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Demographic Translation:

From Period to Cohort Perspective and Back

By

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DEMOGRAPHIC TRANSLATION:
FROM PERIOD TO COHORT PERSPECTIVE AND BACK

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Abstract

When successive birth cohorts of women get their children at progressively lower ages, births that would have occurred during a certain period without changes in the timing, are now “squeezed” into a shorter period. This pushes period fertility up, and the period Total Fertility Rate (TFR) will be inflated, compared to the TFR that would have occurred *without* changes in the timing. In general, even when the number of children per woman is constant over successive birth cohorts, period fertility levels may vary – they are inflated in years in which childbearing is accelerated, and deflated when women postpone childbearing. Thus period fertility cannot be used as a reliable indicator for the level of cohort fertility: period fertility may be “distorted” in times of tempo changes in cohort fertility. At the same time cohort fertility cannot be fully understood without studying periods.

These qualitative links between period and cohort fertility are straightforward. But the detailed interplay between period and cohort fertility, both its quantum (level) and tempo (timing) aspects, can be formalized mathematically. The resulting expressions constitute the core of what has become known as the theory of demographic translation, a term coined by Norman Ryder. This chapter gives a brief general overview of demographic translation theory. It integrates Ryder’s findings from the 1960s that he applied to age-specific fertility, with more recent insights, which can be used for analysing other demographic processes, such as childlessness, first marriage, and divorce.

Note

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1. **The need for both cohort and period analysis**

In his account of fertility levels and trends in England and Wales since the 1930s, John Hobcraft (1996) noted that the mean age at birth fell rapidly after the Second World War. In 1972, women were 26.2 years old on average when they gave birth - about three years younger than in 1945-6 (29.2 years). The decrease corresponds to more than a year per decade. This change in the timing of fertility had an upward effect on period Total Fertility Rates. If successive cohorts of women get their children at progressively lower ages, such that the mean age falls every year by one-tenth of a year, births that would have occurred during a period of ten years without changes in the timing, are now “squeezed” into a period which is one year shorter, Hobcraft argues. This pushes period fertility upwards, and the period TFR will be inflated by about ten per cent, compared to the TFR that would have occurred *without* changes in the timing.

Hobcrafts example shows clearly the point repeatedly stressed by Norman Ryder (1956, 1964, 1980): even with constant completed cohort fertility, period fertility levels may vary – they are inflated in years in which childbearing is accelerated, and deflated when women postpone childbearing. Thus period fertility cannot be used as a reliable indicator for the level of cohort fertility: period fertility may be “distorted” in times of tempo changes in cohort fertility. At the same time cohort fertility cannot be fully understood without studying periods.

These qualitative links between period and cohort fertility are straightforward. But the detailed interplay between period and cohort fertility, both its quantum (level) and tempo (timing) aspects, can be formalized mathematically. The resulting expressions constitute the core of what has become known as the theory of demographic translation, a term coined by Norman Ryder. The starting point is a series of age-specific birth rates for many calendar years. Since the period quantum (TFR) and the cohort quantum (Completed Cohort Fertility CCF) are obtained on the basis of the same age-specific rates, but by summation in different directions (vertically for the TFR, diagonally for the CCF), there must be a relationship between the two quantum measures. In certain cases, when fertility changes show strong regularities (e.g. the TFR falls linearly, while the age pattern is constant), the resulting relationships are very simple mathematical expressions. The purpose of deriving such expressions is to gain insight into the degree of “translational distortion”, in other words to predict the quantum and tempo of cohort fertility, given trends in period fertility, and the other way round.

This Chapter will give a brief general overview of the theory of demographic translation. The theory can be applied not only to fertility, but also to other demographic processes, such as first marriage and divorce. But in any case two conditions have to be fulfilled. First, the quantum and tempo indicators must develop sufficiently smoothly over a long period. Second, the sum of age-specific (or duration-specific) rates in either period or cohort dimension, or a transformation of this sum, must have a clear demographic interpretation. This must also be the case for the moments of the age schedule.

2. **Early expressions by Ryder for the case of age-specific fertility**

Translation formulae can be used in two directions. From a theoretical point of view, one could be interested in expressions for the development of period quantum and tempo indicators, given certain trends in cohort indicators. Such expressions give insight in possible cohort mechanisms behind observed period developments, since period developments are considered a function of cohort developments. But in practice, data for period developments are easier to obtain than those for cohort trends. Therefore it is also useful to take the opposite point of view, and analyse cohort developments on the basis of period developments.

2.1 From cohort to period

Let $m[t,x]$ be a time- and age-specific birth rate, with t and x representing time and age, respectively. We use the term `quantum' to denote the mean number of children per woman in a real or synthetic cohort, and the term `tempo' for the timing of the births event during the life course. When we sum the rates over childbearing ages for calendar year t, the result is the period quantum (TFR): $\Sigma_x\, m[t,x]=A[t]$. Similarly, we define the cohort quantum (CCF) for cohort g as $\Sigma_x\, m[g+x,x]=B[g]$, with $g=t-x$. We introduce for cohort g the age-specific proportions $b[g,x]$ of the sum of cohort rates $B[g]$ by $b[g,x]=m[g+x,x]/B[g]$, and the k'th moment of the age schedule $b[g,x]$ as $M_k[g]=\Sigma_x\, x^k b[g,x]$. These moments describe the tempo of the event: the first moment (k=1) equals the mean age at childbearing, the second moment indicates the width of the age pattern, etc. Next, Taylor series approximations for $B[g-x]$ and $b[g-x,x]$ about $g=t$ result in:

$$B[t-x]=B[t]-xB^{'}[t]+½x^2B^{''}[t]-\ldots, \text{ and}$$
$$b[t-x,x]=b[t,x]-xb^{'}[t,x]+½x^2b^{''}[t,x]-\ldots .$$

