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Reflections on Abatement Modelling

By
Ove Wolfgang

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P. O.Box 1095 Blindern
N-0317 OSLO Norway
Telephone: + 47 22855127
Fax: + 47 22855035
Internet: http://www.sv.uio.no/sosoek/
e-mail: econdep@econ.uio.no

In co-operation with
The Frisch Centre for Economic Research

Gaustadalleén 21
N-0371 OSLO Norway
Telephone: +47 22 95 88 20
Fax: +47 22 95 88 25
Internet: http://www.frisch.uio.no/
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Reflections on Abatement Modelling

Ove Wolfgang

Ragnar Frisch Centre for Economic Research, Gaustadalléen 21, 0349 Oslo, Norway
E-mail: ove.wolfgang@frisch.uio.no

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Abstract: In environmental economics it is common to add non-negative constraints on abatement, or equivalently to constrain emissions by some maximum unabated level. However, in this paper it is argued that these assumptions are both physically wrong and complicating. Also, more seriously, such constraints can rule out optimal solutions. For instance, negative NOx abatement can be optimal from the social planner's point of view since increased emissions give reduced concentrations of polluting ground level ozone in some cases. It is concluded that future contributions can benefit from dropping the constraints on abatement and maximum emissions. It is also shown that marginal abatement costs must have the same sign as abatement to avoid conceptual inconsistencies.

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1. Introduction

Abatement and emissions are two core concepts in environmental economics. Many analyses deal with problems like how large abatement and emissions should be, and how abatement and emissions should be distributed among firms or countries. This paper deals with some methodological aspects of abatement modelling. In particular, the analysis focuses on the adequacy of the common assumptions of non-negative abatement and positive marginal abatement costs. It is argued that the assumption of non-negative abatement should be dropped, while the assumption of positive marginal abatement costs should be modified. There are several advantages with the modified assumptions. First, the modelling is more correct and some conceptual inconsistencies are avoided. Second, the analysis typically gets simpler. Third, optimal solutions can be ruled out in some cases unless assumptions are modified.

Purification, also called end of pipe cleaning, is the most obvious image of abatement. Abatement is done if the dirty smoke is purified. Also, the verb abate means to reduce. This might explain why several authors constrain abatement to be non-negative (e.g., Atkinson and Tietenberg 1991, Batterman and Amann 1991, Tietenberg 1985, Xepapadeas 1997) while others constrain the maximum emissions by some no-control reference level (e.g., Eyckmans 1997, Førsund 1994, Klaassen *et al.* 1994). However, in this paper it is shown that negative abatement is possible. The assumption that firms only can abate non-negatively is therefore wrong. However, this is not a decisive argument against the constraint on abatement since every model is wrong in some sense. The adequacy of the constraint must instead be discussed in relation to specific contexts.

Typically, only one emission type, which is assumed to be a public bad, is considered in the literature. Since at least parts of the damage from emissions are external to the emitting firms in such settings, firms emit too much from the society's point of view.

Therefore, it is optimal for the government to internalise the externality for example by taxing emission with appropriate Pigouvian taxes (Baumol and Oates 1988). Such taxes give firms incentives to reduce their emissions to the social optimal levels, and this implies positive abatement in firms. Clearly, non-negative constraints on abatement do not rule out optimal solutions in such cases, even though negative abatement is a theoretical possibility. On the other hand it seems rather odd to add the constraints in the first place since they have no influence on optimisation. However, in this paper it is shown that the common assumption of positive marginal abatement costs is inconsistent with negative abatement opportunities. It is therefore possible that non-negative constraints on abatement are added *ad hoc* in some contributions to avoid conceptual inconsistencies. However, in this paper also shown that marginal abatement costs have the same sign as the abatement level. It is therefore unnecessary to constrain abatement if the abatement costs are properly defined. It is also argued that the analysis typically gets simpler if the constraints on abatement and maximum emissions are dropped. Taken together with the finding that negative abatement is possible, it is clear that many contributions within the typical context discussed here could benefit from slightly modified assumptions. The constraint on abatement should be dropped, and marginal abatement costs should have the same sign as abatement.

Most frequently the abatement is modelled explicitly. In these analyses abatement is assumed to be costly in monetary terms (e.g., Eyckmans 1997, Hoel 1991, Rubin 1996, Tietenberg 1985) or because resources are limited (e.g., Fisher and Peterson 1976, Kitabatake 1989, Plourde and Yeung 1989). Marginal abatement costs are assumed to be positive in the majority of these contributions. In other analyses emissions are considered to be inputs in production (e.g., Baumol and Oates 1988, Cropper and Oates 1992, Welsch 1993, Zagonari 1998). This modelling approach captures the idea that resources used in abatement activity must be taken away from ordinary production. The abatement activity is therefore modelled implicitly in these analyses, even if the abatement activity itself is swept under the carpet (Førsund 1998). The assumption that emissions have positive marginal productivity is therefore similar to the assumption that marginal abatement costs are positive. In this paper it is therefore argued that emissions can have positive marginal productivity only in a limited interval.

