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Consumption and population age structure

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Consumption and population age structure*

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26. November, 2004

Abstract

In this paper the effects on aggregate consumption of changes in the age distribution of the population are analysed empirically. Economic theories predict that age influences individuals' saving and consumption behaviour. Despite this, age structure effects are rarely controlled for in empirical consumption functions. Our findings suggest that they should. By analysing Norwegian quarterly time series data we find that changes in the age distribution of the population have significant and life cycle consistent effects on aggregate consumption. Furthermore, controlling for age structure effects stabilizes the other parameters of the consumption function and reveals significant real interest rate effects. Simulation experiments show that the numerical effect on the savings rate of age structure changes is substantial when the indirect effects via wealth and income are accounted for.

Keywords: *Consumption, demography, savings, time series models, cointegration.*

JEL classification: *C51, C53, E21, J10.*

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1 Introduction

A key question in economics is whether changes in the age structure of the population affect macroeconomic variables such as aggregate consumption and the savings rate. Economic theory suggests that the influence could be substantial. The life cycle model of Modigliani and his collaborators[1] for instance predicts that individuals' consumption and saving behaviour are functions of their age; an individual borrows as young, saves as middle-aged and dissaves when old. Aggregating up, changes in the age distribution over time could hence induce variations in a nation's private saving rate.

The link between the savings rate and demography, with particular focus on the issue whether the elderly dissave, has been widely explored in the empirical literature. However, no consensus about the impact of a changing age distribution, neither when it comes to the direction nor the size of it, has been reached. At least in part, the discrepancies stem from the use of different types of data and methods.

For example, there is a tendency that studies based on aggregate macroeconomic data report significant, and often numerically important, age structure effects on consumption and savings. Thus, these studies usually confirm the predictions of the life cycle model, finding that savings decrease, or aggregate consumption rises, when the share of elderly persons in the population increases. Among these are Attfield and Cannon (2003), Higgins (1998), Horioka (1997), Masson et al. (1996) and Fair and Dominguez (1991)[2]. On the other hand, studies using household survey data often find no, or only modest, effects on savings of changes in the age distribution of the population, e.g. Parker (1999) and Bosworth et al. (1991).

There are several explanations for the discrepancies between macro and micro evidence of the importance of age effects. Weil (1994) shows that if interactions between generations, such as bequest, are important, one would not expect the estimates from the two types of studies to be the same. Miles (1999), among others, suggests that part of the discrepancy is due to that household survey studies often use savings rates which overestimate the values of pension assets. Furthermore, Deaton and Paxson (2000) emphasise that household surveys data are likely to suffer from sample selection biases as these are based upon households, and not individuals. By taking account of the latter two arguments when estimating a savings-age profile based on UK household survey data, Demery and Duck (2001) derive a savings-age profile which is much more consistent with the life-cycle model.

In this paper we test for age structure effects on aggregate consumption in Norway, by estimating a consumption function which takes account of changes in the age distribution of the population. The model is estimated on quarterly time series data over the period 1968(3)-2000(4). The motivation is twofold. First, it is of importance to investigate the impact of a changing age distribution on Norwegian consumption. Similar to many other developed countries, Norway experienced low birth rates in the interwar years and a baby boom in the first decades after World War II. The small cohorts born in the interwar years are currently in the retiring

[1] Modigliani and Brumberg (1954) and Ando and Modigliani (1963).

[2] Fair and Dominguez (1991) find that prime-age people consume less relative to their income than other age groups on U.S. data. However, they did not find life cycle consistent effects on the aggregate personal savings rate.

phase, while the baby boomers are turning middle-aged. If life cycle saving behaviour applies, the ongoing change in the age distribution will thus put a downward pressure on Norwegian consumption, other things equal. Hence, if an age structure effect on aggregate consumption can be affirmed empirically, it will be of interest for both medium term activity control, and for longer term analysis of private saving and wealth. Using high-quality micro data, Halvorsen (2004) finds strong age effects on Norwegian household savings rates, while cohort effects are weak. This suggest that age structure may be an underlying structural aspect also of aggregate consumption, a possibility which we pursue in this paper.

A second motivation of the paper is methodological and is related to the modelling consequences of demographic changes. A demographic change of some magnitude is an example of a structural change, which can potentially overturn existing macroeconometric relationships, and cause forecast failure. However, it can also lead to new knowledge, which in some instances can be accommodated by revision and extension of an existing model. Hence, the effects of structural changes are not always destructive. From this perspective, the paper investigates the possible incorporation of age structure in an existing consumption function. The incumbent model, specified by Brodin and Nymoen (1992), builds on cointegration between private consumption, income and wealth, a well defined causal structure and has stable over a 15-year period, see Eitrheim et al. (2002) for an analysis. Since both theory and empirical macro studies suggest that changes in the age structure can affect aggregate consumption, it should hence be tested whether changes in the age structure of the population is an omitted variable in the consumption function. Another variable that may influence consumption in the long run is changes in the real interest rate. Hence, we also test whether real interest rates should be included in the consumption function.

To preview our main findings, we find significant and numerically important age structure effects on Norwegian aggregate consumption. The effects are consistent with the life cycle model; consumption falls when the share of middle-aged persons in the population increases, other things equal. An important result in the paper is that the other parameters of the consumption function become more stable when we control for age structure effects. To our knowledge, no other study has affirmed that changes in the age structure can play such a key role in empirical models of this kind. In addition, controlling for the effects of a changing age structure reveal significant effects of changes in the real interest rate on aggregate consumption. Finally, we show, by simulation using a small econometric model of consumption, income and wealth, that changes in the age structure represent important challenges for policy. Specifically, although the medium term effect of an increase in the share of middle-aged persons is an increase in the savings rate, the savings rate may dip in the short run due to a temporary increase in real household wealth.

