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CAN RANDOM COEFFICIENT COBB-DOUGLAS PRODUCTION FUNCTIONS BE AGGREGATED TO SIMILAR MACRO FUNCTIONS ?

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Abstract: Parametric aggregation of heterogeneous micro production technologies is discussed. A four-factor Cobb-Douglas function with normally distributed firm specific coefficients and with log-normal inputs (which agrees well with the available data) is specified. Since, if the number of micro units is large enough, aggregates expressed as arithmetic means can be associated with expectations, we consider conditions ensuring an approximate relation of Cobb-Douglas form to exist between expected output and expected inputs. Similar relations in higher-order moments are also derived. It is shown how the aggregate input elasticities depend on the coefficient heterogeneity and the covariance matrix of the log-input vector and hence vary over time. An implementation based on firm panel data for two manufacturing industries gives estimates of industry level input elasticities and decomposition for expected output. Finally, aggregation errors which emerge when the correct aggregate elasticities are replaced by the expected micro elasticities, are explored.

Keywords: Productivity. Panel Data. Random coefficients. Log-normal distribution. Aggregate production function.

JEL classification: C23, C43, D21, L11

1 Introduction

The production function is usually considered an essentially micro construct, and the existence, interpretation, and stability of a corresponding aggregate function are issues of considerable interest in macro-economic modelling and research, cf. the following quotations: "The benefits of an aggregate production model must be weighted against the costs of departures from the highly restrictive assumptions that underly the existence of an aggregate production function" [Jorgenson (1995, p. 76)] and "An aggregate production function is a function that maps aggregate inputs into aggregate output. But what exactly does this mean? Such a concept has been implicit in macroeconomic analyses for a long time. However, it has always been plagued by conceptual confusions, in particular as to the link between the underlying micro production functions and the aggregate macro production function, *the latter thought to summarize the alleged aggregate technology*" [Felipe and Fisher (2003, p. 209), our italics].[1] Three (related) questions are of particular interest: Can aggregation by analogy, in which estimated micro parameters, or averages of such elasticities, are inserted into a macro function of the same form, give an adequate representation of the 'aggregate technology'? Which are the most important sources of aggregation bias and instability of the macro function over time? Does the heterogeneity of the micro technologies or the spread in the input mix across firms affect the macro parameters, and if so, how? The last question was raised more than 30 years ago by Johansen (1972), in the context of aggregating 'putty-clay' micro technologies to smooth macro functions by utilizing information on the distribution of input coefficients across firms. Panel data is a necessity to examine such issues empirically in some depth. Yet, the intersection between the literature on aggregation and the literature on panel data econometrics is still very small.

Our focus in this paper will be on the three questions raised above, using a rather restrictive parametric specification of the *average* micro technology, based on a four-factor Cobb-Douglas function, with *random coefficients* to represent technological heterogeneity. Although Cobb-Douglas restricts the pattern of input substitution strongly and has to some extent been rejected in statistical tests, the simplicity of this parametric form

[1] A textbook exposition of theoretical properties of production functions aggregated from neo-classical micro functions is given in Mas-Colell, Whinston and Green (1995, Section 5.E).

of the average technology is a distinctive advantage of this functional form over, *e.g.*, Translog or CES. We assume that the random coefficients are jointly normal (Gaussian) and that the inputs are generated by a multivariate log-normal distribution, whose parameters may shift over time. To our knowledge, this is the first study exploring aggregate production functions by using firm-level (unbalanced) panel data in a full random coefficient setting by means of this form of the average micro technology. A model framework which is similar to ours, although denoted as '*cross-sectional* aggregation of log-linear models' (our italics) is considered by van Garderen, Lee and Pesaran (2000, Section 4.2). However, they implement it, not on data from single firms, but on time series data from selected industries (p. 309), which is less consistent with the underlying micro theory. The expectation vector and covariance matrix of the coefficient vector are estimated from panel data for two Norwegian manufacturing industries. Log-normality of the inputs is tested and for the most part not rejected. This, in conjunction with a Cobb-Douglas technology with normally distributed coefficients, allows us to derive interpretable expressions for aggregate production.

From the general literature on aggregation it is known that properties of relationships aggregated from relationships for micro units depend on the average functional form in the micro model, its heterogeneity, the distribution of the micro variables, and the form of the aggregation functions. Customarily, the aggregation functions are arithmetic means or sums. If the number of micro units is large enough to appeal to a statistical law of large numbers and certain additional statistical regularity conditions are satisfied, we can associate the arithmetic mean with the expectation [cf. Fortin (1991, Section 2), Stoker (1993, Section 3), Hildenbrand (1998, Section 2), and Biørn and Skjerpen (2004, Section 2)], which is what we shall do here. However, we will be concerned not only with relationships expressed in terms of expected inputs and output, but also with relationships between higher-order moments.

Given our stochastic assumptions, the marginal distribution of output will not be log-normal. van Garderen, Lee and Pesaran (2000, p. 307) derived a formula for the expectation of output under the same stochastic assumptions as we employ. Their formula can also be generalized to higher order origo moments of output, provided that the moments exist. The existence of these moments is guaranteed by an eigenvalue condition which involves the covariance matrices of the random coefficients and the log-inputs. Examining this condition for each year in the data period, we find that, generally, only the first and second origo moments of output exist.

Besides the exact formulae, approximate expressions for the origo moments of output are considered. An advantage of the latter is that they allow us to derive interpretable relationships between origo moments of output and inputs, construct decompositions of expected output, of interest when for example predicting output changes from changes in inputs, and derive expressions for aggregate elasticities. In order to assess the quality of this approximation, a comparison of estimated origo moments of output based on the approximate formulae with those obtained by the exact formulae.

A main objective of the paper is to compare correctly estimated aggregate input and scale elasticities with elasticities obtained from simple analogy. Since aggregate parameters are in general undefined unless the distribution of the micro variables is restricted in some way, we provide results for the limiting cases where the means of the log-inputs change and their dispersions are preserved, and the opposite case. While the expected micro elasticities, by construction, are time invariant, the correct macro elasticities are allowed to vary. For some elasticities we find a trending pattern. Besides, even when the variation over time is modest there are significant level differences between the aggregate elasticities calculated from the correct formulae and those obtained by analogy, as well as differences in the relative size of the elasticities for the different inputs.

The following sections are organized as follows. The model is presented in Section 2, properties of the theoretical distribution of output are discussed, and approximative expressions for the moments of output are derived. From this we obtain, in Section 3, an approximate aggregate production function and expressions for the correct aggregate input elasticities, in the 'dispersion preserving' and 'mean preserving' cases. In Section 4, the econometric method and data are first described, and then estimates of aggregate input elasticities and other results are presented. Section 5 concludes.

2 Model and output distribution

2.1 Basic assumptions

We consider an n factor Cobb-Douglas production function model for panel data of an arbitrary form, written in log-linear form as

$$(1) \qquad y_{it} = x'_{it}\beta_i + u_{it} = \alpha_i + z'_{it}\gamma_i + u_{it},$$

where i is the firm index, t is the period index, $x_{it} = (1, z'_{it})'$ is an $n+1$ vector (including a one for the intercept) and $\beta_i = (\alpha_i,\, \gamma'_i)'$ is an $n+1$ vector (including the intercept), γ_i denoting the n vector of input elasticities. We interpret z_{it} as $\ln(Z_{it})$, where Z_{it} is the n dimensional input vector, and y_{it} as $\ln(Y_{it})$, where Y_{it} is output, and assume that

$$(2) \qquad x_{it} \sim \mathcal{N}(\mu_{xt}, \Sigma_{xxt}) = \mathcal{N}\left(\begin{bmatrix} 1 \\ \mu_{zt} \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & \Sigma_{zzt} \end{bmatrix}\right),$$

$$(3) \qquad \beta_i \sim \mathcal{N}(\mu_\beta, \Sigma_{\beta\beta}) = \mathcal{N}\left(\begin{bmatrix} \mu_\alpha \\ \mu_\gamma \end{bmatrix}, \begin{bmatrix} \sigma_{\alpha\alpha} & \sigma'_{\gamma\alpha} \\ \sigma_{\gamma\alpha} & \Sigma_{\gamma\gamma} \end{bmatrix}\right),$$

$$(4) \qquad u_{it} \sim \mathcal{N}(0, \sigma_{uu}),$$

$$(5) \qquad x_{it}, \beta_i, u_{it} \ \text{ are stochastically independent},$$

where $\sigma_{\alpha\alpha} = \text{var}(\alpha)$, $\sigma_{\gamma\alpha}$ is the n vector of covariances between α_i and γ_i, and $\Sigma_{\gamma\gamma}$ is the n dimensional covariance matrix of γ_i. The $n+1$ dimensional covariance matrix Σ_{xxt} is singular since x_{it} has a one element, while its n dimensional submatrix Σ_{zzt} is non-singular in general.

The implementation of the model specifies four inputs ($n = 4$): capital (K), labour (L), energy (E), and materials (M), and includes a deterministic linear trend (t), intended to capture the level of the technology. We therefore parameterize (1) as

$$(6) \qquad y_{it} = \alpha^*_i + \kappa t + z'_{it}\gamma_i + u_{it},$$

where α^*_i and γ_i are random parameters specific to firm i, and the trend coefficient κ is assumed to be firm invariant. We let $z_{it} = (z_{Kit}, z_{Lit}, z_{Eit}, z_{Mit})'$, $\gamma_i = (\gamma_{Ki}, \gamma_{Li}, \gamma_{Ei}, \gamma_{Mi})'$, collect all the random coefficients for firm i in the vector $\psi_i = (\alpha^*_i, \gamma'_i)'$, and let

$$\mathsf{E}(\psi_i) = \psi = (\mu^*_\alpha, \mu_K, \mu_L, \mu_E, \mu_M)', \qquad \mathsf{E}[(\psi_i - \psi)(\psi_i - \psi)'] = \Omega,$$

where Ω is a symmetric, but otherwise unrestricted matrix, and $\mathsf{E}(\alpha^*_i) = \mu^*_\alpha$.