In contexts slightly different than the typical discussed so far, there are even stronger arguments to allow negative abatement. In particular, negative abatement can be optimal for both firms and society in some cases. Consider first the case where emissions are a public good rather than a public bad. The idea that emissions can be a public good is contra-intuitive since emissions typically are assumed to produce negative externalities. However, there are examples where emissions are not entirely public bads. For instance, nitrogen oxide emissions are important precursors to the formation of polluting ground level ozone. Still, if there are large initial concentrations of nitrogen oxides, ozone levels can be reduced if emissions are increased (Amann *et al.* 1998; Repetto 1987). Therefore, depending on the initial situation and the environmental ambitious level, extra nitrogen oxide emissions can be both a public good and a public bad.

Figures 1a and 1b are copied from Amann *et al.* (1998). They illustrate how the estimated concentration of ground level ozone, with the EMEP model, typically is affected by proportional European reductions of nitrogen oxides and volatile organic compounds. The curves shows constant ozone concentrations estimates in areas with low (figure 1a) and high (figure 1b) initial nitrogen oxide concentrations. The estimated ozone concentration increases to the right in figure 1a and upward in figure 1b. From figure 1b we can see that the estimated concentration of ground level ozone is increased in some regions if the European nitrogen oxide emissions are reduced proportionally with 20% (factor 1 to factor 0.8). This strongly indicates that increased emissions can give reduced pollution in some cases.

(Figure 1 about here).

In this paper it is demonstrated that negative abatement is optimal from the society's point of view if emissions are a public good on the margin in the optimal solution. The optimal emission tax is shown to be negative (emission subsidy) in such cases, and this leads to negative abatement in firms. Non-negative constraints on abatement can therefore rule out optimal solutions in such cases. It is also shown that negative abatement can be optimal if the environmental problem is caused by several kinds of emissions, which all are public bads. The result is critical dependent on the assumption that some abatement technologies increase some types of emissions in the process of reducing others.

The concept of abatement is defined in section 2, and it is shown that abatement can be negative. From the definition it is also evident that a non-negative constraint on abatement is equivalent to a constraint on maximum emissions. The abatement costs are defined section 3. Given the assumptions of the model, abatement costs are proved to be non-negative, while positive marginal abatement costs are proved to be inconsistent with negative abatement opportunities. Also, the abatement cost function is derived in a formal setting. The analysis shows that marginal abatement costs have the same sign as abatement in the typical case where only one emission type is considered, while the optimal abatement levels have the same sign as the emission tax rate. Optimal taxes are derived in section 4. The analysis shows how the non-negative constraints complicate the analysis in a simple example. It is also shown that the constraints on abatement can rule out the optimal solution if the pollution function is non-monotonic. Interrelated abatement technologies are discussed in section 5, while section 6 concludes the analysis. Two methodological recommendations are offered. First, the standard assumption that firms only can abate non-negatively should be dropped. The assumption is incorrect, and, typically, it also complicates the analysis given that abatement costs are properly defined. In addition, optimal solutions can be ruled out in some cases. Second, if only one kind of emissions is taken into account, marginal abatement costs must have the same sign as abatement to avoid conceptual inconsistencies.

2. Abatement

The concepts of abatement and abatement costs are often defined rather vague in the literature. If abatement is defined, it is typically set equal to the difference between a reference level of emissions and the actual emission level. This reference level of emissions has many names in the literature. Some examples are: Business as usual emissions (Hagem and Westskog 1998), gross aggregate emissions (Falk and Mendelsohn 1993), initial emissions (Benford 1998), initial unrestricted emissions (Klaassen *et al.* 1994), laissez-faire emissions (Welsch 1993), primary emissions (Førsund and Nævdal 1998), reference level emissions (Eyckmans 1997), unabated emissions (Batterman and Amann 1991) and uncontrolled emissions (Tietenberg 1985). In this paper abatement is defined to be the difference between no-control emissions and any other emission level, and the concept of no-control emissions is defined to be the emission levels chosen by firms if no actions are taken by the government to control emissions. The symbols r_i, e_i and e_i^{nc} are firm *i*'s abatement, emissions and no-control emissions respectively.

$$r_i \equiv e_i^{nc} - e_i \tag{1}$$

Notice that no-control emissions are functions of all prices and other profit influencing factors. However, no-control emissions are not functions of governmental emission control variables since they are defined only in no-control.

It is possible to define abatement differently. For instance, Welsch (1993) defines the abatement level in a country to be the difference between the emission level chosen by the government in a Nash equilibrium, where they take emissions from other countries as given, and any other emission level. In cases where emissions give some damage also in the emitters' own country, this implies controls on domestic emissions even if the national abatement level is equal to zero.

All emission-changing actions are potential abatement strategies in (1). Some examples are purification (end-of-pipe cleaning), changed production level, input substitution, changed production process and changed output composition. Also, since the definition of abatement in (1) is given by a difference, abatement can in principle be both positive and negative. However, a non-negative constraint on abatement is added in many contributions:

$$r_i \equiv e_i^{nc} - e_i \geq 0 \tag{2}$$

The adequacy of this constraint is one of the main subjects in this paper. It is argued that the constraint is complicating and superfluous in some cases. Also, more seriously, it can rule out optimal solutions in other cases. However, first it must be established that negative abatement is possible from a technological point of view.