While the intellectual rationale for controlling for age structure effects in a consumption function is clear enough, there is no clear cut operational route. Different candidates for operational definitions of age structure changes have been suggested in the literature, and in Section 2 we motivate our preferred measure. Section 3 explains how the age structure variable, together with a real interest rate variable, represent an extension of the existing Norwegian consumption function. Section 4 presents the econometric evidence for a long run consumption function which

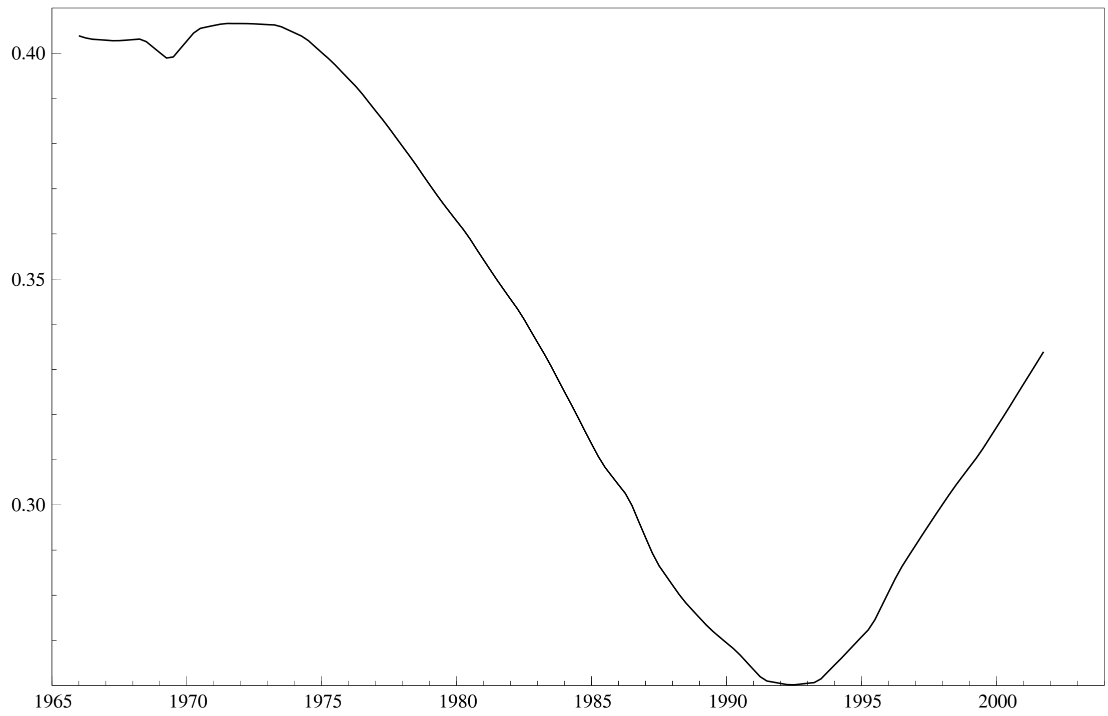

Figure 1: The age structure variable AGE over the period 1965(1)-2000(4).

includes an age structure variable, and derives the corresponding error correction model. A reason for including age structure in a consumption function is to be able to quantify the medium term effects of changes in the age structure on the savings rate. This is done in Section 5, with the aid of an econometric model which accounts for the additional (and indirect) effect of a changing age distribution through wealth accumulation and income. Section 6 concludes.

2 Age structure effects on consumption

Different approaches have been used in the empirical literature to test for age structure effects on aggregate consumption. The life cycle model suggests that the marginal propensity to consume (MPC) vary over the life cycle alongside changes in an individual's preferences, needs and income. On this basis, one would want to estimate an empirical relationship between aggregate consumption, income and wealth, where the MPC is allowed to change with the individuals' age when testing for age structure effects. However, disaggregate age group data for macroeconomic variables are rarely available, and hence when using this type of data one is usually not able to test for age-sensitive MPC's. It is thus more common in empirical macro studies to test for age structure effects on the average propensity to consume. Typically, this is done be including either one or several age structure variables in a regression model. If the variable(s) is (are) significantly different from zero, changes in the age structure of the population is found to have effect on aggregate consumption.

Among the age structure variables, the 'dependency ratio', defined as the number of children and retired persons to those of working age, is often used to represent changes in the age structure.[3] In this paper we use a somewhat different age structure variable, namely the number of persons in the 'prime-saver' age group to the rest of the adult population, suggested and used by McMillan and Baesel (1990).[4] Similar to the 'dependency ratio', this age structure variable is expected to be a life cycle saving measure. As noted by McMillan and Baesel (1990), the ratio of 'prime-savers aged' persons to the rest of the adult population may be a closer approximation to the life cycle ideal than the 'dependency ratio'. The 'prime-savers' are assumed to be the middle-aged age group, having relatively high average earnings at the same time as the size of their households, and hence their needs, are smaller than when they were younger. Furthermore, they are in the pre-retired phase, making it likely for them to save for retirement. The middle-aged persons may therefore have a lower propensity to consume than both those younger and older.

Preliminary estimation on Norwegian aggregate data indicates that it is the age group of 50-66 years old persons which, among the adult population, has the less average propensity to consume.[5] In the remainder of this paper we denote this age group as the prime-savers or the middle-aged, and we include the following age structure variable in the econometric study below:

$$AGE_t = \frac{(Population\ 50-66\ years\ old)_t}{(Population\ 20-49\ years\ old\ and\ 67+\ \ years\ old)_t}.$$

The development of AGE over the sample period 1968(3)-2004(4) is shown in Figure 1.[6] The graph shows that the share of middle-aged persons in Norway was declining from the mid-1970s until the beginning of the 1990s. From then on, AGE has been increasing, as the baby boom generation is turning middle-aged.

Given the prediction that middle-aged persons save more than the rest of the population, AGE is expected to enter the consumption function with a negative coefficient.

3 Extending the consumption function

Up until the beginning of the 1980s, Norwegian private consumption was, according to the consensus view, well represented by a (log-)linear model between total private consumption expenditures and real disposable income in the household sector. The speed of adjustment was estimated to be quick, and hence on annual data a static relationship worked well. This changed however in the aftermath of the credit liberalisation at the beginning of the 1980s, when aggregate consumption

[3]Leff (1969), Masson et al. (1996) and Horioka (1997) are among those using this variable.

[4]McMillan and Baesel (1990) use this variable to identify age structure effects on U.S. real interest rates, income, inflation and unemployment.

