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Risk Externalities in A Payments Oligopoly

By

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Memorandum from

Department of Economics, University of Oslo

No. 10/2000

RISK EXTERNALITIES IN A PAYMENTS OLIGOPOLY*

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Abstract: This paper discusses an oligopoly where firms exert negative externalities upon each other. A theoretical model is developed for a market where these externalities are particularly relevant: the intra-day credit market, which is crucial for the operation of an efficient payments system. A central bank participating actively in this market has two features that distinguishes it from the other banks: first, it is a publicly owned bank and may therefore be considered as maximizing welfare; secondly, it cannot become illiquid, and therefore it does not impose any systemic risk on other banks. The equilibrium outcome in this case is compared to the social optimum and to the equilibrium in a situation where the publicly owned bank is an ordinary one and thus *can* become illiquid.

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1. INTRODUCTION

Although there, in general, is undersupply in the equilibrium of an oligopoly, compared to the social optimum, this outcome may not hold true when the oligopolists' activities exert negative externalities. One commonly discussed case of such negative externalities is pollution.[1] In the present paper, I would like to draw attention to a phenomenon whose relevance may only increase as the globalization of business continues: Firms are financially interdependent, and this interdependence creates externalities: Where trade takes place with a credit, one firm's failure to honor its debt may have repercussions outside its trading partners, *i.e.*, financial interdependence creates negative *risk externalities*.

First, when one firm extends credit to another firm, the first firm does not take into account, in its assessment of this business, how extending this credit affects the other firm's ability to honor its debt to third-parties; this is a *credit-risk* externality. Secondly, and importantly, the first firm does not take into account the effect a failure by the other firm in honoring the debt may affect its own ability to honor its debt to third-parties; this is a *systemic-risk* externality.

Financial interdependence is important in a number of context. However, I focus in this paper on a particular setting where these risk externalities occur among firms operating in the same market: the market for intra-day credit, a form of credit which is crucial in the running of an efficient payments system.

In Section 2 of this paper, I briefly present the payments system and the externalities involved. In Section 3, I present my model of banks participating in this payments system and, in so doing, interact strategically exerting negative risk externalities upon each other. In the model, the demand for payment services is price-sensitive, an assumption which fits the empirical evidence but is not standard in the received payments-system literature. The model of payments-system externalities resembles, and shares the spirit of, one presented earlier by Gelfand and Lindsey (1989). In particular, I treat the externalities exerted by the central bank in the same way they do. However, there are some important differences in the exact modeling of the risk

[1] See, *e.g.*, Goel and Hsieh (1997) on Pigouvian taxes in a Cournot oligopoly, as well as Gulbrandsen and Skeath's (1999) discussion of antitrust policy toward the tobacco industry.

externalities involved. In particular, it is important to treat the externalities as hurting third-parties only, and this has implications for the modeling.

In Section 4, I present three different scenarios: one in which all banks are private and profit-maximizing; one in which one of the banks is a publicly owned bank with the aim of maximizing welfare; and one in which the publicly owned bank in addition also is the central bank. Like Gelfand and Lindsey (1989), I argue that, since the central bank cannot possibly become illiquid, the risk externalities associated with its activities are different from those of the other banks.

In Section 5, I find the socially optimum outcome in this market, account taken, by a social planner, of all externalities. In particular, I establish that, in the case of a central bank, the first-best is for the latter to be larger than the private banks.

In Section 6, I analyze and compare the equilibrium outcomes in the various scenarios outlined in Section 4. In particular, the central bank may have to trade off the more moderate risk externalities it exerts itself with a too high credit level from the private part of the market. Thus, even if the first best calls for the central bank to be the largest, it may well end up the smallest bank in equilibrium.

In this Section, I also compare the equilibrium outcomes and the social optima in the various scenarios. Interestingly, introducing the central bank, thus reducing overall risk externalities in this market, may actually increase the scope for an oversupply of credit in equilibrium, because the private banks get much more aggressive in their extension of intra-day credit when their public counterpart is a benign central bank than when it is just an ordinary bank, albeit a welfare-maximizing one.

Section 7 contains concluding remarks, while the Appendix has the proofs of my results.

2. EXTERNALITIES IN THE PAYMENTS SYSTEM

Payments flow among the banks in a modern economy. Of particular interest here are *large-value* transfers; according to one recent estimate [reported by Chakravorti (2000)], payments through the two large-value funds transfer systems in the US averaged $ 2.7 trillion per day in 1998. Most of these payments are based on a credit extended temporarily, and often implicitly, among the banks until the close of business. Thus, these payment flows necessitate a lot of intra-day (sometimes called "daylight") credit.[2]

Like other banking markets, the intra-day credit market may suffer from distortions caused by the market power of individual banks. In this paper, I present a model of an intra-day credit oligopoly with two idiosyncratic features in addition to the market power of the banks participating:

First, there are negative risk externalities among the banks: Increased credit extended by one bank increases not only its own risk but also the risk of the other banks that are active in this market.

Secondly, the central bank plays a role in this market, a role which has to be modeled properly. In this paper, the central bank differs from the private banks in *two* ways. First, the central bank is a welfare-maximizer, trying to correct as far as possible the combined distortions created by the oligopolistic market structure and the negative risk externalities. Secondly, the central bank, never becoming illiquid, imposes less of a negative externality than a private bank does.

With this latter aspect of the central bank, the analysis makes an interesting twist on the literature of mixed oligopoly [surveyed in De Fraja and Delbono (1990)]: In my analysis, the public firm has the double feature of being both welfare-maximizing in its objectives and comparatively efficient in its technology. When the private-banks-only equilibrium features too much credit compared to the first-best social optimum, because the underproduction tendency of the oligopoly is overturned by the overproduction tendency due to the negative risk externality, the two features of the central bank clash: With the central bank present in the market, it seeks to correct the distortion. But should it produce less than a private bank would, in order to correct the overproduction, or should it produce more in light of it being more efficient? The present

[2] *Small-value* transfers, which are the means by which the average household makes its payments, do not give rise to intra-day credit. Small-value transfers are made either by credit cards, through an automated clearinghouse, or, in the case of the US, by checks. Issues arising with these kinds of payments are discussed by Evans and Schmalensee (1999), Gowrisankaran and Stavins (1999), and McAndrews and Roberts (1997), respectively.

analysis tries to throw some light on these and related issues.

