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Properties of a non-competitive electricity market dominated by hydroelectric power

by

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Abstract

An important conclusion from the literature on hydropower is that if there are no other constraints than the available water reservoirs for a year, and operating costs are ignored, the competitive (and socially optimal) outcome is characterized by the (present value) price being constant through the year. A second important conclusion is that the outcome under monopoly generally will differ from this, provided that the demand functions differ across different days (or other sub-periods) of the year. We show that even if the demand function is the same all days of the year, the monopoly outcome will generally differ from the competitive outcome. The difference is caused by the profit function of a price-setting producer of hydropower being non-concave. This non-concavity can be caused by short-run capacity limits either on exports and imports of electricity, or on the supply of alternative electricity sources.

Keywords: Electricity prices, Hydropower

JEL classification: L12, L13, L94, Q25

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1. Introduction

The deregulation of the electricity power production system in many countries in the last decade has stimulated interest in the possibilities of producers behaving strategically. This is especially so for systems with a significant contribution from hydropower with storage of water. As pointed out in Edwards (2003) 1/3 of all countries in the world depend on hydropower for over 50% of their electricity. For USA the share is 10%, but for countries like New Zealand and Brazil the shares are 80 and 90 % respectively. Norway has a share of 99% and is the sixth largest producer of hydropower in the World, and the largest in Europe. We should also be aware of the importance of regionalized market power. Hydropower may be quite significant in regions within countries with a small national share. The reason for the concern about potential market power abuse of hydro producers is that it is not so easy to detect by regulators because market power is exercised by a reallocation of release of water from the reservoirs compared with what would be the socially desired release pattern. Market power may be used without any spilling of water, which is comparatively easy to detect. The almost costless instantaneous change in hydro generation within the effect capacities makes it perfect for strategic actions in competition with thermal generators with both costs and time lags involved in changing production levels.

The reservoirs play a crucial role for the ability to act strategically over time. In Norway the reservoirs have a capacity of 85% of yearly inflow. In addition there are hydro generation utilising the run of rivers without any storage possibilities making up 20% of yearly normal production. The generation of electricity from stored water in the reservoirs is almost instantaneous and draws insignificant current costs. The capacity of the pipes leading from the dam to the turbines and the turbine installation determines the effect capacity, while the amount of stored water determines the energy capacity. The (vertical) height of the head determines the conversion factor from water to electricity. A finer engineering point is that as water is tapped the conversion efficiency decreases since the height of the head is decreased. As mentioned the costs associated with the level of production of a hydropower system are negligible. Costs are incurred due to maintenance and overseeing the operations and are independent of the level of output. A power station may even be run from a command centre located somewhere else utilising electronic connections, thus a power station may be unmanned.

Optimal short-term utilisation of hydro power systems, with *must-take* run of rivers (water has to be processed as it flows due to lack of storage), independent reservoirs and hydrologically connected ones, is a special subject within engineering. In order to derive rules for practical use large simulation models are usually developed encompassing details about effect (turbine) capacities, energy storage capacities and public regulations of utilisation pattern of water both of dams and water flows off the turbines (due to environmental or other considerations). Most complex models also take into consideration uncertainties of inflow of water to the reservoirs and uncertainty of demand due to outdoor temperature.

Within economics papers dealing with economic aspects of short-run management of hydropower at a manageable analytical level are not so many (see Edwards (2003) for documentation of this observation). In order to focus on key aspects standard simplifications are to disregard hydrologically linked power stations, assume fixed conversion factors from stored water to electricity and disregard uncertainties. Details about operating the dams and limits on release of water from power stations are usually just expressed by minimum and/or maximum constraints on use of water. Maximum constraints on water use will then also capture limited effect capacity, and maximum constraint on storage will capture the limit of the available energy.

An assumption of no binding restriction on the amount of water that can be stored within the yearly cycle of precipitation makes the decision of how to manage the stored water similar to the decision about the utilisation of a finite non-renewable natural resource. However, the limited horizon of one year makes the use of discounting of less interest. It is the discounting that drives Hotelling's rule of the resource price increasing with the rate equal to the rate of social discount. The main focus in hydropower management is on how to distribute the water on the sub periods within a year, i.e. from the most aggregate summer-winter consideration to the detailed hour by hour utilisation during the 24 hours of a day.

We will consider a region (that may be a country) within an area serviced by a common infrastructure of power lines. The power supply within the region may then be supplemented

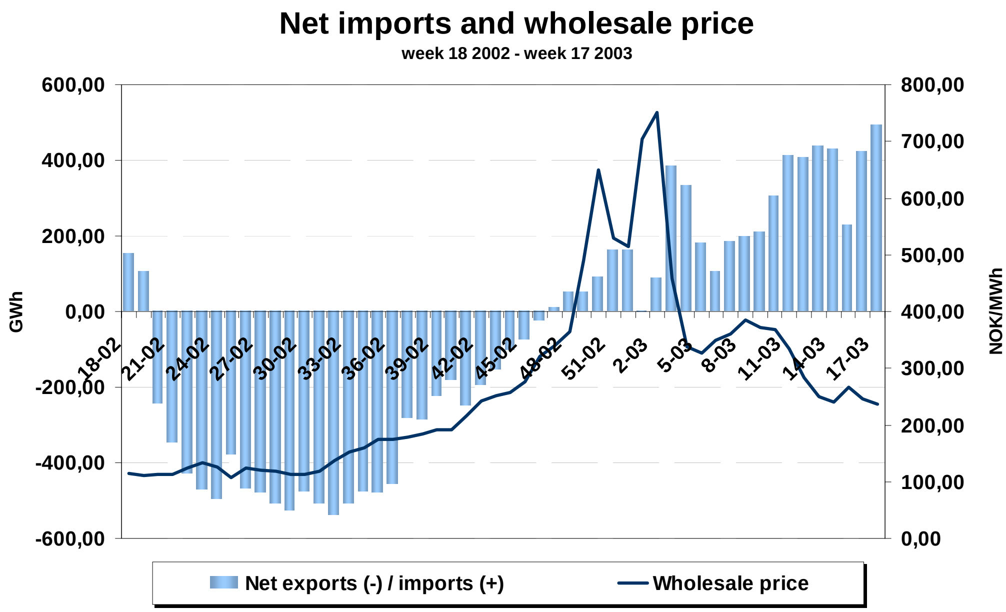

Figure 1. Imports/exports and prices

with imports or part of the regional supply may be exported. The utilisation of hydropower will naturally be influenced by these possibilities.