2.2 The distribution of output

It is convenient first to characterize the distribution of *log-output*. From (1)–(5) it follows that

$$(7)\qquad (y_{it}|\beta_i) \sim \mathcal{N}(\mu_{xt}'\beta_i, \beta_i'\Sigma_{xxt}\beta_i + \sigma_{uu}), \qquad (y_{it}|x_{it}) \sim \mathcal{N}(x_{it}'\mu_\beta, x_{it}'\Sigma_{\beta\beta}x_{it} + \sigma_{uu}),$$

and, by using the law of iterated expectations, that

$$(8)\quad \mu_{yt} = \mathsf{E}(y_{it}) = \mathsf{E}[\mathsf{E}(y_{it}|\beta_i)] = \mu_{xt}'\mu_\beta,$$

$$\begin{aligned}(9)\quad \sigma_{yyt} = \text{var}(y_{it}) &= \mathsf{E}[\text{var}(y_{it}|\beta_i)] + \text{var}[\mathsf{E}(y_{it}|\beta_i)] = \mathsf{E}[\text{tr}(\beta_i\beta_i'\Sigma_{xxt}) + \sigma_{uu}] + \text{var}(\mu_{xt}'\beta_i)\\ &= \text{tr}[\mathsf{E}(\beta_i\beta_i'\Sigma_{xxt})] + \sigma_{uu} + \mu_{xt}'\Sigma_{\beta\beta}\mu_{xt} = \text{tr}[(\mu_\beta\mu_\beta' + \Sigma_{\beta\beta})\Sigma_{xxt}] + \sigma_{uu} + \mu_{xt}'\Sigma_{\beta\beta}\mu_{xt}\\ &= \mu_{xt}'\Sigma_{\beta\beta}\mu_{xt} + \mu_\beta'\Sigma_{xxt}\mu_\beta + \text{tr}(\Sigma_{\beta\beta}\Sigma_{xxt}) + \sigma_{uu}.\end{aligned}$$

The four components of σ_{yyt} represent, respectively, (i) the variation in the coefficients ($\mu_{xt}'\Sigma_{\beta\beta}\mu_{xt}$), (ii) the variation in the log-inputs ($\mu_\beta'\Sigma_{xxt}\mu_\beta$), (iii) the interaction between the variation in the log-inputs and the coefficients [$\text{tr}(\Sigma_{\beta\beta}\Sigma_{xxt})$], and (iv) the disturbance variation (σ_{uu}).

We next characterize the distribution of *output*. Since $Y_{it} = e^{y_{it}} = e^{x_{it}'\beta_i + u_{it}}$, we know from (7) that $(Y_{it}|x_{it})$ and $(Y_{it}|\beta_i)$ are log-normal. From Evans, Hastings, and Peacock (1993, Chapter 25), it therefore follows, for any positive integer r, that

$$(10)\qquad \mathsf{E}(Y_{it}^r|\beta_i) = \mathsf{E}_{x_{it},u}(e^{ry_{it}}|\beta_i) = \exp[r\mu_{xt}'\beta_i + \tfrac{1}{2}r^2(\beta_i'\Sigma_{xxt}\beta_i + \sigma_{uu})],$$

$$(11)\qquad \mathsf{E}(Y_{it}^r|x_{it}) = \mathsf{E}_{\beta_i,u}(e^{ry_{it}}|x_{it}) = \exp[rx_{it}'\mu_\beta + \tfrac{1}{2}r^2(x_{it}'\Sigma_{\beta\beta}x_{it} + \sigma_{uu})],$$

which shows that any *conditional finite-order* moment of output exist. *Marginally,* however, Y_{it} is not log-normal, since $x_{it}'\beta_i$ is non-normal, and some of its finite-order moments may not exist. We now show that a closed form expression for the marginal origo moments of Y exist and can be derived from (10).[2]

From (10) and the law of iterated expectations, from now on, for simplicity, omitting subscripts (i,t),we find that the marginal r'th-order origo moment of Y_{it}, when it exists,

[2]Following a similar argument, the same result can be derived from (11). A proof for the case $r = 1$, similar to the one below, but related to (11) and somewhat longer, is given in van Garderen, Lee, and Pesaran (2000, pp. 306-307).

can be written as

$$(12)\quad \mathsf{E}(Y^r) = \exp[\tfrac{1}{2}r^2\sigma_{uu}]\mathsf{E}_\beta[\exp(r\mu_x'\beta + \tfrac{1}{2}r^2\beta_i'\Sigma_{xx}\beta)]$$
$$= \exp[r\mu_x'\mu_\beta + \tfrac{1}{2}r^2(\mu_\beta'\Sigma_{xx}\mu_\beta + \sigma_{uu})]\mathsf{E}_\delta\{\exp[(r\mu_x' + r^2\mu_\beta'\Sigma_{xx})\delta + \tfrac{1}{2}r^2\delta'\Sigma_{xx}\delta]\},$$

where $\delta = \beta - \mu_\beta \sim \mathcal{N}(0, \Sigma_{\beta\beta})$.

Inserting for the density of δ, $f(\delta) = (2\pi)^{-\frac{n+1}{2}}|\Sigma_{\beta\beta}|^{-\frac{1}{2}}\exp[-\frac{1}{2}\delta'\Sigma_{\beta\beta}^{-1}\delta]$, we find that the last expectation in (12) can be written as

$$(13)\quad H_r = \mathsf{E}_\delta\{\exp[(r\mu_x' + r^2\mu_\beta'\Sigma_{xx})\delta + \tfrac{1}{2}r^2\delta'\Sigma_{xx}\delta]\}$$
$$= \int_{R^{n+1}} \exp[a(r)'\delta + \tfrac{1}{2}r^2\delta'\Sigma_{xx}\delta]f(\delta)d\delta = (2\pi)^{-\frac{n+1}{2}}|\Sigma_{\beta\beta}|^{-\frac{1}{2}}\int_{R^{n+1}} \exp[\lambda_r(\delta)]d\delta,$$

where

$$(14)\quad \lambda_r(\delta) = a(r)'\delta - \tfrac{1}{2}\delta'M(r)\delta, \quad M(r) = \Sigma_{\beta\beta}^{-1} - r^2\Sigma_{xx}, \quad a(r) = r\mu_x + r^2\Sigma_{xx}\mu_\beta.$$

Since an alternative way of writing $\lambda_r(\delta)$ is

$$\lambda_r(\delta) = \tfrac{1}{2}a(r)'M(r)^{-1}a(r) - \tfrac{1}{2}[\delta' - a(r)'M(r)^{-1}]M(r)[\delta - M(r)^{-1}a(r)],$$

and integration goes over R^{n+1}, we can substitute $q = \delta - M(r)^{-1}a(r)$ and rewrite H_r as

$$H_r = |\Sigma_{\beta\beta}|^{-\frac{1}{2}}\exp[\tfrac{1}{2}a(r)'M(r)^{-1}a(r)]\int_{R^{n+1}}(2\pi)^{-\frac{n+1}{2}}\exp[-\tfrac{1}{2}q'M(r)q]dq.$$

The last integral has a formal similarity to the integral over a normal density function, with $M(r)$ occupying the same place as the *inverse* of the covariance matrix of q. Because the latter integral is one for any q and any positive definite $M(r)$, we know that

$$\int_{R^{n+1}}(2\pi)^{-\frac{n+1}{2}}\exp[-\tfrac{1}{2}q'M(r)q]dq = |M(r)^{-1}|^{\frac{1}{2}},$$

if $M(r)$ is positive definite. Hence we can express H_r in closed form as

$$H_r = |\Sigma_{\beta\beta}|^{-\frac{1}{2}}\exp[\tfrac{1}{2}a(r)'M(r)^{-1}a(r)]|M(r)|^{-\frac{1}{2}} = \exp[\tfrac{1}{2}a(r)'M(r)^{-1}a(r)]|\Sigma_{\beta\beta}M(r)|^{-\frac{1}{2}}.$$

Inserting this into (12) yields

$$\mathsf{E}(Y^r) = |M(r)\Sigma_{\beta\beta}|^{-\frac{1}{2}}\exp[r\mu_x'\mu_\beta + \tfrac{1}{2}r^2(\mu_\beta'\Sigma_{xx}\mu_\beta + \sigma_{uu}) + \tfrac{1}{2}a(r)'M(r)^{-1}a(r)].$$

After inserting for $M(r)$ and a from (14), we finally find that the r'th order moment of output can be written in closed form as

$$\text{(15)} \qquad \mathsf{E}(Y^r) = |I_{n+1} - r^2\Sigma_{\beta\beta}\Sigma_{xx}|^{-\frac{1}{2}} \exp[r\mu_x'\mu_\beta + \tfrac{1}{2}r^2(\mu_\beta'\Sigma_{xx}\mu_\beta + \sigma_{uu}) + \tfrac{1}{2}(r\mu_x' + r^2\mu_\beta'\Sigma_{xx})(\Sigma_{\beta\beta}^{-1} - r^2\Sigma_{xx})^{-1}(r\mu_x + r^2\Sigma_{xx}\mu_\beta)].$$

Obviously, the argument above relies on $M(r)$ (and hence $M(r)^{-1}$) being positive definite. Hence,

$$\text{(16)} \qquad H_r \text{ and } \mathsf{E}(Y^r) \text{ exist } \iff \Sigma_{\beta\beta}^{-1} - r^2\Sigma_{xx} \text{ is positive definite.}$$

Intuitively, such a condition must be imposed because both β and z have support from minus to plus infinity. If $\mathsf{E}(Y^r)$ exists, then all lower-order moments also exist. From (14) we have

$$M(r-1) = M(r) + (2r-1)\Sigma_{xx}, \quad r = 2, 3, \dots\ ,$$

so that if $M(r)$ and Σ_{xx} are positive definite, then $M(r-1)$ is also positive definite, since $2r > 1$ and the sum of two positive definite matrices is positive definite.