Berger, *et al.* (1996) provide an account of the various risks and costs involved in the operation of payments systems. In particular, they introduce the risk-cost frontier of a payments system: Risks may be kept down but only at a cost.[3] As discussed by Berger *et al.* (1996), there are many kinds of risk involved in a payment system. In the present analysis, I concentrate on contagion risk: If one bank becomes illiquid, then inter-bank lending and the resulting financial interdependence may cause other banks also to become illiquid.[4]

To be specific, consider a case where a payment system is operated by three banks, called *A*, *B*, and *C*. Suppose a client of bank *B* wants to make a large-value payment to a client of bank *A*. If it accepts this payment to be made, bank *A* extends an intra-day credit to bank *B* by transferring the payment immediately to its client, even though the payment is not finalized, *i.e.* the amount is not transferred from bank *B* to bank *A* until settlement time, which may be at the end of the business day.

When bank *A* considers extending credit to bank *B*, bank *A* takes into account the possibility that bank *B* becomes illiquid before settlement time, as well as the credit risk this possibility imposes on it. This risk to bank *A* that the payment will not be finalized in case the borrower defaults on its obligations during the day is the bank's *internal direct credit risk*.

But extending credit to a borrower increases the probability that also other lenders to the same borrower suffer a loss, since this increases the borrower's debt and therefore, in the likely event that the risk of a bank's failure is positively related to the level of its debt, makes the borrowing bank more susceptible to a default. Bank *A* does not, however, take into account this risk that its credit to bank *B* imposes on bank *C*, who also extends credit to bank *B*; this is the *external direct credit risk* associated with bank *A*'s credit to bank *B*.

And finally, there is still another risk involved: What I call the *systemic risk* relates to losses incurred by third parties when a lender's borrower defaults and the lender then in turn defaults on *its* obligations. Consider the case where bank *A* extends credit to bank *B*. If bank *B*

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[3]See also Angelini and Giannini (1994).

[4]See, *e.g.*, Humphrey (1986), Kaufman (1994), Angelini, *et al.* (1996), and Shen (1997).

becomes illiquid, then bank *A* itself may turn illiquid and bank *C*, extending credit to bank *A* as well, may suffer a credit risk this way; this is the systemic risk associated with bank *A*'s credit to bank *B*.

Below, I analyze the implication of the presence of these risk externalities for the performance of the intra-day credit market, and thus of the payments system, taking explicitly into consideration the effect of strategic interaction among the banks.

In order to focus on these strategic effects on the functioning of a large-value transfer system, I disregard several other important aspects of such payment systems that are discussed elsewhere. These other aspects include: how to design a payment system [Freixas and Parigi (1998), Kahn and Roberds (1998), Kahn, *et. al.* (1999), Roberds (1999), Fujiki, *et al.* (1999)]; how to regulate the participants in a payment system [Rochet and Tirole (1996a), Hancock and Wilcox (1996), and Freixas, *et al.* (1999)]; the scope for self-regulation through peer monitoring and the like [Rochet and Tirole (1996b), Calomiris and Kahn (1996), and Rolnick, *et al.* (1998a, 1998b)]; and relations to monetary policy [Lacker (1997)]. This is not to say that these questions are without importance for the understanding of the strategic interaction. For example, whether the payment is designed as one based on real-time gross settlements or is based on net settlements at the close of the business day, may influence the strategic interaction in ways not explored further in this paper.

An interesting analysis of strategic interaction in a payments system is the one by Angelini (1998) on strategic behavior in a real-time gross settlements system. In order to study such a system, he develops a dynamic model and trace payments over time during a business day. The model that I present below, is designed to focus on the role of the central bank through active participation in the payment system. Therefore, I have chosen to abstract from the dynamics that is at the heart of Angelini's analysis and instead present a static model of the strategic interaction.

The proper role to be played by the central bank is central to many of the other papers cited above. In particular, what the central bank can and should do in terms of regulating and coordinating the activity in the intra-day credit market has undergone careful study in the received literature. However, considering the possibility that the central bank can regulate the intra-day credit market from inside, through its own operation in a market marked by imperfect competition, seems to be a novelty of the present analysis. The analysis may, eventually, throw light on the question why there is so little actual participation by central banks in these markets: When risk externalities are large, my analysis predicts that the central bank's equilibrium participation is zero.

3. A MODEL OF INTRA-DAY CREDIT

My model is a highly stylized picture of what is actually going on in a payments system. In the intra-day credit market, a bank's demand for credit is derived from that of the clients it serves. I assume that the number of lending banks is exogenously fixed. To begin with, all lending banks are assumed to be private. Later on, the issue will be what difference it makes whether one of the banks is publicly owned, with an objective to maximize welfare, and eventually what happens if the publicly owned bank in addition is a central bank that cannot become illiquid (*i.e.*, a lender of last resort).

Let there be n banks who extend credit to each other, where $n \geq 3$,[5] and suppose bank i extends an amount of credit equal to $q_i \geq 0$, $i \in N := \{1, ..., n\}$; note that this consists of a number of *gross* credits extended by a bank when it accepts to perform a payment to one of its own clients from some other bank's client before the payment has been settled between the banks. Let q_{ij} denote the amount of credit extended by bank i to bank j. Throughout, I will assume for simplicity that a bank extends equal amounts of credit to each of the other banks, so that:

$$q_{ij} = \frac{1}{n-1} q_i, \quad i, j \in N, \ i \neq j.$$

In line with practice in most countries, there is no charge on an intra-day loan. The income received by a bank comes from fees paid by the payment-demanding public. The demand for intra-day credit is thus derived from the underlying demand for large-value intra-day payment services. In particular, it is assumed to be given by the following linear inverse demand function:

$$p = p(Q) = a - bQ,$$

[5] With $n = 2$, there would be no externalities involved; see the discussion below.

where $Q := \sum_{i \in N} q_i$ is the total quantity of credit supplied, and a, $b > 0$. This approach contrasts with that of Berger *et al.* (1996), who base their discussion of the risk-cost trade-off in payment systems on the quantity of payment being fixed. Note that p is *not* the inter-bank credit rate but the marginal willingness to pay among consumers of payment services. It seems reasonable to allow the demand for payment services to depend on the functioning of the interbank market.[6]

I consider all *interday*, or long-term, credit as exogenous to the current problem; to ease notation, I normalize long-term credit at zero.