As an illustration the export/import for Norway and the wholesale price of electricity for the period week 18 (starting 29 of April) of the year 2002 to week 17 (ending 27 of April) of the year 2003 (the natural cycle of inflows to the reservoirs) is shown in Figure 1. This was a period with an unprecedented increase in the spot price since the deregulation in 1991 at the end of year 2002 and beginning of the next year. Norway went through a "mini" California crisis with price increasing with a factor of about 7 between lowest summer price and highest winter price. The 64,000 $ question still being investigated by the Norwegian authorities is whether this was due to the use of market power or just the market functioning normally in the face of a significant less precipitation during the fall than expected earlier in the year. We see that while the price level of spring and summer were low the export was considerable, and when the price started to increase sharply in December the imports shoot up. However, the price declined again during the winter months while imports continued to climb and remained high throughout April 2003. This development of exports and prices are consistent both with the use of market power by using up water in the summer season by exporting in order to benefit from a higher price in the winter period, and with price takers benefiting from a higher price on the export market than at home, and then realising that the expectations about inflows of water in the late autumn were wrong. The use of reservoirs in the summer for export may also reflect the capacity limit on reservoirs and expectations about inflows threatening overflow in the autumn.

The use of market power by hydro producers has some characteristics setting it apart from standard analyses of market power. One characteristic is the zero current operating costs. It is only the opportunity cost of water that constitutes operating costs. This implies that a hydro producer with a reservoir always has to solve a dynamic problem, in contrast to the situation for a thermal producer. Coupled with storage of water and sufficient effect capacity a hydro producer can follow other strategies than thermal competitors. The key question is the scheduling of water over the periods, varying from an hour as a unit and considering 24 hours as the decision period, and yearly seasons following the pattern of inflows to the dams. But if spilling of water is to be avoided, a hydro producer cannot simply reduce output in every period to extract rent from the market. The total inflow over a yearly cycle must be used.

Use of market power by hydro producers is covered in Crampes and Moreaux (2001) using simplifying assumptions as discussed above. A two-period model is considered and the standard result of a monopoly following the strategy of equalising marginal revenues of the periods, resulting in a reallocation of water from periods with relative inelastic demand to periods with relatively more elastic demand, is established. A constraint on the transferability of water from one period to the next is not considered. In passing it is mentioned that only identical demand function will maintain equal prices for the two periods in the monopoly case.

Borenstein et al. (2002) investigated the possible use of market power by hydro producers when thermal capacities are also present at the backdrop of the California crisis. The formal model is the same as the model in Bushnell (2003) dealing with strategic scheduling of the hydro producer with different assumptions about the behaviour of the thermal producers. When a monopolist controls thermal capacities, the equalisation of the marginal revenue rule over the periods is confirmed. Period-specific demand functions are assumed, generally implying that the electricity price differs across periods.

An important conclusion from the literature discussed above is that if there are no other constraints than the available water reservoirs for a year, and operating costs are ignored[2], the competitive (and socially optimal) outcome is characterized by the price being constant through the year[3]. A second important conclusion is that the outcome under monopoly generally will differ from this, as the monopolist will equalize marginal revenue across periods. If demand elasticities differ across periods (at any given price), this implies that the price will vary across periods under monopoly. The purpose of the present paper is to show that even if the demand function is same all days of the year, the monopoly outcome will generally differ from the competitive outcome.

In our model the monopoly outcome will be identical to the competitive outcome if either there is no trade between the region considered and outside regions, or if trade can take place at an exogenous foreign price and there are no capacity limits on trade. However, for the more relevant case with trade possibilities but capacity constraints on imports and exports, we show that the monopoly outcome will generally differ from the competitive outcome. The reason for this is that the monopolist will exploit these constraints by exporting as much as the constraint permits on some days of the year, and restricting output so buyers import as much as the trade constraint permits on other days.

The rest of the paper is organized as follows. Section 2 gives a brief description of the socially optimal use of water reservoirs in a hydroelectric power system. This outcome is also the competitive equilibrium. Section 3 presents the equilibrium for the simplest possible monopoly case. The analysis is extended by introducing various complications in Sections 4-6: Section 4 discusses the case of a dominant firm with a competitive fringe. In Section 5 we relax our assumptions that the export/import price of electricity is fixed and that there are no transmission costs. Finally, demand fluctuations through the year are introduced in Section 6. Section 7 concludes.

[2] Introducing a constant unit operating cost would not change this result.
[3] Strictly speaking, it is the present value price that is constant through the year. However, for such a short period as a year, the difference between a zero interest rate and a positive interest at a normal yearly rate is negligible. To simplify notation, we therefore set the interest rate equal to zero in this paper.

2. Socially optimal use of water reservoirs

Assume that demand for electricity is given by $f(p)$ each day of the year, where p is the price of electricity. In Section 6 we shall consider the more realistic case where demand varies over the year. Setting costs equal to zero, the social value of electricity production x on any day is given by

$$(1)\ U(x) = \int_0^x f^{-1}(s)ds$$

Total production of electricity over the year is given by the sum of precipitation of the year. We ignore the fact that in the beginning of the year this amount is uncertain, although we briefly discuss the consequences of uncertainty in the end of Section 3.

