40) \quad \pi(X, K) - B(X) = 1 - \frac{4K}{(1+K)^2} (1-X) - h \frac{X}{1-X}$$

When the hydro monopolist chooses X before K is chosen, it chooses X so that (40) is maximized subject to the constraint (36). Solving numerically, we find $X=0.449$ and $K=0.404$. All other variables follow from our equations (30)-(35), see the column denoted *monopoly II* in table 1.

Table 1: Lon-run equilibrium values of quantities, prices and profits

	<i>Competitive equilibrium</i>	<i>Monopoly I</i>	<i>Monopoly II</i>
X	0.500	0.375	0.449
K	0.250	0.444	0.404
$X+K$	0.750	0.819	0.853
$p^0=c+b$	1.333		
λ	(0.640)	0.852	0.820
x^*	(0.375)	0.278	0.298
p^*	(1.600)	1.385	1.425
θ^*	(0.556)	0.866	0.785
π^0	0.667		
$\pi(X,K)$	(0.680)	0.468	0.548
$B(X)$	0.333	0.200	0.271
$\pi^0 - B(X)$	0.333		
$\pi(X,K) - B(X)$	(0.347)	0.268	0.277

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