The period sum of rates $A[t]=\Sigma_x\, m[t,x]=\Sigma_x\, b[t-x,x]B[t-x]$ can be written as follows:

$$A[t]=\sum_x \{B[t]-xB^{'}[t]+½x^2B^{''}[t]-\ldots\}\{b[t,x]-xb^{'}[t,x]+½x^2b^{''}[t,x]-\ldots\}=$$
$$=B[t]\sum_x \{b[t,x]-xb^{'}[t,x]+½x^2b^{''}[t,x]-\ldots\}-B^{'}[t]\sum_x x\{b[t,x]-xb^{'}[t,x]+½x^2b^{''}[t,x]-\ldots\}+$$
$$+½B^{''}[t]\sum_x x^2\{b[t,x]-xb^{'}[t,x]+½x^2b^{''}[t,x]-\ldots\}.$$

The first derivative with respect to time of the moment $M_k[g]$ is $M_k^{'}[g]=\sum x^k b^{'}[g,x]$, and likewise for higher order derivatives. Hence we find for $A[t]$ that

$$A[t]=B[t]\{1-M_1^{'}[t]+½M_2^{''}[t]-\ldots\}-B^{'}[t]\{M_1[t]-M_2^{'}[t]+½M_3^{''}[t]-\ldots\}+$$
$$+½B^{''}[t]\{M_2[t]-M_3^{'}[t]+½M_4^{''}[t]-\ldots\}-\ldots, \text{ or}$$

(1) $$A[t]=\sum_{i=0}^{\infty}\left\{\frac{(-1)^i B^{(i)}[t]}{i!}\sum_{j=i}^{\infty}\frac{(-1)^{(j-i)}M_j^{(j-i)}[t]}{(j-i)!}\right\}.$$

The upper index (i) for $B[t]$ and $M[g]$ indicates the i-th derivative with respect to time (with the convention that $B^{(0)}[t]=B[t]$ and likewise for $M[t]$), and the lower index for $M[t]$ indicates the moment (with the convention that $M_0[t]=1$). Expression (1) is slightly more general than the corresponding expression derived by Ryder (1964, 1980). Unlike we did, Ryder assumed that the rates $m[g,x]$ are a polynomial function of cohort g, with degree n. This means that Ryder could limit the sums to at most n terms, whereas expression (1) has infinitely many. In the derivation of expression (1) we have not used any polynomial assumption, but these will be introduced (for the moments, not for the rates) below. We will study three extremely simple special cases in somewhat more detail. First it is assumed that both the cohort quantum and the age pattern of cohort fertility are constant, while only the mean age changes linearly. This would be the case when women born in successive generations on average would have the same number of children, while the curve of age-specific birth rates shifts progressively towards higher or lower ages. The second case is one with constant cohort tempo and a linear change in the cohort quantum. In this case all rates grow or diminish from one cohort to the next with the same relative amount. The third case combines linear changes in both cohort quantum and mean age. Although these cases are clearly unrealistic, they provide nonetheless insight in the basic mechanisms of translation.

Constant cohort quantum and a linear change in the cohort mean age

Constant cohort quantum implies that $B[g]=B$ for each cohort g, and hence the first and all higher order derivatives of $B[g]$ vanish. When the cohort mean age $M_1[g]$ follows a straight line, its first derivative is constant while its second and higher order derivatives are all zero. The assumption of a constant age pattern implies that second and higher order moments are constant, so that their derivatives are zero. With these assumptions, expression (1) simplifies into

(2) $$A[t] = A = B\left(1 - M_1^{'}\right).$$

Thus the TFR is also constant in this case, and it equals the CCF multiplied by one minus the annual change in the mean age. When the mean age falls by one-tenth of a year from one cohort to the next, the TFR is ten per cent higher than the CCF, and the translational distortion is ten per cent. In general, when childbearing is progressively concentrated in younger ages, the period TFR is inflated, other things being equal. See also Ryder (1964, 76), Pressat (1983, 102-103) and Wunsch and Termote (1978, 62-63).

Constant cohort mean age and a linear change in the cohort quantum

With constant cohort mean age and a linear change in the cohort quantum, the second and higher order derivatives of $B[g]$ and all derivatives for the moments in expression (1) vanish. In that case one obtains

$$A[t] = B[t] - B^{'}.M_1 .$$

Thus the TFR in year t equals the CCF for the cohort born in year t, minus the slope in the CCF times the mean age. In other words, the TFR changes linearly as well. With a mean age of 30 years, say, and a CCF that falls by 0.05 child per woman each generation, the TFR is higher by 0.15 children, compared to the CCF. Since the CCF falls by B'=0.05 child per generation, the decrease is $M_1.B'$=0.15 children over 30 generations. In other words, the TFR in year t equals the CCF for cohort t-30. More generally,

(3) $$A[t] = B[t - M_1],$$

as expected, see also Pressat (1983, 102). This simple relationship justifies common practice among demographers to shift the CCF curve over a distance of between 25-30 years, when it is plotted in one graph together with the period TFR.

