MEMORANDUM

No 36/99

Statistical Discrimination and the Returns to Human Capital and Credentials

By
Christian Brinch

Department of Economics
University of Oslo
List of the last 10 Memoranda:

<table>
<thead>
<tr>
<th>No</th>
<th>Title</th>
</tr>
</thead>
<tbody>
<tr>
<td>34</td>
<td>By Ove Wolfgang: Reflections on Abatement Modelling. 30 p.</td>
</tr>
<tr>
<td>33</td>
<td>By Jon Strand: Effects of Progressive Taxes under Decentralized Bargaining and Heterogeneous Labor. 29 p.</td>
</tr>
<tr>
<td>32</td>
<td>By Tor Jakob Klette: Efficiency in the provision of municipal nursing – and home care services: The Norwegian experience. 28 p.</td>
</tr>
<tr>
<td>31</td>
<td>By Geir B. Asheim: Proper Consistency. 21 p.</td>
</tr>
<tr>
<td>29</td>
<td>By Jon Strand: Labor unions versus individualized bargaining with heterogeneous labor. 36 p.</td>
</tr>
<tr>
<td>28</td>
<td>By Jon Vislie: The role of foreign ownership in domestic environmental regulation under Asymmetric. 19 p.</td>
</tr>
</tbody>
</table>

A complete list of this memo-series is available in a PDF® format at:
http://www..sv.uio.no/sosoek/memo/
Statistical Discrimination and the Returns to

Human Capital and Credentials

Christian Brinch

Department of Economics, University of Oslo

December 13, 1999

1The paper is part of the project Qualifications, education, and productivity at the Ragnar Frisch Centre for Economic Research, Oslo, financed by the Norwegian Research Council, under the programme Competence, Education and Value Creation, grant 120652/520. Thanks to Kåre Bævre, John Dagsvik, Pål Longva, Oddbjørn Raaum, Christian Riis and Steinar Strøm for valuable comments at different stages.

2P.O. Box 1095 Blindern, N - 0317 Oslo, Norway, telephone: (47) 22 85 51 27, fax: (47) 22 85 50 35, E-mail: c.n.brinch@econ.uio.no
Abstract

The theory of statistical discrimination predicts lower returns to investments in human capital prior to labor market entry for minority groups if such investments are not directly observable to future employers. Lower returns lead to lower optimum levels of human capital and lower average wages for minority groups. This explanation of a persistent wage gap is extended to a model where individual investments in education lead to both higher human capital and observable credentials. The predictions about the returns to human capital and average wages carry over, and new predictions about wage distributions and the returns to credentials are derived.

(JEL: J31, J71)
1 Introduction

How can different wages for equally productive workers persist over time in a competitive labor market? This question dates back at least to the work of Becker (1957), who explains different wages for equally productive workers as a consequence of “tastes” for discrimination among employers, coworkers and customers. However, as Arrow (1973) pointed out, the theory of discrimination due to tastes primarily explains labor force segregation and has difficulties explaining persistent wage differences.

Another explanatory scheme, the theory of statistical discrimination, started with Phelps (1972), Arrow (1973) and Spence (1973). The idea behind the theory of statistical discrimination is that information problems in the labor market lead to equilibria with different wages for equally productive workers, even though individual agents do not try to act discriminatorily. The approach of Phelps, which will be discussed here, was rather different from the approaches of Arrow and Spence, who explained discriminatory outcomes as one of several possible equilibria in game theoretic models of the labor market.1

In the original Phelps (1972) model of statistical discrimination, employers observe only an imperfect estimate of a worker’s productivity, a test score.

1For a more recent contribution following Arrow and Spence, see Coate and Loury (1993).
Such a test score need not be interpreted literally, but only as an instrument for modelling imperfect observation. Employers set the wage equal to the expected productivity of a worker, conditional on the test score. The expected productivity is a weighted average between the test score and average productivity within a group, defined on the basis of some easily observable characteristic. First, this means that if the average productivity is lower in a minority group than in the majority group, then minority group workers will earn expected wages that are lower than equally productive majority group workers. Second, and more important in the following, less reliable test scores for minority group workers give wages that are to a larger extent determined by the average productivity within a group than by individual productivity.