For simplicity we consider a competitive firm that pays emission taxes, and there are no other governmental controls on emissions. Only one emission type is considered, and the emission level is assumed to increase linearly with the level of production. However, emissions can be reduced by some non-negative end-of-pipe purification technologies. These two emission-changing actions are sufficient to demonstrate the points of this section. Both production and purification costs are assumed to be increasing and convex functions. Emissions are assumed to influence on individuals utility, for instance through negative health effects from pollution, but profits in a given firm is assumed to be independent of the emission levels chosen by other firms. The firm maximises profits, and it is assumed that both profits and emissions are strictly positive in the optimal solution.

The symbols in (3) are profits (π), product price (p), emission tax rate (t), emission intensity of production (b>0), production level (x), amount purified (a), production costs (s), purification costs (*d*), shadow price on emissions (λ) and shadow price on minimum purification (υ). The index *i* is dropped since only one firm is considered.

$$\begin{aligned} &\pi(p,t,b) = max_{x,a,e}\left\{px - s(x) - d(a) - te\right\} \\ &\quad subject\ to\ \ e = bx - a \quad (\lambda) \\ &\quad and \quad a \geq 0 \qquad (\upsilon) \end{aligned} \tag{3}$$

No-control emissions are given by optimal emissions when t=0. Faced with a zero tax rate, the firm adjusts production so that marginal production costs equals the output price, and it abstains from costly purification. The symbol x^{nc} is the no-control production level.

$$e^{nc} = bx^{nc} \quad \text{where} \quad x^{nc} = \left\{x{:}p = s'(x)\right\} \tag{4}$$

Using the definition of abatement in (1) and $e=bx-a$, the abatement level can be written as:

$$r = e^{nc} - e = b\left(x^{nc} - x\right) + a \tag{5}$$

The firm has two potential abatement technologies: Output level adjustments and purification. Is negative abatement possible from a technological point of view? The answer must be yes since abatement is negative for any production level larger than x^{nc} if no purification is done. Can negative abatement be optimal too? The first order conditions for the optimisation problem in (3) are given by (6) and (7) since $\lambda=t$.

$$p - s'(x) = tb \tag{6}$$

$$d'(a) = t + \upsilon \quad \text{where } a,\upsilon \geq 0 \text{ and } a\upsilon = 0 \tag{7}$$

From (6) it follows that the product price is larger than marginal production costs if $t>0$. Since production costs are convex, and $p=s'(x^{nc})$, this implies $x^{nc}>x(t>0)$. The firm produces less when emissions are taxed since the production cause emissions. Also, production is decreased for any increase in the tax rate. Optimal purification is given by (7). If $t \leq d'(0)$, then $\upsilon = d'(0) - t \geq 0$ and a=0. However, if $t > d'(0)$ then υ=0 and purification is carried out to the point where marginal purification costs equals the emission tax rate $d'(a > 0) = t$

Consequently, purification is increased if the tax rate is increased and larger than $d'(0)$ initially.

The abatement level is given by (5). A positive tax implies positive abatement since the production level is reduced from no-control and purification is non-negative. Also, a larger emission tax implies larger abatement.

Now we consider the case where t<0. Later in this paper it is shown why it can be optimal for the government to tax emissions negatively in some cases. However, these considerations are less important in this section since the firm is assumed to react optimally to any *t*. A negative *t* can be interpreted as an emission subsidy.

From (6) it follows that the product price is less than marginal production costs if $t<0$. Since production costs are convex, and $p=s'(x^{nc})$, this implies $x^{nc}<x(t<0)$. The firm produces more than no-control levels when emissions are subsidised. Also, increased emission subsidy gives increased production levels. Since the left hand side in (7) is positive and $t<0$, we know that $\upsilon>0$. Consequently, from $a\upsilon=0$, it is not optimal to carry out any purification. The abatement level, given by (5), is therefore negative in the optimal solution when $t<0$. In total it is therefore shown that (a) optimal abatement has the same sign as the emission tax rate, and (b) the level of abatement increases monotonically in the emission is tax rate.

$$sign(r) = sign(t) \quad \text{and} \quad \frac{dr}{dt} > 0 \tag{8}$$

Since firms choose negative abatement if the government subsidise emissions, it is of interest to find out if the optimal governmental tax is negative in some cases. This is done in section 4. However, first we derive firms’ abatement cost functions. These functions are convenient to use when optimal tax policies are derived.

3. Abatement Costs

An important feature with the definition of abatement in (1) is that every emission-changing action is a potential abatement strategy. The abatement costs must be defined wide enough to reflect this too. A starting point for deriving the abatement costs can therefore be to consider the change in profits, for the optimally adjusted firm, caused by emission controls. However, it is also important to distinguish abatement costs from other costs of regulation. For instance, emission taxes and net buying of emission permit are costs of regulation, but not abatement cost. On the contrary, firms can reduce their abatement if they, for instance, buy more emission permits. The abatement costs must therefore be adjusted for emission tax expenses and for net expenses in emission permit markets. Firms can also have other costs of regulation, e.g. administrative costs, monitoring costs and lobbying expenses. However, such costs are not considered in this paper.

The productivity in a given firm can in some cases be increased if other firms reduce their emissions. The governmental controls on emissions can therefore have an indirect effect on the firm's profits if other firms are regulated too. This indirect effect on profits should also be distinguished from the abatement costs. The concept of abatement costs can therefore be defined to be firms' loss in profits caused by the governmental controls on emissions, minus emission tax expenses and net expenses in emission permit markets plus any positive externality on firms' profits caused by the emission controls. This definition of abatement costs is wide enough to reflect the costs of all emission changing actions, and it distinguishes abatement costs from other major costs and benefits from regulation.