The sum of precipitation is denoted by X^*. Average electricity production per year is thus given by $x^*=X^*/365$. Ignoring discounting through the year, the socially optimal distribution of the total amount of electricity is simply to have the same electricity production all days of the year, i.e. $x_t = x^*$ for all t. Formally, this follows from the fact that $U(x)$ is concave, so that $x_t = x^*$ for all t is the solution to the optimisation problem

$$(2)\ \max \sum_t U(x_t) \text{ s.t. } \sum_t x_t \leq X^*$$

This outcome is also a competitive outcome. If the price is equal to $p^*=f^{-1}(x^*)$ on all days, producers cannot do better than to supply x^* all days of the year.

3. Monopoly

Assume now that a monopolist owns all of the power generating capacity. The profit per day of the monopolist is then given by $\pi(x) = x f^{-1}(x)$. If this function is concave and x^* is lower than the profit maximizing output level x^m, as in Figure 2, the socially optimal outcome ($x_t = x^*$ for all t) is also the profit maximizing outcome for the monopolist. Formally, the solution to (2) does not change when $U(x)$ is replaced by $\pi(x)$, as long as $\pi(x)$is concave. The case where $x^* > x^m$ is trivial, and will not be considered further. The assumption that $\pi(x)$ is concave, however, is crucial to the result that the monopolist's production profile is socially optimal. This is illustrated in Figure 3. If the monopolist in this case chooses constant

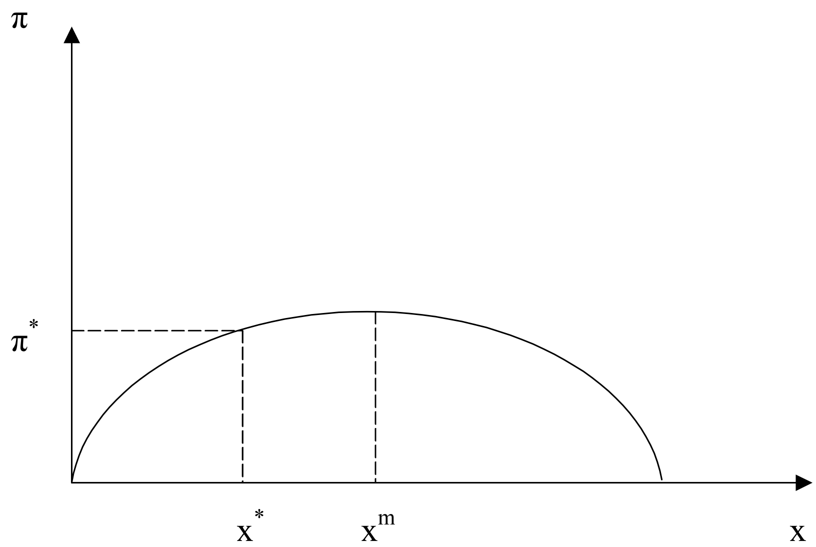

Figure 2

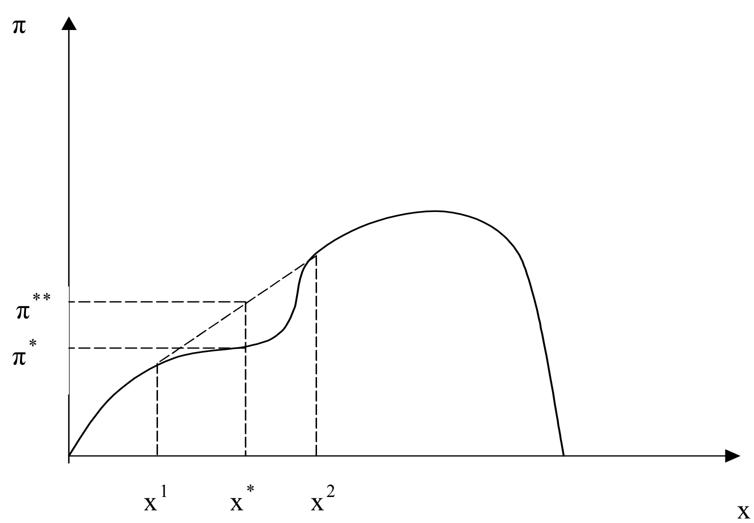

Figure 3

production (= x^*) throughout the year, it would get a profit π^* per day. However, the monopolist can do better. By producing x^1 some days of the year and x^2 on the remaining days, and choosing the number of days with each output level so that average output is x*, the monopolist achieves an average profit of π^{**} per day (see Figure 3).

It is well known that the demand function may have such properties that the profit function of the monopolist is non-concave as in Figure 3. But is there any particular reason to believe that this is the case in an electricity market? At least for the Norwegian electricity market there is an important feature making this a very realistic possibility. Assume that electricity can be traded with neighbouring regions at an exogenous price p^0. Export and import of electricity require transmission cables, and these will have some maximal capacity. In the short run this capacity is given, denote this capacity limit by c. Throughout the paper we assume that the transmission cables are owned by profit maximizing price takers (or a government agency that behaves in the same way). Until Section 5 we assume that there are no short-run transmission costs.

With the assumptions above, the demand function facing the domestic monopolist will no longer be given by $f(p)$, but instead by $f(p)-c$ for $p > p^0$ and $f(p)+c$ for $p < p^0$, see Figure 4a. The corresponding profit function is illustrated in Figure 4b[4].

We shall from now on assume that

(3) $x^* < f\left(p^0\right)+c$

This means that in the social optimum (and competitive equilibrium) the constant electricity price will be p^0 or higher. If $x^* > f(p^0)-c$, the competitive price will be p^0 throughout the year. In this case the exact production profile throughout the year in the social optimum is not completely determined, but on all days it must be possible to satisfy the domestic demand $f(p^0)$ without violating the constraints on the export/import capacity. If $x^* \leq f(p^0)-c$, production will be x* on each day of the year, imports will be at the capacity limit throughout the year, and the constant price will be $f^{-1}(x^*+c)$ in the social optimum. The owners of the transmission cables will in this case earn rents equal to $f^{-1}(x^*+c) - p^0$ per day.