Aigner and Cain (1977) pointed out that although the original model of statistical discrimination may explain why workers from different groups with otherwise equal observable characteristics receive different wages, it does not explain systematic discrimination against some groups. Workers from one group on average receive wages equal to the average productivity in the group. The authors invoke the assumption of risk averse employers, which together with less reliable test scores for minority group workers explain differences in average wages. They admit, however, that this does not seem a plausible explanation of large persistent wage differences. A more reasonable
explanation of how less reliable test scores for minority group workers may lead to wage differences, due to Rotschild and Stiglitz (1982), is that the productivity of a worker depends on job assignments, and that this will be more efficient if the employer has a more reliable estimate of potential worker productivity.

Lundberg and Startz (1983) showed that less reliable test scores lead to lower returns to human capital investments prior to labor market entry. Workers from minority groups, assumed to have less reliable test scores, will choose lower investments in human capital and receive lower wages on average. However, this theory does not take into account that human capital investments prior to labor market entry may to a large extent be observable through credentials from educational institutions. It is therefore not clear what the predictions of this theory would be with respect to education, certainly an important part of human capital investments prior to labor market entry.

Here I extend the theory of Lundberg and Startz (1983) to take into account credentials from the educational system. The results of investments in education are decomposed into two separate products, human capital and credentials. Credentials are the aspects of an education that are easily observable for future prospective employers, like the number of completed school years and the grades achieved in school. Credentials do not by themselves
lead to higher worker productivity. Human capital is the aspect of education that makes a worker more productive. The human capital of a worker is not directly observable to future prospective employers, but is partially observed through a test score, an unbiased estimate of human capital. Workers are heterogeneous in their ability to produce human capital and credentials, and they can choose different combinations of these outputs. Employers set wages equal to the expected human capital of workers, conditional on test scores and credentials.

In the next section, I specify a simple model of individual investments in human capital and credentials prior to labor market entry. This is a hybrid of the human capital and sorting models of education. In section three, the predictions of this model with respect to statistical discrimination, interpreted as minority group workers having a less reliable test score, is examined in some detail. Section four concludes with a summary of results and a short discussion of the potential empirical applicability of the model.

2 Theoretical model

The outline of the model is as follows: Prior to entering the labor market, workers decide how much to invest in human capital and in credentials. The objective is to maximize expected wages net of investment costs. When
workers enter the labor market, employers observe the credentials of workers and test scores, unbiased estimates of the human capital of workers. Workers are paid wages equal to what employers perceive as the expected human capital of the workers. The investment decisions of the workers depend on the wage schedules in test scores and credentials chosen by the employers. The employers’ wage schedules depend on the optimum worker decisions. As a solution concept for the model a rational expectations equilibrium is chosen. In this equilibrium the investment decisions of workers and the wage schedules offered by employers are consistent.

The model is a hybrid of the human capital and sorting models of education. Human capital models of education emphasize the role of education in increasing the productivity of workers, while sorting models emphasize the role of education as a way of providing information about workers in a labor market with information problems.\textsuperscript{2} Sorting models easily lead to perfect sorting of individuals. With perfect sorting, credentials would enable employers to predict human capital perfectly, and no test score would be necessary. In this model, independent variation in individual ability to produce credentials ensures that the sorting is not perfect. Credentials carry information about the human capital of a worker, but do not enable firms to

\\textsuperscript{2}Sorting models is a common term for signalling and screening models of education. For a discussion of human capital and sorting, see Weiss (1995). For another example of a hybrid of such models, see Weiss (1983).
predict human capital perfectly.