[5]This is consistent with Attfield and Cannon (2003)'s findings on UK data, while Fair and Dominguez (1991) report the lowest propensity to consume among the age groups which are 10 to 15 years younger on U.S. data.

[6]The population data are from Statistics Norway. See Appendix A for more information on data.

rose sharply relative to income.[7] Subsequently the existing empirical consumption functions broke down. Respecification of the Norwegian consumption function by including a broad measure of household wealth succeeded, however, in accounting for the breakdown ex post, see Brodin and Nymoen (1992), denoted B&N hereafter.[8] In more detail, B&N's results, based on quarterly data from 1968(1) to 1989(4), can be summarised in four points:

1. *Cointegration.* The log-linear relationship between the three variables C (total private consumption), Y (real disposable income), and W (net household wealth in real terms)

$$\log C_t = \text{ Constant } + \beta_1 \log Y_t + \beta_2 \log W_t, \tag{1}$$

   constituted a cointegrating relationship. In the equation, subscript t denotes the time period, while β_1 and β_2 denote the cointegration parameters for y_t and w_t, respectively. The appendix contains detailed definitions of the variables C, Y and W.

2. *Weak exogeneity.* Income and wealth were weakly exogenous to the cointegration parameters.

3. *Invariance.* Estimation of the marginal models for income and wealth showed evidence of structural breaks. The joint occurrence of a stable conditional model (the consumption function) and unstable marginal models for the conditioning variables is evidence of within sample invariance of the coefficients of the conditional model and hence super exogenous conditioning variables (income and wealth).[9]

4. *Speed of adjustment* is relatively fast, both with respect to revaluation of wealth and changes in income.

Recently, Eitrheim et al. (2002), denoted EJN hereafter, showed that these features of the B&N model remain more or less unchanged when the consumption function is estimated on the extended sample 1968(3)-1998(4); consumption, income and wealth were seen to cointegrate, the latter two variables were found to be weakly exogenous with respect to the cointegrating parameters and the recursive estimates of the cointegration coefficients for income and wealth were stable over the sample period. Furthermore, the adjustment speed towards equilibrium was still relative fast, although lower than reported in B&N.

However, at the turn of the century, something seems to be happening to the consumption function. Table 1 shows estimated versions of (1) over EJN's

[7] Similar developments took place in e.g. the UK and the other Scandinavian countries, see Muellbauer and Murphy (1990), Berg (1994) and Lehmussaari (1990).

[8] The role of a broad wealth measure in Norwegian consumption functions has been confirmed in several other specifications, using different definitions of consumer expenditure (non-durables, non-housing, etc.) and different measures of income and wealth, see e.g. Brubakk (1994) and Frøiland (1999).

[9] The result of invariance has been corroborated by Jansen and Teräsvirta (1996) using an alternative method based on smooth transition models.

Table 1: Cointegration vectors estimated on different sub-samples.

1968(3)-1998(4)

$$\widehat{\log C_t} = \underset{(0.04)}{0.72} \log Y_t + \underset{(0.03)}{0.18} \log W_t$$

1968(3)-2000(4)

$$\widehat{\log C_t} = \underset{(0.06)}{0.75} \log Y_t + \underset{(0.05)}{0.13} \log W_t$$

sample, and a sample ending in 2000(4). Although the changes in $\hat{\beta}_1$ and $\hat{\beta}_2$ are quite moderate, the standard errors (below the estimates) almost doubles, and this signals that the evidence of cointegration between consumption, income and wealth becomes weaker when data for 1999 and 2000 are included in the sample. An interpretation is that equation (1) subsumes an age variable in the constant term, which has been undetected empirically because the samples used so far has been dominated by colinearity with the trend in income and wealth. As shown in Figure 1, the age composition of the Norwegian population has behaved like a trend over large parts of the sample period. This can have hidden age composition effects in the consumption function over a long period of time, making the function stable as long as the age structure development have co-moved with either income or wealth (or with a linear combination of the two).

Another variable that may influence consumption in the long run is the real interest rate. The income effect of changes in interest rates is already included in the B&N consumption function via the income variable. However, there may also be substitution effects from interest rate changes; an increase in real interest rates makes consumption today more expensive relative to tomorrow's consumption, and, hence, consumption is expected to decline. From 1984(1) the after-tax real interest rate variable, RR_t, is calculated as

$$RR_t = RLB_t(1 - \tau_t) - \Delta_4 cpi_t,$$

where RLB_t = average nominal interest rate on bank loans, τ_t = marginal income tax rate for households, and $\Delta_4 cpi_t$ = change in annual consumer price inflation. RR_t is set to zero before 1984, because of strict credit market restrictions,[10] which prevented interest rate movements to have significant effects on saving.

We next attempt to establish an extended consumption function, where the effects of changes in the age structure and real interest rates are accounted for. Because of the focus on demography, we chose to express the extended consumption function in per adult capita terms, where aggregate consumption, income and wealth variables are divided by the adult population, defined to be those of 20 years or older. However, let it be said that the results obtained does not depend in any substantive way on the per capita formulation.

In the outset, we assume that both the age structure variable and real interest rates are weakly exogenous, and test the hypothesis that the following relationship

[10] Norwegian credit markets were gradually deregulated from 1984.

Table 2: Johansen tests of cointegration.

VAR system of order: 5 Range: 1968(3) - 2000(4) Endogenous variables : (log of) c y w Exogenous variables: AGE RR $Trend$ Deterministic variables: VAT $STOP$ $Const$ $CS1$ $CS2$ $CS3$				
Eigenvalues	Hypotheses (of rank) and trace test			
λ_i	H_0	H_1	λ_{trace}	*90%*[1]
0.18	$r = 0$	$r \geq 1$	52.1	52.6
0.13	$r \leq 1$	$r \geq 2$	26.0	32.5
0.05	$r \leq 2$	$r \geq 3$	7.30	15.7

[1)] The critical values are taken from Table 2 in Harbo et al. (1998), allowing for two exogenous variables in the system.

constitute a cointegrating relationship:

$$\begin{aligned} \log c_t &= \beta_1 \log y_t + \beta_2 \log w_t - \beta_3 AGE_t - \beta_4 RR_t + Const, \\ \beta_j &> 0,\ j = 1, 2, \text{ and } \beta_i \geq 0,\ i = 3, 4. \end{aligned} \tag{2}$$

Small letters denote the corresponding variable in per adult capita terms ($c_t = C_t/N_t$, etc, where N_t is the population of age 20 or older) in equation (2).