The technological costs to a bank of extending an amount q of credit is given by: $cq^2/2$, where $c > 0$. In addition, there are costs associated with the risk of the intra-day credit that is extended. I assume that banks are risk neutral and that the expected loss per unit of credit is increasing in the amount extended: $dq/2$. This implies that a bank's total internal direct credit risk is: $dq^2/2$, where $d > 0$.

In our partial-equilibrium framework, welfare is the sum of consumer and producer surpluses, account taken of the externalities.[7] There are two kinds of externalities. First, there is the external direct credit risk: Each dollar lent by a bank to another bank is a cost not only to the lender, but to any third bank as well. In particular, for each unit of credit extended to bank k by other banks, there is a credit risk imposed on bank j equal to $dq_j/2$. Thus, the external direct credit risk that bank i imposes on bank j by extending credit to bank k equals:

$$\frac{d}{2} q_{ik} q_j \; = \; \frac{d}{2} \frac{1}{n - 1} q_i q_j.$$

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[6]The assumption that p depends on Q, or, vice versa, that demanded payment quantity depends on the price charged for payments, is consistent with the empirical evidence that is available for the price-sensitivity of small-value payments [Humphrey, *et al.* (1996); Humphrey, *et al.* (1999)]. There do not seem to exist any similar studies on large-value payments. However, there is no reason to believe results would be much different. Linearity is assumed here for simplicity. While there is no reason to believe payment demand is price sensitive for very low prices, linearity and a finite b may be a good local approximation when firms have market power as they have here.

[7]My way of modeling these externalities follows closely that of Gelfand and Lindsey (1989), although they do not correct for the fact that externalities are suffered by third banks only; see below.

For each third bank *j*, there are $(n - 2)$ banks *k* through which this externality works. Thus, the external direct credit risk that bank *i* imposes on bank *j* is:

$$\frac{d}{2}\frac{n-2}{n-1}q_i q_j,$$

so that the external credit risk imposed by the lending activity of bank *i* is, all in all,

$$\frac{d}{2}\frac{n-2}{n-1}q_i \sum_{j \neq i} q_j.$$

Similarly, the external credit risk imposed upon bank *j* by the collective lending activity by other banks is:

$$\frac{d}{2}\frac{n-2}{n-1}q_j \sum_{i \neq j} q_i.$$

Secondly, there is the systemic risk, which operates in the same way as the external direct credit risk.[8] In order to simplify, I assume that the strength of the two externalities is the same, *i.e.*, I use the risk parameter *d* also for the systemic risk. This means that the systemic risk caused by the lending activity of bank *i* is another

$$\frac{d}{2}\frac{n-2}{n-1}q_i \sum_{j \neq i} q_j.$$

As above, the systemic risk imposed upon bank *j* by all other banks' lending activity is:

$$\frac{d}{2}\frac{n-2}{n-1}q_j \sum_{i \neq j} q_i.$$

Total social risk, *i.e.*, internal and external direct credit risk and systemic risk over all banks, now equals:

$$\frac{d}{2}\sum_i q_i^2 + d\frac{n-2}{n-1}\sum_i \sum_{j \neq i} q_i q_j.$$

With the specified inverse demand, total gross benefit from a credit quantity Q equals $(aQ - bQ^2/2)$. Thus, welfare is:

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[8]As we shall see later, introducing a lender of last resort will imply a distinction between the two kinds of external risk.

$$W := a\left(\sum_{j \in N} q_j\right) - \frac{b}{2}\left(\sum_{j \in N} q_j\right)^2 - \frac{c}{2}\left(\sum_{j \in N} q_j^2\right) - \frac{d}{2}\left(\sum_{j \in N} q_j^2\right) - d\frac{n-2}{n-1}\left(\sum_{i \in N} \sum_{\substack{j \in N \\ j \neq i}} q_i q_j\right).$$

For simplicity, I assume that all revenue from performing payment services accrues to the bank of the client *receiving* the payment, *i.e.*, to the bank extending intra-day credit. In calculating its payoff, private bank *i* takes into account revenues, technological costs, the internal direct credit risk, and the credit-risk externalities imposed on him by the other lenders. Thus, bank *i*'s profit is:

$$\pi_i = \left(a - b\sum_{j \in N} q_j\right) q_i - \frac{c}{2} q_i^2 - \frac{d}{2} q_i^2 - d\frac{n-2}{n-1}\left(\sum_{\substack{j \in N \\ j \neq i}} q_j\right) q_i.$$

Note that, although the external credit risk and systemic risk are imposed upon a bank by the activities of other banks, the extent to which each bank is exposed to these externalities can be decreased by the bank reducing its own activity in the market.

4. THREE DIFFERENT SCENARIOS

Although the activities of the central bank is the ultimate focus of interest in this analysis, I present three different scenarios of the payments system and the intra-day credit market. In the first scenario, all the *n* banks operating in the payment system are private and thus maximizing profits. This scenario is of interest in its own right, because the analysis of it highlights the contrast between the tendency towards undersupply because of oligopoly and the tendency towards oversupply because of negative externalities.

In the second scenario, one of the banks is made into a publicly owned bank with the aim of maximizing welfare. Apart from this difference in objective, this bank is like the other banks; in particular, it is as likely as any other bank to become illiquid. The existence of a publicly owned bank is interesting for at least two reasons. First, the intra-day credit market is very difficult to regulate from above, particularly in a net-settlement payments system, where there is little supervision of credit flows during the business day. In such a case, it is interesting to see if there is anything to gain from "regulating from within" with the help of a welfare-maximizing participant in the market.

Secondly, there do in fact exist publicly owned banks in many countries: The central bank is typically publicly owned, but in addition also other parts of the banking industry may be so. It may be doubtful whether such banks perform differently from their private counterparts; it may, in fact, be that the government keeps the public bank for enhancement of fiscal revenue rather than for welfare maximization. Presently, however, I will argue that using the public bank as an instrument for regulating the intra-day credit market from within is an alternative worth considering.