Since the government is assumed to control emissions only through the emission tax rate and since profits in a given firm is assumed to be independent of the emission levels chosen by other firms, the abatement costs is given by profit loss minus emission tax expenses. The symbol c is a firm's abatement costs and π^{nc} is profits in absence governmental emission-controls. The index i is dropped since only one firm is considered.

$$c = \pi^{nc} - \pi - te \tag{9}$$

Can the abatement costs be negative in some cases? Assume constant prises and let the input-output vector X be optimal for the firm for a given governmental control on emissions. The corresponding profit is π and total taxes are equal to te. Since the firm can choose X also in absence of governmental controls, with the corresponding profit $\pi+te$, we know that $\pi^{nc} \geq \pi+te$. Consequently, from (9), abatement costs are non-negative for any governmental regulation.

Notice that the assumption of constant prises is necessary to reach this conclusion. However, relative prices are typically affected by emission taxes. Profits can therefore, in principle, be increased in some firms if the government control emissions. Such general equilibrium effects are not considered in this paper. Instead it is assumed that prices are constant and determined on a world market.

The derivative of the abatement cost function with respect on abatement gives the marginal abatement costs. Notice that no-control emissions are constant in (9) since all prices and the emission intensity of production are assumed to be constant. Consequently, from (1), dr=-de.

$$\frac{dc(r)}{dr} = -\frac{dc\left(e^{nc} - e\right)}{de} = \frac{d\pi}{de} + t \tag{10}$$

The second term from the right in (10) must be equal to zero for any tax rate when emissions are optimally adjusted. Otherwise, larger profits are obtainable at a different emission level. This is also seen from the optimisation in (3). Using the envelope theorem (e.g., Varian 1992), we derive:

$$\frac{d\pi}{de} = -t + \lambda \tag{11}$$

Since $\lambda=t$ it follows directly that the left-hand side in (11) is equal to zero. Consequently, marginal abatement costs are equal to the emission tax rate when the emission level is optimally adjusted.

$$\frac{dc(r)}{dr}=t \tag{12}$$

From (8) and (12) it follows that marginal abatement costs have the same sign as optimal abatement.

$$sign\left(\frac{dc}{dr}\right)=sign(r) \quad \Leftrightarrow \quad \frac{dc}{dr}\begin{cases} >0 \mid r=e^{nc}-e>0 \\ =0 \mid r=e^{nc}-e=0 \\ <0 \mid r=e^{nc}-e<0 \end{cases} \tag{13}$$

The curvature of the abatement cost function is found from the differentiation of (12) with respect on *t*:

$$\frac{d^2c}{dtdr}=1 \quad \Rightarrow \quad \frac{d^2c}{dr^2}=\left(\frac{dr}{dt}\right)^{-1}>0 \tag{14}$$

The abatement cost curve is convex since abatement is monotonically increasing in the emission tax rate. The properties of the abatement cost function, given by (13) and (14), are presented graphically in figure 2. Notice that the slope of the marginal cost curve is reduced in the first point where it is optimal to use purification abatement technology in addition to reduced production. See McKitrick (1999) for further details.

(Figure 2 about here).

It is quite common to assume positive marginal abatement costs. However, it is easy to show that this leads into conceptual inconsistencies when negative abatement is allowed. The marginal abatement costs are given by (10). If marginal abatement costs are positive, and the emission tax rate is equal to zero, profits can be increased from no-control through increased emissions. However, this is a contradiction since no-control emission per definition maximise profits when the tax rate is zero.

The contradiction is avoided if abatement is constrained to be non-negative. It is therefore possible that non-negative constraints on abatement are added *ad hoc* in some contributions to avoid conceptual inconsistencies. However, if the abatement costs are properly defined it is not necessary to constrain abatement *ad hoc*.

The contradiction can also be avoided by assuming non-negative rather than positive marginal abatement costs. However, it is evident that there must be some costs associated with emission levels much larger than no-control, even though emission taxes are equal to zero. Consequently, marginal abatement cannot be non-negative in the whole domain of real numbers.

Positive marginal abatement costs are assumed without any constraints on abatement in Falk and Mendelsohn (1993), Hagem and Westskog (1998), Hoel (1991), Rubin (1996) and Segerson and Miceli (1998). However, with the exception of Rubin (1996), which follows Montgomery (1972), it is hard to prove any inconsistencies since the concepts of abatement and abatement costs are defined rather vague in these contributions. Still one might ask why these questionable assumptions are made. The most probable explanation is that the authors thought of non-negative abatement only. It is also possible that they had different definitions of the concepts abatement and abatement costs in mind.

As noted in section 1, implicit abatement modelling is allowed if emissions are treated as inputs in production. The argument was that the production of other commodities is reduced if scarce resources are used to abate emissions. However, it is per definition counter-productive for the firm to emit more than no-control levels in absence of governmental controls. The marginal productivity of emissions must therefore have the same sign as abatement if the emissions are the only inputs of production. If prices are fixed, and the economical value of damages from pollution is measured separately, this result must valid also for the aggregate production in a country. In Welsch (1993) it is assumed that gross (national) output increase if emissions are increased, and emissions are constrained by “limited production capacity”. Limited production capacity does of course limit no-control emissions. However, the terminology is questionable since it is reasonable to assume that national emissions are far from maximised in no-control.