4 Cointegration and exogeneity

The underlying modelling assumption used by the previous studies is that (log of) c_t, y_t, and w_t are integrated of degree one, $\mathsf{I}(1)$ in a common notation. In this subsection we test the hypothesis that the extended relationship in equation (2) is a cointegrating relationship. Unless the cointegration by the earlier studies is spurious, finding equation (2) to be cointegrating then logically requires that AGE_t and RR_t are integrated of degree zero, $\mathsf{I}(0)$. Another interpretation, which can also be reconciled with the existing evidence, is that the two variables are $\mathsf{I}(1)$ but cointegrated. We build on the first interpretation, for the following reasons. First, being a rate, AGE_t has finite variance by construction, although from Figure 1, the characteristic root(s) of AGE will be close to unity empirically, at least on samples ending before 1990. Second, the $\mathsf{I}(0)$ interpretation does not rule out non-stationarity in the form of deterministic shifts in mean, which is particularly relevant for the real interest rate.

To test for cointegration, we apply the Johansen methodology, see Johansen (1988) and Johansen (1995). A vector autoregressive (VAR) model of the log of the three variables c_t, y_t, and w_t, with five lags is formulated. To take account of the two new variables introduced above, we condition on AGE_{t-1} and RR_{t-1}, which are regarded as weakly exogenous variables in the system. For the purposes of testing for the rank, the two variables are taken as $\mathsf{I}(1)$. In accordance with Harbo et al. (1998)'s suggestions for partial systems, a deterministic trend is also included in the system. In addition, a constant term, centred seasonal dummies ($CS1$, $CS2$ and

Figure 2: Recursive trace test statistics, with and without AGE and RR in the information set.

$CS3$), and the two dummy variables VAT and $STOP$, which capture the effects of the introduction of the VAT in 1970 and of a wage and price freeze in 1978-79, respectively, enter the system unrestrictedly.

Trace test statistics for the sample period 1968(3)-2000(4) are reported in Table 2. The critical values for the trace test are taken from Harbo et al. (1998). Although these values take into account that the system is partial, the correct distribution of the test statistics is unknown as it will also depends on the dummy variables which we have included in the system. The trace test statistics λ_{trace} in the table, for the null hypothesis of no cointegration ($r = 0$) is close to the 10% critical value. The statistic testing the null hypothesis $r \leq 1$ is further away from its relevant critical value. Hence, although the formal evidence is not very strong, we proceed on the basis of at most one cointegrating vector.

The deterministic trend is insignificant in the $\mathsf{I}(0)$ system, and it is therefore excluded from the VAR model in the following analysis. Conditioning on $r = 1$, we now turn to test for weak exogeneity of y_t and w_t in the system. The χ^2-distributed statistic for testing the restrictions that the loading coefficients α_y and α_w equal zero has a *p-value* of 0.11. Hence, the restriction is not rejected at the 5% level, and y_t and w_t can be considered as weakly exogenous for the cointegrating parameter. Equation (2) can therefore be regarded as a long run consumption function. The estimated cointegration parameters of the system is given in equation (3), with standard errors in parentheses.

$$\log c_t = \underset{(0.04)}{0.66} \log y_t + \underset{(0.02)}{0.13} \log w_t - \underset{(0.14)}{0.51} AGE_t - \underset{(0.27)}{0.76} RR_t + Const. \tag{3}$$

The income and wealth elasticities in equation (3) are a little lower than in the EJN model, but the associated standard errors also decline when AGE and RR are included in the equation. The two new coefficients in the consumption function are both significant at a 1% level, and have, as expected, negative signs. The interpretation of the age structure coefficient is that consumption decreases by 0.51 percent when the number of middle-aged persons relative to the rest of the adult population increases with one percentage point. The model predicts that an 0.01 point increase in AGE reduces consumption by 0.51 percent. Hence, if income is unaffected, the savings rate is predicted to increase by as much. Similarly, consumption is expected to decrease by 0.76 percent when after-tax real interest rates increases with one percentage point.

Among the two new explanatory variables, it is the age structure variable which is most robust. Hence, it is necessary to include AGE in the model in order to obtain significant real interest rates effects, while AGE is still significant on a 10% level when RR is left out of the information set. However, both the impact and the significance of the age structure variable also increase when changes in real interest rates are controlled for. One interpretation of this interdependence is that the effects on aggregate consumption of changes in these two variables have counteracted each other. A glance at the development in AGE and RR over the sample period, plotted in Figure 1, supports this view. With the exception of the last few years AGE and RR have moved in opposite directions. Hence, as both variables enter the extended consumption function with negative signs, their separate effects on consumption may have been hidden. At the end of the 1990s, on the other hand, both variables moved in the same direction. And, it is when this period is included in the sample period that the evidence of cointegration between consumption, income and wealth become weaker.

Figure 2 shows that the recursive trace statistics of the extended model is markedly more stable than the trace statistic of the B&N model. The trace statistic for the B&N relationship falls visibly as the change in correlation structure between y, w and AGE and RR is beginning to mark its influence on the results. As noted above, it is the age structure variable which is the key variable in "rebuilding" the cointegrating relationship of the variables in the system.

Figure 3 plots the recursive estimates of the cointegration parameters of equation (3). Compared to the recursive plots of the income and wealth elasticities in the EJN model estimated on the sample 1968(3)-2000(4), the new income and wealth elasticites are stable over the last 20 years. Also the recursive plots of the two new coefficients in the consumption function are relatively stable, being inside $\pm$ two standard errors for most of the period.

Having identified a revised cointegrating relationship, we next turn to the specification of an equilibrium correction model for $\Delta \log c_t$, conditional on the exogeneity of w_t and y_t, and with an equilibrium correction term as given by equation (3). Apart from those restrictions, the general single equation model which makes up the starting point of a general-to-specific search procedure contains the same lags and dummies that we used in the cointegration analysis. The preferred model is shown in Table 3. Note that the equilibrium correction term, $EqCM_t$, is consistent with equation (3).