I therefore, in this scenario, perform the following exercise: I transform one of the private banks of the previous scenario into a publicly owned, welfare-maximizing one; thus, there are now $(n - 1)$ private banks and one public one. Apart from the ownership status and the difference in the banks' objectives that this entails, the banks are identical.

In the third and final scenario, I take into account that the publicly owned bank, if it is also the central bank, cannot become illiquid and therefore imposes less external risks on other banks than a private one does. To be specific, there is no systemic risk associated with the credit it extends. With a central bank present, therefore, the systemic risk becomes

$$\frac{d}{2}\frac{n-2}{n-1}\sum_{i \in B} q_i \sum_{j \neq i} q_j,$$

where B is the set of private banks. Thus, welfare is now:

$$\begin{aligned} W^C := {} & a\left(\sum_{j \in N} q_j\right) - \frac{b}{2}\left(\sum_{j \in N} q_j\right)^2 - \frac{c}{2}\left(\sum_{j \in N} q_j^2\right) - \frac{d}{2}\left(\sum_{j \in N} q_j^2\right) \\ & - d\frac{n-2}{n-1}\left(\sum_{i \in B} \sum_{\substack{j \in N \\ j \neq i}} q_i q_j\right) - \frac{d}{2}\frac{n-2}{n-1}\left(\sum_{j \in B} q_C q_j\right), \end{aligned}$$

where q_C is the amount of credit extended by the central bank.

Similarly, the profit of each private bank $i \in B$ becomes:

$$\pi_i^C = \left(a - b\sum_{j \in N} q_j\right)q_i - \frac{c}{2}q_i^2 - \frac{d}{2}q_i^2 - d\frac{n-2}{n-1}\left(\sum_{\substack{j \in B \\ j \neq i}} q_j\right)q_i - \frac{d}{2}\frac{n-2}{n-1}q_C q_i.$$

Again, each private bank's profit is reduced by the externality imposed on it by other banks. However, this externality is lower from the lending activity of the central bank, which imposes external credit risk only.

I suggest that the homogeneous-good model of Section 3 is applicable also in this scenario, even though the banks involved now are heterogeneous. The banks differ only with respect to their systemic-risk properties. It is unlikely that consumers of payment services are concerned about the systemic risk associated with a particular bank's credit activity. Thus, from the viewpoint of the consumers, these banks provide homogeneous payment services.

5. SOCIAL OPTIMUM

In a discussion of the social optimum, there are only two different cases to consider. When it comes to efficiency, the banks' objectives are irrelevant. Thus, I need only distinguish between the case with ordinary banks only and the case with a central bank present.

In the latter case, not surprisingly, the central bank with its more benign externality property performs the leading role: The effect of the central bank not causing any systemic risk is that this bank should provide a larger portion of total credit than any private bank. Recall that the qs measure *gross* credit extended. Thus, it is not possible for any bank to supply a negative amount, *i.e.*, to turn itself into a borrower in gross terms. When the risk externalities are sufficiently large, the social optimum, therefore, calls for the central bank to be the only bank active in the payments system:

Proposition 1: (i) With identical banks in the intra-day market, the social optimum is that each of the n banks supplies:

$$q_P^* = \frac{a}{bn + c + d(2n - 3)}.$$

(ii) Let q^*_{Cc}, q^*_{Cp}, and Q^*_C denote the socially optimum amount of credit offered by the central bank, a private bank, and in total, respectively, in the case with $(n - 1)$ private banks and one central bank. Define:

$$D^*_C := d[8b + n^2(8c - d)] + 4[c(bn + c) - 2d(3c - d)](n - 1).$$

There exists a $d_1 > 2c$, possibly infinite such that:[9]

(a) if $d < d_1$, then the social optimum is given by:

$$q^*_{Cc} = \frac{2a}{D^*_C}\left[2c(n-1) + d(n^2 - 5n + 8)\right],$$

$$q^*_{Cp} = \frac{2a}{D^*_C}\left[2c(n-1) - d(n-4)\right], \text{ and}$$

$$Q^*_C = \frac{4a}{D^*_C}\left[cn(n-1) + 2d\right];$$

(b) if $d \geq d_1$, then private banks do not extend credit in social optimum, *i.e.*, $q^*_{Cp} = 0$, and

$$q^*_{Cc} = Q^*_C = \frac{a}{b+c+d}.$$

Part (ii) of this Proposition indicates that the central bank has a larger role to play in social optimum than its private counterparts, exactly because it exerts less externalities. If the risk externalities are large enough, then the externality advantage of the central bank is so large that the social optimum is for the central bank to be a monopolist. For smaller levels of the risk externalities, the increasing marginal technological costs make it too expensive, in terms of social efficiency, to load all business over from the private to the central bank. Note that $d_1 > 2c$; thus, a simple condition for an interior solution, with private banks present in optimum, is $d/c \leq 2$.

Clearly, the public bank extends more credit in optimum when it is a central bank than when it is an ordinary bank, since the former imposes a smaller externality on other banks than the latter; thus: $q^*_{Cc} > q^*_P$. Total optimum credit supply also increases from the previous case, since, by making the public bank a central bank, the total social credit risk is reduced; thus, $Q^*_C > Q^*_P := nq^*_P$. However, when the risk externality is sufficiently large relative to the technological costs, it is optimum to let the central bank take on so much of the credit that the private banks' credit supply actually decreases at the introduction of the former. In particular, by comparing parts (i) and (ii) of the above Proposition, I have:

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[9]The exact definition of d_1 is in the proof of Proposition 1, in the Appendix.

Corollary to Proposition 1: If

$$\frac{d}{c} > \frac{2}{3n - 8},$$

then each of the $(n - 1)$ private banks extends less credit in social optimum if the public bank is a central bank than if it is an ordinary bank, *i.e.*: $q^*_{Cp} < q^*_P$.

6. EQUILIBRIUM ANALYSIS

In the analysis of equilibrium, both externality properties and objectives matter. Thus, I must distinguish between all three of the scenarios outlined in Section 4.