[4] To keep the discussion as simple as possible, we assume that the marginal revenue corresponding to the demand function in Figure 4a is negative immediately to the right of the point C. Formally, this means that $p^0f'(p^0)+f(p^0)+c > 0$.

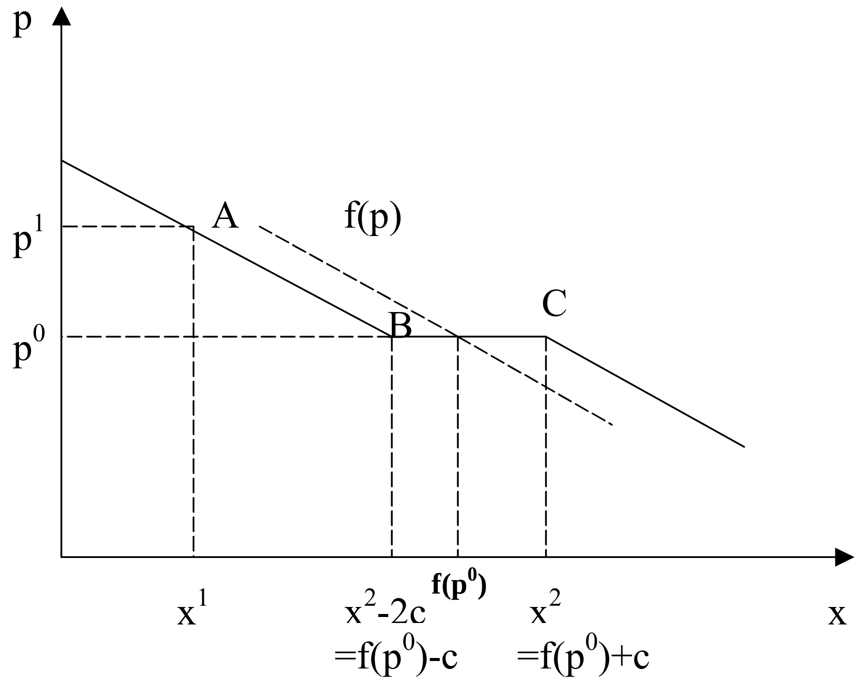

Figure 4a

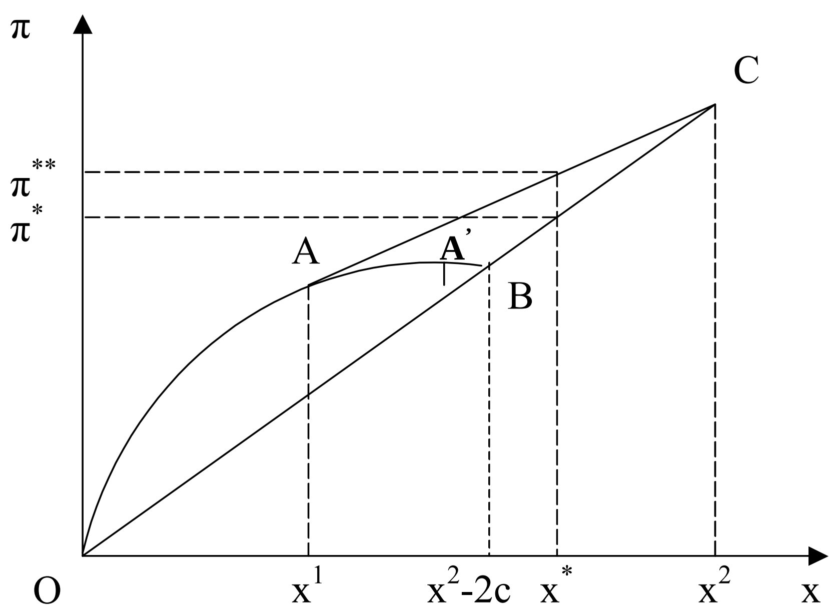

Figure 4b

If the monopolist chooses a socially optimal production profile, its profit will be $\pi^* = p^c x^*$, where p^c is the competitive (and socially optimal) price. Consider first the case where $p^c = p^0$, i.e. $x^* \geq f(p^0)-c$ (as in Figure 4b). In this case the monopolist can increase its average profits per day to π^{**} by producing x^1 some days of the year and x^2 on the remaining days, and choosing the number of days with each output level so that average output is x^* (see Figure 4b). The same is true if $p^c > p^0$, as long as $p^c < p^1$, see Figure 4a and 4b. If $p^c \geq p^1$, i.e. $x^* \leq x^1$, it is clear from Figure 4b that the monopolist cannot do better than to have a constant production equal to x*. This latter case will not be discussed any more in the rest of the paper.

Denote the number of days with the high price p^1 by T. T is determined by

(4) $Tx^1 + (365 - T)x^2 = X^* \left(= 365x^*\right)$

The monopolist's total profit through the year is $365\pi^{**}$ no matter which T dates it chooses to have the low production level x^1. However, if we introduce a small discount rate through the year, the best strategy is to have the high profit days early and the low profit days late in the year. From Figure 4b it is clear that this means that during the first 365-T days of the year the monopolist will produce x^2, and then produce x^1 on the remaining T days. It is clear from (4) that T is smaller the larger is the total precipitation X^*.