2.1 Workers’ investments in human capital and credentials

First consider the workers’ investment decisions. Prior to labor market entry, individual workers invest in human capital, $h$, and credentials, $s$. Both human capital and credentials are continuous variables. Workers are heterogeneous in their capacity to produce human capital and credentials. For each worker two random variables, $x$ and $y$, are drawn. These variables are interpreted as initial endowments of ability, where $x$ is relevant to the production of human capital and credentials, and $y$ is relevant to the production of credentials only. Let $x$ and $y$ have independent normal distributions, both with expectation 0 and with variances 1 and $\sigma_y^2$.\(^3\)

Assume a quadratic cost function

$$c(h, s) = \frac{1}{2} a (h - x)^2 + \frac{1}{2} b (s - y - h)^2,$$  \hspace{1cm} (1)

with $a, b > 0$. This particular cost function can be considered a generalization of the human capital cost function in Lundberg and Startz (1983). Human

\(^3\)The assumption of independence for $x$ and $y$ is not crucial, what is crucial is that these variables do not have a large negative covariance, see appendix A. Setting the variance of $x$ to one is an innocent normalization.
capital is measured as the value of the productive capacity of the worker in any future job. The measurement unit of credentials is arbitrary. Because of this arbitrariness, the cost function above may be derived from more general quadratic cost functions, leaving out irrelevant terms. In the function above the measurement unit of credentials is normalized to a scale such that one unit of human capital gives exactly one free unit of credentials.

Workers maximize expected wages net of production costs in education. Conjecture that workers are faced with a schedule of expected wages, $w^e$ that is linear in $h$ and $s$,

$$w^e(h, s) = A + Bh + Cs,$$

(2)

where $A, B$ and $C$ are constants to be determined in equilibrium. The first order conditions of the solution to this problem are given by

$$B = a(h - x) - b(s - y - h)$$

(3)

and

$$C = b(s - y - h).$$

(4)

---

4I show that there is a unique equilibrium with a linear schedule of expected wages in human capital and credentials. I do not show that there are no other equilibria.
The first order conditions trivially imply the global maximum as the expected net wage function is strictly concave.

Convenient solutions to these equations are given by

\[ h = x + \frac{B + C}{a} \]  \hspace{1cm} (5)

and

\[ s = h + \frac{C}{b} + y. \]  \hspace{1cm} (6)

The investments in human capital are increasing in the returns to human capital, \( B \), and in the returns to credentials, \( C \), as investments in human capital lead to increases in both. Denote the term \( C/b \), to be determined in equilibrium, as “excess investments in credentials.” Excess investments in credentials are increasing in the returns to credentials. Total investments in credentials are increasing in both the returns to human capital and the returns to credentials.

### 2.2 Employers’ wage schedules

When hiring a worker, an employer observes two measures of the worker’s human capital, the worker’s level of credentials and in addition a test score,
The test score is an unbiased estimate of the worker’s human capital,

\[ t = h + \varepsilon, \quad (7) \]

where \( \varepsilon \) is a random variable drawn from a normal distribution with expectation 0 and variance \( \sigma_{\varepsilon}^2 \), independent of the distributions of \( x \) and \( y \). In the following, I will refer to \( \sigma_{\varepsilon}^{-2} \) and \( \sigma_{y}^{-2} \) as the reliability of the test score and the reliability of credentials, as estimates of a worker’s human capital. The employer pays a wage equal to the expected human capital, conditional on credentials and test score. Equations (5), (6) and (7) now imply that employers will regard credentials, human capital and test scores as trivariate normal distributed variables. Denote the average human capital, derived from (5) as

\[ \mu_h = \frac{B + C}{a}. \quad (8) \]

From the formula for conditional expectations in a trivariate normal distribution, the expected human capital, conditional on test score and credentials, is now given by

\[
\begin{align*}
\text{w}(t, s) &= E \left( h | h + \varepsilon = t, h + \frac{C}{b} + y = s \right) \\
 &= \mu_h + \sigma_{\varepsilon}^{-2} t + \sigma_{y}^{-2} \left( s - \frac{C}{b} \right) \\
 &= \frac{\mu_h + \sigma_{\varepsilon}^{-2} t + \sigma_{y}^{-2} \left( s - \frac{C}{b} \right)}{1 + \sigma_{\varepsilon}^{-2} + \sigma_{y}^{-2}}.
\end{align*}
\]
The expression has a simple interpretation. The employers pay workers a weighted average of population average of human capital, actual test score and credentials, where credentials are measured as a difference from excess investments in credentials. More reliable test scores lead to higher weight for the test score and more reliable credentials lead to higher weight for the credentials. When one indicator of human capital becomes more reliable, the weight of the other indicator and the weight of the population average decrease.