6.1. PRIVATE BANKS ONLY

The case when all the banks operating in the interbank market are private highlights some essential features of an intra-day credit market.

Proposition 2: Consider the case where n identical private banks extend credit on the intra-day credit market.

(i) In equilibrium, each bank supplies

$$q_P^e = \frac{a}{b(n + 1) + c + d(n - 1)}.$$

(ii) Credit is oversupplied (undersupplied) in equilibrium, relative to the social optimum, if:

$$d > (<) \frac{b}{n - 2}.$$

There are two contrasting forces at play producing this result. First, there is the effect of imperfect competition, leading in isolation to an undersupply of credit. Second, there is the effect of risk externalities (external direct risk and systemic risk, combined), leading in isolation to an oversupply of credit: Although each bank does take into account how it can reduce the effect of other banks' externalities on its own profit, it does not account for the externalities it itself imposes on the other banks. An increase in n, the number of banks, decreases the downward distortion caused by imperfect competition and increases the upward one caused by the risk externalities. An increase in d increases the distortion due to risk externalities.[10] Thus, even if there is imperfect competition in the payments market, there is not necessarily an undersupply of credit in this market, due to the risk externalities involved.

6.2. INTRODUCING A PUBLIC BANK

The next scenario has a publicly owned, welfare-maximizing bank pitted against $(n - 1)$ private ones; thus, the number of banks is kept the same as in the previous subsection. Apart from the ownership status and the difference in the banks' objectives that this entails, the banks are (still) identical.

In cases where private banks, if left alone, would oversupply credit, it becomes relevant for the public firm to reduce total credit supply by supplying less credit than a private bank would. I have the following result:

Proposition 3: Let q^e_{Oo}, q^e_{Op}, and Q^e_O denote the equilibrium amount of credit offered by the public bank, a private bank, and in total, respectively, in the case of $(n - 1)$ identical private banks and one publicly owned, welfare-maximizing bank. Define:

$$D_O := n^2(b + d)(c - d) + (n - 1)[(b - c + 2d)^2 + c(2b + d)] + b(5c + 2d),$$

and assume $d < c$.

(i)(a) If

$$d < \min\left[c,\ \frac{(b + c)(n - 1)}{n(n - 2)}\right],$$

then, in equilibrium, each private bank lends

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[10]As indicated in the Introduction, the observation that a monopoly or an oligopoly with negative externalities may overproduce relative to the social optimum is well known from other contexts, such as pollution. Häckner and Nyberg (1996), like I do, discuss an oligopoly with negative externalities between firms. In their model, however, the negative externality works through the demand side, while here, it works through costs.

$$q^e_{Op} = \frac{a}{D_O}[c(n - 1) + d],$$

the public bank lends

$$q^e_{Oo} = \frac{a}{D_O}[(b + c)(n - 1) - dn(n - 2)],$$

and total credit supply equals

$$Q^e_O = \frac{a}{D_O}[(b + c)(n - 1) + (c - d)(n - 1)^2 + dn].$$

(b) If

$$\frac{(b + c)(n - 1)}{n(n - 2)} \leq d \leq c,$$

then, in equilibrium, each private bank lends

$$q^e_{Op} = \frac{a}{bn + c + d(n - 2)},$$

the public bank is inactive,

$$q^e_{Oo} = 0,$$

and total credit supply equals

$$Q^e_O = \frac{a(n - 1)}{bn + c + d(n - 2)}.$$

(ii) There exists a $d_2 > 0$ such that, in equilibrium, credit is oversupplied (undersupplied) relative to the social optimum if:[11]

$$d > (<) \min\left[d_2, \frac{(b + c)(n - 1)}{n(n - 2)}\right].$$

Although the public bank does obtain a reduction in total credit by reducing its own credit supply, the merit of such a policy is lessened by the other banks stealing part of its business. The latter effect may override the former, so that the public bank supplies a positive amount of credit even if there is an oversupply in total; this occurs if

[11] The exact definition of d_2 is in the proof of Proposition 3, in the Appendix.

$$d \in \left(d_2,\ \min\left[c,\ \frac{(b + c)(n - 1)}{n(n - 2)} \right] \right),$$

In particular, the higher the technological costs, c, are, the more expensive it is, on the margin, to shift the extension of credit from the public bank over to the private banks.

For sufficiently high risk externalities, though, the public bank does get more concerned with the latter than with the oligopoly problem, and extends less credit than what each of its private banks does. Comparing expressions in Proposition 3 above, I have:[12]

Corollary to Proposition 3: In the case of $(n - 1)$ identical private banks and one publicly owned, welfare-maximizing bank, the public bank extends a larger (smaller) amount of credit in equilibrium than a private bank if:

$$d < (>) \frac{b}{n - 1}.$$

6.3. WHEN THE PUBLIC BANK IS ALSO THE CENTRAL BANK

I turn, finally, to the case where the public bank not only is a welfare-maximizer but also is the central bank and as such is not able to become illiquid. While this creates an asymmetry among the banks, with the central bank exerting less externalities on other banks than any of the private banks does, the central bank may still suffer from a lack of instruments in this case: If risk externalities are high and a reduction in credit is called for, the central bank may end up with a lower activity level than its private counterpart, despite its superior externality properties. I have:

Proposition 4: Let q^e_{Cc}, q^e_{Cp}, and Q^e_C denote the equilibrium amount of credit offered by the central bank, a private bank, and in total, respectively, in the case with $(n - 1)$ private banks and one central bank. Define:

[12] Another analysis of a publicly owned firm in a non-cooperative game with externalities is provided by Poyago-Theotoky (1998), discussing an R&D game with spill-overs, *i.e.*, a case of positive externalities. She does not consider the case of asymmetric externalities, as I do below.

$$D_C^e := 4(n - 1)[b^2 + c^2 + bc(n + 1) + cd(n - 1)] + d[4b(2n - 1) + 4c + dn^2].$$

(i) (a) If

$$d < \frac{2(b + c)(n - 1)}{n(n - 3)},$$

then, in equilibrium, each private bank lends

$$q_{Cp}^e = \frac{2a}{D_C^e}[2c(n - 1) + dn],$$

the central bank lends

$$q_{Cc}^e = \frac{2a}{D_C^e}[2(b + c)(n - 1) - dn(n - 3)],$$

and total credit supply equals

$$Q_C^e = \frac{4a}{D_C^e}[(b + cn)(n - 1) + dn].$$

(b) If

$$d \geq \frac{2(b + c)(n - 1)}{n(n - 3)},$$

then, in equilibrium, the central bank is inactive and the outcome is the same as the one reported in Proposition 3(i)(b).