The bottom part of the table contains the following diagnostic tests; the mul-

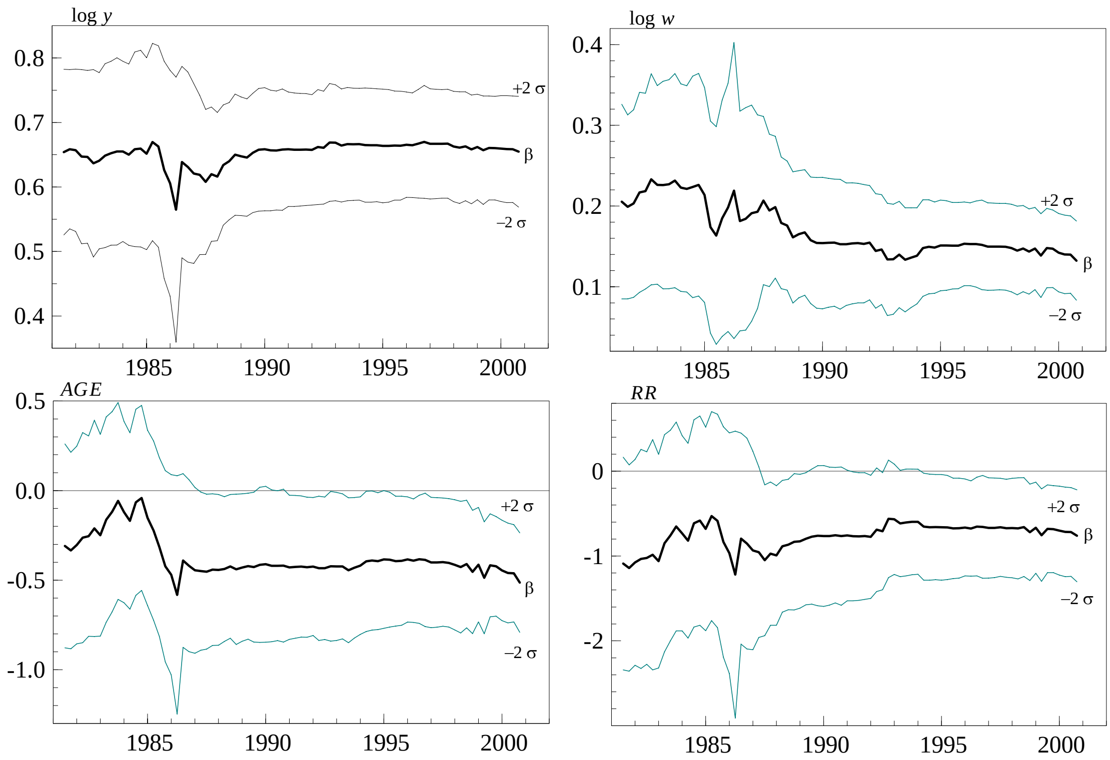

Figure 3: Recursive estimates of the coefficients of the extended cointegrated consumption function, with ± 2 standard errors, denoted σ. Full sample estimates correspond to results in equation (3).

tiple correlation coefficient (R^2), the residual standard error ($\hat{\sigma}$), the $\mathsf{F}_{\mathsf{Null}}$ test of the null hypothesis of "no relationship", the F distributed tests of residual autocorrelation ($\mathsf{F}_{\mathsf{AR}(1-5)}$) and autoregressive conditional heteroscedasticity ($\mathsf{F}_{\mathsf{ARCH}(1-4)}$). In addition, we report the Doornik and Hansen (1994) Chi-square test of residual non-normality ($\chi^2_{\mathsf{normality}}$), see Doornik and Hendry (1999). The numbers in brackets are p-values for the respective null hypotheses, implying that none of the diagnostic tests are significant.

Figure 4 shows the stability of the model over the period 1980(1)-2000(4). The six first graphs show the recursively estimated elasticities in the same order as in Table 3, with ± 2 estimated coefficient standard errors. The last three graphs show first the 1-step residuals with ± 2 residual standard errors, $\pm 2\sigma$ in the graph, the sequence of 1-step Chow statistics scaled with their 1% critical levels, and finally the recursive break-point Chow-tests (also with the one-off 1% level indicated). All graphs show a high degree of stability.

Table 3: Equilibrium correction model of consumption

$$
\begin{aligned}
\Delta \log c_t \;=\; & -\underset{(0.06)}{0.23}\;\Delta \log c_{t-1} + \underset{(0.05)}{0.28}\;\Delta \log c_{t-4} + \underset{(0.05)}{0.29} \\
& + \underset{(0.04)}{0.26}\;\Delta \log y_t + \underset{(0.05)}{0.16}\;\Delta \log w_{t-1} - \underset{(0.07)}{0.44}\;EqCM_t \\
& + \underset{(0.07)}{0.15}\;STOP_t + \underset{(0.01)}{0.08}\;VAT_t \\
& - \underset{(0.01)}{0.07}\;CS1_t - \underset{(0.01)}{0.05}\;CS2_t - \underset{(0.01)}{0.04}\;CS3_t \\
EqCM_t \;=\; & \log c_{t-1} - 0.66 \log y_{t-1} - 0.13 w_t + 0.51 AGE_{t-1} + 0.76 RR_{t-1}
\end{aligned}
$$

Notes

The sample is 1968 (3) to 2000 (4), 130 observations. Estimation is by OLS. Standard errors are in parentheses below the parameter estimates.