Most abatement strategies, for instance input substitution, typically involve changed use of several factors of production. The marginal productivity of emissions must therefore be interpreted slightly different than marginal abatement costs if other inputs of production are taken into account too. However, the assumption of positive marginal productivity of emissions, in the whole domain of positive emissions, leads directly into contradictions in this case too if emissions are unconstrained and no-control emissions well defined.

4. Optimal Taxes

In section 2 it was shown that competitive firms abate a positive level if emission taxes are positive, and *vice versa*. Therefore, this section focuses on optimal governmental taxation. It is assumed that the emissions e_i give pollution P, which influence directly on consumers' utility, for instance through negative health effects. For simplicity, the dynamic nature of many environmental problems is not taken into account. Instead, the government considers a static problem of minimising total abatement costs $\sum c_i$ plus the monetary value of damages from pollution $D(P)$. The damage function is assumed to be increasing and convex.

There are n competitive firms in the country. All firms emit a polluting substance, but they can reduce their emissions at certain costs, which satisfies (13) and (14). There are no other market failures in the economy, and all prices, except emission taxes, are constant and determined on a world market.

$$min_{e_i}\left\{\sum_{i=1}^{n}c_i\left(e_i^{nc}-e_i\right)+D\left(P\left(e_1,....,e_n\right)\right)\right\} \tag{15}$$

Since firms' total costs of regulation are given by their profit loss, it can be argued that emission taxes should be added in (15). However, the net effects of these financial transfers are equal to zero since the government receives the revenues from emission taxes.

Assuming positive emissions from all firms in the optimal solution, the necessary conditions for an optimal solution is given by (16). The symbols are firm *i*'s marginal abatement costs (c_i'), the derivative of the damage function (*D*') and the partial derivative of the pollution function with respect on firm *i*'s emissions (P_i').

$$c_i' = D'P_i' \quad i=1,\ldots,n \tag{16}$$

Firms' marginal abatement costs in the optimal solution are equal to monetary value of the additional damage caused by a marginal emission-increase from the respective firms. From (12) we also know that marginal abatement costs are equal to the emission tax rates when firms are optimally adjusted. The optimal solution is therefore generated in a decentralised economy if the government choose the following tax rates:

$$t_i = D'P_i' \quad i=1,\ldots,n \tag{17}$$

From (17) it follows that the optimal emission tax not necessarily is the same for all firms. There are for instance several examples of environmental problems where the locations of emitting sources are important for their polluting effect (see for instance Alcamo *et al.* 1990 and Amann *et al.* 1998), and this implies that P_i', and therefore also optimal t_i, varies between firms. However, for other environmental problems, for instance the greenhouse effect, the locations of emitting sources are typically assumed to be unimportant. In the special case where all emissions have the same polluting effect, implying P_i'=P_j' $\forall i,j$, it follows from (17) that the optimal emission tax rate is the same for all firms.

Now we turn to the consequences of constraining abatement. First we consider the typical case where the level of pollution is increased for any emission increases, implying

P_i'>0 $\forall i$. Since D'>0 and P_i'>0 $\forall i$, we know from (17) that $t_i > 0 \; \forall i$. Consequently, from (8), all firms abate positively in the optimal solution. Clearly, non-negative constraints on abatement are superfluous in such cases, but they do not rule out optimal solutions.

Suppose that non-negative constraints on firms' abatement are added to (15) even though it has been shown that optimal abatement is positive for every firm without these constraints. In this case the necessary conditions for an optimal solution are given by (18), where the shadow price on the non-negative constraint on firm i's abatement is given by the symbol μ_i.

$$c_i' = D'P_i' + \mu_i \quad \text{where } \mu_i \geq 0,\ \mu_i(e_i^{nc} - e_i) = 0\ \forall i \tag{18}$$

Marginal abatement costs must be strictly positive for all firms since the right hand side in (18) is strictly positive. Therefore, from (13), $r_i = e_i^{nc} - e_i > 0 \; \forall i$. Consequently, from the complementary slackness condition in (18), $\mu_i = 0 \; \forall i$. The necessary conditions for optimal solutions with non-negative constraints on abatement, given by (18), are therefore reduced to the necessary conditions for optimal solutions without these constraints, given by (16). However, some analysing was required to obtain this simplification. Also, the simplification is necessary to solve the constrained optimisation problem properly. It follows that the non-negative constraints on abatement are both complicating and superfluous if abatement costs are properly defined and the typical case where P_i'>0 $\forall i$ are considered. Taken together with the finding that negative abatement is possible, these findings are sufficient to recommend the proposed changes in the shape and domain of the abatement cost function summarised by (13). However, now we turn to an even stronger support for these changes: Non-negative constraints on abatement can rule out optimal solutions in some cases.

From figure 1b we can see that reduced nitrogen oxide emissions give increased concentrations of polluting ground level ozone in some cases. Therefore, we should not expect P_i' to be positive in all cases. Consider the case where at least one P_i' is negative in the optimal solution. Additional emissions from the corresponding firm is therefore a public good rather than a public bad on the margin. Therefore, from (17), the optimal emission tax rate is negative for this firm, and from (8) it follows that the firm abates negatively. The intuition of this result is simple. If a firm has positive abatement and P_i' is negative for this firm, both total abatement costs and pollution levels are reduced if emissions are increased from this firm. This is of course inconsistent with initial optimality.