Linear changes in cohort quantum and cohort mean age

When both the CCF and the cohort mean age change linearly, expression (1) becomes

(4a) $$A[t] = B[t](1 - M_1^{'}) - B^{'}.M_1[t], \text{ or equivalently}$$

(4b) $$A[t] = B[\gamma] - M_1^{'}B[t], \text{ with } \gamma = \gamma[t] = t - M_1[t].$$

Thus an increasing CCF and a fall in the mean age imply, according to expression (4a), an additional inflation of the TFR by an amount equal to $B^{'}.M_1[t]$, compared to the situation in which only the mean age falls (expression (2)). This is close to the situation in England and Wales during the years of the baby boom: the CCF of cohorts born in the period 1905-1935 showed a near linear increase, from ca. 1.8 to 2.4 children per woman (Festy 1979). The consequence was that the TFR in the years 1935-1965 also rose, see expression (4b). However, the TFR was *extra* high (by an amount of $-M^{'}_{1}.B[t]$) because the mean age for cohorts 1910-1935 fell by two years, cf. also the period indicators mentioned in the introduction. Other baby boom countries where an increase in CCF for women born in the first few decades of this century went together with a fall in the cohort mean age, are Denmark, Sweden, Finland, Norway, France, Netherlands, Switzerland, Canada, Australia, and the United States (white women), see Festy (1979, 121, 141).

The general expressions

Expression (1) gives the period quantum as a function of cohort quantum and tempo indicators. Ryder derived similar expressions for the period mean age and the period variance. He noted that these formulae are special cases of a very general expression, derived as follows (see also Yntema 1977, 163).

Denote the k-th period moment of the fertility rates $m[t,x]$ by $V_k[t]=\Sigma x^k m[t,x]$. Similarly, define the k-th cohort moment of those rates by $W_k[g]=\Sigma x^k m[t+x,x]$, $g=t+x$. Then $V_0[t]$ is simply the TFR, or the period sum of rates in year t, also written as $A[t]$. Also, $W_0[g]= B[g]$ is the CCF for cohort g. The difference between the moments $M_k[g]$ introduced above and the moments $V_k[t]$ and $W_k[g]$ is, that the latter are non-normalized, in the sense that the zero-th moment is unequal one. In contrast, M_0=1. Hence we find the following relationship between normalized and non-normalized cohort moments:

$$M_k[g]= W_k[g]/W_0[g],\ k=0, 1, 2, \ldots$$

and similarly for the period moments.

With these definitions for the non-normalized period and cohort moments, Taylor series approximation of $m[t+x,x]$ about t leads to

(5) $$V_k[t] = \sum_{i=0}^{\infty} \frac{(-1)^i}{i!} W_{k+i}^{(i)}[t] .$$

Indeed, substituting k=0 in expression (5) leads to expression (1).

We will use (5) to derive an expression for $N_1[t]$, the period mean age at childbearing, as a function of cohort quantum and tempo indicators. Substituting k=1 in expression (5), and writing $W_{1+i}[g]$ as $M_{1+i}[g].W_0[g]= M_{1+i}[g].B[g]$, we find that

(6) $$N_1[t] = V_1[t]/V_0[t] = \frac{1}{A[t]} \sum_i \frac{(-1)^i}{i!} \left(\frac{\partial^i}{\partial g^i} M_{1+i}[g]B[g] \right)_{g=t}$$

Let us consider the special case of a linear change in the cohort quantum, and constant cohort tempo. This means that all second and higher order derivatives in the expression for $N_1[t]$ vanish, whereas the moments $M_k[g]$ are independent of g. This results in to

$$N_1[t] = \frac{1}{A[t]} \left\{ M_1 B[t] - M_2 B' \right\}$$

$$= \frac{M_1}{A[t]} B[t - \frac{M_2}{M_1}] .$$

On the basis of expression (3) we find that

(7) $$N_1[t] = M_1 \frac{B[t - {M_2}/{M_1}]}{B[t - M_1]} .$$

Thus we see that the period mean age equals the cohort mean age times an adjustment factor ("distortion factor"), which depends on two other factors: the development of the cohort quantum, and the relationship between the first and the second moment of the cohort fertility age schedule. Since for any distribution the second moment equals $M_2 = (M_1)^2 + \sigma^2$, where σ^2 represents the variance of the distribution, we have that $M_2 \geq (M_1)^2$. The equality sign holds when the variance is zero, in other words when all women get their children at the mean age M_1. In this extreme case, the distortion factor equals one and the period mean age is constant, equal to the cohort mean age. For $M_2 > (M_1)^2$ the distortion factor is larger/smaller than one, when cohort fertility falls/increases. Thus under the assumptions stated, the period mean age in year t is *higher* than the cohort mean age for women born in year t when the CCF *falls*, and vice versa.

Expression (5) may also be used to analyse the change in the variance of the period fertility age schedule, as a consequence of changes in cohort quantum and tempo.