The wage schedule (9) implies that the average wage for the whole population will be equal to average human capital, \( \mu_h \), as this is the mean of all the components of the weighted average.

Since credentials are measured as a difference from excess investment in credentials and average wages are independent of the level of excess investment in credentials, all workers would be better off if there were no excess investments in credentials. Excess investments in credentials is an extra cost incurred on each worker due to an externality. When a worker increases his expected wage by spending resources on investments in credentials, this increase in expected wage is at the expense of other workers’ expected wages.
2.3 Equilibrium

The expected wage schedule for workers is derived from (9) by noting that to the worker, the expected test score is \( h \),

\[
w^e(h, s) = \frac{\mu_h + \sigma_h^{-2}h + \sigma_y^{-2}(s - \frac{C}{B})}{1 + \sigma_h^{-2} + \sigma_y^{-2}}. \tag{10}
\]

The expected wage schedule is linear in \( h \) and \( s \), as conjectured in equation (2). The solution is the unique linear rational expectations equilibrium, and the equilibrium values \( B \), \( C \) and \( A \) follow as

\[
B = \frac{\sigma_h^{-2}}{1 + \sigma_h^{-2} + \sigma_y^{-2}}, \tag{11}
\]

\[
C = \frac{\sigma_y^{-2}}{1 + \sigma_h^{-2} + \sigma_y^{-2}}, \tag{12}
\]

\[
A = \frac{\mu_h - \frac{\sigma_y^{-2}}{1 + \sigma_h^{-2} + \sigma_y^{-2}}}{1 + \sigma_h^{-2} + \sigma_y^{-2}}. \tag{13}
\]

In equilibrium, the returns to human capital, \( B \), are increasing in the reliability of the test score and decreasing in the reliability of credentials. The returns to credentials, \( C \), are increasing in the reliability of credentials and
decreasing in the reliability of the test score.

Substituting the equilibrium values of $B$ and $C$, into (5) and (6), the optimum level of human capital is

$$h = x + \frac{1}{a} \frac{\sigma^{-2}_\varepsilon + \sigma^{-2}_y}{1 + \sigma^{-2}_\varepsilon + \sigma^{-2}_y},$$  \hspace{1cm} (14)$$

and the optimum level of credentials is

$$s = h + \frac{1}{b} \frac{\sigma^{-2}_y}{1 + \sigma^{-2}_\varepsilon + \sigma^{-2}_y} + y.$$ \hspace{1cm} (15)$$

Human capital investments are increasing in the reliability of both the test score and credentials. More reliable test scores and credentials provides means for employers to observe the increased human capital investments. Human capital investments are increasing in the reliability of credentials even though the returns to human capital are decreasing in the reliability of credentials. The reason for this is the complementarities in the production of human capital and credentials, modelled as one free unit of credentials for every unit of human capital.

Excess investments in credentials (the middle term in (15)) are decreasing in the reliability of the test score and increasing in the reliability of credentials. The total investments in credentials are increasing in the reliability
of credentials, as more reliable credentials increase both the human capital investments and the excess investments in credentials. The effect of more reliable test scores on the total investments in credentials is ambiguous, as more reliable test scores increase human capital investments but decrease excess investments in credentials.

The model developed here is a hybrid of the human capital and sorting models of education. The human capital aspect of the model is the ability of workers to invest in human capital to increase their productive value. The sorting aspect of the model is the ability of workers to invest in credentials to signal to employers that their human capital is high. The sorting aspect of the model involves an inefficiency, as workers overinvest in credentials. All workers would earn the same if they had agreed on setting excess investments in credentials to zero, but the individual worker faces positive returns to excess investments in credentials.

The model have some interesting limiting cases. If both the test score and the credentials are completely unreliable, that is, if both $\sigma^2_y$ and $\sigma^2_\varepsilon$ approach infinity, all workers are paid the average productivity, and no workers invest in either human capital or credentials. If only credentials are reliable ($\sigma^2_\varepsilon$ approaches infinity), workers face wage schedules that are only increasing in credentials. Still, some human capital investments occur as instrumental to the investments in credentials. Wages are higher than in the case with
completely unreliable credentials, though the average wage of the workers net of investment costs may well be lower. If only the test score is reliable ($\sigma^2_y$ approaches infinity), there are no excess investments in credentials, and the model collapses to the model of Lundberg and Startz (1983), a human capital model of investments in education with imperfectly observed human capital investments.