$\hat{\sigma} = 1.47\%$	$\mathsf{R}^2 = 0.96$	$\mathsf{F}_{\mathsf{Null}} = 314.5[0.000]$
$\mathsf{F}_{\mathsf{AR}(1-5)} = 0.39[0.85]$	$\mathsf{F}_{\mathsf{ARCH}(1-4)} = 0.77[0.54]$	$\chi^2_{\mathsf{normality}} = 0.72[0.70]$

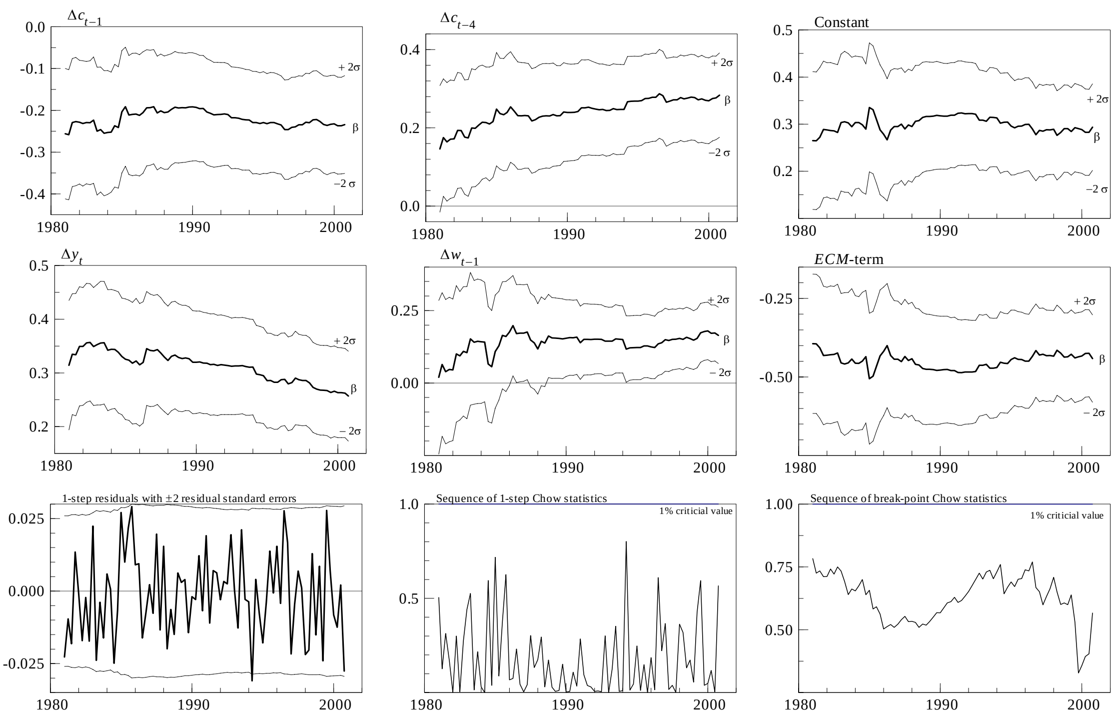

Figure 4: Recursive OLS estimates of the equilibrium correction model of $\Delta \log c_t$.

5 The dynamic effects of age structure

The cointegration analysis showed that the estimated effect of a changing age distribution on consumption and the savings rate is numerically as well as statistically significant. The speed of adjustment of consumption following a demographic "shock" appears also to be relatively sharp. However, these results are only partial, since changes in the age structure also can affect income and wealth. Moreover, even if the effect of AGE_t is isolated to the long run consumption function, there may be dynamic interaction between consumption, income and wealth which will affect the adjustment speed. In other words, if the purpose is to use the consumption function in policy analysis, $\Delta \log y_t$ and $\Delta \log w_t$ cannot be taken as strongly exogenous without further investigation. In this section we therefore use a model of all three variables, $\Delta \log c_t$, $\Delta \log y_t$ and $\Delta \log w_t$, to illustrate how the effects of age structure on other variables than consumption may play a role for the response of the savings rate. Building on cointegration, we first formulate a VAR (in terms of stationary variables), and then an identified economic model of the VAR.

In order to formulate a satisfactory statistical system of that includes $\Delta \log y_t$ and $\Delta \log w_t$, we extend the information set used hitherto by the change in the unemployment rate (ΔU_t), and the change in real government expenditure ($\Delta \log CO_t$). Moreover, since the cointegration analysis showed that $EqCM_t$ does not contain predictive power for $\Delta \log y_t$ or $\Delta \log w_t$, we need to include separate trends for these variables in the model. The trends are *ad hoc*, and are constructed by the use of the Hodrick Prescott filter. In the model reported in Table 5 below, the de-trended series appear as $(\log Y - ytr)_{t-1}$ and $(\log W - wtr)_{t-1}$. The deterministic terms in the multi-equation model are the same as above. Finally, in the following it is useful to "de-mean" the cointegration relationship. Hence, instead of $EqCM_t$ we use

$$eqcm_t = \log c_{t-1} - 0.66 \log y_{t-1} - 0.13 \log w_t + 0.51 AGE_{t-1} + 0.76 RR_{t-1} - 0.66. \quad (4)$$

where 0.66 is the mean of the long-run relationship over the sample period 1968(3)-2000(4).

We first estimate the reduced form system (only lags of the three endogenous variables are used in the right hand side). Diagnostic tests for this unrestricted system are reported in the left hand side column of Table 4. Below the estimated resid-

Table 4: Diagnostics for the unrestricted reduced form and the model in Table 5.

	Unrestricted system	*Model in Table 5*
	72 parameters	29 parameters. FIML estimation
$\hat{\sigma}_c$	1.50%	1.49%
$\hat{\sigma}_y$	2.03%	1.95%
$\hat{\sigma}_w$	2.44%	2.58%
$\mathsf{F}_{\mathsf{AR}(1-5)}$	1.39[0.06]	2.37[0.00]
$\chi^2_{\mathsf{normality}}$	0.99[0.99]	3.93[0.69]
$\mathsf{F}_{\mathsf{HET}x^2}$	0.68[0.99]	0.99[0.51]
χ^2_{overid}		47.755[0.2856] for 43 restrictions

The numbers in [ ] are p-values. The sample is 1968(3)-2000(4), 130 observations.

ual standard errors of each variable (denoted $\hat{\sigma}_c$, $\hat{\sigma}_y$, $\hat{\sigma}_w$) the table shows three diagnostic tests based on the residual vector $\hat{\varepsilon}_t$, for residual autocorrelation ($\mathsf{F}_{\mathsf{AR}(1-5)}$), departure from normality ($\chi^2_{\mathsf{normality}}$) and heteroscedasticity due to squares of the regressor ($\mathsf{F}_{\mathsf{HET}x^2}$), all tests are vector versions of the single equation diagnostics used above, see Doornik and Hendry (2001). Although the test of (vector) residual autocorrelation has a relatively high p-value, due to the residuals of the income and wealth equations, we can proceed *as if* the disturbances are normally distributed, and test different restrictions on the system, with the aim of obtaining a model with fewer parameters than the 72 in the unrestricted reduced form.