(ii) In the case where, in equilibrium, both the central bank and the private banks are active, *i.e.* where the condition in (i)(a) holds, credit is oversupplied in equilibrium, relative to the social optimum, if and only if, in addition to the above condition,

$$\left[\sqrt{\alpha^2 + 8bcn(n - 2)} - \alpha\right]\frac{n - 1}{2n(n - 2)} < d < \frac{2c(n - 1)}{n - 4},$$

where $\alpha := b(n - 4) + 2c(n - 1)(n - 2)$.

Notice in Proposition 4(i) that, contrary to the case of subsection 6.2, when the public bank is a central bank, there is no need for a condition to ensure equilibrium stability. The reason is that equilibrium non-stability is related to conflicts of interests among the firms in the market. This conflict is most striking in the case of one bank being welfare-maximizing but otherwise identical to the other banks (subsection 6.2). When the welfare-maximizing bank, in addition, exerts less risk externalities from its lending activities than the other banks (this subsection), the conflict is reduced, since also the private banks benefit in part from the central bank having a large share of the market.

Comparing Proposition 4 with Proposition 1(ii), we see how differently an increase in the risk externality parameter *d* affects the first-best optimum and the mixed-oligopoly equilibrium. In optimum, an increase in the systemic risk implies the central bank taking on a larger share of the market. In equilibrium, instead, the central bank has to reduce its activities in order to correct for the oversupply from the private banks. In fact, in the extreme case of

$$d > \max\left[d_1,\ \frac{2(b + c)(n - 1)}{n(n - 3)}\right],$$

the first best calls for the central bank to be the only bank in the market, while in equilibrium, the central bank is, quite to the contrary, the only bank that is inactive. More generally, I can compare expressions in Proposition 4(i) to obtain:

Corollary to Proposition 4: In equilibrium, the central bank extends a smaller (larger) amount of credit in equilibrium than a private bank if:

$$d > (<)\ \frac{2b(n - 1)}{n(n - 2)}.$$

This Corollary addresses the question asked earlier, in the presentation of the risk externalities in Section 2: Will the central bank extend more or less credit than the private banks? It has incentives to extend more credit because its credit activities constitute less social risk than that of the others, and because of the traditional oligopoly undersupply effect. The former of these is strong when there are many private banks, while the latter is strong when there are few of them. But it also has incentives to extend less credit in order to correct for the risk externalities emanating from the private banks’ credit activities. When the risk is large enough, this latter effect will surely dominate and the central bank, despite its lower externalities, ends up with a lower activity level than the others. An ordinary public bank, on the other hand, is less likely to end up with a higher quantity than the private banks, since it does not have an externality advantage over them as the central bank does. By comparing expressions in Propositions 3 and 4, I find:[13]

Proposition 5: Suppose $d < c$.

(i) If

$$\frac{b}{n-1} < d < \min\left[\frac{2b(n-1)}{n(n-2)},\ c\right],$$

then, in equilibrium, the public bank is larger than the private banks when it is also the central bank, but smaller than them when it is an ordinary bank.

(ii) If

$$\frac{(b+c)(n-1)}{n(n-2)} < d < \min\left[\frac{2(b+c)(n-1)}{n(n-3)},\ c\right],$$

then the publicly owned bank is active in equilibrium if it is also a central bank but not if it is an ordinary bank.

Proposition 5 indicates that a central bank has a larger role to play in this game than an ordinary, publicly owned bank has, not only in the first-best optimum, but also in the second-best equilibrium: In general, a central bank extends more credit than an ordinary public bank. And more particularly, there are cases where an ordinary public bank would choose not to operate in order to counteract an oversupply by the private banks, but where the beneficial externality properties of a central bank would make room for its active participation. This difference comes about partly because there are less overall externalities when one of the banks is a central bank, which calls for increased total credit, and partly because the central bank's credit is particularly beneficial for welfare. The change in private banks' behavior as we move from a regime with an ordinary public bank to one with a central bank is caused by the banks' quantities being strategic substitutes, in the sense of Bulow *et al.* (1985): If the public bank expands its output, then the private banks contract theirs, and vice versa.

[13]The restriction $d < c$, although not applied in Proposition 4, is made here in order to compare the results there with those of Proposition 3, where this restriction is imposed to ensure equilibrium stability.

However, the way the private banks and the public bank react to an increase differs between the two cases of an ordinary public bank and a central bank. Because the introduction of the central bank into the market reduces the systemic risk, the private banks react to an increase in risk in a much more aggressive way than in the case when the public bank share their risk properties. Actually, this amounts to an overreaction on the part of the private banks. Alternatively, compared to its private counterparts, the central bank, with its benign risk externalities, chooses to hold on to its extension of credit, as the risk increases, to a much larger extent than if it were an ordinary bank. The decrease in externalities caused by the introduction of the central bank is asymmetric, which calls for a decrease in private banks' credit in social optimum, a decrease which is too hard to implement for a central bank with only one instrument, as in my model, *viz*. its own credit activity. The outcome is that, even though the case of a central bank has less risk externalities overall and therefore the social optimum calls for a higher level of total credit than in the case of an ordinary public bank, an oversupply of equilibrium credit may occur with the central bank, even in cases where, with an ordinary public bank, there would be an undersupply of credit in equilibrium.[14, 15]

7. CONCLUDING REMARKS

This paper, contrary to most of the existing literature on the payments system, suggests that considering a large-value payment system as an interbank credit oligopoly is a worthwhile exercise and presents an analysis of strategic interaction in such a market. Particular emphasis is

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[14]In particular, if

$$\left[\sqrt{\alpha^2 + 8bcn(n-2)} - \alpha\right]\frac{n-1}{2n(n-2)} < d < \min\left[d_2,\ c,\ \frac{(b+c)(n-1)}{n(n-2)}\right],$$

then there is undersupply in equilibrium if the public bank is an ordinary bank but oversupply of credit in equilibrium if it is the central bank.