2.3 Approximations to the origo moments of output

We now present a way of obtaining, from (12), an approximate formula for $\mathsf{E}(Y^r)$. Neglecting in the integral the effect of the dispersion in $\delta'\Sigma_{xx}\delta = \mathrm{tr}[\delta\delta'\Sigma_{xx}]$ by replacing it by its expectation, $\mathrm{tr}[\Sigma_{\beta\beta}\Sigma_{xx}]$, we get, provided that (16) holds, the following analytical approximation to the r'th origo moment of output:

$$\begin{aligned} \text{(17)} \qquad \mathsf{E}(Y^r) \approx G_r(Y) &= \exp[r\mu_x'\mu_\beta + \tfrac{1}{2}r^2(\mu_\beta'\Sigma_{xx}\mu_\beta + \mathrm{tr}[\Sigma_{\beta\beta}\Sigma_{xx}] + \sigma_{uu})] \\ &\quad \times\ \exp[\tfrac{1}{2}(r\mu_x' + r^2\mu_\beta'\Sigma_{xx})\Sigma_{\beta\beta}(r\mu_x + r^2\Sigma_{xx}\mu_\beta)] \\ &= \exp[r\mu_x'\mu_\beta + \tfrac{1}{2}r^2(\mu_\beta'\Sigma_{xx}\mu_\beta + \mu_x'\Sigma_{\beta\beta}\mu_x + \mathrm{tr}[\Sigma_{\beta\beta}\Sigma_{xx}] + \sigma_{uu}) \\ &\quad + r^3\mu_\beta'\Sigma_{xx}\Sigma_{\beta\beta}\mu_x + \tfrac{1}{2}r^4\mu_\beta'\Sigma_{xx}\Sigma_{\beta\beta}\Sigma_{xx}\mu_\beta], \end{aligned}$$

since $\mathrm{var}(a'\delta) = a'\Sigma_{\beta\beta}a$. When (16) does not hold, this approximation, of course, makes no sense. When applying this approximation we eliminate both the square root of the inverse of the determinant $|I_{n+1} - r^2\Sigma_{\beta\beta}\Sigma_{xx}|$ and all terms involving $\Sigma_{\beta\beta}^{-1}$ from the function. This is an obvious simplification when we use the function to derive *and, more importantly, interpret* expressions for the aggregate input elasticities below. A price we have to pay is that in deriving (17) we neglect the variance of a term occurring in the exponent. Since the exponential function is convex, intuition says that $G_r(Y)$ is likely to underestimate $\mathsf{E}(Y^r)$. The numerical calculations in Section 4 will support this intuition.

We can then, using (8) and (9), write the analytical approximation to $\mathsf{E}(Y^r)$ as

$$G_r(Y) = \Phi_r(y)\Gamma_r\Lambda_r, \tag{18}$$

where

$$\Phi_r(y) = \exp[r\mu_y + \tfrac{1}{2}r^2\sigma_{yy}] \tag{19}$$

is the 'first-order' approximation we would have obtained if we had proceeded as if y were normally and Y were log-normally distributed marginally [cf. (8) and (9)], and

$$\Gamma_r = \exp[r^3\mu_x'\Sigma_{\beta\beta}\Sigma_{xx}\mu_\beta], \qquad \Lambda_r = \exp[\tfrac{1}{2}r^4\mu_\beta'\Sigma_{xx}\Sigma_{\beta\beta}\Sigma_{xx}\mu_\beta] \tag{20}$$

can be considered correction factors to this first-order approximation.[3]

3 An approximate aggregate production function in origo moments

We next derive an approximate relationship between $\mathsf{E}(Y^r)$ and $\mathsf{E}(Z^r)$ to be used, for $r = 1$, in examining aggregation biases when representing the aggregate variables by their arithmetic means. In doing this, we recall that $e^{\mathsf{E}[\ln(Y)]}$ and $e^{\mathsf{E}[\ln(Z_i)]}$ correspond to the geometric means, and $\mathsf{E}(Y)$ and $\mathsf{E}(Z_i)$ to the arithmetic means of the output and the i'th input, respectively. We first assume r arbitrarily large, still assuming that (16) is satisfied, and afterwards discuss the case $r = 1$ in more detail.

3.1 A Cobb-Douglas production function in origo moments

Let

$$\begin{aligned} \theta_{yr} = \ln[G_r(Y)] - r\mu_y &= \ln[\Phi_r(y)] + \ln[\Gamma_r] + \ln[\Lambda_r] - r\mu_x'\mu_\beta \\ &= \tfrac{1}{2}r^2\sigma_{yy} + r^3\mu_x'\Sigma_{\beta\beta}\Sigma_{xx}\mu_\beta + \tfrac{1}{2}r^4\mu_\beta'\Sigma_{xx}\Sigma_{\beta\beta}\Sigma_{xx}\mu_\beta, \end{aligned} \tag{21}$$

[3] The approximation leading to (18) follows by applying the law of iterated expectations on (10). Proceeding in a similar way from (11) would have given a similar approximation, although with a different Λ_r component; see Biørn, Skjerpen and Wangen (2003, Sections 3.1 and 6.3).

which can be interpreted as an approximation to $\ln[\mathsf{E}(Y^r)] - \mathsf{E}[\ln(Y^r)]$, and let Z_i denote the i'th element of Z, *i.e.*, the i'th input, and $z_i = \ln(Z_i)$. Since $z_i \sim \mathcal{N}(\mu_{zi}, \sigma_{zizi})$, where μ_{zi} is the i'th element of μ_z and σ_{zizi} is the i'th diagonal element of Σ_{zz} [cf. (2)], we have

$$(22) \qquad \mathsf{E}(Z_i^r) = \mathsf{E}(e^{z_i r}) = \exp(\mu_{zi} r + \tfrac{1}{2}\sigma_{zizi} r^2), \quad r = 1, 2, \dots \;; \quad i = 1, \dots, n.$$

Let $\mu_{\gamma i}$ be the i'th element of μ_γ, *i.e.*, the expected elasticity of the i'th input. Since (22) implies $e^{\mu_{zi}\mu_{\gamma i} r} = \exp(-\frac{1}{2}\sigma_{zizi} r^2 \mu_{\gamma i})[\mathsf{E}(Z_i^r)]^{\mu_{\gamma i}}$, it follows from (17) and (21) that

$$(23) \qquad G_r(Y) = e^{\mu_\alpha r} A_r \prod_{i=1}^{n} [\mathsf{E}(Z_i^r)]^{\mu_{\gamma i}},$$

where

$$(24) \qquad A_r = \exp\left(\theta_{yr} - \tfrac{1}{2} r^2 \sum_{i=1}^{n} \sigma_{zizi}\mu_{\gamma i}\right) = \exp(\theta_{yr} - \tfrac{1}{2} r^2 \mu_\gamma' \sigma_{zz}),$$

and $\sigma_{zz} = \text{diagv}(\Sigma_{zz})$.[4] Eq. (23) can be interpreted (approximately) as a *Cobb-Douglas function in the r'th origo moments of* Y *and* $Z_1, \dots, Z_n$ with exponents equal to the expected micro elasticities $\mu_{\gamma 1}, \dots, \mu_{\gamma n}$ and an intercept $e^{\mu_\alpha r}$, adjusted by the factor A_r. The latter depends, via θ_{yr}, on the first and second moments of the log-input vector z and the coefficient vector β and σ_{uu} [cf. (8), (9) and (21)].

For $r = 1$, (23) gives

$$(25) \qquad G_1(Y) = e^{\mu_\alpha} A_1 \prod_{i=1}^{n} [\mathsf{E}(Z_i)]^{\mu_{\gamma i}}.$$

Seemingly, this equation could be interpreted as a Cobb-Douglas function in the arithmetic means $\mathsf{E}(Y)$ and $\mathsf{E}(Z_1), \dots, \mathsf{E}(Z_n)$, with elasticities coinciding with the expected micro elasticities $\mu_{\gamma 1}, \dots, \mu_{\gamma n}$ and an intercept e^{μ_α} adjusted by the factor A_1. However, we will show that, due to the randomness of the micro coefficients, in combination with the non-linearity of the micro production function, the situation is not so simple.

3.2 Aggregation by analogy and aggregation biases in output and in input elasticities

Assume that we, instead of (25), represent the aggregate production function simply by

$$(26) \qquad \widehat{\mathsf{E}(Y)} = e^{\mu_\alpha} \prod_{i=1}^{n} [\mathsf{E}(Z_i)]^{\mu_{\gamma i}}.$$

[4] We let 'diagv' before a square matrix denote the column vector containing its diagonal elements.