The right-hand column in Table 4 shows diagnostics of a model of the system, consisting of the estimated equations shown in Table 5 and identities. The model, estimated by FIML, corresponds to a set of restrictions on the system. Without any restrictions the model's structure is unidentified, but in Table 5 we have an over-identified structure, and we are particularly interested in whether this model is a valid and parsimonious representation of the system—whether it is an encompassing model. A natural test statistic is the likelihood ratio test of the over-identifying restrictions, see Hendry et al. (1988). This test statistic is denoted χ^2_{overid} in Table 4, and it shows that the 43 restrictions separating the unrestricted system from the model is statistically acceptable, with a rather high p-value. We note that the residual autocorrelation test for the model is significant. This is due to the restrictions imposed on the income and wealth equations, and reflects that these variables are determined, in part, by variables elsewhere in the economy. Hence, the significant residual autocorrelation is a reminder that the results based on our small system remain tentative, and that a larger macroeconomic model may be needed to be able to quantify the net effects of age structure with a greater confidence.

Turning to the individual equations, we first note that the consumption function is almost identical to the single equation results above. The only difference is that the intercept is omitted, which reflects that once the mean is subtracted from the equilibrium correction term, there is no trend in the consumption level, which is reasonable. The second equation, for $\Delta \log Y_t$ contains the lagged de-trended $\log Y_t$ variable, and the lags of consumption and wealth growth. Current and lagged unemployment and government expenditure are also significant in the equation for $\Delta \log Y_t$.

The wealth equation has the lagged interest rate on the right hand side, as well as $\Delta \log W_{t-1}$ and the two de-trended series. The negative coefficient of the real interest rate is reasonable, since the wealth variable includes the real value of housing. Importantly, ΔAGE_{t-1} enters the with a numerically huge coefficient. Apparently, real wealth increases when the share of middle-aged persons increases. Although the statistical precision of this effect leaves something to be asked, the model demonstrates the importance of checking for indirect effects of age structure, before assessing the the effects on consumption in a wider macro setting.

Table 5: A dynamic model of the consumption, income and wealth.

$$\Delta \log c_t = -\underset{(0.06)}{0.23}\ \Delta \log c_{t-1} + \underset{(0.05)}{0.26}\ \Delta \log c_{t-4} + \underset{(0.05)}{0.34}\ \Delta \log y_t$$
$$+\underset{(0.05)}{0.20}\ \Delta \log w_{t-1} - \underset{(0.07)}{0.50}\ eqcm_t + \underset{(0.01)}{0.09}\ VAT_t + \underset{(0.07)}{0.13}\ STOP_t$$
$$\Delta \log(Y_t) = \underset{(0.002)}{0.01} - \underset{(0.08)}{0.64}\ (\log(Y) - ytr)_{t-1} + \underset{(0.07)}{0.27}\ \Delta \log c_{t-1}$$
$$-\underset{(0.06)}{0.28}\ \Delta \log W_{t-1} + \underset{(0.05)}{0.43}\ \Delta \log(CO)_t - \underset{(0.68)}{1.71}\ \Delta U_t - \underset{(0.58)}{2.23}\ \Delta U_t$$
$$\Delta \log W_t = \underset{(0.004)}{0.02} + \underset{(0.09)}{0.18}\ \Delta \log W_{t-1} - \underset{(0.12)}{0.31}\ RR_{t-1}$$
$$+\underset{(0.09)}{0.22}\ (\log Y - ytr)_{t-1} - \underset{(0.05)}{0.20}\ (\log W - wtr)_{t-1}$$
$$+\underset{(1.41)}{3.84}\ \Delta AGE_{t-1}$$

Notes

The sample is 1968 (3) to 2000 (4), 129 observations. Estimation is by FIML. Standard errors are in parentheses below the parameter estimates. Each equation also contain three centred seasonal dummies (not shown).
See Table 4 for residual standard errors, diagnostic tests, and encompassing tests.

The upper panel in Figure 5 shows the cumulated dynamic effects on annual consumption growth of a one percentage point increase in $AGE \cdot 100$, e.g., from 30% to 31%. Note that in the third period after the change in AGE, consumption growth is above the baseline. This is due to the inclusion of ΔAGE_{t-1} in the equation for $\Delta \log W_t$. The corresponding effect on the savings rate, according to the estimated model, is a 1.3 percentage point reduction. However, as the effect via $\Delta \log W_t$ dies out, the equilibrium correcting effect in the consumption function takes over. Already, in the fourth quarter after the exogenous change, the savings rate is back at its baseline value, and it quickly approaches the 0.5 point increase consistent with the single equation results of section 4.

In the sample period used in this study AGE has increased by 7 percentage points. Of course, the historical increase in AGE covers several years, see Figure 1, and the dynamic effects will be more complicated (and drawn out in time) than our stylized scenario indicates. That said, if the model captures the main mechanisms triggered by the age structure change, the effect may be an increase in the savings rate by 4 percentage points.