2.2 From period to cohort

When cohort developments are considered a function of period developments, expressions similar to those given above can be derived. To that end we introduce for period *t* the age-specific proportions $a[t,x]$ of the sum of period rates $A[t]$ by $a[t,x]=m[t,x]/A[t]$, and the *k*'th moment of the age schedule $a[t,x]$ as $N_k(t)=\Sigma_x x^k a(t,x)$. Nest, Taylor series approximations for $A[g+x]$ and $a[g+x,x]$ about $t=g$ result in the following expression for the CCF of cohort *g*:

(1') $$B[g]=\sum_{i=0}^{\infty}\left\{\frac{A^{(i)}[g]}{i!}\sum_{j=i}^{\infty}\frac{N_j^{(j-i)}[g]}{(j-i)!}\right\}.$$

Assuming constant period quantum and a linear change in the period mean age results in

(2') $$B[g]=B=A\left(1+N_1^{'}\right).$$

Similarly, we find for constant period tempo and a linear change in period quantum that

(3') $$B[g]=A[g+N_1].$$

When both quantum and tempo are linear, the result is

(4a') $$B[g]=A[g](1+N_1^{'})+A^{'}.N_1[g]\text{, or equivalently}$$

(4b') $$B[g]=A[\tau]+N_1^{'}A[g]\text{, with } \tau=\tau[g]=g+N_1[g].$$

Expression (2') tells us, as expected, that the CCF is lower than the TFR by a factor 10 per cent in case the period mean age falls by one year per decade, and the TFR is constant. Comparison of expressions (2) and (2') shows that $\left(1-M_1^{'}\right)\left(1+N_1^{'}\right)=1$, or $M_1^{'}=N_1^{'}/(1+N_1^{'})$. Thus under the assumptions stated and for a small change in the period mean age, the period and cohort mean age have the same slope. Using expressions (3) and (3') it is easily verified that period and cohort mean ages are equal, when period and cohort quantum develop linearly, and the mean ages are constant over time.

The counterpart of the general expression (5) is

(5') $$W_k[t]=\sum_{i=0}^{\infty}\frac{1}{i!}V_{k+i}^{(i)}[t],$$

which, after normalizing the moments, indeed simplifies to expression (1') for $k=0$.

Before we turn to the case of non-repeatable events, it should be noted that assumptions of a linear trend in quantum or tempo indicators are primarily used for mathematical convenience, since higher order derivatives in the relevant expressions vanish. When trends are non-linear, polynomials of increasing order may be used instead. While the mathematics still remain tractable, such polynomials cannot describe actual trends accurately over a long period, since they tend to result in unrealistically large positive or negative values in the long run. In practice, two solutions are followed. First, one may fit a low-degree polynomial over successive rather short intervals (Calot 1992) – in other words, the polynomial is said to hold only locally. A second solution is to use a bounded non-linear function, for instance a logistic or a periodic curve, see De Beer (1982) and Foster (1990).

3. **Expressions for non-repeatable events**

The expressions in Section 2 apply to age-specific fertility. They are based on the fact that when age-specific rates, either for a given calendar year or a given cohort, are summed over all ages, the sum reflects the quantum of the process. This is typically the case for repeatable events, such as childbearing irrespective of parity. The rates are additive, because a woman who gives birth to a child remains at the risk for a new birth (except for a short period immediately after delivery). In other words, the denominator of the rate is not affected by the event.

In contrast, we have *non-repeatable* events such as childbearing broken down by birth order, or first marriage, or emigration. Events of this type are usually characterized by means of occurrence-exposure rates (o-e rates, "taux de 1e catégorie")[1]. An o-e rate expresses the risk of the event in question, for instance births of a certain order relative to all women of the corresponding parity, or first marriages relative to the number of never-married persons. When the o-e rates for such an event are age-specific, the sum of the rates over all ages does not reflect the quantum of the process in question[2]. For instance, while the quantum of first birth usually takes on values of between 80 and 95 per cent (implying that 5-20 per cent of the women remain childless), the rate sum is typically between 1.5 and 2.5.

Nevertheless for some non-repeatable events the translational distortion *can* be analysed along the lines sketched above, because the rate sum for these events, after an appropriate transformation, can be interpreted as the quantum of the process in question. As an example, consider the case of births of order one. Write the age-specific occurrence-exposure rate as $m[t,x]$, as before, and the sum over all fertile ages for cohort g as $B(g)$. Then a traditional life table calculation shows that the cohort quantum of the process, expressed as the proportion of women in that cohort who at the end of the reproductive period ever had experienced a first birth, can be written as[3]

$$Q_c[g] = 1\text{-}\exp(-\Sigma\, m[g+x,x]). \qquad (8)$$

[1] Sometimes, incidence rates (also called frequencies, or "taux de 2e catégorie") are used. Incidence rates are additive, but they exaggerate period distortions. See Section 4 below.
[2] In certain cases, the rate sum is a reasonable approximation for the quantum, see below.
[3] Expression (8) assumes piecewise constant intensities.

A similar expression holds for the period quantum of the process. Hence the translation expressions given in the previous section can be applied to non-repeatable events as well, provided the exponential transformation of the predicted rate sum is performed. For instance, in case the period rate sum of o-e rates for first births is constant, while the first moment follows a straight line, expression (2') may be used to predict the cohort rate sum. Next, expression (8) predicts the cohort proportion of women who ever gave birth to a first child, see for instance Keilman (1994), Keilman and Van Imhoff (1995), and - for time-continuos expressions - Calot (1992). Empirical illustrations will be given in Section 5. A number of points should be noted.