This can be said to mimic the *aggregation by analogy* often used by macro-economists and macro model builders. The resulting *aggregation error in output*, when we approximate $\mathsf{E}(Y)$ by $G_1(Y)$, is

$$(27) \qquad \epsilon(Y) = G_1(Y) - \widehat{\mathsf{E}(Y)} = (A_1 - 1)e^{\mu_\alpha} \prod_{i=1}^{n} [\mathsf{E}(Z_i)]^{\mu_{\gamma i}}.$$

Representing the aggregate Cobb-Douglas production function by (26) will bias not only its intercept, but also its derived input elasticities, because A_1 in (25) responds to changes in μ_z and Σ_{zz}. We see from (8), (9) and (21) that when $\Sigma_{\gamma\gamma}$ is non-zero, a change in μ_z affects not only expected log-output, μ_y, but also its variance σ_{yy}, as well as Γ_1. Eqs. (8), (9), and (17) imply

$$(28) \qquad \ln[G_1(Y)] = \mu_y + \tfrac{1}{2}\sigma_{yy} + \mu_x' \Sigma_{\beta\beta}\Sigma_{xx}\mu_\beta + \tfrac{1}{2}\mu_\beta' \Sigma_{xx}\Sigma_{\beta\beta}\Sigma_{xx}\mu_\beta.$$

Using the fact that $\Delta \ln[\mathsf{E}(Z)] = \Delta(\mu_z + \frac{1}{2}\sigma_{zz})$ [cf. (22)], we show in Appendix B that

$$(29) \quad \frac{\partial \ln[G_1(Y)]}{\partial \ln[\mathsf{E}(Z)]} = \begin{cases} \mu_\gamma + \sigma_{\gamma\alpha} + \Sigma_{\gamma\gamma}(\mu_z + \Sigma_{zz}\mu_\gamma) & \text{when } \Sigma_{zz} \text{ is constant,} \\ \operatorname{diagv}[\mu_\gamma\mu_\gamma' + \Sigma_{\gamma\gamma} + \mu_\gamma\mu_\gamma'\Sigma_{zz}\Sigma_{\gamma\gamma} & \text{when } \mu_z \text{ and the} \\ \quad + \Sigma_{\gamma\gamma}\Sigma_{zz}\mu_\gamma\mu_\gamma' & \quad \text{off-diagonal elements of} \\ \quad + 2\mu_\gamma(\sigma_{\gamma\alpha}' + \mu_z'\Sigma_{\gamma\gamma})] & \quad \Sigma_{zz} \text{ are constant.} \end{cases}$$

From this expression we can not uniquely define and measure an exact aggregate i'th input elasticity, $(\partial \ln[G_1(Y)])/(\partial \ln[\mathsf{E}(Z_i)])$. The two parts of (29) are limiting cases, the former may be interpreted as a vector of *dispersion preserving* aggregate input elasticities, the latter as a vector of *mean preserving* elasticities. Anyway, μ_γ provides a biased measure of the aggregate elasticity vector. The bias vector implied by the dispersion preserving macro input elasticities, obtained from the first part of (29), is

$$(30) \qquad \epsilon(\mu_\gamma) = \sigma_{\gamma\alpha} + \Sigma_{\gamma\gamma}(\mu_z + \Sigma_{zz}\mu_\gamma),$$

The bias vector for the mean preserving elasticities can be obtained from the second part in a similar way.

Dispersion preserving elasticities may be of more practical interest than mean preserving ones, since constancy of the *variance* of the log-input i, *i.e.*, σ_{zizi}, implies constancy of the *coefficient of variation* of the untransformed input i. This will be the situation when the i'th input of all micro units change proportionally.[5] This follows from the fact that coefficient of variation of Z_i is [cf. (22) and Evans, Hastings, and Peacock (1993, Chapter 25)]

$$v(Z_i) = \frac{\text{std}(Z_i)}{\mathsf{E}(Z_i)} = (e^{\sigma_{zizi}} - 1)^{\frac{1}{2}}\,, \tag{31}$$

which shows that constancy of σ_{zizi} implies constancy of $v(Z_i)$.

4 Estimation, data, and empirical results

4.1 Econometric method and data

The unknown parameters are estimated by Maximum Likelihood (ML), using the PROC MIXED procedure in the SAS/STAT software [see Littell *et al.* (1996)] and imposing positive definiteness of Ω as an *a priori* restriction. Details are given in Appendix A. In this particular application we have used the ML estimation results in Biørn, Lindquist and Skjerpen (2002, cf. Section 2 and Appendix A). The data are from an unbalanced panel for the years 1972–1993 from two Norwegian manufacturing industries, *Pulp and paper* (2823 observations, 237 firms) and *Basic metals* (2078 observations, 166 firms). A further description is given in Appendix C. The estimates of ψ, Ω, and the expected scale elasticity $\mu = \sum_j \mu_j$ are given in Tables 1 and 2.

4.2 Tests of the normality of the log-input distribution

Since this study relies on log-normality of the inputs, we will comment briefly on tests of this hypothesis. Using univariate statistics which depend on skewness and excess kurtosis, Biørn, Skjerpen and Wangen (2003, Appendix D) report, for each year in the sample period, tests of normality of the log-inputs. In most cases, the result is non-rejection at the five per cent level. However, for Pulp and paper, there is some evidence of non-normality, especially at the start of the sample period. This is most pronounced for energy and materials, and normality is rejected at the 1 per cent level in the years 1972–1976. Despite these irregularities, we proceed by imposing normality of all log-inputs as a simplifying assumption.

[5]The mean preserving elasticities relate to a more 'artificial' experiment in which $\mathsf{E}[\ln(Z_i)]$ is kept fixed and $v(Z_i)$ is increased, *i.e.*, $\text{std}(Z_i)$ is increased relatively more than $\mathsf{E}(Z_i)$.

4.3 Decompositions of the origo moments of output

Table 3 presents decompositions of the estimated $\ln[G_1(Y)]$, corresponding to (18), which may interesting in predicting annual changes in industry output from changes in inputs. Table 4 gives similar results for approximate second-order moment, $\ln[G_2(Y)]$, The latter may be useful in decomposing industry output volatility into contributions from the volatility of industry inputs. The last column in each panel gives the corresponding results based on the exact formula. The first column Table 3, $\ln[\Phi_1(y)]$, can be interpreted as the log of expected output if output were log-normally distributed, which is not in accordance with our assumptions.

Adding to $\ln[\Phi_1(y)]$ the correction factors $\ln(\Gamma_1)$, which is uniformly negative (column 2), and $\ln(\Lambda_1)$, which is uniformly positive (column 3), generally reduces the discrepancy between results based on this 'first-order approximation' formula and the correct results (column 5). The only exception is Basic metals in the ultimate year. The approximation formula performs better for the first-order than for the second-order moment.

Table 5 carries the decomposition a step further, by decomposing $\ln[\Phi_1(y)]$ and $\ln[\Phi_2(y)]$ into five sub-components. Comparing the first and sixth columns in this table shows the large downward bias that would have followed by the naive way of representing the expectation of a log-normal variable, say W, by $e^{\mathsf{E}[\ln(W)]}$. All sub-components [1]–[5] contribute positively to $\ln[\Phi_1(y)]$ and $\ln[\Phi_2(y)]$.

For the first-order moment, the largest contribution comes from μ_y (column [1]), and the next largest, $\frac{1}{2}\mu_x'\Sigma_{\beta\beta}\mu_x$, from the variation in the random coefficients (column [2]). Smaller contributions are due to the variation in log-inputs, $\frac{1}{2}\mu_\beta'\Sigma_{xx}\mu_\beta$ (column [3]), the interaction term, $\frac{1}{2}\mathrm{tr}(\Sigma_{\beta\beta}\Sigma_{xx})$ (column [4]), and the the disturbance variance, $\frac{1}{2}\sigma_{uu}$ (column [5]). Regarding the second-order moments, the contribution from the random variation in coefficients, $2\mu_x'\Sigma_{\beta\beta}\mu_x$ (column [2]), exceeds the contribution from the expectation term, $2\mu_y$ (column [1]), for Basic metals.

4.4 Scale and input elasticities

Table 6 reports summary statistics for the industry elasticities obtained by the two hypothetic experiments, giving dispersion preserving and mean preserving industry elasticities, respectively; cf. (29). The underlying year specific elasticities are given in Table 7.

In both industries and for all the years, the estimated micro *scale elasticity* is smaller than the dispersion preserving scale elasticity, and larger than the mean preserving scale elasticity, but the aggregation biases are not dramatic. In Pulp and paper, the mean of the estimated dispersion preserving scale elasticity is 1.16, the estimated expected micro elasticity is 1.07 and the mean preserving elasticity is 0.9. The corresponding figures for Basic metals are 1.15, 1.11 and 1.00, respectively. The annual variation in the industry elasticities is rather small.

However, when turning to the components of the scale elasticities, i.e., the input elasticities, we find more important aggregation biases. The estimate of the expected firm-level *capital elasticity* is 0.25 in Pulp and paper, which exceeds both the dispersion preserving and the mean preserving industry elasticities. The mean of the latter is 0.19, the former increases from 0.16 in 1972 to 0.20 in 1993. In Basic metals the dispersion preserving capital elasticity increases from about 0.05 in the first years to 0.15 in 1992, and then decreases substantially in the final year, 1993. Also, the mean preserving capital elasticity is relatively low in 1993, in the other years it is slightly higher than the expected firm elasticity which is 0.12. The estimate of the expected firm-level *labour elasticity* is 0.17 in Pulp and paper and 0.27 in Basic metals. The mean preserving elasticity is approximately constant in both industries, while the dispersion preserving elasticity decreases: for Pulp and paper from 0.19 in 1972 to 0.12 at the end of the data period, and for Basic metals from 0.36 to 0.19. For Pulp and paper the *energy elasticity* has the lowest estimate among the expected firm elasticities, 0.09. The dispersion preserving and mean preserving elasticities are about 0.12 and 0.03, respectively, and show almost no variation over time. For Basic metals the industry energy elasticity varies more, around a mean which is somewhat less than the expected firm elasticity. The *materials elasticity* is the largest elasticity according to all the three measures. In both sectors the dispersion preserving energy elasticity exceeds the expected firm elasticity; the mean preserving elasticity is substantially lower.