Notice that $T > 0$ is implied by the inequality in (3). Moreover, $T < 365$ as long as we disregard the case of $x^* \leq x^1$ (or $X^* \leq 365x^1$, cf. the discussion above). Notice also that as long as (3) is valid, changes in X^* (and thus in x^*) affect only T, and not x^1 and x^2. This means that if there is some uncertainty regarding X^* in the beginning of the year, this does not necessarily have any consequence for the monopolist's decisions. Assume e.g. that in the beginning of the year X^* is uncertain, but that it for sure will lie in the interval $[X^L, X^H]$. The monopolist then knows (from (4) with X^* replaced by X^L and X^H) that T will be in the interval $[T^L, T^H]$. The optimal outcome will in this case consist of at least T^L high-price days and at least $(1 - T^H)$ low-price days. During the year, the monopolist will obtain more and more information about the total amount of precipitation of the year. In Norway, one will typically have good knowledge of X^* by late November, as most of the relevant precipitation after that date comes as snow and is thus only relevant for power production in the next year. If the monopolist knows for sure what X^* is no later than the date $365 - T^H + T^L$ the initial uncertainty of X^* therefore has no consequence for the monopolist's total profit (for a negligible interest rate). Consider the stylised case in which the true value of X^* is revealed exactly at the date 365 - $T^H + T^L$. The optimal strategy for the monopolist in this case is to first have 365 - T^H low-price days (producing x^2), and then T^L high-price days (producing x^1). At this point the true value of X^* is revealed, and the remaining period of T^H - T^L days is as before split into low-price and high-price days. The lengths of these two sub-periods depend on the realisation of X^*.

So far, we have assumed that there is no capacity constraint limiting the production the monopolist can have on any day. Assume now that there is such a limit L. The limit is only binding if $L < x^2(=f(p^0)+c)$, which we therefore assume is the case. If this limit is so small that $L < x^*$ the monopolist's optimisation problem is trivial, the monopolist should simply produce at its capacity limit all days of the year. Similarly, if $x^* < L < x^2 - 2c$, it is clear from Figure 3b that the best the monopolist can do is to produce x^* all days of the year. The interesting case is when $L - 2c < L < x^2$ and $L > x^*$. In this case we have a situation similar to the one discussed above. The difference is that the point C in Figure 3a and 3b is now determined by L instead of by x^2, and therefore lies further to the left. The point B is the same as before. However, it is easy to see from Figure 3b that the point A must lie further to the right the smaller is L. Production on high-price days is therefore higher with a capacity constraint than without, and higher the lower the capacity limit L is. Similarly, the electricity price on high-price days is lower with a capacity constraint than without, and lower the lower the capacity limit L is. Provided $-dx^1/dL < 1$ (which seems reasonable although it does not hold for all demand functions), it is clear from (4) (with x^2 replaced by L) that T will be lower the lower is L. Introducing a capacity limit on daily production thus reduces the price on high-price days, and also most likely reduces the number of high-price days.

4. Competitive fringe

In Section 3 we considered the case of a pure monopoly (but with the possibility of electricity imports). A more realistic description of an electricity market is a market with one dominant firm together with a competitive fringe. The production of the fringe may be limited by two possible constraints. One constraint is given by the total precipitation of the year, X^C, corresponding to X^* in the previous sections. The second possible constraint is a capacity limit on how much the fringe can produce per day. Denote this limit by L^C. The simplest (but not very realistic) case is when this latter limit is always binding, which will be the case if $365L^C < X^C$. When this inequality holds (and operating costs as before are ignored), it is optimal for the fringe to produce L^C each day of the year, no matter what the price is (as long as it is positive). For this case the description of the monopolist's behaviour in Section 3 remains valid, except that $f(p)$ now stands for total domestic demand minus L^C.

The opposite case from the one above is the case where the capacity limit L^C is so large that it will never be binding. We shall discuss this case in detail in the present section. In the end of the section we will briefly mention how our results must be modified if the constraint L^C is binding for some days of the year.

Using D and C as superscripts for "dominant firm" and "competitive fringe", respectively, we have (in obvious notation) $X^* = X^D + X^C$. Competitive suppliers will obviously want to use the water reservoir they have (= X^C) to produce electricity on the days when the electricity prices are highest. We shall assume that the fringe always has correct predictions of what the future price will be. If $X^C \geq 365(f(p^0) - c)$, the market price will therefore be p^0 on all days: A higher price on any day cannot be an equilibrium, as all fringe producers would like to produce on these days. With $X^C \geq 365(f(p^0) - c)$, we would thus get excess supply on such days.

The interesting case in when $X^C < 365(f(p^0) - c)$. It is then possible to have $p > p^0$ on some days. Below we give a formal derivation of the optimisation problem of the dominant firm for this case.

At prices above p^0 the demand facing the domestic suppliers (dominant firm and fringe) is $f(p\ ) - c$. The price facing domestic suppliers is thus $p=g(x)$ where $g(x)= f^{\ -1}(x+c)$. On each day, the dominant firm will either produce $x^2 = f(p^0) + c$ at the price p^0 or $x^1 - X^C/T$ at the price $g(x^1)$, where x^1 as before is total domestic production on that day. Notice that for a given total domestic production x^1 on any day, the production of the dominant firm is lower the fewer such low-output/high-price days there are, since the fringe produces all its electricity on these days.

The optimisation problem of the dominant firm is to choose x^1 and T to solve the following problem:

(5) $\max\left[Tx^1 g\left(x^1\right)+\left(365-T\right)x^2 p^0\right]-X^c g\left(x^1\right)$

s.t. $Tx^1+\left(365-T\right)x^2 \leq X^*$

The term in square brackets is total industry profit. The dominant firm's profit is equal to this total profit minus the profit of the fringe, which is $X^C g(x^1)$.

An interior solution of the maximization problem above satisfies

(6) $\pi'\left(x^1\right)\equiv g\left(x^1\right)+x^1 g'\left(x^1\right)=\lambda+\frac{X^c}{T}g'\left(x^1\right)$

(7) $\frac{x^2 p^0 - x^1 g\left(x^1\right)}{x^2-x^1}=\lambda$

It is useful first to consider the case without a competitive fringe, i.e. $X^C = 0$. In this case x^1 is determined so that the marginal revenue (i.e. the slope of *OAB* at *A* in Figure 4b) is equal to λ, which is equal to the slope of the line *AC* in Figure 4b. Introducing a competitive fringe reduces the r.h.s. of (6), since $g' < 0$. The equilibrium with a competitive fringe is thus at a point such as *A'* in Figure 3b, where the slope of *OAB* at *A'* is less than the slope of the (not drawn) line *A'C*. Total domestic production x^1 thus increases with X^C, while the price p^1 on high-price days is declining in X^C. From the constraint in (5) it also follows that as X^C, and thus x^1, goes up, the number of high-price days T must also go up for a given value of X^*. A reallocation of some water reservoirs from a monopolist to a competitive fringe (X^D down, X^C up, $X^*= X^D+X^C$ unchanged) must therefore increase T. If on the other hand X^C goes up without X^D going down, X^* will increase. Such an increase in X^* will (cet. par.) reduce T. The combined effect on T of an increase in both x^1 and X^* is ambiguous.