- Because of the arithmetics of the life table, the mean age of the process does not coincide with the first moment of the age schedule of o-e rates (Keilman 1994, 343). Translation formulas for the mean age or other tempo indicators are not known of - only quantum expressions have been derived for non-repeatable events. However, it will often be reasonable to assume that the slope in the mean age may be approximated by the slope of the first moment.
- Although the procedure sketched in this Section may be applied to such non-repeatable events as birth of the first child, first marriage, emigration, and many others, some events can *not* be analysed this way. Examples are births of order two or higher by age of the mother, and remarriage of divorced persons by age. The risk population of such processes may not only decrease, but increase as well (for instance caused by births of the previous order, or divorce). Such an increase is impossible in the case of first births or first marriage. As a consequence, the multistate life table that traces the fertility or nuptiality history of the real or synthetic cohort over its life course results in intractable matrix expressions for the quantum of these events. Another example is age-specific mortality. Since everyone dies, the quantum for mortality is 100 per cent, and the interest is solely in the tempo aspects. As noted in the previous point, expressions for tempo indicators are unknown.
- For low-intensity processes, the rate sum in expression (8) is small. In such cases, the rate sum itself approximates the quantum reasonably well, and all the expressions derived in Section (2) apply. Rate sums up to 0.2 are up to 10 per cent higher than the corresponding quantum values. Examples of low-intensity processes are long distance migration, outmigration from large areas, and divorce in Mediterranean countries (broken down by marriage duration).

4. **The Bongaarts/Feeney method for tempo adjustment of period fertility**

Bongaarts and Feeney (1998) have proposed a method which corrects age- and parity-specific period fertility for distortions caused by tempo changes, see also Bongaarts (1999). The purpose of the method is to obtain a tempo-free TFR, i.e. a TFR that would have been observed in year t if the age pattern of fertility had been the same as that in year t-1. Starting point is the birth order-specific TFR for year t defined as $\mathrm{TFR}_p[t]=\Sigma f_p[t,x]$, where the fertility rate $f_p[t,x]$ expresses the number of births of order p by mothers aged x in year t, relative to the number of women aged x irrespective of parity. Assuming a constant shape of the age schedule of fertility (i.e. women of all ages defer or advance their births to the same extent), an adjusted TFR is computed as $\mathrm{TFR}_p^{'}[t]= \mathrm{TFR}_p[t]/(1-\Delta M_{1,p}[t])$, where $\Delta M_{1,p}[t]$ is the annual change in the mean age at childbearing for parity p. Summing the results for different birth orders gives the overall tempo-free total fertility $\mathrm{TFR}^{'}[t]= \Sigma\mathrm{TFR}_p^{'}[t]$.

Note the differences with Ryder's approach described in Section 2. First, Ryder did not include birth order. Second, he sees the tempo distortion in period fertility as caused by changes in cohort tempo. In contrast, Bongaarts and Feeney (B&F) assume that all changes are period driven. They do not attempt to predict cohort fertility, since they assume that period-by-period changes are independent of age and cohort. In spite of these differences, for small changes in the mean age, the B&F approach (given birth order) gives the same result as Ryder's expression (2'), although the interpretations of the two results are entirely different.

The B&F adjustment procedure is attractive as it is based on period data only. But it has two major weaknesses (Van Imhoff and Keilman 1999; Kohler and Philipov 1999; Lesthaeghe and Willems 1999). First, the method is based on fertility rates unsuitable for the purpose of tempo adjustment. The rates used by B&F (known as incidence rates, frequencies, or "taux de 2e catégorie) express the number of births of order *p* by mothers aged *x* in year *t*, relative to the number of women aged *x irrespective of parity*. The use of such rates in a period perspective introduces *extra* tempo distortions, compared to o-e rates. When age-specific incidence rates for first births (for example) are summed for a given year, one erroneously assumes that the share of childless women at the end of one age interval is equal to that share at the start of the next interval. This is not necessarily the case, since the age intervals refer to different cohorts. The stronger the tempo changes between cohorts, the more the shares for subsequent ages differ, and the effect of tempo distortions is exaggerated. Quantum measures based on occurrence-exposure rates of the type used in Section 3 do not display this kind of bias. Second, the constant shape assumption underlying the B&F method is not supported by the data for European countries: the age schedule is not only shifted towards higher or lower ages, but its shape changes as well. Hence period-by-period changes are not independent of age and cohort, contrary to what B&F assume.[4] Thus period changes *are* dependent on cohort, and a pure "period quantum" is an untenable concept: cohort data are necessary to understand period effects fully (just as cohort effects cannot be fully understood without studying periods).

5. **Numerical illustrations**

5.1 Childlessness in the Netherlands

Dutch population forecasts from the mid-1990s assumed a percentage childless among women born in the 1970s and 1980s equal to 25 per cent (De Beer 1997). Indeed, the period level of first births suggested a proportion women with at least one child equal to 75 per cent, see Figures 1a and 1b. The fall in the proportion mothers between cohort 1945 and cohort 1955 is real, but the further decline for later cohorts depends on the extrapolation method used, compare the two lines "extrapolated with 1992 rates" and "censored after 1992" in Figure 1b. The period sum of first birth rates (ages 15-39) in Figure 1a fluctuated around a level of 1.4 since 1981 (with a slight tendency to increase), which led to a more or less constant proportion mothers from that year. However, the period first moment rose sharply during the years 1975-1992, with a slope equal to 0.14 years per year. Dutch women postponed the birth of their first child, and

[4] Kohler and Philipov (1999) extend the B&F-approach to allow for a changing variance of the fertility schedule.

this depressed the period quantum. Although the forecasters acknowledged this postponement effect, it was stronger than they believed in the mid-1990s. Indeed, the forecast published in 1999 assumes lower childlessness than previously: 20 per cent, instead of 25 per cent (De Beer 1999). Translation theory could have predicted this in 1993 already. Assuming a constant period rate sum from 1981 onwards, and a linear first moment, expression (2') predicts the cohort rate sum for the cohorts born in 1981 and later as 1.4*(1+0.14)=1.60, which implies a cohort quantum equal to 80 per cent (expression (8)), and thus 20 per cent childlessness.