It is shown that optimal abatement can be negative for some firms in those cases where the pollution-function is non-monotonic. The incorrect and complicating non-negative constraints on abatement can therefore exclude optimal solutions in these cases. For the ozone example, Wolfgang (1998) shows that optimal nitrogen oxide abatement may be negative in the optimal solution if no-control nitrogen oxide concentrations are large.

5. Interrelated Abatement

Increased emissions are typically expected to give increased pollution, even though there are examples of the opposite. However, this section shows that optimal abatement can be negative for some kinds of emissions even though all emissions are optimally taxed public bads.

Consider the case where firms emit z kinds of emissions, which all are public bads that contribute to the pollution level P and the corresponding damage D. The damage function is an increasing and convex in the pollution level $D(P)$, while the pollution level is an increasing and convex function in total emission levels of various kinds $P\left(\sum e_{i1},\ldots,\sum e_{iz}\right)$. No qualitative conclusions would change if location specific factors also had been taken into account.

The government is assumed to control emissions only through emission taxes, and firms have several abatement opportunities. First, all z emission types $(e_1,\ldots,e_z)$ are assumed to increase linearly with the level of production (*x*) in firms, but the emission intensity of production ($b_1,\ldots,b_z$) is not the same for all emissions. Second, there are non-negative abatement effort opportunities ($a_j \geq 0$) for all emissions. In addition, there is a non-negative abatement effort opportunity $(\tilde{a} \geq 0)$ that increases some kinds of emissions in the process of reducing others. The symbol g_j shows how much the emissions of type *j* are reduced if the abatement effort $\tilde{a}$ is increased by one unit. Some g_j are positive while others are negative. The emissions from a given firm is given by (19), where the firm index *i* is dropped.

$$e_j = b_j x - a_j - g_j \tilde{a} \quad j=1,\ldots,z \tag{19}$$

The costs of the various abatement efforts are assumed to be increasing and convex functions of the respective effort levels. The symbol d_j is the cost of the abatement effort a_j, while $\tilde{d}$ is the cost of the abatement effort $\tilde{a}$. The firms are assumed to be competitive, and both profits and emissions are assumed to be strictly positive in the optimal solution.

$$\begin{gathered}\pi(t_1,\ldots,t_z) = max_{x,a_j,\tilde{a},e}\left\{px - s(x) - \sum_{j=1}^{z} d_j(a_j) - \tilde{d}(\tilde{a}) - \sum_{j=1}^{z} t_j e_j\right\} \\ subject\ to\ \ e_j = b_j x - a_j - g_j \tilde{a} \qquad j=1,\ldots,z\end{gathered} \tag{20}$$

No-control emissions are defined to be the optimal emission levels for profit maximising firms in absence of governmental controls on emissions. Since the government is assumed to control emissions only through the tax rate on various emissions, all variables are adjusted to no-control levels if $t_j = 0$ $\forall j$. The abatement costs are given by firm’s loss in profits from no-control, caused by emission taxes, minus the tax payment.

$$c = \pi^{nc} - \pi - \sum_{j=1}^{z} t_j e_j \tag{21}$$

Stated differently, profits are equal to no-control profits minus abatement costs and emission taxes. Also, since firms maximise profits and no-control profits are constant, this implies that firms minimise abatement costs plus emission taxes.

$$min_{e_j}\left\{c\left(e_1^{nc}-e_1,\ldots,e_z^{nc}-e_z\right)+\sum_{j=1}^{z}t_je_j\right\} \tag{22}$$

The various emission levels are related through the production level and the abatement effort $\tilde{a}$. The shape of the abatement cost function c reflects this. Also, the domain of possible emission vectors can, in principle, be restricted in (22). However, all positive emission vectors are in principle available for the firm considered in this section. The necessary condition for an optimal solution with positive emissions is therefore given by (23).

$$\frac{dc}{dr_j}=-\frac{dc}{de_j}=t_j \quad j=1,\ldots,z \tag{23}$$

The firm adjusts its emissions so that marginal abatement costs are equal to the respective tax rates in the optimal solution. The government uses this information when it minimises total abatement costs plus damages from pollution:

$$min_{e_{ij}}\left\{\sum_{i=1}^{n}c_i\left(e_{i1}^{nc}-e_{i1},\ldots,e_{iz}^{nc}-e_{iz}\right)+D\left(P\left(\sum_{i=1}^{n}e_{i1},\ldots,\sum_{i=1}^{n}e_{iz}\right)\right)\right\} \tag{24}$$

The optimal solution satisfies (25) since all kinds of emissions are assumed to be positive from all firms in the optimal solution. The symbol c'_{ij} is the partial derivative of firm *i*'s abatement costs with respect on abatement of type *j*, while P'_j is the partial derivative of the pollution function with respect on emissions of type *j*.

$$c'_{ij}=D'P'_j>0 \quad \forall i,j \tag{25}$$

Marginal abatement costs for a given emission type is the same for all firms since it is assumed that only total emission levels matters. However, marginal abatement costs are in general not the same for all emission types since they may have different marginal effect on pollution. From (23), we know that firms adjust their respective emissions so that marginal abatement costs are equal to the governmental tax rates. Consequently, optimal taxes are given by (26).