Since both the mean and the covariance matrix of the log-input vector usually change from year to year, the assumptions underlying the dispersion and mean preserving elasticities are unrealistic. It may therefore be worthwhile to consider *intermediate cases* in which a weighting of the two extremes is involved. Some experiments along these lines suggest, contrary to what might be expected, that the expected micro elasticity is not invariably closer to this weighted aggregate elasticity than it is to the closest of the two extremes.

Overall, the above results give ample evidence against using ‘raw’ micro elasticities to represent macro parameters. As basis for analyzing productivity issues, for instance estimating total factor productivity growth, and comparing growth rates across countries, annually as well as for longer periods, and for other policy related issues, they are potentially misleading.

5 Conclusions

This paper has been concerned with the aggregation of micro Cobb-Douglas production functions to the industry level when the firm specific production function parameters and the log-inputs are assumed to be independent and multinormally distributed. We have provided analytical approximations for the expectation and the higher-order origo moments of output, as well as conditions for the existence of such moments. These existence conditions turn out to be rather strong in the present case: only the first and second moments exist. To some extent, this may be due to our simplifying normality assumption, so that products of two vectors, both with support extending from minus to plus infinity, will enter the exponent of the expression for the moments of output. This suggests that investigating truncated distributions, in particular for the coefficients, may be interesting topics for further research. Relaxation of normality and/or truncation is, however, likely to increase the analytical and numerical complexity of the aggregation procedures.

We have shown how an industry level production function, expressed as a relationship between expected output and expected inputs, can be derived and how to quantify discrepancies between correctly aggregated input and scale elasticities and expected elasticities obtained from micro data. Obviously, the correctly aggregated coefficients are not strict technology parameters, as they also depend on the coefficient heterogeneity and the covariance matrix of the log-input vector; cf. the quotation from Felipe and Fisher (2003, p. 209) in the introduction. Our empirical decompositions give evidence of this. To indicate the possible range, as aggregate elasticities are undefined unless the input distribution is restricted, we have provided results for the limiting 'dispersion preserving' and 'mean preserving' cases. However, the experiment underlying our definition of the mean preserving elasticities is one in which the variances of the log-input distribution, but none of the covariances, are allowed to change. This simplifying assumption may have affected some of the above conclusions. An interesting modification may be to assume that the correlation matrix of the log-input vector, rather than the covariances, is invariant when the variances change.

Different decompositions of the log of expected output have been demonstrated. One of the components, a kind of a first-order approximation, is what we would have obtained if we had proceeded as if output were log-normal marginally. The numerical values when additional terms are included, are closer to the correct ones. The dispersion preserving scale elasticity is substantially higher than the expected micro scale elasticity for both industries and in all the years. For the mean preserving counterpart the differences are smaller. For Pulp and paper the micro elasticity exceeds the aggregate elasticity in all years. It is worth noting that the ranking of the industry level and the mean firm level input elasticities do not coincide, and the former is allowed to change over time.

An assumption not put into question in this paper is zero correlation between the production function parameters and the log-inputs. An interesting extension would be to relax this assumption, for instance to model the correlation. Simply treating all parameters as fixed and firm specific would, however, imply wasting a substantial part of the sample, since a minimal time series length is needed to estimate firm specific fixed parameters properly. Whether an extension of our approach to more flexible micro technologies, like the CES, the Translog, or the Generalized Leontief production functions, is practicable is an open question. Probably, it will be harder to obtain useful analytical approximations for expected output, from which aggregate parameters can be derived, and since both Translog and Generalized Leontief contain second-order terms, the problems related to the non-existence and numerical stability of higher-order origo moments of output, if normality of coefficients and log-inputs is retained, are likely to become more severe. In such cases it may be more promising to abandon the assumption that parameters and log-input variables are generated from specific parametric distributions.

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Table 1: Estimates of parameters in the micro CD production functions

	Pulp and paper		Basic metals	
Parameter	Estimate	St.err.	Estimate	St.err.
μ_α^*	-2.3021	0.2279	-3.1177	0.2702
κ	0.0065	0.0013	0.0214	0.0021
μ_K	0.2503	0.0344	0.1246	0.0472
μ_L	0.1717	0.0381	0.2749	0.0550
μ_E	0.0854	0.0169	0.2138	0.0374
μ_M	0.5666	0.0309	0.4928	0.0406
μ	1.0740	0.0287	1.1061	0.0324

Table 2: Covariance matrix of firm specific coefficients

Pulp and paper					
	α^*	γ_K	γ_L	γ_E	γ_M
α^*	5.9336				
γ_K	-0.4512	0.1147			
γ_L	-0.7274	-0.0559	0.1515		
γ_E	0.3968	-0.4197	-0.3009	0.0232	
γ_M	0.3851	-0.6029	-0.4262	0.1437	0.1053
Basic metals					
	α^*	γ_K	γ_L	γ_E	γ_M
α^*	3.5973				
γ_K	-0.0787	0.1604			
γ_L	-0.6846	-0.5503	0.1817		
γ_E	0.3040	-0.6281	0.1366	0.1190	
γ_M	0.1573	0.1092	-0.3720	-0.6122	0.1200

Variances on the main diagonal and correlation coefficients below.

Table 3: Decomposition of $\ln[G_1(Y)]$

Year	$\ln[\Phi_1(y)]$	$\ln(\Gamma_1)$	$\ln(\Lambda_1)$	$\ln[G_1(Y)]$	$\ln[\mathsf{E}(Y)]$
Pulp and paper					
1972	5.8221	-0.1336	0.2241	5.9126	5.9410
1973	5.8886	-0.1301	0.2300	5.9885	6.0188
1974	6.0399	-0.1524	0.3090	6.1965	6.2423
1975	5.9314	-0.1559	0.3047	6.0802	6.1234
1976	5.9413	-0.1622	0.3085	6.0876	6.1304
1977	5.8042	-0.1485	0.2434	5.8991	5.9265
1978	5.8231	-0.1493	0.2450	5.9188	5.9472
1979	5.9353	-0.1263	0.2336	6.0426	6.0717
1980	5.9900	-0.1171	0.2125	6.0854	6.1116
1981	6.0488	-0.1184	0.2246	6.1550	6.1833
1982	6.0927	-0.1067	0.2440	6.2301	6.2656
1983	6.1363	-0.0792	0.1826	6.2397	6.2635
1984	6.1470	-0.0747	0.1779	6.2502	6.2735
1985	6.2474	-0.0752	0.1854	6.3577	6.3830
1986	6.2395	-0.0844	0.1923	6.3475	6.3722
1987	6.2426	-0.0742	0.2013	6.3696	6.3967
1988	6.2859	-0.0641	0.2126	6.4344	6.4668
1989	6.2919	-0.0647	0.1961	6.4232	6.4517
1990	6.2698	-0.0790	0.2047	6.3955	6.4243
1991	6.3335	-0.0687	0.2040	6.4688	6.4999
1992	6.1708	-0.0398	0.1114	6.2424	6.2568
1993	6.2397	-0.0725	0.1334	6.3007	6.3160
Basic metals					
Year	$\ln[\Phi_1(y)]$	$\ln(\Gamma_1)$	$\ln(\Lambda_1)$	$\ln[G_1(Y)]$	$\ln[\mathsf{E}(Y)]$
1972	6.5750	-0.1186	0.2111	6.6675	6.6993
1973	6.6588	-0.1394	0.2302	6.7497	6.7792
1974	6.6042	-0.1211	0.1863	6.6694	6.6922
1975	6.7501	-0.1205	0.2440	6.8736	6.9049
1976	6.5689	-0.1446	0.2158	6.6401	6.6623
1977	6.6196	-0.1544	0.2387	6.7038	6.7243
1978	6.3791	-0.1339	0.1805	6.4258	6.4395
1979	6.7926	-0.1217	0.2084	6.8793	6.8972
1980	6.8043	-0.1425	0.1984	6.8602	6.8761
1981	6.7392	-0.1455	0.1898	6.7836	6.7979
1982	6.6320	-0.1415	0.1793	6.6698	6.6854
1983	6.7103	-0.1255	0.1757	6.7605	6.7816
1984	6.9263	-0.1300	0.1756	6.9718	6.9891
1985	6.8837	-0.1081	0.1482	6.9238	6.9505
1986	6.9838	-0.1014	0.1438	7.0261	7.0468
1987	7.1013	-0.1042	0.1664	7.1635	7.1920
1988	7.2176	-0.0732	0.1591	7.3036	7.3327
1989	7.1698	-0.0767	0.1480	7.2411	7.2733
1990	7.2243	-0.0766	0.1528	7.3004	7.3313
1991	7.2103	-0.1095	0.1645	7.2653	7.2878
1992	7.0697	-0.0712	0.0941	7.0927	7.1097
1993	7.1142	-0.3304	0.2801	7.0639	7.1722