5.2 Divorce in Norway

Figure 2a shows period proportions divorced in Norway. The underlying data are occurrence-exposure rates for divorce, broken down by marriage cohort and marriage duration (Mamelund et al., 1997). The year of divorce equals marriage year plus marriage duration. For each year, divorce rates were summed across marriage durations (up to 60 years in the original data), and next expression (8) resulted in period proportions divorced. The period proportion for year t is interpreted as the proportion of a fictitious marriage cohort that has experienced divorce by duration 60, given the divorce rates for the year t. Until the beginning of the 1970s, proportions divorced were very low in Norway, and the rate sum in Figure 2a is only slightly higher than the proportion, as expected.

The first moment fluctuated between 13 and 15 years since 1950, while the rate sum was more or less linear between 1970 and 1993. When we assume a constant first moment equal to 14 years, and a linear rate sum, expressions (2') and (8) predict the proportion divorced for marriage cohort g as the period proportion in year g+14. Figure 2b shows that the fit is remarkably good. Thus in spite of the fact that no actual Norwegian marriage cohorts have ever had a proportion divorced which exceeds 25 per cent, it is not unlikely that couples married in the last half of the 1960s will be the first ones to experience such a high share.

6. **Conclusion**

Translation theory provides expressions for the relationships between period and cohort quantum and tempo. The expressions can be applied to age-specific fertility, spanning several years and birth cohorts. With a slight modification, they can also be used to study other events, such as first marriage by age, divorce by marriage duration, emigration by age, or the birth of the first child by mother's age.

One of two perspectives can be adopted. First, the interest may be in period trends, and in tracing the effects of changing cohort behaviour on those period trends. Applied to the case of age-specific fertility, the formulae show that when the Completed Cohort Fertility (CCF) is constant, a fall in the cohort mean age at birth results in an inflated period Total Fertility Rate (TFR), because women accelerate childbearing and births are "squeezed" into shorter periods. The TFR is pushed upwards even stronger when the CCF rises, in addition to the decrease in the mean age. In contrast, the TFR falls when women postpone childbearing, i.e. when the cohort mean age rises, together with a constant or falling CCF. When the CCF and the cohort mean age move in the same direction, there are two opposite forces, and it is an empirical question whether the TFR is inflated or deflated. The second perspective is to take observed period trends as given, and to use translation theory to infer cohort developments. For example, the theory predicts, as expected, that the CCF is lower than the TFR by a factor 10 per cent in case the period mean age falls by one year per decade, and the TFR is constant.

From a mathematical point of view the two perspectives are symmetric. For age-specific fertility, however, there are strong empirical differences (and probably for other phenomena as well). The period TFR shows much larger annual fluctuations than the cohort CCF does. Thus it is relatively easy to predict the CCF on the basis of period quantum and tempo indicators. With "relatively easy" we mean here that the model is more parsimonious than one that explains the TFR on the basis of cohort indicators. For instance, Calot (1992) analyses age-specific fertility in France for the years 1900-1980 and cohorts 1870-1950. He finds that period moments that develop linearly with time describe the CCF accurately. But when he takes the opposite perspective and fits cohort moments to polynomials of increasing order when explaining the TFR, the accuracy he obtains is much less, even with fourth-degree polynomials (and the accuracy does not improve for higher order polynomials). Similarly, Foster (1990) concludes that period-based models provide a more parsimonious description of the observed patterns of age-specific fertility in eight countries in Europe and Northern America than cohort-based models do.

The primary use of translation theory is to improve the formal demographic analysis of historical developments. Consider, for instance, the baby boom in Western countries, i.e. an increase in the TFR starting in the 1930s, and next a plateau in the 1950s and 1960s. Translation theory explains this trend by the fact that cohorts born in the first three or four decades of this century got increasingly larger numbers of children, while, at the same time, they accelerated childbearing. When quantum and tempo indicators develop according to a straight line over several decades, the resulting translation expressions are simple. This is seldom the case, in particular for period indicators. This makes translation theory less useful for predicting the behaviour of cohorts, which the theory sees as a function of trends in period indicators.