[15]Note that Proposition 4(ii), on the over- versus undersupply of credit in equilibrium compared to the first-best, only covers the case where the central bank is active in equilibrium. For levels of *d* so high that only private banks prevail in equilibrium, the picture is more complicated and results less instructive and thus not reported here.

put on how a central bank, with more benign objectives *and* externalities than other banks, performs in the equilibrium of such an oligopoly. Thus, the analysis may also, more generally, have a bearing on the understanding of the performance of publicly owned firms in this kind of mixed oligopolies: The present analysis shows how the market equilibrium is affected by introducing a public firm that is not only a welfare maximizer but whose production carries a lower social cost than that of the private firms in the market.

An analysis of the payments system limited in scope to the social optimum would conclude that the central bank ought to engage heavily as a regular participant in the interbank credit market. The present analysis shows the importance of performing an equilibrium analysis as well: For sufficiently high risk externalities and an inability, on the part of the central bank, to instruct each private bank its extension of credit, the correct thing for the central bank to do is to stay out and stick to the role as supervisor and coordinator.

The analysis presented here *is* limited, though, in claiming that the central bank is a welfare-maximizer. Although this seems a natural position to take on present-day central banks, accounts from history indicate that central banking may be profitable in some circumstances; see, *e.g.*, the discussion in Rolnick, *et al.* (1998a, 1998b). A possible extension of the research report here could, therefore, be to the case where the central bank keeps its superior externality properties but where, in contrast to the present analysis, it otherwise is a private bank with the objective of maximizing profit.

The modeling of institutional details is, quite deliberately, kept to a minimum in this work. Among the aspects that should be introduced into the formal analysis in future work is the role of the central bank as both an active participant in the market and a regulatory body. It would also be fruitful to go beyond the present modeling of credit risk as a cost parameter and instead model more explicitly the lack of information among banks about each other. And finally, one may extend the present analysis to a dynamic model of interbank credit over the course of the day, for example along the lines of Angelini (1998).

APPENDIX

This Appendix contains all proofs that are not given in the text.

Proof of Proposition 1:(i) From the welfare expression in Section 3, I get the first-order condition for bank i's optimum amount of credit:

$$\frac{\mathrm{d}W}{\mathrm{d}q_i} = a - b\left(\sum_{j \in N} q_j\right) - cq_i - dq_i - 2d\frac{n-2}{n-1}\left(\sum_{\substack{j \in N \\ j \neq i}} q_j\right) = 0.$$

By symmetry, $q_i = Q/n$, for each i. Thus, the above equation reduces to:

$$a - bnq - cq - d(2n - 3) = 0,$$

from which I obtain the expression for q_P^*.

(ii) The relevant welfare expression in this case is W^C in Section 4. Because of the symmetry of the private banks, the first-order condition for a social optimum with respect to the central bank's credit quantity is:

$$a - b[q_c + (n-1)q_p] - cq_c - dq_c - d(n-2)q_p - \frac{d}{2}(n-2)q_p, \qquad \text{(A1)}$$

while the first-order condition with respect to each private bank's quantity is:

$$a - b[q_c + (n-1)q_p] - cq_p - dq_p - d\frac{n-2}{n-1}[q_c + 2(n-2)q_p] - \frac{d}{2}\frac{n-2}{n-1}q_c.$$

Supposing an interior solution, I can now solve the following system of first-order conditions to find welfare maximum:

$$\begin{pmatrix} (b + c + d) & \left[b(n-1) + \frac{3(n-2)}{2}d\right] \\ \left(b + \frac{3(n-2)}{2(n-1)}d\right) & \left[b(n-1) + c + \frac{2n^2 - 7n + 7}{n-1}d\right] \end{pmatrix} \begin{pmatrix} q_{Cc} \\ q_{Cp} \end{pmatrix} = \begin{pmatrix} a \\ a \end{pmatrix},$$

where the first (second) equation is the first-order condition with respect to the quantity of the central bank, q_{Cc} (the quantity of a private bank, q_{Cp}). The solution is the one outlined in part (ii)(a) of the Proposition. The determinant of the coefficient matrix is D_C^*, given in the

Proposition. Second-order conditions for an optimum require that $D^*_C > 0$, which holds if $3 \leq n \leq 6$ or, for $n \geq 7$, if

$$d \leq \frac{2\left[\tau + \sqrt{\tau^2 + c(bn + c)(n - 1)(n^2 - 8n + 8)}\right]}{n^2 - 8n + 8}, \tag{A2}$$

where $\tau := 2[b + c(n^2 - 3n + 3)]$. In addition, an interior solution requires that quantities be non-negative. Together, these two conditions imply the single condition $d < d_1$, where:

$$d_1 = \min\left[\frac{2\left[\tau + \sqrt{\tau^2 + c(bn + c)(n - 1)(n^2 - 8n + 8)}\right]}{n^2 - 8n + 8}, \frac{2c(n - 1)}{n - 4}\right], \text{ if } n \geq 7$$

$$d_1 = \frac{2c(n - 1)}{n - 4}, \text{ if } 5 \leq n \leq 6, \text{ and}$$

$d_1 = \infty$, if $3 \leq n \leq 4$.