Table 4: Decomposition of $\ln[G_2(Y)]$

Pulp and paper					
Year	$\ln[\Phi_2(y)]$	$\ln(\Gamma_2)$	$\ln(\Lambda_2)$	$\ln[G_2(Y)]$	$\ln[\mathsf{E}(Y^2)]$
1972	15.7045	-1.0688	3.5857	18.2214	21.4873
1973	15.8728	-1.0408	3.6798	18.5118	22.0487
1974	16.7528	-1.2192	4.9443	20.4779	27.3060
1975	16.4704	-1.2468	4.8747	20.0983	26.6425
1976	16.5261	-1.2974	4.9357	20.1643	26.7637
1977	15.7070	-1.1880	3.8939	18.4128	22.3059
1978	15.7740	-1.1946	3.9203	18.4996	22.4563
1979	15.9206	-1.0106	3.7379	18.6479	22.3683
1980	15.8562	-0.9369	3.3999	18.3193	21.4657
1981	16.0864	-0.9472	3.5935	18.7327	22.2065
1982	16.3489	-0.8535	3.9044	19.3998	23.6732
1983	15.9355	-0.6338	2.9221	18.2239	20.5925
1984	15.9089	-0.5973	2.8457	18.1573	20.4216
1985	16.1904	-0.6013	2.9670	18.5561	21.0330
1986	16.1914	-0.6748	3.0775	18.5940	21.2485
1987	16.2786	-0.5938	3.2204	18.9051	21.8326
1988	16.4840	-0.5127	3.4019	19.3733	22.7633
1989	16.3544	-0.5177	3.1375	18.9742	21.8505
1990	16.3813	-0.6320	3.2756	19.0249	22.0874
1991	16.5299	-0.5499	3.2642	19.2442	22.3479
1992	15.2931	-0.3184	1.7822	16.7569	17.7799
1993	15.6447	-0.5797	2.1349	17.1999	18.5189
Basic metals					
Year	$\ln[\Phi_2(y)]$	$\ln(\Gamma_2)$	$\ln(\Lambda_2)$	$\ln[G_2(Y)]$	$\ln[\mathsf{E}(Y^2)]$
1972	20.1521	-0.9489	3.3775	22.5807	24.9982
1973	20.5013	-1.1149	3.6837	23.0702	24.8810
1974	19.8479	-0.9692	2.9816	21.8603	23.1716
1975	20.9268	-0.9639	3.9046	23.8675	25.8393
1976	20.0598	-1.1567	3.4520	22.3552	23.8486
1977	20.3634	-1.2352	3.8186	22.9468	24.4927
1978	19.2740	-1.0711	2.8884	21.0912	21.9435
1979	20.4934	-0.9736	3.3345	22.8542	24.0145
1980	20.2736	-1.1398	3.1746	22.3084	23.3635
1981	20.0626	-1.1638	3.0372	21.9360	22.8382
1982	19.7675	-1.1319	2.8683	21.5039	22.3777
1983	19.8954	-1.0042	2.8118	21.7029	22.7532
1984	20.2629	-1.0404	2.8094	22.0319	22.8900
1985	19.7910	-0.8651	2.3718	21.2977	22.6453
1986	19.9254	-0.8115	2.3001	21.4140	22.2983
1987	20.5655	-0.8332	2.6616	22.3939	23.8639
1988	20.6914	-0.5853	2.5458	22.6519	24.1436
1989	20.3009	-0.6138	2.3672	22.0542	23.5474
1990	20.5871	-0.6129	2.4445	22.4187	23.9313
1991	20.5945	-0.8760	2.6322	22.3508	23.4116
1992	19.0666	-0.5692	1.5062	20.0036	20.4955
1993	a)	a)	a)	a)	a)

a) The moment does not exist.

Table 5: Decomposition of $\ln[\Phi_1(y)]$ and $\ln[\Phi_2(y)]$

Pulp and paper							
Year	[1]	[2]	[3]	[4]	[5]	$\ln[\Phi_1(y)]$	$\ln[\Phi_2(y)]$
1972	3.7920	3.3244	0.4262	0.2688	0.0408	5.8221	15.7045
1973	3.8408	3.3520	0.4322	0.2706	0.0408	5.8886	15.8728
1974	3.7034	3.9220	0.4318	0.2784	0.0408	6.0399	16.7528
1975	3.6275	3.8954	0.3946	0.2768	0.0408	5.9314	16.4704
1976	3.6195	3.9310	0.3918	0.2800	0.0408	5.9413	16.5261
1977	3.7550	3.4344	0.3738	0.2496	0.0408	5.8042	15.7070
1978	3.7592	3.4454	0.3862	0.2554	0.0408	5.8231	15.7740
1979	3.9103	3.3606	0.3970	0.2514	0.0408	5.9353	15.9206
1980	4.0518	3.1770	0.3966	0.2618	0.0408	5.9900	15.8562
1981	4.0545	3.2906	0.3986	0.2586	0.0408	6.0488	16.0864
1982	4.0110	3.4624	0.3898	0.2706	0.0408	6.0927	16.3489
1983	4.3047	2.9884	0.3908	0.2432	0.0408	6.1363	15.9355
1984	4.3396	2.9464	0.3998	0.2278	0.0408	6.1470	15.9089
1985	4.3996	3.0046	0.4196	0.2306	0.0408	6.2474	16.1904
1986	4.3833	3.0366	0.4094	0.2256	0.0408	6.2395	16.1914
1987	4.3458	3.1428	0.3908	0.2192	0.0408	6.2426	16.2786
1988	4.3297	3.2454	0.3878	0.2384	0.0408	6.2859	16.4840
1989	4.4065	3.0962	0.4002	0.2334	0.0408	6.2919	16.3544
1990	4.3489	3.1534	0.4064	0.2410	0.0408	6.2698	16.3813
1991	4.4020	3.1612	0.4046	0.2564	0.0408	6.3335	16.5299
1992	4.6950	2.3084	0.3978	0.2046	0.0408	6.1708	15.2931
1993	4.6571	2.4752	0.4356	0.2136	0.0408	6.2397	15.6447
Basic metals							
Year	[1]	[2]	[3]	[4]	[5]	$\ln[\Phi_1(y)]$	$\ln[\Phi_2(y)]$
1972	3.0739	5.9854	0.5354	0.3826	0.0986	6.5750	20.1521
1973	3.0670	6.1622	0.5546	0.3684	0.0986	6.6588	20.5013
1974	3.2845	5.6614	0.5516	0.3278	0.0986	6.6042	19.8479
1975	3.0368	6.4540	0.5192	0.3548	0.0986	6.7501	20.9268
1976	3.1078	5.9772	0.5148	0.3316	0.0986	6.5689	20.0598
1977	3.0574	6.2398	0.4806	0.3054	0.0986	6.6196	20.3634
1978	3.1213	5.6736	0.4760	0.2672	0.0986	6.3791	19.2740
1979	3.3385	6.0502	0.4828	0.2766	0.0986	6.7926	20.4934
1980	3.4717	5.7764	0.5052	0.2850	0.0986	6.8043	20.2736
1981	3.4472	5.7212	0.4862	0.2782	0.0986	6.7392	20.0626
1982	3.3802	5.6322	0.4946	0.2782	0.0986	6.6320	19.7675
1983	3.4730	5.5748	0.4974	0.3040	0.0986	6.7103	19.8954
1984	3.7212	5.5218	0.5112	0.2788	0.0986	6.9263	20.2629
1985	3.8718	5.0792	0.5294	0.3164	0.0986	6.8837	19.7910
1986	4.0049	5.0532	0.5224	0.2836	0.0986	6.9838	19.9254
1987	3.9199	5.4118	0.5460	0.3064	0.0986	7.1013	20.5655
1988	4.0896	5.3138	0.5490	0.2948	0.0986	7.2176	20.6914
1989	4.1892	4.9344	0.5934	0.3348	0.0986	7.1698	20.3009
1990	4.1550	5.1612	0.5820	0.2968	0.0986	7.2243	20.5871
1991	4.1233	5.1818	0.5856	0.3080	0.0986	7.2103	20.5945
1992	4.6061	3.9846	0.5972	0.2468	0.0986	7.0697	19.0666
1993	4.2915	4.1004	0.7892	0.6570	0.0986	7.1142	19.8738

[1]: μ_y; [2]: $\mu'_x \Sigma_{\beta\beta} \mu_x$; [3]: $\mu'_\beta \Sigma_{xx} \mu_\beta$; [4]: $tr(\Sigma_{\beta\beta}\Sigma_{xx})$; [5]: σ_{uu}.

$\ln[\Phi_1(y)] = [1] + 0.5 \times ([2] + [3] + [4] + [5])$.

$\ln[\Phi_2(y)] = 2 \times ([1] + [2] + [3] + [4] + [5])$.