If there is no independent individual variation in the ability to produce credentials, ($\sigma^2_y$ approaches zero), the credentials are completely reliable and enable employers to predict human capital perfectly, the model exhibits perfect sorting. There are excess investments in credentials and only investments in human capital that are instrumental to investments in credentials.

If the test score is completely reliable ($\sigma^2_z$ approaches zero), human capital is perfectly observed, workers are paid according to their productivity and there are no excess investments in credentials. The model is then a standard human capital variant.

3 Statistical Discrimination

In this section, I will examine in some detail the predictions of the model, interpreted as a model of statistical discrimination. As is usual in the literature on statistical discrimination, I assume that employers are better at
assessing the productivity of workers from the majority group than they are at assessing the productivity of workers from minority groups. As group membership is easily observable, a separate wage schedule will exist for each group, each determined as in the model above. Statistical discrimination can then be analyzed as comparative statics with respect to the reliability of the test score.

I will first restate the basic predictions of the model with respect to statistical discrimination and then turn to prediction for wage distributions, both unconditional and conditional on the level of credentials.

3.1 Basic predictions

From (11) and (12), the returns to human capital are increasing in the reliability of the test score, and the returns to credentials are decreasing in the reliability of the test score. As minority groups are associated with less reliable test scores, workers from these groups face expected wage schedules that are steeper in credentials and flatter in human capital.

It is seen from (14) that the different expected wage schedules lead minority group workers to choose lower human capital investments. Further, it is seen from (15) that minority group workers will choose higher excess investments in credentials. Minority group workers are then worse off both
in the sense of receiving lower average wages and in the sense of spending more resources on excess investments in credentials.

The results of statistical discrimination on the optimum level of credentials is ambiguous. For any given level of credentials, however, workers from minority groups will have lower average human capital and thus lower average wages, as they have higher excess investments in credentials.

### 3.2 Predictions about wage distributions

The model above leads to normal wage distributions. This is not meant to be taken literally, the assumption of normality is evoked for expository reasons. Still, the implications for the mean and variances of the wage distributions may carry over to more realistic wage distributions. For the study of the unconditional wage distributions, the following transformation of (9) will be useful,

\[
w = \mu_h + \frac{\left( \sigma_y^{-2} + \sigma_y^{-2} \right) x + \sigma_e^{-2} z + \sigma_y^{-2} y}{1 + \sigma_e^{-2} + \sigma_y^{-2}}. \tag{16}
\]

The expected wage, given only information on group membership is equal to the average level of human capital in the group, \( \mu_h \). This follows easily from
(14),

\[ E(w) = \frac{1}{a} \frac{\sigma_{\varepsilon}^{-2} + \sigma_{Y}^{-2}}{1 + \sigma_{\varepsilon}^{-2} + \sigma_{Y}^{-2}}, \tag{17} \]

which is decreasing in \( \sigma_{\varepsilon}^2 \). The variance of the wage is given by

\[ \text{Var}(w) = \frac{\sigma_{\varepsilon}^{-2} + \sigma_{Y}^{-2}}{1 + \sigma_{\varepsilon}^{-2} + \sigma_{Y}^{-2}}, \tag{18} \]

which is clearly increasing in the reliability of the test score. Thus, the model predicts that minority groups will have wage distributions with a lower mean and also a lower variance.

The majority and the minority groups do not necessarily choose exactly the same level of credentials on average. What matters to employers when setting wages is the level of credentials measured as a difference from the group specific population average. The wage distributions conditional on this corrected level of credentials is easy to derive as the corrected level of credentials can be expressed as

\[ s - C/b - \mu_h = x + y, \tag{19} \]
and the wage schedule (16) is equivalent to

\[ w = \mu_h + \frac{\sigma^2_x}{1 + \sigma^2_x + \sigma^2_y} (x + \varepsilon) + \frac{\sigma^2_y}{1 + \sigma^2_x + \sigma^2_y} (x + y). \] (20)

The expected wage, given the level of credentials measured from the group specific population average is

\[ E\left( w|s - \frac{C}{b} - \mu_h \right) = \mu_h + \frac{1}{1 + \sigma^2_y} \left( s - \frac{C}{b} - \mu_h \right). \] (21)

Thus, the average wage, given the level of credentials measured from group specific average, is not only higher for majority groups, but the difference in average wages is constant over different levels of credentials.