When $D^*_C \leq 0$, which can only occur if $n \geq 7$, optimum is one of two possible corner solutions: either the central bank alone and $q_{Cp} = 0$; or $(n - 1)$ private banks and $q_{Cc} = 0$. Comparing the welfare levels obtainable in each of these two cases, the former case is found to be preferred to the latter if $d \geq c(n - 2)/(n - 4)$, a condition which is implied by $D^*_C \leq 0$. Thus, when the solution is not interior, social optimum is the central bank alone. Finally, note that, clearly, $d_1 > 2c$ if the right-hand side of (A2) is. This expression is decreasing in n, and as n goes to infinity, it goes to

$$2\left[2c + \sqrt{4c^2 + bc}\right] > 8c.$$

Thus, $d_1 > 2c$. *QED.*

Proof of Proposition 2: From the profit expression in Section 3, I get each bank's first-order condition:

$$\frac{d\pi_i}{dq_i} = a - b\left(\sum_{j \in N} q_j\right) - bq_i - cq_i - dq_i - d\frac{n - 2}{n - 1}\left(\sum_{\substack{j \in N \\ j \neq i}} q_j\right) = 0.$$

Again, by symmetry, all qs are the same and equal to Q/n. Thus, I write the first-order condition as:

$$a - b(n + 1)q - cq - d(n - 1) = 0,$$

and I obtain the expression for q_P^e in part (ii). Comparing this expression with the one for social optimum in Proposition 1(i), the condition for equilibrium oversupply, $q_P^e > q_P^*$, is obtained. *QED*.

Proof of Proposition 3: (i) (a) Each private bank maximizes its payoff with respect to own credit, while the public bank maximizes welfare with respect to its credit. Thus, following the proof of Proposition 1(i), and since the $(n - 1)$ private banks are symmetric, I can write the welfare-maximizing public bank's first-order condition as:

$$a - b[(n - 1)q_p + q_o] - cq_o - dq_o - 2d(n - 2)q_p = 0$$

Similarly, I can write each private bank's first-order condition as:

$$a - b[(n - 1)q_p + q_o] - bq_p - cq_p - dq_p - d\frac{n - 2}{n - 1}[(n - 2)q_p + q_o] = 0$$

The two equations can be combined into the following system:

$$\begin{pmatrix} \left(b + \dfrac{d(n - 2)}{n - 1}\right) & \left(bn + c + \dfrac{d(n - 2)^2}{n - 1}\right) \\ (b + c + d) & [b(n - 1) + 2d(n - 2)] \end{pmatrix} \begin{pmatrix} q_{Oo} \\ q_{Op} \end{pmatrix} = \begin{pmatrix} a \\ a \end{pmatrix},$$

where the first (second) equation is the first-order condition of a private (the public) bank. Equilibrium stability requires $D_O > 0$ [see Dixit (1986)], which is met here under the maintained assumption $d \leq c$. The solution to the equation system is given in the Proposition, and it holds if all the qs are non-negative; the condition for this is the one given. The quantity of total credit equals $q_{Oo}^e + (n - 1)q_{Op}^e$.

(b) The optimum amount of credit for the public bank is now zero. Taking this into account in the first-order condition of a private bank, the expression in the Proposition is obtained. The quantity of total credit equals $(n - 1)q_{Op}^e$.

(ii) First, I check the case of part (i)(b), supposing

$$d \geq \frac{(b + c)(n - 1)}{n(n - 2)}.$$

Comparing $(n - 1)q_{Op}^e$ with nq_{Oo}^*, I find that there is oversupply if the weaker condition

$$d \geq \frac{bn + c}{n^2 - 3n + 2}$$

holds; the latter condition is weaker because, in the two righthand-side expressions, the denominator of the latter is less than or equal to that of the former (since $n \geq 3$), and its numerator is greater. Thus, there is always oversupply in the case of part (i)(b).

In the case of part (i)(a), where

$$d < \min\left[c, \frac{(b+c)(n-1)}{n(n-2)}\right],$$

I compare $q^e_{Oo} + (n-1)q^e_{Op}$ with nq^*_{Oo} and find that there is oversupply if and only if $d > d_2$, where

$$d_2 := \frac{\beta + \gamma + \sqrt{\beta^2 + \gamma^2 - 4b(n-3)(n^2 + 10n + 1)}}{2(n-1)(n-3)},$$

with $\beta := 2(n-1)^2$ and $\gamma := b(n-3)$, is the lower root solving the equation $q^e_{Oo} + (n-1)q^e_{Op} = nq^*_{Oo}$ with respect to d; the higher root does not satisfy the above restriction. *QED.*

Proof of Proposition 4: (i) The central bank's first-order condition in part (a) is identical to (A1) above. From a private bank's profit in the case of a central bank, π^C_i in Section 4, each private bank's first-order condition is, by symmetry among the private banks, given by:

$$a - b[(n-1)q_p + q_c] - bq_p - cq_p - dq_p - d\frac{(n-2)^2}{n-1}q_p - \frac{d}{2}\frac{n-2}{n-1}q_c = 0.$$

Supposing non-negative quantities in equilibrium, the equilibrium is thus found by solving the following system:

$$\begin{pmatrix} (b+c+d) & \left(b(n-1) + \frac{3}{2}(n-2)d\right) \\ \left(b + \frac{n-2}{2(n-1)}d\right) & \left(bn + c + \frac{n^2 - 3n + 3}{n-1}d\right) \end{pmatrix} \begin{pmatrix} q_{Cc} \\ q_{Cp} \end{pmatrix} = \begin{pmatrix} a \\ a \end{pmatrix},$$

where the first (second) equation is the first-order condition of the central bank, q_{Cc} (a private bank, q_{Cp}). The resulting outcome is the one outlined. Quantities are non-negative if the numerator of q^e_{Cc} is, which gives rise to the condition distinguishing parts (i)(a) and (b).

In order to show part (ii), given the supposition that both the equilibrium outcome is interior, I first compare the total equilibrium supply of part (i)(a) with the optimum total supply in Proposition 1(ii)(a), which covers the subcase where the social optimum is interior, too. This comparison shows that there is oversupply in this case if and only if

$$\left[\sqrt{\alpha^2 + 8bcn(n-2)} - \alpha\right]\frac{n-1}{2n(n-2)} < d < \frac{2c(n-1)}{n-4},$$

where α is defined in the Proposition. The second inequality is always satisfied when social optimum is interior, by Proposition 1(ii) and the definition of d_1 in the proof of the latter. Secondly, I compare the total equilibrium supply of part (i)(a) with the optimum supply in Proposition 1(ii)(b), for the subcase when the optimum is for the central bank to be alone. This comparison shows that there is oversupply in this case if and only if

$$d < \frac{2c(n-1)}{n-4}.$$

Combining the two cases, we find that the former condition holds in both subcases, thus proving the statement in the Proposition. *QED*.

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