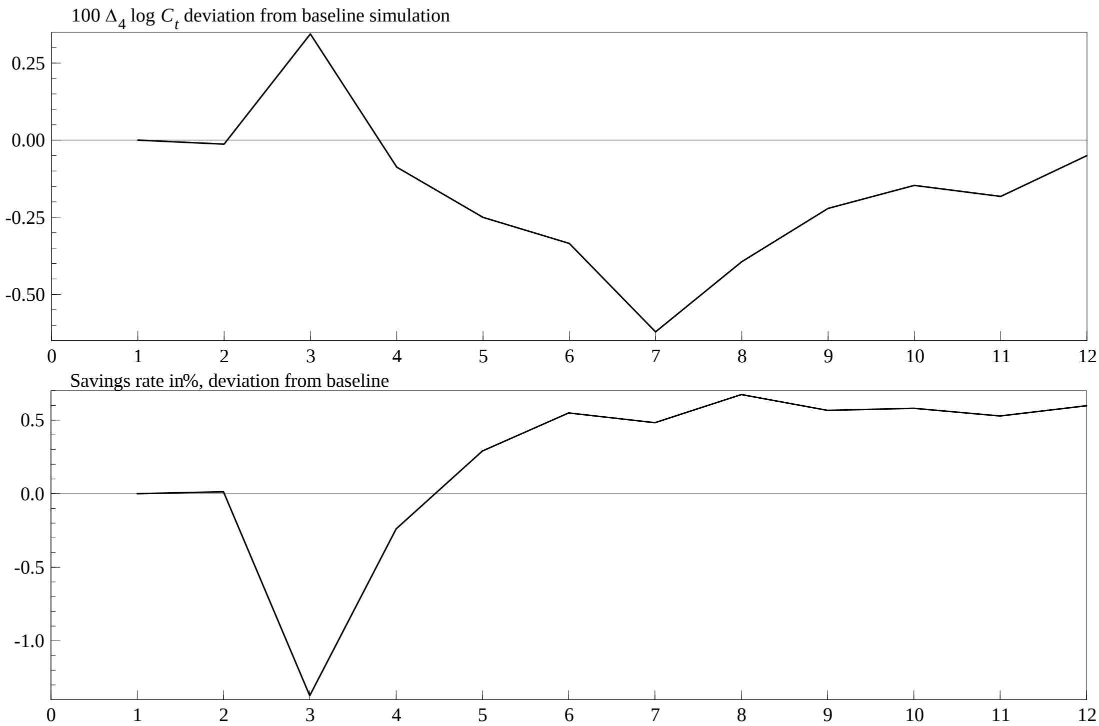

Figure 5: Upper panel: effect of a 1 point permanent rise in AGE on $\Delta_4 \log C_t$. Lower panel: effect of a 1 point permanent rise in AGE on the savings rate, s.

6 Concluding remarks

In this paper we have investigated the empirical relationship between aggregate consumption and the age structure of the population in Norway. The analysis is based on aggregate time series data, and age structure changes are represented by the ratio of number of middle-aged persons, defined as those between 50 and 66 years of age, to the rest of the adult population. Similar to what other studies using aggregate data report, we found that changes in the age structure of the population have significant and numerically important effects on Norwegian aggregate consumption. More specifically, we found that aggregate consumption decreases when the share of the middle aged persons in the population increases. Our results give hence support to the life cycle model. An important finding in the paper is that controlling for age structure effects stabilize the parameters of the consumption model when data up to year 2000 are included in the estimation sample. Although several studies have reported significant age structure effects on aggregate consumption in different countries, none have, to our knowledge, emphasized that a changing age distribution can play such a key role in the consumption function. Furthermore, controlling for age structure effects reveal significant real interest rate effects on aggregate consumption.

Our findings imply that changes in the age distribution of the population can have important consequences on domestic demand and households' saving. The predicted estimated medium term effect of the recent change in population age structure is an increase in the savings rate. However, as illustrated by the simulation exercise, the adjustment process is unlikely to be smooth, and may even contain sign reversals. Clearly, such complicated dynamics represents a challenge to an economic policy aiming at stabilization of savings behaviour.

Although the analysis in this paper have been restricted to Norwegian data, there are good reasons to believe that the results also apply to other countries. And, identifying age structure effects on consumption may be of increasing importance in many countries. Several developed countries experienced a baby boom in the first decades after World War II. The share of middle-aged, prime savers relative to the rest of the population is hence also on the rise in these countries. If our findings on Norwegian data apply to these countries, the changing age distribution would then put a downward pressure on consumption in the Western world in the first coming years; a pattern the empirical models then would not identify if age structure effects not are controlled for.

An issue discussed in this paper is the role of demography in forecasting. In contrast to most of the economic explanatory variables, changes in the age structure of the population is known with approximate certainty in the short to medium run. Whether this property of the age structure variable, or demography in general, can be used to improve forecasts of consumption and GDP growth, represents interesting topics for future research.

A Data definitions

All data are quarterly and seasonally unadjusted. Throughout the paper small letters indicate the logarithmic values of the variables. The data are taken from Norges Bank's model database RIMINI (the spring 2003 version of the database). In addition to variable descriptions, the table below contains the source for each of the variables.

Symbol	Definitions and sources
AGE	$\frac{\text{Population 50-66 years old}}{\text{Population 20-49 years old + Population above 66 years old}}$. *Source: Statistics Norway.*
C	Private consumption expenditure (incl. ideal organizations), fixed 2000 prices. *Source: Statistics Norway.*
CO	Public consumption expenditure, fixed 2000 prices. *Source: Statistics Norway.*
CPI	Consumer price index (2000=1). *Source: Statistics Norway.*
N	The population of age 20, or older. *Source: Statistics Norway.*
PC	Price deflator for total private consumption expenditure. (2000=1). *Source: Statistics Norway.*
RR	Marginal after-tax real interest rates for households 1968(3)-1985(4): zero 1986(1)-2000(4): $RLB(1-\tau) - \Delta_4 cpi_t$, where RLB = average interest rate on households' bank loans, and τ= marginal income tax rate for households. *Sources: Statistics Norway and Norges Bank.*
$STOP$	Income policy dummy; constructed for catching up the inflationary pressures which were being built up during the wage and price freeze in 1978. It takes non-zero values in the quarters from 1979(1) to 1980(1), and is zero elsewhere. See Brodin and Nymoen (1992) for details.
U	Unemployment rate. Source: *Statistics Norway and Norges Bank.*
VAT	Dummy for the introduction of VAT. Takes the values 1 in 1969(4) and-1 in 1970(1).
W	Real household wealth; nominal household wealth (financial and housing wealth) deflated by PC. *Sources: Statistics Norway and Norges Bank.*
Y	Households' real disposable income; nominal disposable income deflated by PC. *Source: Statistics Norway.*

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