$$t_j = D'P_j' > 0 \quad j=1,\ldots,z \tag{26}$$

The emission taxes are positive for all emission types since the pollution function is monotonically increasing in all emissions. Now we reconsider the firm's optimisation problem, given that the government follows the optimal taxation strategy. The firm maximises the profits in (20), and the necessary conditions for optimal solutions with positive emissions are given by (27)-(29). The symbols υ_j and $\tilde{\upsilon}$ are shadow prices on minimum abatement efforts.

$$p - s' = \sum_{j=1}^{z} t_j b_j \tag{27}$$

$$d_j'(a_j) = t_j + \upsilon_j \quad a_j, \upsilon_j \geq 0 \text{ and } a_j \upsilon_j = 0 \;\; \forall j \tag{28}$$

$$\tilde{d}'(\tilde{a}) = \sum_{i=1}^{z} t_j g_j + \tilde{\upsilon} \quad \tilde{a}, \tilde{\upsilon} \geq 0 \text{ and } \tilde{a}\tilde{\upsilon} = 0 \tag{29}$$

From (27) it follows that the price equals marginal production costs in no-control. Also, since the left-hand side is positive in (28) and (29), we know that υ and $\tilde{\upsilon}$ must be strictly positive in no-control. Consequently, from $a_j \upsilon_j = 0 \;\forall j$ and $\tilde{a}\tilde{\upsilon} = 0$, we know that $a_j=0 \;\forall j$ and $\tilde{a} = 0$ in no-control. No-control emissions are therefore given by (30).

$$e_j^{nc} = b_j x^{nc} \quad j=1,\ldots,z \tag{30}$$

From (27) we know that $p>s'(x)$ since all taxes are positive. This implies $x<x^{nc}$ since $p=s'(x^{nc})$ and $s''>0$. The level of production is reduced when emissions are taxed since the production cause emissions. The separate purification effort levels and the corresponding shadow prices on minimum effort levels are given by (31).

$$\text{If}\quad \begin{array}{lll} d_j'(0) < t_j & & a_j > 0 \text{ and } \upsilon_j = 0 \text{ and } d_j'(a_j) = t_j \\ d_j'(0) = t_j & \text{then} & a_j = 0 \text{ and } \upsilon_j = 0 \\ d_j'(0) > t_j & & a_j = 0 \text{ and } \upsilon_j = d_j'(0) - t_j > 0 \end{array} \qquad (31)$$

The firm carries out separate purification efforts only if marginal purification costs at zero purification effort is less than the corresponding tax rate. For the purification efforts that change all emission types, the optimal effort level and shadow price on minimum purification is given by (32):

$$\text{If}\quad \begin{array}{lll} \tilde{d}'(0) < \sum_j g_j t_j & & \tilde{a} > 0 \text{ and } \tilde{u} = 0 \text{ and } \tilde{d}'(\tilde{a}) = \sum_j g_j t_j \\ \tilde{d}'(0) = \sum_j g_j t_j & \text{then} & \tilde{a} = 0 \text{ and } \tilde{\upsilon} = 0 \\ \tilde{d}'(0) > \sum_j g_j t_j & & \tilde{a} = 0 \text{ and } \tilde{\upsilon} = d_j'(0) - \sum_j g_j t_j > 0 \end{array} \qquad (32)$$

Since the government is assumed to follow the optimal taxation strategy, given by (26), we know that $\sum_j g_j t_j = D'\sum_j g_j P'_j$, where the latter sum shows the effect on pollution from a marginal increase in $\tilde{a}$. Therefore, from (32) and $\tilde{d}' > 0$, the firm never choose $\tilde{a} > 0$ unless this abatement strategy reduces the pollution level on the margin in the optimal solution. The firm's abatement of the various emission types are given by (33):

$$r_j \equiv e_j^{nc} - e_j = b_j\left(x^{nc} - x\right) + a_j + g_j\tilde{a} \quad j=1,\ldots,z \qquad (33)$$

Since $b_j\left(x^{nc} - x\right) > 0$, $a_j \geq 0$ and $\tilde{a} \geq 0$, abatement is positive for those emission types where $g_j \geq 0$. Also, abatement is positive for all emissions if $\tilde{a} = 0$. However, assume that $\tilde{a} > 0$ in the optimal solution, and consider one of those emission types where $g_j < 0$. In this case, $g_j\tilde{a}$ contribute negatively on abatement in (33). Also, if the abatement strategy $\tilde{a}$ is cheap enough compared to the other abatement strategies (x^{nc}-x and a_j), then the term $g_j\tilde{a}$ decides the sign of abatement for this particular emission type. Optimal abatement in firms can be therefore be negative for some emissions types, even though all emissions are positively taxed public bads. Also, assuming identical firms, it is easy to show that the total abatement level in a country also can be negative for some emission types in the optimal solution.

6. Summary and Conclusions

Some methodological aspects of standard abatement modelling are considered in this paper. In particular, the analysis focuses on the adequacy of the common assumptions of non-negative abatement and positive marginal abatement costs.

The concepts of abatement and abatement costs are usually defined rather vague or not at all in many contributions, even though these are central concepts in environmental economics. A sound foundation for the concept of abatement is therefore provided in section 2. Abatement is defined to be the difference between no-control emissions and any other emission level, while no-control emissions are the emission levels chosen by profit maximising firms in absence on governmental controls on emissions. Clearly, negative abatement is possible since firms can emit more than no-control levels.