Assume now that there is a binding constraint L^C on how much the fringe can produce on any day. This implies that the fringe no longer can sell all of its production on the high-price days. The fringe's profit (last term in (5)) is therefore changed from $X^C g(x^1)$ to $TL^C g(x^1)$ + $(TL^C - X^C)g(x^1)$. Solving the maximization problem given by (5) with this change gives us

(8) $\pi'\left(x^1\right)\equiv g\left(x^1\right)+x^1 g'\left(x^1\right)=\lambda+L^c g'\left(x^1\right)+L^C\frac{g\left(x^1\right)-p^0}{x^2-x^1}$

instead of (6), while (7) is unchanged. The last term in (8) is positive. Moreover, $L^C < X^C/T$, so $L^C g'(x^1) > X^C g'(x^1)/T$. The terms after λ in (8) are therefore larger than the corresponding term in (6). The equilibrium point A' must therefore be further to the left when there is a binding capacity constraint than when there is no such constraint. From the previous discussion we therefore have the following result: Introducing or reducing a capacity limit L^C on fringe production has the effect of increasing the price on high-price days, but also reducing the number of high-price days.

With a capacity limit L^C on fringe production, it is no longer obvious that the point A' lies to the right of A in Figure 3b. If

$$(9) \qquad \frac{g\left(x^1\right) - p^0}{x^2 - x^1} > -g'\left(x^1\right)$$

at the equilibrium point, a comparison of (8) with (6) reveals that the point A' will lie to the left of A. Consider the case of L^C small, so that the equilibrium point A' will be close to A, whatever side of A it is. From Figure 4a we see that the left hand side of (9) at the point A is equal to the slope (measured positively) of the un-drawn line from A to C, while the right hand side of (9) at the point A is the steepness (measured positively) of the line AB at A. Clearly, if the demand function is linear (as in Figure 3a) or convex, the inequality (9) cannot hold. However, if the demand function is concave, this inequality may hold, and it is more likely to hold the lower the trade capacity limit c is (since the line from A to C is steeper the lower is c). If the inequality (9) holds at the equilibrium, this means that the introduction of a competitive fringe will increase the price on high-price days. However, the number of high-price days will go down (since a lower x^1 and a higher X^* both give a lower value of T, cf. the constraint in (5)).

5. Endogenous export/import price and transportation costs

In this and the next Section, we return to the case of a pure monopolist. So far, we have assumed that the international price of electricity is given, and there are no transmission costs of export or import of electricity. We shall modify this in the current Section. We first consider the effects of endogenizing the export/import price.

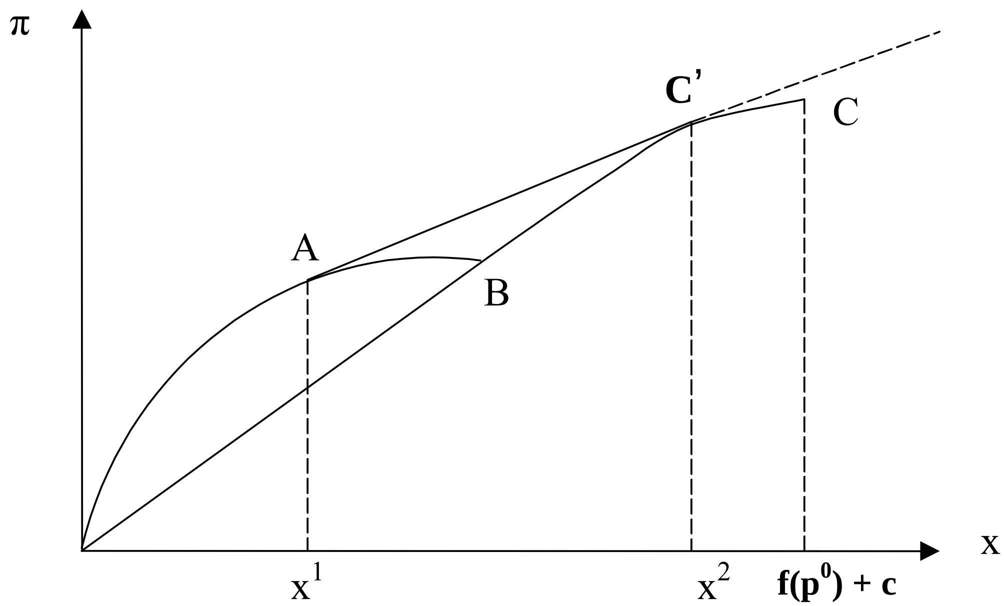

Figure 5

Assume now that the price p^0 is lower the higher are net exports. This means that the horizontal line BC in Figure 4a now instead will be downward sloping. In Figure 4b, the line BC will now be concavely curved instead of straight. If BC is sufficiently flat, i.e. the curvature of BC is modest; there will be no changes in our results. If however BC is somewhat steeper, we will get a situation as described in Figure 5 instead of the situation described in Figure 4b. The only difference is that the production on high-output days, x^2, is no longer equal to $f(p^0)+c$, but instead determined endogenously by the convex envelope of the curve OABC. All of our results remain valid if $x^* < x^2$, except that the export capacity in the present case is not fully utilized on the high-output days. If $x^* \geq x^2$ (which now is possible even if (3) holds), the monopolist's optimal strategy will simply be to produce x^* all days of the year.