Table 6: Comparison of different type of elasticities

Type of elasticity	Expected micro elasticity	Dispersion preserving macro elasticity			Mean preserving macro elasticity		
		Minimum	Mean	Maximum	Minimum	Mean	Maximum
Pulp and paper							
Scale	1.07	1.13	1.16	1.19	0.86	0.90	0.93
Capital	0.25	0.15	0.18	0.20	0.18	0.19	0.20
Labour	0.17	0.12	0.16	0.19	0.19	0.19	0.20
Energy	0.09	0.12	0.12	0.13	0.03	0.03	0.04
Materials	0.57	0.67	0.70	0.72	0.44	0.48	0.52
Basic metals							
Scale	1.11	1.11	1.15	1.19	0.96	1.00	1.02
Capital	0.12	0.01	0.10	0.16	0.11	0.16	0.18
Labour	0.27	0.15	0.25	0.36	0.24	0.27	0.31
Energy	0.21	0.14	0.17	0.22	0.17	0.19	0.26
Materials	0.49	0.60	0.63	0.68	0.34	0.38	0.41

Table 7: Dispersion and mean preserving macro input and scale elasticities

	Pulp and paper									
	Dispersion preserving					Mean preserving				
	Capital	Labour	Energy	Materials	Scale	Capital	Labour	Energy	Materials	Scale
1972	0.160	0.193	0.120	0.695	1.168	0.192	0.190	0.034	0.483	0.900
1973	0.156	0.186	0.125	0.699	1.166	0.192	0.188	0.035	0.481	0.896
1974	0.151	0.194	0.126	0.715	1.185	0.187	0.192	0.035	0.508	0.923
1975	0.163	0.191	0.125	0.703	1.181	0.183	0.191	0.035	0.515	0.925
1976	0.170	0.182	0.124	0.704	1.179	0.184	0.191	0.035	0.518	0.928
1977	0.192	0.192	0.119	0.669	1.172	0.192	0.195	0.034	0.480	0.901
1978	0.184	0.181	0.122	0.679	1.166	0.188	0.197	0.034	0.484	0.903
1979	0.173	0.177	0.125	0.690	1.165	0.187	0.195	0.035	0.482	0.899
1980	0.180	0.174	0.121	0.688	1.163	0.189	0.197	0.034	0.471	0.891
1981	0.183	0.160	0.122	0.696	1.162	0.189	0.194	0.034	0.477	0.895
1982	0.172	0.152	0.124	0.714	1.162	0.179	0.192	0.035	0.504	0.910
1983	0.186	0.148	0.120	0.701	1.155	0.188	0.191	0.034	0.477	0.889
1984	0.180	0.146	0.121	0.705	1.152	0.188	0.190	0.034	0.475	0.887
1985	0.178	0.141	0.123	0.713	1.154	0.189	0.191	0.034	0.474	0.887
1986	0.184	0.149	0.123	0.702	1.159	0.190	0.194	0.034	0.469	0.886
1987	0.183	0.144	0.123	0.709	1.159	0.185	0.192	0.034	0.486	0.896
1988	0.180	0.127	0.128	0.720	1.154	0.181	0.187	0.035	0.499	0.902
1989	0.181	0.127	0.127	0.716	1.151	0.183	0.188	0.035	0.486	0.893
1990	0.183	0.127	0.126	0.714	1.151	0.184	0.190	0.035	0.484	0.892
1991	0.185	0.120	0.129	0.716	1.150	0.183	0.189	0.035	0.487	0.895
1992	0.194	0.115	0.122	0.697	1.127	0.186	0.185	0.034	0.459	0.865
1993	0.198	0.117	0.122	0.697	1.134	0.195	0.189	0.033	0.444	0.862
	Basic metals									
	Dispersion preserving					Mean preserving				
	Capital	Labour	Energy	Materials	Scale	Capital	Labour	Energy	Materials	Scale
1972	0.008	0.360	0.206	0.606	1.180	0.167	0.270	0.186	0.379	1.002
1973	0.028	0.358	0.183	0.618	1.187	0.169	0.280	0.182	0.375	1.005
1974	0.056	0.325	0.169	0.627	1.177	0.177	0.260	0.174	0.383	0.994
1975	0.044	0.326	0.172	0.641	1.183	0.164	0.268	0.184	0.405	1.022
1976	0.049	0.326	0.189	0.610	1.174	0.161	0.278	0.195	0.371	1.004
1977	0.049	0.327	0.199	0.598	1.174	0.161	0.285	0.187	0.386	1.019
1978	0.096	0.266	0.187	0.604	1.152	0.173	0.254	0.178	0.396	1.002
1979	0.072	0.275	0.195	0.618	1.159	0.167	0.262	0.183	0.400	1.011
1980	0.073	0.290	0.196	0.603	1.162	0.165	0.276	0.191	0.366	0.998
1981	0.106	0.276	0.183	0.595	1.160	0.172	0.267	0.183	0.370	0.992
1982	0.140	0.240	0.165	0.606	1.151	0.175	0.258	0.180	0.379	0.991
1983	0.126	0.224	0.176	0.618	1.144	0.165	0.260	0.187	0.392	1.004
1984	0.116	0.237	0.183	0.613	1.148	0.165	0.268	0.191	0.371	0.995
1985	0.153	0.219	0.144	0.632	1.149	0.172	0.262	0.179	0.378	0.992
1986	0.143	0.213	0.161	0.627	1.144	0.169	0.259	0.186	0.374	0.988
1987	0.159	0.202	0.139	0.649	1.149	0.171	0.261	0.178	0.390	1.000
1988	0.150	0.190	0.139	0.667	1.145	0.170	0.253	0.179	0.403	1.004
1989	0.139	0.185	0.138	0.679	1.141	0.160	0.266	0.186	0.401	1.013
1990	0.144	0.175	0.147	0.672	1.138	0.163	0.257	0.187	0.398	1.006
1991	0.123	0.188	0.170	0.654	1.135	0.155	0.269	0.198	0.384	1.007
1992	0.154	0.145	0.174	0.640	1.113	0.163	0.243	0.204	0.353	0.963
1993	0.062	0.185	0.223	0.641	1.110	0.109	0.306	0.257	0.336	1.008

APPENDIX A: Details on the ML estimation

We consider our unbalanced panel data set [cf. Appendix C] as a data set where the firms are observed in at least 1 and at most P years, and arrange the observations in groups according to the time series lengths [a similar ordering is used in Biørn (2004)]. Let N_p be the number of firms which are observed in p years (not necessarily the same and consecutive), let (ip) index the i'th firm among those observed in p years, and let t index the observation number ($t = 1, \ldots, p$). The production function (1), can be written as

$$\text{(A.1)} \qquad y_{(ip)t} = x'_{(ip)t}\beta_{(ip)} + u_{(ip)t}, \qquad p = 1, \ldots, P;\ i = 1, \ldots, N_p;\ t = 1, \ldots, p,$$

where $\beta_{(ip)}$ is the coefficient vector of firm (ip). Inserting $\beta_{(ip)} = \mu_\beta + \delta_{(ip)}$ we get

$$\text{(A.2)} \qquad y_{(ip)t} = x'_{(ip)t}\mu_\beta + \psi_{(ip)t}, \qquad \psi_{(ip)t} = x'_{(ip)t}\delta_{(ip)} + u_{(ip)t}.$$

Stacking the p realisations from firm (ip) in $y_{(ip)} = [y_{(ip)1}, \ldots, y_{(ip)p}]'$, $X_{(ip)} = [x_{(ip)1}, \ldots, x_{(ip)p}]$, $u_{(ip)} = [u_{(ip)1}, \ldots, u_{(ip)p}]'$, and $\psi_{(ip)} = [\psi_{(ip)1}, \cdots, \psi_{(ip)p}]'$, we can write (A.2) as

$$\text{(A.3)} \qquad y_{(ip)} = X'_{(ip)}\mu_\beta + \psi_{(ip)}, \qquad \psi_{(ip)} = X'_{(ip)}\delta_{(ip)} + u_{(ip)}.$$

where

$$\text{(A.4)} \qquad \psi_{(ip)}|X_{(ip)} \sim \mathcal{N}(0, \Omega_{(ip)}), \qquad \Omega_{(ip)} = X'_{(ip)}\Omega X_{(ip)} + \sigma_{uu}I_p,$$

and I_p is the p-dimensional identity matrix. The log-likelihood function is therefore

$$\text{(A.5)} \quad \mathcal{L} = -\frac{m}{2}\ln(2\pi) - \frac{1}{2}\sum_{p=1}^{P}\sum_{i=1}^{N_p}\{\ln|\Omega_{(ip)}| + [y_{(ip)} - X'_{(ip)}\mu_\beta]'\Omega_{(ip)}^{-1}[y_{(ip)} - X'_{(ip)}\mu_\beta]\},$$

where $m = \sum_{p=1}^{P} pN_p$. The ML estimators of $(\mu_\beta, \sigma_{uu}, \Omega)$ follow by maximising $\mathcal{L}$. The solution may be simplified by concentrating $\mathcal{L}$ over μ_β and maximising the resulting function with respect to σ_{uu} and the unknown elements of Ω.

APPENDIX B: Proof of Eq. (29)

The first three components of $\ln[G_1(Y)]$, as given by (28), respond to changes in μ_z and the last three elements respond to changes in Σ_{zz}. Inserting in (9) from (2) and (3), we obtain

$$\sigma_{yy} = \sigma_{\alpha\alpha} + 2\mu_z\sigma_{\gamma\alpha} + \mu'_z\Sigma_{\gamma\gamma}\mu_z + \mu'_\gamma\Sigma_{zz}\mu_\gamma + \mathrm{tr}(\Sigma_{\gamma\gamma}\Sigma_{zz}) + \sigma_{uu},$$
$$\mu'_x\Sigma_{\beta\beta}\Sigma_{xx}\mu_\beta = \sigma'_{\gamma\alpha}\Sigma_{zz}\mu_\gamma + \mu'_z\Sigma_{\gamma\gamma}\Sigma_{zz}\mu_\gamma, \qquad \mu'_\beta\Sigma_{xx}\Sigma_{\beta\beta}\Sigma_{xx}\mu_\beta = \mu'_\gamma\Sigma_{zz}\Sigma_{\gamma\gamma}\Sigma_{zz}\mu_\gamma.$$