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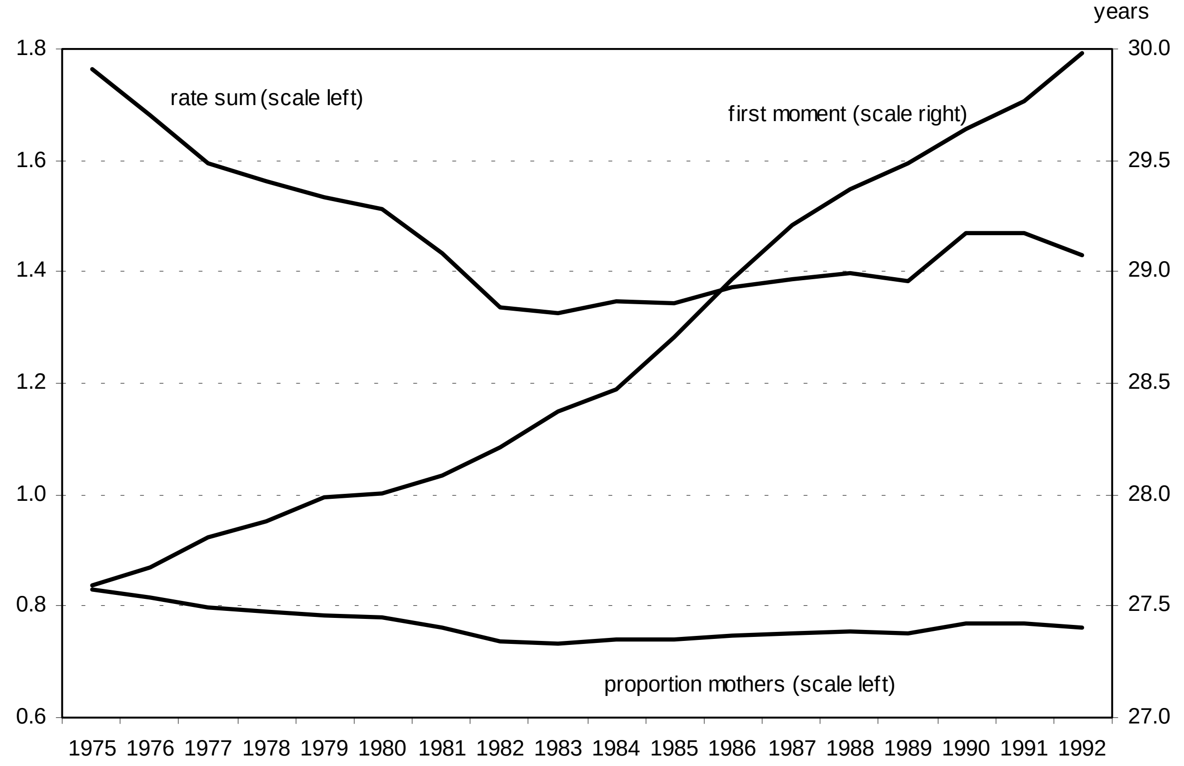
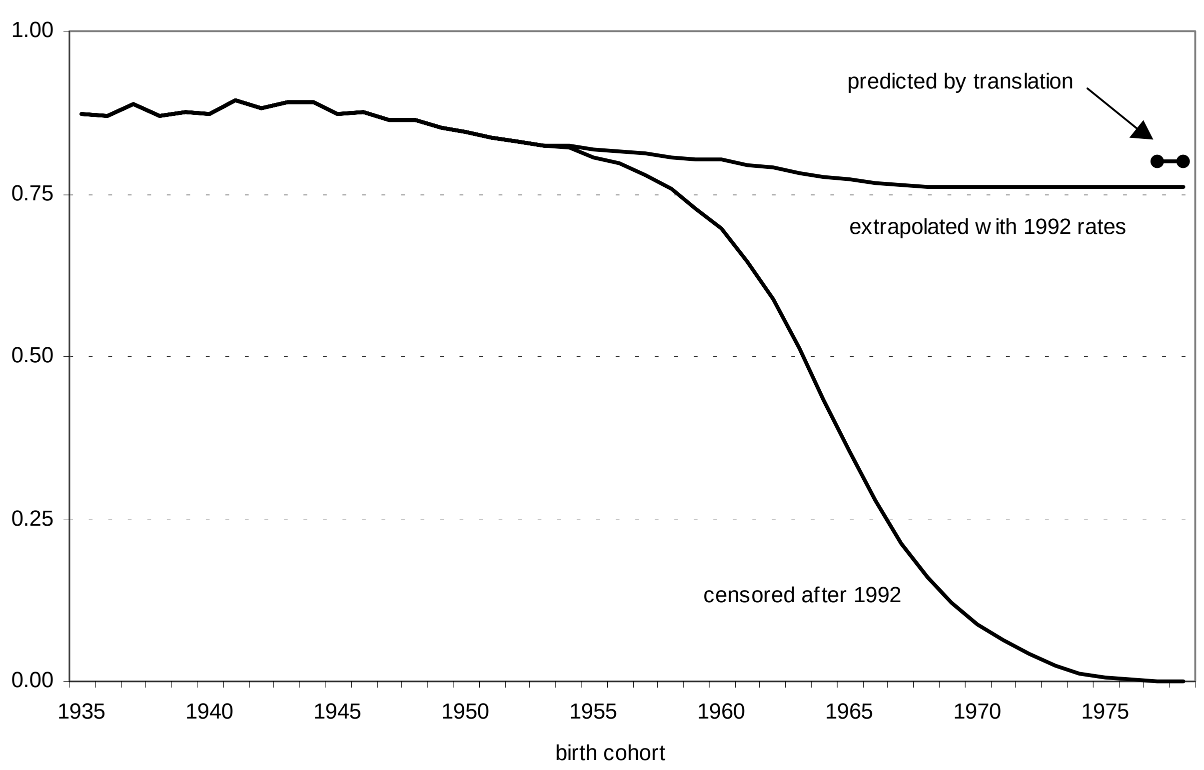
Figure 1a. Period quantum and tempo, first births, the Netherlands



Source: Bosveld (1996).