The variance of the wage, given the level of credentials as a difference from group specific average is

\[ Var\left( w|s - \frac{C}{b} - \mu_h \right) = \left( \frac{\sigma^2_x}{1 + \sigma^2_x + \sigma^2_y} \right) \left( \frac{1}{\sigma^2_y + 1} \right), \] (22)

which is clearly increasing in the reliability of the test score.

The results about wage distributions can be summarized in that minority groups have lower average wages and lower wage variance. This also holds for the wage distributions conditional on the level of credentials as a difference from the average level of credentials within a group.
4 Discussion and Conclusions

Lundberg and Startz (1983) showed how returns to human capital investments are reduced by statistical discrimination. However, their model did not take into account that credentials from the educational system can be an important signal to employers about the productivity of a worker. Their explanation have been extended here by allowing for production of credentials as well as unobservable human capital in education. The original result about the returns to human capital investments holds. Groups faced with “statistical discrimination” have lower returns to investments in unobservable human capital and greater returns to investments in credentials. This results in lower optimum levels of human capitals and lower average wages, even though human capital investments are instrumental to investments in credentials. In addition, minority workers will waste more resources on unproductive credentials in education. The explanation of different wages for workers with the same productive potential provided here seems more satisfactory than the original explanation by Lundberg and Startz (1983), as the human capital investments in the explanation considered here can more easily be interpreted as investments in education prior to labor market entry.

Turning to empirical matters, the explanation of wage differences between groups offered here might well be important, for example with respect to the
black-white wage gap in the US. According to Neal and Johnson (1996), an ability test measuring the skills and abilities of teenagers as they were prepared to leave high school is able to explain the full wage gap between young black and white women and much of the gap for men. Though they attribute this difference to intergenerational transfers of human capital, it is equally consistent with the explanation provided by the model developed here. Neal and Johnson aim to show that their results are not explicable by statistical discrimination of the type proposed by Lundberg and Startz (1983), as a regression of low wages on their test score does not indicate significant differences in returns to skill between groups. This result is equally inconsistent with the model in this paper. However, the log wage specification in Neal and Johnson means that equal returns to skills can be interpreted as higher monetary returns to skills for whites, as whites on average have higher wages. Only further empirical analyses may decide this issue. I hope the model provided here will contribute to clarifying such analyses, as it is easier to relate to education than earlier models of statistical discrimination and also provide more testable predictions.
A Appendix: Dependent ability endowments

In the main body of the paper, it is assumed that the initial ability endowments $x$ and $y$ are independently distributed. The purpose of this appendix is to show that this is not a crucial assumption.

First note that equations (5) and (6) imply that

$$corr(h, s) = \frac{1}{\sqrt{1 + \sigma_y^2}}$$

(23)

Thus, all positive correlations between human capital and credentials can be accommodated in the model in the main body of the paper through variations in $\sigma_y^2$.

Next, if $x$ and $y$ are dependent, though not perfectly correlated, normal variables, it is possible to write $y$ as $\sigma_{xy}x + w$, where $w$ is a random normal variable that is independent of $x$, and $\sigma_{xy}$ is the covariance between $x$ and $y$. From (5) and (6) we find that

$$s = \frac{B + C}{a} + \frac{C}{b} + (1 + \sigma_{xy})x + w.$$  

(24)

As long as this does not imply a negative correlation between human capital and credentials, this can easily be accommodated in the model in the main body of the paper by a proper choice of $\sigma_y^2$. 

21
From (24), noting that human capital is perfectly correlated with \( x \), credentials and human capital are positively correlated if and only if

\[
\sigma_{xy} > -1. \tag{25}
\]

Thus, a positive or a small negative covariance between \( x \) and \( y \) can be reproduced in the model in the main body of the paper through changes in parameters.

**References**