Differentiating the various terms in (28) with respect to μ_z and Σ_{zz} [using Lütkepohl (1996, Section 10.3.2, eqs. (2), (5) and (21))], we get

$$\text{(B.1)}\quad \frac{\partial \mu_y}{\partial \mu_z} = \frac{\partial(\mu_x'\mu_\beta)}{\partial \mu_z} = \frac{\partial(\mu_z'\mu_\gamma)}{\partial \mu_z} = \mu_\gamma,$$

$$\text{(B.2)}\quad \frac{\partial \sigma_{yy}}{\partial \mu_z} = 2\frac{\partial(\mu_z'\sigma_{\alpha\gamma})}{\partial \mu_z} + \frac{\partial(\mu_z'\Sigma_{\gamma\gamma}\mu_z)}{\partial \mu_z} = 2(\sigma_{\gamma\alpha} + \Sigma_{\gamma\gamma}\mu_z),$$

$$\text{(B.3)}\quad \frac{\partial(\mu_x'\Sigma_{\beta\beta}\Sigma_{xx}\mu_\beta)}{\partial \mu_z} = \frac{\partial(\mu_z'\Sigma_{\gamma\gamma}\Sigma_{zz}\mu_\gamma)}{\partial \mu_z} = \Sigma_{\gamma\gamma}\Sigma_{zz}\mu_\gamma,$$

$$\text{(B.4)}\quad \frac{\partial \sigma_{yy}}{\partial \Sigma_{zz}} = \frac{\partial(\mu_\gamma'\Sigma_{zz}\mu_\gamma)}{\partial \Sigma_{zz}} + \frac{\partial \mathrm{tr}(\Sigma_{\gamma\gamma}\Sigma_{zz})}{\partial \Sigma_{zz}} = \mu_\gamma\mu_\gamma' + \Sigma_{\gamma\gamma},$$

$$\text{(B.5)}\quad \frac{\partial(\mu_x'\Sigma_{\beta\beta}\Sigma_{xx}\mu_\beta)}{\partial \Sigma_{zz}} = \frac{\partial(\sigma_{\gamma\alpha}'\Sigma_{zz}\mu_\gamma)}{\partial \Sigma_{zz}} + \frac{\partial(\mu_z'\Sigma_{\gamma\gamma}\Sigma_{zz}\mu_\gamma)}{\partial \Sigma_{zz}}$$
$$= \frac{\partial \mathrm{tr}(\sigma_{\gamma\alpha}'\Sigma_{zz}\mu_\gamma)}{\partial \Sigma_{zz}} + \frac{\partial \mathrm{tr}(\mu_z'\Sigma_{\gamma\gamma}\Sigma_{zz}\mu_\gamma)}{\partial \Sigma_{zz}} = \mu_\gamma\sigma_{\gamma\alpha}' + \mu_\gamma\mu_z'\Sigma_{\gamma\gamma},$$

$$\text{(B.6)}\quad \frac{\partial(\mu_\beta'\Sigma_{xx}\Sigma_{\beta\beta}\Sigma_{xx}\mu_\beta)}{\partial \Sigma_{zz}} = \frac{\partial(\mu_\gamma'\Sigma_{zz}\Sigma_{\gamma\gamma}\Sigma_{zz}\mu_\gamma)}{\partial \Sigma_{zz}} = \frac{\partial \mathrm{tr}(\mu_\gamma'\Sigma_{zz}\Sigma_{\gamma\gamma}\Sigma_{zz}\mu_\gamma)}{\partial \Sigma_{zz}}$$
$$= \mu_\gamma\mu_\gamma'\Sigma_{zz}\Sigma_{\gamma\gamma} + \Sigma_{\gamma\gamma}\Sigma_{zz}\mu_\gamma\mu_\gamma'.$$

It follows from (28) and (B.1)–(B.6), that

$$\text{(B.7)}\quad \frac{\partial \ln[G_1(Y)]}{\partial \mu_z} = \mu_\gamma + \sigma_{\gamma\alpha} + \Sigma_{\gamma\gamma}(\mu_z + \Sigma_{zz}\mu_\gamma),$$

$$\text{(B.8)}\quad \frac{\partial \ln[G_1(Y)]}{\partial \Sigma_{zz}} = \tfrac{1}{2}(\mu_\gamma\mu_\gamma' + \Sigma_{\gamma\gamma} + \mu_\gamma\mu_\gamma'\Sigma_{zz}\Sigma_{\gamma\gamma} + \Sigma_{\gamma\gamma}\Sigma_{zz}\mu_\gamma\mu_\gamma') + \mu_\gamma(\sigma_{\gamma\alpha}' + \mu_z'\Sigma_{\gamma\gamma}).$$

Since, from (22), $\Delta \ln[\mathsf{E}(Z)] = \Delta(\mu_z + \frac{1}{2}\sigma_{zz})$, we have

$$\text{(B.9)}\quad \frac{\partial \ln[G_1(Y)]}{\partial \ln[\mathsf{E}(Z)]} = \begin{cases} \mu_\gamma + \sigma_{\gamma\alpha} + \Sigma_{\gamma\gamma}(\mu_z + \Sigma_{zz}\mu_\gamma) & \text{when } \Sigma_{zz} \text{ is constant,} \\ \mathrm{diagv}[\mu_\gamma\mu_\gamma' + \Sigma_{\gamma\gamma} + \mu_\gamma\mu_\gamma'\Sigma_{zz}\Sigma_{\gamma\gamma} & \text{when } \mu_z \text{ and the} \\ \quad + \Sigma_{\gamma\gamma}\Sigma_{zz}\mu_\gamma\mu_\gamma' & \quad \text{off-diagonal elements of} \\ \quad + 2\mu_\gamma(\sigma_{\gamma\alpha}' + \mu_z'\Sigma_{\gamma\gamma})] & \quad \Sigma_{zz} \text{ are constant,} \end{cases}$$

which completes the proof.

APPENDIX C: Data

The data are from the years 1972–1993 and represent two Norwegian manufacturing industries, *Pulp and paper* and *Basic metals*. Table C.1, classifying the observations by the number of years, and Table C.2, sorting the firms by the calendar year in which they are observed, shows the unbalanced structure of the data set. There is a negative trend in the number of firms for both industries.

The primary data source is the Manufacturing Statistics database of Statistics Norway, classified under the Standard Industrial Classification (SIC)-codes 341 Manufacture of paper and paper products (Pulp and paper, for short) and 37 Manufacture of basic metals (Basic metals, for short). Both firms with contiguous and non-contiguous time series are included.

In the description below, MS indicates firm data from the Manufacturing Statistics, NNA indicates that the data are from the Norwegian National Accounts and are identical for firms classified in the same National Account industry.

Y: Output, 100 tonnes (MS)

$K = KB + KM$: Total capital stock (buildings/structures plus machinery/transport equipment), 100 000 1991-NOK (MS,NNA)

L: Labour input, 100 man-hours (MS)

E: Energy input, 100 000 kWh, electricity plus fuels (excl. motor gasoline) (MS)

$M = CM/QM$: Input of materials (incl. motor gasoline), 100 000 1991-NOK (MS,NNA)

CM: Total material cost (incl. motor gasoline) (MS)

QM: Price of materials (incl. motor gasoline), 1991=1 (NNA)

Output: The firms in the Manufacturing Statistics are in general multi-output firms and report output of a number of products measured in both NOK and primarily tonnes or kg. For each firm, an aggregate output measure in tonnes is calculated. Hence, rather than representing output in the two industries by deflated sales, which may be affected by measurement errors [see Klette and Griliches (1996)], our output measures are actual output in physical units, which are in several respects preferable.

Capital stock: The calculations of capital stock data are based on the perpetual inventory method assuming constant depreciation rates. We combine firm data on gross investment with fire insurance values for each of the two categories Buildings and structures and Machinery and transport equipment from the MS. The data on investment and fire insurance are deflated using industry specific price indices of investment goods from the NNA (1991=1). In both industries, the depreciation rate is set to 0.02 for Buildings and structures and 0.04 for Machinery and transport equipment. For further documentation, see Biørn, Lindquist and Skjerpen (2000, Section 4, and 2003).

Table C.1: Number of firms by number of replications

	Pulp and paper		Basic metals	
p	N_p	$N_p p$	N_p	$N_p p$
22	60	1320	44	968
21	9	189	2	42
20	5	100	4	80
19	3	57	5	95
18	1	18	2	36
17	4	68	5	85
16	6	96	5	80
15	4	60	4	60
14	3	42	5	70
13	4	52	3	39
12	7	84	10	120
11	10	110	7	77
10	12	120	6	60
09	10	90	5	45
08	7	56	2	16
07	15	105	13	91
06	11	66	4	24
05	14	70	5	25
04	9	36	6	24
03	18	54	3	9
02	5	10	6	12
01	20	20	20	20
Sum	237	2823	166	2078

p = no. of observations per firm.

N_p = no. of firms observed p times

Table C.2: Number of firms by calendar year

Year	Pulp and paper	Basic metals
1972	171	102
1973	171	105
1974	179	105
1975	175	110
1976	172	109
1977	158	111
1978	155	109
1979	146	102
1980	144	100
1981	137	100
1982	129	99
1983	111	95
1984	108	87
1985	106	89
1986	104	84
1987	102	87
1988	100	85
1989	97	83
1990	99	81
1991	95	81
1992	83	71
1993	81	83
Sum	2823	2078

Other inputs: From the MS we get the number of man-hours used, total electricity consumption in kWh, the consumption of a number of fuels in various denominations, and total material costs in NOK for each firm. The different fuels are transformed to the common denominator kWh by using estimated average energy content of each fuel [Statistics Norway (1995, p. 124)]. This enables us to calculate aggregate energy use in kWh for each firm. Total material costs is deflated by the price index (1991=1) of material inputs from the NNA. This price is identical for all firms classified in the same National Account industry.

Observations with missing values of output or inputs have been removed. This reduced the number of observations by 6–8 per cent in the two industries.

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