Consider again the case of an exogenous foreign price p^0. However, assume that there is a transmission cost t per unit of electricity exported or imported. Instead of the horizontal line BC in Figure 4a we now get a situation as illustrated in Figure 6a. The corresponding revenue

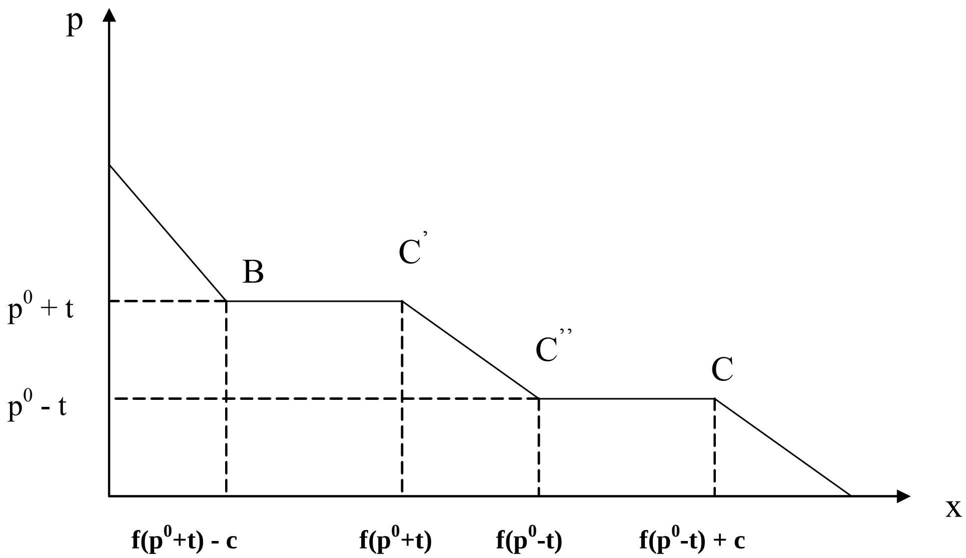

Figure 6a

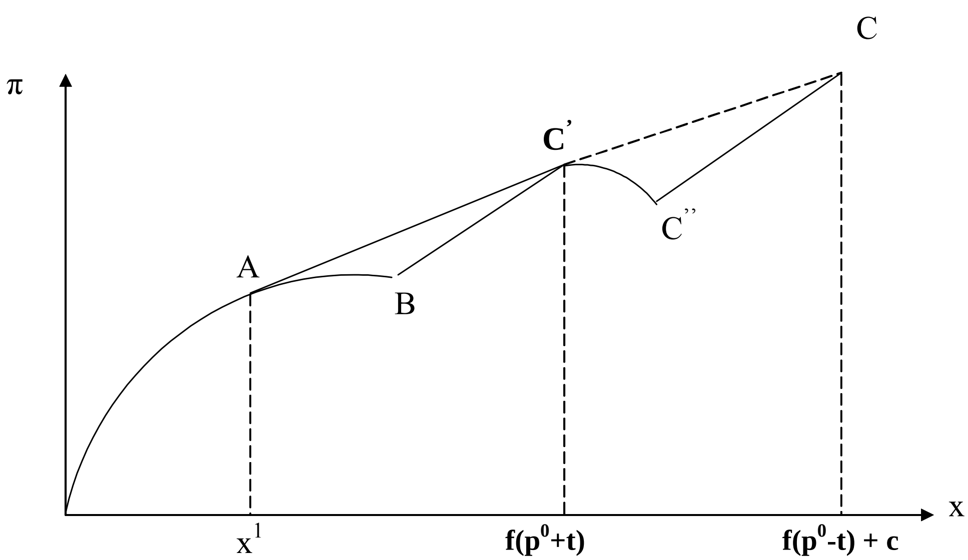

Figure 6b

function is illustrated in Figure 6b.[5] If t is sufficiently small, the results from Section 3 remain valid.[6] If t is sufficiently large, however, as in Figure 6b, the outcome will be different from what we found in Section 3. The outcome will depend on the size of x^*.

Instead of (3) we now assume that

(3') $\quad x^* < f\left(p^0 - t\right) + c$

If $x^* < f(p^0+t)$ we have a situation similar to the one discussed in Section 3, except that on high-output days production is now only $f(p^0+t)$. In other words, in this case the monopolist will never export any electricity. If $x^* > f(p^0+t)$, it is clear from Figure 6b that the optimal strategy for the monopolist will be to produce $f(p^0+t)$ on T days of the year, and $f(p^0-t)+c$ on the remaining days. The value of T is determined in the same manner as in Section 3. In this case there will thus never be any import of electricity.

6. Demand variations over the year

The demand for electricity varies over the days of the year. The most important variation in Norway is the variation in electricity demand for heating, which obviously varies with the outside temperature. The simplest way to model this is to split our year into two periods, "summer" and "winter", with demand functions $f^S(p)$ and $f^W(p)$. We assume that total reservoirs are not high enough to satisfy total domestic demand and fill the export capacity at the price p^0, i.e.

(10) $\quad X^* < D^S\left[f^s\left(p^0\right) + c\right] + D^w\left[f^w\left(p^0\right) + c\right]$

where D^S is the number of summer days and D^W is the number of winter days. This assumption corresponds to the second inequality in (3).

In a social optimum (and competitive equilibrium) the total water reservoirs would be divided between the two periods so that the electricity price was equal in the two periods. Whether

[5] To keep the discussion as simple as possible, we assume that the marginal revenue corresponding to the demand function in Figure 6a is negative immediately to the right of the point C'. Formally, this means that $(p^0+t)f'(p^0+t)+f(p^0+t)>0$. If this were not the case, a production level $x \in (f(p^0+t), f(p^0-t))$ could be optimal for the monopolist on some days.

[6] This will be the case if t is so small that the line going from A to C in Figure 6b lies above the line going from A to C'.

this common summer and winter price is p^0 or higher depends on how large the total reservoirs are.