Abatement costs are defined in section 3. It is equal to firms' loss in profits caused by the governmental controls on emissions, minus emission tax expenses and net expenses in emission permit markets plus any positive externality on firms' profits caused by the emission controls. This definition is wide enough to reflect an important aspect of abatement: Every emission-changing action is a potential abatement strategy. In addition, the definition distinguishes abatement costs from some other major costs and benefits of regulation. Abatement costs are shown to be non-negative if prices are constant and the benefits from regulation are received by other than the emitting firms. Also, the common assumption of positive marginal abatement costs is proved to be inconsistent with negative abatement possibilities. It is therefore possible that non-negative constraints on abatement are added *ad hoc* in some contributions to avoid conceptual inconsistencies. However, the analysis shows that marginal abatement costs has the same sign as abatement in the typical case where only one emission type is considered. It is therefore unnecessary to constrain abatement if abatement costs are properly defined. It is also shown that firms adjust their emissions so that abatement has the same sign as the emission tax rate. Negative abatement is therefore optimal for firms if the emission taxes are negative.

Optimal taxes are derived in section 4. It is shown how the non-negative constraints complicate the analysis in a simple example. The non-negative constraints on abatement are also superfluous in the typical case where increased emissions generate larger pollution levels and only one emission type is considered. It is therefore rather odd to add the constraints even though they do not rule out the optimal solution. Taken together with the finding that negative abatement is possible, these findings are sufficient to recommend the changes in the shape and domain of the abatement cost function. The non-negative constraint on abatement should be dropped, and the marginal abatement costs should have the same sign as the level of abatement in the typical case where only one emission type is considered.

Reduced nitrogen oxide emissions give increased ground level ozone concentrations in some cases. Therefore, emissions are not only public bads in all cases. If the emissions from some firms are public goods on the margin, then the optimal emission taxes are negative for these firms. This was shown in section 4. Consequently, the corresponding firms emit more than no-control levels, and this implies negative abatement. Optimal solutions can therefore be ruled in some cases if non-negative constraints on abatement are added.

Increased emissions are typically expected to give increased pollution, even though there are examples of the opposite. However, if there exist abatement technologies that increase some emission types in the process of reducing others, then optimal abatement can be negative also in those cases where all emissions are positively taxed public bads.

Non-negative constraints on abatement can therefore rule out optimal solutions in such cases too. This is shown in section 5.

From the findings in this paper, two methodological recommendations are offered: The standard assumption of non-negative abatement should be dropped and the common assumption of positive marginal abatement costs should be modified. In the typical case where only one kind of emission is considered, marginal abatement costs should always have the same sign as the level of abatement. There are several advantages with the modified assumptions. First, the modelling is more correct and some conceptual inconsistencies are avoided. Second, the analysis typically gets simpler. Third, optimal solutions can be ruled out in some cases unless assumptions are modified. Future contributions can therefore benefit from these modifications.

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Figure 1:Typical ozone concentration estimates

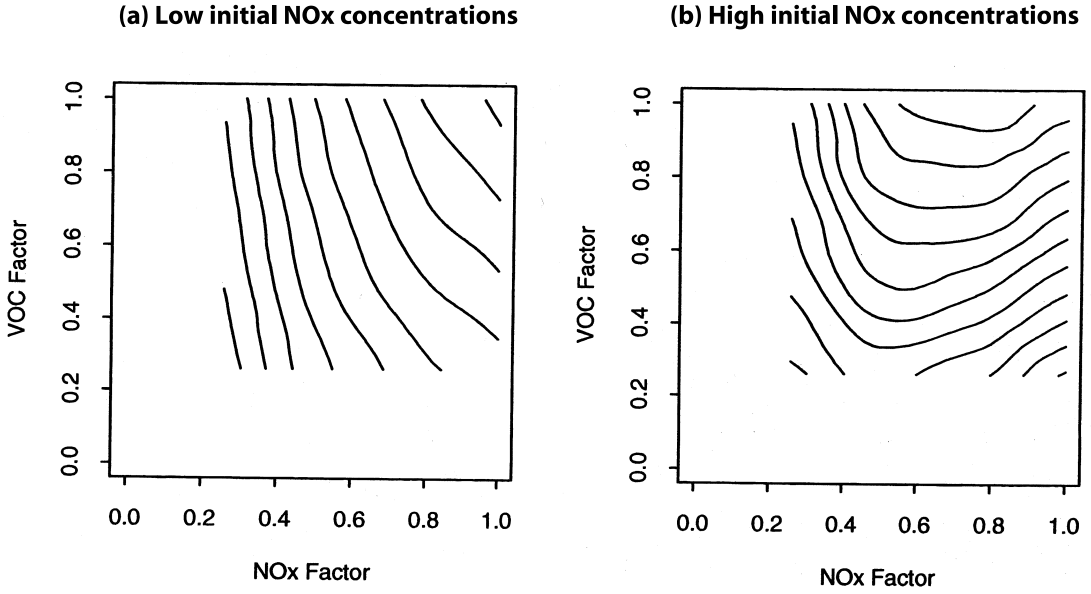

Figure 2: The abatement cost function