Figure 1b. Percentage mothers by cohort, the Netherlands



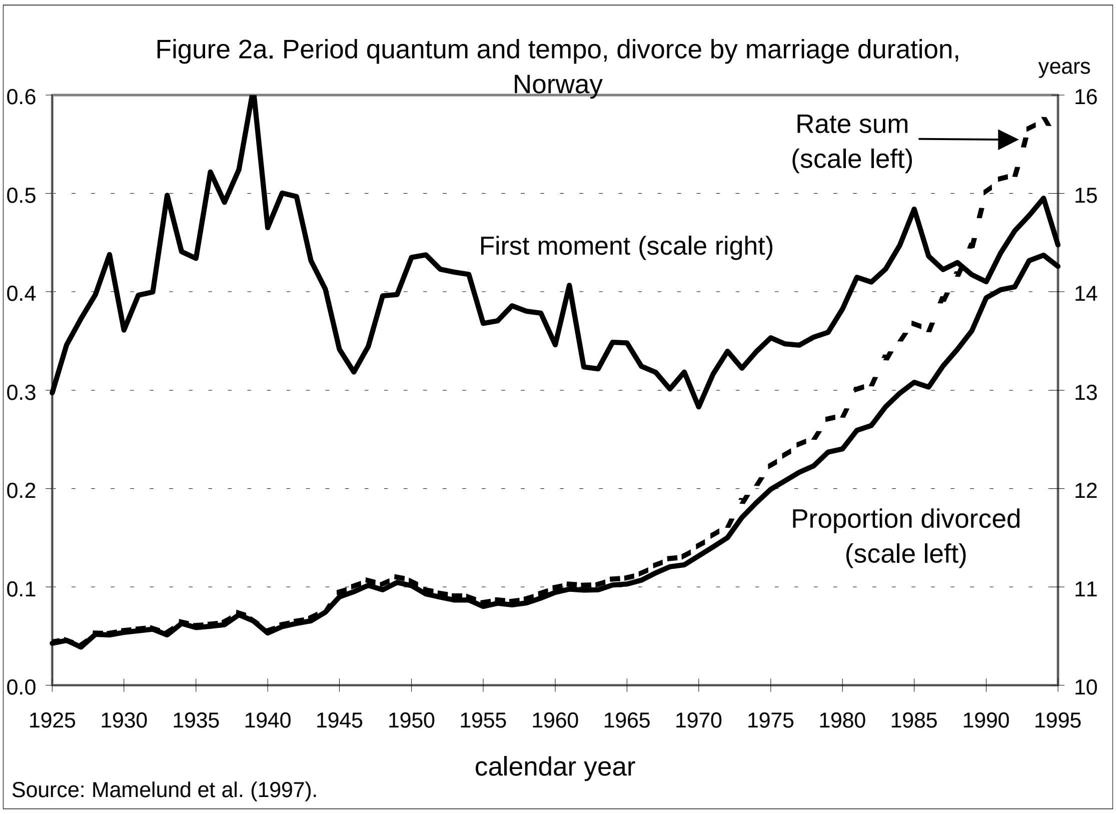


Figure 2a. Period quantum and tempo, divorce by marriage duration, Norway

Source: Mamelund et al. (1997).

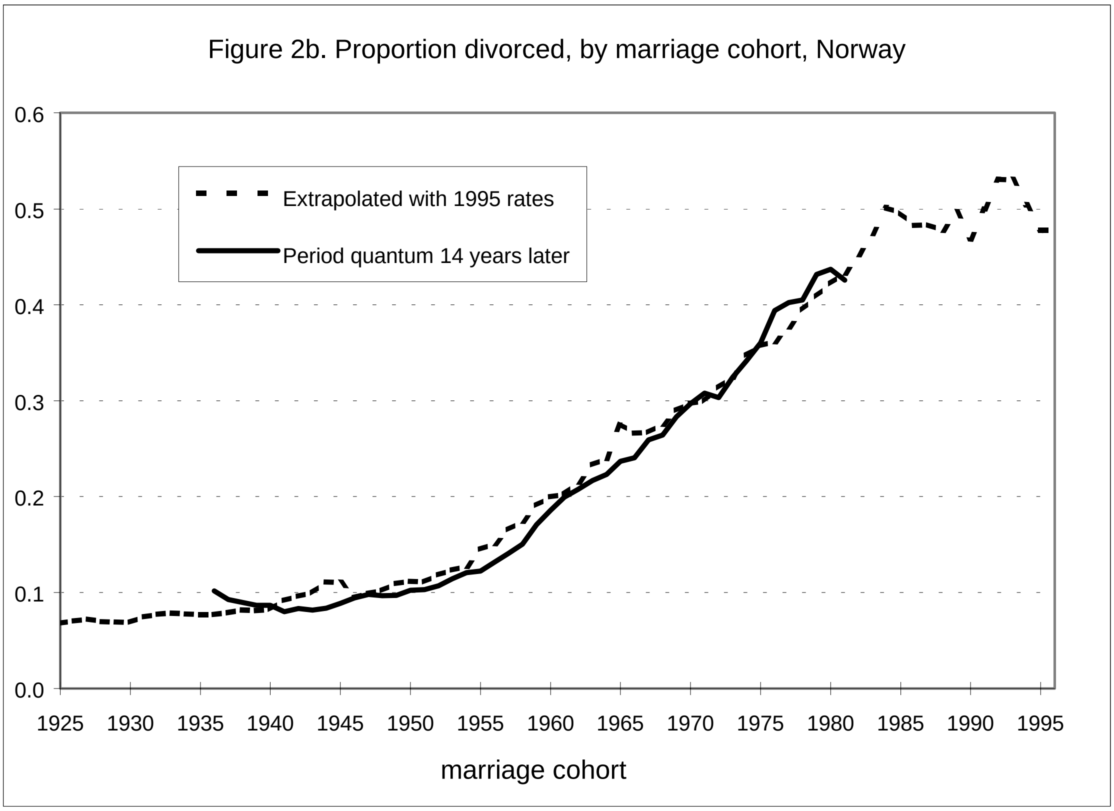


Figure 2b. Proportion divorced, by marriage cohort, Norway