A monopolist will divide the total reservoirs into summer electricity production X^S and winter electricity production X^W so that the following optimization problem is solved:

$$(11) \qquad \max D^s \pi^s \left(\frac{X^s}{D^s} \right) + D^w \pi^w \left(\frac{X^w}{D^w} \right)$$

$$\text{s.t.} \quad X^s + X^w \leq X^*$$

where the functions $\pi^j(x^j)$ for $j = S,W$ are the average daily profit functions for the two periods. In this section we let these functions represent the curve *OAC* in Figure 4b.

Solving (11) gives us

$$(12) \qquad \pi^{s}{'}\left(x^s\right) = \pi^{w}{'}\left(x^w\right)$$

i.e. marginal profits should be equalized in the two periods.

In each period we will have a curve *OAC* as in Figure 4b. Except by coincidence, the slopes of the lines *AC* will differ between summer and winter. We shall assume that the *AC* slope is highest in the summer: In the Appendix we have shown that a sufficient condition for this to be the case is that the relative difference between winter and summer demand is non-declining in price (i.e. that at any given price, the demand elasticity, measured positively, is lower or equal in the winter than in the summer).

To interpret (12), it is useful to distinguish between three cases:

Case 1: small total reservoirs

In this case reservoirs are so small that output in both of the periods is to the left of point *B* in Figure 4b. Both summer and winter are characterized by prices (and production) being constant throughout each period, and higher than p^0 in both periods. In both periods imports are as high as the transmission capacity allows. Marginal profits are equal in the two periods, cf. (12), implying that prices will be highest in the period with the lowest demand elasticity.

Case 2: medium total reservoirs

In this case average production is somewhere on the line segment *AC* during the summer, and in the winter at the point on *OA* where the tangency is equal to the winter *AC* slope. Marginal profits are thus equalized across periods also in this case, cf. (12). During the summer prices are initially p^0, but then rise to a higher level later. To begin with during the summer electricity is exported as much as capacity limits allow, while later in the summer electricity is imported as much as capacity allows. During the winter prices are constant, and higher than p^0. Winter imports are as high as the transmission capacity allows.

Case 3: large total reservoirs

In this case we are somewhere on the line segment *AC* during both summer and winter. Since the winter *AC* is flatter than the summer *AC*, this seems to contradict (12). However, at *C* the profit function is not differentiable, with the right derivative being lower than the left derivative. The outcome is therefore characterized by being at *C* during the summer, and somewhere along *AC* during the winter. The winter situation is therefore in this case just like the summer situation was in case 2. The summer situation is in the present case characterized by a constant price equal to p^0, a constant production, and electricity being exported as much as capacity limits allow.

7. Concluding remarks

Previous literature (as discussed in the Introduction) has demonstrated that we should expect the development of electricity prices over the year to be different in a non-competitive market than under perfect competition. In a situation where hydropower plays a dominant role, this difference is explained by the demand function being non-stationary over the year. In this paper we have argued that there may be a difference between the non-competitive and the competitive outcome even if the demand function is stationary.

A possible objection against our model is that there are very few countries or regions that rely completely on hydropower plus exported electricity. As mentioned in Section 4, however, the demand function we have used may be interpreted as total demand minus electricity production from other sources, which may include both nuclear and thermal electricity. As long as the supply of these electricity sources is given by a supply function that is increasing in the electricity price, our analysis covers this more general case.

In our analysis the possibility of electricity import and export, with a capacity limit on trade, played a crucial role. However, this modelling of trade can be given a different interpretation. Assume there are no import or export possibilities. Moreover, assume that for some of the electricity sources other than hydropower, short-run supply is not given by an upward sloping supply curve, but by an inverse L supply function. The horizontal part of the inverse L is short-run unit costs, while the vertical part represents a capacity limit, which is given in the short run. We can interpret our model as describing this case, with p^0 representing the unit cost, $2c$ representing the capacity limit, and $f(p)+c \equiv F(p)$ representing total demand minus the supply from producers that have a standard upward sloping supply curve. Except for the discussion in Section 5, all of our results are valid also for such an electricity market.

It should be clear from the discussion above that our analysis and results are valid also for regions and countries where hydropower is not as dominant as in e.g. Brazil, New Zealand or Norway.

References

Borenstein, S., Bushnell, J. B. and Wolak, F. A. (2002): "Measuring market inefficiencies in California's restructured wholesale electricity market," *American Economic Review* 92(5), 1376-1405.

Bushnell, J (2003): A mixed complementarity model of hydro – thermal electricity competition in the Western U.S., *Operations Research* 2003, 51(1) January-February

Crampes, C. and M. Moreaux (2001): "Water resource and power generation," *International Journal of Industrial Organization* 19, 975-997.

Edwards, B. K. (2003): *The economics of hydroelectric power*, Cheltenham, UK / Northampton, MA, USA: Edward Elgar.

Johnsen, T. A. (2001): "Hydropower generation and storage, transmission constraints and market power," *Utilities Policy* 10, 63-73.

Appendix: The effect of a positive demand shift

Consider a demand increase that has the property that the relative increase in demand is non-declining in price. Notice that both a multiplicative demand increase and a constant positive shift have this property. To see what happens when demand increases in this way, consider first the hypothetical case of a multiplicative increase both in demand and in trade capacity c. Clearly, this would simply blow up all curves in Figure 4b proportionally, leaving all slopes unchanged. In particular, the slope AC would remain unchanged. The actual demand increase we are considering differs from this hypothetical change in two ways. First, c remains unchanged. But this means that AC must be flatter than it was for the hypothetical change. Second, if demand increases relatively more for high than for low prices (i.e. more the further to the left in Figure 3b we are), the derivative of the profit function OAB must be smaller at any given value of x than if the demand change was proportional. This will make the line AC even flatter. It is thus clear that a demand increase of the type assumed must make the line AC in Figure 4b less steep.